A Welfare Analysis of Global Patent Protection in a Model with Endogenous Innovation and Foreign Direct Investment

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Abstract

This paper constructs a North–South quality ladder model in which foreign direct investment (FDI) is determined by the endogenous location choice of firms and examines analytically how strengthening patent protection in the South affects welfare in the South. Strengthening patent protection increases the South’s welfare by enhancing innovation and FDI, but also allows the firms with patents to charge higher prices for their goods, which decreases welfare. However, the model shows that the former positive welfare effect overcomes the latter negative one, and introducing the strictest form of patent protection in the South, that is, harmonizing patent protection in the South with that in the North, may maximize welfare in the South as well as in the North.

Keywords: foreign direct investment, innovation, intellectual property rights protection, welfare analysis

JEL classification: F43, O33, O34, O40

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1 Introduction

Recently, many developing countries have been pressed to strengthen intellectual property rights (IPR) protection. An agreement on Trade-Related Aspects of Intellectual Property Rights (the TRIPs agreement) claims that all World Trade Organization (WTO) member countries should adopt a set of minimum standards on IPR, including patents, copyrights, etc., and strengthening IPR protection is often a requirement for developing countries to enter the WTO. However, most developing countries seem to be afraid that stronger domestic protection of IPR may damage their economies. Some empirical studies show that strengthening IPR protection in developing countries tends to cause an income transfer from developing countries that have only a few or no patents to developed countries, which have many patents.\(^1\) However, in order to judge whether strengthening IPR protection in developing countries is beneficial or harmful in reality, it is important to examine how strengthening IPR protection in developing countries affects their welfare, not their income. Is strengthening IPR protection in developing countries really harmful to their welfare?

The present paper examines how strengthening patent protection in a developing country affects its welfare, considering all the effects through changes of endogenous variables. To do this, we use a North–South quality-ladder model in which both innovation and technology transfers are endogenous. In our model, the main mode of technology transfer is assumed to be foreign direct investment (FDI). Generally, there are some modes of technology transfers that occur from a developed country (hereafter referred to as the North) to a developing country (hereafter, the South), such as FDI, licensing, illegal imitation, and outsourcing. FDI is one of the most important modes of such technology transfer and accounts for the largest share of all modes of technology transfer in most developing countries, including China and Brazil.

The present analysis obtains the following two main results. First, the model shows that strengthening patent protection in the South enhances FDI and innovation and raises the wage rate in the South. The reason for these results is as follows. Strengthening patent protection in the South enhances FDI because it enables multinationals to charge higher prices and obtain higher profits. Moreover, the en-

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\(^1\)McCalman (2001) estimated income transfers brought about by patent harmonization as a result of the TRIPs agreement. His result implies that only a few developed countries, including the United States (US) could benefit from cross-country income transfers by strengthening patent protection, whereas all other countries actually lose some of their income as a result; for instance, the net transfer from Brazil amounts to 28% of GDP. Moreover, Yang and Maskus (2001a) and Park and Lippoldt (2005) examined how US receipts of royalties and license fees depend on IPR protection in the recipient countries and showed that strengthening IPR protection has statistically significant positive influences on licensing receipts.
hancement of FDI further promotes innovation in the North because it reduces the labor demand of the production sectors in the North and directs more labor resources to research and development (R&D). On the other hand, an increase in multinationals resulting from the strengthening of patent protection causes an increase in labor demand in the South and thus raises the wage rate in the South.

Second, using the results of the above positive analysis, the present model shows that strengthening patent protection in the South increases welfare in both the South and the North. That is, we show that strengthening patent protection in the South can be a Pareto-improving policy for the North and the South. Moreover, we obtain the following important result for an assessment of global patent protection: harmonizing patent protection policy in the South with that existing in the North— that is, applying the strictest patent protection—can maximize welfare in the South. This result implies that, in contrast to the developing countries’ apprehension that stronger IPR protection damages their welfare, patent harmonization is beneficial to developing countries with only a few patents.

In our model, strengthening patent protection in the South affects welfare through three channels, as follows. The first channel is through enhancing innovation: strengthening patent protection promotes innovation and consequently raises welfare. The second channel is through the change in nominal spending: as mentioned above, strengthening patent protection raises the wage rate in the South and thus increases the nominal spending of Southern consumers, which raises welfare in the South. Meanwhile, strengthening patent protection lowers the wage rate in the North and thus decreases the nominal spending of Northern consumers, which reduces welfare in the North. The third channel is through changing prices of goods: the sign of this effect is indeterminate because strengthening patent protection affects the prices of goods positively and negatively. In more detail, the third channel can be decomposed into the following three effects. First, there is a welfare effect that occurs (a) through promoting FDI: strengthening patent protection lowers the prices of some goods by increasing the proportion of goods produced by multinationals in the South, which produce cheaper goods than do the firms located in the North. Therefore, a rise in the proportion of FDI firms raises welfare. The second welfare effect caused by the change in prices occurs (b) through raising the wage in the South: strengthening patent protection raises the wage rate in the South and enables production firms to charge higher prices for their goods because it raises the marginal cost of rival firms, which consequently reduces welfare. The last effect is the welfare effect that occurs (c) through reducing competition: strengthening patent protection allows the multinationals to charge higher prices for their goods because it reduces competition with non-patentees, which reduces welfare. As a result of this analysis, we show that the positive welfare effects can overcome the negative welfare effects.
In the theoretical literature on technology transfer, a number of earlier studies have examined how strengthening IPR protection affects innovation and FDI. Such studies include Helpman (1993), Lai (1998), Glass and Saggi (2002), Glass and Wu (2007), and Mondal and Gupta (2008). However, with the exception of Helpman (1993), none of the above studies has conducted welfare analyses, mainly because the equilibrium paths in their models are complicated.

One of the few studies dealing with the welfare effect of IPR protection in developing countries is Helpman (1993), which consisted of two welfare analyses: first, a welfare analysis in a North–South model where the only mode of technology transfer is illegal imitation; and second, a welfare analysis in a model where the means of technology transfer is FDI. The former analysis examined how lowering the probability of imitation by Southern firms of Northern products—which is caused by introducing tighter IPR in the South—affects welfare levels in both the South and the North. The results showed that tighter IPR reduces welfare in the South mainly because of the hampering of innovation. The latter analysis, which is more relevant to the present paper in that it deals with FDI, showed a similar result to the first analysis without FDI; that is, tighter IPR in the South necessarily reduces welfare in the South.

Why does the result of the present paper contrast with this pessimistic result shown in the FDI model of Helpman? The main reason is that the present paper assumes that innovation is determined endogenously, whereas Helpman’s FDI model assumes for simplicity that innovation is exogenous. By introducing the endogenous determination of innovation, our model can capture the important positive welfare effect of strengthening IPR protection, that is, the welfare effect that occurs through enhancing innovation, which is not taken into account in Helpman’s FDI model. The main results of the present paper imply that the negative conclusion regarding the welfare effect of strengthening patent protection in the South may change significantly when the welfare effect that occurs through innovation is taken into consideration.

We must briefly note the setting of patent protection in the present paper. There are two instruments of patent policies: patent length and patent breadth. Patent length refers to the duration for which a patentee can sell the patented product monopolistically, whereas patent breadth refers to the scope of products that patentees can prevent firms without patents from producing and selling. The present paper focuses

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2 As well as FDI, licensing can play an important role in technology transfer in the development process, as has occurred in Korea, Taiwan, and Japan, for example. Some studies have constructed North–South growth models where the mode of technology transfer is not FDI but licensing; see, for example, Yang and Maskus (2001b) and Tanaka et al. (2007, 2008).

3 Extending the model of Helpman (1993), Grinols and Lin (2006) analyzed the welfare effects of strengthening patent protection in the South. However, the equilibrium paths in their model are so complex that their analysis relies on numerical analysis. In addition, in contrast to the present model, their model does not include FDI.
on the effects of extending patent breadth to evaluate analytically the welfare effect of the stronger patent protection in the South. In contrast to the present paper and other studies that use the imitation rate as a parameter of IPR protection, Dinopoulos and Kottaridi (2008) focused their analysis on the effect of changing patent length.\(^4\) They analyzed the effects of patent harmonization, whereby the South’s patent protection was strengthened to the same level as the North’s patent protection, and obtained the important result that patent harmonization raises the long-run growth rate and improves the relative wage in the South. However, they did not evaluate the welfare effects of strengthening patent protection because of the complexity of the model with a finite patent length.\(^5\) In contrast, by keeping the model as simple as possible, we can examine welfare on the equilibrium path fully and obtain clearer results on the welfare effects of strengthening patent protection through the change in patent breadth.

The rest of the paper is structured as follows. Section 2 describes the model. In section 3, we derive the equilibrium path of the model and show that strengthening patent protection promotes both innovation and FDI. In section 4, we consider the effect of stronger patent protection on the welfare of consumers in both the South and the North. Section 5 provides concluding remarks.

2 The Model

We develop a dynamic general equilibrium model such that FDI is introduced into a quality-ladder model, in contrast to Lai (1998), where FDI is introduced into a variety-expansion model. Our model has the same basic structure as Grossman and Helpman (1991, Ch. 12).

Consider an economy consisting of two countries, North and South, which are denoted by \(N\) and \(S\), respectively. The population size of country \(i \in \{N, S\}\) is given by \(L_i\) and each agent supplies one unit of his or her labor inelastically at each point of time. There is a continuum of goods, indexed by \(\omega \in [0, 1]\), that are produced in the North or the South. Each product is classified by a countable infinite number of “generations” \(j = 0, 1, \cdots\) and each generation progresses one step ahead if innovation occurs in the industry. Therefore, product \(\omega\) of generation \(j\) can be produced after the \(j\)th innovation in industry \(\omega\). As described later, innovation occurs as a result of successful R&D efforts by firms. We assume that

\(^4\)Most of the existing studies dealing with patent protection, such as Kwan and Lai (2003), have used the imitation rate as a parameter of patent protection.

\(^5\)In the theoretical analysis of patent length, the dynamic property of the equilibrium paths tends to become rather complicated. For example, Futagami and Iwaisako (2007) investigated analytically the characteristics of the equilibrium paths of the economy with a finite patent length and showed that, even if the production structure is a simple \(AK\) type, the equilibrium paths exhibit oscillations.
products of different generations have different “qualities” from each other, and the quality of product \( \omega \) of generation \( j \) is provided by \( q_j(\omega) = \lambda^j \), where the increment of quality between generation \( j \) and \( j + 1 \), \( \lambda > 1 \), is identical for all products. We choose our units appropriately so that the generation number is zero and the quality is equal to unity for all goods at time \( t = 0 \).

2.1 Consumers

Consumers living in country \( i \in \{N, S\} \) have the following lifetime utility:

\[
U_i = \int_0^\infty e^{-\rho t} \log u_{i,t} dt,
\]

where \( \rho \) is a common subjective discount rate and \( \log u_{i,t} \) represents instantaneous utility at time \( t \). We specify the instantaneous utility function as:

\[
\log u_{i,t} = \int_0^1 \log \left[ \sum_j q_j(\omega) d_{j,t}^i(\omega) \right] d\omega,
\]

where \( d_{j,t}^i(\omega) \) denotes the individual’s consumption of good \( \omega \) of generation \( j \) at time \( t \).\(^6\) The representative consumer maximizes his or her lifetime utility (1) under the following budget constraint:

\[
\int_0^\infty e^{-\int_0^t r_s ds} E_{i,t} dt = A_{i,0} + \int_0^\infty e^{-\int_0^t r_s ds} w_{i,t} dt,
\]

where \( r_t \) is the interest rate that consumers in both countries face at time \( t \), \( A_{i,0} \) is the initial asset holdings of a consumer in country \( i \), and \( w_{i,t} \) denotes the wage in country \( i \). The term \( E_{i,t} \) represents the flow of spending at time \( t \), namely:

\[
E_{i,t} = \int_0^1 \left[ \sum_j p_{j,t}(\omega) d_{j,t}^i(\omega) \right] d\omega,
\]

where \( p_{j,t}(\omega) \) is the price of product \( \omega \) of generation \( j \) at time \( t \).

This consumer’s utility maximization problem can be solved in two stages. In the first stage, the consumer allocates his or her spending \( E_{i,t} \) to maximize \( \log u_{i,t} \), given prices at time \( t \). To solve this static problem, the consumer allots identical expenditure shares to all products. Then, for each product,

\(^6\)In this model, we implicitly assume that the product with a quality level that lies between the latest generation and the second-latest generation can be potentially produced and consumed as well as the latest-generation product. However, as mentioned below in subsection 2.2, owing to the pricing behavior of the firms that possess the patent on the latest-generation product, only the product of the latest generation is produced and consumed in each goods sector. Thus, we describe the instantaneous utility as (2) as consumers cannot consume a product with a level of quality that lies between the latest generation and the second-latest generation.
the consumer chooses the single generation \( j = J_t(\omega) \) that carries the lowest quality-adjusted price \( p_{j,t}(\omega)/q_{j,t}(\omega) \). This implies the following static demand function:

\[
d_{j,t}(\omega) = \begin{cases} 
E_{i,t}/p_{j,t}(\omega) & \text{for } j = J_t(\omega), \\
0 & \text{otherwise}.
\end{cases}
\]

In the second stage, the consumer chooses the time pattern of spending to maximize his or her lifetime utility (1). This intertemporal utility maximization requires that \( \dot{E}_{i,t}/E_{i,t} = rt - \rho \). By taking the aggregate spending as the numeraire, we normalize \( E_t = E_N,tL_N + E_S,tL_S = 1 \) for all \( t \) so that the interest rate \( r_t \) always corresponds to the subjective discount rate \( \rho \).

### 2.2 Production

We assume that each economy has a single primary production factor—labor. The amounts of total labor supplied in the North and the South are constant and given by \( L_N \) and \( L_S \), respectively. As in most related studies, we assume that labor is not mobile between the North and the South. Labor is devoted to the production of goods in both the North and the South. In addition, in the North, labor is devoted to innovative activities to develop a higher quality product. We assume that state-of-the-art products cannot be invented in the South.

If a Northern firm succeeds in inventing a state-of-the-art good, it can take out a patent for the good in both countries and supply the good monopolistically. In contrast with the typical setting adopted by, for example, Grossman and Helpman (1991), we assume that firms in country \( i \in \{N, S\} \) other than the inventor of the latest-generation product have the technological capacity to make the product of a quality that lies between the latest generation and the second-latest generation by imitating the product without undertaking R&D efforts if and only if the inventor is located in country \( i \). However, the existence of the patent legally guards the inventor from imitation. Thus, as mentioned below, the highest quality that the other firms can produce depends on the degree of patent protection in the country.

In the present paper, we consider that the inventor of a latest-generation product can select the location of production, i.e., it determines whether to produce the good in the North or shift production to the South by undertaking FDI. In particular, we assume that the Northern firm can shift production from the North to the South instantaneously without any cost if the firm chooses to undertake FDI.\(^8\) If the firm elects to shift production to the South, the firm can use Southern labor, which is cheaper than

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\(^7\)This normalization is a convenient method for examining the dynamic behavior of the economy. See Grossman and Helpman (1991, Ch. 12).

\(^8\)We can extend the model to include the cost of FDI. However, the results remain almost unchanged.
Northern labor, which allows the firm to obtain higher profits at each point of time. However, a firm that chooses to undertake FDI is faced with more intense competition from potential rivals than is a patentee located in the North because patent protection is assumed to be weaker in the South than in the North. We assume that a firm can freely export its product from one country to the other without incurring any transportation costs or tariffs.

Before considering how the patentees decide whether to undertake FDI, we must consider how governments protect patents in the North and the South. Generally, there are two instruments influencing the degree of patent protection. One is the patent length, which represents how long the patentee can produce and sell the product exclusively. The other is the patent breadth, which refers to the scope of products that the patentee can prevent other firms from producing and selling. In the quality-ladder model, products of different qualities within the same product line are perfectly substitutable, and thus patent breadth represents the degree of quality that the government permits other producers to produce. In reality, governments control both policy variables. However, for simplicity, we assume that the patent length is fixed and infinite and that governments control the degree of patent protection by using only the patent breadth.

In the present paper, we consider the patent breadth as follows. When the state-of-the-art quality of product $\omega$ is given by $q_j(\omega)$, firms other than the patentee of the state-of-the-art quality cannot legally produce product $\omega$ with a higher quality than $q_j(\omega)/\beta$, where $\beta \in [1, \lambda]$. Then, $\beta$ can be interpreted as representing the patent breadth in country $i$. In this setting, a higher $\beta$ implies a broader patent breadth: if $\beta$ is equal to $\lambda$, then patent protection in country $i$ is at its maximum; if $\beta$ is equal to one, then patent protection in country $i$ is nonexistent.

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9Strictly speaking, the concept of patent breadth includes leading breadth and lagging breadth: leading breadth specifies the level of superiority of a product (compared with the patented product) that producers without the patent are legally permitted to produce and sell; whereas lagging breadth specifies how inferior a product must be compared with the patented product for the producers without the patent to legally produce and sell it. The definition of patent breadth that we use in the present paper corresponds to lagging breadth. O’Donoghue and Zweimuller (2004) examined how a change of leading breadth affects innovation and welfare in a closed economy. On the other hand, similarly to the present paper, Li (2001) analyzed the effect on innovation of changing lagging breadth in a quality-ladder model. However, he focused on the analysis of a closed economy.

10Judd (1985), Iwaisako and Futagami (2003), and Futagami and Iwaisako (2007) examined how changing patent length affects social welfare. As shown by Futagami and Iwaisako (2007), the equilibrium paths under finite patent length become more complicated.

11We can consider a patent breadth that is broader than $\lambda$. However, $\beta > \lambda$ means that the patent of the state-of-the-art quality excludes even the production of the good of the second-latest generation on the same product line, that is, the product invented by the previous innovator. Such a broad patent breadth seems to be somewhat unrealistic. Thus, we assume that the patent breadth, $\beta = \lambda$, is the strictest patent protection in our analysis.
Under the rules of patent policy, the pricing strategy of a firm operating in country \( i \) depends on the patent breadth in that country. The optimal price level for the firm with the patent of a state-of-the-art good is such that the other firms cannot earn any positive profit by entering the market for the good. That is, the leader firm chooses to adopt a limit pricing strategy. To put it concretely, the patentee of the latest generation of product \( \omega \), the quality of which is equal to \( \lambda_j \), adopts a pricing strategy such that the quality-adjusted price of the good is not higher than the quality-adjusted price charged by the other producers. If the patentee is operating in country \( i \), the other producers can legally produce product \( \omega \) with quality \( \lambda_j / \beta_i \) at most. Therefore, if the patentee charges a price \( p \) that satisfies \( p/\lambda_j \leq p'/(\lambda_j / \beta_i) \), where \( p' \) denotes the price set by the other producers, then the patentee can exclude the other producers from the market. Because the lowest price that the other producers can charge is equal to their marginal cost, the limit price of the patentee is given by \( p = \beta_i MC \), where \( MC \) denotes the marginal cost of the other firms. This implies that a broader patent breadth enables the patentee to charge a higher price; in particular, when \( \beta_i \) takes the highest value, \( \lambda \), the patentee can raise the price to \( \lambda MC \), whereas when \( \beta_i \) takes the lowest value, 1, the patentee must lower the price to the level of marginal cost.\(^{12}\)

Under the patent breadth policy mentioned above, we derive the optimal pricing strategy and the profit of the patent holders producing in the North and the South, respectively. In the rest of the paper, we refer to the patent holder firms producing in the North and the patent holder firms shifting production to the South as “Northern leaders” and “multinationals”, respectively. First, let us consider the pricing behaviors of Northern leaders. We assume that patent protection is strictest in the North; that is, the patent breadth is broadest, \( \beta_N = \lambda \), in the North. Then, the Northern firms other than the patent holder of the latest-generation product \( \omega \) are prohibited from producing product \( \omega \) above the quality level of the second-latest generation product. Therefore, the strongest potential rival of the patent holder of the latest-generation product \( \omega \) is necessarily the patent holder of the second-latest-generation product that chooses to operate in the South. Letting \( w_S \) denote the wage in the South, the marginal cost of the strongest rival is equal to \( w_S \). Thus, patentees of the latest-generation products that decide to produce their good in the North set their prices to \( p_N = \lambda w_S \) as a result of the optimal strategy. Thereby, the instantaneous profit of Northern leaders becomes:

\[
\pi_N = (\lambda w_S - w_N) \frac{1}{\lambda w_S} = 1 - \frac{w_N}{\lambda w_S},
\]

\(^{12}\)Originally, Gilbert and Shapiro (1990) assumed that the patent authority could raise the profit yielded by the patentee by widening the patent breadth and, hence, they identified the size of the profit flow with the extent of the patent breadth. Similarly, Goh and Olivier (2002) assumed that the patent authority could indirectly raise the legal marginal cost of producing a patented good illegally by widening patent breadth, and they identified the extent of this legal cost with the extent of the patent breadth.
where $w_N$ denotes the wage of Northern labor.

Second, let us consider the pricing behaviors of multinationals. If the patent protection is strong enough in the South as well as in the North, then the optimal price for multinationals in the South, $p_F$, is the same as that of Northern firms: $p_F = \lambda w_S$. However, the patent breadth may be narrower in the South than in the North. Suppose that the patent breadth in the South $\beta_S$ takes a value of $\beta \in [1, \lambda]$. That is, Southern firms other than the multinationals are permitted to produce the product with a quality $\lambda j / \beta$ when the state-of-the-art quality that the multinationals produces is given by $\lambda j$. Then, multinationals are obliged to cut their prices to $p_F = \beta w_S (\leq \lambda w_S)$, which is lower than $p_N$ except in the case of maximum patent protection. Note that the price that multinationals can charge depends on the extent of the patent breadth in the South, as in Goh and Olivier (2002). If the Southern government extends the patent breadth, the other Southern firms can produce only lower quality products. In consequence, multinationals can charge a higher price when the patent breadth is extended, that is, when the patent protection becomes stricter in the South. From the pricing rule, the profit flow of multinationals is given by:

$$\pi_F = \left( \beta w_S - w_S \right) \frac{1}{\beta w_S} = 1 - \frac{1}{\beta} \left( \geq \pi_N \right).$$

As this equation shows, the higher $\beta$ is, the larger is the profit flow to the multinationals. Consequently, the stronger is the patent protection in the South, then the stronger is the incentive to shift production to the South.

### 2.3 R&D and FDI

Next, we consider the behaviors of R&D firms. Following Grossman and Helpman (1991), we assume an R&D process as follows: if a Northern firm devotes $a_N \beta t$ units of Northern labor for a time interval of length $dt$ to research on product $\omega$, it succeeds in developing the next generation product $\omega$ with probability $\beta t dt$. If a firm succeeds in developing the new generation of a good, then it can take out a patent for that generation of product. For a finite size of R&D activities in equilibrium, the expected gain from R&D must not exceed the cost of R&D. Thus, letting $v_{N,t}$ denote the market value of the patent, we have:

$$v_{N,t} \leq w_{N,t} a_N$$

with equality whenever $I_t > 0$,

$$v_{N,t} \leq w_{N,t} a_N \quad \text{with equality whenever } I_t > 0,$$

where $I_t$ denotes the innovation rate in the entire economy at time $t$, which is common to every industry.

Once a Northern firm succeeds in inventing a new-generation good, the firm can become a multinational by shifting production to the South without any cost. Therefore, as long as both Northern leader
firms and multinationals exist in equilibrium, the market values are equal for the Northern leader firms and the multinational firms; that is, the following equality holds at each point of time:

\[ v_{N,t} = v_{F,t}, \]  

where \( v_{F,t} \) denotes the market value of the multinationals.

Finally, we turn our attention to the no-arbitrage conditions. Shareholders of a Northern leader firm earn dividends \( \pi_N dt \) and capital gains \( \dot{v}_N dt \) over a time interval of length \( dt \). Moreover, the Northern leader firm is exposed to the risks of being leapfrogged by development of the next-generation good by another Northern firm at the innovation rate \( I_t \) over the time interval. Thus, shareholders are faced with a capital loss of amount \( v_N \) with a probability \( I_t dt \). Therefore, we obtain the no-arbitrage condition between the stocks of the patentee of a state-of-the-art product in the Northern market and a riskless asset as follows:\(^{13}\)

\[ r_t v_{N,t} = \pi_{N,t} + \dot{v}_{N,t} - I_t v_{N,t}. \]  

Next, shareholders of a multinational earn dividends \( \pi_F dt \) and capital gains \( \dot{v}_F dt \) over a time interval of length \( dt \). The multinational is also exposed to the risks of being leapfrogged by a Northern firm at the innovation rate \( I_t \). Thus, its shareholders are faced with a capital loss of amount \( v_F \) with a probability \( I_t dt \). Then, the no-arbitrage condition between the stocks of a multinational and a riskless asset is:

\[ r_t v_{F,t} = \pi_F + \dot{v}_{F,t} - I_t v_{F,t}. \]  

2.4 Labor Market

First, we consider the labor market in the South. Southern labor is demanded for production by multinationals that have patents for the state-of-the-art products. We let \( n_{F,t} \) denote the measure of industries in which multinationals produce the state-of-the-art products. As each multinational demands \( 1/(\beta w_{S,t}) \) units of Southern labor, the aggregate labor demand of the multinationals is given by \( n_{F,t}/(\beta w_{S,t}) \). Therefore, the labor market clearing condition in the South becomes:

\[ \frac{n_{F,t}}{\beta w_{S,t}} = L_S. \]  

Next, we consider the labor market in the North. Northern labor is devoted not only to production but also to R&D activities. Letting \( n_{N,t} \) represent the measure of industries in which Northern firms produce the

\(^{13}\)If the Northern firm shifts production to the South and becomes a multinational, the gain is given by \( (v_{F,t} - v_{N,t}) \); however, this is zero from (7). Hence, even if we consider the choice of FDI by the Northern firms, the no-arbitrage condition remains unchanged.
state-of-the-art quality products, the labor demand for production in the North is given by \( n_{N,t} / (\lambda w_{S,t}) \).

In addition, because R&D firms target all of the goods, the labor demand for R&D activities is given by \( a_N I_t (n_{F,t} + n_{N,t}) \). Noting that \( n_{F,t} + n_{N,t} = 1 \), the labor market clearing condition in the North is:

\[
\frac{n_{N,t}}{\lambda w_{S,t}} + a_N I_t = L_N. \tag{11}
\]

### 3 Market Equilibrium Paths

In this section, we derive the equilibrium path of the economies. In subsection 3.1, we show how the measure of firms that elect to undertake FDI, \( n_{F,t} \), the wage rate in the South, \( w_{S,t} \), and the innovation rate, \( I_t \), are determined when the market value of firms, \( v_{N,t} (= v_{F,t}) \), is given. Then, in subsection 3.2, we show how the equilibrium value of \( v_{N,t} \) is determined and, consequently, how innovation and FDI are determined in the market equilibrium.

#### 3.1 The Determination of FDI and Innovation for Given Market Values of Firms

First, the labor market equilibrium in the South, (10), determines the wage rate in the South, \( w_{S,t} \), as follows:

\[
w_{S,t} = \frac{n_{F,t}}{\beta L_S}. \tag{12}
\]

This shows that, as more multinationals produce in the South, the labor demand in the South becomes larger, and thus the wage rate in the South rises.

Next, we consider the equilibrium determination of FDI. As (7) holds at each point of time, we obtain 
\( \dot{v}_{F,t} / v_{F,t} = \dot{v}_{N,t} / v_{N,t} \). Substituting the no-arbitrage conditions (8) and (9) into this equation yields:

\[
\pi_F = \pi_{N,t}. \tag{13}
\]

From (7), the market values are equal between the Northern leaders and the multinationals at each point of time and, therefore, we let \( v_t \) denote the market value of firms, that is, \( v_t \equiv v_{N,t} = v_{F,t} \) in the rest of the paper. As long as the innovation rate is positive, (6) holds with equality, that is, \( w_{N,t} = v_t / a_N \).

Substituting this and (12) into (4), the profit of the Northern leader is given by:

\[
\pi_{N,t} = 1 - \frac{\beta}{\lambda} L_S \frac{v_t}{a_N n_{F,t}}. \tag{14}
\]

Substituting (5) and (14) into (13), we obtain the equilibrium measure of multinationals as follows:

\[
n_{F,t} = \frac{\beta^2}{\lambda a_N} L_S v_t, \tag{15}
\]
when \( v_t \) is given. The mechanism by which the measure of multinationals is determined is illustrated in Figure 1. The profit of the multinationals, \( \pi_F \), does not depend on \( n_{F,t} \) and, therefore, is represented by the horizontal line. On the other hand, the profit of the Northern leader, \( \pi_{N,t} \), is an increasing function of \( n_{F,t} \) and is represented by an upward curve in Figure 1. The reason for this is because an increase in the measure of multinationals raises the wage rate in the South, as shown in (12). A rise in the wage rate in the South causes a rise in the marginal cost of the nearest rivals of the Northern leaders. It enables the Northern leaders to raise the prices of their goods, \( p_{N,t} (= \lambda w_{S,t}) \), which in turn raises their profit flow, \( \pi_{N,t} \). The equilibrium measure of the multinationals, \( n_{F,t} \), is determined by the intersection of the two curves.

By using this figure, we find that strengthening patent protection—that is, a rise in \( \beta \)—affects the determination of \( n_{F,t} \) in the following two ways. First, strengthening patent protection raises the profit flow of the multinationals and shifts the \( \pi_F \) line upward. Second, strengthening patent protection reduces the labor demand in the South and lowers the wage in the South, as shown in (12). A fall in the wage in the South causes a fall in the marginal cost of the nearest rival of the Northern leaders and, thus, the Northern leaders have to lower the prices of their goods. As a result, strengthening patent protection shifts the \( \pi_N \) curve downward. Because both of these effects raise \( n_{F,t} \), strengthening patent protection in the North promotes multinationalization.

At the same time that the measure of the multinationals is determined, the wage rate in the South is
determined as follows:

\[ w_{S,t} = \frac{\beta}{\lambda a_N} v_t, \tag{16} \]

where we use (12) and (15). According to this equation, strengthening patent protection (higher \( \beta \)) raises the wage rate in the South. This is because tighter patent protection increases the measure of multinationals, as shown in (15). An increase in the measure of multinationals raises labor demand for production in the South. Although the stronger patent protection in the South has a negative effect on the Southern wage as a result of the decrease in the labor demand by each multinational, the positive effect occurring through the increase in the measure of multinationals overcomes this negative effect. Thus, stronger patent protection raises the wage in the South.

Finally, we consider the determination of the innovation rate, \( I_t \). The wage rate in the South determines the price of a good produced by the Northern leader, as \( p_{N,t} = \lambda w_{S,t} = \beta v_t / a_N \) by using (16). Then, the quantity and the labor demand of this good is given by \( x_{N,t} = 1 / p_{N,t} = a_N / \beta v_t \). Substituting this and (15) into the Northern labor market clearing condition (11), we obtain the innovation rate as follows:

\[ I_t = \frac{L_N + (\beta/\lambda)L_S}{a_N} - \frac{1}{\beta v_t}, \tag{17} \]

when \( v_t \) is given. This equation shows that, given \( v_t \), strengthening patent protection (an increase in \( \beta \)) raises the innovation rate. The reason for this is as follows: as mentioned above, strengthening patent protection raises the profit flow of the multinationals and reduces that of the Northern leaders. These effects induce more firms to shift production to the South. In addition, strengthening patent protection raises the wage in the South and the good price, which allows the Northern leaders to charge higher prices and, consequently, reduces the labor demand of each good sector, \( x_{N,t} \). Because of these two effects, stronger patent protection in the South decreases the total demand for labor in the Northern production of goods, \((1 - n_{F,t})x_{N,t}\). This leads to an increase in the labor devoted to R&D in the North, so that the equilibrium innovation rate rises.

### 3.2 The Equilibrium Path of FDI and Innovation

So far, we have derived the equilibrium values of \( n_{F,t} \) and \( I_t \) for a given value of \( v_t \). Finally, using these values, we derive the equilibrium dynamics of the market value of a firm, \( v_t \). Substituting (17) into (8) or (9), we obtain the equilibrium dynamics of \( v_t \) as follows:

\[ \dot{v}_t = \left[ \frac{L_N + (\beta/\lambda)L_S}{a_N} + \rho \right] v_t - 1. \]
This differential equation has the unique steady state, \( v \), which is given by:

\[
v = \frac{a_N}{L_N + (\beta/\lambda) L_S + \rho a_N}.
\]  

(18)

This unique steady state \( v \) is unstable, thus \( v_t \) diverges to positive or negative infinity if \( v_t \) takes any value except \( v \). Then, the equilibrium value of \( v_t \) must immediately jump to \( v \) at the initial point of time, because \( v_t \) is jumpable. Because \( v_t \) becomes constant over time, we can show that the other variables are also constant over time; that is, the present North–South economy has no transitional dynamics and has tractable features that enable an examination of the effects of policies. Note that \( v \) depends negatively on \( \beta \), as shown in (18). This is because strengthening patent protection causes capital losses to rise, owing to leapfrogging by other firms, which occurs as a result of the promotion of innovation.

By substituting the equilibrium value of \( v \) into (15) and (17), we can derive the equilibrium values of \( n_{F,t} \) and \( I_t \) as follows:

\[
\begin{align*}
n_{F,t} &= \frac{(\beta^2/\lambda)L_S}{L_N + (\beta/\lambda) L_S + \rho a_N}, \\
I_t &= \frac{\beta - 1}{\beta} \frac{L_N + (\beta/\lambda) L_S}{a_N} - \frac{\rho}{\beta}.
\end{align*}
\]

Differentiating these with respect to \( \beta \) yields:

\[
\begin{align*}
\frac{dn_{F,t}}{d\beta} &= \frac{\beta (\beta^2)/\lambda L_S}{L_N + \rho a_N + (\beta/\lambda) L_S} \left[ 2 - \frac{(\beta/\lambda) L_S}{L_N + \rho a_N + (\beta/\lambda) L_S} \right] > 0, & (19) \\
\frac{dI_t}{d\beta} &= \frac{1}{\beta^2} \left[ \frac{L_N}{a_N} + \rho \right] + \frac{L_S}{\lambda a_N} > 0. & (20)
\end{align*}
\]

From (19) and (20), we can show that both \( n_{F,t} \) and \( I_t \) are increasing functions of \( \beta \). Because both the measure of the multinationals and the innovation rate depend positively on \( v \), as shown in (15) and (17), a rise in \( \beta \) has negative effects on FDI and innovation through the decrease in \( v \). However, the direct positive effects of a rise in \( \beta \), shown in (15) and (17), overcome these indirect negative effects. Consequently, strengthening patent protection in the South necessarily promotes FDI and innovation in the North.

Although we implicitly assume that the equilibrium is an interior solution in the above analysis, we must also consider the case of a corner solution where either \( n_{F,t} = 1 \) or \( I_t = 0 \) holds.\(^{14}\) First, we consider the equilibrium where all firms possessing patents move to the South. If the patent protection is sufficiently strong in the South, that is, if \( \beta \) is sufficiently high (or close to \( \lambda \)), all firms that succeed

\(^{14}\)An equilibrium with no multinationals \((n_{F,t} = 0)\) is not possible because the Southern labor market cannot clear.
invention may choose to become multinationals and shift their production to the South. In this case, no firm engages in production in the North, so that the measure of multinationals, $n_{F,t}$, is equal to one. In order to exclude such an extreme case, we assume that $(\lambda - 1)L_S < L_N + \rho a_N$. If the values of the parameters satisfy this inequality, $n_{F,t}$ is less than one even under the strictest patent protection in the South ($\beta_S = \lambda$).

By contrast, when the patent protection is sufficiently weak in the South, that is, $\beta_S$ is sufficiently low, no firm may conduct R&D ($I_t = 0$). Specifically, innovation intensity $I_t$ takes a value of zero if:

$$\beta \leq \beta_{\text{min}} \equiv 2^{-1}(1 - \lambda L_N/L_S) + \sqrt{2^{-2}(1 - \lambda L_N/L_S)^2 + \lambda(1 - \lambda L_N + \rho a_N)/L_S}. $$

As there is no innovation, the Northern labor market equilibrium (11) becomes $(1 - n_{F})/(\lambda w_S) = L_N$. As there are both Northern leaders and multinationals in this case, (13) continues to hold. Using these relations and the Southern labor market equilibrium, we obtain the equilibrium values of the measure of multinationals and the Southern and the Northern wage rates as follows: $n_F = \beta L_S/(\lambda L_N + \beta L_S)$, $w_S = (\lambda L_N + \beta L_S)^{-1}$, and $w_N = \lambda/\beta(\lambda L_N + \beta L_S)$. Furthermore, if $\beta_{\text{min}} > \lambda$, no firm conducts R&D even under the strictest patent protection in the South. In order to exclude such an extreme case, we assume that the values of the parameters satisfy $(\lambda - 1)(L_N + L_S) > \rho a_N$.

Taking into account the possibility of the corner solution such that $I_t = 0$, we obtain the equilibrium values of $n_{F,t}$ and $I_t$ as follows:

$$n_F = \begin{cases} 
\frac{(\beta/\lambda)L_S}{L_N + (\beta/\lambda)L_S + \rho a_N} & \text{if } \beta_{\text{min}} \leq \beta \leq \lambda, \\
\beta L_S/\lambda L_N + \beta L_S & \text{if } 1 \leq \beta \leq \beta_{\text{min}}.
\end{cases}$$ (21)

$$I = \begin{cases} 
\frac{\beta - 1}{\beta} L_N + (\beta/\lambda)L_S - \rho \frac{\rho}{\beta} a_N & \text{if } \beta_{\text{min}} \leq \beta \leq \lambda, \\
0 & \text{if } 1 \leq \beta \leq \beta_{\text{min}}.
\end{cases}$$ (22)

where $n_F$ and $I$ are the equilibrium values of $n_{F,t}$ and $I_t$, respectively. In the rest of the paper, the variables without subscript “t” represent the values at the equilibrium.

From (21) and (22), we can summarize the results about the effects of strengthening patent protection on innovation and FDI in the following proposition:

**Proposition 1** Suppose that parameters are in the region where innovation is positive, that is, $\beta > \beta_{\text{min}}$. Then, strengthening patent protection promotes both innovation and FDI.
The result that strengthening patent protection in the South enhances FDI accords with the results of empirical studies; for instance, Lee and Mansfield (1996) estimated the relation between the volume of FDI flows and the strength of IPR protection and found that they are positively correlated.\footnote{Some theoretical studies obtained a similar result to ours: Vishwasrao (1994) and Zigic (1998) showed that weaker protection of patents in the South may lead to reductions in technology transfers in a partial equilibrium; and Lai (1998) showed this tendency in a dynamic general equilibrium.}

Moreover, we can derive the equilibrium wage rates $w_{S,t}$ and $w_{N,t}$ as follows:

\begin{align}
  w_{S} &= \begin{cases} 
  \frac{\beta}{\lambda} & \text{if } \beta_{\text{min}} \leq \beta \leq \lambda, \\
  \frac{1}{\lambda L_N + \beta L_S} & \text{if } 1 \leq \beta \leq \beta_{\text{min}}, 
  \end{cases} \\
  w_{N} &= \begin{cases} 
  \frac{1}{\lambda L_N + \beta L_S} & \text{if } \beta_{\text{min}} \leq \beta \leq \lambda, \\
  \frac{1}{\lambda L_N + \beta L_S} & \text{if } 1 \leq \beta \leq \beta_{\text{min}}.
  \end{cases}
\end{align}

We can summarize the results about the wage rate in the South and that in the North in the following proposition:

**Proposition 2** Suppose that parameters are in the region where innovation is positive, that is, $\beta > \beta_{\text{min}}$. Then, strengthening patent protection raises the wage rate in the South and lowers the wage rate in the North.

The reason for the results is as follows: an increase in the measure of multinationals, induced by strengthening patent protection in the South, raises the demand for labor in the South, and consequently raises the wage in the South. On the other hand, strengthening patent protection enhances innovation and therefore reduces the value of the Northern firms and multinationals. From the zero profit condition in R&D, the reduction in the reward for innovation must bring about a decrease in the cost of R&D, which lowers the wage in the North.

### 4 Welfare Analysis

In the previous section, we show that strengthening patent protection raises the relative wage in the South. However, the most important concern for the Southern government is the domestic consumers’ welfare: if strengthening patent protection in the South improves the South’s welfare, the Southern government will be eager to carry the policy into action. If not, it has an incentive to relax patent protection. Thus, in this section, we examine the welfare effects of strengthening patent protection in the South.
First, we derive the aggregate spending of the representative consumer living in country \( i \in \{N, S\} \).

As mentioned in subsection 2.1, because we take the total spending as the numeraire, that is, \( E_t = 1 \) for all \( t \), the interest rate is equal to the subjective discount rate, \( r_t = \rho \). Then, the per capita spending of a consumer living in each country is constant over time from the intertemporal utility maximization of each consumer. Therefore, we let \( E_i \) denote the spending of a consumer in country \( i \). Because spending levels and the wage rate in each country are constant over time, the intertemporal budget constraint (3) is reduced to:

\[
E_i = \rho A_{i,0} + w_i, \quad i \in \{N, S\}. \tag{25}
\]

Multiplying both sides of these budget constraints by the population and adding them, we obtain \( \rho(A_{N,0}L_N + A_{S,0}L_S) + w_N L_N + w_S L_S = 1 \), where we use \( E_t \equiv E_{N,t}L_N + E_{S,t}L_S = 1 \). Letting \( A_0 \) denote the total initial asset holdings, that is, \( A_0 \equiv A_{N,0}L_N + A_{S,0}L_S \), we can derive the value of \( A_0 \) as follows:

\[
A_0 = \frac{1 - (w_N L_N + w_S L_S)}{\rho}. \tag{26}
\]

Because country \( i \)'s share of asset holdings must be given as the initial conditions, we let \( \zeta \in [0, 1] \) denote the share of the asset holdings by Northern consumers, that is, \( \zeta \equiv A_{i,0}L_N/A_0 \). Substituting (26) into (25), we can derive the equilibrium values of \( E_N \) and \( E_S \) as follows:

\[
E_N = \zeta \frac{1 - w_S L_S}{L_N} + (1 - \zeta)w_N, \tag{27}
\]

\[
E_S = (1 - \zeta) \frac{1 - w_N L_N}{L_S} + \zeta w_S. \tag{28}
\]

Next, we rewrite the instantaneous utility as follows:

\[
\log u_{i,t} = \int_0^1 \log \lambda^h(\omega) d_i^h(\omega) d\omega
= (\log \lambda) \int_0^1 J_t(\omega) d\omega + \int_0^1 \log d_i^t(\omega) d\omega. \tag{29}
\]

Because the latest generation of a product is always set at the lowest quality-adjusted price in the product line, \( J_t(\omega) \) corresponds to the generation number of the latest generation of product \( \omega \). Thus, the first term of (29) is equal to \( \log \lambda \) times the total number of innovations obtained in all industries by time \( t \). In the present model, the rate of innovation is constant over time and thus we can rewrite this term easily as follows:

\[
(\log \lambda) \int_0^1 J_t(\omega) d\omega = (\log \lambda) It. \tag{30}
\]
The second term of (29) can be rewritten as follows:

\[
\int_0^1 \log d_i^t(\omega) d\omega = n_F \log d_{F,t}^i + (1 - n_F) \log d_{N,t}^i,
\]

where \(d_{F,t}^i\) and \(d_{N,t}^i\) denote the demand for the good produced by multinationals and the Northern leaders, respectively. Moreover, using \(d_{F,t}^i = E_i/p_F = E_i/(\beta w_S)\) and \(d_{N,t}^i = E_i/p_N = E_i/(\lambda w_S)\), we obtain:

\[
\int_0^1 \log d_i^t(\omega) d\omega = \log E_i - \log w_S - (\log \beta) n_F - (\log \lambda)(1 - n_F). \tag{31}
\]

From (1) and (29) - (31), the welfare of each consumer in country \(i = N, S\) is given by:

\[
U_i(\beta) = \frac{1}{\rho} \left[ \frac{\log \lambda}{\rho} I + \frac{\log E_i}{E_i} - \log w_S - (\log \beta) n_F - (\log \lambda)(1 - n_F) \right], \tag{32}
\]

where \(U_i(\beta)\) denotes the welfare of each consumer in country \(i\) when the patent breadth in the South is equal to \(\beta\). This shows that the welfare of an individual depends on the innovation rate, nominal spending, the wage in the South, which in turn determines the prices of goods, the measure of multinationals, and the patent breadth in the South. The difference between the welfare levels of a Northern individual and a Southern individual is only the difference in nominal spending, \(E_i\).

In order to examine whether extending patent breadth in the South raises welfare, we differentiate (32) with respect to breadth \(\beta\). The derivative of \(U_i(\beta)\) is given by:

\[
\frac{dU_i(\beta)}{d\beta} = \frac{1}{\rho} \left\{ \frac{\log \lambda}{\rho} \frac{dI}{d\beta} + \frac{1}{E_i} \frac{dE_i}{d\beta} \right. + \left. \begin{array}{c}
(\log \lambda - \log \beta) \frac{dn_F}{d\beta} \\
- \frac{1}{w_S} \frac{dw_S}{d\beta} \\
- \frac{1}{n_F} \frac{dF}{d\beta}
\end{array} \right\} \right\} \tag{33}
\]

As shown on the right-hand side (RHS) of (33), extending patent breadth in the South affects the welfare of both countries through the following three channels. The first channel is the welfare effect that occurs through enhancing innovation, which is indicated by the first term on the RHS of (33). As shown in Proposition 1, extending patent breadth promotes innovation and raises welfare. We refer to this effect as the innovation-enhancing effect. This effect has a positive influence on the welfare of both countries.

The second channel is the welfare effect that occurs through the change in nominal spending, which is indicated by the second term on the RHS of (33). As shown later, extending patent breadth raises nominal spending in the South, but reduces nominal spending in the North. Thus it affects the welfare of both countries. We refer to this effect as the nominal spending effect. The last channel is the welfare effect through the competition-reducing effect. This effect has a negative influence on the welfare of both countries.
effect that occurs *through changing the prices of goods*, which is indicated by the three terms in square 
brackets on the RHS of (33).

The sign of the sum of the three terms is indeterminate because extending patent breadth has both 
positive and negative effects on the prices of goods in the following ways. First, extending patent breadth 
increases the proportion of goods that multinationals produce, as shown in Proposition 1. Because patent 
breadth is narrower in the South than in the North, the price of the goods that multinationals produce 
($\beta w^S_n$) is cheaper than the price of the goods that the Northern leaders produce ($\lambda w^S_n$). This means that 
extension patent breadth improves the welfare of both countries through a rise in the proportion of FDI 
firms, $n_F$. We refer to this positive welfare effect as the *FDI-promoting effect*, which is indicated by the 
first term in the square brackets. Second, from (23), extending patent breadth raises the wage rate in the 
South. This causes a rise in the marginal cost of followers, which allows the Northern leaders and the 
multinationals to charge higher prices and thus reduces the welfare of both countries. We refer to this 
negative effect as the *marginal cost effect*, which is indicated by the second term in the square brackets. 
Finally, extending patent breadth in the South enables the multinationals to raise the price of their goods 
directly, which reduces welfare. This is because extending patent breadth permits the Southern firms 
other than the multinationals to produce only goods of lower quality. This means that the multinationals 
can out-compete the other firms even if the multinationals charge a higher price for their goods. For this 
reason, extending patent breadth reduces the welfare of both countries. We refer to this negative effect, 
which is shown by the last term in the square brackets, as the *competition-reducing effect*. If the positive 
welfare effects outweigh the negative welfare effects, we can show that strengthening patent protection 
in the South raises welfare. As shown in the subsequent subsections, whether this is the case depends on 
the values of the parameters.

### 4.1 The Effect on the South’s Welfare

First, we explore the effect of strengthening patent protection on the South’s welfare and derive the 
values of the parameters that cause the strictest patent protection to maximize the welfare of Southern 
consumers. In the rest of this subsection, we focus our analysis on the case where consumers in the 
South have no assets at the initial point of time, that is, $\zeta = 1$, as a benchmark case. In the final part, we 
analyze the generalized case where $\zeta \neq 1$.

From (28), we obtain $E_S = w_S$ when $\zeta = 1$ and thus the *nominal spending effect* and the *marginal 
cost effect*, which are shown by the second and fourth terms on the RHS of (33), respectively, cancel each
other. Then, the derivative of $U_S(\beta)$ becomes a simpler form as follows:

$$\frac{dU_S(\beta)}{d\beta} = \frac{1}{\rho} \left[ \frac{\log \lambda}{\rho} \frac{dI}{d\beta} \right. \quad \text{innovation-enhancing effect}
\left. \begin{array}{c}
+ (\log \lambda - \log \beta) \frac{dn_F}{d\beta} \\
\frac{1}{\beta} n_F
\end{array} \right] \quad \text{FDI-promoting effect}
\left. \begin{array}{c}
- \frac{1}{\beta} n_F
\end{array} \right] \quad \text{competition-reducing effect} \right]. \quad (34)$$

As discussed in Appendix A, we can show that the innovation-enhancing effect, which is indicated by the first term on the RHS in (34), necessarily overcomes the competition-reducing effect, which is indicated by the third term of the equation, for all $\beta \in [\beta_{min}, \lambda]$ if and only if the parameters satisfy

$$\frac{(\log \lambda)^2}{\lambda^2} \left( L_N + \lambda L_S + \rho a_N \right) \left( \rho a_N \right)^{-1} \geq L_S \left( L_N + L_S + \rho a_N \right)^{-1}. \quad \text{Therefore, we can guarantee that the strictest patent protection maximizes welfare in the South if and only if that inequality holds because the second term on the RHS in (34) is necessarily nonnegative and becomes zero when $\beta = \lambda$. We can summarize the results of the welfare analysis as follows:}^{16}

**Proposition 3** Suppose that consumers in the South have no assets at the initial point of time, that is, $\zeta = 1$. Then, the strictest patent protection in the South ($\beta_S = \lambda$) maximizes the welfare of consumers in the South if and only if the parameters satisfy $(\log \lambda)\lambda^2 \left( L_N + \lambda L_S + \rho a_N \right) \left( \rho a_N \right)^{-1} \geq L_S \left( L_N + L_S + \rho a_N \right)^{-1}.$

As can be seen easily, when the Northern labor, $L_N$, is larger, the condition in Proposition 3 tends to hold. Therefore, Proposition 3 implies that the strictest patent protection improves the welfare of consumers in the South if labor is more abundant in the North. Why can strengthening patent protection raise the welfare of Southern consumers in such a case? More abundant labor in the North intensifies the promotion of the innovation effect of strengthening patent protection, $dI/d\beta$, which is indicated by the LHS of the condition in Proposition 3. In addition, the factor decreases the proportion of multinationals, $n_F$, and, consequently, weakens the competition-reducing effect of strengthening patent protection.

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16In Propositions 3–5, we suppose that the degree of patent protection in the South is not weaker than the level of protection below which Northern R&D activities cease, that is, $\beta \in [\beta_{min}, \lambda]$. For some values of parameters, a low value of $\beta$ that prevents innovation could maximize welfare. However, if the cost of innovation is sufficiently low, such a case is impossible. For instance, assuming $a_N < (\lambda - 1)(L_N + L_S)^2 \left[ (\lambda + 1)(L_N + L_S) + \lambda L_S \right]^{-1} \rho^{-1}$, we can show that any low value of $\beta$ that prevents innovation will not maximize the welfare of the Southern consumer nor that of the Northern consumer. The proof is available on request.
which is indicated by the RHS of the condition in Proposition 3. Thus, more abundant labor in the North intensifies the positive welfare effect of strengthening patent protection.

In addition, as proved in Appendix A, the condition in Proposition 3 necessarily holds irrespective of the size of labor in the North, as long as \( L_S/a_N \) is so large that \( L_S/a_N \geq \lambda \left[ (\log \lambda)(2\lambda - 1) \right]^{-1} \rho \). Therefore, Proposition 3 shows that the strictest patent protection improves the welfare of consumers in the South if labor is more abundant in the South and if the productivity of R&D is higher. The reason is as follows. More abundant labor in the South and higher productivity of R&D intensify the promotion of the innovation effect of strengthening patent protection, \( dI/d\beta \), while they also increase the proportion of multinationals, \( n_F \), and consequently, intensify the competition-reducing effect of strengthening patent protection. Thus, more abundant labor in the South and higher productivity of R&D have two opposing effects on welfare. However, if \( L_S/a_N \) is so large as to satisfy \( L_S/a_N \geq \lambda \left[ (\log \lambda)(2\lambda - 1) \right]^{-1} \rho \), the former positive effect overcomes the latter negative one.

The result obtained in Proposition 3 is important in the following two aspects. First, it seems to be widely recognized that stronger patent protection lowers welfare in the South. However, our result contrasts with this intuitive recognition and shows that, far from being harmful, stronger patent protection is beneficial to the South. In other words, this paper provides a rationale for strengthening patent protection, which is a policy implemented in reality in many developing countries.

Second, our result contrasts with that of Helpman (1993), the seminal paper that examines the effect of strengthening IPR protection on welfare. Helpman examined the welfare effect of strengthening IPR protection both in an endogenous innovation model, where the only mode of technology transfer was illegal imitation, and in an exogenous innovation model, where FDI was the mode of technology transfer. Helpman concluded that stronger IPR protection in the South necessarily damaged welfare in the South in both models. We obtain some important implications from our results that contrast with the results of Helpman’s two models. First, comparing our model and Helpman’s result from his model where imitation is the only mode of technology transfer, we can infer that the effect of strengthening IPR protection on the South’s welfare depends on what the main mode of technology transfer is: when the main mode of technology transfer is FDI, strengthening IPR protection is likely to raise the South’s welfare. Second, by comparing our model and Helpman’s result from his model that includes FDI, we can show that taking the innovation-enhancing effect into account may reverse the sign of the total welfare effect from strengthening IPR protection: endogenizing innovation in the model is essential to an appropriate assessment of the welfare effect of strengthening IPR protection.
4.2 The Effect on the North’s Welfare

Next, we examine how strengthening patent protection in the South affects the North’s welfare. To do this, we first show that the effect on the North’s welfare of extending patent breadth is independent of the initial distribution of assets, $\zeta$, by examining the nominal spending effect and the marginal cost effect. From (23), (24), and (27), $E_N/w_S$ becomes:

$$E_N/w_S = \left(1 + \zeta \frac{\rho a_N}{L_N}\right) \frac{\lambda}{\beta},$$

and the effect of extending patent breadth on $\log(E_N/w_S)$ is given by:

$$1 \frac{dE_N}{E_N} - 1 \frac{dw_S}{w_S} = -\frac{1}{\beta} < 0. \quad (35)$$

This shows that the effect of extending patent breadth on $E_N/w_S$, that is, the sum of the nominal spending effect and the marginal cost effect is independent of the initial distribution of assets, $\zeta$. Because the innovation rate, $I$, and the measure of multinationals, $n_F$, are independent of $\zeta$, the magnitudes of the innovation-enhancing effect, the FDI-promoting effect, and the competition-reducing effect are determined without regard to $\zeta$. Thus, we find that the effect on the North’s welfare, $dU_N(\beta)/d\beta$, is independent of $\zeta$.

Using (33) and (35), we find the effect of strengthening patent protection on the North’s welfare as follows:

$$\frac{dU_N(\beta)}{d\beta} = \frac{1}{\rho} \left[ \log \frac{\lambda}{\rho} \frac{dI}{d\beta} \right]_{\text{innovation-enhancing effect}} + \left( \log \lambda - \log \beta \right) \frac{dn_F}{d\beta} \left[ \frac{1}{\beta} \right]_{\text{FDI-promoting effect}} - \frac{1}{\beta} n_F \left[ \text{competition-reducing effect} \right].$$

If we verify that this is positive, we can show that strengthening patent protection in the South necessarily raises the North’s welfare. As shown in Appendix B, $dU_N(\beta)/d\beta$ is positive for all $\beta \in [\beta_{\text{min}}, \lambda]$ as long as the parameters satisfy an inequality. We can summarize the results of the analysis of the North’s welfare as the following proposition:

**Proposition 4** The strictest patent protection in the South ($\beta_S = \lambda$) maximizes the welfare of consumers in the North if the parameters satisfy $L_N \geq \left[ \frac{\lambda}{(\log \lambda) - 1} \right] \rho a_N$. 

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The condition in Proposition 4 is stricter than the condition imposed in Proposition 3. Hence, we can show that strengthening patent protection in the case where ζ = 1 makes both the South and the North better off as long as the parameters satisfy the condition in Proposition 4. In other words, as long as labor resource in the North is abundant enough to satisfy the condition, or as long as the productivity of R&D is sufficiently high to satisfy the condition, then harmonizing the Southern standard of patent protection with the Northern one by strengthening patent protection in the South is a Pareto-improving policy.

With respect to the effect on the welfare of consumers in the North, the results of the present paper agree with that of Helpman’s FDI model. In his exogenous innovation model, Helpman concluded that tightening IPR protection when the main mode of technology transfer is FDI benefits the North if the imitation rate in the South is sufficiently small. Meanwhile, our endogenous innovation model implies that maximum patent protection in developing countries is globally optimal for the consumers in the North if the labor resource of the North is sufficiently large and the productivity of R&D is sufficiently high. This result implies that Helpman’s conclusion on the welfare of the North will not change even if one takes into account the innovation-enhancing effect of strengthening IPR protection. In that sense, the result obtained in Proposition 4 complements Helpman’s conclusion on the welfare of the North.

4.3 The Generalization of the Initial Distribution of Assets

Finally, in order to generalize the result obtained in Proposition 3, we show that the strictest patent protection can maximize the welfare of consumers in the South even in the case where ζ ≠ 1. When ζ ≠ 1, the nominal spending effect and the marginal cost effect do not cancel each other out and we have to consider these welfare effects of strengthening patent protection. From (23) and (28), $E_S/w_S$ becomes:

$$\frac{E_S}{w_S} = 1 + (1 - \zeta)\frac{\lambda \rho a_N}{L_S \beta},$$

and the effect of strengthening patent protection on $\log(E_S/w_S)$ is given by:

$$\frac{1}{E_S} \frac{dE_S}{d\beta} - \frac{1}{w_S} \frac{dw_S}{d\beta} = \frac{1 - \zeta}{\beta L_S / (\lambda \rho a_N)} + \frac{1}{1 - \zeta \beta} < 0.$$  (36)

17By substituting $L_N = [\lambda / (\log \lambda) - 1] \rho a_N$ into the inequality in Proposition 3, we can confirm that the inequality holds necessarily if $L_N = [\lambda / (\log \lambda) - 1] \rho a_N$. Thus, the inequality in Proposition 3 holds whenever $L_N > [\lambda / (\log \lambda) - 1] \rho a_N$, because the LHS of the inequality of the condition in Proposition 3 is an increasing function of $L_N$ and the RHS is a decreasing function of $L_N$. That is, if the parameters satisfy the inequality in Proposition 4, then the parameters also satisfy the inequality in Proposition 3.
This means that when the initial asset level of the Southern consumer is larger, that is, \(1 - \zeta\) is larger, the negative effect of strengthening patent protection on \(\log(E_S/w_S)\) becomes larger. Therefore, the total effect of strengthening patent protection on the welfare of the Southern consumer is more likely to be negative as the initial asset level of the Southern consumer is larger. However, as shown in Appendix C, we can show that the strictest patent protection maximizes the welfare of the Southern consumer for any \(\zeta \in [0, 1]\) if the parameters satisfy the condition given in Proposition 4. Combining this result with the result that we obtained in Proposition 4, we can obtain the following proposition.

**Proposition 5** The strictest patent protection in the South \(\beta_S = \lambda\) maximizes the welfare of consumers both in the South and in the North for any initial asset distribution between the North and the South if the parameters satisfy \(L_N \geq [\lambda/(\log \lambda) - 1] \rho a_N\).

Proposition 5 shows that as long as labor resource in the North is abundant enough to satisfy the condition, or as long as the productivity of innovation is high enough to satisfy the condition, then harmonizing the Southern standard of patent protection with the Northern one by strengthening patent protection in the South is a Pareto-improving policy irrespective of the distribution of assets between the North and the South. In Proposition 3, we suppose that the consumer in the South has no assets initially. As seen in (28), holding no assets initially means that no assets are held at each point of time, and the result of the proposition seems to be dependent on the restriction of the distribution of assets. However, if the inequality in Proposition 5 holds, we can show that the strictest patent protection in the South is optimal, regardless of the distribution of assets among countries. That is, even if the consumer in the South holds some assets, the strictest patent protection in the South can maximize the welfare of each consumer in the South.

5 Conclusion

The present paper constructs a North–South quality-ladder model, where FDI is the main channel of technology transfer, and conducts not only a positive analysis but also a welfare analysis. Despite the fact that welfare analysis is the most important factor for assessing policies, few previous theoretical studies on IPR protection in developing countries have conducted such an analysis, mainly because the equilibrium paths are so complex that they cannot evaluate the welfare effect of strengthening IPR protection. However, by focusing the analysis on patent breadth, the present paper examines analytically how strengthening patent protection in the South affects welfare in the South. As a result of the analysis,
we have shown that strengthening patent protection can raise welfare not only in the North but also in
the South.

This result stands in contrast to Helpman (1993), the pioneering study that examined the effect of
stronger IPR protection on welfare. Helpman concluded that stronger IPR protection in the South nec-
essarily damages welfare in the South, regardless of whether the mode of technology transfer is illegal
imitation or FDI. However, our analysis differed markedly from Helpman’s result in this respect. Thus,
our result provides a theoretical basis for strengthening patent protection in the South.

Appendix A: Proof of Proposition 3

In this appendix, we prove Proposition 3 by showing that

\[
\frac{dU_S}{d\beta} = \frac{1}{\rho} \left\{ \frac{\log \lambda}{\rho} \left[ \frac{1}{\beta^2} \left( \frac{L_N}{a_N} + \rho \right) + \frac{L_S}{\lambda a_N} \right] \right. \\
+ (\log \lambda - \log \beta) \left( \frac{\beta/\lambda}{L_N + \rho a_N + (\beta/\lambda)L_S} \right) \frac{L_N + \rho a_N + (\beta/\lambda)L_S}{L_N + \rho a_N + (\beta/\lambda)L_S} \right\}.
\]

We define \( f(\beta) \) as the sum of the first term and the third term in the curly bracket of (37):

\[
f(\beta) \equiv \frac{\log \lambda}{\rho} \frac{dI}{d\beta} - \frac{1}{\beta^n_F}
= \frac{\log \lambda}{\rho} \left[ \frac{1}{\beta^2} \left( \frac{L_N}{a_N} + \rho \right) + \frac{L_S}{\lambda a_N} \right] - \frac{L_N + \rho a_N + (\beta/\lambda)L_S}{L_N + \rho a_N + (\beta/\lambda)L_S}.
\]

If \( f(\beta) \) is positive, then the RHS of (37) is also positive because the second term is always nonnegative.
As it is straightforward to show that \( f'(\beta) < 0 \), it is ensured that \( f(\beta) \geq 0 \) for all \( \beta \in [\beta_{\min}, \lambda] \) if \( f(\lambda) \geq 0 \). Because the second term in the RHS of (37) is always nonnegative, this implies that \( dU_S(\beta)/d\beta \) is
nonnegative for all \( \beta \in [\beta_{\min}, \lambda] \), that is, the strictest patent protection \( (\beta_S = \lambda) \) maximizes the welfare.
of the consumer in the South if \( f(\lambda) \geq 0 \). On the other hand, if \( f(\lambda) < 0 \), the strictest patent protection (\( \beta_S = \lambda \)) does not maximize the welfare of the consumer in the South, because the second term in the RHS of (37) is zero when \( \beta = \lambda \), and consequently \( dU_S(\lambda)/d\beta \) is negative. As a result, the strictest patent protection (\( \beta_S = \lambda \)) maximizes the welfare of the consumer in the South if and only if \( f(\lambda) \geq 0 \).

By the definition, \( f(\lambda) \) is given by:

\[
f(\lambda) = \frac{\log \lambda}{\rho \lambda^2} \left( \frac{L_N}{a_N} + \rho + \frac{L_S}{a_N} \right) - \frac{L_S}{L_N + \rho a_N + L_S}. \tag{38}
\]

Rewriting \( f(\lambda) \geq 0 \), we obtain the condition in Proposition 3, \((\log \lambda)\lambda^{-2}(L_N + \lambda L_S + \rho a_N)(\rho a_N) \geq L_S(L_N + L_S + \rho a_N)^{-1}\).

Finally we show that the condition given in Proposition 3 holds irrespective of the size of labor in the North if labor in the South satisfies \( L_S \geq \lambda[(\log \lambda)(2\lambda - 1)]^{-1}\rho a_N \). Note that \( f(\lambda) \) is an increasing function of \((L_N/a_N + \rho)\). Because we focus our analysis on the equilibrium where \( n_F < 1, L_N/a_N + \rho > (\lambda - 1)L_S/a_N \) necessarily holds. Therefore, if \( f(\lambda) > 0 \) when \( L_N/a_N + \rho = (\lambda - 1)L_S/a_N \), then \( f(\lambda) \) is necessarily positive. Substituting \( L_N/a_N + \rho = (\lambda - 1)L_S/a_N \) into (38) yields the following inequality:

\[
L_S \geq \frac{\lambda}{(\log \lambda)(2\lambda - 1)}\rho a_N.
\]

**Appendix B: Proof of Proposition 4**

In this appendix, we prove Proposition 4 by showing that \( dU_N(\beta)/d\beta \) is positive for any value of \( \beta \in [\beta_{\text{min}}, \lambda] \) if the parameters satisfy \( L_N \geq [\lambda/(\log \lambda) - 1] \rho a_N \).

Using \( f(\beta) \) that is defined in Appendix A, we can rewrite \( dU_N(\beta)/d\beta \) as follows:

\[
\frac{dU_N(\beta)}{d\beta} = \frac{1}{\rho} \left[ \frac{\log \lambda}{\rho} \frac{dI}{d\beta} - \frac{1}{\beta} + (\log \lambda - \log \beta) \frac{dn_F}{d\beta} - \frac{1}{\beta} n_F \right]
\]

\[
= \frac{1}{\rho} \left[ f(\beta) - \frac{1}{\beta} + (\log \lambda - \log \beta) \frac{dn_F}{d\beta} \right]. \tag{39}
\]

The last term on the RHS of (39) is necessarily positive and thus \( dU_N(\beta)/d\beta > 0 \) if \( f(\beta) - \beta^{-1} \) is positive. By the definition of \( f(\beta) \), we find that the following relation must hold:

\[
f(\beta) - \frac{1}{\beta} = \frac{\log \lambda}{\rho} \left( \left[ \frac{L_N}{a_N} + \rho \right] \frac{1}{\beta^2} + \frac{L_S}{\lambda a_N} \right) - \frac{(\beta/\lambda)L_S}{L_N + \rho a_N + (\beta/\lambda)L_S} - \frac{1}{\beta} \]

\[
> \frac{\log \lambda}{\rho} \left( \left[ \frac{L_N}{a_N} + \rho \right] \frac{1}{\beta^2} + \frac{L_S}{\lambda a_N} \right) - \frac{(\beta/\lambda)L_S}{L_N + \rho a_N} - \frac{1}{\beta} \]

\[
= \frac{1}{\beta} \left[ L_N + \rho a_N + \frac{\beta^2}{\lambda} L_S \right] \left[ \frac{\log \lambda}{\rho a_N} - \frac{1}{L_N + \rho a_N} \right]. \tag{40}
\]
If we assume that $L_N \geq [\lambda/(\log \lambda) - 1] \rho a_N$, then the parameters satisfy the following relation for all $\beta \in [1, \lambda]$:

$$\frac{\log \lambda}{\rho a_N \beta} - \frac{1}{L_N + \rho a_N} \geq 0.$$  

Therefore, from (39) and (40), we can conclude that $dU_N(\beta)/d\beta > 0$ for any value of $\beta \in [\beta_{min}, \lambda]$ if the parameters satisfy $L_N \geq [\lambda/(\log \lambda) - 1] \rho a_N$.

**Appendix C: Proof of Proposition 5**

In this appendix, we prove Proposition 5, which states that the strictest patent protection in the South maximizes the welfare of consumers both in the South and in the North for any initial asset distribution if $L_N \geq [\lambda/(\log \lambda) - 1] \rho a_N$ is satisfied. Because the proof on the welfare of the Northern consumer is given in Appendix B, it is sufficient for the proof to show that $dU_S(\beta)/d\beta > 0$ for any values of $\beta \in [\beta_{min}, \lambda]$ and $\zeta \in [0, 1]$ if the parameters satisfy $L_N \geq [\lambda/(\log \lambda) - 1] \rho a_N$.

Using $f(\beta)$ that is defined in Appendix A, we can rewrite $dU_S(\beta)/d\beta$ for any $\zeta \in [0, 1]$ as follows:

$$
\frac{dU_S(\beta)}{d\beta} = \frac{1}{\rho} \left[ \frac{\log \lambda}{\beta} \frac{dI}{d\beta} + \left( \frac{1}{E_S} \frac{dE_S}{d\beta} - \frac{1}{w_S} \frac{dw_S}{d\beta} \right) + (\log \lambda - \log \beta) \frac{dn_F}{d\beta} - \frac{1}{\beta} n_F \right] \\
= \frac{1}{\rho} \left[ f(\beta) + \left( \frac{1}{E_S} \frac{dE_S}{d\beta} - \frac{1}{w_S} \frac{dw_S}{d\beta} \right) + (\log \lambda - \log \beta) \frac{dn_F}{d\beta} \right] \\
> \frac{1}{\rho} \left[ f(\beta) - \frac{1}{\beta} + (\log \lambda - \log \beta) \frac{dn_F}{d\beta} \right],
$$

(41)

where the last inequality holds because $\frac{1}{E_S} \frac{dE_S}{d\beta} - \frac{1}{w_S} \frac{dw_S}{d\beta}$ is larger than $-\beta^{-1}$, which can be shown directly from (36). Note that the RHS of (41) is equal to $dU_N(\beta)/d\beta$. Therefore, using the same proof as in Appendix B, we can show that $dU_S(\beta)/d\beta > dU_N(\beta)/d\beta > 0$ for any values of $\beta \in [\beta_{min}, \lambda]$ and $\zeta \in [0, 1]$ if the parameters satisfy $L_N \geq [\lambda/(\log \lambda) - 1] \rho a_N$. 

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References


Appendix D (Not Intended for Publication): Proof of the Assertion in Footnote 16

In this appendix, we prove the assertion in footnote 16. It is sufficient for the proof to show that $U_i(\lambda)$ is higher than $U_i(\bar{\beta})$ for any $\bar{\beta} \in [1, \beta_{\text{min}}]$ if $a_N < (\lambda - 1)(L_N + L_S)^2 [(\lambda + 1)(L_N + L_S) + \lambda L_S]^{-1} \rho^{-1}$.

To verify this, we first compute the welfare of each consumer in the case of no innovation, that is, $U_i(\bar{\beta})$ for $\bar{\beta} \in [1, \beta_{\text{min}}]$. Substituting (23) and (24) into (27) and (28), we have:

$$E_N = \frac{\zeta(\bar{\beta} - 1)(\lambda L_N + \bar{\beta} L_S) + \lambda L_N}{\beta L_N(\lambda L_N + \bar{\beta} L_S)}$$

(42)

and

$$E_S = \frac{(1-\zeta)(\bar{\beta} - 1)(\lambda L_N + \bar{\beta} L_S) + \bar{\beta} L_S}{\beta L_S(\lambda L_N + \bar{\beta} L_S)}$$

(43)

for $\bar{\beta} \in [1, \beta_{\text{min}}]$. Thus, from (21)–(23), (32), and (42), we can compute $U_N(\bar{\beta})$ for $\bar{\beta} \in [1, \beta_{\text{min}}]$ as follows:

$$U_N(\bar{\beta}) = \frac{1}{\rho} \left\{ \log \frac{\zeta(\bar{\beta} - 1)(\lambda L_N + \bar{\beta} L_S) + \lambda L_N}{\beta L_N(\lambda L_N + \bar{\beta} L_S)} \right. \
+ \frac{\bar{\beta} L_S}{\lambda L_N + \bar{\beta} L_S} \left( \log \lambda - \log \bar{\beta} \right) - \log \lambda \right\}.$$

Similarly, from (21)–(23), (32), and (43), $U_S(\bar{\beta})$ for $\bar{\beta} \in [1, \beta_{\text{min}}]$ can be derived as follows:

$$U_S(\bar{\beta}) = \frac{1}{\rho} \left\{ \log \frac{(1-\zeta)(\bar{\beta} - 1)(\lambda L_N + \bar{\beta} L_S) + \bar{\beta} L_S}{\beta L_S(\lambda L_N + \bar{\beta} L_S)} \right. \
+ \frac{\bar{\beta} L_S}{\lambda L_N + \bar{\beta} L_S} \left( \log \lambda - \log \bar{\beta} \right) - \log \lambda \right\}.$$

Next, we derive the welfare of each consumer in the case where $\beta = \lambda$, that is, $U_i(\lambda)$. By substituting (23) and (24) into (27) and (28), the equilibrium values of each consumer’s spending in the case where $\beta = \lambda$ are given as follows:

$$E_N = \frac{1}{L_N + L_S + \rho a_N} \left( \frac{\zeta \rho a_N}{L_N} + 1 \right)$$

(44)

and

$$E_S = \frac{1}{L_N + L_S + \rho a_N} \left[ \frac{(1-\zeta)\rho a_N}{L_S} + 1 \right].$$

(45)

Therefore, substituting (21)–(23) and (44) into (32) yields:

$$U_N(\lambda) = \frac{1}{\rho} \left\{ \log \lambda \left[ \frac{\lambda - 1}{\lambda} \frac{L_N + L_S}{a_N} - \frac{\rho}{\lambda} \right] + \log \left( \frac{\zeta \rho a_N}{L_N} + 1 \right) - \log \lambda \right\}.$$
In a similar way, from (21)–(23), (32), and (45), we obtain:

\[ U_S(\lambda) = \frac{1}{\rho} \left\{ \log \frac{\lambda - 1}{\lambda} \frac{L_N + L_S - \rho}{a_N} \frac{\rho}{\lambda} \right\} + \log \left[ \frac{(1 - \zeta)\rho a_N}{L_S} + 1 \right] - \log \lambda \].

Now, we compare \( U_i(\bar{\beta}) \) with \( U_i(\lambda) \) to confirm the assertion in footnote 16. By subtracting \( U_N(\bar{\beta}) \) from \( U_N(\lambda) \), we have:

\[ U_N(\lambda) - U_N(\bar{\beta}) = \frac{1}{\rho} \left\{ \log \frac{\lambda - 1}{\lambda} \frac{L_N + L_S - \rho}{a_N} \frac{\rho}{\lambda} \right\} + \log \left( \frac{\zeta \rho a_N}{L_N} + 1 \right) \]

\[ - \log \frac{\zeta(\bar{\beta} - 1)(\lambda L_N + \beta L_S) + \lambda L_N}{\beta L_N} - \frac{\beta L_S}{\lambda L_N + \beta L_S} (\log \lambda - \log \bar{\beta}) \].

Note that the sum of the second term and the third term in (46) satisfies the following inequality:

\[ \log \left( \frac{\zeta \rho a_N}{L_N} + 1 \right) - \log \frac{\zeta(\bar{\beta} - 1)(\lambda L_N + \beta L_S) + \lambda L_N}{\beta L_N} \]

\[ = \log \frac{\zeta \rho a_N \bar{\beta} + \beta L_N}{\zeta(\bar{\beta} - 1)(\lambda L_N + \beta L_S) + \lambda L_N} \]

\[ \geq \log \frac{\zeta(\bar{\beta} - 1)(\lambda L_N + \beta L_S) + \lambda L_N}{\zeta(\bar{\beta} - 1)(\lambda L_N + \beta L_S) + \lambda L_N} \]

\[ = \log \bar{\beta} - \log \lambda, \]

where the inequality of the third line uses the relation that \( \rho a_N \geq \lambda^{-1}(\bar{\beta} - 1)(\lambda L_N + \beta L_S) \) must hold because \( (\beta - 1)\beta^{-1}[L_N + (\beta/\lambda)L_S]a_N^{-1} - \rho \beta^{-1} < 0 \) for \( \beta \leq \beta_{\text{min}} \) from (22). Thus, \( U_N(\lambda) - U_N(\bar{\beta}) \) must satisfy the following inequality for any \( \bar{\beta} \in [1, \beta_{\text{min}}] \):

\[ U_N(\lambda) - U_N(\bar{\beta}) \geq \frac{1}{\rho} \left\{ \log \frac{\lambda - 1}{\lambda} \frac{L_N + L_S - \rho}{a_N} \frac{\rho}{\lambda} \right\} + \log \bar{\beta} - \log \lambda \]

\[ - \frac{\beta L_S}{\lambda L_N + \beta L_S} (\log \lambda - \log \bar{\beta}) \]

\[ = \frac{1}{\rho} \left\{ \log \lambda \left[ \frac{\lambda - 1}{\lambda} \frac{L_N + L_S - \rho}{a_N} - \frac{1}{\lambda} - 1 - \frac{L_S}{\rho a_N} \right] \right\} \]

\[ + \left( \log \bar{\beta} \right) \left[ 1 + \frac{\beta L_S}{\lambda L_N + \beta L_S} \right] \]

\[ > 0, \]

where the last inequality uses the condition that \( a_N < (\lambda - 1)(L_N + L_S)^2 [(\lambda + 1)(L_N + L_S) + \lambda L_S)^{-1} \rho^{-1} \). Therefore, we can conclude that the welfare of each Northern consumer in the case where \( \beta = \lambda \) is higher than the welfare in the case of no innovation, if \( a_N < (\lambda - 1)(L_N + L_S)^2 [(\lambda + 1)(L_N + L_S) + \lambda L_S)^{-1} \rho^{-1} \).

In the same way, we can show that \( U_S(\lambda) - U_S(\bar{\beta}) \) is positive if the condition is satisfied. Subtracting \( U_S(\bar{\beta}) \) from \( U_S(\lambda) \), we obtain:

\[ U_S(\lambda) - U_S(\bar{\beta}) = \frac{1}{\rho} \left\{ \log \frac{\lambda - 1}{\lambda} \frac{L_N + L_S - \rho}{a_N} \frac{\rho}{\lambda} \right\} + \log \left[ \frac{(1 - \zeta)\rho a_N}{L_S} + 1 \right] \]

\[ - \log \frac{(1 - \zeta)(\bar{\beta} - 1)(\lambda L_N + \beta L_S) + \beta L_S}{\beta L_S} - \frac{\beta L_S}{\lambda L_N + \beta L_S} (\log \lambda - \log \bar{\beta}) \].
The sum of the second term and the third term in (47) satisfies the following inequality:

\[
\log \left[ \frac{(1 - \zeta)\rho a_N}{L_S} + 1 \right] - \log \left( \frac{(1 - \zeta)(\beta - 1)(\lambda L_N + \beta L_S) + \beta L_S}{\beta L_S} \right)
\]

\[
= \log \left( \frac{(1 - \zeta)\rho a_N + \beta L_S}{(1 - \zeta)(\beta - 1)(\lambda L_N + \beta L_S) + \beta L_S} \right)
\]

\[
\geq \log \left( \frac{1 - \zeta}{(1 - \zeta)((\beta - 1)(\lambda L_N + \beta L_S) + \beta L_S)} \right)
\]

\[
> \log \beta - \log \lambda,
\]

where the first inequality uses the relation that \( \rho a_N \geq \lambda^{-1}(\beta - 1)(\lambda L_N + \beta L_S) \) must hold because \( (\beta - 1)\beta^{-1}[L_N + (\beta/\lambda)L_S]a_N^{-1} - \rho \beta^{-1} < 0 \) for \( \beta \leq \beta_{\text{min}} \) from (22). Thus, \( U_S(\lambda) - U_S(\bar{\beta}) \) must satisfy the following inequality for any \( \bar{\beta} \in [1, \beta_{\text{min}}] \):

\[
U_S(\lambda) - U_S(\bar{\beta}) > \frac{1}{\rho} \left\{ \log \lambda \left[ \frac{\lambda - 1}{\lambda} \frac{L_N + L_S}{a_N} - \frac{\rho}{\lambda} \right] + \log \bar{\beta} - \log \lambda \right. \\
- \frac{\bar{\beta} L_S}{\lambda L_N + \beta L_S} (\log \lambda - \log \bar{\beta}) \right.
\]

\[
= \frac{1}{\rho} \left\{ (\log \lambda) \left[ \frac{\lambda - 1}{\lambda} \frac{L_N + L_S}{\rho a_N} - \frac{1}{\lambda} - 1 - \frac{\bar{\beta} L_S}{\lambda L_N + \beta L_S} \right] \right.
\]

\[
+ (\log \bar{\beta}) \left( 1 + \frac{\bar{\beta} L_S}{\lambda L_N + \beta L_S} \right) \right\}
\]

\[
> 0,
\]

where the last inequality uses the condition that \( a_N < (\lambda - 1)(L_N + L_S)^2 [((\lambda + 1)(L_N + L_S) + \lambda L_S)]^{-1} \rho^{-1} \).

That is, the welfare of each Southern consumer in the case where \( \beta = \lambda \) is also higher than the welfare in the case of no innovation, if \( a_N < (\lambda - 1)(L_N + L_S)^2 [((\lambda + 1)(L_N + L_S) + \lambda L_S)]^{-1} \rho^{-1} \).

Thus, we have been able to confirm that when the value of \( \beta \) is so low that no R&D is undertaken, this does not maximize the welfare of each consumer if \( a_N < (\lambda - 1)(L_N + L_S)^2 [((\lambda + 1)(L_N + L_S) + \lambda L_S)]^{-1} \rho^{-1} \).