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Marital Status and Derived Pension Rights: A Political Economy Model of Public Pensions with Borrowing Constraints

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Abstract

This paper develops an overlapping-generation model featuring four types of households: single female, single male, one-breadwinner couple and two-breadwinner couple. The paper considers majority voting over public pension in the presence of derived pension rights for one-breadwinner couples. In an economy with a low intertemporal elasticity of substitution, borrowing-constrained one-breadwinner couples may prefer a lower tax rate than do other types of households, although the former attain a higher benefit-to-cost ratio of public pension than do others. Changes in the gender wage gap, the level of derived pension rights, and the fraction of two-breadwinner couples produce an inverse U-shaped relationship between the relevant variable and the tax rate.

Keywords: Borrowing constraint; Marital status; Gender wage gap; Derived pension rights; Political economy

JEL Classification: D72, H55, J12

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1 Introduction

Most OECD (Organization for Economic Co-operation and Development) countries offer pension benefits for non-working spouses and divorcees. The benefits, called derived pension rights, include (i) the survivors' benefits for widows; (ii) the benefits for divorced spouses; and (iii) the spousal benefits as a supplement to a worker's benefit (Choi, 2006). These benefits imply that derived pension rights have an intra-generational redistribution component from working singles and two-breadwinner couples to one-breadwinner couples. Thus, recent pension reforms in many OECD countries that attempt to link contributions and benefits more closely (OECD, 2011) may foment an intra-generational conflict over pension policy. In addition, recent economic trends such as reductions in the gender wage gap (see Weichselbaumer and Winter-Ebmer, 2005, and the references therein) and increases in the fraction of two-breadwinner couples due to the increased labor force participation rates of females (Jaumotte, 2003) are expected to affect the pension preferences of households and thus the resulting pension policy.

The role of derived pension rights is expected to increase over time because of an aging population and the longer life expectancy of women (Choi, 2006). However, few theoretical studies focus on these pension rights; the exceptions are the works of Leroux and Pestieau (2010) and Leroux, Pestieau, and Racionero (2011). Leroux and Pestieau (2010) consider an economy composed of couples who maximize the joint lifetime utility of a husband and a wife. A husband always works regardless of his labor productivity, while a wife chooses to work or not depending on her reservation wage. Under this framework, Leroux and Pestieau (2010) demonstrate an interaction between a wife's labor supply decision and pension policy preferences, and they show that a pension system with derived pension rights is likely to emerge as a voting equilibrium outcome.

Leroux, Pestieau, and Racionero (2011) assume that the degree of derived pension rights is fixed. They allow for the presence of single males and females and examine how the degree of derived pension rights affects tax burden policy preferences for public pension and thus a resulting pension system via voting. Their results are as follows: (1) a reduction of derived pension rights results in a smaller tax burden for public pension; and (2) an increase in the share of two-breadwinner couples have two opposing effects on the pension burden, where the net effect is positive or negative depending on the other economic factors.

The results in Leroux, Pestieau, and Racionero (2011) provide significant policy predictions for public pension. However, their results heavily depend on the following two assumptions: quasi-linear utility and no borrowing constraint. The first assumption, which is often adopted in the political economy analysis of social security (see, for example, Conde-Ruiz and Galasso, 2003, 2004, 2005; Borck, 2007), abstracts away the effect of the intertemporal elasticity of substitution on a household's decisions concerning saving and voting. The second assumption allows for borrowing against future pension benefits, which is hard to support from the empirical viewpoint (Diamond and Hausman, 1984; Mulligan and Sala-i-Martin, 1999). The aim of this paper is to relax these two assumptions in the framework of Leroux, Pestieau, and Racionero (2011) and to provide new insight to derived pension rights from the viewpoint of political economy.

For the purpose of analysis, the current paper develops a two-period overlapping-generation model based on that of Leroux, Pestieau, and Racionero (2011). The economy consists of four types of households: single female, single male, one-breadwinner couple, and two-breadwinner couples. Agents work in youth and retire in old age, except females who belong to one-breadwinner couples. Females are assumed to have a longer life span than males but earn lower wages. Each household saves part of its labor income and consumes the rest in youth; it consumes the return from savings and pension benefits in old age.

The pension system is assumed to be Beveridgean: benefits and tax rates are common to all working agents. Females who belong to one-breadwinner couples can also receive benefits as derived pension rights, but the amount is limited because of a lack of contribution. At the beginning of each period, all agents (including the young and the old) vote on tax rates; the equilibrium tax rate and the resulting pension benefit are determined via majority voting. Old households prefer a higher tax rate than do any young households because the former bear no tax burden. The decisive voter belongs to the young generation because the population size of the old is smaller than that of the young given the death of some males at the end of youth. We focus on a situation in which a household that prefers the highest tax rate among the young becomes a decisive voter.

As mentioned above, the current framework differs from that of Leroux, Pestieau, and Racionero (2011) in the following two ways. First, the preferences of each household are represented by a utility function with a constant intertemporal elasticity of substitution. Second, each household is unable to borrow against its future pension benefits. It is known that the borrowing constraint associated with a low intertemporal elasticity of substitution produces an "ends-against-the-middle" equilibrium in which the old and the middle-income young form a coalition in favor of a higher tax rate, while the low- and the high-income young agents favor a lower tax rate if a decisive voter is borrowing-constrained (Casamatta et al., 2000; Conde-Ruiz and Profeta, 2007; Cremer et al., 2007; Arawatari and Ono, 2011).

The current paper introduces the abovementioned mechanism into the framework of Leroux, Pestieau, and Racionero (2011) and shows the following two results. First, in an economy with a high intertemporal elasticity of substitution, one-breadwinner couples are more likely to be decisive voters (that is, to prefer the highest tax rate among the young) when the level of derived pension rights is higher: this result is identical to that of Leroux, Pestieau, and Racionero (2011). The result is intuitive because a higher level of derived pension rights is more likely to yield a higher benefit-to-burden ratio for one-breadwinner couples than for any other type of household. However, in an economy with a low intertemporal elasticity of substitution, one-breadwinner couples may prefer a lower, rather than a higher, tax rate than would single females because of the presence of borrowing constraints. Borrowing-constrained one-breadwinner couples want to choose a low tax rate to keep their after-tax income level as high as possible. There is then a voting equilibrium in which single females and the old form a coalition against the others.

Second, the gender wage gap, the level of derived pension rights, and the share of two-breadwinner couples produce monotone effects on the equilibrium tax rate when the intertemporal elasticity of substitution is high, but they create non-monotone effects when the elasticity is low. Specifically, when the elasticity is low, a reduction of the gender wage gap, a reduction of derived pension rights, and an increase in the share of two-breadwinner couples create an inverse U-shaped relationship between the relevant variable and the tax rate. Near the maximum of the inverse U-shaped curve, the decisive voter is borrowing-unconstrained on one side and borrowing-constrained on the other side. These non-monotone effects, which were not shown in Leroux, Pestieau, and Racionero (2011), are derived by the presence of a borrowing constraint associated with a low intertemporal elasticity of substitution.

The organization of this paper is as follows. Section 2 describes the economic environment. Section 3 demonstrates the utility maximization of singles, one-breadwinner couples, and two-breadwinner couples. Section 4 presents the political institution and pension policy preferences of the young and the old. Section 5 characterizes the political equilibrium. Section 6 performs a comparative statics analysis and shows how gender wage gap, derived pension rights, and the share of two-breadwinner couples affect the equilibrium pension policy. Section 7 provides concluding remarks. Proofs are provided in Appendix.

2 The Economic Environment

Consider a discrete time economy in which time is denoted by $t = 0, 1, 2, \cdots$. The economy comprises overlapping generations of individuals, each of whom lives two periods: youth and old age. Each generation is composed of a continuum of agents. Specifically, in each generation, there are males and females; the size of each gender population is normalized to unity. Thus, the total population size of each generation is two.

Each generation consists of four different categories of households: single male, single female, one-breadwinner couple, and two-breadwinner couples. The fraction of singles is $1-\varphi \in (0,1)$, and the fraction of couples is φ. Among couples, a fraction $1-\mu \in (0,1)$ has one breadwinners; the remaining fraction μ is composed of two breadwinners. Therefore, in each generation, the population size of each type of household is as follows: the size of single males is $1-\varphi$; the size of single females is $1-\varphi$; the size of one-breadwinner couples is $2\varphi(1-\mu)$; and the size of two-breadwinner couples is $2\varphi\mu$. The fraction of each type of household is fixed over time. For simplicity, marriage decisions are abstracted away from the analysis. The structure of each generation is depicted in Figure 1.

[**Figure 1** is around here.]

Each agent is endowed with one unit of labor in youth and retires in old age. Males supply labor whatever their marital status, while only females who are single or belong to two-breadwinner couples supply labor. Females who belong to one-breadwinner couples do not supply labor; instead, they devote it to home production and leisure, both of which are assumed not to have an effect on utility or household income.

In this economy, there are two types of heterogeneity between males and females: wage and longevity. These types are characterized by the parameter pairs (w^m, π^m) for males and (w^f, π^f) for females such that

$$\begin{cases} (w^m, \pi^m) = (w, \pi), \ \pi \in (0,1); \\ (w^f, \pi^f) = (\alpha w, 1), \ \alpha \in (0,1), \end{cases}$$

where w^i $(i = f, m)$ represents the wage, and π^i represents the probability of surviving in old age. The term $\alpha \in (0,1)$ represents the gender wage gap, and the term $\pi \in (0,1)$ represents the longevity difference between males and females. It is assumed that females have a longer life span than males but obtain a lower wage.

Individuals contribute to the pension system during youth and receive a pension benefits in old age. Following the convention in the literature, we present the efficiency loss of taxation by assuming convex costs of collecting taxes (see, for example, Casamatta, Cremer, and Pestieau, 2000; Bellettini and Berti Ceroni, 2007; Borck, 2007; Cremer et al., 2007). The actual tax revenue from the young is therefore given by

$$\tau(1-\tau)\left[w + \alpha w(1-\varphi) + \alpha w\varphi\mu\right],$$

where the terms w, $\alpha w(1-\varphi)$, and $\alpha w\varphi\mu$ in the square brackets correspond to the contributions by males, single females, and females who belong to two-breadwinner couples, respectively. The term $(1-\tau)$ is the distortionary factor. The assumption of distortionary taxation is made solely to ensure an interior solution to preferred tax rates and otherwise plays no role.

In the current environment, males supply labor whatever their marital status, while only females who are single or belong to two-breadwinner couples supply labor; females who belong to one-breadwinner couples do not contribute to the pension system. Let p denote pension benefits for contributors; let γp denote pension benefits for non-contributors, where $\gamma \in [0,1]$ represents the level of derived pension rights. The total pension payments are

$$p \cdot [\pi + (1-\varphi) + \varphi\mu + \gamma\varphi(1-\mu)].$$

The pension benefit for males is $p\pi$, rather than p, because their length of life in old age is assumed to be $\pi \in [0,1)$.

Under the assumption of a balanced budget, the government budget constraint becomes

$$p = w\chi(\cdot)\tau(1-\tau), \tag{1}$$

where

$$\chi(\cdot) \equiv \frac{1+\alpha(1-\varphi+\varphi\mu)}{\pi+1-\varphi+\varphi\mu+\gamma\varphi(1-\mu)}.$$

The parameter $\chi(\cdot)$ corresponds to the benefit-to-burden ratio of public pensions in the economy. The tax rate τ is determined via majority voting, whereas the degree of derived pension rights γ is assumed to be fixed at the constitutional level. Voting over γ will be discussed in Section 7.

3 Economic Decisions

Let $j = f, m, c1$, and $c2$ denote single females, single males, one-breadwinner couples, and two-breadwinner couples, respectively. In this section, we illustrate the economic decisions on saving by each type of a young household. Any old agent makes no economic decision because his/her saving is predetermined in youth.

3.1 Singles

Each single agent is assumed to receive utility from private consumption. The lifetime utility function of a type-j ($j = f, m$) single young agent is specified by:

$$\max U^j = \frac{(c^j)^{1-\sigma}-1}{1-\sigma} + \beta \cdot \frac{(d^j)^{1-\sigma}-1}{1-\sigma},$$

where c^j is consumption in youth, d^j is consumption in old age, $\beta \in (0,1)$ is a discount factor, and $\sigma(>0)$ is the inverse of the intertemporal elasticity of substitution. A lower

$1/\sigma$ implies a lower intertemporal elasticity of substitution.[1]

Type-j's ($j = f, m$) individual budget constraints in youth and in old age are given by, respectively,

$$c^j + s^j \leq (1-\tau)w^j,$$
$$d^j \leq Rs^j + \pi^j p,$$

where s^j is saving, τ is the income tax rate, $R(> 1)$ is the gross interest rate, and p is the per capita pension benefit. If $j = f$, then $w^f = \alpha w$ and $\pi^f = 1$; if $j = m$, then $w^m = w$ and $\pi^m = \pi$.

Throughout this paper, we assume borrowing constraints, that is

$$s^j \geq 0.$$

This constraint precludes the possibility of borrowing when young against future pension benefits (Diamond and Hausman, 1984; Mulligan and Sala-i-Martin, 1999).

A type-j young agent maximizes his/her utility subject to his/her budget constraint and the borrowing constraint. When $s^j > 0$, the first-order condition for an interior solution is $d^j = (\beta R)^{1/\sigma} c^j$. This condition determines an interior solution of saving by a type-j agent. By taking the borrowing constraint into account, the saving function of a type-j agent becomes

$$s^j = \max\left\{0, \frac{(\beta R)^{1/\sigma}}{R + (\beta R)^{1/\sigma}}\left[(1-\tau)w^j - \frac{\pi^j p}{(\beta R)^{1/\sigma}}\right]\right\},\ j = f, m. \tag{2}$$

The saving function (2) and the government budget constraint (1) imply that there is a critical rate of tax such that

$$s^f > 0 \Leftrightarrow \tau < \hat{\tau}^f \equiv \frac{\alpha (\beta R)^{1/\sigma}}{\chi(\cdot)};$$
$$s^m > 0 \Leftrightarrow \tau < \hat{\tau}^m \equiv \frac{(\beta R)^{1/\sigma}}{\pi \chi(\cdot)}.$$

A type-j ($= f, m$) agent saves a part of his/her disposable income in youth when the tax rate is below his/her critical value. However, when the tax rate is above the critical value, a type-j agent faces a borrowing constraint and can save nothing in youth. The critical value for single females, $\hat{\tau}^f$, is lower than that for single males, $\hat{\tau}^m$, because single

[1] For $j = m$, the second-period utility might be more appropriately written as $\pi\beta\{(d^j)^{1-\sigma} - 1\}/(1-\sigma)$ since π is interpreted as the probability of surviving to the second period of life. Following Borck (2007), we assume away the effect of π on the second-period utility for the tractability of analysis.

males obtain higher wages and live shorter than do single females.

With the saving function and the private and government budget constraints, we can obtain the consumption functions of a type-j $(= f, m)$ agent in youth and in old age. We use the functions to obtain indirect utility functions of single females and males, denoted by V^f and V^m, respectively:

$$V^f = \begin{cases} V^f_{s>0} & \text{if} \quad \tau < \hat{\tau}^f \equiv \alpha \left(\beta R\right)^{1/\sigma} / \chi(\cdot) \\ V^f_{s=0} & \text{if} \quad \tau \geq \hat{\tau}^f \equiv \alpha \left(\beta R\right)^{1/\sigma} / \chi(\cdot) \end{cases},$$

where

$$V^f_{s>0} \equiv \frac{1}{1-\sigma}\left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma}\left[(1-\tau)\alpha w + \frac{w\chi(\cdot)}{R}\tau(1-\tau)\right]^{1-\sigma} - \frac{1+\beta}{1-\sigma},$$

$$V^f_{s=0} \equiv \frac{1}{1-\sigma}\left((1-\tau)\alpha w\right)^{1-\sigma} + \frac{\beta}{1-\sigma}\left\{w\chi(\cdot)\tau(1-\tau)\right\}^{1-\sigma} - \frac{1+\beta}{1-\sigma};$$

and

$$V^m = \begin{cases} V^m_{s>0} & \text{if} \quad \tau < \hat{\tau}^m \equiv \left(\beta R\right)^{1/\sigma} / \pi\chi(\cdot) \\ V^m_{s=0} & \text{if} \quad \tau \geq \hat{\tau}^m \equiv \left(\beta R\right)^{1/\sigma} / \pi\chi(\cdot) \end{cases},$$

where

$$V^m_{s>0} \equiv \frac{1}{1-\sigma}\left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma}\left[(1-\tau)w + \frac{\pi w\chi(\cdot)}{R}\tau(1-\tau)\right]^{1-\sigma} - \frac{1+\beta}{1-\sigma},$$

$$V^m_{s=0} \equiv \frac{1}{1-\sigma}\left((1-\tau)w\right)^{1-\sigma} + \frac{\beta}{1-\sigma}\left\{\pi w\chi(\cdot)\tau(1-\tau)\right\}^{1-\sigma} - \frac{1+\beta}{1-\sigma}.$$

The function $V^j_{s>0}$ $(j = f, m)$ denotes the indirect utility of a type-j young household when it saves some portion of income, and $V^j_{s=0}$ denotes the indirect utility when it is faced with a borrowing constraint and saves nothing. The term in square brackets in the equation of $V^j_{s>0}$ represents the lifetime income; the first and the second terms on the right-hand side in the equation for $V^j_{s=0}$ represents the utilities of consumption in youth and in old age, respectively; the constant term, $(1+\beta)/(1-\sigma)$, summarizes the parameters unrelated to the political decision on taxes.

3.2 One-breadwinner Couples

We next consider consumption decisions by couples. Following Leroux, Pestieau, and Racionero (2011), we adopt the unitary model of the household that has only one set of preferences. Under this specification, spouses play cooperatively and share their resources over their lifecycle.

A one-breadwinner couple chooses consumption and saving to maximize the following household utility function:

$$U^{c1} = 2 \cdot \left\{ \frac{(c^{c1})^{1-\sigma} - 1}{1-\sigma} + \beta \cdot \frac{(d^{c1})^{1-\sigma} - 1}{1-\sigma} \right\}$$

subject to the budget constraints,

$$\begin{aligned} 2c^{c1} + s^{c1} &\leq (1-\tau)w, \\ 2d^{c1} &\leq Rs^{c1} + (\pi + \gamma)p, \end{aligned}$$

and the borrowing constraint, $s^{c1} \geq 0$. The superscript "$c1$" indicates a couple with one breadwinner.

In the first period of life, a husband works and earns the after-tax wage income, $(1-\tau)w$. A one-breadwinner couple consumes a part of the after-tax wage and saves the rest for old-age consumption. In the second period of life, the couple obtains the return from savings, Rs^{c1}, the pension benefit paid to the husband, πp, and the derived pension benefit paid to the wife, γp.

When $s^{c1} > 0$, the first-order condition for an interior solution is $d^{c1} = (\beta R)^{1/\sigma} c^{c1}$. This condition determines an interior solution of saving by a one-breadwinner couple. By taking the borrowing constraint into account, the saving function of the one-breadwinner couple becomes

$$s^{c1} = \max \left\{ 0, \frac{(\beta R)^{1/\sigma}}{R + (\beta R)^{1/\sigma}} \left[(1-\tau)w - \frac{(\pi+\gamma)p}{(\beta R)^{1/\sigma}} \right] \right\} \tag{3}$$

The saving function (3) and the government budget constraint (1) imply that there is a critical tax rate for one-breadwinner couples such that

$$s^{c1} > 0 \Leftrightarrow \tau < \hat{\tau}^{c1} \equiv \frac{(\beta R)^{1/\sigma}}{(\pi + \gamma)\chi(\cdot)}.$$

With the saving function and the private and government budget constraints, we can obtain the consumption functions in youth and in old age. We use these functions to derive a type-$c1$'s indirect utility function:

$$V^{c1} = \begin{cases} V^{c1}_{s>0} & \text{if} \quad \tau < \hat{\tau}^{c1} \equiv \frac{(\beta R)^{1/\sigma}}{(\pi+\gamma)\chi(\cdot)} \\ V^{c1}_{s=0} & \text{if} \quad \tau \geq \hat{\tau}^{c1} \equiv \frac{(\beta R)^{1/\sigma}}{(\pi+\gamma)\chi(\cdot)} \end{cases},$$

where

$$V^{c1}_{s>0} \equiv \frac{1}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma}\left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma}\left[(1-\tau)w+\frac{(\pi+\gamma)w\chi(\cdot)}{R}\tau(1-\tau)\right]^{1-\sigma}-\frac{2\,(1+\beta)}{1-\sigma};$$

$$V^{c1}_{s=0} \equiv \frac{1}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma}((1-\tau)w)^{1-\sigma}+\frac{\beta}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma}\{(\pi+\gamma)w\chi(\cdot)\tau(1-\tau)\}^{1-\sigma}-\frac{2\,(1+\beta)}{1-\sigma}.$$

The function $V^{c1}_{s>0}$ denotes the indirect utility of a type-$c1$ household when it saves in youth, and $V^{c1}_{s=0}$ denotes the indirect utility when it is faced with a borrowing constraint and saves nothing in youth. The interpretation of each term in these equations follows that offered for singles.

3.3 Two-breadwinner Couples

Under the assumption of a unitary model of households, the utility function of a two-breadwinner couple is given by

$$U^{c2} = 2\cdot\left\{\frac{(c^{c2})^{1-\sigma}-1}{1-\sigma}+\beta\cdot\frac{(d^{c2})^{1-\sigma}-1}{1-\sigma}\right\},$$

where the superscript "$c2$" indicates a couple with two breadwinners. The couple chooses consumption and saving to maximize its lifetime utility subject to the budget constraints

$$2c^{c2}+s^{c2} \leq (1-\tau)(w+\alpha w),$$
$$2d^{c2} \leq Rs^{c2}+(\pi+1)p,$$

and the borrowing constraint, $s^{c2} \geq 0$.

In the first period of life, both husband and wife work. The husband earns the after-tax wage income, $(1-\tau)w$, while the wife earns $(1-\tau)\alpha w$. They consume a part of their joint wage income and save the rest for their old-age consumption. In the second period of life, they obtain the return from saving, Rs^{c2}, the pension benefit for the husband, πp, and the pension benefit for the wife, p.

As in the case of one-breadwinner couples, we can derive the indirect utility function of two-breadwinner couples as follows. First, we obtain an interior first-order condition of the utility maximization problem, $d^{c2} = (\beta R)^{1/\sigma} c^{c2}$. Then, we substitute the private budget constraints into this first-order condition and take the borrowing constraint into account to obtain the saving function

$$s^{c2} = \max\left\{0, \frac{(\beta R)^{1/\sigma}}{R+(\beta R)^{1/\sigma}}\left[(1-\tau)(1+\alpha)w-\frac{(\pi+1)p}{(\beta R)^{1/\sigma}}\right]\right\}.$$

Finally, we use the saving function and private and government budget constraints to obtain the following indirect utility function:

$$V^{c2} = \begin{cases} V^{c2}_{s>0} & \text{if} \quad \tau < \hat{\tau}^{c2} \equiv \frac{(1+\alpha)(\beta R)^{1/\sigma}}{(\pi+1)\chi(\cdot)} \\ V^{c2}_{s=0} & \text{if} \quad \tau \geq \hat{\tau}^{c2} \equiv \frac{(1+\alpha)(\beta R)^{1/\sigma}}{(\pi+1)\chi(\cdot)} \end{cases},$$

where

$$V^{c2}_{s>0} \equiv \frac{1}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma}\left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma}\left[(1-\tau)(1+\alpha)w + \frac{(\pi+1)w\chi(\cdot)}{R}\tau(1-\tau)\right]^{1-\sigma} - \frac{2(1+\beta)}{1-\sigma};$$

$$V^{c2}_{s=0} \equiv \frac{1}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma}((1-\tau)(1+\alpha)w)^{1-\sigma} + \frac{\beta}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma}\{(\pi+1)w\chi(\cdot)\tau(1-\tau)\}^{1-\sigma} - \frac{2(1+\beta)}{1-\sigma}.$$

The function $V^{c2}_{s>0}$ denotes the indirect utility of a type-$c1$ household when it saves in youth, and $V^{c2}_{s=0}$ denotes the indirect utility when it is faced with a borrowing constraint and saves nothing. The interpretation for each term in the equations follows that offered for singles.

4 The Political Institution and Policy Preferences

The tax rate τ is determined by individuals through a political process of a majority voting. Elections take place every period and all individuals alive, both young and old, cast a ballot over τ. The tax preferences of young individuals are represented by the indirect utility functions presented in the previous section. The tax preferences of old agents are determined by the size of pension because their saving when young is predetermined and has no critical effect on voting behavior. Every individual has zero mass, and thus no individual vote can change the outcome of the election. We thus assume individuals vote sincerely.

The majority voting game is intrinsically dynamic because it describes the interaction among successive generations. To address this feature, we assume full commitment, that is, once-and-for-all voting. That is, voters determine the constant sequence of the parameters: $\tau_t = \tau_{t+1} = \tau$ for all t, where τ_t denotes the tax rate in period t (see, for example, Casamatta, Cremer and Pestieau, 2000; Cond-Ruiz and Profeta, 2007). We can view the full commitment solution as the solution that includes intergenerational interaction because the full commitment solution can be supported as the subgame perfect equilibrium in repeated voting (see, for example, Conde-Ruiz and Galasso, 2003, 2005; Poutvaara, 2006).

Given the stationary environment, the current model presents a static voting game. Therefore, the median voter theorem can be applied to the voting game. To find the voting equilibrium, we need to show that tax preferences of voters are single-peaked. As for the tax preferences of old voters, their objective is to maximize their pension benefits regardless of their marriage status, labor supply, and saving. Although the benefit levels differ between old agents, the factor related to political decision is common to all old agents and is specified by the Laffer curve $\tau(1-\tau)$. Thus, the tax preferences of the old are single-peaked; their preferred tax rate, denoted by τ^{oj}, is $\tau^{oj} = 1/2\ \forall j = f, m, c1, c2$.

4.1 Policy Preferences of the Young

Next, let us consider the preferences of the young. To show that the preferences of a young agent who belongs to a type-j ($j = f, m, c1, m2$) household are single peaked, we should note that the following three properties hold. First, $\partial^2 V^j_{s>0}/\partial\tau^2 < 0$ and $\partial^2 V^j_{s=0}/\partial\tau^2 < 0$ hold; that is, $V^j_{s>0}$ and $V^j_{s=0}$ are single peaked. Second, the indirect utility V^j of a young agent in a type-j household is continuous at $\tau = \hat{\tau}^j$:

$$V^j_{s>0}\big|_{\tau=\hat{\tau}^j} = V^j_{s>0}\big|_{\tau=\hat{\tau}^j},\ j = f, m, c1, c2.$$

Third, the slope of $V^j_{s>0}$ at $\tau = \hat{\tau}^j$ is equivalent to that of $V^j_{s=0}$ at $\tau = \hat{\tau}^j$:

$$\left.\frac{\partial V^j_{s>0}}{\partial\tau}\right|_{\tau=\hat{\tau}^j} = \left.\frac{\partial V^j_{s=0}}{\partial\tau}\right|_{\tau=\hat{\tau}^j},\ j = f, m, c1, c2.$$

The detail of the calculation is given in Appendix 8.1.

The three properties imply that V^j has a unique local maximum. In what follows, we investigate the tax rates preferred by single females, single males, agents who belong to one-breadwinner couples, and agents who belong to two-breadwinner couples, respectively.

First, consider a preferred tax rate by a single female, young agent. Suppose that she is borrowing-unconstrained. She chooses τ to maximize $V^f_{s>0}$. The first derivative of $V^f_{s>0}$ with respect to τ is given by

$$\begin{aligned}\frac{\partial V^f_{s>0}}{\partial\tau} = & \left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma}\cdot\left[(1-\tau)\alpha w + \frac{w\chi(\cdot)}{R}\tau(1-\tau)\right]^{-\sigma} \\ & \times\left\{-\alpha w + \frac{w\chi(\cdot)}{R}(1-2\tau)\right\},\end{aligned}$$

where the first term within the braces shows the marginal pension burden, and the second term shows the marginal pension benefit. The borrowing-unconstrained single female chooses a τ that balances the marginal cost and benefit in terms of utility. That is, she chooses a τ that attains $\partial V^f_{s>0}/\partial\tau = 0$, which is equivalent to

$$1 - 2\tau = \frac{\alpha R}{\chi(\cdot)}. \tag{4}$$

Alternatively, suppose that a single female is borrowing-constrained. She chooses τ to maximize $V^f_{s=0}$. The first derivative of $V^f_{s=0}$ with respect to τ is

$$\frac{\partial V^f_{s=0}}{\partial\tau} = (-1)(\alpha w)^{1-\sigma}(1-\tau)^{-\sigma} + \beta\left(w\chi(\cdot)\right)^{1-\sigma}\left(\tau(1-\tau)\right)^{-\sigma}(1-2\tau),$$

where the first term on the right-hand side shows the marginal pension burden and the second term shows the marginal pension benefit. The borrowing-constrained single female chooses a τ that balances the marginal cost and benefit in terms of utility. That is, she chooses a τ that attains $\partial V^f_{s=0}/\partial\tau = 0$, which is equivalent to

$$1 - 2\tau = \frac{1}{\beta}\left(\frac{\alpha}{\chi(\cdot)}\right)^{1-\sigma}(\tau)^{\sigma}. \tag{5}$$

With (4) and (5), the preferred tax rate by a single female is summarized as follows:

$$\begin{aligned} LHS &\equiv 1 - 2\tau \\ &= RHS^f \equiv \begin{cases} \frac{\alpha}{\chi(\cdot)}R & \text{if } \tau < \hat{\tau}^f \equiv \alpha\left(\beta R\right)^{1/\sigma}/\chi(\cdot), \\ \frac{1}{\beta}\left(\frac{\alpha}{\chi(\cdot)}\right)^{1-\sigma}(\tau)^{\sigma} & \text{if } \tau \geq \hat{\tau}^f \equiv \alpha\left(\beta R\right)^{1/\sigma}/\chi(\cdot). \end{cases} \end{aligned} \tag{6}$$

The terms LHS and RHS^f represent the *marginal efficiency loss of taxation* and the *marginal cost-to-benefit ratio of taxation in terms of utility*, respectively. Single females choose the tax rate that balances these terms. In the next subsection, we provide an interpretation of the ratio and explain its role in the determination of a preferred tax rate by each type of agent.

Next, consider the preference of a single male young agent over τ. His preferred tax rate satisfies $\partial V^m_{s>0}/\partial\tau = 0$ if he is borrowing-unconstrained; it satisfies $\partial V^m_{s=0}/\partial\tau = 0$ if he is borrowing-constrained. As in the case of a single female, we find that his preferred tax rate satisfies

$$\begin{aligned} LHS &\equiv 1 - 2\tau, \\ &= RHS^m \equiv \begin{cases} \frac{1}{\pi\chi(\cdot)}R & \text{if } \tau < \hat{\tau}^m \equiv \left(\beta R\right)^{1/\sigma}/\pi\chi(\cdot) \\ \frac{1}{\beta}\left(\frac{1}{\pi\chi(\cdot)}\right)^{1-\sigma}(\tau)^{\sigma} & \text{if } \tau \geq \hat{\tau}^m \equiv \left(\beta R\right)^{1/\sigma}/\pi\chi(\cdot) \end{cases} \end{aligned} \tag{7}$$

The difference between single females and males is twofold: the parameter α, representing the gender wage gap, appears on RHS^f, which expresses the marginal cost-to-benefit ratio of taxation for single females as in (6); the parameter π, representing the gender longevity difference, appears on RHS^m, which expresses the marginal cost-to-benefit ratio of taxation for single males as in (7). Given these two differences, preferred tax rates differ between the two types of singles.

As in the case of singles, we can derive the conditions that determine the preferred tax rates by one-breadwinner and two-breadwinner couples, respectively. A preferred tax rate by an agent who belongs to a one-breadwinner couple satisfies $\partial V^{c1}_{s>0}/\partial\tau = 0$ if $\tau < \hat{\tau}^{c1}$ and $\partial V^{c1}_{s=0}/\partial\tau = 0$ if $\tau \geq \hat{\tau}^{c1}$. Thus, his/her preferred tax rate is determined by the following condition:

$$\begin{aligned} LHS &\equiv 1 - 2\tau \\ &= RHS^{c1} \equiv \begin{cases} \frac{1}{(\pi+\gamma)\chi(\cdot)} R & \text{if} \quad \tau < \hat{\tau}^{c1} \equiv (\beta R)^{1/\sigma} / (\pi+\gamma)\chi(\cdot); \\ \frac{1}{\beta}\left(\frac{1}{(\pi+\gamma)\chi(\cdot)}\right)^{1-\sigma} (\tau)^{\sigma} & \text{if} \quad \tau \geq \hat{\tau}^{c1} \equiv (\beta R)^{1/\sigma} / (\pi+\gamma)\chi(\cdot). \end{cases} \end{aligned} \tag{8}$$

Similarly, a preferred tax rate by an agent who belongs to a two-breadwinner couple satisfies $\partial V^{c2}_{s>0}/\partial\tau = 0$ if $\tau < \hat{\tau}^{c2}$ and $\partial V^{c2}_{s=0}/\partial\tau = 0$ if $\tau \geq \hat{\tau}^{c2}$. Thus, his/her preferred tax rate is determined by the following condition:

$$\begin{aligned} LHS &\equiv 1 - 2\tau \\ &= RHS^{c2} \equiv \begin{cases} \frac{1+\alpha}{(\pi+1)\chi(\cdot)} R & \text{if} \quad \tau < \hat{\tau}^{c2} \equiv (1+\alpha)(\beta R)^{1/\sigma} / (\pi+1)\chi(\cdot), \\ \frac{1}{\beta}\left(\frac{1+\alpha}{(\pi+1)\chi(\cdot)}\right)^{1-\sigma} (\tau)^{\sigma} & \text{if} \quad \tau \geq \hat{\tau}^{c2} \equiv (1+\alpha)(\beta R)^{1/\sigma} / (\pi+1)\chi(\cdot). \end{cases} \end{aligned} \tag{9}$$

4.2 The Role of Marginal Cost-to-benefit Ratio of Taxation

The conditions (6), (7), (8), and (9) determine the preferred tax rates by single females, single males, one-breadwinner couples, and two-breadwinner couples, respectively. Each type of young agent chooses his/her preferred tax rate to equate the marginal efficiency loss of taxation, represented by the left-hand side, to the marginal cost-to-benefit ratio of taxation in terms of utility, represented by the right-hand side; he/she prefers a lower tax rate as the ratio becomes higher.

To understand the intuition behind the abovementioned property, let us consider the single female's condition, (6), as an example:

$$\underbrace{1 - 2\tau}_{LHS} = \underbrace{\begin{cases} \frac{\alpha}{\chi(\cdot)} R & \text{if} \quad \tau < \hat{\tau}^{f} \equiv \alpha (\beta R)^{1/\sigma} / \chi(\cdot), \\ \frac{1}{\beta}\left(\frac{\alpha}{\chi(\cdot)}\right)^{1-\sigma} (\tau)^{\sigma} & \text{if} \quad \tau \geq \hat{\tau}^{f} \equiv \alpha (\beta R)^{1/\sigma} / \chi(\cdot). \end{cases}}_{RHS^f} \tag{6}$$

The term $\alpha/\chi(\cdot)$ appearing on the right-hand side represents the marginal cost-to-benefit ratio of taxation *in terms of goods*; the right-hand side itself expresses the marginal cost-to-benefit ratio of taxation *in terms of utility.* A single female agent prefers a tax rate that equates the marginal efficiency loss of taxation, denoted by LHS, to the marginal cost-to-benefit ratio of taxation in terms of utility, denoted by RHS^f.

To understand the role of the ratio in terms of utility, we next focus on the parameter α, representing the gender wage gap, appearing on the right-hand side, for illustrative purposes. There are two opposing effects of an increase in α on the ratio. First, given a tax rate, an increase in α imposes a further tax burden. This burden gives a single female an incentive to choose a lower tax rate, resulting in a negative effect on the preferred tax rate. Second, an increase in α augments wage income for single females and thus pension benefits in old age via the Bismarckian factor. This augmentation gives a single young female an incentive to choose a higher tax rate from the viewpoint of maintaining the utility of consumption, resulting in a positive effect on the preferred tax rate via the term $\chi(\cdot)$.

When a single young female is borrowing-unconstrained, she can reallocate income freely across periods. Because of this intertemporal reallocation of income, the positive effect is compensated for by the negative effect regardless of the degree of intertemporal elasticity of substitution. Therefore, an increase in α results in a higher marginal cost-to-benefit ratio in terms of goods, a higher marginal cost-to-benefit ratio in terms of utility, and thus a lower preferred tax rate when a single young female is borrowing-unconstrained.

When a single female is borrowing-constrained, the positive effect is not necessarily compensated for by the negative one. The borrowing-constrained single female wants to consume more when young, but her demand is restricted by the borrowing constraint. In this situation, the borrowing-constraint individual attaches a large weight to the utility gain of an increase in her wage. This effect might lead to a situation in which the positive effect overcomes the negative one, resulting in a lower, rather than a higher, marginal cost-to-benefit ratio in terms of utility and thus a higher preferred tax rate in response to an increase in α.

Which effect outweighs the other depends on the degree of the intertemporal elasticity of substitution. A lower elasticity results in a smaller change of consumption in response to a change in the interest rate. In other words, a lower elasticity implies a stronger incentive for single females to smooth consumption over periods. Because of this incentive, the borrowing-constrained single females attach a larger weight to the positive effect on youthful consumption as the interest-rate elasticity becomes lower.

The net effect of the two opposing forces depends on the intertemporal elasticity of substitution, $1/\sigma$. When the elasticity is high, that is, $1/\sigma \geq 1$, the net effect on the tax is negative. An increase in α results in a higher marginal cost-to-benefit ratio in terms of utility and thus a lower preferred tax rate by single females. In contrast, when the elasticity is low, that is, $1/\sigma < 1$, the net effect is positive. An increase in α results in a higher marginal cost-to-benefit ratio in terms of goods but a lower marginal cost-to-benefit ratio in terms of utility and thus a higher preferred tax rate by single females.

To close section, we note that the tax rates preferred by the young are lower than those preferred by the old, who choose $\tau = 1/2$. This property results in that the decisive voter with respect to τ belongs to the young generation because the population size of the young is larger than that of the old given the death of some males in early life. Given this result, we focus on young agents' preferences over τ and consider the determination of τ in majority voting in the next section.

5 Political Equilibrium

This section characterizes the political equilibrium of the majority voting. In what follows, an "agent" implies a "young agent" if not otherwise specified.

5.1 Political Environment

We impose the following assumption to proceed with the analysis.

Assumption 1: (i) $1 < \alpha(1+\pi)$ and (ii) $\max\left\{\frac{1-\pi}{4(1-\mu)}, \frac{1-\pi}{4\mu}\right\} < \varphi < \frac{1+\pi}{2}$.

The first assumption, $1 < \alpha(1+\pi)$, enables us to illustrate a variety of cases. Under this assumption, there are two critical values of γ:

$$\gamma = \frac{1-\alpha\pi}{1+\alpha}, \frac{1-\alpha\pi}{\alpha}.$$

With these two critical values, the order of critical tax rates that determine the status of saving becomes as follows:

$$\begin{array}{llll} \text{Case (a)} & \hat{\tau}^f < \hat{\tau}^{c2} \leq \hat{\tau}^{c1} < \hat{\tau}^m & \text{if} & 0 \leq \gamma \leq \frac{1-\alpha\pi}{1+\alpha}, \\ \text{Case (b)} & \hat{\tau}^f \leq \hat{\tau}^{c1} < \hat{\tau}^{c2} < \hat{\tau}^m & \text{if} & \frac{1-\alpha\pi}{1+\alpha} < \gamma \leq \frac{1-\alpha\pi}{\alpha}, \\ \text{Case (c)} & \hat{\tau}^{c1} \leq \hat{\tau}^f < \hat{\tau}^{c2} < \hat{\tau}^m & \text{if} & \frac{1-\alpha\pi}{\alpha} < \gamma \leq 1. \end{array} \tag{10}$$

Case (c) is available if and only if Assumption 1(i) holds. We will investigate who becomes a decisive voter for each case.

The second assumption, $\max\left\{\frac{1-\pi}{4(1-\mu)}, \frac{1-\pi}{4\mu}\right\} < \varphi < \frac{1+\pi}{2}$, ensures that one who prefers the highest tax rate among the young becomes the decisive voter. To understand the argument stemming from Assumption 1(ii), suppose that a young agent who belongs to a type-j ($j = f, m, c1, c2$) household prefers the highest tax rate among the young. He/she becomes a decisive voter if the size of the old plus the size of young agents who belong to the type-j household are more than a half of the population. Based on the argument above, a single young female or male becomes a decisive voter if

$$(1-\varphi) + (1+\pi) > \frac{3+\pi}{2};$$

a young agent who belongs to a one-breadwinner couple becomes a decisive voter if

$$2\varphi(1-\mu) + (1+\pi) > \frac{3+\pi}{2};$$

a young agent who belongs to a two-breadwinner couple becomes a decisive voter if:

$$2\varphi\mu + (1+\pi) > \frac{3+\pi}{2}.$$

For each condition, the first term on the left-hand side shows the size of young agents who prefer the highest tax rate among the young, the second term, $1+\pi$, shows the size of the old, and the term on the right-hand side, $(3+\pi)/2$, shows a half the population size. The above-mentioned three conditions are summarized as in Assumption 1(ii). Figure 2 depicts the set of (π, φ) that satisfies Assumption 1(ii).

[**Figure 2** about here.]

Hereafter, we focus on the parameter σ, which represents the inverse of the intertemporal elasticity of substitution and consider two cases separately: a high elasticity case ($1/\sigma \geq 1$ in Subsection 5.2) and a low elasticity case ($1/\sigma < 1$ in Subsection 5.3). We adopt this classification because the order of preferences for the tax rate depends critically on the degree of elasticity, as shown below. For each case, we will show the existence and uniqueness of an equilibrium of the majority voting game and find conditions that determine the decisive voter.

5.2 The Case of a High Intertemporal Elasticity of Substitution: $1/\sigma \geq 1$

Figure 3 illustrates the conditions (6), (7), (8), and (9) that determine the preferred tax rates by single females, single males, one-breadwinner couples, and two-breadwinner couples, respectively, for the case of $1/\sigma \geq 1$. Specifically, panels (a), (b), and (c) correspond to the cases (a), (b), and (c) in (10), respectively. The left-hand side of each condition, denoted by LHS, is decreasing in τ, independent of the type, and featured with

$LHS|_{\tau=0} = 1$ and $LHS|_{\tau=1/2} = 0$. In contrast, the right-hand side of each condition, denoted by RHS^j ($j = f, m, c1$ and $c2$), is non-decreasing in τ, dependent on the type of a household, and characterized by

$$\begin{array}{llll} \text{Case (a):} & RHS^f \leq RHS^{c2} \leq RHS^{c1} \leq RHS^m & \text{if} & 0 \leq \gamma \leq \frac{1-\alpha\pi}{1+\alpha}, \\ \text{Case (b):} & RHS^f \leq RHS^{c1} \leq RHS^{c2} \leq RHS^m & \text{if} & \frac{1-\alpha\pi}{1+\alpha} < \gamma \leq \frac{1-\alpha\pi}{\alpha}, \\ \text{Case (c):} & RHS^{c1} \leq RHS^f \leq RHS^{c2} \leq RHS^m & \text{if} & \frac{1-\alpha\pi}{\alpha} < \gamma \leq 1, \end{array}$$

where equality holds if and only if $1/\sigma = 1$ and $s^j = 0$. The kink point of $\tau = \hat{\tau}^j$ implies that a type-j household can save part of its income if $\tau < \hat{\tau}^j$ and nothing if $\tau \geq \hat{\tau}^j$.

[**Figure 3** about here.]

The crossing point of LHS and RHS^j determines the tax rate preferred by a young agent who belongs to a type-j household. Under Assumption 1(ii), the decisive voter is the one who prefers the highest tax rate among the young. From the observation in Figure 3, we can conclude that the decisive voter is a single female ($j = f$) agent if $\gamma \leq (1 - \alpha\pi)/\alpha$ (see panels (a) and (b)); this voter is an agent who belongs to a one-breadwinner couple ($j = c1$) if $\gamma > (1 - \alpha\pi)/\alpha$ (see panel (c)).

Proposition 1. *Suppose that $1/\sigma \geq 1$ holds. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(i) *a type-f, single female agent if $\gamma \leq \frac{1-\alpha\pi}{\alpha}$;*

(ii) *a type-$c1$ agent who belongs to a one-breadwinner couple otherwise.*

The result established in Proposition 1 has the following two features. First, a single male or an agent who belongs to a two-breadwinner couple does not become a decisive voter. Such agents' marginal cost-to-benefit ratios of taxation in terms of utility are always higher than those of the other two types of households. This result implies that single males and two-breadwinner couples prefer a lower tax rate than do other two types of young agents. The decisive voter therefore is a single female or an agent who belongs to a one-breadwinner couple.

Second, which household becomes a decisive voter depends on α, π and γ that represent the gender wage gap, life expectancy of men, and the fraction of derived pension rights, respectively. Suppose that the gender wage gap is high (i.e., α is low), the life expectancy of men (π) is low, and the level of derived pension rights (γ) is low such that $\gamma \leq (1 - \alpha\pi)/\alpha$. Then, the marginal cost-to-benefit ratio of taxation in terms of utility for single females is lower than that for one-breadwinner couples because the former owe less tax burden whereas the latter receive lower pension benefits. Therefore, single females prefer a higher tax rate than do one-breadwinner couples and thus become decisive voters if $\gamma \leq (1-\alpha\pi)/\alpha$.

5.3 The Case of a Low Intertemporal Elasticity of Substitution: $1/\sigma < 1$

This subsection considers an economy with a low intertemporal elasticity of substitution such that $1/\sigma < 1$. The decisive voter over τ may differ from that of the previous case; the order of preferred tax rates may change depending on the fraction of each household type, the gender wage gap, the life expectancy of men, and the level of derived pension rights. To determine the decisive voter over τ, we recall the conditions (6), (7), (8), and (9) that determine the preferred tax rates by four types of households. The graphs of these conditions when $1/\sigma < 1$ are illustrated in Figure 4.

[**Figure 4** about here.]

The main difference from the previous subsection is that RHS^j and RHS^k $(j \neq k)$ intersect at some tax rate $\tau \in (0, 1/2)$. This difference is because in an economy with $1/\sigma < 1$, the slope of RHS^j becomes steeper the lower the marginal cost-to-benefit ratio becomes when a type-j household is borrowing-constrained. To investigate in detail the property of the political equilibrium when $1/\sigma < 1$, we hereafter consider in turn three cases, (a), (b), and (c), classified according to the level of derived pension right as in (10), and then summarize the results of the three cases to provide a global characterization of the political equilibrium.

5.3.1 Low Level of Derived Pension Rights: $0 \leq \gamma \leq \frac{1-\alpha\pi}{1+\alpha}$

Panel (a) of Figure 4 illustrates the conditions (6), (7), (8), and (9) that determine the preferred tax rates of the four types of households when the level of derived pension rights is low such that $\gamma \in [0, (1-\alpha\pi)/(1+\alpha)]$. As depicted in the figure, there are three critical values of τ, $\tilde{\tau}^{f,c2} \in \left(\hat{\tau}^f, \hat{\tau}^{c2}\right)$, $\tilde{\tau}^{c2,c1} \in (\hat{\tau}^{c2}, \hat{\tau}^{c1})$, and $\tilde{\tau}^{c1,m} \in (\hat{\tau}^{c1}, \hat{\tau}^m)$, such that RHS^f and RHS^{c2} intersect at $\tau = \tilde{\tau}^{f,c2}$, RHS^{c2} and RHS^{c1} cross at $\tau = \tilde{\tau}^{c2,c1}$, and RHS^{c1} and RHS^m intersect at $\tau = \tilde{\tau}^{c1,m}$. By direct calculation, we obtain

$$\tilde{\tau}^{f,c2} \equiv \left(\frac{(1+\alpha)R\beta}{(\pi+1)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \left(\frac{\alpha}{\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}, \quad \tilde{\tau}^{c2,c1} \equiv \left(\frac{R\beta}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \left(\frac{1+\alpha}{(\pi+1)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}},$$

$$\tilde{\tau}^{c1,m} \equiv \left(\frac{R\beta}{\pi\chi(\cdot)}\right)^{\frac{1}{\sigma}} \left(\frac{1}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}.$$

The tax rate preferred by a type-j household is determined by the crossing point of LHS and RHS^j. Given the assumption of household distribution in Assumption 1(ii), the decisive voter over τ is the one who prefers the highest tax rate among the young households. Based on the illustration in Panel (a) of Figure 4, the decisive voter over τ when $1/\sigma < 1$ and $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$ is determined as follows.

Lemma 1. *Suppose that $1/\sigma < 1$ and $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$ hold. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(j) *a type-f single female agent if* $\chi(\cdot) \leq 2\left(\frac{(1+\alpha)R\beta}{\pi+1}\right)^{\frac{1}{\sigma}} (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{(1+\alpha)R}{\pi+1}$;

(ii) *a type-$c2$ agent who belongs to a two-breadwinner couple, otherwise.*

Proof. See Appendix 8.2.

The result in Lemma 1 indicates that (i) a type-m or type-$c1$ agent never becomes a decisive voter; and (ii) the decisive voter is a type-f single female agent if the gender wage gap is small and the life expectancy of men is high such that $\chi(\cdot) \leq 2\left(\frac{(1+\alpha)R\beta}{\pi+1}\right)^{\frac{1}{\sigma}} (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{(1+\alpha)R}{\pi+1}$; otherwise, the decisive voter is a type-$c2$ agent who belongs to a two-breadwinner couple.

To understand the mechanism behind the first result, suppose that a type-m or type-$c1$ agent is a decisive voter. Based on the observation in Panel (a) of Figure 4, the equilibrium tax rate must be high such that $\tau > \tilde{\tau}^{c2,c1}$. For this high tax rate, the marginal cost-to-benefit ratios of taxation in terms of utility for type-m and type-$c1$ agents, which are increasing in the tax rate, are always larger than the marginal efficiency loss of taxation, $1 - 2\tau$, which is decreasing in the tax rate. The marginal ratio and efficiency loss of taxation are never balanced for $\tau > \tilde{\tau}^{c2,c1}$ for type-m and type-$c1$ agents. Therefore, neither of the agents becomes a decisive voter.

Next, to understand the mechanism behind the second result, recall the condition that produces an equilibrium in which a type-f single female agent becomes a decisive voter in Lemma 1(i). The condition is rewritten as

$$\frac{1 - \frac{\alpha}{1+\alpha}\varphi(1-\mu)}{1 - \frac{\varphi(1-\mu)(1-\gamma)}{\pi+1}} \leq 2\left(R\beta\right)^{\frac{1}{\sigma}} (\pi+1)^{1-\frac{1}{\sigma}} \left(\frac{\alpha}{1+\alpha}\right)^{\frac{\sigma-1}{\sigma}} + R, \tag{11}$$

where the left-hand side is decreasing in α and π, and the right-hand side is increasing in α and π. Therefore, the condition (11) states that a type-f agent is more likely to become a decisive voter when α and π are higher.

Single females owe greater tax burden as α becomes higher (i.e., as their wage becomes higher), and two-breadwinner couples receive greater pension benefits as π becomes higher (i.e., as men live longer). The marginal cost-to-benefit ratio of taxation *in terms of goods* for single females is more likely to be higher than that for two-breadwinner couples as α and π become higher. When the elasticity is low such that $1/\sigma < 1$, the relation is reversed in terms of utility as argued in Section 4. Therefore, when α and π are high such that (11) holds, the marginal cost-to-benefit ratio of taxation in terms of utility for single females is lower than that for two-breadwinner couples. Therefore, a type-f single female agent prefers a higher tax rate than does a type-$c2$ agent who belongs to a two-breadwinner couple.

5.3.2 Medium Level of Derived Pension Right: $\frac{1-\alpha\pi}{1+\alpha} < \gamma \leq \frac{1-\alpha\pi}{\alpha}$

Panel (b) of Figure 4 illustrates the conditions (6), (7), (8), and (9) that determine the preferred tax rates by the four types of households when the level of derived pension right is medium such that $\gamma \in ((1-\alpha\pi)/(1+\alpha), (1-\alpha\pi)/\alpha]$. As depicted in the figure, there are three critical values of τ, $\tilde{\tau}^{f,c1} \in \left(\hat{\tau}^f, \hat{\tau}^{c1}\right)$, $\tilde{\tau}^{c1,c2} \in (\hat{\tau}^{c1}, \hat{\tau}^{c2})$, and $\tilde{\tau}^{c2,m} \in (\hat{\tau}^{c2}, \hat{\tau}^m)$, such that RHS^f and RHS^{c1} intersect at $\tau = \tilde{\tau}^{f,c1}$, RHS^{c1} and RHS^{c2} cross at $\tau = \tilde{\tau}^{c1,c2}$, and RHS^{c2} and RHS^m intersect at $\tau = \tilde{\tau}^{c2,m}$. By direct calculation, we obtain

$$\tilde{\tau}^{f,c1} \equiv \left(\frac{R\beta}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \left(\frac{\alpha}{\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}, \quad \tilde{\tau}^{c1,c2} \equiv \left(\frac{R\beta(1+\alpha)}{(\pi+1)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \left(\frac{1}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}},$$
$$\tilde{\tau}^{c2,m} \equiv \left(\frac{R\beta}{\pi\chi(\cdot)}\right)^{\frac{1}{\sigma}} \left(\frac{1+\alpha}{(\pi+1)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}.$$

As in the previous case, we can characterize the political equilibrium for the case of $1/\sigma < 1$ and $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \frac{1-\alpha\pi}{\alpha}\right]$ as follows.

Lemma 2. *Suppose that $1/\sigma < 1$ and $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \frac{1-\alpha\pi}{\alpha}\right]$ hold. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(j) *a type-f single female agent if $\chi(\cdot) \leq 2\left(\frac{R\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}} (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{R}{\pi+\gamma}$;*

(ii) *a type-$c1$ agent who belongs to a one-breadwinner couple otherwise.*

Proof. See Appendix 8.3.

The main difference from the previous case is that a type-$c2$ agent cannot be a decisive voter; instead, a type-$c1$ agent can be a decisive voter under a certain condition. A change of the decisive voter is due to that, in the current case, a type-$c1$ agent attains a higher pension benefit because of a higher level of derived pension right relative to the previous case. This relationship results in a lower marginal cost-to-benefit ratio of taxation in terms of goods but a higher marginal cost-to-benefit ratio of taxation in terms of utility for a type-$c1$ agent compared to a type-$c2$ agent. Therefore, a type-$c1$ agent prefers a higher tax rate than does a type-$c2$ agent.

In the current case, the decisive voter is a type-f or a type-$c1$ agent. The former becomes a decisive voter if the condition in Lemma 2(i) holds. The left-hand side of the condition is decreasing in α and π, while the right-hand side is increasing in α and π. An interpretation similar to that of the previous case applies to the current case. For higher α and π, a type-f, single female agent is more likely to become a decisive voter.

5.3.3 High Level of Derived Pension Right: $\frac{1-\alpha\pi}{\alpha} < \gamma \leq 1$

Panel (c) of Figure 4 illustrates the conditions (6), (7), (8), and (9) that determine the preferred tax rates by four types of households when the level of derived pension rights is high such that $\gamma \in ((1-\alpha\pi)/\alpha, 1]$. As depicted in the figure, there are three critical values of τ, $\tilde{\tau}^{c1,f} \in \left(\hat{\tau}^{c1}, \hat{\tau}^{f}\right)$, $\tilde{\tau}^{f,c2} \in \left(\hat{\tau}^{f}, \hat{\tau}^{c2}\right)$, and $\tilde{\tau}^{c2,m} \in (\hat{\tau}^{c2}, \hat{\tau}^{m})$, such that RHS^{c1} and RHS^{f} intersect at $\tau = \tilde{\tau}^{c1,f}$, RHS^{f} and RHS^{c2} intersect at $\tau = \tilde{\tau}^{f,c2}$, and RHS^{c2} and RHS^{m} intersect at $\tau = \tilde{\tau}^{c2,m}$. By direct calculation, we obtain

$$\tilde{\tau}^{c1,f} \equiv \left(\frac{\alpha R\beta}{\chi(\cdot)}\right)^{\frac{1}{\sigma}} \left(\frac{\alpha}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}, \ \tilde{\tau}^{f,c2} \equiv \left(\frac{(1+\alpha)R\beta}{(\pi+1)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \left(\frac{\alpha}{\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}},$$
$$\tilde{\tau}^{c2,m} \equiv \left(\frac{R\beta}{\pi\chi(\cdot)}\right)^{\frac{1}{\sigma}} \left(\frac{1+\alpha}{(\pi+1)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}.$$

As in the previous two cases, we can characterize the political equilibrium for the case of $1/\sigma < 1$ and $\gamma \in ((1-\alpha\pi)/\alpha, 1]$ as follows:

Lemma 3. *Suppose that $1/\sigma < 1$ and $\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, 1\right]$ hold. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(j) *a type-$c1$ agent who belongs to a one-breadwinner couple if* $\chi(\cdot) \leq 2\left(\alpha R\beta\right)^{\frac{1}{\sigma}} \left(\frac{1}{\pi+\gamma}\right)^{\frac{\sigma-1}{\sigma}} + \alpha R$;

(ii) *a type-f single female agent otherwise.*

Proof. See Appendix 8.4.

A set of possible decisive voters is unchanged from case (b): the decisive voter is a type-f or a type-$c1$ agent depending on α and π. However, the qualitative impact of these parameters on the determination of the decisive voter is completely reversed. In case (b), a type-f agent is more likely to be a decisive voter for higher α and π. In contrast, she is more likely to be a decisive voter for lower α and π in the current case.

The difference between the two cases is due to the different levels of derived pension right. The level of derived pension right in the current case is higher than that in case (b). This difference implies that when α and π are high such that both types of agents are borrowing-unconstrained the marginal cost-to-benefit ratio of taxation for type-$c1$ agents is lower than that for type-f agents in terms of goods as well as utility. Therefore, type-$c1$ agents prefer the highest tax rate among the young and become the decisive voter if α and π are high such that the condition in Lemma 3(i) holds. In contrast, when α and π are low such that both types of agents are borrowing-constrained, the order of the ratios in terms of utility is reversed because of a low intertemporal elasticity of substitution. The ratio for type-f agents becomes lower than that for the type-$c1$ agents; the former prefer the highest tax rate among the young and become decisive voters.

5.3.4 A Decisive Voter When $1/\sigma < 1$

The results established so far are summarized as follows.

Proposition 2. *Suppose that $1/\sigma < 1$ holds. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(i) *a type-$c1$ agent who belongs to a one-breadwinner couple if $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \frac{1-\alpha\pi}{\alpha}\right]$ and $\chi(\cdot) > 2\left(\frac{R\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}}(\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{R}{\pi+\gamma}$; or if $\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, 1\right]$ and $\chi(\cdot) \leq 2(\alpha R\beta)^{\frac{1}{\sigma}}\left(\frac{1}{\pi+\gamma}\right)^{\frac{\sigma-1}{\sigma}} + \alpha R$;*

(ii) *a type-$c2$ agent who belongs to a two-breadwinner couple if $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$ and $\chi(\cdot) > 2\left(\frac{(1+\alpha)R\beta}{\pi+1}\right)^{\frac{1}{\sigma}}(\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{(1+\alpha)R}{\pi+1}$;*

(iii) *a type-f single female agent otherwise.*

Proof. It is immediate from Lemmas 1-3.

The result in Proposition 2 is different than that in Proposition 1 in the following three ways. First, an agent who belongs to a two-breadwinner couple never becomes a decisive voter when $1/\sigma \geq 1$, but he/she can be a decisive voter when $1/\sigma < 1$ (Lemma 1 and Proposition 2(ii)). Second, a single female agent is more likely to be a decisive voter for lower α and π when $1/\sigma \geq 1$, but she is more likely to be a decisive voter for higher α and π when $1/\sigma < 1$ and $\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, 1\right]$ (Lemma 3). The mechanism behind these two properties has been discussed in the previous subsections.

The third property is that the fraction of two-breadwinner couples, denoted by μ, has no effect on the determination of a decisive voter when $1/\sigma \geq 1$, but it has a crucial effect on the determination of a decisive voter when $1/\sigma < 1$. The result in Proposition 2(ii) states that for an economy with $1/\sigma < 1$, a type-$c2$ agent who belongs to a two-breadwinner couple becomes a decisive voter if (a) the level of derived pension right is set at the low level, and (b) the benefit-burden ratio of public pensions in the economy, denoted by $\chi(\cdot)$, is high. Because $\chi(\cdot)$ is increasing in μ, the latter condition implies that a type-$c2$ agent is more likely to be a decisive voter when the fraction of type-$c2$ agents is larger in the economy.

6 Gender Wage Gap, Derived Pension Rights and the Fraction of Two-breadwinner Couples

Given the characterization of the political equilibrium in Section 5, we investigate how the tax rate changes in response to recent trends in developed economies: a reduction of the gender wage gap, a reduction of derived pension rights, and an increase in the fraction of two-breadwinner couples. The aim of the analysis is to predict the direction of future change in the tax rate in response to these trends. The analysis also aims to explore the roles of the borrowing constraint and the interest-rate elasticity of consumption on the determination of the tax rate.

6.1 A Reduction of the Gender Wage Gap

First, we investigate the effect of a reduction of the gender wage gap on the determination of the tax rate.

Proposition 3: *Consider an increase in α that implies a reduction of the gender wage gap.*

(i) *In an economy with $1/\sigma \geq 1$, a reduction of the gender wage gap does not increase the tax rate if $\gamma \leq (1-\alpha\pi)/\alpha$ where the decisive voter is a type-f agent; it does not decrease the tax rate if $\gamma > (1-\alpha\pi)/\alpha$, where the decisive voter is a type-$c1$ agent.*

(ii) *In an economy with $1/\sigma < 1$ where the decisive voter is a type-j ($j = f, c1$, or $c2$) agent, a reduction of the gender wage gap locally produces an inverse U-shaped relationship between α and τ around $\hat{\tau}^j$.*

Proof. See Appendix 8.5.

[**Figure 5** about here.]

To understand the mechanism behind the result in Proposition 3, we first consider the effect of an increase in α on the marginal cost-to-benefit ratio of taxation in terms of goods. The ratios for $j = f, c1$ and $c2$, denoted by RHS^f_{goods}, RHS^{c1}_{goods} and RHS^{c2}_{goods}, are given by, respectively,

$$RHS^f_{goods} \equiv \frac{\alpha}{\chi(\cdot)}, RHS^{c1}_{goods} \equiv \frac{1}{(\pi+\gamma)\chi(\cdot)}, RHS^{c2}_{goods} \equiv \frac{1+\alpha}{(\pi+1)\chi(\cdot)}.$$

After some calculation, we obtain

$$\frac{\partial RHS^{f}_{goods}}{\partial \alpha} > 0, \frac{\partial RHS^{c1}_{goods}}{\partial \alpha} < 0, \frac{\partial RHS^{c2}_{goods}}{\partial \alpha} > 0.$$

A reduction of the gender wage gap (i.e., an increase in α) increases the marginal cost-to-benefit ratio of taxation in terms of goods for type-f and type-$c2$ agents, whereas it decreases the ratio for type-$c1$ agents. The difference is due to that single females and two-breadwinner couples owe an additional tax burden due to an increase in females' wage, whereas one-breadwinner couples owe no additional burden. Because increased tax revenue is returned to all types of agents as public pension benefits, single females and two-breadwinner couples pay more than they receive, whereas one-breadwinner couples pay nothing but receive additional benefits. Therefore, a reduction of the gender wage gap results in a higher cost-to-benefit ratio of taxation in terms of goods for type-f and type-$c2$ agents, whereas it results in a lower cost-to-benefit ratio for type-$c1$ agents.

Figure 5 illustrates the effects of a reduction of the gender wage gap on the equilibrium tax rate. When $1/\sigma \geq 1$, the marginal cost-to-benefit ratio of taxation in terms of goods, RHS^{j}_{goods}, is positively related to that in terms of utility, RHS^{j}. Given this positive relation, a reduction of the gender wage gap results in a higher RHS^{f} and thus in a lower preferred tax rate by type-f agents. In contrast, a reduction of the gender wage gap results in a lower RHS^{c1} and thus in a higher preferred tax rate by type-$c1$ agents. An exception is the case of $1/\sigma = 1$ and $s^{j} = 0$ in which α has no effect on RHS^{j}: a reduction of the gender wage gap has no effect on the equilibrium tax rate.

When $1/\sigma < 1$, this intuitive result still holds as long as an agent is borrowing-unconstrained. However, the opposite result holds when an agent is borrowing-constrained because RHS^{j}_{goods} is negatively related to RHS^{j} if $1/\sigma < 1$ and $s^{j} = 0$; a reduction of the gender wage gap results in a lower RHS^{j} ($j = f, c2$) and thus in a higher preferred tax rate by type-f and type-$c2$ agents, whereas it results in a higher RHS^{c1} and thus in a lower preferred tax rate by type-$c1$ agents. Therefore, there is an inverse U-shaped relationship between α and τ^{j} around the critical value of the tax, $\hat{\tau}^{j}$, that divides the status of saving.

6.2 A Reduction of Derived Pension Rights

Next, we consider the effect of a reduction of derived pension rights on the determination of the tax rate.

Proposition 4: *Consider a decrease in γ that implies a reduction of derived pension rights.*

(i) *In an economy with* $1/\sigma \geq 1$*, a reduction of derived pension rights does not decrease the tax rate if* $\gamma \leq (1-\alpha\pi)/\alpha$ *where the decisive voter is a type-*f *agent; it does not increase the tax rate if* $\gamma > (1-\alpha\pi)/\alpha$ *where the decisive voter is a type-*$c1$ *agent.*

(ii) *In an economy with* $1/\sigma < 1$ *where the decisive voter is a type-*j *(*$j = f, c1$*, or* $c2$*) agent, a reduction of derived pension rights produces an inverse U-shaped relationship between* γ *and* τ *around* $\hat{\tau}^j$*.*

Proof. See Appendix 8.5.

To understand the result in Proposition 4, we first consider the effect of the derived pension rights on the marginal cost-to-benefit ratio of taxation in terms of goods, RHS^j_{goods}:

$$(-1)\frac{\partial RHS^f_{goods}}{\partial \gamma} < 0, (-1)\frac{\partial RHS^{c1}_{goods}}{\partial \gamma} > 0, (-1)\frac{\partial RHS^{c2}_{goods}}{\partial \gamma} < 0.$$

We multiply the derivatives by (-1) to demonstrate the qualitative effect of a decrease in γ. A reduction of derived pension rights (i.e., a decrease in γ) increases the pension benefits for type-f and type-$c2$ agents and thus lowers their cost-to-benefit ratio of taxation in terms of goods. In contrast, such a reduction decreases the pension benefits for type-$c1$ agents and thus raises their cost-to-benefit ratio of taxation in terms of goods.

When $1/\sigma \geq 1$, the marginal cost-to-benefit ratio of taxation in terms of goods, RHS^j_{goods}, is positively related to that in terms of utility, RHS^j. This property gives type-f agents an incentive to choose a higher tax rate but gives type-$c1$ agents an incentive to choose a lower tax rate in response to a reduction of derived pension rights. An exception is the case of $1/\sigma = 1$ and $s^j = 0$ in which γ has no effect on RHS^j. When $1/\sigma < 1$, the abovementioned result still holds as long as an agent is borrowing-unconstrained, but the opposite result holds when an agent is borrowing-constrained. The two opposing effects result in an inverse U-shaped relationship between γ and τ^j around the critical value of the tax, $\hat{\tau}^j$. The mechanism behind this result follows that shown in the case of a reduction of gender wage gap.

6.3 An Increase in the Fraction of Two-breadwinner Couples

Finally, we examine the effect of the share of two-breadwinner couples on the equilibrium tax rate.

Proposition 5: *Consider an increase in* μ *that implies an increase in the fraction of two-breadwinner couples.*

(i) *In an economy with* $1/\sigma \geq 1$, *an increase in the fraction of two-breadwinner couples does not increase the tax rate if* $\gamma \leq (1-\alpha\pi)/(1+\alpha)$; *it does not decrease the tax rate if* $\gamma > (1-\alpha\pi)/(1+\alpha)$.

(ii) *In an economy with* $1/\sigma < 1$ *where the decisive voter is a type-*j *(*$j = f, c1,$ *or* $c2$*) agent, an increase in the fraction of two-breadwinner couples* (a) *locally produces an inverse U-shaped relationship between* μ *and* τ *around* $\hat{\tau}^j$ *if* $\gamma \neq (1-\alpha\pi)/(1+\alpha)$; (b) *has no effect on the equilibrium tax if* $\gamma = (1-\alpha\pi)/(1+\alpha)$.

Proof. See Appendix 8.5.

The result in Proposition 5 can be understood by focusing on the effects of μ on RHS^j_{goods} ($j = f, c1, c2$). The effects are summarized as

$$\frac{\partial RHS^f_{goods}}{\partial \mu} \gtreqless 0, \frac{\partial RHS^{c1}_{goods}}{\partial \mu} \gtreqless 0, \frac{\partial RHS^{c2}_{goods}}{\partial \mu} \gtreqless 0 \text{ if and only if } \gamma \lesseqgtr \frac{1-\alpha\pi}{1+\alpha}.$$

The marginal cost-to-benefit ratio of taxation in terms of goods, RHS^j_{goods} ($j = f, c1, c2$), is affected by an increase in μ via the benefit-to-burden ratio of public pension, denoted by $\chi(\cdot)$, in the following two ways. On one hand, an increase in μ results in an increase of tax revenue from the working females and thus in an increase per capita pension benefit. This effect is observed in the numerator of $\chi(\cdot)$ in (1), which yields a negative effect on RHS^j_{goods}. On the other hand, an increase in μ implies a larger size of two-breadwinner couples and a smaller size of one-breadwinner couples. Because the level of pension rights is larger for working females than for non-working females, an increase in μ results in a smaller per capita pension benefits. This effect is observed in the denominator of $\chi(\cdot)$ in (1), which yields a positive effect on RHS^j_{goods}.

As observed in (1), the former effect is independent of γ, whereas the latter effect is dependent of γ. Specifically, the latter effect becomes larger as γ becomes smaller. Therefore, the latter effect overcomes the former one when the level of derived pension rights is low such that $\gamma < (1-\alpha\pi)/(1+\alpha)$. That is, RHS^j_{goods} is increased by an increase in μ if $\gamma < (1-\alpha\pi)/(1+\alpha)$. The opposite result holds if $\gamma > (1-\alpha\pi)/(1+\alpha)$. The two opposing effects are offset by each other if $\gamma = (1-\alpha\pi)/(1+\alpha)$.

When $1/\sigma \geq 1$, RHS^j_{goods} is positively related to RHS^j. An increase in μ results in a higher RHS^j_{goods}, a higher RHS^j and thus a lower preferred tax rate by a type-j agent if $\gamma < (1-\alpha\pi)/(1+\alpha)$. The opposite result holds if $\gamma > (1-\alpha\pi)/(1+\alpha)$. When $1/\sigma < 1$, the abovementioned result still holds as long as an agent is borrowing-unconstrained, but the opposite result holds when an agent is borrowing-constrained. This effect creates an inverse U-shaped relationship between μ and τ^j around the critical value of the tax, $\hat{\tau}^j$.

7 Concluding Remarks

This paper developed an overlapping-generation model based on that of Leroux, Pestieau and Racionero (2011): the model includes four types of households: single female, single male, one-breadwinner couple and two-breadwinner couple. The paper introduced a borrowing constraint into their model and generalized the model by assuming a utility function with a constant intertemporal elasticity of substitution. Under this generalized framework, we consider majority voting over public pension policy in the presence of derived pension rights, and investigate how the borrowing constraint and intertemporal elasticity of substitution affect the preferences of each household over pension and the resulting equilibrium pension policy.

The paper showed the following two results. First, in an economy with a high intertemporal elasticity of substitution, one-breadwinner couples are more likely to be a decisive voter (that is, to prefer the highest tax rate among the young) when the level of derived pension rights is higher: this result is identical to that of Leroux, Pestieau and Racionero (2011). However, in an economy with a low intertemporal elasticity of substitution, one-breadwinner couples may prefer a lower, rather than a higher, tax rate than do single females because of the presence of borrowing constraints. The paper showed that a borrowing constraint associated with a low intertemporal elasticity of substitution critically affects the order of tax rates preferred by the four types of households.

Second, the gender wage gap, the level of derived pension rights, and the fraction of two-breadwinner couples produce monotone effects on the equilibrium tax rate when the intertemporal elasticity of substitution is high. However, these factors create an inverse U-shaped relationship between the relevant variable and the tax rate when the elasticity is low. The non-monotone effects were derived via a borrowing constraint associated with a low intertemporal elasticity of substitution.

Throughout the analysis, we assumed that the degree of derived pension rights is fixed. This assumption can be relaxed by assuming a structure-induced Nash equilibrium of voting (for example, Conde-Ruiz and Galasso, 2003; 2005; Casamatta, Cremer and Pestieau, 2005; Conde-Ruiz and Profeta, 2007; Bethencourt and Galasso, 2008). In this voting equilibrium, one-breadwinner couples prefer a full derived pension right, whereas others prefer no right. Thus, the full derived pension right is realized if the size of one-breadwinner couples is larger than a half of the population; no derived pension right is realized otherwise. However, in the real world, the degree of the derived pension right is set between these two extreme solutions. There is a need to add some institutional feature to demonstrate a more realistic situation: this task is left as future work.

8 Appendix

8.1 Single-peaked Preferences of the Young

In this appendix, we prove that preferences of a type-f young agent are single peaked. The proof applies to other types of young agents.

The proof proceeds as follows. First, we show that both $V^f_{s>0}$ and $V^f_{s=0}$ are single peaked over τ. Then we demonstrate that $\partial V^f_{s>0}/\partial\tau = \partial V^f_{s=0}/\partial\tau$ and $V^f_{s>0} = V^f_{s=0}$ hold at $\tau = \hat{\tau}^f$, implying that V^f has a unique local maximum over the whole range of τ and thus that V^f is single peaked over τ.

The first and the second derivatives of $V^{y,j}_{s>0}$ and $V^{y,j}_{s=0}$ with respect to τ are

$$\begin{aligned}
\frac{\partial V^f_{s>0}}{\partial\tau} &= \left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma} \cdot \left[(1-\tau)\alpha w + \frac{w\chi(\cdot)}{R}\tau(1-\tau)\right]^{-\sigma} \cdot \left\{-\alpha w + \frac{w\chi(\cdot)}{R}(1-2\tau)\right\};\\
\frac{\partial^2 V^f_{s>0}}{\partial\tau^2} &= \left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma} \cdot (-\sigma) \cdot \left[(1-\tau)\alpha w + \frac{w\chi(\cdot)}{R}\tau(1-\tau)\right]^{-\sigma-1} \cdot \left\{-\alpha w + \frac{w\chi(\cdot)}{R}(1-2\tau)\right\}^2\\
&+ \left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma} \cdot \left[(1-\tau)\alpha w + \frac{w\chi(\cdot)}{R}\tau(1-\tau)\right]^{-\sigma} \cdot (-2)\frac{w\chi(\cdot)}{R}\\
&< 0;\\
\frac{\partial V^f_{s=0}}{\partial\tau} &= [(1-\tau)\alpha w]^{-\sigma} \cdot (-\alpha w) + \beta \cdot [w\chi(\cdot)\tau(1-\tau)]^{-\sigma} \cdot w\chi(\cdot) \cdot (1-2\tau);\\
\frac{\partial^2 V^f_{s=0}}{\partial\tau^2} &= (-\sigma) \cdot [(1-\tau)\alpha w]^{-\sigma-1} \cdot (\alpha w)^2\\
&- \sigma\beta \cdot [w\chi(\cdot)\tau(1-\tau)]^{-\sigma-1} \cdot (w\chi(\cdot))^2 \cdot (1-2\tau)^2\\
&- 2\beta \cdot [w\chi(\cdot)\tau(1-\tau)]^{-\sigma} \cdot w\chi(\cdot)\\
&< 0.
\end{aligned}$$

The functions $V^f_{s>0}$ and $V^f_{s=0}$ are single peaked over τ because the second derivatives are negative.

Next, we show that $\partial V^f_{s>0}/\partial\tau = \partial V^f_{s=0}/\partial\tau$ and $V^f_{s>0} = V^f_{s=0}$ hold at $\tau = \hat{\tau}^f$. By direct calculation, we have:

$$
\begin{aligned}
V_{s>0}^{y,j}\Big|_{\tau=\hat{\tau}^f} &= V_{s=0}^{y,j}\Big|_{\tau=\hat{\tau}^f} \\
&= \frac{1}{1-\sigma}\cdot\frac{R+(\beta R)^{1/\sigma}}{R}\cdot\left(1-\frac{\alpha(\beta R)^{1/\sigma}}{\chi(\cdot)}\right)^{1-\sigma}\cdot(\alpha w)^{1-\sigma}-\frac{1+\beta}{1-\sigma}, \\
\left.\frac{\partial V_{s>0}^{y,j}}{\partial\tau}\right|_{\tau=\hat{\tau}^f} &= \left.\frac{\partial V_{s=0}^{y,j}}{\partial\tau}\right|_{\tau=\hat{\tau}^f} \\
&= \left(1-\frac{\alpha(\beta R)^{1/\sigma}}{\chi(\cdot)}\right)^{-\sigma}\cdot(\alpha w)^{1-\sigma}\cdot\left[-1+\frac{\chi(\cdot)}{\alpha R}+\frac{2}{R}(\beta R)^{1/\sigma}\right].
\end{aligned}
$$

With this result and the single-peakedness of $V_{s>0}^{y,j}$ and $V_{s=0}^{y,j}$ over τ, we can conclude that V^f has a unique local maximum with respect to τ over the whole range of τ. Specifically, V^f is maximized at $\tau = \arg\max V_{s>0}^f$ if $\arg\max V_{s>0}^f < \hat{\tau}^f$; it is maximized at $\tau = \arg\max V_{s=0}^f$ otherwise.

■

8.2 Proof of Lemma 1

1st step. We first show that a type-j ($j = c1, m$) agent cannot be a decisive voter. To show this, suppose that an agent who belongs to a type$-c1$ or type$-m$ household is a decisive voter. From Panel (a) of Figure 4, it must be that $LHS > RHS^{c2}$ at $\tau = \tilde{\tau}^{c2,c1}$; that is,

$$1-2\tilde{\tau}^{c2,c1} > \frac{R}{(\pi+\gamma)\chi(\cdot)}.$$

By rearranging this condition, we obtain:

$$
\begin{aligned}
&1-2\left(\frac{R\beta}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{1}{\sigma}}\left(\frac{1+\alpha}{(\pi+1)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}} > \frac{R}{(\pi+\gamma)\chi(\cdot)} \\
&\Leftrightarrow (\pi+\gamma)\chi(\cdot) > 2(\pi+\gamma)\left(\frac{R\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}}\left(\frac{1+\alpha}{\pi+1}\right)^{\frac{\sigma-1}{\sigma}} + R. \qquad (12)
\end{aligned}
$$

The left-hand side of (12), $(\pi+\gamma)\chi(\cdot)$, is less than one under the current assumption of $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$.[2] Conversely, the right-hand side is greater than one under the assumption

[2]The condition $(\pi+\gamma)\chi(\cdot) \gtreqless 1$ is rewritten as

$$
\begin{aligned}
&(\pi+\gamma)\chi(\cdot) \gtreqless 1 \\
&\Leftrightarrow [(-1)(1-\pi\alpha)+\gamma(1+\alpha)]\cdot[1-\varphi(1-\mu)] \lesseqgtr 1
\end{aligned}
$$

where the left-hand side in the second line is negative because $(-1)(1-\pi\alpha)+\gamma(1+\alpha) < 0$ holds by the current assumption of $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$. Thus, we obtain $(\pi+\gamma)\chi(\cdot) < 1$.

of $R > 1$. Therefore, the condition (12) fails to hold, implying that an agent who belongs to a type$-j$ $(j = c1, m)$ household never becomes a decisive voter.

2nd step. Given the result in the first step, a candidate for the decisive voter is an agent who belongs to a type$-f$ or type$-c2$ household. From Panel (a) of Figure 4, a type-f agent becomes a decisive voter if and only if

$$1 - 2\tilde{\tau}^{f,c2} \leq \frac{(1+\alpha)R}{(\pi+1)\chi(\cdot)},$$

that is, if and only if

$$\chi(\cdot) \leq 2\left(\frac{(1+\alpha)R\beta}{\pi+1}\right)^{\frac{1}{\sigma}}(\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{(1+\alpha)R}{\pi+1}.$$

Otherwise, a type-$c2$ agent becomes a decisive voter.

■

8.3 Proof of Lemma 2

1st step. We first show that a type-j $(j = c2, m)$ household cannot be a decisive voter. To show this, suppose that a type$-j(= c2, m)$ be a decisive voter. From Panel (b) of Figure 4, it must be that $LHS > RHS^{c1}$ at $\tau = \tilde{\tau}^{c1,c2}$, i.e.,

$$1 - 2\tilde{\tau}^{c1,c2} > \frac{R(1+\alpha)}{(\pi+1)\chi(\cdot)}.$$

By rearranging this condition, we obtain

$$\frac{(\pi+1)\chi(\cdot)}{1+\alpha} > \frac{\pi+1}{1+\alpha}\cdot 2\left(\frac{R\beta(1+\alpha)}{\pi+1}\right)^{\frac{1}{\sigma}}\left(\frac{1}{\pi+\gamma}\right)^{\frac{\sigma-1}{\sigma}} + R. \tag{13}$$

The left-hand side of (13) is less than one under the current assumption of $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \frac{1-\alpha\pi}{\alpha}\right]$, while the right-hand side is greater than one under the assumption of $R > 1$. Therefore, (13) fails to hold, implying that a type$-j$ $(j = c2, m)$ agent never becomes a decisive voter.

2nd step.

Given the result in the first step, a decisive voter is an agent who belongs to a type-f or type-$c1$ household. From Panel (b) of Figure 4, a type-f agent becomes a decisive voter if and only if

$$1 - 2\tilde{\tau}^{f,c1} \leq \frac{R}{(\pi+\gamma)\chi(\cdot)},$$

or

$$\chi(\cdot) \leq 2 \left(\frac{R\beta}{\pi + \gamma} \right)^{\frac{1}{\sigma}} (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{R}{\pi + \gamma}.$$

Otherwise, a type$-c1$ agent becomes a decisive voter.

■

8.4 Proof of Lemma 3

1st step.

We first show that a type$-j(= c2, m)$ agent cannot be a decisive voter. To show this, suppose that a type$-j(= c2, m)$ agent is a decisive voter. From Panel (c) of Figure 4, it must be that $LHS > RHS^f$ at $\tau = \tilde{\tau}^{f,c2}$, i.e.,

$$1 - 2\tilde{\tau}^{f,c2} > \frac{(1+\alpha)R}{(\pi+1)\chi(\cdot)},$$

which is rewritten as

$$\frac{(\pi+1)\chi(\cdot)}{1+\alpha} > 2 \left(\frac{(1+\alpha)R\beta}{(\pi+1)\chi(\cdot)} \right)^{\frac{1}{\sigma}} (\alpha)^{\frac{\sigma-1}{\sigma}} \frac{\pi+1}{1+\alpha} + R. \tag{14}$$

The left-hand side of (14) is less than one under the assumption of $\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, 1 \right]$, while the right-hand side is greater than one under the assumption of $R > 1$. Therefore, (14) fails to hold, implying that a type$-j(= c2, m)$ agent never becomes a decisive voter.

2nd step.

Given the result in the first step, a decisive voter is a type$-j$ $(= c1, f)$ agent. A type-$c1$ agent becomes a decisive voter if and only if

$$1 - 2\tilde{\tau}^{c1,f} \leq \frac{\alpha R}{\chi(\cdot)},$$

or

$$\chi(\cdot) \leq 2 (\alpha R\beta)^{\frac{1}{\sigma}} \left(\frac{1}{\pi + \gamma} \right)^{\frac{\sigma-1}{\sigma}} + \alpha R.$$

Otherwise, a type$-f$ agent is a decisive voter.

■

8.5 Proof of Propositions 3-5

First, we determine the effects of α, γ and μ on the marginal cost-to-benefit ratio of taxation in terms of goods. The ratios for $j = f, c1$ and $c2$, denoted by RHS^f_{goods},

RHS^{c1}_{goods} and RHS^{c2}_{goods}, are

$$RHS^{f}_{goods} \equiv \frac{\alpha}{\chi(\cdot)}, RHS^{c1}_{goods} \equiv \frac{1}{(\pi+\gamma)\chi(\cdot)}, RHS^{c2}_{goods} \equiv \frac{1+\alpha}{(\pi+1)\chi(\cdot)}.$$

After some calculation, we obtain the following properties of RHS^{j}_{goods} ($j = f, c1, c2$):

$$\frac{\partial RHS^{f}_{goods}}{\partial \alpha} > 0, \frac{\partial RHS^{c1}_{goods}}{\partial \alpha} < 0, \frac{\partial RHS^{c2}_{goods}}{\partial \alpha} > 0; \tag{15}$$

$$\frac{\partial RHS^{f}_{goods}}{\partial \gamma} > 0, \frac{\partial RHS^{c1}_{goods}}{\partial \gamma} < 0, \frac{\partial RHS^{c2}_{goods}}{\partial \gamma} > 0; \tag{16}$$

$$\frac{\partial RHS^{f}_{goods}}{\partial \mu} \gtreqless 0, \frac{\partial RHS^{c1}_{goods}}{\partial \mu} \gtreqless 0, \frac{\partial RHS^{c2}_{goods}}{\partial \mu} \gtreqless 0 \text{ if and only if } \gamma \lesseqgtr \frac{1-\alpha\pi}{1+\alpha}. \tag{17}$$

In what follows, we denote τ^{j} ($j = f, c1, c2$) as the tax rate preferred by a type-j agent.

8.5.1 The case of $1/\sigma \geq 1$

Consider the political equilibrium in an economy with $1/\sigma \geq 1$. Suppose that $\gamma \leq (1-\alpha\pi)/\alpha$ holds: the decisive voter is a type-f agent (Proposition 1(i)). The optimality condition for a type-f agent, given by (6), indicates that a higher RHS^{f}_{goods} results in a lower preferred tax rate except in the case of $1/\sigma = 1$ and $s^{f} = 0$:

$$\begin{cases} \frac{\partial \tau^{f}}{\partial RHS^{f}_{goods}} = 0 \text{ if } 1/\sigma = 1 \text{ and } s^{f} = 0, \\ \frac{\partial \tau^{f}}{\partial RHS^{f}_{goods}} < 0 \text{ otherwise.} \end{cases}$$

With the property of RHS^{f}_{goods} in (15) - (17), we obtain the following result:

$$\frac{\partial \tau^{f}}{\partial \alpha} \leq 0, \frac{\partial \tau^{f}}{\partial \gamma} \leq 0,$$

and

$$\frac{\partial \tau^{f}}{\partial \mu} \lesseqgtr 0 \Leftrightarrow \gamma \lesseqgtr \frac{1-\alpha\pi}{1+\alpha}.$$

Next, suppose that $\gamma > (1-\alpha\pi)/\alpha$ holds: the decisive voter is a type-$c1$ agent (Proposition 1(ii)). The optimality condition for a type-$c1$ agent, given by (8), indicates that a higher RHS^{c1}_{goods} results in a lower preferred tax rate except the case of $1/\sigma = 1$ and $s^{c1} = 0$:

$$\begin{cases} \frac{\partial \tau^{c1}}{\partial RHS^{c1}_{goods}} = 0 \text{ if } 1/\sigma = 1 \text{ and } s^{c1} = 0, \\ \frac{\partial \tau^{c1}}{\partial RHS^{c1}_{goods}} < 0 \text{ otherwise.} \end{cases}$$

With the property of RHS^{c1}_{goods} in (15) - (17) and the assumption of $\gamma > (1-\alpha\pi)/\alpha$, we obtain the following result:

$$\frac{\partial \tau^{c1}}{\partial \alpha} \geq 0, \frac{\partial \tau^{c1}}{\partial \gamma} \geq 0, \frac{\partial \tau^{c1}}{\partial \mu} \geq 0.$$

8.5.2 The case of $1/\sigma < 1$

Consider the political equilibrium in an economy with $1/\sigma < 1$. The decisive voter in the current case is a type-f, type-$c1$ or type-$c2$ agent (Proposition 2). The optimality condition for a type-j agent is given by (6) for $j = f$, (8) for $j = c1$ and (9) for $j = c2$. Because we here consider the effects of α, γ and μ around the critical value $\hat{\tau}^j$ defined in Section 3, we calculate the effects of these parameters on $\hat{\tau}^j$ as follows:

$$\frac{\partial \hat{\tau}^f}{\partial \alpha} > 0, \frac{\partial \hat{\tau}^{c1}}{\partial \alpha} < 0, \frac{\partial \hat{\tau}^{c2}}{\partial \alpha} > 0; \tag{18}$$

$$\frac{\partial \hat{\tau}^f}{\partial \gamma} > 0, \frac{\partial \hat{\tau}^{c1}}{\partial \gamma} < 0, \frac{\partial \hat{\tau}^{c2}}{\partial \gamma} > 0; \tag{19}$$

$$\frac{\partial \hat{\tau}^f}{\partial \mu} \gtreqless 0, \frac{\partial \hat{\tau}^{c1}}{\partial \mu} \gtreqless 0, \frac{\partial \hat{\tau}^{c2}}{\partial \mu} \gtreqless 0 \text{ if and only if } \gamma \lesseqgtr \frac{1-\alpha\pi}{1+\alpha}. \tag{20}$$

(i) The effect of α on the equilibrium tax rate: Proof of Proposition 3(ii) Consider first the equilibrium where the decisive voter is a type-f agent. Suppose that α is initially given such that type-f's preferred tax rate is $\tau = \hat{\tau}^f$. We denote α^f as the α that makes a type-f young agent choose $\tau = \hat{\tau}^f$.

With the property of RHS^f_{goods} in (15) and the property of $\hat{\tau}^f$ in (18), we find a positive real number $\varepsilon (> 0)$ around α^f such that the type-f agent is borrowing-constrained for $\alpha \in (\alpha^f - \varepsilon, \alpha^f)$ and borrowing-unconstrained for $\alpha \in [\alpha^f, \alpha^f + \varepsilon)$ as illustrated in Panel (b) of Figure 5. The equilibrium tax rate satisfies the following condition:

$$LHS \equiv 1 - 2\tau = RHS^f \equiv \begin{cases} \frac{1}{\beta}\left(\frac{\alpha}{\chi(\cdot)}\right)^{1-\sigma}(\tau)^{\sigma} & \text{for} \quad \alpha \in (\alpha^f - \varepsilon, \alpha^f], \\ \frac{\alpha}{\chi(\cdot)}R & \text{for} \quad \alpha \in \left(\alpha^f, \alpha^f + \varepsilon\right). \end{cases}$$

Given the properties in (15) and (18), we can illustrate the effects of an increase in α on RHS^f and $\hat{\tau}^f$ as in the panel (c) of Figure 5. The illustration leads to the following result:

$$\begin{array}{ll} \frac{\partial \tau^f}{\partial \alpha} > 0 & \text{for} \quad \alpha \in (\alpha^f - \varepsilon, \alpha^f), \\ \frac{\partial \tau^f}{\partial \alpha} < 0 & \text{for} \quad \alpha \in \left(\alpha^f, \alpha^f + \varepsilon\right). \end{array}$$

The result shows that an increase in α locally produces an inverse U-shaped relationship between α and τ^f around $\tau = \hat{\tau}^f$.

The analysis and result apply to the equilibrium in which the decisive voter is a type-$c2$ agent because the effects of α on RHS^{j}_{goods} and $\hat{\tau}^{j}$ are qualitatively similar between the two types of agents, as demonstrated in (15) and (18).

Next, consider the equilibrium where the decisive voter is a type-$c1$ agent. Suppose that α is initially given such that type-$c1$'s preferred tax rate is $\tau = \hat{\tau}^{c1}$. We denote α^{c1} as the α that makes a type-$c1$ young agent choose $\tau = \hat{\tau}^{c1}$.

With the property of RHS^{c1}_{goods} in (15) and the property of $\hat{\tau}^{c1}$ in (18), we find a positive real number $\varepsilon(>0)$ around α^{c1} such that the type-$c1$ agent is borrowing-unconstrained for $\alpha \in (\alpha^{c1} - \varepsilon, \alpha^{c1})$ and borrowing-constrained for $\alpha \in [\alpha^{c1}, \alpha^{c1} + \varepsilon)$ as illustrated in Panel (c) of Figure 5. The equilibrium tax rate satisfies the following condition:

$$LHS \equiv 1 - 2\tau = RHS^{c1} \equiv \begin{cases} \frac{1}{(\pi+\gamma)\chi(\cdot)} R & \text{for} \quad \alpha \in (\alpha^{c1} - \varepsilon, \alpha^{c1}); \\ \frac{1}{\beta}\left(\frac{1}{(\pi+\gamma)\chi(\cdot)}\right)^{1-\sigma} (\tau)^{\sigma} & \text{for} \quad \alpha \in [\alpha^{c1}, \alpha^{c1} + \varepsilon). \end{cases}$$

Given the properties in (16) and (19), we can illustrate the effects of an increase in α on RHS^{c1}_{goods} and $\hat{\tau}^{c1}$ as in Panel (b) of Figure 5. The illustration leads to the following result:

$$\begin{array}{l} \frac{\partial \tau^{c1}}{\partial \alpha} > 0 \quad \text{for} \quad \alpha \in (\alpha^{c1} - \varepsilon, \alpha^{c1}), \\ \frac{\partial \tau^{c1}}{\partial \alpha} < 0 \quad \text{for} \quad \alpha \in (\alpha^{c1}, \alpha^{c1} + \varepsilon). \end{array}$$

The result shows that an increase in α locally produces an inverse U-shaped relationship between α and τ^{c1} around $\tau = \hat{\tau}^{c1}$.

(ii) The effect of γ on the equilibrium tax rate: Proof of Proposition 4(ii)
Suppose that the decisive voter is a type-j ($j = f, c1, c2$) agent. Suppose that γ is initially given such that type-j's preferred tax rate is $\tau = \hat{\tau}^{j}$. We denote γ^{j} as the γ that makes a type-j agent choose $\tau = \hat{\tau}^{j}$.

Under the abovementioned situation, suppose that an *increase* in γ around γ^{j} locally produces an inverse U-shaped relationship between γ and type-j's preferred tax rate. This assumption implies that a *decrease* in γ around γ^{j} locally produces an inverse U-shaped relationship between γ and type-j's preferred tax rate. Therefore, it is sufficient to show the effect of an increase in γ on the preferred tax rate by the decisive voter.

The analysis of the effect of α applies to the current analysis because the effects of γ on RHS^{j}_{goods} and $\hat{\tau}^{j}$ are qualitatively similar to those of α on RHS^{j}_{goods} and $\hat{\tau}^{j}$. Therefore, we obtain the result described in Proposition 4(ii).

(iii) The effect of μ on the equilibrium tax rate: Proof of Proposition 5(ii)
Suppose that $\gamma < (1 - \alpha\pi)/(1 + \alpha)$ holds. The decisive voter is a type-f or type-$c2$ agent (Lemma 1). The effects of μ on RHS^{j}_{goods} and $\hat{\tau}^{j}$ ($j = f, c2$) are qualitatively similar

to those of α on RHS^j_{goods} and $\hat{\tau}^j$ $(j = f, c2)$. We can apply the analysis and result in Proposition 3(ii) to the current case.

Next, suppose that $\gamma > (1 - \alpha\pi)/(1 + \alpha)$ holds. The decisive voter is a type-f or a type-$c1$ agent (Lemmas 2 and 3). The effects of μ on RHS^j_{goods} and $\hat{\tau}^j$ $(j = f, c1)$ are qualitatively similar to those of α on RHS^{c1}_{goods} and $\hat{\tau}^{c1}$. We can apply the analysis and result in Proposition 3(ii) to the current case.

Finally, suppose that $\gamma = (1 - \alpha\pi)/(1 + \alpha)$ holds. The parameter μ has no effect on RHS^j_{goods} and $\hat{\tau}^j$. A change in μ has no effect on the equilibrium tax rate.

■

References

[1] Arawatari, R., and Ono, T., 2011. The political economy of social security in a borrowing-constrained economy. *Journal of Public Economic Theory*, forthcoming. Available at http://www.accessecon.com/pubs/JPET/tempPDF/JPET-10-00157_0_0_2144890_temp.pdf

[2] Bellettini, G., and Berti Ceroni, C., 2007. Income distribution, borrowing constraints and redistributive policies. *European Economic Review* 51, 625–645.

[3] Bethencourt, C., and Galasso, V., 2008. Political complements in the welfare state: Health care and social security. *Journal of Public Economics* 92, 609–632.

[4] Borck, R., 2007. On the choice of public pensions when income and life expectancy are correlated. *Journal of Public Economic Theory* 9, 711–725.

[5] Casamatta, G., Cremer, H., and Pestieau, P., 2000. The political economy of social security. *Scandinavian Journal of Economics* 102, 503–522.

[6] Casamatta, G., Cremer, H., and Pestieau, P., 2005. Voting on pensions with endogenous retirement age. *International Tax and Public Finance* 12, 7–28.

[7] Choi, J., 2006. The role of derived rights for old-age income security of women. OECD social, employment, and migration working papers No. 43. Available at http://www.oecd.org/dataoecd/3/55/37817844.pdf

[8] Conde-Ruiz, J.I., and Galasso, V., 2003. Early retirement. Review of Economic Dynamics 6, 12–36.

[9] Conde-Ruiz, J.I., and Galasso, V., 2004. The macroeconomics of early retirement. *Journal of Public Economics* 88, 1849-1869.

[10] Conde-Ruiz, J.I., and Galasso, V., 2005. Positive arithmetic of the welfare state. *Journal of Public Economics* 89, 933–955.

[11] Conde-Ruiz, J.I., and Profeta, P., 2007. The redistributive design of social security systems. *Economic Journal* 117, 686–712.

[12] Cremer, H., De Donder, P., Maldonado, D., and Pestieau, P., 2007. Voting over type and generosity of a pension system when some individuals are myopic. *Journal of Public Economics* 91, 2041–2061.

[13] Diamond, P.A., Hausman, J.A., 1984. Individual retirement and savings behavior, *Journal of Public Economics* 23, 81–114.

[14] Jaumotte, F., 2003. Female labour force participation: past trends and main determinants in OECD countries. OECD Economics Department Working Papers, No. 376.

[15] Leroux, M.L., and Pestieau, P., 2010. The political economy of derived pension rights. CORE Discussion Papers 2010048, Universite catholique de Louvain, Center for Operations Research and Econometrics (CORE).

[16] Leroux, M.L., Pestieau, P., and Racionero, M., 2011. Voting on pensions: Sex and marriage. *European Journal of Political Economy* 27, 281–296.

[17] Mulligan, C.B., Sala-i-Martin, X., 1999. Social security in theory and practice (I): facts and political theories. NBER Working Papers 7118, National Bureau of Economic Research.

[18] OECD, 2011. *Pensions at a Glance 2011: Retirement-Income Systems in OECD and G20 Countries.* OECD, Paris.

[19] Poutvaara, P., 2006. On the political economy of social security and public education. *Journal of Population Economics* 19, 345–365.

[20] Weichselbaumer, D., and Winter-Ebmer, R., 2005. A meta-analysis of the international gender wage gap. *Journal of Economic Surveys* 19, 479–511.

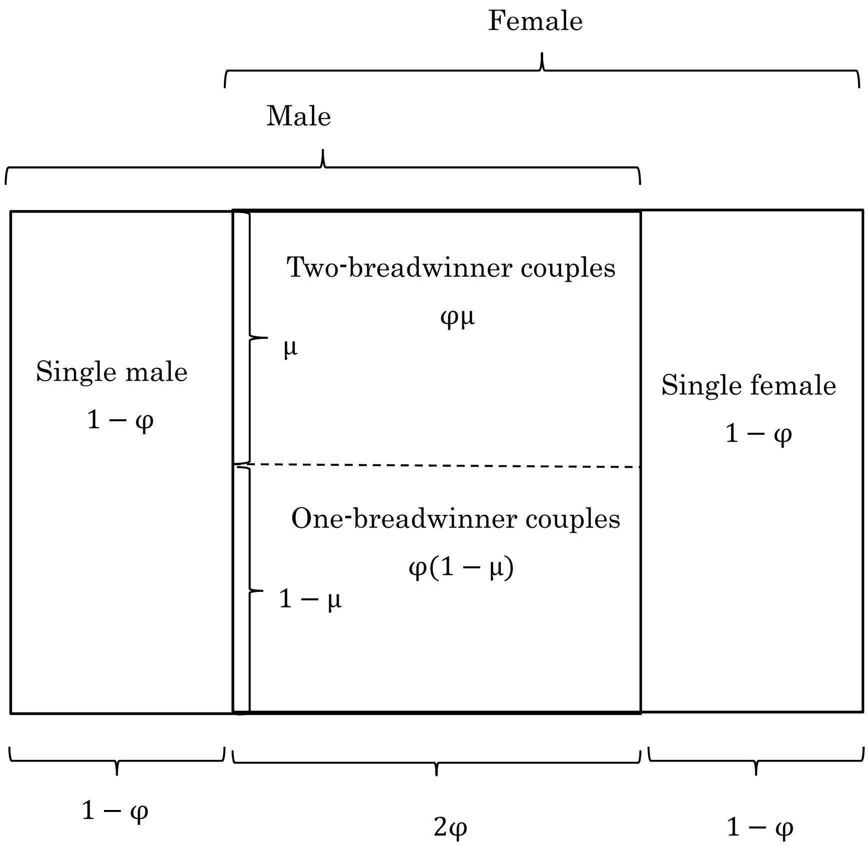


Figure 1. The structure of each generation

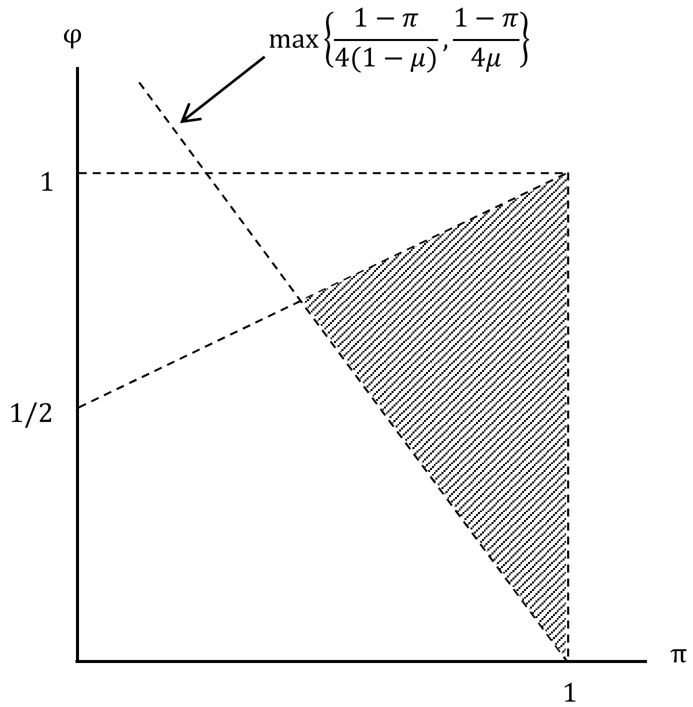


Figure 2. A set of π and φ satisfying Assumption 1.

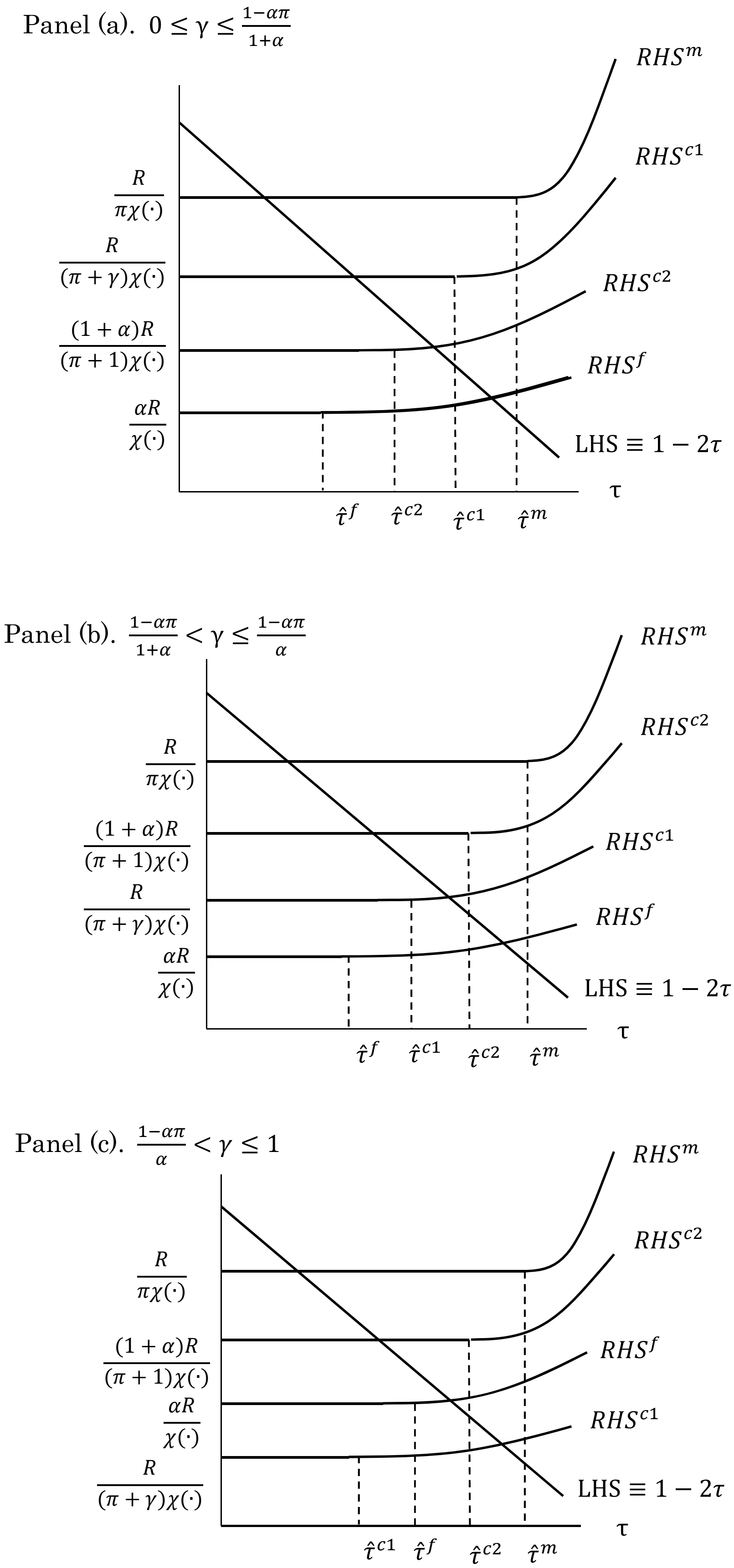
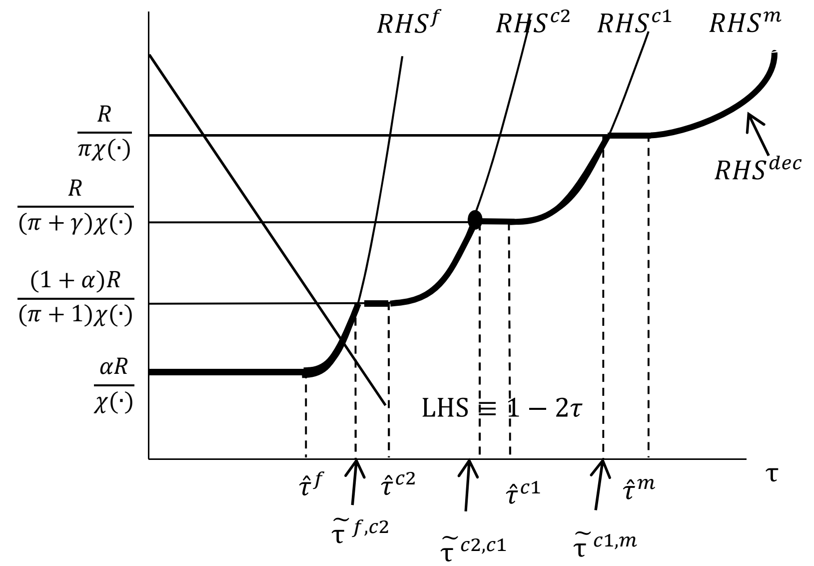
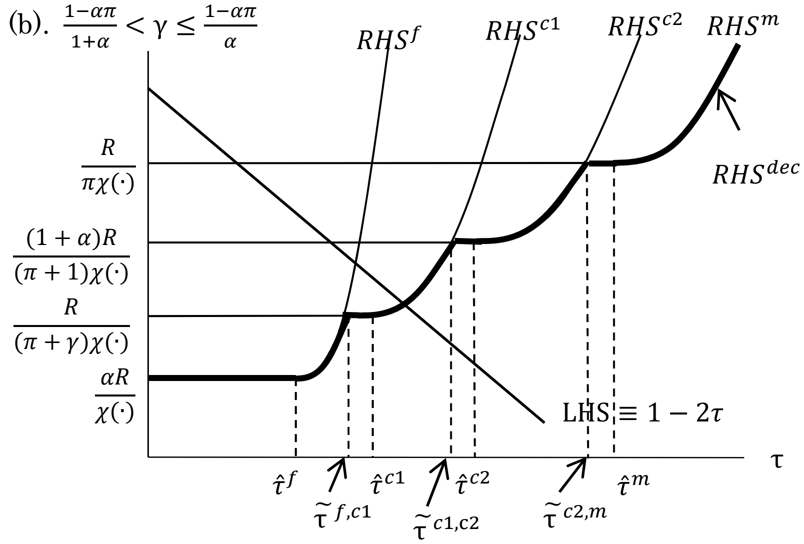


Figure 3. The preferred tax rates when $1/\sigma \geq 1$. Panel (a) is the case of $0 \leq \gamma \leq (1-\alpha\pi)/(1+\alpha)$; panel (b) is the case of $(1-\alpha\pi)/(1+\alpha) < \gamma \leq (1-\alpha\pi)/\alpha$; and panel (c) is the case of $(1-\alpha\pi)/\alpha < \gamma \leq 1$.

Panel (a). $0 \leq \gamma \leq \frac{1-\alpha\pi}{1+\alpha}$



Panel (b). $\frac{1-\alpha\pi}{1+\alpha} < \gamma \leq \frac{1-\alpha\pi}{\alpha}$



Panel (c). $\frac{1-\alpha\pi}{\alpha} < \gamma \leq 1$

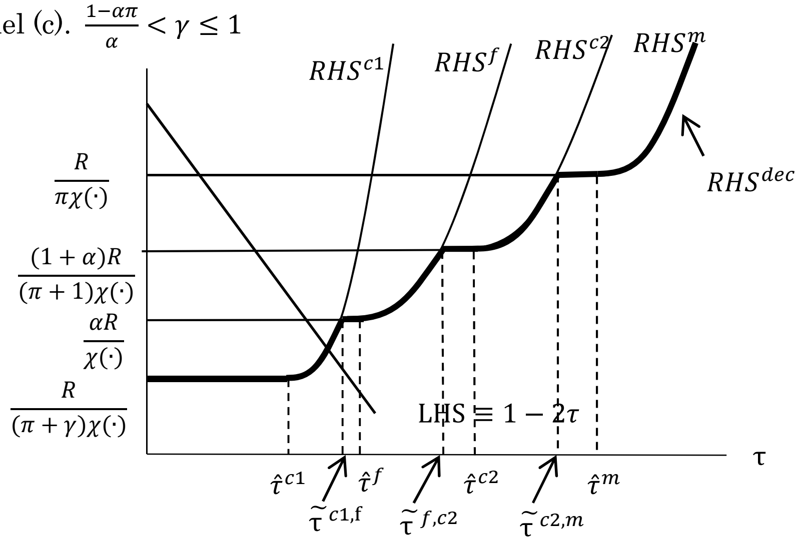
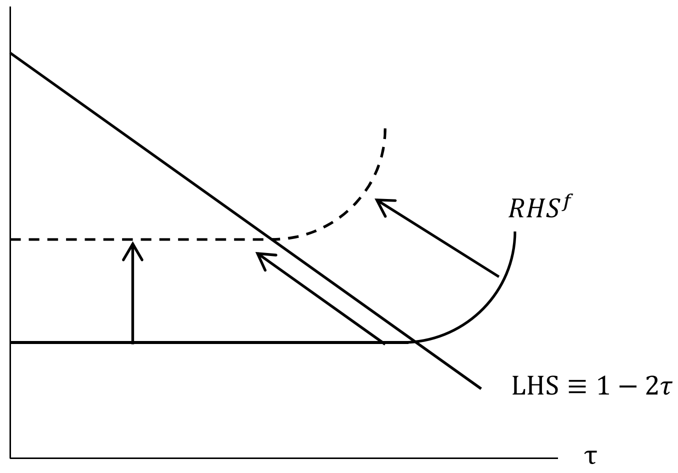
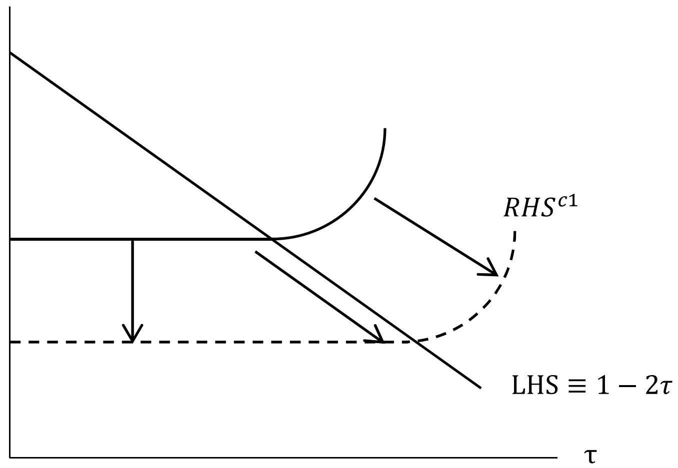


Figure 4. The preferred tax rates when $\frac{1}{\sigma} < 1$. Panel (a) is the case of $0 \leq \gamma \leq (1-\alpha\pi)/(1+\alpha)$; panel (b) is the case of $(1-\alpha\pi)/(1+\alpha) < \gamma \leq (1-\alpha\pi)/\alpha$; and panel (c) is the case of $(1-\alpha\pi)/\alpha < \gamma \leq 1$.

Panel (a). Case of $1/\sigma \geq 1$ and $\gamma \leq (1-\alpha\pi)/\alpha$



Panel (b). Case of $1/\sigma \geq 1$ and $\gamma > (1-\alpha\pi)/\alpha$



Panel (c). Case of $\frac{1}{\sigma} < 1$

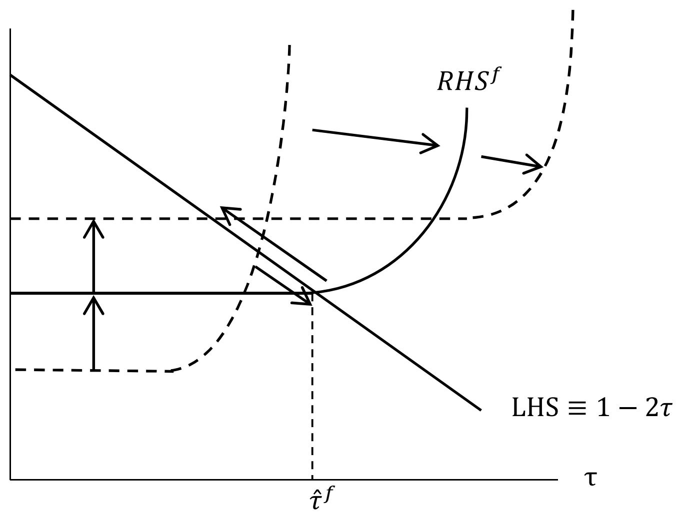


Figure 5. The effect of an increase in α on the equilibrium tax rate. Panel (a) is the case of $1/\sigma \geq 1$ and $\gamma \leq (1-\alpha\pi)/\alpha$; panel (b) is the case of $1/\sigma \geq 1$ and$\gamma > (1-\alpha\pi)/\alpha$; and panel (c) is the case of $1/\sigma < 1$.