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Secure Implementation in Queueing Problems

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Abstract

This paper studies secure implementability (Saijo, T., T. Sjöström, and T.Yamato (2007) "Secure Implementation," Theoretical Economics 2, pp.203-229) in queueing problems. Our main result shows that the social choice function satisfies strategy-proofness and strong non-bossiness (Saijo, Sjöström, and Yamato, 2007), both of which are necessary for secure implementation, if and only if it satisfies constancy on the domains that satisfy weak indifference introduced in this paper. This result implies that only constant social choice functions are securely implementable on weakly indifferent domains in queueing problems. Weak indifference is weaker than minimal richness (Fujinaka, Y. and T. Wakayama (2008) "Secure Implementation in Economies with Indivisible Objects and Money," Economics Letters 100, pp.91-95). Our main result illustrates that secure implementation is too difficult in queueing problems since many reasonable domains satisfy weak indifference, for example, convex domains.

Key words: Secure implementation, Dominant strategy implementation, Nash implementation, Strategy-proofness, Queueing problems.

JEL classification: C44, C72, D61, D71, D82

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1 Introduction

1.1 Background

In this paper, we consider the following situations: agents have to queue up to enjoy a service which cannot be consumed by more than one agent simultaneously. [1] Examples of such situations are the use of large-scaled experimental installations, event sites, and so forth. Organ transplantation is another example. In a queue, each agent has to wait her turn; waiting is a cost for her and such a cost can be described as a unit waiting cost, that is, the cost of waiting for one agent to finish enjoying a service. [2] Therefore, the total waiting cost for each agent is calculated as follows: the number of agents preceding her times her unit waiting cost. In this paper, we assume that monetary transfers among agents are allowed. [3] In fact, each agent has a linear utility function: her utility level is equal to her monetary transfers minus her total waiting cost. In such environments, we study the problems of allocating positions in a queue to agents with monetary transfers. These problems are called **queueing problems**.

In allocating positions in a queue to agents with monetary transfers, we adhere to several criteria. For example, cost minimization is widely accepted. To achieve cost minimization, we need to know each agent's unit waiting cost which is private information to her. To make each agent reveal this private information, we construct certain mechanisms. Mechanism design theory, especially implementation theory, has studied which mechanisms are suitable for resolving such allocation problems. [4]

When we consider the structure of mechanisms, manipulability of agents is an important problem: certain agents might manipulate the outcome of the mechanism in their favor. Such a manipulation might induce a non-optimal outcome. **Strategy-proofness** is a standard property for non-manipulability. [5] This property requires that the truthful revelation is a weakly dominant strategy for each agent in the direct revelation mechanism associated with the social choice function. A social choice function is one that associates an allocation with agents' private information. This function characterizes certain optimal outcome according to the information. A direct revelation mechanism associated with a social choice function is a mechanism in which (i) the set of strategy profiles is equivalent to the domain of the function and (ii) the game form is equivalent to the function.

Strategy-proofness is a desirable property but has a shortcoming: the strategy-proof mechanism might have a Nash equilibrium which induces a non-optimal outcome. This problem is solved by **secure implementation** (Saijo, Sjöström, and Yamato, 2007) which requires that there exists a mechanism in which (i) each dominant strategy equilibrium induces an optimal allocation and (ii) each Nash equilibrium also induces an optimal allocation, that is, double implementation in dominant strategy equilibria and Nash equilibria. [6] This concept is considered to be a benchmark for constructing mechanisms which

[1] Mitra (2005) and Chun and Heo (2008) consider the situations in which there exist several services.

[2] Note that it is implicitly assumed that each agent has a constant unit waiting cost. Moreover, we assume that each agent's unit waiting cost might be different from other agents.

[3] Note that monetary transfers include not only financial transactions between any two agents but also discriminations of the usage fee.

[4] Mechanism design theory consists of implementation theory and realization theory. See Jackson (2001, 2003) and Maskin and Sjöström (2002) for implementation theory and Hurwicz and Reiter (2006) for realization theory.

[5] See Barberà (2010) for the relationship between strategy-proofness and implementation theory.

[6] See Mizukami and Wakayama (2007) and Saijo, Sjöström, and Yamato (2007) for dominant strategy implementation and Maskin (1977), Repullo (1987), and Saijo (1988) for Nash implementation.

work well in laboratory experiments. [7] In certain environments, the possibility of secure implementation is studied: voting environments (Saijo, Sjöström, and Yamato, 2007; Berga and Moreno, 2009), public good economies (Saijo, Sjöström, and Yamato, 2007; Nishizaki, 2011), the problems of providing a divisible and private good with monetary transfers (Saijo, Sjöström, and Yamato, 2007; Kumar, 2009), the problems of allocating indivisible and private goods with monetary transfers (Fujinaka and Wakayama, 2008), Shapley-Scarf housing markets (Fujinaka and Wakayama, 2011), and allotment economies with single-peaked preferences (Bochet and Sakai, 2010). [8] These studies illustrate how difficult it is to find securely implementable social choice functions with desirable properties.

Queueing problems are special cases of sequencing problems in which certain agents might enjoy a servicing time which is different from those of other agents. [9] Moreover, sequencing problems are special cases of quasi-linear environments. In the environments, Groves mechanisms (Groves, 1973) are well-known for a class of direct revelation mechanisms which satisfy strategy-proofness and decision-efficiency. [10] Decision-efficiency requires that the allocation assigned by the social choice function maximizes total welfare of the group. It is also well-known that Groves mechanisms are the only direct revelation mechanisms that satisfy strategy-proofness and decision-efficiency on the domains that satisfy smooth connectedness (Holmström, 1979). [11] Unfortunately, Green and Laffont (1979) show that Groves mechanisms rarely satisfy budget-balance. [12] Budget-balance requires that the social choice function assigns an allocation in which there exists no monetary transfer from outside the group or wasted within the group. [13] Since the combination of decision-efficiency with budget-balance is equivalent to Pareto-efficiency in quasi-linear environments, the above results show how difficult it is to construct strategy-proof and Pareto-efficient direct revelation mechanisms.

Since queueing problems are special cases of quasi-linear environments, the same results as Holmström (1979) hold in queueing problems. [14] However, there exist strategy-proof and decision-efficient mechanisms which satisfy more desirable properties including budget-balance in queueing problems than in broader environments. In queueing problems, equally distributed pairwise pivotal rules (Suijs,

[7] See Cason, Saijo, Sjöström, and Yamato (2006) for experimental results on secure implementation.

[8] See also Saijo, Sjöström, and Yamato (2003) for theoretical results on secure implementation.

[9] The constancy of unit waiting costs assumed in queueing problems implies that all agents enjoy the same servicing time. Scheduling problems are also special cases of sequencing problems. See Mendelson and Whang (1990), Suijs (1996), Mitra (2002), Hain and Mitra (2004), Mishra and Rangarajan (2007), and Chun (2011) for sequencing problems and Moulin (2007, 2008) for scheduling problems.

[10] The pivotal mechanism (Clarke, 1971) and the second-price auction (Vickrey, 1961) are included in the class. See also Groves and Loeb (1975) for Groves mechanisms and Tideman and Tullock (1976) and Moulin (1986) for the pivotal mechanism.

[11] See also Green and Laffont (1977), Walker (1978), and Suijs (1996) for the uniqueness of Groves mechanisms in term of domain-richness condition. Moreover, see Hashimoto and Saitoh (2010) for a domain expansion of the pivotal mechanism and Saitoh and Serizawa (2008) and Sakai (2008) for domain expansions of the second-price auction.

[12] Similar results are obtained by Groves and Ledyard (1977), Walker (1980), and Hurwicz and Walker (1990) in non-excludable public good economies, Ohseto (2000) in the problems of allocating an indivisible good, and Schummer (2000) in the problems of allocating heterogeneous indivisible goods. For domain-richness conditions for the existence of budget-balanced Groves mechanisms, see Groves and Loeb (1975), Green and Laffont (1979), Laffont and Maskin (1980), Tian (1996), and Liu and Tian (1999) in non-excludable public good economies and Mitra and Sen (2010) in the problems of allocating heterogeneous indivisible goods.

[13] See Rob (1982) and Mitsui (1983) for the relationship between budget-balance and the number of agents.

[14] See also Dolan (1978) for a class of direct revelation mechanisms which satisfy strategy-proofness and decision-efficiency in queueing problems.

1996), which is a subclass of Groves mechanisms, satisfy budget-balance. [15] Kayı and Ramaekers (2010) show that equally distributed pairwise pivotal rules are the only direct revelation mechanisms that satisfy strategy-proofness, decision-efficiency, budget-balance, and equal treatment of equals in queueing problems. [16] Equal treatment of equals requires that any two agents' utility levels assigned by the social choice function should be equal when they have an equal unit waiting cost. Moreover, Hashimoto and Saitoh (2011) show the relationship between decision-efficiency and anonymity which is stronger than equal treatment of equals in queueing problems. [17] Anonymity requires that any two agents' utility levels assigned by the social choice function should be exchanged when their unit waiting costs are exchanged. On the other hand, Mitra and Mutuswami (2011) show that the k-pivotal mechanisms (Mitra and Mutuswami, 2011) are the only direct revelation mechanisms that satisfy pairwise strategy-proofness, decision-efficiency, equal treatment of equals, and weak linearity in queueing problems. [18] Pairwise strategy-proofness is stronger than strategy-proofness but weaker than weak group strategy-proofness. [19] Weak linearity is a linearity property for monetary transfers.

1.2 Motivation

Unfortunately, almost all previous studies show negative results on secure implementation: there rarely exists a non-trivial securely implementable social choice function. On the basis of these results, investigating which environment has a non-trivial securely implementable social choice function is an interesting topic. In this paper, we conduct such an investigation into queueing problems.

It is well-known that there rarely exists a social choice function which satisfies strategy-proofness, decision-efficiency, and budget-balance in quasi-linear environments. However, in queueing problems which are special cases of quasi-linear environments, there exist social choice functions which satisfy the above properties. [20] This means that strategy-proofness in queueing problems is much weaker than in broader environments. On the basis of this relationship, we study the possibility of secure implementation in queueing problems since strategy-proofness is necessary for secure implementation.

[15] This name is given by Kayı and Ramaekers (2010). See Mitra (2001) for a characterization of a domain-richness condition for the existence of direct revelation mechanisms that satisfy strategy-proofness, decision-efficiency, and budget-balance in queueing problems.

[16] Kayı and Ramaekers (2010) also study certain properties of social choice correspondences which assign a non-empty set of allocations. See also Maniquet (2003) and Chun (2006a) for studies of social choice correspondences and the Shapley value in queueing problems. Note that equal treatment of equals can be replaced by symmetry in the result of Kayı and Ramaekers (2010). Symmetry requires that the allocation that is constructed by exchanging the only two consumption bundles for agents with an identical unit waiting cost in the allocation assigned by the social choice correspondence should be also assigned. By definition, we know that there exists no social choice function that satisfies symmetry.

[17] By the results of Kayı and Ramaekers (2010) and Hashimoto and Saitoh (2011), we have an alternative characterization of equally distributed pairwise pivotal rules. See Chun (2006b) for the relationship between decision-efficiency and envy-freeness (Foley ,1967) which is stronger than anonymity in queueing problems.

[18] The k-pivotal mechanism is equivalent to the pivotal mechanism when $k = n$, where n is the number of agents.

[19] Note that pairwise strategy-proofness is stronger than effective pairwise strategy-proofness (Serizawa, 2006).

[20] This fact is due to the linearity of utility functions.

1.3 Related Literature

This paper is most closely related to the one written by Fujinaka and Wakayama (2008), that studies the problems of allocating indivisible and private goods with monetary transfers. [21] They show that only constant social choice functions are securely implementable when the domain satisfies minimal richness (Fujinaka and Wakayama, 2008). Since queueing problems are special cases of their ones, we have the same constancy result in queueing problems if the domain satisfies minimal richness by their result. However, there exist many reasonable domains that do not satisfy minimal richness in queueing problems. This implies the possibility of the existence of non-constant securely implementable social choice functions in queueing problems.

1.4 Our Result

Our main result shows that on many reasonable domains, only constant social choice functions satisfy strategy-proofness and strong non-bossiness (Saijo, Sjöström, and Yamato, 2007), both of which are necessary for secure implementation, in queueing problems. In fact, we show a constancy result on the domains that satisfy **weak indifference**, a new domain-richness condition introduced in this paper. Note that weak indifference is weaker than minimal richness which implies a constancy result on secure implementation in broader environments.

This paper is organized according to the following sections. In Section 2, our model is introduced. We define properties of social choice functions related to secure implementability in Section 3 and domain-richness conditions in Section 4. Certain preliminary results on properties of social choice functions are shown in Section 5. In Section 6, we show our main result. Section 7 concludes this paper. Appendix shows the relationship between weak indifference and certain domain-richness conditions.

2 Model

We study the problems of allocating positions in a queue to agents with monetary transfers. Let $I \equiv \{1,\ldots,n\}$ ($n \geq 2$) be a set of **agents**. Let $\sigma \equiv (\sigma_i)_{i\in I} \in I^n$ be a **queue**, where, for each $i \in I$, σ_i is the **position for agent *i* in the queue σ**. Note that each position is an indivisible and private good.

Each agent can consume the only one position with a positive or negative amount of money. For each $i \in I$, let $(\sigma_i, t_i) \in I \times \mathbb{R}$ be a **consumption bundle for agent *i***, where t_i is a **monetary transfer for agent *i***. Let $t \equiv (t_i)_{i\in I} \in \mathbb{R}^n$ be a profile of monetary transfers and $(\sigma, t) \in I^n \times \mathbb{R}^n$ be a profile of consumption bundles, called an **allocation**. Let

$$Z \equiv \left\{ (\sigma,t) \in I^n \times \mathbb{R}^n \;\middle|\; \sigma_i \neq \sigma_j \text{ for each } i,j \in I \text{ with } i \neq j \text{ and } \sum_{k\in I} t_k \leq 0 \right\}$$

be the set of **feasible allocations**. [22]

[21] See Svensson (1983) and Alkan, Demange, and Gale (1991) for the problems of allocating indivisible and private goods with monetary transfers.

[22] In a feasible allocation, the sum of monetary transfers should be non-positive. Our results do not depend on this non-positivity. This requirement is generically assumed in queueing problems.

Each agent has a linear utility function. For each $i \in I$, let $c_i \in \mathbb{R}_{++} \equiv \{x \in \mathbb{R} \mid x > 0\}$ be a **unit waiting cost for agent *i*** and $C_i \subseteq \mathbb{R}_{++}$ be a set of unit waiting costs for agent i. For each $i \in I$, let $u\colon I \times \mathbb{R} \times C_i \to \mathbb{R}$ be the **utility function for agent *i*** such that for each $(\sigma_i, t_i) \in I \times \mathbb{R}$ and each $c_i \in C_i$,

$$u(\sigma_i, t_i; c_i) \equiv -(\sigma_i - 1)c_i + t_i.$$

Let $C \equiv \prod_{i \in I} C_i$ be the **domain** and $c \equiv (c_i)_{i \in I} \in C$ be a profile of unit waiting costs. For each $i \in I$, let $c_{-i} \equiv (c_j)_{j \neq i} \in C_{-i} \equiv \prod_{j \neq i} C_j$ be a profile of unit waiting costs for agents other than agent i.

An allocation is assigned by a social choice function for each profile of unit waiting costs. Let $f\colon C \to Z$ be a **social choice function**. For each $c \in C$, let $(\sigma(c), t(c)) \in Z$ be the allocation associated with the social choice function f at the profile of unit waiting costs c and $(\sigma_i(c), t_i(c)) \in I \times \mathbb{R}$ be the consumption bundle for agent $i \in I$ in the allocation $(\sigma(c), t(c))$.

Remark 1. Our model is a special case of the one presented by Fujinaka and Wakayama (2008). The difference from their model is the number of each good and the existence of a "null" good. In our model, the number of each position is equal to one and each agent necessarily consumes a position. On the other hand, in their model, the number of each object is equal to or more than one and each agent does not necessarily consume an object. There exists another difference in utility functions. In our model, each agent's valuation of a position decreases in the order and the marginal valuation is constant. On the other hand, in their model, each agent's valuation of an object is almost unrestrictive. This difference strongly affects our results.

3 Properties of Social Choice Functions

Saijo, Sjöström, and Yamato (2007) introduce secure implementation which is identical with double implementation in dominant strategy equilibria and Nash equilibria. [23] They show that the social choice function is **securely implementable** if and only if it satisfies **strategy-proofness**, **strong non-bossiness**, and the **outcome-rectangular property** (Saijo, Sjöström, and Yamato, 2007). [24] In this paper, we study securely implementable social choice functions in queueing problems. Especially, we focus on social choice functions which satisfy strategy-proofness and strong non-bossiness.

Strategy-proofness requires that the truthful revelation is a weakly dominant strategy for each agent in the direct revelation mechanism associated with the social choice function.

Definition 1. The social choice function f satisfies **strategy-proofness** if and only if for each $c, c' \in C$ and each $i \in I$,

$$-(\sigma_i(c_i, c'_{-i}) - 1)c_i + t_i(c_i, c'_{-i}) \geq -(\sigma_i(c'_i, c'_{-i}) - 1)c_i + t_i(c'_i, c'_{-i}).$$

[23] See Saijo, Sjöström, and Yamato (2007) for the definition of secure implementation.

[24] Strong non-bossiness is called non-bossiness in their paper. They characterize securely implementable social choice functions by strategy-proofness and the rectangular property (Saijo, Sjöström, and Yamato, 2007) and show that the rectangular property is equivalent to strong non-bossiness and the outcome-rectangular property. See Saijo, Sjöström, and Yamato (2007) for the definitions of the rectangular property and the outcome-rectangular property and Mizukami and Wakayama (2008) for an alternative characterization of securely implementable social choice functions in terms of a stronger version of monotonicity (Maskin, 1977). See also Berga and Moreno (2009) for an alternative characterization of minmax rules in single-peaked voting environments in terms of strong non-bossiness.

Strong non-bossiness requires that each agent cannot change the outcome by her deviation while maintaining her utility level in the direct revelation mechanism associated with the social choice function.

Definition 2 (Saijo, Sjöström, and Yamato, 2007)**.** The social choice function f satisfies **strong non-bossiness** if and only if for each $c, c' \in C$ and each $i \in I$, if

$$-(\sigma_i(c_i, c_{-i}) - 1)c_i + t_i(c_i, c_{-i}) = -(\sigma_i(c'_i, c_{-i}) - 1)c_i + t_i(c'_i, c_{-i}),$$

then

$$(\sigma(c_i, c_{-i}), t(c_i, c_{-i})) = (\sigma(c'_i, c_{-i}), t(c'_i, c_{-i})).$$

By definition, strong non-bossiness is stronger than non-bossiness (Satterthwaite and Sonnenschein, 1981). [25] This property is also stronger than quasi-strong non-bossiness (Mizukami and Wakayama, 2007; Saijo, Sjöström, and Yamato, 2007) which is necessary for dominant strategy implementation. [26]

Constancy requires that any revelations are not reflected on the outcome in the direct revelation mechanism associated with the social choice function.

Definition 3. The social choice function f satisfies **constancy** if and only if for each $c, c' \in C$,

$$(\sigma(c), t(c)) = (\sigma(c'), t(c')).$$

4 Domain-Richness

In the problems of allocating indivisible and private goods with monetary transfers, where each agent has a quasi-linear utility function, Fujinaka and Wakayama (2008) show that if the domain satisfies **minimal richness**, then only constant social choice functions are securely implementable.

Definition 4 (Fujinaka and Wakayama, 2008)**.** The domain C satisfies **minimal richness** if and only if for each $i \in I$, each $c'_i, c''_i \in C_i$, each $\sigma'_i, \sigma''_i \in I$, and each $T \in \mathbb{R}$, if

$$(\sigma''_i - \sigma'_i)c''_i < T < (\sigma''_i - \sigma'_i)c'_i,$$

then there exists $c_i \in C_i$ such that

(i) $(\sigma''_i - \sigma'_i)c_i = T$,

(ii) $(\sigma''_i - \sigma_i)c_i \leq (\sigma''_i - \sigma_i)c'_i$ for each $\sigma_i \in I \setminus \{\sigma'_i, \sigma''_i\}$.

Since our model is a special case of the one presented by Fujinaka and Wakayama (2008), only constant social choice functions are securely implementable in our model if the domain satisfies minimal richness by their result. However, Example 1 shows that many reasonable domains do not satisfy minimal richness in our model.

[25] The social choice function f satisfies **non-bossiness** if and only if for each $c, c' \in C$ and each $i \in I$, if $(\sigma_i(c_i, c_{-i}), t_i(c_i, c_{-i})) = (\sigma_i(c'_i, c_{-i}), t_i(c'_i, c_{-i}))$, then $(\sigma(c_i, c_{-i}), t(c_i, c_{-i})) = (\sigma(c'_i, c_{-i}), t(c'_i, c_{-i}))$.

[26] The social choice function f satisfies **quasi-strong non-bossiness** if and only if for each $c, c' \in C$ and each $i \in I$, if $-(\sigma_i(c_i, c''_{-i}) - 1)c_i + t_i(c_i, c''_{-i}) = -(\sigma_i(c'_i, c''_{-i}) - 1)c_i + t_i(c'_i, c''_{-i})$ for each $c''_{-i} \in C_{-i}$, then $(\sigma(c_i, c_{-i}), t(c_i, c_{-i})) = (\sigma(c'_i, c_{-i}), t(c'_i, c_{-i}))$. Saijo, Sjöström, and Yamato (2007) call this property weak non-bossiness. Mizukami and Wakayama (2007) and Saijo, Sjöström, and Yamato (2007) independently show that the social choice function is dominant strategy implementable if and only if it satisfies strategy-proofness and quasi-strong non-bossiness.

Example 1. Let $i \in I$ and $C_i = \mathbb{R}_{++}$. Moreover, let $c_i' = 3$, $c_i'' = 1$, $\sigma_i' = 1$, $\sigma_i'' = 2$, and $T = 2$. In this case, we have

$$(\sigma_i'' - \sigma_i')c_i'' = 1 < T < 3 = (\sigma_i'' - \sigma_i')c_i'.$$

Let $c_i \in C_i$ be such that $(\sigma_i'' - \sigma_i')c_i = T$, that is, $c_i = 2$. This implies that condition (i) in Definition 4 holds. On the other hand, if $c_i = 2$, then

$$(\sigma_i'' - \sigma_i)c_i = -2 > -3 = (\sigma_i'' - \sigma_i)c_i'$$

for $\sigma_i = 3$. This implies that condition (ii) in Definition 4 does not hold.

Example 1 implies that there exist the other quasi-linear environments that Fujinaka and Wakayama (2008) do not study, that is, we have the possibility of the existence of non-constant securely implementable social choice functions in certain quasi-linear environments. However, our main result implies that on many reasonable domains, only constant social choice functions are securely implementable in queueing problems. In fact, we show that if the domain satisfies the following condition, called **weak indifference**, then any social choice function satisfying strategy-proofness and strong non-bossiness also satisfies constancy.

Definition 5. The domain C satisfies **weak indifference** if and only if for each $i \in I$, each $c_i', c_i'' \in C_i$, each $\sigma_i', \sigma_i'' \in I$, and each $T \in \mathbb{R}$, if

$$(\sigma_i'' - \sigma_i')c_i'' < T < (\sigma_i'' - \sigma_i')c_i',$$

then there exists $c_i \in C_i$ such that

$$(\sigma_i'' - \sigma_i')c_i = T.$$

By definition, weak indifference is weaker than minimal richness. In our model, weak indifference is equivalent to convexity. [27] By bringing this relationship together with the result of Holmström (1979), we know that weak indifference is stronger than smooth connectedness in our model. Moreover, we know that smooth connectedness is stronger than convexity, that is, smooth connectedness is equivalent to convexity in our model. [28] This implies that in our model, weak indifference is also equivalent to smooth connectedness.

Remark 2. Fujinaka and Wakayama (2008) show the possibility of the existence of non-constant securely implementable social choice functions on the domains that satisfy monotonic closedness (Schummer, 2000). [29] In our model, there exists no monotonically closed domain except for the case of $n = 2$ due to the linearity of utility functions. [30]

[27] See Appendix for the relationship between weak indifference and convexity.

[28] See Appendix for the relationship between smooth connectedness and convexity.

[29] In quasi-linear environments, Schummer (2000) shows a constancy result on bribe-proofness (Schummer, 2000) on monotonically closed domains. Bribe-proofness is stronger than strategy-proofness. He also shows that only all-dictatorial (Schummer, 2000) social choice functions satisfy bribe-proofness on smoothly connected domains. All-dictatorship requires that each agent is actually a dictator on the range of the social choice function.

[30] See Appendix for the existence of monotonically closed domains and the relationship between weak indifference and monotonic closedness in the case of $n = 2$.

5 Preliminary Results

In what follows, we show certain preliminary results on strategy-proofness in our model. Note that all preliminary results are irrespective of domain-richness conditions. For simplicity of notation, let $\sigma_i \equiv \sigma_i(c_i, c_{-i})$, $\sigma_i' \equiv \sigma_i(c_i', c_{-i})$, $\sigma_i'' \equiv \sigma_i(c_i'', c_{-i})$ and $t_i \equiv t_i(c_i, c_{-i})$, $t_i' \equiv t_i(c_i', c_{-i})$, $t_i'' \equiv t_i(c_i'', c_{-i})$ for each $c, c' \in C$ and each $i \in I$.

Lemma 1 shows that each agent's monetary transfer depends on her position in the queue if the social choice function satisfies strategy-proofness.

Lemma 1. *Suppose that the social choice function f satisfies* ***strategy-proofness****. For each $c, c' \in C$ and each $i \in I$, if $\sigma_i = \sigma_i'$, then $t_i = t_i'$.*

Proof. Suppose, by contradiction, that there exist $c, c' \in C$ and $i \in I$ such that $\sigma_i = \sigma_i'$ and $t_i \neq t_i'$. If $t_i < t_i'$, then we have $-(\sigma_i - 1)c_i + t_i < -(\sigma_i' - 1)c_i + t_i'$. This is a contradiction to **strategy-proofness**. If $t_i > t_i'$, then we have $-(\sigma_i - 1)c_i' + t_i > -(\sigma_i' - 1)c_i' + t_i'$. This is also a contradiction to **strategy-proofness**. $\square$

Remark 3. Lemma 1 corresponds to Claim 1 in Proposition 1 of Fujinaka and Wakayama (2008).

By Lemma 1, we know that for each $c, c' \in C$ and each $i \in I$, if $-(\sigma_i - 1)c_i + t_i \neq -(\sigma_i' - 1)c_i + t_i'$, then $\sigma_i \neq \sigma_i'$ when the social choice function f satisfies strategy-proofness.

Lemma 2 shows that if there exists a unit waiting cost such that some two different consumption bundles are indifferent in terms of utility level, then the position associated with the unit waiting cost is in between the two positions if the social choice function satisfies strategy-proofness. In Lemma 2, we use the following notation: for each $i \in I$, each $c_i \in C_i$, each $(\sigma_i, t_i) \in I \times \mathbb{R}$, and each $\sigma_i' \in I$, let

$$\boldsymbol{t}_i(\sigma_i'; (\sigma_i, t_i), c_i) \equiv (\sigma_i' - \sigma_i)c_i + t_i.$$

This implies $-(\sigma_i - 1)c_i + t_i = -(\sigma_i' - 1)c_i + \boldsymbol{t}_i(\sigma_i'; (\sigma_i, t_i), c_i)$, that is, $(\sigma_i', \boldsymbol{t}_i(\sigma_i'; (\sigma_i, t_i), c_i))$ is indifferent to (σ_i, t_i) for agent i with c_i.

Lemma 2. *Suppose that the social choice function f satisfies* ***strategy-proofness****. For each $c, c' \in C$ and each $i \in I$, if $\sigma_i < \sigma_i'$ and there exists $c_i'' \in C_i$ such that $-(\sigma_i - 1)c_i'' + t_i = -(\sigma_i' - 1)c_i'' + t_i'$, then $\sigma_i \leq \sigma_i'' \leq \sigma_i'$.*

Proof. Suppose, by contradiction, that there exist $c, c' \in C$ and $i \in I$ such that $\sigma_i < \sigma_i'$, $-(\sigma_i - 1)c_i'' + t_i = -(\sigma_i' - 1)c_i'' + t_i'$ for some $c_i'' \in C_i$, and $\sigma_i'' < \sigma_i$ or $\sigma_i' < \sigma_i''$.

We consider the case of $\sigma_i'' < \sigma_i$. By the hypothesis, we have

$$c_i'' = \frac{t_i' - t_i}{\sigma_i' - \sigma_i}. \tag{1}$$

By the definition of $\boldsymbol{t}_i$, we have

$$c_i = \frac{\boldsymbol{t}_i(\sigma_i'; (\sigma_i, t_i), c_i) - t_i}{\sigma_i' - \sigma_i}. \tag{2}$$

By the definition of $\boldsymbol{t}_i$ and **strategy-proofness**, we have $-(\sigma_i' - 1)c_i + t_i' < -(\sigma_i' - 1)c_i + \boldsymbol{t}_i(\sigma_i'; (\sigma_i, t_i), c_i)$, that is,

$$t_i' < \boldsymbol{t}_i(\sigma_i'; (\sigma_i, t_i), c_i).^{31} \tag{3}$$

[31] Note that the equality does not hold. If it holds, then we have $c_i = c_i''$ by (1) and (2). This implies that f should be a correspondence since we consider the case of $\sigma_i'' < \sigma_i$.

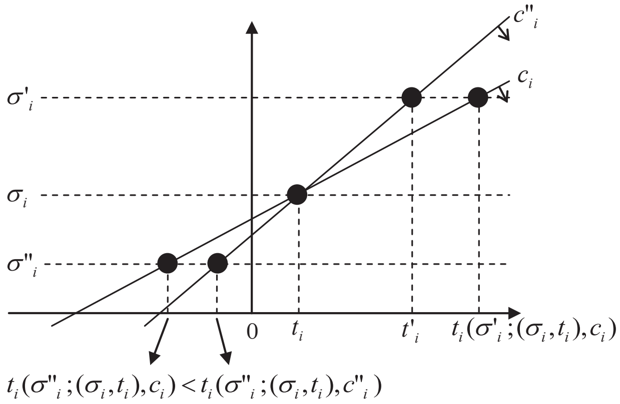

Figure 1: Proof of Lemma 2

By (1), (2), (3), and $\sigma_i < \sigma_i'$, we have $c_i'' < c_i$. Since we consider the case of $\sigma_i'' < \sigma_i$, this implies $(\sigma_i'' - \sigma_i)c_i'' + t_i > (\sigma_i'' - \sigma_i)c_i + t_i$, that is,

$$\boldsymbol{t}_i(\sigma_i''; (\sigma_i, t_i), c_i) < \boldsymbol{t}_i(\sigma_i''; (\sigma_i, t_i), c_i''). \tag{4}$$

By the definition of $\boldsymbol{t}_i$ and **strategy-proofness**, we have $-(\sigma_i''-1)c_i + t_i'' \leq -(\sigma_i''-1)c_i + \boldsymbol{t}_i(\sigma_i''; (\sigma_i, t_i), c_i)$, that is,

$$t_i'' \leq \boldsymbol{t}_i(\sigma_i''; (\sigma_i, t_i), c_i). \tag{5}$$

Moreover, by the definition of $\boldsymbol{t}_i$ and **strategy-proofness**, we have $-(\sigma_i''-1)c_i'' + t_i'' \geq -(\sigma_i''-1)c_i'' + \boldsymbol{t}_i(\sigma_i''; (\sigma_i, t_i), c_i'')$, that is,

$$\boldsymbol{t}_i(\sigma_i''; (\sigma_i, t_i), c_i'') \leq t_i''. \tag{6}$$

By (5) and (6), we have $\boldsymbol{t}_i(\sigma_i''; (\sigma_i, t_i), c_i'') \leq \boldsymbol{t}_i(\sigma_i''; (\sigma_i, t_i), c_i)$. This is a contradiction to (4) (see Figure 1).

Similarly, we have a contradiction to **strategy-proofness** in the case of $\sigma_i' < \sigma_i''$. □

Remark 4. There exists no result of Fujinaka and Wakayama (2008) corresponding to Lemma 2. Lemma 2 strongly depends on the decreasingness of utility functions in positions, which is not assumed in their model.

6 Main Result

By bringing preliminary results on strategy-proofness together with strong non-bossiness and weak indifference, we show our main result. In line with the previous section, for simplicity of notation, let $\sigma_i \equiv \sigma_i(c_i, c_{-i})$, $\sigma_i' \equiv \sigma_i(c_i', c_{-i})$, $\sigma_i'' \equiv \sigma_i(c_i'', c_{-i})$, $\sigma_i^* \equiv \sigma_i(c_i^*, c_{-i})$, $\sigma_i^{**} \equiv \sigma_i(c_i^{**}, c_{-i})$ and $t_i \equiv t_i(c_i, c_{-i})$, $t_i' \equiv t_i(c_i', c_{-i})$, $t_i'' \equiv t_i(c_i'', c_{-i})$, $t_i^* \equiv t_i(c_i^*, c_{-i})$, $t_i^{**} \equiv t_i(c_i^{**}, c_{-i})$ for each $c, c' \in C$ and each $i \in I$.

Theorem. *Suppose that the domain C satisfies* ***weak indifference****. The social choice function f satisfies* ***strategy-proofness*** *and* ***strong non-bossiness*** *if and only if it satisfies* ***constancy****.*

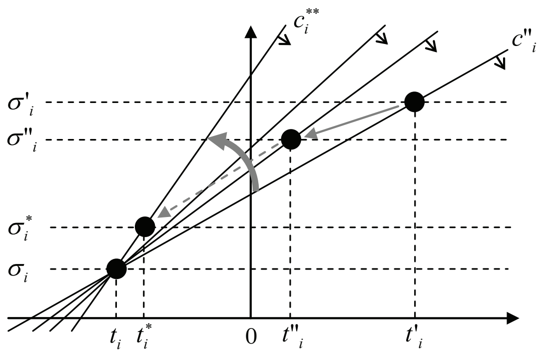

Figure 2: Proof of Theorem

Proof. Since the "if" part is obvious, we only prove the "only if" part.

Let $c, c' \in C$. To prove $(\sigma(c), t(c)) = (\sigma(c'), t(c'))$, we firstly show

$$(\sigma(c_i, c_{-i}), t(c_i, c_{-i})) = (\sigma(c'_i, c_{-i}), t(c'_i, c_{-i}))$$

for each $i \in I$. Suppose, by contradiction, that there exists $j \in I$ such that $(\sigma(c_j, c_{-j}), t(c_j, c_{-j})) \neq (\sigma(c'_j, c_{-j}), t(c'_j, c_{-j}))$. By **strong non-bossiness**, this implies $-(\sigma_j - 1)c_j + t_j \neq -(\sigma'_j - 1)c_j + t'_j$. By **strategy-proofness**, this implies

$$-(\sigma_j - 1)c_j + t_j > -(\sigma'_j - 1)c_j + t'_j. \tag{7}$$

By Lemma 1, this implies $\sigma_j \neq \sigma'_j$, that is, $\sigma_j < \sigma'_j$ or $\sigma_j > \sigma'_j$. We consider the case of $\sigma_j < \sigma'_j$. Since $(\sigma(c_j, c_{-j}), t(c_j, c_{-j})) \neq (\sigma(c'_j, c_{-j}), t(c'_j, c_{-j}))$, by **strong non-bossiness**, it implies $-(\sigma_j - 1)c'_j + t_j \neq -(\sigma'_j - 1)c'_j + t'_j$. By **strategy-proofness**, this implies

$$-(\sigma_j - 1)c'_j + t_j < -(\sigma'_j - 1)c'_j + t'_j. \tag{8}$$

By (7) and (8), we have $(\sigma'_j - \sigma_j)c'_j < t'_j - t_j < (\sigma'_j - \sigma_j)c_j$. Since C satisfies **weak indifference**, this implies that there exists $c''_j \in C_j$ such that $(\sigma'_j - \sigma_j)c''_j = t'_j - t_j$, that is,

$$-(\sigma_j - 1)c''_j + t_j = -(\sigma'_j - 1)c''_j + t'_j. \tag{9}$$

Since we consider the case of $\sigma_j < \sigma'_j$, by Lemma 2, this implies $\sigma_j \leq \sigma''_j \leq \sigma'_j$. If $\sigma''_j = \sigma_j$, then, by (9) and **strong non-bossiness**, we have $\sigma_j = \sigma'_j$. This is a contradiction. Similarly, we have a contradiction if $\sigma''_j = \sigma'_j$. Therefore, we know

$$\sigma_j < \sigma''_j < \sigma'_j.$$

By applying the above argument to the left inequality repeatedly, we can find $c^*_j, c^{**}_j \in C_j$ such that $\sigma_j < \sigma^*_j$ and $-(\sigma_j - 1)c^{**}_j + t_j = -(\sigma^*_j - 1)c^{**}_j + t^*_j$, where there exists no position between σ_j and σ^*_j induced by a unit waiting cost for agent i given c_{-j} since each position is indivisible (see Figure 2). In this case, we have $\sigma^{**}_j = \sigma_j$ or $\sigma^{**}_j = \sigma^*_j$. By **strong non-bossiness**, these imply $\sigma_j = \sigma^*_j$. This is a contradiction. Similarly, we have a contradiction in the case of $\sigma_j > \sigma'_j$.

Without loss of generality, let $i = 1$. Therefore, we have

$$(\sigma(c_1, c_{-1}), t(c_1, c_{-1})) = (\sigma(c'_1, c_{-1}), t(c'_1, c_{-1})). \tag{10}$$

By the same argument stated above, we also have

$$(\sigma(c'_1,c_2,c_{-1,2}),t(c'_1,c_2,c_{-1,2})) = (\sigma(c'_1,c'_2,c_{-1,2}),t(c'_1,c'_2,c_{-1,2})), \tag{11}$$

where $c_{-1,2}$ is a profile of unit waiting costs for agents other than agents 1 and 2. By (10) and (11), we have $(\sigma(c_1,c_2,c_{-1,2}),t(c_1,c_2,c_{-1,2})) = (\sigma(c'_1,c'_2,c_{-1,2}),t(c'_1,c'_2,c_{-1,2}))$. By sequentially replacing c_j by c'_j for each $j \neq 1,2$ in this manner, we finally prove $(\sigma(c),t(c)) = (\sigma(c'),t(c'))$. □

Remark 5. Theorem does not depend on the finiteness of the number of positions, which is used to prove Claim 3 in Proposition 1 of Fujinaka and Wakayama (2008). This difference arises from whether Lemma 2 holds.

The above theorem is tight. Example 2 shows that if the domain does not satisfy weak indifference, then there exist non-constant securely implementable social choice functions. Example 3 shows that strategy-proofness is necessary for Theorem and Example 4 shows that strong non-bossiness is necessary for Theorem.

Example 2. Suppose that $I = \{1,2\}$ and $C_i = \mathbb{R}_{++} \setminus \{2\}$ for each $i \in I$. In this case, the domain C does not satisfy weak indifference: if there exist $i \in I$ such that $c'_i = 3$, $c''_i = 1$, $\sigma'_i = 1$, and $\sigma''_i = 2$ and $T = 2$, then we have $(\sigma''_i - \sigma'_i)c''_i = 1 < T < 3 = (\sigma''_i - \sigma'_i)c'_i$ and $(\sigma''_i - \sigma'_i)c_i = c_i \neq 2 = T$ for each $c_i \in C_i$.

Let f be the social choice function such that for each $c \in C$,

$$((\sigma_1(c),t_1(c)),(\sigma_2(c),t_2(c))) = \begin{cases} ((1,-1),(2,1)) & \text{if } c_1 > 2, \\ ((2,1),(1,-1)) & \text{if } c_1 < 2. \end{cases}$$

We consider strategy-proofness. Let $c,c' \in C$. If $c_1 > 2$, then we have $-(\sigma_1(c_1,c_2)-1)c_1 + t_1(c_1,c_2) = -1 \geq -(\sigma_1(c'_1,c_2)-1)c_1 + t_1(c'_1,c_2)$. If $c_1 < 2$, then we have $-(\sigma_1(c_1,c_2)-1)c_1 + t_1(c_1,c_2) = -c_1 + 1 \geq -(\sigma_1(c'_1,c_2)-1)c_1 + t_1(c'_1,c_2)$. Since any allocation associated with f depends only on agent 1's unit waiting cost, we know that f satisfies strategy-proofness (see Figure 3).

We consider strong non-bossiness. By the argument about strategy-proofness, we know that any allocation associated with f depends only on agent 1's unit waiting cost and the hypothesis of strong non-bossiness is realized only when agent 1's consumption bundle does not change. Therefore, we know that f satisfies strong non-bossiness.

Example 3. Suppose that $I = \{1,2\}$ and $C_i = \mathbb{R}_{++}$ for each $i \in I$. In this case, the domain C satisfies weak indifference since C satisfies convexity which is stronger than weak indifference.

Let f be the social choice function such that for each $c \in C$,

$$((\sigma_1(c),t_1(c)),(\sigma_2(c),t_2(c))) = \begin{cases} ((1,1),(2,-1)) & \text{if } c_1 \geq 2, \\ ((2,-1),(1,1)) & \text{if } c_1 < 2. \end{cases}$$

By the same argument as Example 2, we know that f satisfies strong non-bossiness. We consider strategy-proofness. Let $c,c' \in C$ be such that $c_1 < 2$ and $c'_1 \geq 2$. In this case, we have $-(\sigma_1(c_1,c'_2) - 1)c_1 + t_1(c_1,c'_2) = -c_1 - 1 < 1 - (\sigma_1(c'_1,c'_2)-1)c_1 + t_1(c'_1,c'_2)$. This implies that f does not satisfy strategy-proofness (see Figure 3).

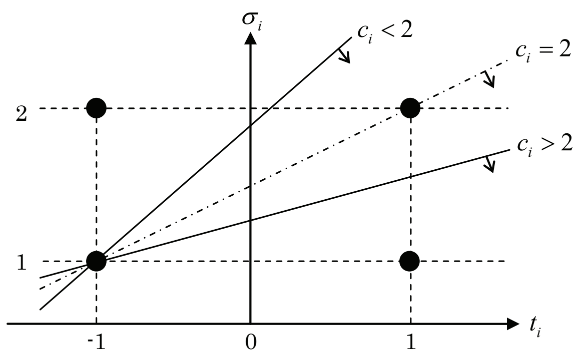

Figure 3: Examples 2, 3, and 4

Example 4. Suppose that $I = \{1,2\}$ and $C_i = \mathbb{R}_{++}$ for each $i \in I$. By the same argument as Example 3, we know that the domain C satisfies weak indifference.

Let f be the social choice function such that for each $c \in C$,

$$((\sigma_1(c),t_1(c)),(\sigma_2(c),t_2(c))) = \begin{cases} ((1,-1),(2,1)) & \text{if } c_1 \geq 2, \\ ((2,1),(1,-1)) & \text{if } c_1 < 2. \end{cases}$$

By the same argument as Example 2, we know that f satisfies strategy-proofness. We consider strong non-bossiness. Let $c, c' \in C$ be such that $c_1 = 2$ and $c'_1 < 2$. In this case, we have $-(\sigma_1(c_1,c_2)-1)c_1 + t_1(c_1,c_2) = -1 = -(\sigma_1(c'_1,c_2)-1)c_1 + t_1(c'_1,c_2)$ and

$$\begin{aligned} ((\sigma_1(c_1,c_2),t_1(c_1,c_2)),(\sigma_2(c_1,c_2),t_2(c_1,c_2))) &= ((1,-1),(1,1)) \\ &\neq ((2,1),(1,-1)) \\ &= ((\sigma_1(c'_1,c_2),t_1(c'_1,c_2)),(\sigma_2(c'_1,c_2),t_2(c'_1,c_2))). \end{aligned}$$

This implies that f does not satisfy strong non-bossiness (see Figure 3).

Obviously, constant social choice functions are securely implementable. Therefore, by bringing Theorem together with a characterization of securely implementable social choice functions by Saijo, Sjöström, and Yamato (2007), we have the following constancy result on secure implementation.

Corollary 1. *Suppose that the domain satisfies **weak indifference**. The social choice function is **securely implementable** if and only if it satisfies **constancy**.*

7 Conclusion

This paper studies secure implementability in queueing problems. In the problems of allocating indivisible and private goods with monetary transfers, where each agent has a quasi-linear utility function, Fujinaka and Wakayama (2008) show that if the domain satisfies minimal richness, then only constant social choice functions are securely implementable. Since our model is a special case of their one, we know that only constant social choice functions are securely implementable if the domain satisfies minimal richness by their results. However, many reasonable domains do not satisfy minimal richness in our model. In this paper, we show that only constant social choice functions satisfy strategy-proofness and strong non-bossiness, both of which are necessary for secure implementation, on weakly indifferent domains which are less restrictive than minimally rich domains. Our main result implies that secure implementation is too difficult in queueing problems since many reasonable domains satisfy weak indifference, for example, convex domains. Moreover, by applying the observations of Cason, Saijo, Sjöström, and Yamato (2006), our main result suggests that almost all strategy-proof direct revelation mechanisms do not work well in queueing problems.

Our main result shows certain domain-richness conditions for the constancy of securely implementable social choice functions. On the other hand, it remains to show domain-richness conditions for the existence of non-constant securely implementable social choice functions. However, our main result implies that it is difficult to find such conditions that are reasonable in the economic sense.

Appendix: Relationship between Weak Indifference and Certain Domain-Richness Conditions

In this paper, we introduce a new domain-richness condition, called weak indifference. By definition, this condition is weaker than minimal richness. In what follows, we show a relationship among weak indifference and certain domain-richness conditions other than minimal richness: convexity, smooth connectedness, and monotonic closedness.

First, we consider the relationship between weak indifference and convexity.

Definition 6. The domain C satisfies **convexity** if and only if for each $i \in I$, each $c'_i, c''_i \in C_i$, and each $\lambda \in [0,1]$, there exists $c_i \in C_i$ such that

$$c_i = \lambda c'_i + (1-\lambda)c''_i.$$

Fact 1 shows that convexity is stronger than weak indifference in our model. Since this fact is obvious, the proof is omitted.

Fact 1. *If the domain C satisfies* ***convexity****, then it satisfies* ***weak indifference****.*

Fact 2 shows that weak indifference is stronger than convexity in our model.

Fact 2. *If the domain C satisfies* ***weak indifference****, then it satisfies* ***convexity****.*

Proof. Let $i \in I$, $c'_i, c''_i \in C_i$, and $\lambda \in [0,1]$. We consider the following two cases: $\lambda = 0$ or 1 and $\lambda \in (0,1)$. If $\lambda = 0$ or 1, then we have $\lambda c'_i + (1-\lambda)c''_i \in C_i$ since $c'_i, c''_i \in C_i$. If $\lambda \in (0,1)$, then we consider the following three subcases: $c'_i = c''_i$, $c'_i < c''_i$, and $c'_i > c''_i$. If $c'_i = c''_i$, then we also have $\lambda c'_i + (1-\lambda)c''_i \in C_i$ since $c'_i, c''_i \in C_i$. In what follows, we consider the subcases of $c'_i < c''_i$ and $c'_i > c''_i$ when $\lambda \in (0,1)$.

Let $\sigma'_i, \sigma''_i \in I$ and $T \in \mathbb{R}$ be such that $\sigma'_i < \sigma''_i$ and $(\sigma''_i - \sigma'_i)\{\lambda c'_i + (1-\lambda)c''_i\} = T$. Since $\lambda \in (0,1)$, if $c'_i < c''_i$, then we have

$$(\sigma''_i - \sigma'_i)c'_i = T\frac{c'_i}{\lambda c'_i + (1-\lambda)c''_i} < T < T\frac{c''_i}{\lambda c'_i + (1-\lambda)c''_i} = (\sigma''_i - \sigma'_i)c''_i.$$

Since C satisfies **weak indifference**, there exists $c_i \in C_i$ such that $(\sigma''_i - \sigma'_i)c_i = T$ and $c_i = \lambda c'_i + (1-\lambda)c''_i$. Similarly, we can show that there exists $c_i \in C_i$ such that $c_i = \lambda c'_i + (1-\lambda)c''_i$ if $c'_i > c''_i$. □

By Facts 1 and 2, we know that weak indifference is equivalent to convexity in our model. Next, we consider the relationship between smooth connectedness and convexity.

Definition 7. The domain C satisfies **smooth connectedness** if and only if for each $i \in I$ and each $c_i', c_i'' \in C_i$, there exists $v\colon I \times [0,1] \to \mathbb{R}$ such that for each $\sigma_i \in I$,

(i) there exists $c_i^\theta \in C_i$ such that $v(\sigma_i;\theta) = -(\sigma_i - 1)c_i^\theta$ for each $\theta \in [0,1]$,

(ii) $v(\sigma_i;0) = -(\sigma_i - 1)c_i'$ and $v(\sigma_i;1) = -(\sigma_i - 1)c_i''$,

(iii) $v(\sigma_i;\cdot)$ is differentiable on $[0,1]$, [32]

(iv) there exists $K \in \mathbb{R}_{++}$ such that $|\partial v(\sigma_i;\theta)/\partial\theta| \leq K$ for each $\theta \in [0,1]$.

Holmström (1979) shows that convexity is stronger than smooth connectedness in quasi-linear environments. Since our model is one of quasi-linear environments, this relationship also holds in our model. Moreover, Fact 3 shows that smooth connectedness is stronger than convexity in our model.

Fact 3. *If the domain C satisfies **smooth connectedness**, then it satisfies **convexity**.*

Proof. Let $i \in I$, $c_i', c_i'' \in C_i$, and $\lambda \in [0,1]$. Since C satisfies **smooth connectedness**, there exists $v\colon I \times [0,1] \to \mathbb{R}$ such that for each $\sigma_i \in I$, (i) there exists $c_i^\theta \in C_i$ such that $v(\sigma_i;\theta) = -(\sigma_i - 1)c_i^\theta$ for each $\theta \in [0,1]$, (ii) $v(\sigma_i;0) = -(\sigma_i - 1)c_i'$ and $v(\sigma_i;1) = -(\sigma_i - 1)c_i''$, and (iii) $v(\sigma_i;\cdot)$ is differentiable on $[0,1]$. Without loss of generality, we assume $v(\sigma_i;0) \leq v(\sigma_i;1)$. Let $\sigma_i \in I$. By (ii), we have

$$\lambda v(\sigma_i;0) + (1-\lambda)v(\sigma_i;1) = -(\sigma_i - 1)\{\lambda c_i' + (1-\lambda)c_i''\}. \tag{12}$$

By (iii), we know that $v(\sigma_i;\cdot)$ is continuous on $[0,1]$. This implies that

$$[v(\sigma_i;0), v(\sigma_i;1)] \subseteq v(\sigma_i;[0,1]). \tag{13}$$

By (12) and (13), there exists $\theta \in [0,1]$ such that

$$v(\sigma_i;\theta) = -(\sigma_i - 1)\{\lambda c_i' + (1-\lambda)c_i''\}. \tag{14}$$

By (i), there exists $c_i^\theta \in C_i$ such that

$$v(\sigma_i;\theta) = -(\sigma_i - 1)c_i^\theta. \tag{15}$$

By (14) and (15), we have $c_i^\theta = \lambda c_i' + (1-\lambda)c_i''$. □

By Fact 3 and the result of Holmström (1979), we know that smooth connectedness is equivalent to convexity in our model. According to the relationships stated above, we have the following corollary in our model.

Corollary 2. *The following statements are equivalent: (i) the domain satisfies **weak indifference**, (ii) the domain satisfies **smooth connectedness**, and (iii) the domain satisfies **convexity**.*

Finally, we consider the relationship between weak indifference and monotonic closedness.

[32]The existence of one-sided derivatives is assumed only at the endpoints.

Definition 8. The domain C satisfies **monotonic closedness** if and only if for each $i \in I$, each $c'_i \in C_i$, and each $\sigma'_i \in I$, there exists $c_i \in C_i$ such that

$$(\sigma'_i - \sigma_i)c_i < (\sigma'_i - \sigma_i)c'_i \text{ for each } \sigma_i \in I \setminus \{\sigma'_i\}.$$

If $n \geq 3$, then many reasonable domains do not satisfy monotonic closedness in our model. Let $i \in I$ and $C_i = \mathbb{R}_{++}$. Moreover, let $c'_i \in C_i$ and $\sigma'_i = 2$. In this case, for each $c_i \in C_i$ with $c_i \geq c'_i$, there exists $\sigma_i \in I \setminus \{2\}$ such that $(2-\sigma_i)c_i \geq (2-\sigma_i)c'_i$, that is, $\sigma_i = 1$. On the other hand, for each $c_i \in C_i$ with $c_i < c'_i$, there exists $\sigma_i \in I \setminus \{2\}$ such that $(2-\sigma_i)c_i \geq (2-\sigma_i)c'_i$, for example, $\sigma_i = 3$. In fact, Fact 4 shows that there exists no domain that satisfies monotonic closedness in our model when $n \geq 3$. [33] Since the proof of this fact is similar to the above argument, it is omitted.

Fact 4. *Suppose $n \geq 3$. There exists no domain that satisfies* ***monotonic closedness****.*

Fact 4 implies that monotonic closedness is stronger than weak indifference in our model when $n \geq 3$. Moreover, this fact implies that weak indifference is not stronger than monotonic closedness in our model when $n \geq 3$. If $n = 2$, then many reasonable domains satisfy monotonic closedness in our model, for example, $C = \mathbb{R}^2_{++}$. However, since such convex domains also satisfy weak indifference, only constant social choice functions are securely implementable on the domains by Corollary 1.

References

Alkan, A., G. Demange, and D. Gale (1991) "Fair Allocation of Indivisible Goods and Criteria of Justice," *Econometrica* 59, pp.1023-1039.

Barberà, S. (2010) "Strategy-Proof Social Choice," Barcelona Economics Working Paper Series 420, Universitat Autònoma de Barcelona.

Berga, D. and B. Moreno (2009) "Strategic Requirements with Indifference: Single-Peaked versus Single-Plateaued Preferences," *Social Choice and Welfare* 32, pp.275-298.

Bochet, O. and T. Sakai (2010) "Secure Implementation in Allotment Economies," *Games and Economic Behavior* 68, pp.35-49.

Cason, T., T. Saijo, T. Sjöström, and T.Yamato (2006) "Secure Implementation Experiments: Do Strategy-Proof Mechanisms Really Work?" *Games and Economic Behavior* 57, pp.206-235.

Chun, Y. (2006a) "A Pessimistic Approach to the Queueing Problem," *Mathematical Social Sciences* 51, pp.171-181.

Chun, Y. (2006b) "No-Envy in Queueing Problems," *Economic Theory* 29, pp.151-162.

Chun, Y. (2011) "Consistency and Monotonicity in Sequencing Problems," *International Journal of Game Theory* 40, pp.29-41.

[33] Fact 4 depends on the linearity of utility functions strongly.

Chun, Y. and E. J. Heo (2008) “Queueing Problems with Two Parallel Servers,” *International Journal of Economic Theory* 4, pp.299-315.

Clarke, E. H. (1971) “Multipart Pricing of Public Goods,” *Public Choice* 11, pp.17-33.

Dolan, R. J. (1978) “Incentive Mechanisms for Priority Queuing Problems,” *Bell Journal of Economics* 9, pp.421-436.

Foley, D. (1967) “Resource Allocation and the Public Sector,” Yale Economic Essays 7, pp.45-98.

Fujinaka, Y. and T. Wakayama (2008) “Secure Implementation in Economies with Indivisible Objects and Money,” *Economics Letters* 100, pp.91-95.

Fujinaka, Y. and T. Wakayama (2011) “Secure Implementation in Shapley-Scarf Housing Markets,” *Economic Theory* 48, pp.147-169.

Green, J. and J. J. Laffont (1979) *Incentives in Public Decision Making*, North Holland.

Groves, T. (1973) “Incentives in Teams,” *Econometrica* 41, pp.617-631.

Groves, T. and J. Ledyard (1977) “Some Limitations of Demand Revealing Processes,” *Public Choice* 29, pp.107-124.

Groves, T. and M. Loeb (1975) “Incentives and Public Inputs,” *Journal of Public Economics* 4, pp.211-226.

Hain, R. and M. Mitra (2004) “Simple Sequencing Problems with Interdependent Costs,” *Games and Economic Behavior* 48, pp.271-291.

Hashimoto, K. and H. Saitoh (2010) “Domain Expansion of the Pivotal Mechanism,” *Social Choice and Welfare* 34, pp.455-470.

Hashimoto, K. and H. Saitoh (2011) “Strategy-Proof and Anonymous Rule in Queueing Problems: A Relationship between Equity and Efficiency,” *Social Choice and Welfare*, available online at `http://www.springerlink.com/content/th704g5706407n7h/`.

Holmström, B. (1979) “Groves’ Scheme on Restricted Domains,” *Econometrica* 47, pp.1137-1144.

Hurwicz, L. and M. Walker (1990) “On the Generic Nonoptimality of Dominant-Strategy Allocation Mechanisms: A General Theorem That Includes Pure Exchange Economies,” *Econometrica* 58, pp.683-704.

Hurwicz, L. and S. Reiter (2006) *Designing Economic Mechanisms*, Cambridge University Press.

Jackson, M. O. (2001) “A Crash Course in Implementation Theory,” *Social Choice and Welfare* 18, pp.655-708.

Jackson, M. O. (2003) “Mechanism Theory,” in *Encyclopedia of Life Support Systems*, edited by U. Derigs, EOLSS Publishers.

Kayı, Ç. and E. Ramaekers (2010) "Characterizations of Pareto-Efficient, Fair, and Strategy-Proof Allocation Rules in Queueing Problems," *Games and Economic Behavior* 68, pp.220-232.

Kumar, R. (2009) "Secure Implementation in Production Economies," mimeo.

Laffont, J. J. and E. Maskin (1980) "A Differential Approach to Dominant Strategy," *Econometrica* 48, pp.1507-1520.

Liu, L. and G. Tian (1999) "A Characterization of the Existence of Optimal Dominant Strategy Mechanisms," *Review of Economic Design* 4, pp.205-218.

Maniquet, F. (2003) "A Characterization of the Shapley Value in Queueing Problems," *Journal of Economic Theory* 109, pp.90-103.

Maskin, E. (1977) "Nash Equilibrium and Welfare Optimality," mimeo, the revised version appeared in *Review of Economic Studies* 66 (1999) pp.23-38.

Maskin, E. and T. Sjöström (2002) "Implementation Theory," in *Handbook of Social Choice and Welfare*, edited by K. Arrow, A. Sen, and K. Suzumura, North Holland.

Mendelson, H. and S. Whang (1990) "Optimal incentive compatible priority pricing for M/M/1 queue," *Operations Research* 38, pp.870-883.

Mishra, D. and B. Rangarajan (2007) "Cost Sharing in a Job Scheduling Problem," *Social Choice and Welfare* 29, pp.369-382.

Mitra, M. (2001) "Mechanism Design in Queueing Problems," *Economic Theory* 17, pp.277-305.

Mitra, M. (2002) "Achieving the First Best in Sequencing Problems," *Review of Economic Design* 7, pp.75-91.

Mitra, M. (2005) "Incomplete Information and Multiple Machine Queueing Problems," *European Journal of Operational Research* 165, pp.251-266.

Mitra, M. and A. Sen (2010) "Efficient Allocation of heterogeneous Commodities with Balanced Transfers," *Social Choice and Welfare* 35, pp.29-48.

Mitra, M. and S. Mutuswami (2011) "Group Strategyproofness in Queueing Models," *Games and Economic Behavior* 72, pp.242-254.

Mitsui, T. (1983) "Asymptotic Efficiency of the Pivotal Mechanism with General Project Space," *Journal of Economic Theory*, 31, pp.318-331.

Mizukami, H. and T. Wakayama (2007) "Dominant Strategy Implementation in Economic Environments," *Games and Economic Behavior* 60, pp.307-325.

Mizukami, H. and T. Wakayama (2008) "Secure Implementation: An Alternative Characterization," Working Paper No. 238, University of Toyama.

Moulin, H. (1986) "Characterization of the Pivotal Mechanism," *Journal of Public Economics* 31, pp.53-78.

Moulin, H. (2007) "On Scheduling Fees to Prevent Merging, Splitting and Transferring of Jobs," *Mathematics of Operations Research* 32, pp.266-283.

Moulin, H. (2008) "Proportional Scheduling, Split-Proofness, and Merge-Proofness," *Games and Economic Behavior* 63, pp.567-587.

Nishizaki, K. (2011) "Secure Implementation in Discrete and Excludable Public Good Economies," *Osaka Economic Papers* 61, pp.48-56.

Ohseto, S. (2000) "Strategy-Proof and Efficient Allocation of an Indivisible Good on Finitely Restricted Preference Domains," *International Journal of Game Theory* 29, pp.365-374.

Repullo, R. (1987) "A Simple Proof of Maskin's Theorem on Nash-Implementation," *Social Choice and Welfare* 4, pp.39-41.

Rob, R. (1982) "Asymptotic Efficiency of the Demand Revealing Mechanism," *Journal of Economic Theory* 28, pp.207-220.

Saijo, T. (1988) "Strategy Space Reduction in Maskin's Theorem: Sufficient Conditions for Nash Implementation," *Econometrica* 56, pp.693-700.

Saijo, T., T. Sjöström, and T.Yamato (2003) "Secure Implementation: Strategy-Proof Mechanisms Reconsidered," RIETI Discussion Paper 03-E-019.

Saijo, T., T. Sjöström, and T.Yamato (2007) "Secure Implementation," *Theoretical Economics* 2, pp.203-229.

Saitoh, H. and S. Serizawa (2008) "Vickrey Allocation Rule with Income Effect," *Economic Theory* 35, pp.391-401.

Sakai, T. (2008) "Second Price Auctions on General Preference Domains: Two Characterizations," *Economic Theory* 37, pp.347-356.

Satterthwaite, M. A. and H. Sonnenschein (1981) "Strategy-Proof Allocation Mechanisms at Differentiable Points," *Review of Economic Studies* 48, pp.587-597.

Schummer, J. (2000) "Manipulation through Bribes," *Journal of Economic Theory* 91, pp.180-198.

Serizawa, S. (2006) "Pairwise Strategy-Proofness and Self-Enforcing Manipulation," *Social Choice and Welfare* 26, pp.305-331.

Suijs, J. (1996) "On Incentive Compatibility and Budget Balancedness in Public Decision Making," *Economic Design* 2, pp.193-209.

Svensson, L. G. (1983) "Large Indivisibles: An Analysis with Respect to Price Equilibrium and Fairness," *Econometrica* 51, pp.939-954.

Tian, G. (1996) “On the Existence of Optimal Truth-Telling Dominant Mechanisms,” *Economics Letters* 53, pp.17-24.

Tideman, T. N. and G. Tullock (1976) “A New and Superior Process for Making Social Choices,” *Journal of Political Economy* 84, pp.1145-1159.

Vickrey, W. (1961) “Counterspeculation, Auctions, and Competitive Sealed Tenders,” *Journal of Finance* 16, pp.8-37.

Walker, M. (1978) “A Note on the Characterization of Mechanisms for the Revelation of Preferences,” *Econometrica* 46, pp.147-152.

Walker, M. (1980) “On the Nonexistence of a Dominant Strategy Mechanism for Making Optimal Public Decisions,” *Econometrica* 48, pp.1521-1540.