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Dynamic Analysis of Outsourcing and FDI

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Abstract

This paper constructs a North-South endogenous growth model to investigate how the organizational forms of final good firms evolve. Initially, the final good firms in the North outsource the intermediate goods to Northern firms and produce in the North. As the economy develops, final good firms produce the final good in the North and outsource the production of intermediate goods to Southern firms. As the economy develops more, they produce the final good in the South and outsource the production of intermediate goods to Southern firms.

Keywords: Outsourcing, FDI, North-South, R&D, Economic Growth
JEL classification: F43, O31

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1 Introduction

Recently, trade costs and communication costs are decreasing rapidly. Firms in the developed countries have various strategies: some firms outsource the production of intermediate goods to the developing countries and other firms produce the final goods in the developing countries.[1] I investigate why some firms outsource the production of the intermediate goods and other firms produce the final goods in the developing countries?

Researchers have used static models to investigate these issues. Grossman and Helpman (2002) investigated choices between outsourcing and vertical integration in a closed economy. Antràs and Helpman (2004) analyzed how final good producers with different productivity levels choose whether to outsource the production of the intermediate goods. They showed that the final good producers' strategy crucially depends on their productivity. However, although these studies have suggested some important results about the final good firms' strategies, they analyze only static equilibria.

In contrast to these static analyses, Naghavi and Ottaviano (2008) used a North-South endogenous growth model with offshoring to investigate the extent to which final good producers outsource the production of intermediate goods to the South. They analyzed two types of equilibrium: one in which all of the producers outsource the production of intermediate goods, and another in which none of them outsource. In addition, Gao (2007) developed an endogenous growth model and analyzed the relationship between trade costs and the location where the intermediate goods are produced. He showed that, as the trade cost falls, the number of intermediate goods produced in the South increases.

Morita (2008) constructed an endogenous growth model of the variety-expansion type to investigate the dynamic choice problem of final good producers. I explored transition paths and the point at which final good producers obtained goods from foreign suppliers rather than from the domestic suppliers. In this paper, I generalize that model to allow Northern firms not only to outsource intermediate goods to

[1] Canon Inc. outsources the production of intermediate goods to China. On the other hand, Toyota produces the pickup truck in the Thailand and outsources the production of the intermediate goods to the Thailand firms. Suzuki produces motorbike in the India and they outsources 90 percent of production of the intermediate goods to the Indian firms.

either the North or South, but also to base the production of final goods in either the North or South. This enables me to investigate why final good firms in developed countries did not outsource the production of intermediate goods to suppliers in the developing countries a several years ago and why they do now. I show that, as the trade cost decreases, Northern firms not only outsource intermediate goods to the South but also move their production of final goods to the South.

In this model, I assume that it is costless to trade intermediate goods within a country but not between countries. In addition, I assume that when the final good firms produce the final good in the South, they bear a management cost because they have to monitor the Southern workers. The final good firms choose the organization form from which they can earn a larger profit. I show why the organizational forms of the final good firms change as the economy develops. Suppose that, initially, the final good firms produce the final good in the North and outsource the production of intermediate goods to the Northern firms. As the economy develops, the cost of trade shrinks and the final good firms outsource the production of intermediate goods to the Southern firms. Then, as the economy develops more, the final good firms outsource to the South and also produce the final good in the South.

The remainder of the paper is structured as follows. Section 2 introduces the model. In Section 3, I derive the equilibrium path of the model and prove that there exists a unique equilibrium path. Section 4 concludes.

2 The Model

I develop a dynamic general equilibrium model where final good producers outsource the production of intermediate goods to intermediate producers either in their home country or in foreign countries and they produce the final good in their home country or in foreign countries. Our model has a similar structure to the model of Grossman and Helpman (1991, Ch. 3).

The world economy consists of two countries, the North and the South denoted by $l \in \{N, S\}$. The population size in the world is one. Each individual lives forever and is endowed with L^l units of labor services, which are supplied inelastically at each point of time. There exist three types of goods: a homogeneous good, intermediate goods, and final goods. The homogeneous good can be produced only in the South, and the final goods can be produced in both countries. The intermediate goods can be produced in either country. Individuals consume the homogeneous good and the final goods. R&D activities can be conducted only in the North.

The final good firms outsource the production of intermediate goods either to the Northern firms or to the Southern firms, and produce the final good in the North or in the South in order to maximize their profits. Therefore, there are four possible strategies. First, the final good firms outsource the intermediate goods to the Northern firms and produce the final good in the North. Second, they outsource to the Southern firms and produce the final good in the North. Third, they outsource to the Northern firms and produce the final good in the South. Finally, they outsource to the Southern firms and produce the final good in the South. In the first case, there are three sectors in the North: the R&D sector, the final good sector, and the intermediate sector. In the South, there is only one sector: the homogeneous good sector. In the second case, there are two sectors in the North: the R&D sector and the final good sector. In the South, there are two sectors: the homogeneous good sector and the intermediate sector. In the third case, there are two sectors in the North: the R&D sector and the intermediate sector. In the South, there are also two sectors: the homogeneous good sector and the final good sector. Finally, in the fourth case, there is only one sector in the North: the R&D sector. In the South, there are now three sectors: the homogeneous good sector, the intermediate sector, and the final good sector.

2.1 Individuals

Individuals in both countries have identical preferences:

$$\int_t^\infty e^{-\rho(\nu-t)}U(\nu)d\nu, \quad 0<\rho<1, \tag{1}$$

where ρ is the constant subjective discount rate. $U(\nu)$ is the instantaneous utility per person at time ν and is specified as follows:

$$U(\nu)=y(\nu)+\frac{1}{\mu}X(\nu)^{\mu}, \quad 0<\mu<1, \tag{2}$$

where μ is a parameter, $y(\nu)$ stands for consumption of the homogeneous good at time ν, and $X(\nu)$ is a composite good at time ν that is made up of differentiated final goods. For simplicity, I drop the time index ν from all the variables. A composite good X is given by:

$$X = \left[\int_0^n x(i)^\alpha di \right]^{\frac{1}{\alpha}}, \qquad 0 < \alpha < 1, \tag{3}$$

where $x(i)$ represents consumption of different final goods i and n denotes the number of final goods. $\frac{1}{1-\alpha}$ is the elasticity of substitution between any two varieties of final goods in a given sector. If α is close to one, the goods are nearly perfect substitutes and the sector is highly competitive. If α is close to zero, the goods are distinct products and the sector becomes monopolistic. I assume that $\alpha > \mu$ in the following analysis.

The utility maximization problem of the individual can be solved in two steps. The first step is to solve the following static optimization problem:

$$\max_{x, x(i)} \quad y + \frac{1}{\mu} X^\mu,$$
$$\text{subject to} \quad y + \int_0^n P(i) x(i) di = E,$$

where the homogeneous good is chosen to be the numeraire, $P(i)$ stands for the price of the product i, and E is the total expenditure. From the first-order condition, I can obtain the following inverse demand function:

$$P(i) = X^{\mu-\alpha} x(i)^{\alpha-1}, \; i \in [0, n]. \tag{4}$$

The second step is to solve the intertemporal optimization problem. From (4), the indirect utility function is given by:

$$U = E - \left(1 - \frac{1}{\mu}\right) \left[\int_0^n P(i)^{\frac{\alpha}{\alpha-1}} di \right]^{\frac{\mu(\alpha-1)}{\alpha(\mu-1)}}. \tag{5}$$

As is clear from the indirect utility function (5), the marginal utility of expenditures is constant. The market interest rate at time t, $r(t)$, must be equal to the subjective discount rate:

$$r(t) = \rho \qquad \text{for all } t. \tag{6}$$

2.2 Production

I denote the wage rate in the North by w_N, and the wage rate in the South by w_S, and assume that the wage rate in the North is larger than the wage rate in the South, $w_N > w_S$. Four sectors exist in the world: a homogeneous good sector, an intermediate good sector, a final good sector, and an R&D sector.

2.2.1 The homogeneous good sector

Production of the homogeneous good uses labor only. The homogeneous good is assumed to be produced only in the South. The production of one unit of the homogeneous good requires one unit of Southern labor. I assume that the homogeneous good market is perfectly competitive. Thus, the price of the homogenous good becomes equal to the wage rate in the South. As the homogeneous good is chosen to be the numeraire, the wage rate in the South becomes unity, that is, $w_S = 1$. I assume that the homogeneous good can be traded costlessly.

2.2.2 The intermediate goods sector

The intermediate goods can be produced in the both countries. Production of one unit of each intermediate good requires one unit of labor. I assume that perfect competition prevails in the intermediate goods markets in both countries. Thus, the marginal cost of intermediate goods produced in the North is the wage rate in the North, $w_N > 1$, and the marginal cost of intermediate goods produced in the South is the wage rate in the South, $w_S = 1$. I assume that no transportation cost is incurred in transferring the intermediate goods when the final good firms and the intermediate good firms exist in the same country, but the final good firms have to pay an iceberg cost τ ($\tau > 1$) to obtain intermediate goods from the other country. Therefore, the price of intermediate goods facing the final good firms is as follows:

$$p^m_{N,N} = w_N, \tag{7}$$

$$p^m_{N,S} = \tau, \tag{8}$$

$$p^m_{S,N} = \tau w_N, \tag{9}$$

$$p^m_{S,S} = 1, \tag{10}$$

where $p^m_{j,k}$ is the price of intermediate goods when final good firms are in country j, $j \in \{N, S\}$ and they obtain the intermediate goods from the suppliers in country k, $k \in \{N, S\}$. Hereafter, if there are two subscripts, the first subscript denotes where final good firms are located, and the second subscript denotes where the final good firms outsource the production of intermediate goods.

2.2.3 The final goods sector

Production of each final good requires labor and a variety-specific intermediate good. When the final good firms exist in the North, they use Northern labor and Northern intermediate goods. I assume that their unit cost function is as follows:

$$c_N(w_N, p^m_{N,k}) = w_N^{1-\beta}(p^m_{N,k})^\beta, k \in \{N, S\}, \tag{11}$$

where $p^m_{N,k}$ is the price of the intermediate good produced in country k when final good firms are in the North, β is constant, and $0 < \beta < 1$. On the other hand, when the final good firms produce the final good in the South, I assume that they have to hire not only Southern workers but also Northern workers in order to manage and train Southern workers. The final good firms have to incur the cost because Northern workers have to cross international borders and live in the South, which has a different culture and language from the North, in order not only to produce but also to monitor the Southern labor. Specifically, I assume that management costs take the iceberg form. Hence, the cost of Northern labor is ϵw_N, $\epsilon > 1$. Then, I assume that the unit cost function of the final good firms is as follows:

$$\begin{aligned} c_S(\epsilon w_N, p^m_{S,k}) &= A(\epsilon w_N)^{\gamma(1-\beta)} w_S^{(1-\gamma)(1-\beta)}(p^m_{S,k})^\beta, \\ &= A(\epsilon w_N)^{\gamma(1-\beta)}(p^m_{S,k})^\beta, k \in \{N, S\}, \end{aligned} \tag{12}$$

where $p^m_{S,k}$ is the price of the intermediate good produced in country k when the final good firms are in the South, γ is a constant parameter, and $0 < \gamma < 1$. A is also a constant parameter.

By using (4), I can obtain the revenue of each final good producer as follows:

$$\begin{aligned} R_{j,k}(i) =& P(i)x(i), \\ =& X_{j,k}^{\mu-\alpha} x_{j,k}(i)^\alpha, \quad j, k \in \{N, S\}, \ i \in [0, n]. \end{aligned} \tag{13}$$

where $R_{j,k}$ is the revenue of the final good producer, $X_{j,k}$ is the composite good and $x_{j,k}(i)$ is the consumption of different final goods i. I consider the profit maximization problem of the final good producers in both countries. In the Appendix, I derive the profit function when the final good firms exist in the North and in the South. Using (7), (8), (9), and (10), I can write the profit functions as follows:

$$\pi_{N,N} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu}{1-\mu}} (1-\alpha), \tag{14}$$

$$\pi_{N,S} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu(1-\beta)}{1-\mu}} \tau^{\frac{-\beta\mu}{1-\mu}} (1-\alpha), \tag{15}$$

$$\pi_{S,N} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} (\tau w_N)^{\frac{-\beta\mu}{1-\mu}} (1-\alpha), \tag{16}$$

$$\pi_{S,S} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} (1-\alpha). \tag{17}$$

Comparing (16) to (17), I can show that the profit of the final good firms in the South when they outsource the production of intermediate goods to the Southern firms is larger than that when they outsource the production of the intermediate goods to the Northern firms, that is, $\pi_{S,S} > \pi_{S,N}$, because $\tau w_N > 1$. Therefore, when they are in the South, the final good firms never outsource the production of intermediate goods to the Northern firms. I obtain the following lemma.

Lemma 1. *When the final good firms are in the South, they never outsource the production of intermediate goods to the Northern firms.*

I label the economy the *North Regime* when the final good firms are in the North and outsource the production of intermediate goods to the Northern firms; the *Outsourcing Regime* when the final good firms are in the North and outsource the production of intermediate goods to the Southern firms; and the *FDI Regime* when the final good firms are in the South and outsource the production of intermediate goods to the Southern firms.

2.3 The R&D sector

The R&D activities of the present model follow Grossman and Helpman (1991, Ch. 3). I assume that there is no R&D sector in the South. Thus, the firm in the South cannot innovate any new variety of final good and imitate any existing variety of final good. The final good producers in the North enter into the R&D

race and finance the cost of R&D by issuing equity in the stock market. The equity is bought by individuals who live in either country. The stock value of the final good producers at time t is equal to the discounted sum of its profit stream from time t. Suppose that the final good firms is first in one regime and they next move to another regime at time s and further to another regime at time s'. Then, the stock value of the final good producers at time t is given by:

$$v_{j,k} = \int_t^s e^{r(\tau)(\tau-t)}\pi_{j,k}d\tau + e^{r(s)(s-t)}\int_s^{s'} e^{r(\tau)(\tau-s)}\pi_{j',k'}d\tau \\ + e^{r(s')(s'-t)}\int_{s'}^{\infty} e^{r(\tau)(\tau-s')}\pi_{j'',k''}d\tau,\ j,j',j'',k,k',k'' \in \{N,S\}, \quad (18)$$

where $v_{j,k}$ is the stock value. Differentiating (18) with respect to time t yields the following no-arbitrage conditions:

$$\rho v_{N,N} = \pi_{N,N} + \dot{v}_{N,N}, \quad (19)$$

$$\rho v_{N,S} = \pi_{N,S} + \dot{v}_{N,S}, \quad (20)$$

$$\rho v_{S,S} = \pi_{S,S} + \dot{v}_{S,S}. \quad (21)$$

The final good producers hire labor to develop blueprints. I assume that knowledge spillovers in the R&D activities exist; that is, the more innovations that have been created previously, the lower is the cost of innovating. I assume that L^A units of labor for R&D activities for a time interval dt produce a new variety of final good according to the following:

$$dn = \frac{L^A n}{a}dt. \quad (22)$$

The cost of the R&D activities is $w_N L^A dt$ because the R&D sector is located only in the North. Producing the blueprints creates a value for the final good producers of $v_{j,k}dn$ because each blueprint has a market value of $v_{j,k}$. I assume that there is free entry into the R&D race. Therefore, the following free-entry condition must hold:

$$v_{j,k} \leq \frac{aw_N}{n}, \quad \text{with equality whenever } \dot{n} \equiv \frac{dn}{dt} > 0,\ j,k \in \{N,S\}. \quad (23)$$

2.4 The labor market

Labor market equilibrium requires that labor supply equals to labor demand. Under the *North Regime*, the demand for labor in the North comes from the intermediate sector, the final good sector, and the R&D sector. In the South, the demand for labor comes only from the homogeneous good sector. Therefore, the labor market equilibrium conditions are as follows:

$$
\begin{aligned}
L^N &= L^A + L^M_{N,N} + L^F_{N,N}, \\
L^S &= y_{N,N},
\end{aligned}
$$

where $y_{j,k}$ is the labor demand for production of the homogeneous good in the South, $L^M_{j,k}$ is the labor demand for production of the intermediate goods in the North, and $L^F_{j,k}$ is the labor demand for the production of the final goods when the final good firms producing in the j country obtain the intermediate goods from the k country. Under the *Outsourcing Regime*, the demand for labor in the North comes from the R&D sector and the final good sector. In the South, the demand for labor comes from the homogeneous good sector and the intermediate goods sector. Therefore, the labor market equilibrium conditions are as follows:

$$
\begin{aligned}
L^N &= L^A + L^F_{N,S}, \\
L^S &= y_{N,S} + L^M_{N,S}.
\end{aligned}
$$

Finally, under the *FDI Regime*, the demand for labor in the North comes from the R&D sector and the final good sector. In the South, the demand for labor comes from the homogeneous good sector, the intermediate goods sector and the final good sector. Therefore, the labor market equilibrium conditions are as follows:

$$
\begin{aligned}
L^N &= L^A + L^F_{S,S}, \\
L^S &= y_{S,S} + L^M_{S,S} + L^{F,S}_{S,S},
\end{aligned}
$$

where $L^F_{S,S}$ is the labor demand for the production of the final good in the South.

In the Appendix, I derive the labor demand in each sector and under each regime. Under the *North Regime*, I can rewrite the labor market clearing conditions in both countries using (A.11), (A.12), (A.13), and the price of intermediate goods, (7), as follows:

$$
\text{North} \qquad L^N = \frac{a\dot{n}}{n} + n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} w_N^{\frac{-\mu}{1-\mu}}, \tag{24}
$$

$$
\text{South} \qquad L^S = E - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu}{1-\mu}}. \tag{25}
$$

Using (8), (A.11), (A.12), and (A.13), I can rewrite the labor market clearing conditions of the *Outsourcing Regime* as follows:

$$\text{North} \qquad L^N = \frac{a\dot{n}}{n} + n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}}(1-\beta)\alpha^{\frac{1}{1-\mu}} w_N^{\frac{\beta\mu-1}{1-\mu}} \tau^{\frac{-\beta\mu}{1-\mu}}, \tag{26}$$

$$\text{South} \qquad L^S = E + n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \left(\alpha\beta - \tau\right) \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu(1-\beta)}{1-\mu}} \tau^{\frac{\mu(1-\beta)-1}{1-\mu}}. \tag{27}$$

Finally, using (10), (A.14), (A.15), (A.16), and (A.17), I can rewrite the labor market clearing conditions of the *FDI Regime* as follows:

$$\text{North} \qquad L^N = \frac{a\dot{n}}{n} + n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha\gamma(1-\beta) \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}-1}, \tag{28}$$

$$\text{South} \qquad L^S = E - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} \left[1 - \alpha\beta - \alpha(1-\gamma)(1-\beta)\right]. \tag{29}$$

3 The Equilibrium Path

In this section, I examine the dynamics of the economy. First, I consider the dynamic behaviors of *North Regime*, *Outsourcing Regime*, and *FDI Regime* separately. Finally, I integrate these dynamic behaviors.

3.1 Dynamic behavior under *FDI Regime*

In *FDI Regime*, the equilibrium conditions are (6), the no-arbitrage condition, (21), the free-entry condition, (23), and the labor market clearing condition, (28). From the labor market clearing condition, I can derive the differential equation for the number of firms, n, as follows:

$$\dot{n} = \frac{n}{a}\left[L^N - \alpha\gamma(1-\beta) n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}-1}\right]. \tag{30}$$

In the Appendix, I derive the differential equation for the wage rate in the North, w_N as follows:

$$\dot{w_N} = \left(\rho + \frac{L^N}{a}\right) w_N - \frac{1}{a} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} \left(1 - \alpha + \frac{\alpha\gamma(1-\beta)}{\epsilon}\right). \tag{31}$$

These two equations, (30) and (31), constitute the dynamic system of *FDI Regime*. Then, the equation for the $\dot{n} = 0$ locus is represented by:

$$w_N = \epsilon^{-1} \left(\frac{\alpha\gamma(1-\beta)}{L^N}\right)^{\frac{1-\mu}{\Phi}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{\Phi}} n^{\frac{\mu(1-\alpha)}{\alpha\Phi}}, \tag{32}$$

where $\Phi = 1 - \mu + \mu\gamma(1-\beta)$. The $\dot{n} = 0$ locus is concave and $\lim_{n\to\infty} \frac{\partial w_N}{\partial n} = 0$. On the other hand, the equation for the $\dot{w_N} = 0$ locus is given by:

$$w_N = \epsilon^{-1} \left(\frac{\epsilon(1-\alpha) + \alpha\gamma(1-\beta)}{a\rho + L^N} \right)^{\frac{1-\mu}{\Phi}} \left(\frac{A}{\alpha} \right)^{\frac{-\mu}{\Phi}} n^{\frac{\mu(1-\alpha)}{\alpha\Phi}}. \tag{33}$$

The $\dot{w_N} = 0$ locus has a similar shape to the $\dot{n} = 0$ locus. Thus, the origin is the only intersection point of the two curves, $\dot{n} = 0$ and $\dot{w_N} = 0$. Figure 1 depicts the phase diagram for this system on the $(n,\, w_N)$ plane. When $\frac{\alpha a \rho}{1-\alpha} < L^N$ holds, the $\dot{w_N} = 0$ locus is above the $\dot{n} = 0$ locus for any $w_N > 0$. Hereafter, I focus on $\frac{\alpha a \rho}{1-\alpha} < L^N$.[2]

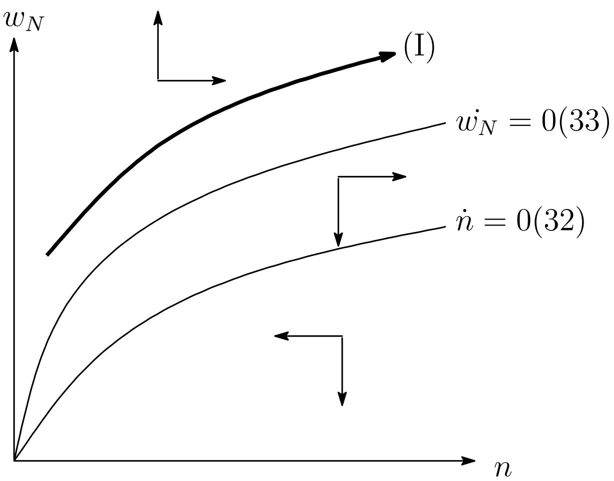

Figure 1: Phase diagram under *FDI Regime*

I show the saddle path stability in *FDI Regime*. Let us define $z_{SS} \equiv n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} (\epsilon w_N)^{\frac{-\Phi}{1-\mu}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}}$. Then, differentiating z_{SS} with respect to time t and using (30) and (31) yields the following differential equation for z_{SS}:

$$\begin{aligned} \frac{\dot{z_{SS}}}{z_{SS}} &= \frac{\mu(1-\alpha)}{\alpha(1-\mu)} \left(\frac{\dot{n}}{n} \right) - \frac{\Phi}{1-\mu} \left(\frac{\dot{w_N}}{w_N} \right) \\ &= \left[\frac{\mu - \alpha - \alpha\mu\gamma(1-\beta)}{\alpha(1-\mu)} \frac{L^N}{a} - \frac{\Phi}{1-\mu}\rho \right] + \frac{\Gamma_{SS} z_{SS}}{a(1-\mu)}, \end{aligned}$$

where

$$\Gamma_{SS} \equiv (1-\mu)\left[\epsilon(1-\alpha) + \alpha\gamma(1-\beta)\right] + \mu\gamma(1-\beta)\left[(\epsilon-1)(1-\alpha) + \alpha\gamma(1-\beta)\right] > 0.$$

[2]Even if $\frac{\alpha a \rho}{1-\alpha} < L^N$ does not hold, the following discussion does not lose its generality.

From $\epsilon > 1$, the $\frac{\dot{z_{SS}}}{z_{SS}} = 0$ schedule is a straight line through the negative intercept with a positive slope in Figure 2. There is a unique steady state at E. The steady state value is as follow:

$$z_{SS}^* = \frac{[\mu - \alpha(\mu + \Phi)]\, L^N - a\alpha\rho\Phi}{\alpha\Gamma_{SS}}. \tag{34}$$

Because the steady state E is unstable and z_{SS} is jump variable, z_{SS} jumps to z_{SS}^* at the initial time. In the steady state, the relationship between the number of final goods, n, and the wage rate in the North, w_N becomes as follow:

$$w_N = \epsilon^{-1} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{\Phi}} (z_{SS}^*)^{\frac{-1+\mu}{\Phi}} n^{\frac{\mu(1-\alpha)}{\alpha\Phi}}. \tag{35}$$

This equation represents the stable saddle path. Therefore, the dynamic equilibrium follows the stable saddle path shown by the solid locus with arrow (I) in the Figure 1.

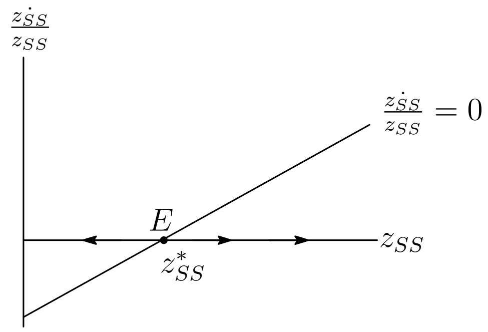

Figure 2: Phase diagram about z_{SS}

3.2 Dynamic behavior under *North Regime*

In *North Regime*, the equilibrium conditions are (6), the no-arbitrage condition, (19), the free-entry condition, (23), and the labor market clearing condition, (24). From the labor market clearing condition, I can derive the differential equation for the number of final goods, n, as follows:

$$\dot{n} = \frac{n}{a}\left[L^N - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} w_N^{\frac{-1}{1-\mu}}\right]. \tag{36}$$

Using (6), (19), (23), and (36), I can obtain the differential equation for the wage rate in the North, w_N, in the same way as (31). Then, the differential equation for the wage rate in the North is

$$\dot{w_N} = \left(\rho + \frac{L^N}{a}\right) w_N - \frac{1}{a} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu}{1-\mu}}. \tag{37}$$

These two equations, (36) and (37), constitute the dynamic system of *North Regime*. The equation for the $\dot{n} = 0$ locus is given by the following:

$$w_N = \left(\frac{1}{L^N}\right)^{1-\mu} \alpha n^{\frac{\mu(1-\alpha)}{\alpha}}. \tag{38}$$

The $\dot{n} = 0$ locus is concave and $\lim_{n\to\infty} \frac{\partial w_N}{\partial n} = 0$. On the other hand, the equation for the $\dot{w_N} = 0$ locus is given by:

$$w_N = \left(a\rho + L^N\right)^{-(1-\mu)} \alpha^{\mu} n^{\frac{\mu(1-\alpha)}{\alpha}}. \tag{39}$$

The $\dot{w_N} = 0$ locus has a similar shape to the $\dot{n} = 0$ locus. Thus, the origin is the only intersection point of the two curves $\dot{n} = 0$ and $\dot{w_N} = 0$. Figure 3 depicts the phase diagram for this system on the (n, w_N) plane when the inequality $\frac{\alpha a\rho}{1-\alpha} < L^N$ holds.

Similarly to *FDI Regime*, I can obtain the stable saddle path (See Appendix). In the steady state, the relationship between the number of final good, n, and the wage rate in the North, w_N becomes as follow:

$$w_N = \left(\frac{A}{\alpha}\right)^{-\mu} \left(\frac{(\alpha - \mu + \alpha\mu)L^N + a\alpha\rho}{\alpha\left\{1 - \mu(1-\alpha)\right\}}\right)^{-1+\mu} n^{\frac{\mu(1-\alpha)}{\alpha}}.$$

This equation represents the stable saddle path. Therefore, the dynamics in *North Regime* follows the stable saddle path shown by the arrow (II) in the Figure 3.

3.3 Dynamic behavior under the *Outsourcing Regime*

I focus on the economy in which the final good firms outsource the production of intermediate goods to the Southern firms and produce in the North. The equilibrium conditions are (6), the no-arbitrage condition, (20), the free-entry condition, (23), and the labor market clearing condition, (26). In the same way as in the former section, I can derive the $\dot{n} = 0$ locus and $\dot{w_N} = 0$ locus. From the labor market clearing condition, I can derive the differential equation for the number of firms, n, as follows:

$$\dot{n} = \frac{n}{a}\left[L^N - \alpha^{\frac{1}{1-\mu}}(1-\beta) n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} w_N^{\frac{-(1-\beta\mu)}{1-\mu}} \tau^{\frac{-\beta\mu}{1-\mu}}\right]. \tag{40}$$

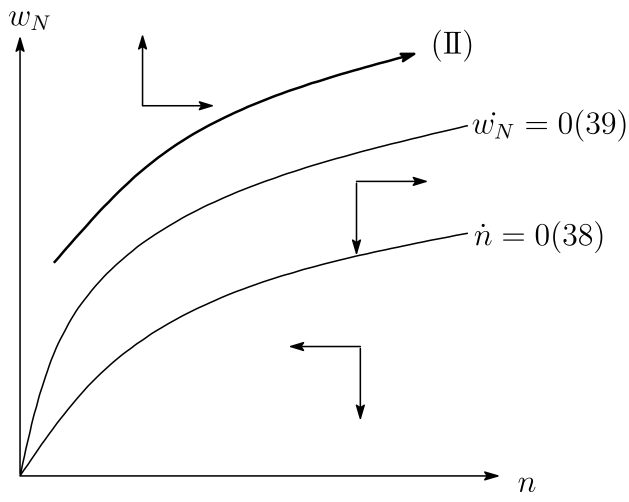

Figure 3: Phase diagram under *North Regime*

In the same way as (31), I can obtain the differential equation for the wage rate in the North, w_N, as follows:

$$\dot{w_N} = \left(\rho + \frac{L^N}{a}\right) w_N - \frac{1-\alpha\beta}{a} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu(1-\beta)}{1-\mu}} \tau^{\frac{-\beta\mu}{1-\mu}}. \tag{41}$$

These two equations, (40) and (41), constitute the dynamic system of *Outsourcing Regime*. Then, the equation for the $\dot{n} = 0$ locus is represented by:

$$w_N = \left(\frac{1-\beta}{L^N}\right)^{\frac{1-\mu}{1-\beta\mu}} \alpha^{\frac{1}{1-\beta\mu}} \tau^{\frac{-\beta\mu}{1-\beta\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\beta\mu)}}. \tag{42}$$

The $\dot{n} = 0$ locus is concave and $\lim_{n\to\infty} \frac{\partial w_N}{\partial n} = 0$. On the other hand, the equation for the $\dot{w_N} = 0$ locus is given by:

$$w_N = \left(\frac{1-\alpha\beta}{a\rho + L^N}\right)^{\frac{1-\mu}{1-\beta\mu}} \alpha^{\frac{\mu}{1-\beta\mu}} \tau^{\frac{-\beta\mu}{1-\beta\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\beta\mu)}}. \tag{43}$$

The $\dot{w_N} = 0$ locus has a similar shape to the $\dot{n} = 0$ locus. Thus, the origin is the only intersection point of the two curves, $\dot{n} = 0$ and $\dot{w_N} = 0$. Figure 4 depicts the phase diagram for this system on the $(n,\, w_N)$ plane. When $\frac{\alpha a\rho}{1-\alpha} < L^N$ holds, the $\dot{w_N} = 0$ locus is above the $\dot{n} = 0$ locus for any $w_N > 0$.

Similarly to former regimes, I can obtain the stable saddle path (See Appendix). In the steady state, the relationship between the number of final good, n, and the wage rate in the North, w_N becomes as follow (See appendix):

$$z^*_{NS} = \frac{\left[(\alpha - \mu + \alpha\mu(1-\beta))L^N + (1-\beta\mu)a\alpha\rho\right]}{\alpha\Gamma_{NS}}.$$

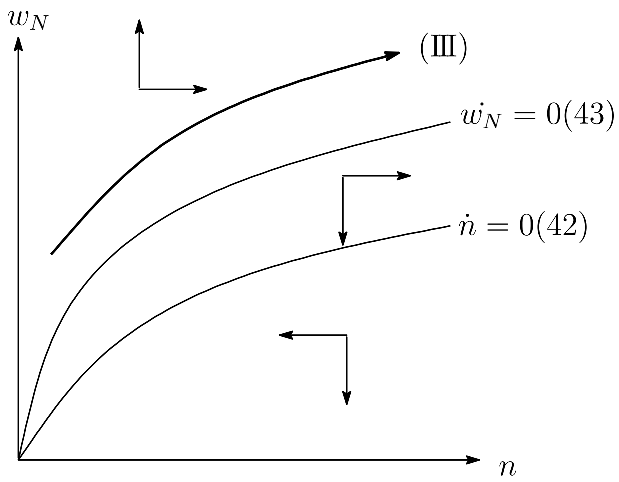

Figure 4: Phase diagram under *Outsourcing Regime*

This equation represents the stable saddle path. Therefore, the dynamics in *Outsourcing Regime* follows the stable saddle path shown by the arrow (Ⅲ) in the Figure 4.

3.4 Boundaries between the three regimes

The final good firms have to decide where they produce the final goods and outsource the production of the intermediate goods. Then, the final good firms compare levels of the profit in each regime and choose a regime where they can earn the largest profit. I investigate the condition under which the profit in *North Regime* become the same as the profits in *Outsourcing Regime*; that is $\pi_{N,N} = \pi_{N,S}$. From (14) and (15), the boundary condition between *North Regime* and *Outsourcing Regime* is given by:

$$w_N = \tau. \tag{44}$$

Next, I show the condition under which the profit in *North Regime* become the same as the profits in *FDI Regime*; that is $\pi_{N,N} = \pi_{S,S}$. From (14) and (17), the boundary condition between *North Regime* and *FDI Regime* becomes as follow:

$$w_N = A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}, \tag{45}$$

where $\Psi = 1 - \gamma(1 - \beta)$ and $0 < \Psi < 1$. Finally, from (15) and (17), I investigate the boundary condition between *Outsourcing Regime* and *FDI Regime* as follows:

$$w_N = A^{\frac{1}{\Psi-\beta}} \tau^{\frac{-\beta}{\Psi-\beta}} \epsilon^{\frac{\gamma}{1-\gamma}}. \tag{46}$$

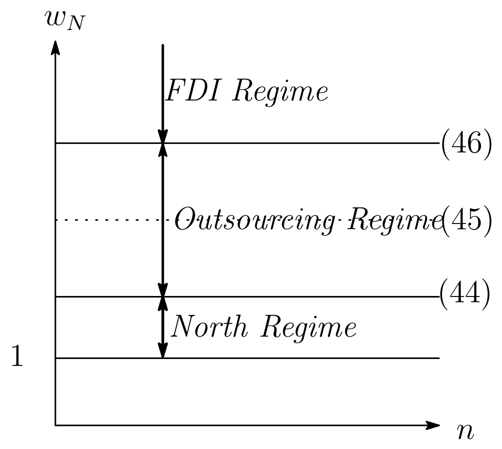

Figure 5: $\tau < A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$

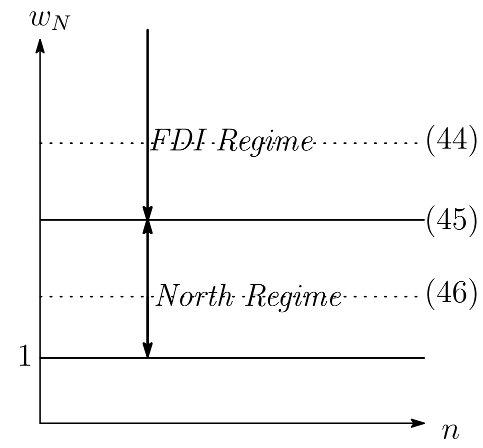

Figure 6: $\tau \geq A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$

From the three boundary conditions, (44), (45), and (46), I can obtain the following proposition.

Proposition 1. *When the transportation cost is small and the management cost is large, that is,* $\tau < A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$*, the economy has the three regimes, North Regime, Outsourcing Regime, and FDI Regime On the other hand, when the transportation cost is large and the management cost is small, that is,* $\tau \geq A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$*, the economy has only two regimes, North Regime and FDI Regime.*

Proof: See the Appendix.

Thus, when the inequality, $\tau < A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$, holds, the economy has three regimes, that is, *North Regime*, *Outsourcing Regime*, and *FDI Regime*. When the wage rate in the North is low, Figure 5 shows that the economy is in *North Regime*. As the wage rate in the North increases, the economy turns into *Outsourcing Regime*. Finally, a further increase in the wage rate in the North get the economy turn into *FDI Regime*. When the trade cost is sufficiently small, outsourcing become easier for the final good firms. Increases in the management cost and in the parameter of cost function, A, mean that the cost of FDI increases compared to outsourcing. Then, the final good firms outsource the intermediate goods to the Southern firms. On the other hand, when the inequality, $\tau \geq A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$, holds, Figure 6 shows that the economy has only a *North Regime* and an *FDI Regime*. When the wage rate in the North is high, the economy is in *FDI Regime*.

Lemma 2. *When* $\tau < A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$ *holds, a decrease in the trade cost and an increase in* *the management cost enlarge the area of Outsourcing Regime. On the other hand, when* $\tau \geq A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$ *holds, an increase in the management cost enlarges the area of North Regime and an increase in the trade cost does not affect the sizes of those regimes.*

Proof: See the Appendix.

When $\tau < A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$, the economy has the three Regimes. A decrease in the trade cost means that the cost of outsourcing decreases and firms outsource the intermediate goods to the foreign countries easier. An increase in the management cost means that the cost of FDI is increasing and the firms prefer outsourcing to FDI. When $\tau \geq A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$, the economy has the two Regimes. Then, if the management cost increases, the cost of FDI increases and final good firms prefer *North Regime* to *FDI Regime*. On the other hand, an increase in the trade cost does not affect the behavior of final good firms. Because in *North Regime* and *FDI Regime*, final good firms does not freight the intermediate goods to cross the border, the final good firms does not incur the trade cost.

3.5 The equilibrium path from *North Regime* to another regime

In the previous subsections, I explored the dynamic behaviors in *North Regime*, *Outsourcing Regime*, and *FDI Regime*, and showed the boundary conditions among the regimes. Now, I integrate the three phase diagrams of these regimes into one. When the inequality, $\tau < A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$, holds, the economy has the three regimes, that is, *North Regime*, *Outsourcing Regime* and *FDI Regime*. I can depict the phase diagram for the system on the (n, w_N) plane. Figures 1, 3, 4, and 5 are integrated into Figure 7. In Figure 7, the arrow $ABCD$ show the traditional dynamics. These results are stated as the following proposition.

Proposition 2. *Suppose that the economy is initially in North Regime. When the economy has the three regimes, that is, when* $\tau < A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$, *and* $\frac{\alpha a\rho}{1-\alpha} < L^N$ *holds, there exists an equilibrium path along which the economy enters into FDI Regime, passing through Outsourcing Regime.*

Suppose that the initially number of firms is sufficient small. In this economy, the firms have rational expectation. The firms expect that the economy become

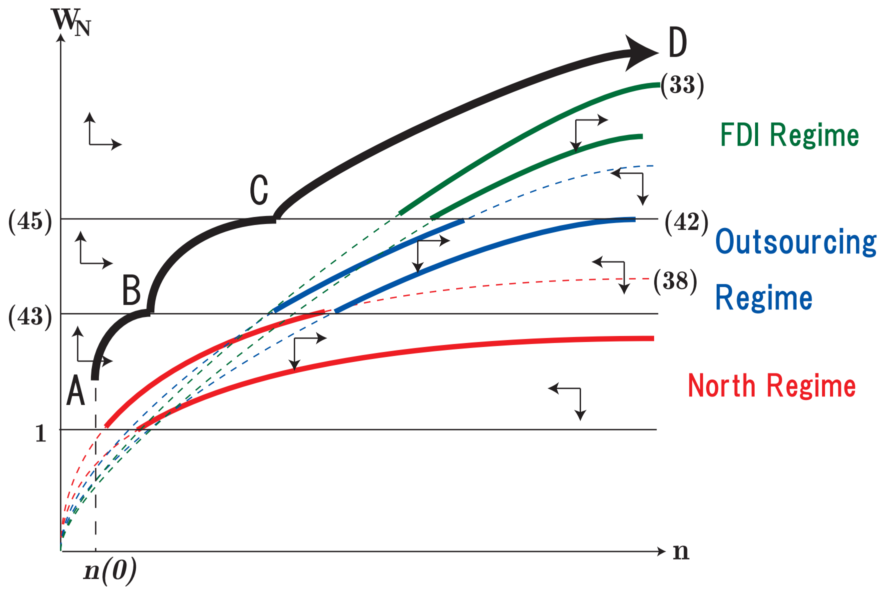

Figure 7: Phase diagram when $\tau < A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$ and $\frac{\alpha a\rho}{1-\alpha} < L^N$

FDI Regime and the economy follows the stable saddle path in *FDI Regime* in the future. To follow the stable saddle path in *FDI Regime*, the economy follows the divergent path in *Outsourcing Regime* to reach the intersection point C of the stable saddle path with the boundary line between *Outsourcing Regime* and *FDI Regime*. Then, to follow the divergent path in *Outsourcing Regime*, the economy follows the divergent path in *North Regime* to reach the intersection point B of the divergent path in *Outsourcing Regime* with the boundary line between *North Regime* and *Outsourcing Regime*.

On the other hand, when the inequality, $\tau \geq A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$, the economy has two regimes, *North Regime* and *FDI Regime*. I can depict the phase diagram similarly and Figures 1, 3, 4, and 6 are integrated into Figure 8. In Figure 8, the arrow EFG show the traditional dynamics. These results are stated as the following proposition.

Proposition 3. *When the economy has the two regimes, that is $\tau \geq A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$, and $\frac{\alpha a\rho}{1-\alpha} < L^N$ holds, there exists an equilibrium path along which the economy evolves from North Regime to FDI Regime.*

Suppose that the initially number of firms is sufficient small. The firms expect that the economy becomes *FDI Regime* and the economy follows the stable saddle

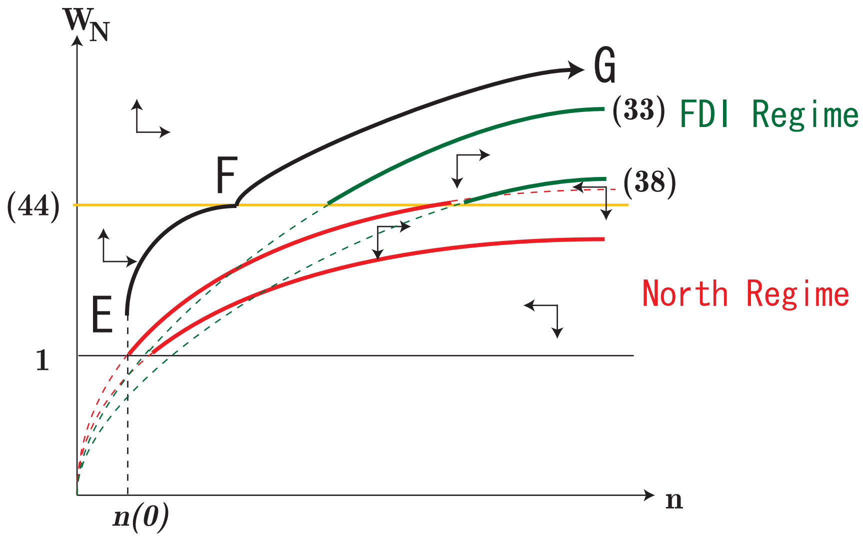

Figure 8: Phase diagram when $\tau \geq A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$ and $\frac{\alpha a\rho}{1-\alpha} < L^N$

path in *FDI Regime* in the future. To follow the stable saddle path in *FDI Regime*, the economy follows the divergent path in *North Regime* to reach the intersection point F of the stable saddle path with the boundary line between *North Regime* and *FDI Regime*.

4 Conclusion

This paper constructs a North-South endogenous growth model to investigate how the organizational forms of final good firms transform. The final good firms have four strategies, based on the regions in which they produce the final good and to which outsource the production of the intermediate goods. This paper shows why the organizational forms of the final good firms change as the economy develops and why, decade ago, the final good firms did not adopt a strategy of obtaining intermediate goods from abroad. Suppose that, initially, the final good firms obtain the intermediate goods from Northern firms and produce in the North. When the trade cost is low, as the economy develops, final good firms produce the final good in the North and outsource the production of intermediate goods to the Southern firms. Then, as the economy develops more, the final good firms produce the final good in the South and outsource the production of the intermediate goods to the Southern firms. When the trade cost is high, the final good firms never outsource the production of intermediate goods to the foreign country.

A Appendix

A.1 Derivation of the profit functions

First, I focus on the case where the final good firms are in the North. Then, I consider the profit-maximization problem of the final good producers in the North. Thus, the final good producer i maximizes profits as follows:

$$\pi_{N,k}(i) = X_{N,k}^{\mu-\alpha} x_{N,k}(i)^{\alpha} - w_N^{1-\beta}(p_{N,k}^m)^{\beta} x_{N,k}(i),\ k \in \{N, S\},\ i \in [0, n], \tag{A.1}$$

where $\pi_{N,k}$ is the operating profit of the final good firms in the North when they obtain the intermediate goods from country k. The profit-maximizing output is given by:

$$x_{N,k}(i) = \alpha^{\frac{1}{1-\alpha}} X_{N,k}^{\frac{\alpha-\mu}{\alpha-1}} w_N^{\frac{1-\beta}{\alpha-1}} (p_{N,k}^m)^{\frac{\beta}{\alpha-1}}, k \in \{N, S\}. \tag{A.2}$$

By substituting (A.2) into (3), I obtain:

$$X_{N,k} = n^{\frac{1-\alpha}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} w_N^{\frac{-(1-\beta)}{1-\mu}} (p_{N,k}^m)^{\frac{-\beta}{1-\mu}}, k \in \{N, S\}. \tag{A.3}$$

Using (A.2) and (A.3), I can obtain the following output and the profit functions when the final good firms produce the final good in the North:

$$x_{N,k}(i) = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} w_N^{\frac{-(1-\beta)}{1-\mu}} (p_{N,k}^m)^{\frac{-\beta}{1-\mu}}, \tag{A.4}$$

$$\pi_{N,k} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu(1-\beta)}{1-\mu}} (p_{N,k}^m)^{\frac{-\beta\mu}{1-\mu}} (1-\alpha),\ k \in \{N, S\},\ i \in [0, n]. \tag{A.5}$$

The profits of the final good producers decrease with the wage rate in the North w_N and the number of the final good firms n.

Next, I turn to the case where the final good firms are in the South and derive the profit function of the final good firms similarly to the above case. The final good producer i maximizes the following profits:

$$\pi_{S,k}(i) = X_{S,k}^{\mu-\alpha} x_{S,k}(i)^{\alpha} - A(\epsilon w_N)^{\gamma(1-\beta)}(p_{S,k}^m)^{\beta} x_{S,k},\ k \in \{N, S\},\ i \in [0, n]. \tag{A.6}$$

The profit-maximizing output is given by:

$$x_{S,k}(i) = \left(\frac{A}{\alpha}\right)^{\frac{1}{\alpha-1}} X_{S,k}^{\frac{\alpha-\mu}{\alpha-1}} (\epsilon w_N)^{\frac{\gamma(1-\beta)}{\alpha-1}} (p_{S,k}^m)^{\frac{\beta}{\alpha-1}}, k \in \{N, S\}. \tag{A.7}$$

By substituting (A.7) into (3), I obtain:

$$X_{S,k} = n^{\frac{1-\alpha}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-1}{1-\mu}} (\epsilon w_N)^{\frac{-\gamma(1-\beta)}{1-\mu}} (p^m_{S,k})^{\frac{-\beta}{1-\mu}}, k \in \{N, S\}. \tag{A.8}$$

Using (A.7) and (A.8), I can obtain the following output and the profit functions when the final good firms produce the final good in the North:

$$x_{S,k}(i) = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-1}{1-\mu}} (\epsilon w_N)^{\frac{-\gamma(1-\beta)}{1-\mu}} (p^m_{S,k})^{\frac{-\beta}{1-\mu}}, \tag{A.9}$$

$$\pi_{S,k} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} (p^m_{S,k})^{\frac{-\beta\mu}{1-\mu}} (1-\alpha),\ k \in \{N, S\},\ i \in [0, n]. \tag{A.10}$$

The profits of the final good producers decrease with the wage rate in the North w_N and the number of the final good firms n.

A.2 Derivation of the labor market equilibrium condition

To derive the labor demand of the final goods sector and the intermediate goods sector when the final good firms are in the North, I apply Shepard's lemma on (11) and use (A.4), which yields:

$$\begin{aligned} L^F_{N,k} &= \int_0^n \frac{\partial c_N(w_N, p^m_{N,k})}{\partial w_N} x_{N,k}(i) di \\ &= n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} (1-\beta) \alpha^{\frac{1}{1-\mu}} w_N^{\frac{\beta\mu-1}{1-\mu}} (p^m_{N,k})^{\frac{-\beta\mu}{1-\mu}}, \end{aligned} \tag{A.11}$$

$$\begin{aligned} L^M_{N,k} &= \int_0^n \frac{\partial c_N(w_N, p^m_{N,k})}{\partial p^m_{N,k}} x_{N,k}(i) di \\ &= n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \beta \alpha^{\frac{1}{1-\mu}} w_N^{\frac{-\mu(1-\beta)}{1-\mu}} (p^m_{N,k})^{\frac{\mu(1-\beta)-1}{1-\mu}},\ k \in \{N, S\}. \end{aligned} \tag{A.12}$$

Then, the production of one unit of the homogeneous good requires one unit of labor. Thus, production of the homogeneous good equals the labor demand. I can write the production of the homogeneous good by using the budget constraint, (4), (A.3), and (A.4).

$$\begin{aligned} y_{N,k} &= E - \left[\int_0^n P(i) x(i) di\right]^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \\ &= E - \left[\int_0^n x(i)^\alpha X_{N,k}^{\frac{\alpha(\mu-\alpha)}{\alpha-1}} di\right] \\ &= E - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu(1-\beta)}{1-\mu}} (p^m_{N,k})^{\frac{-\beta\mu}{1-\mu}},\ k \in \{N, S\}. \end{aligned} \tag{A.13}$$

On the other hand, to derive the labor demand of the final goods sector and the intermediate goods sector when the final good firms are in the South, I apply Shepard's lemma on (12) and use (A.9), which yields

$$
\begin{aligned}
L_{S,k}^{F} &= \int_0^n \frac{\partial c_S(w_N, p_{S,k}^m)}{\partial(\epsilon w_N)} x_{S,k}(i)di \\
&= n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha\gamma(1-\beta) \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}-1} (p_{S,k}^m)^{\frac{-\beta\mu}{1-\mu}}, \qquad \text{(A.14)}
\end{aligned}
$$

$$
\begin{aligned}
L_{S,k}^{F,S} &= \int_0^n \frac{\partial c_S(w_N, p_{S,k}^m)}{\partial(w_S)} x_{S,k}(i)di \\
&= n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha(1-\gamma)(1-\beta) \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} (p_{S,k}^m)^{\frac{-\beta\mu}{1-\mu}}, \qquad \text{(A.15)}
\end{aligned}
$$

$$
\begin{aligned}
L_{S,k}^{M} &= \int_0^n \frac{\partial c_S(w_N, p_{S,k}^m)}{\partial p_{S,k}^m} x_{S,k}(i)di \\
&= n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha\beta \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu(1-\beta)}{1-\mu}} (p_{S,k}^m)^{\frac{-\beta\mu}{1-\mu}-1}, \ k \in \{N, S\}. \qquad \text{(A.16)}
\end{aligned}
$$

Then, I can obtain the production of the homogeneous good by using the budget constraint, (4), (A.8), and (A.9).

$$
\begin{aligned}
y_{S,k} &= E - \left[\int_0^n P(i)x(i)di\right]^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \\
&= E - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu(1-\beta)}{1-\mu}} (p_{S,k}^m)^{\frac{-\beta\mu}{1-\mu}}, \ k \in \{N, S\}. \qquad \text{(A.17)}
\end{aligned}
$$

A.3 Derivation of the differential equation (31)

I derive the differential equation for the wage rate in the North. I differentiate the Free entry condition (23) by t as follows:

$$
\dot{v_{SS}} = \frac{a\dot{w_N}n - aw_N\dot{n}}{n^2}. \qquad \text{(A.18)}
$$

Substituting (6) and (A.18) into (21), I obtain:

$$
\rho\frac{aw_N}{n} = \pi_{SS} + \frac{a\dot{w_N}}{n} - \frac{aw_N}{n}\frac{\dot{n}}{n}. \qquad \text{(A.19)}
$$

Then, I substitute (17) and (30) into (A.19) and I can obtain the differential equation for the wage rate in the North as follows:

$$
\begin{aligned}
\dot{w_N} &= (\rho + \frac{L^N}{a})w_N - \frac{1-\alpha}{a}(\frac{A}{\alpha})^{\frac{-\mu}{1-\mu}}(\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \\
&- \frac{\alpha\gamma(1-\beta)}{a\epsilon}(\frac{A}{\alpha})^{\frac{-\mu}{1-\mu}}(\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \\
&= (\rho + \frac{L^N}{a})w_N - \frac{1}{a}(\frac{A}{\alpha})^{\frac{-\mu}{1-\mu}}(\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}}(1-\alpha+\frac{\alpha\gamma(1-\beta)}{\epsilon}). \qquad \text{(A.20)}
\end{aligned}
$$

A.4 Proof of the stable saddle paths in ***North Regime*** and ***Outsourcing Regime***

First, I show the proof of the stable saddle path in *North Regime*. Suppose that I define $z_{NN} \equiv n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} w_N^{\frac{-1}{1-\mu}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}}$. Then, differentiating z_{NN} with respect to time t and using (36) and (37) yields the following differential equation for z_{NN}:

$$
\begin{aligned}
\frac{\dot{z_{NN}}}{z_{NN}} &= \frac{\mu(1-\alpha)}{\alpha(1-\mu)}\left(\frac{\dot{n}}{n}\right) - \frac{1}{1-\mu}\left(\frac{\dot{w_N}}{w_N}\right) \\
&= \left[\left(\frac{(\mu-\alpha)-\alpha\mu}{\alpha(1-\mu)}\right)\frac{L^N}{a} - \frac{\rho}{1-\mu}\right] + \frac{1-\mu(1-\alpha)}{1-\mu}\left(\frac{z_{NN}}{a}\right).
\end{aligned}
$$

From $\mu < \alpha$, the first parentheses is negative. Then, the $\frac{\dot{z_{NN}}}{z_{NN}} = 0$ schedule is a straight line through the negative intercept with slope $\frac{\alpha}{1-\mu} > 0$ in figure 9. There is a unique steady state at $E^{'}$. The steady state value is as follow:

$$
z_{NN}^{*} = \frac{(\alpha - \mu + \alpha\mu)L^N + a\alpha\rho}{\alpha\left\{1-\mu(1-\alpha)\right\}}. \qquad \text{(A.21)}
$$

Because the steady state $E^{'}$ is unstable and z_{NN} is jump variable, z_{NN} jumps to z_{NN}^{*} at the initial time. In the steady state, the relationship between the number of final goods, n, and the wage rate in the North, w_N, becomes as follow:

$$
w_N = \left(\frac{A}{\alpha}\right)^{-\mu}\left(\frac{(\alpha-\mu+\alpha\mu)L^N + a\alpha\rho}{\alpha\left\{1-\mu(1-\alpha)\right\}}\right)^{-1+\mu} n^{\frac{\mu(1-\alpha)}{\alpha}}. \qquad \text{(A.22)}
$$

Therefore, the dynamic equilibrium follows the stable saddle path shown by the solid locus with arrow II in the Figure 3.

Next, I show the saddle path stability in *Outsourcing Regime* in the same way. Suppose I define $z_{NS} \equiv n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} w_N^{\frac{-1+\beta\mu}{1-\mu}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} \tau^{\frac{-\beta\mu}{1-\mu}}$. Then, differentiating z_{NS} with respect to time t and using (40) and (41) yields the following differential equation

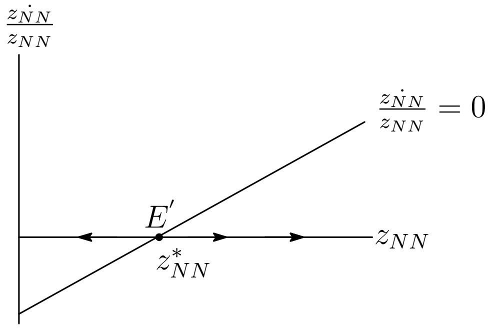

Figure 9: Phase diagram about z_{NN}

for z_{NS}:

$$\frac{\dot{z_{NS}}}{z_{NS}} = \frac{\mu(1-\alpha)}{\alpha(1-\mu)}\left(\frac{\dot{n}}{n}\right) - \frac{1-\beta\mu}{1-\mu}\left(\frac{\dot{w_N}}{w_N}\right)$$
$$= \left[\left(\frac{(\mu-\alpha)-\alpha\mu(1-\beta)}{\alpha(1-\mu)}\right)\frac{L^N}{a} - \frac{(1-\beta\mu)\rho}{1-\mu}\right] + \frac{\Gamma_{NS} z_{NS}}{a(1-\mu)},$$

where

$$\Gamma_{NS} \equiv (1-\beta\mu)(1-\alpha\beta) - \mu(1-\alpha)(1-\beta)$$
$$> (1-\alpha)(1-\beta\mu-\mu(1-\beta)) = (1-\alpha)(1-\mu) > 0.$$

From $\mu < \alpha$ and $1-\alpha < 1-\alpha\beta$, the $\frac{\dot{z_{NS}}}{z_{NS}} = 0$ schedule is a straight line through the negative intercept with a positive slope in Figure 10. There is a unique steady state at E''. The steady state value is as follow:

$$z^*_{NS} = \frac{\left[(\alpha-\mu+\alpha\mu(1-\beta))L^N + (1-\beta\mu)a\alpha\rho\right]}{\alpha\Gamma_{NS}}. \tag{A.23}$$

Because the steady state E'' is unstable and z_{NS} is jump variable, z_{NS} jumps to z^*_{NS} at initial time. In the steady state, the relationship between the number of final goods, n, and the wage rate in the North, w_N, becomes as follow:

$$w_{NS} = \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\beta\mu}} (z^*_{NS})^{\frac{-1+\mu}{1-\beta\mu}} \tau^{\frac{\beta\mu}{1-\beta\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\beta\mu)}}. \tag{A.24}$$

Therefore, the dynamic equilibrium follows the stable saddle path shown by the arrow Ⅲ in the Figure 4.

A.5 Proof of proposition 1

Suppose that the boundary condition between the *North Regime* and the *FDI Regime* is bigger than the condition between the *North Regime* and the *Outsourcing Regime*

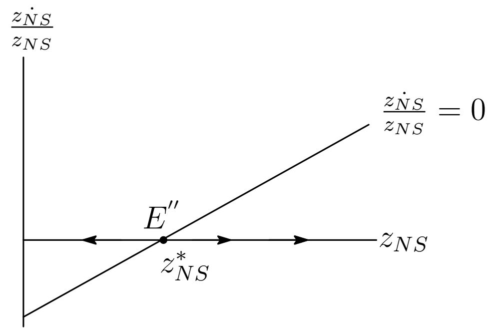

Figure 10: Phase diagram about z_{NS}

in terms of the wage rate in the North, w_N. Using (44) and (45), this inequality holds: $\tau < A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$. Then, I compare the boundary condition between the *North Regime* and the *FDI Regime*, (45), to the condition between the *Outsourcing Regime* and the *FDI Regime*, (45), as follows:

$$\begin{aligned}
&A^{\frac{1}{\Psi-\beta}}\tau^{\frac{-\beta}{\Psi-\beta}}\epsilon^{\frac{\gamma}{1-\gamma}} - A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}} \\
>&A^{\frac{1}{\Psi-\beta}}\epsilon^{\frac{\gamma}{1-\gamma}}\left[A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}\right]^{\frac{-\beta}{\Psi-\beta}} - A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}} \\
=&A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}\left[A^{\frac{1}{\Psi-\beta}}\epsilon^{\frac{\gamma}{1-\gamma}}\left\{A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}\right\}^{\frac{-\Psi}{\Psi-\beta}} - 1\right] \\
=&0,
\end{aligned}$$

where I derive the above inequality to use $\tau < A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$. If this inequality holds, the economy experiences the three regimes. On the other hand, if the inverse inequality holds, the economy has only two regimes, that is, the *North Regime* and the *FDI Regime*.

A.6 Proof of Lemma 2

When $\tau < A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$ holds, the boundary line is (44) and (46). Differentiating (46) with respect to τ and ϵ yields

$$\frac{\partial}{\partial\tau}\left(A^{\frac{1}{\Psi-\beta}}\tau^{\frac{-\beta}{\Psi-\beta}}\epsilon^{\frac{\gamma}{1-\gamma}}\right) = \frac{\beta}{\Psi-\beta}A^{\frac{1}{\Psi-\beta}}\tau^{\frac{-\Psi}{\Psi-\beta}}\epsilon^{\frac{\gamma}{1-\gamma}} < 0, \tag{A.25}$$

$$\frac{\partial}{\partial\epsilon}\left(A^{\frac{1}{\Psi-\beta}}\tau^{\frac{-\beta}{\Psi-\beta}}\epsilon^{\frac{\gamma}{1-\gamma}}\right) = \frac{\gamma}{1-\gamma}A^{\frac{1}{\Psi-\beta}}\tau^{\frac{-\beta}{\Psi-\beta}}\epsilon^{\frac{\gamma}{1-\gamma}-1} > 0. \tag{A.26}$$

When $\tau \geq A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}$ holds, the boundary line is (45). Differentiating (45) with respect to ϵ yields

$$\frac{\partial}{\partial\epsilon}\left(A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}\right) = \frac{1-\Psi}{\Psi}A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}-1} > 0. \tag{A.27}$$

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