Optimal Structure of Monetary Policy Committees

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Abstract

This paper explores an optimal personnel organization problem of monetary policy committees. First, I construct an analytically tractable model for monetary policy analysis which starts from decision-making in the monetary policy committee. Using the model, I investigate the relationship between preference heterogeneity among the committee members and the optimal structure of the monetary policy committee. The result shows that it is optimal in general cases to appoint not only inflation-minded (hawkish) persons but also output-minded (dovish) persons. This is a justification for the fact that the actual monetary policy committees (e.g. MPC of Bank of England and FOMC) usually consist of both type members as the empirical researches suggest.

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1 Introduction

How should the monetary policy committees be organized? This paper addresses this problem. Since about a decade ago, the central banks of many countries have established the formal committees for decision-making on monetary policy one after another: England, Japan, Sweden, Brazil and so on. This trend of central banking by committees brings some important problems on monetary policy design to macroeconomists. \footnote{Blinder (2004, 2007) provides a brief survey of this issue.} The next three are representative ones. Why have the committees replaced the single policy makers in most central banks? How should the committees collect the members’ views and adjust the differences of their opinions? Who should be chosen for members of the committees? In this paper, I focus on the third. \footnote{I will show that the answer for the third solves the first.}

The question is significant in terms of the actual institutions. In most countries, the procedure for appointing members of the monetary policy committee is one of legal mandates. The Monetary Policy Committee of Bank of England consists of nine members. Five are from the inside of Bank of England and four from the outside. FOMC of the Federal Reserve System consists of twelve members. Five are district bank presidents, the insiders, and seven are politically appointed governors, the outsiders. Thus, the optimal personnel organization problem of monetary policy committees is one of the objects of monetary policy design. This paper analyze it in view of information imperfectness and coordination behavior of the monetary policy committee.

There are empirical studies on a related topic which provide the interesting facts of the actual monetary policy committees. The monetary policy committees often consist of the outsiders and insiders and the outsiders are apt to prefer more flexible monetary policy than the insiders. Bhattacharjee and Holly (2006) reports that in the Monetary Policy Committee of Bank of England, the outsiders tend to be more output-minded than the insiders according to the voting records of the meetings. Meade and Sheets (2005) find the empirical fact that in FOMC, the district bank presidents (the insiders) seems more inflation-minded than the governors (the outsiders) on average.

Considering the facts above, this paper gives the next answer for the optimal personnel organization problem. It is optimal in general cases to appoint not only inflation-minded (hawkish) persons but also output-minded (dovish) persons. The mechanism for this is as follows. Under imperfect information, along with coordination behavior, the monetary policy committee fails to set the optimal level of nominal interest rate corresponding to the economic state. It causes inefficient volatility of the demand side of the macro economy. This also brings excessive volatility of inflation through the AS relation. The
output-minded committee members play the role of mitigating these effects, while the inflation-minded members balance monetary policy by stabilizing expected and current inflation.

This paper’s argument is related to another problem. Despite the facts above, the popular belief, which may be based on the established theory, is that central bankers should concern primarily with inflation stabilization. Since the seminal work by Rogoff (1985), many theoretical studies focus on the optimal delegation of monetary policy to single policy makers. In various economic environments, they suggest that appointing conservative central bankers, who place on higher weights on the real objectives (e.g. output and unemployment) relative to inflation than society, improves social welfare under the discretionary policy regimes. ³ Adding to the theoretical studies, the discussions on practice of central banking usually emphasis on importance of conservatism and it is widely regarded as common sense on contemporary central banking.

So, why is the actual monetary policy not delegated to only (single) conservative central bankers? This papers’ main result answers this. Under imperfect information, to accommodate control error of the demand side of the economy, it is desirable in view of social welfare to include output-minded persons in the monetary policy committee to some extent.

There is a few existing studies on the appointments of the monetary policy committees. Waller (1992) analyzes the endogenous appointment of the monetary policy committee by modeling the situation that the dovish and hawkish political parties bargain over designating the monetary policy committee. Faust (1996) tries to explain theoretically why the FOMC is a compound of the district bank presidents and politically appointed governors by showing that preference heterogeneity among the FOMC members lowers inefficient inflation under peculiar voting structure. However, as Blinder (2007) claims, these existing literature does not clarify why the monetary policy committees replace single policy makers and why the actual central banks do not just follow the Rogoff’s (1985) suggestion and appoint only a single conservative central banker. This paper’s result gives an answer for these questions by showing the merit of preference heterogeneity among the members and especially, the welfare-improving role of the output-minded members.

In the methodological aspect, this paper provides a way to model monetary policy by committee easily. To conduct a simple welfare analysis, I construct a tractable forward-looking model that starts from decision-making of monetary policy committees. The model consists of two parts. One part is a macroeconomic model which gives macro-

³See, for examples, Adam and Billi (2008), Svensson (1997), Vestin (2006), Walsh (2003a) and etc.
conomic consequences of decision-making by committees and their welfare evaluation. I adopt a basic New Keynesian model as the underlying macroeconomic model since it has a rigorous micro foundation and is used for a benchmark model for contemporary monetary policy analysis. However, according to the analysis later, it will appear that the approach in this paper is applicable to a wide class of forward-looking macroeconomic models. The other part is a microeconomic model which describes the process of decision-making in the committee. I assume that the committee members play a variant of the beauty contest game, which was exploited by Morris and Shin (2002), in the decision-making on setting nominal interest rate. This simple structure makes it possible to model a strategic situation in the monetary policy committee and shows that the framework of this paper applies to the class of the games that have linear equilibrium strategies as in many applied game-theoretic studies.

In the literature, several works deal with monetary policy by committees. Gelrach-Kristen (2006) uses a (backward-looking) traditional Keynesian model and compares the macroeconomic consequences of alternative aggregation rules of the committee members’ votes on economic conditions, which are honestly declared according to their information without any strategic manipulations. Sibert (2003) and Mihov and Sibert (2006) use a Lucas type Phillips curve and analyze the role of reputation for monetary policy by committees in view of communication with the private sector. Therefore, the technical innovation of this paper is that it gives the way to analyze the game structures in monetary policy committees and connect it simply to various contemporary macroeconomic models.

The rest of this paper is organized as follows. Section 2 sets up a benchmark model and describes equilibrium. As the benchmark, I treat the case of homogeneous committee members. Section 3 analyzes the optimal delegation of monetary policy in the benchmark model, which reveals the desirable type of central bankers in the economic environment this paper assumes. Section 4 extends the benchmark model to the case of heterogeneous committee members which the empirical studies report. Based on the results in Section 3, I analyze the optimal personnel organization of monetary policy committees. Section 5 concludes.

2 Benchmark Model

This section provides a detail explanation of the benchmark model and the description of equilibrium. The model can be partitioned into the two parts. One is a macroeconomic model, which determines the consequences of monetary policy. The other is a microe-
economic model that describes the process of decision-makings in the monetary policy committee. By connecting the two models, we can discuss the welfare implication and the design of the committee structure.

2.1 Macroeconomic Model

As the underlying macroeconomic model, I adopt a basic New Keynesian model. The model consists of the following two dynamic equations

\[ x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + u_t, \]  
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \]

together with a monetary policy rule. Here, \( x_t, i_t, \pi_t, u_t \) and \( e_t \) denote output gap, nominal interest rate, inflation rate, demand shock and cost shock in period \( t \), respectively.

Parameters \( \sigma, \beta, \) and \( \kappa \) are positive constants, where \( \sigma \) is the degree of constant relative risk aversion, \( \beta \) is the discount factor and \( \kappa \) is the impact of one unit of output gap on inflation. The symbol \( E_t \) denotes a mathematical expectation conditioned on information available to the private agents in period \( t \). I assume that \( \{u_t\}_{t=0}^{\infty} \) and \( \{e_t\}_{t=0}^{\infty} \) follow AR(1) processes such that

\[ u_t = \rho_u u_{t-1} + \varphi_t, \]
\[ e_t = \rho_e e_{t-1} + \psi_t, \]

where \( \rho_u, \rho_e \in [0, 1] \) and \( \varphi_t \) and \( \psi_t \) follow independently the normal distributions with mean 0 and variance \( \sigma_{\varphi}^2 \) and \( \sigma_{\psi}^2 \), respectively.

Equation (1), the dynamic IS curve, represents the AD relation that is derived from the Euler equation of household. Equation (2), the New Keynesian Phillips curve, is the AS relation which is a linear approximation of the firms’ optimization condition and the dynamic equation of average price under price-stickiness. The parameter \( \kappa \) depends on the structural parameters which represent the technology of the firms, the degree of price-stickiness and so on. If necessary, we specify the function form of the household utility function and the micro structure behind the New Keynesian Phillips curve but for the time being, we treat \( \kappa \) itself as one of the structural parameters for simplicity.

One of the reason why we use the New Keynesian model is that the model has good and manageable property in view of modern central banking in that the macroeconomic dynamics is determined once a setting rule of the central bank’s policy instrument, \( i_t \), is specified. Many central banks today, as widely known, adopt nominal interest rate

\[ 4 \text{For detail of a micro foundation of New Keynesian models, see Ch.5 of Walsh (2003b).} \]
as their policy instruments. So that, the most important part of decision-making in the monetary policy committee is determination of policy interest rate. The New Keynesian model is appropriate to incorporate it. The other reason is that welfare analysis is meaningful in the New Keynesian framework because it has rigorous micro foundations and thus clarifies the source of inefficiency in the market economy and the goal and role of monetary policy.

Along the line of Woodford (2002), the social loss function is endogenously determined as the second order approximation of household’s utility function and its function form is proportional to

\[ E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2), \]

where \( \lambda \) is the weight which society places on output gap relative to inflation as disutility. The parameter \( \lambda \) depends on the structural parameters in the way that is determined once we assume a particular micro foundation of the New Keynesian model. I treat, however, \( \lambda \) as a structural parameter for analytical ease unless a specification is needed.  

Since the purpose of this paper is to analyze an optimal institution design problem of monetary policy committees, I focus on only the average performance of monetary policy. Thus, I reset the social loss function to

\[ L \equiv V[\pi] + \lambda V[x], \quad (3) \]

where \( V[\pi] \) and \( V[x] \) are asymptotic variances of inflation rate and output gap respectively.

### 2.2 Monetary Policy Committee

Next, I set up the decision-making process of the monetary policy committee in this model. The committee consists of \( N \geq 2 \) members. Here, assume that all members are (ex-ante) homogenous in this benchmark model. To make a decision on monetary policy, they receive noisy signals on the economic condition. Given realizations of signals, any

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5 The results of this paper are invariant as long as we specify a form of \( \lambda \) which depends on a usual micro foundation of the New Keynesian model.

6 The average social loss \( L \) can be obtained by

\[ L = \lim_{\beta \to 1} (1 - \beta) E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2). \]
committee member $j$ votes a level of nominal interest rate, $i^j$, to maximize his/her own payoff and then all voting rates are aggregated by a specific rule.

First, let us see the detail of the informational structure of the monetary policy committee. As in the actual monetary policy procedure, I assume the situation that decision-making on policy instrument setting in each period must be done with noisy information about the economic state in the concerned period before it reveals. In the end of period $t - 1$, each member receives the two kinds of noisy signals on innovations of demand shock and cost shock in period $t$. One is common and the other is idiosyncratic signal. The common signals on innovations of demand shock and cost shock are of the standard form such that

\[
\varphi^c_t = \varphi_t + \mu_t, \\
\psi^c_t = \psi_t + \nu_t, 
\]

where $\mu_t$ and $\nu_t$ are independent and serially uncorrelated noises which are normally distributed with mean zero and variance $\sigma^2_\mu$ and $\sigma^2_\nu$, respectively. Each member knows the distribution of $\mu_t$ and $\nu_t$ and that realizations of $\varphi^c_t$ and $\psi^c_t$ are common to everyone. The common signals can be interpreted as well-balanced recognition among economists on the future economic condition or the result of discussions in the committee which usually depend on suggestion of the report by the research staff of the central bank. In actual, the monetary policy committees usually begin with staff reports on economic conditions. For example, every meeting of FOMC begins with a staff report and discussion on the economic condition.

Besides, there is probably idiosyncrasy among the committee members’ assessments on future economic developments. Each committee member $j$ receives idiosyncratic signal on innovations of demand shock and cost shock of the form such that

\[
\varphi^j_t = \varphi_t + \varepsilon^j_t, \\
\psi^j_t = \psi_t + \eta^j_t, 
\]

where $\varepsilon^j_t$ and $\eta^j_t$ are independent and serially uncorrelated noises which are normally distributed with mean zero and variance $\sigma^2_\varepsilon$ and $\sigma^2_\eta$, respectively. They know the distribution of the noise terms and that these are common to them but do not know realizations of the others’ idiosyncratic signals. That is, idiosyncratic signals are private information of each member. For simplicity, I assume that similar to the private agents, the central bank

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7 In this paper, to keep the model simple, I do not deal with the issues of communication.
8 Chappell, McGregor and Vermilyea (2005) explains this point precisely.
can observe innovations of demand shock and cost shock in period \( t \) once they realize in the beginning of period \( t \).

Second, consider the behavior of the committee members. Assume that any member \( j \) votes a level of nominal interest rate in period \( t \), \( \hat{i}^j_t \), to minimize the following loss function

\[
l^j_t = E_{t-1} \left[ (1 - r)(\hat{i}^j_t - \hat{i}^*_t)^2 + r \left( \hat{i}^j_t - \frac{1}{N-1} \sum_{k \neq j} i^k_t \right)^2 \right],
\]

where \( r \in [0, 1] \), \( \hat{i}^*_t \) is the level of interest rate in period \( t \) which would be set in optimal monetary policy under perfect information and \( E_{t-1} \) is a mathematical expectation conditioned on information available to member \( j \) in the end of period \( t - 1 \). The meaning of this function form is that each member concerns with the weighted sum of accuracy of his/her vote on interest rate (the first term) and the distance between it and the average of the others members’ votes (the second term). Therefore, using the available information, the committee members seek to balance the two objectives: to spot the genuine optimal level of interest rate and to coordinate with the others. Thus, the parameter \( r \) is interpreted as a measure of a motive for coordination or strategic complementarity. The presence of the coordination motive among the committee members reflects that they dislike standing out in the committee in that they care the position in the committee, the relationship with the other members and the release of the voting record.

In this paper, I mean optimal policy for optimal discretionary policy. Indeed, it might be natural to adopt optimal commitment policy but there is no analytical solution in the policy regime. Besides, the policy implication generated from introducing the decision-making of the monetary policy committee does not change in the both regimes. So, I adopt optimal discretionary policy and keep the model analytically tractable. As usually seen in the New Keynesian literature, equilibrium interest rate in optimal discretionary monetary policy under perfect information, \( \hat{i}^*_t \), is the solution of the following linear-quadratic problem

\[
\begin{align*}
\min \quad & \pi^2_t + \lambda^c x^2_t, \\
\text{s.t.} \quad & x_t = E_t x_{t+1} - \frac{1}{\sigma}(\hat{i}_t - E_t \pi_{t+1}) + u_t, \\
& \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t,
\end{align*}
\]

where the parameter \( \lambda^c > 0 \) is the relative weight that the committee members place on output gap. Note that it is not necessarily identical to society’s weight. This preference

\[^9\text{Thus, except the process of decision-making on policy instrument setting, this paper’s model is a model with perfect information.}\]
parameter can be thought to represent the underlying preference of the committee members in that it determines their attitudes on desirable trade-off of monetary policy. The solution of the problem above is \( i_t^* = \sigma u_t + \Phi e_t, \)

where \( \Phi = \frac{\lambda c \rho_e + (1 - \rho_e) \sigma \kappa}{\lambda c (1 - \beta \rho_e) + \kappa^2}. \)

The game described above is a variant of the beauty contest framework exploited by Morris and Shin (2002). In this setting, unlike Morris and Shin (2002), the target each member seeks is not the true state of the economy but a variable determined by it. However, the framework of Morris and Shin (2002) is applicable to this paper's model because of linearity of the target, \( i_t^* \), with respect to the state, \( (u_t, e_t) \).

Finally, I set the aggregation rule of individual votes to the arithmetic mean:

\[
i_t = \frac{1}{N} \sum_{j=1}^{N} i_t^j.
\]

This rule is quite simple and not so realistic in terms of the actual institutions but can easily grasp an aspect of compromise in the monetary policy committee. Since the goal of this paper is not to find the optimal aggregation rule but to investigate the optimal personnel organization of monetary policy committee, we use this simple aggregation rule as a starting point of analysis of monetary policy committee. We may adopt the weighted average rule for a generalization of the arithmetic mean but it does not change the results of this paper.

### 2.3 Equilibrium

To calculate the equilibrium of the whole model, I first derive the equilibrium of the subgame which describes the decision-making of the monetary policy committee. The first order condition of each committee member \( j \)'s problem is

\[
i_t^j = (1 - r) E_{t-1}^j i_t^* + r E_{t-1}^j \frac{\sum_{k \neq j} i_t^k}{N - 1}.
\]

Along the line of Morris and Shin (2002), consider the following linear strategy of each committee member \( j \)'s voting on interest rate

\[
i_t^j = \sigma \left[ \rho_u u_{t-1} + \gamma_u \varphi_t^j + (1 - \gamma_u) \psi_t^j \right] + \Phi \left[ \rho_e e_{t-1} + \gamma_e \psi_t^j + (1 - \gamma_e) \psi_t^e \right],
\]

\(10\)See Ch.11 of Walsh (2003b) for a detail explanation on the issue.
where \( \gamma_u \in [0, 1] \) and \( \gamma_e \in [0, 1] \) are undetermined coefficients. \(^{11}\) Let 
\[
 p_\mu = \sigma_\mu^{-2}, p_\nu = \sigma_\nu^{-2}, p_\pi = \sigma_\pi^{-2}, p_\eta = \sigma_\eta^{-2}.
\]
I interpret them as measures of the precision of the corresponding signals. Using 
\( p_\mu, p_\nu, p_\pi, p_\eta \), for each \( j \),
\[
 E_{t-1}^j i_{t}^{*} = \sigma\left[ p_\mu u_{t-1} + \frac{p_\pi}{p_\pi + p_\mu} \varphi^j_t + \frac{p_\mu}{p_\pi + p_\mu} \varphi^c_t \right] + \Phi\left[ p_\nu \tilde{e}_{t-1} + \frac{p_\eta}{p_\eta + p_\nu} \psi^j_t + \frac{p_\nu}{p_\eta + p_\nu} \psi^c_t \right],
\]
\[
 E_{t-1}^j i_{t}^{k} = \sigma\left[ p_\mu u_{t-1} + \gamma_u \left( \frac{p_\pi}{p_\pi + p_\mu} \varphi^j_t + \frac{p_\mu}{p_\pi + p_\mu} \varphi^c_t \right) + (1 - \gamma_u) \varphi^c_t \right] + \Phi\left[ p_\nu \tilde{e}_{t-1} + \gamma_e \left( \frac{p_\eta}{p_\eta + p_\nu} \psi^j_t + \frac{p_\nu}{p_\eta + p_\nu} \psi^c_t \right) + (1 - \gamma_e) \psi^c_t \right],
\]
for \( k \neq j \).

Thus, substituting these equations into the right-hand side of (5) and comparing the coefficients of (6), the undetermined coefficients turn out to be
\[
 \gamma_u = \frac{(1 - r)p_\pi}{p_\mu + (1 - r)p_\pi}, \quad \gamma_e = \frac{(1 - r)p_\eta}{p_\nu + (1 - r)p_\eta},
\]
which give the solution of the subgame of the decision-making by the monetary policy committee. Here, I investigate the properties of the response coefficient above.

**Remark 1**
\[
 \frac{\partial \gamma_u}{\partial r} < 0, \quad \frac{\partial \gamma_e}{\partial r} < 0, \quad \frac{\partial \gamma_u}{\partial p_\pi} > 0, \quad \frac{\partial \gamma_e}{\partial p_\pi} > 0, \quad \frac{\partial \gamma_u}{\partial p_\mu} < 0, \quad \frac{\partial \gamma_e}{\partial p_\mu} < 0.
\]

This is parallel to the analysis of Morris and Shin (2002). Since the importance of common information rises when the motive for coordination becomes larger, each committee member then places a higher weight on the common signals relative to the private signals: 
\[
 \frac{\partial \gamma_u}{\partial r} < 0, \quad \frac{\partial \gamma_u}{\partial p_\pi} > 0. \quad \text{Since private signals (common signals, resp.) become more informative relative to common signals (private signals) when the precisions of private signals (common signals) increase, each member is tend to depend on private signals (common signals) more:} \quad \frac{\partial \gamma_u}{\partial p_\mu} > 0, \quad \frac{\partial \gamma_u}{\partial p_\pi} > 0, \quad \frac{\partial \gamma_e}{\partial p_\nu} < 0, \quad \frac{\partial \gamma_e}{\partial p_\pi} < 0.
\]

Next, let us see equilibrium nominal interest rate and the consequent macroeconomic dynamics. Using the solution of the subgame in the monetary policy committee, nominal interest rate set in equilibrium is determined by the aggregation rule (4) as follows.
\[
 i_t = \sigma[u_t + \gamma_u \tilde{e}_t + (1 - \gamma_u)\mu_t] + \Phi[\epsilon_t + \gamma_e \tilde{\eta}_t + (1 - \gamma_e)\mu_t]
\]
\[
 = i_t^* + \sigma[\gamma_u \tilde{e}_t + (1 - \gamma_u)\mu_t] + \Phi[\gamma_e \tilde{\eta}_t + (1 - \gamma_e)\mu_t],
\]
(7)

\(^{11}\)According to the setting of this paper, this is enough to find equilibrium linear strategy. For a detail of the calculation of equilibrium in global games with finite players, see Calvó-Armengol and de Martí Beltran (2009).
where $\tilde{\varepsilon}_t = \frac{1}{N} \sum_{j=1}^{N} \varepsilon_j$ and $\tilde{\eta}_t = \frac{1}{N} \sum_{j=1}^{N} \eta_j$ are the average of private signals on innovations of demand shock and cost shock. The second and third terms of (7) represent the effects of imperfect information on interest rate setting in the monetary policy committee.

Now, macro dynamics of the model economy is given by AD relation, (1), AS relation, (2), and monetary policy rule, (7). By the method of undetermined coefficients, we find the solution of the system above. Equilibrium output gap and inflation rate are

$$x_t = -\frac{\kappa}{\lambda^c(1 - \beta \rho_e) + \kappa^2} e_t - \left[ \gamma_u \varepsilon_t + \frac{\gamma_e \Phi}{\sigma} \eta_t + (1 - \gamma_u) \mu_t + \frac{(1 - \gamma_e) \Phi}{\sigma} \nu_t \right], \quad (8)$$

$$\pi_t = \frac{\lambda^c}{\lambda^c(1 - \beta \rho_e) + \kappa^2} e_t - \kappa \left[ \gamma_u \varepsilon_t + \frac{\gamma_e \Phi}{\sigma} \eta_t + (1 - \gamma_u) \mu_t + \frac{(1 - \gamma_e) \Phi}{\sigma} \nu_t \right]. \quad (9)$$

Under perfect information, equilibrium output gap and inflation rate in the equations above are identical to those of the basic New Keynesian model with discretionary monetary policy. The effect of coordination vanishes if information is perfect since every member shares the same information set with the others under perfect information. 13

Calculating asymptotic variances of output gap and inflation rate, social loss in equilibrium is obtained by (3). 14

**Proposition 1** In the benchmark model, asymptotic variances of output gap and inflation rate and social loss are

$$V[x] = \left[ \frac{\kappa}{\lambda^c(1 - \beta \rho_e) + \kappa^2} \right]^2 \sigma^2_{\psi} \frac{1}{1 - \rho^2_e} + \frac{1}{N} \left[ \frac{\gamma_u^2 \sigma^2}{\varepsilon} + \left( \frac{\gamma_e \Phi}{\sigma} \right)^2 \sigma^2_{\eta} \right], \quad (10)$$

$$V[\pi] = \left[ \frac{\lambda^c}{\lambda^c(1 - \beta \rho_e) + \kappa^2} \right]^2 \sigma^2_{\psi} \frac{1}{1 - \rho^2_e} + \frac{\kappa^2}{N} \left[ \frac{\gamma_u^2 \sigma^2}{\varepsilon} + \left( \frac{\gamma_e \Phi}{\sigma} \right)^2 \sigma^2_{\eta} \right]$$

$$+ \left[ (1 - \gamma_u)^2 \sigma^2_{\mu} + \left( \frac{(1 - \gamma_e) \Phi}{\sigma} \right)^2 \sigma^2_{\nu} \right], \quad (11)$$

$$L = \left[ \frac{\lambda^c}{\lambda^c(1 - \beta \rho_e) + \kappa^2} \right]^2 \sigma^2_{\psi} \frac{1}{1 - \rho^2_e} + \lambda \left( \frac{\kappa}{\lambda^c(1 - \beta \rho_e) + \kappa^2} \right)^2 \left( \frac{\sigma^2_{\psi}}{1 - \rho^2_e} \right).$$

12 For a detail of the calculation, see Appendix A.
13 In the case of perfect information, $\tilde{\varepsilon}_t = \tilde{\eta}_t = \mu_t = \nu_t = 0$ holds. Then, in (8) and (9),

$$x_t = -\frac{\kappa}{\lambda^c(1 - \beta \rho_e) + \kappa^2} e_t, \quad \pi_t = \frac{\lambda^c}{\lambda^c(1 - \beta \rho_e) + \kappa^2} e_t,$$

which are equilibrium output gap and inflation rate in the basic New Keynesian model with discretionary optimal policy. Ch.11 of Walsh (2003b) provides the derivation and an explanation of the issue.
14 Appendix A provides a derivation of asymptotic variances of output gap and inflation rate.
\begin{equation*}
+ \frac{\kappa^2 + \lambda}{N} \left[ \gamma_0^2 \sigma_e^2 + \left( \frac{\gamma e \Phi}{\sigma} \right)^2 \sigma_n^2 \right] + \left( \kappa^2 + \lambda \right) \left[ \left( 1 - \gamma e \right)^2 \sigma_\mu^2 + \left( \frac{1 - \gamma e \Phi}{\sigma} \right)^2 \sigma_\nu^2 \right].
\end{equation*}

This seems somewhat complicated but the meaning is clear. The macroeconomic volatility presented in (10) and (11) is decomposed into the three parts as follows. The first term is due to cost shock, the second term is due to noisy common information and the third term is due to noisy private information. Therefore, the cause of the first term is a fundamental element and those of the second and third term are non-fundamental elements.

Immediately, we obtain the following assertion which provides a welfare implication about the committee size $N$ in this benchmark case.

**Corollary 1** In this benchmark model, asymptotic variances of output gap and inflation rate are decreasing in the size of the monetary policy committee. Thus social welfare is increasing in it.

This is intuitively plausible. The larger the committee becomes, the more accurate the averages of idiosyncratic information on the two shocks become since idiosyncratic noises of information among the members are absorbed by averaging. In fact, the second term of (10) and (11) show that an increase of the committee size reduces the part of macroeconomic volatilities due to idiosyncratic noises in the same order as $N$. Since it is not costly to enlarge the monetary policy committee in the setting of this section, a finite optimal size of the monetary policy committee does not exist. This mechanism for Corollary 1 is the same as Condorcet’s (1785) jury theorem. The part of social loss which noisy private signals generate goes to 0 as the committee size goes to infinity. It indicates that the averages of private signals on demand shock and cost shock converge to realizations of the two shocks in probability. In fact, in this model, the two important assumptions of Condorcet’s jury theorem hold. First, the assumption that enlarging the size of the monetary policy committee is costless means that so is getting more information. Second, the aggregation of information is costless since private signals are used without strategic manipulations in this model.

Let us discuss the value of information. The next corollary shows the relationship between the precision of private signals and social welfare.

**Corollary 2**

\[ \frac{\partial L}{\partial p_\varepsilon} \leq 0, \quad \frac{\partial L}{\partial p_\eta} \leq 0. \]
Corollary 2 indicates that an increase of the precisions of common signals always improve social welfare. While precise common information stimulates the coordination motive of the committee members, precise private information does not so. Hence, a rise in the quality of private signal has only the positive effect that it raises the accuracy of the members’ forecast on demand shock and cost shock. The relationship between the precision of private signals and social welfare is as below.

**Corollary 3**

$$\frac{\partial L}{\partial p_\mu} \leq 0 \iff \frac{p_\mu}{p_\epsilon} \geq (1 - r) \left[ 1 - \frac{2}{N} (1 - r) \right],$$

$$\frac{\partial L}{\partial p_\nu} \leq 0 \iff \frac{p_\nu}{p_\eta} \geq (1 - r) \left[ 1 - \frac{2}{N} (1 - r) \right].$$

Corollary 3 asserts that the marginal values of common signals are non-negative when common signals are sufficiently precise relative to private signals and vice versa. It might seem curious that the anti-transparency result holds but we can explain this as follows. Raising the precisions of common signals has two effects on the behavior of the committee members. One is a direct effect such that the committee members can get more precise information on economic developments and forecast more exactly the target level of nominal interest rate, $i^*_t$. The other is an indirect effect such that the committee members become more dependent on common signals as Remark 1 shows and it increases the inefficiency of interest rate setting due to noisiness of common signals, which brings larger macroeconomic volatility. Clearly, the former is positive one and the latter is negative one. When the precisions of common signals are low, the marginal effect of the latter is relatively large and thus social welfare decreases as the common signals become more precise.

### 3 Optimal Delegation Problem of Monetary Policy Committees

This section deals with the optimal delegation problem of monetary policy, which has been one of the most important issues of monetary policy analysis. Since the seminal work by Rogoff (1985), many researches on this topic support for optimality of conservative central

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15The mechanism for Corollary 2 presented below is similar to literature of global games, for example, Morris and Shin (2002).
bankers under various environments. As shown later, introducing decision-makings by committees, however, makes a difference in this topic. We first analyze two limiting cases to understand it easily.

3.1 Optimality of Conservatism: Limiting Cases

3.1.1 The Case of Serial Uncorrelation

Next, we consider the case where cost shock is serially uncorrelated and information of each committee member is imperfect, i.e., $e_t = \psi_t$ and $\sigma^2_\mu, \sigma^2_\nu, \sigma^2_\xi, \sigma^2_\eta > 0$. In this case, the social loss function is

$$ L = \left( \left( \frac{\sigma - \kappa \Phi_0}{\sigma} \right)^2 + \lambda \left( \frac{\Phi_0}{\sigma} \right)^2 \right) \sigma^2_\psi + \frac{\kappa^2 + \lambda}{N} \left[ \gamma_u \sigma^2_\xi + \left( \frac{\gamma_e \Phi_0}{\sigma} \right)^2 \sigma^2_\eta \right] $$

$$ + (\kappa^2 + \lambda) \left( 1 - \gamma_u \right)^2 \sigma^2_\mu + \left( \frac{1 - \gamma_e \Phi_0}{\sigma} \right)^2 \sigma^2_\eta, \tag{12} $$

where $\Phi_0 = \frac{\sigma \kappa}{\sigma + \kappa}$. As in the previous case, the first order condition, $\frac{\partial L}{\partial \lambda} = 0$, gives the optimal weight $\lambda^*$ such that

$$ \lambda^* = \lambda + (1 - \gamma_e) \frac{2(\kappa^2 + \lambda)}{\sigma^2_\eta} + (1 - \gamma_u) \frac{2 \gamma_e \sigma^2_\eta}{\sigma^2_\psi}, \tag{13} $$

We obtain the following proposition immediately from (15).

**Proposition 2** In the benchmark model without serial correlation of cost shock, the optimal weight on output gap is higher than society’s weight. That is, it is optimal to appoint a monetary policy committee which consists of more flexible policy makers than society.

An elemental cause of this result is imperfectness of information. By $\rho_e = 0$, equilibrium output gap and inflation in this case are

$$ x_t = -\frac{\Phi_0}{\sigma} e_t - \left[ \gamma_u \tilde{\epsilon}_t + \frac{\gamma_e \Phi_0}{\sigma} \tilde{\eta}_t + (1 - \gamma_u) \mu_t + \frac{(1 - \gamma_e) \Phi_0}{\sigma} \nu_t \right], $$

$$ \pi_t = \frac{\sigma - \kappa \Phi_0}{\sigma} e_t - \kappa \left[ \gamma_u \tilde{\epsilon}_t + \frac{\gamma_e \Phi_0}{\sigma} \tilde{\eta}_t + (1 - \gamma_u) \mu_t + \frac{(1 - \gamma_e) \Phi_0}{\sigma} \nu_t \right]. $$

The first terms of equilibrium output gap and inflation rate in the expression above are identical to those of the case of perfect information. The second terms of them are generated from information imperfectness (and coordination motive) in the interest rate setting. Note that the second term of inflation rate is as $\kappa$ times as that of output gap.
and $\kappa$ is the slope of the New Keynesian Phillips curve. Thus, we find that the control error of interest rate due to imperfect information affects output gap directly in the AD relation, (1), and then impacts inflation through the AS relation, (2).

The effect mentioned above leads to non-fundamental macroeconomic volatility: the second and third terms of (12). So that, to reduce the volatility, it is beneficial to appoint central bankers who place higher weights on output gap. This appears the second and third term in (13). The first term of (13) reflects the famous result in the literature that sharing the relative weight with society is optimal under perfect information. To see this, by (12), we have enough to solve the equation of $\lambda^c$

$$\frac{\partial}{\partial \lambda^c} \left\{ \left( \frac{\sigma - \kappa \Phi_0}{\sigma} \right)^2 + \lambda \left( \frac{\Phi_0}{\sigma} \right)^2 \right\} = 0.$$  

The solution is $\lambda^c = \lambda$. Therefore, the optimal weight given by (13) balances the benefit of heightening the weight to reduce the economic volatility due to common and cross-sectional noise of the members’ information and its cost: excessive variance of inflation which the accompanying weak response to cost shocks brings.

Remark that by the analysis above, information imperfectness require flexible inflation targeters for the committee members through the channel of control error of nominal interest rate.

Let us investigate the relationships between the optimal weight and the parameters, which help us analyze the general case.

**Corollary 4** If the information of the committee members on the cost shock is imperfect, i.e., $\sigma^2_\nu > 0$, $\sigma^2_\eta > 0$, then

$$\frac{\partial \lambda^*}{\partial r} > 0, \quad \frac{\partial \lambda^*}{\partial \lambda} > 0, \quad \frac{\partial \lambda^*}{\partial \kappa} > 0, \quad \frac{\partial \lambda^*}{\partial \sigma^2_\psi} < 0.$$

When coordination motive among the committee members rises, their dependency on common information, $1 - \gamma_e$, also rise and inefficiency due to the common noise increases. So that, $\frac{\partial \lambda^*}{\partial r} > 0$. When the society’s weight on output gap is high, the central bank is required to place a correspondingly high weight: $\frac{\partial \lambda^*}{\partial \lambda} > 0$. When an impact of output gap to inflation rises, control error of the demand side due to common and cross-sectional noise of information induces more excessive inflation. Thus, $\frac{\partial \lambda^*}{\partial \kappa} > 0$. When the innovation in the cost shock becomes larger, the marginal cost of raising the weight on output gap (and thus weakening the response to the cost shock) increases, which leads to a lower optimal weight: $\frac{\partial \lambda^*}{\partial \sigma^2_\psi} < 0$.

I review the detail of this issue in the next subsection.
Next, let us see the relationships between the parameters on informational structure and the optimal weight.

**Corollary 5**

\[
\frac{\partial \lambda^*}{\partial p_\eta} < 0, \quad (14)
\]

\[
\frac{\partial \lambda^*}{\partial p_\nu} \leq 0 \quad \text{iff} \quad \frac{p_\nu}{p_\eta} \geq (1-r) \left[1 - \frac{2}{N}(1-r)\right]. \quad (15)
\]

According to (13), we find that both of changes in the parameters \(p_\eta\) and \(p_\nu\) have three effects on the optimal weight \(\lambda^*\):

\[
\frac{\partial \lambda^*}{\partial p_\eta} = (\kappa^2 + \lambda) \left[ -2(1 - \gamma_e) \frac{\partial \gamma_e}{\partial p_\eta} \frac{p_\nu^{-1}}{\sigma_\psi^2} + 2 \gamma_e \frac{\partial \gamma_e}{\partial p_\eta} \frac{p_\eta^{-1}}{\sigma_\psi^2} - \frac{\gamma_e^2}{N} \frac{p_\eta^{-2}}{\sigma_\psi^2} \right], \quad (16)
\]

\[
\frac{\partial \lambda^*}{\partial p_\nu} = (\kappa^2 + \lambda) \left[ -2(1 - \gamma_e) \frac{\partial \gamma_e}{\partial p_\nu} \frac{p_\nu^{-1}}{\sigma_\psi^2} - (1 - \gamma_e) \gamma_e \frac{p_\nu^{-2}}{\sigma_\psi^2} + 2 \gamma_e \frac{\partial \gamma_e}{\partial p_\nu} \frac{p_\eta^{-1}}{\sigma_\psi^2} \right]. \quad (17)
\]

Consider the meaning of (16). Note that \(\frac{\partial \gamma_e}{\partial p_\eta} > 0\). The first term of the bracketed part is the indirect negative effect which is made by the decrease of dependency of interest rate setting on the common signal. The second term is the positive effect which is made by the increase of dependency of interest rate setting on the private signal. The third term is the direct negative effect due to the decrease of economic volatility generated by noisiness of the private signal. We can easily find that the third term dominates the second term as long as \(r < 1\). 17 Thus, the optimal weight is always decreasing in the precision of the private signal on cost shock.

The second assertion of Corollary 5, (15), is also intuitive. Note that \(\frac{\partial \gamma_e}{\partial p_\nu} < 0\). The first term of the bracketed part of (17) is a positive effect owing to an increase of the members’ dependency on common signals. As in (16), the second and third terms are the direct and indirect negative effects respectively. However, contrary to (16), the positive effect can dominate the two negative effects when, for example, \(N\) is large and the third effect is small. Recall that Corollary 3 suggests that when the precision of the common signal is sufficiently small, an increase of dependency on it generates a dominantly large volatility, which requires more flexible monetary policy. This is because the optimal weight can be increasing in the precision of the common signal on cost shock when it is sufficiently noisy.

17When \(r = 1\), \(\gamma_e = 0\) and thus the second and third term vanish. Even in this case, the result does not change by the negativity of the first term.
Corollary 6 The optimal weight on output gap is decreasing in the size of the committee.

The mechanism which generates this result is similar to that of Corollary 1. When the size of the monetary policy committee becomes larger, the cross-sectional noise of the members’ information is absorbed by averaging the individual votes. The marginal benefit of raising the relative weight on output gap then decreases, which makes the optimal weight lower.

3.1.2 The Case of Perfect information

To begin with, consider the case of perfect information, i.e., $\sigma_e^2 = \sigma_\eta^2 = \sigma_\mu^2 = \sigma_\nu^2 = 0$. The coordination motive is irrelevant in this case, so that the model is identical to the popular version of the New Keynesian model with perfect information. This case had been analyzed in literature but let us review it briefly for comparison with the result of this paper.

The results of Clarida et al. (1999) indicate that in the basic New Keynesian model with perfect information, if the cost shock $\{e_t\}_{t=0}^\infty$ is serially correlated, then the optimal weight on output gap under discretionary monetary policy is smaller than the social preference $\lambda$. In other words, it is optimal to appoint a central banker who dislikes inflation more than society. In fact, by $\sigma_e^2 = \sigma_\eta^2 = \sigma_\mu^2 = \sigma_\nu^2 = 0$, social loss is reduced to

$$L = \left[\left(\frac{\kappa}{\lambda^c(1-\beta\rho_e) + \kappa^2}\right)^2 + \lambda\left(\frac{\lambda^c}{\lambda^c(1-\beta\rho_e) + \kappa^2}\right)^2\right] \frac{\sigma_\psi^2}{1-\rho_e^2}. $$

The first order condition $\frac{\partial L}{\partial \lambda^c} = 0$ gives the optimal weight $\lambda^*$ such that 18

$$\lambda^* = (1-\beta\rho_e)\lambda,$$

which implies that $\lambda^* = \lambda$ when $\rho_e = 0$ and $\lambda^* < \lambda$ when $\rho_e > 0$. The intuition for this result is as follows. Since $e_{t+j} = \rho_e^j e_t + \sum_{i=0}^{j} \rho_e^i \psi_{t+j-i}$, if $\rho_e > 0$, the future values of the cost shocks can be partially forecast by public. Noting that in the basic New Keynesian model, equilibrium inflation in inflation targeting under discretion is given by 19

$$\pi_{t+j} = \frac{\lambda^c}{\lambda^c(1-\beta\rho_e) + \kappa^2}\rho_e^j e_t,$$

the response coefficient is increasing in $\lambda^c$. Since the expected inflation rate in period $t+j$ at period $t$ is

$$E_t\pi_{t+j} = \frac{\lambda^c}{\lambda^c(1-\beta\rho_e) + \kappa^2}\rho_e^j e_t,$$

18Vestin (2006) also derives the optimal weight on output gap in the case of perfect information.

19For the derivation, see Chapter 11 of Walsh (2003).
the role above of $\lambda^c$ becomes larger as $\rho_e$ increases. Hence, intuitively, the rational agents, who know that a conservative central banker (i.e., one with lower $\lambda^c$) will stabilize inflation harder, expect stable future inflation. This behavior contributes to stabilizing current inflation. Clearly, it disappears if $\rho_e = 0$.

Summing up, the existence of serial correlation of cost shock supports for appointing conservative committee members through the channel of stabilization of inflation expectations.

### 3.2 Optimality of Conservatism: The General Case

The general case where information is imperfect and cost shock is serially correlated is quite complex but we can analyze it with help of the limiting cases presented above.

For simplicity, we assume that $\rho_e \kappa < \sigma(1-\rho_e)(1-\beta\rho_e)$. \(^{20}\) This condition is equivalent to $\frac{\partial \Phi}{\partial \lambda^c} < 0$. That is, we consider only the case where conservative central bankers react to cost shock harder.

**Proposition 3** Assume that the social loss function $L$ is convex with respect to $\lambda^c$ and its critical point is positive. \(^{21}\) Then, the optimal weight on output gap is

$$\lambda^* = \lambda + \frac{[\sigma\kappa(1-\rho_e) + \rho_e\lambda] \Lambda - \beta\rho_e \kappa \lambda}{\kappa - \rho_e \Lambda},$$

where \(\Lambda = \frac{(\lambda + \kappa^2)}{\sigma^2} [\sigma(1-\beta\rho_e)(1-\rho_e) - \rho_e \kappa] \left[ \frac{\gamma_e^2}{N} \cdot \frac{\sigma^2_y}{\sigma^2_{\psi} / (1-\rho_e^2)} + (1-\gamma_e)^2 \frac{\sigma^2_{\psi}}{\sigma^2_{\psi} / (1-\rho_e^2)} \right].$$

Besides, it is optimal to appoint more (less, resp.) flexible inflation targeters when information about cost shock is sufficiently noisy (accurate, resp.).

**Proof.**

The proof is so long but straightforward. It is available on request.

Proposition 3 asserts that the parameter $\lambda^*$ can be both lower and higher than $\lambda$. Recall the two limiting cases. Serial correlation of cost shock brings expectations effect and lowers the optimal weight on output gap. Contrary, imperfectness of the members’ information generates extra economic volatility and requires flexible monetary policy makers. Since the optimal weight in the general case balances the both effect, it is lower (higher, resp.) than society’s weight when the former (the latter) dominates the latter (the former). Since $\Lambda$ is small (large) when $\sigma^2_{\eta}$ and/or $\sigma^2_{\nu}$ are small (large), I obtain the following remark.

\(^{20}\) This condition holds for usual calibrations of the New Keynesian literature.

\(^{21}\) The assumption is equivalent to that $\kappa - \rho_e \Lambda > 0$. The parameter $\Lambda$ is presented below.
Remark 2 *Inflation-minded committee members improve social welfare if their information on economic shocks is accurate. Otherwise, appointing output-minded committee members is optimal.*

4 Optimal Structure of the Committee

This section studies optimal structure of the committee members. In practice of monetary policy, it is often pointed out that there is preference heterogeneity among the members of the monetary policy committees. This may be because each member has different background, career and thus principle for goal of monetary policy, especially on trade-off between inflation and real objectives.

I abstract from serial correlation of demand shock and cost shock since it is not essential to understand the role of imperfect information. However, the approach I will take is applicable to the case where demand shock and cost shock are serially correlated. The qualitatively similar results will be obtained in the case where the shock is serially correlated by a few modification.

4.1 Preference Heterogeneity of the Members

Assume that $L$ members of the monetary policy committee place more weight on inflation than the other members. Without loss of generality, we assume that the weight on output gap of every member $j \in \{1, 2, ..., L\}$ is $\lambda^h$, that of the other member $k \in \{L + 1, L + 2, ..., N\}$ is $\lambda^d$ and $\lambda^h < \lambda^d$. \(^{22}\) I call $\{1, 2, ..., L\}$ and $\{L + 1, L + 2, ..., N\}$ "Hawk group" and "Dove group" respectively. For convenience, put $H = \{1, 2, ..., L\}$ and $D = \{L + 1, L + 2, ..., N\}$. For the time being, I assume that $N \geq 4$ and $2 \leq L \leq N - 2$, which ensure that both the groups have at least two members and the beauty contest frame games described below make sense. To address why the monetary policy committees are usually amalgams of both the groups, this assumption will be relaxed later by considering a non-strategic situation where the coordination motive among the members does not exist.

I set up a beauty contest game in which ex ante heterogeneity among the committee members exists. The modification of the members’ preference generates heterogeneity

\(^{22}\)Since I do not treat the issues of communication in this paper, this preference heterogeneity does not cause Crawford and Sobel (1982) type strategic information transmission.
of the problem they face since the targets of the members are not homogenous. By \( \rho_a = \rho_c = 0 \), the levels of interest rate in period \( t \) in optimal policy under perfect information for the hawkish and dovish members are respectively

\[
\begin{align*}
    i_t^{hs} &= \sigma u_t + \Phi^h e_t, \\
    i_t^{ds} &= \sigma u_t + \Phi^d e_t,
\end{align*}
\]

where \( \Phi^h = \frac{\sigma \kappa}{\lambda^h} \) and \( \Phi^d = \frac{\sigma \kappa}{\lambda^d} \). Equations (18) and (19) display the role of heterogeneity of underlying preferences among the committee members. It makes a difference between their responses to cost shock. The members of Hawk group react to cost shock harder than the members of Dove group: \( \Phi^h > \Phi^d \).

Since the target rates are different, I set up the problems of the committee members in both groups separately. I reset the loss function of each member \( j \in H \) to

\[
L^j_t = E_{t-1}^j \left[ (1 - r)(i_t^j - i_t^{hs})^2 + r \left( i_t^j - \frac{1}{L-1} \sum_{m \in H(\{j\})} i_t^{m}\right)^2 \right]
\]

and that of each member \( k \in D \) to

\[
L^k_t = E_{t-1}^k \left[ (1 - r)(i_t^k - i_t^{ds})^2 + r \left( i_t^k - \frac{1}{N - L - 1} \sum_{m \in D(\{k\})} i_t^{m}\right)^2 \right].
\]

The parameter \( r \) represents the degree of coordination motive with the other members in the same group. In this setting, I assume the situation that each members of both groups cares the position in the same group and the relationship with her fellows.

All committee members in the both groups are assumed to minimize their own loss function above. The first order conditions for both the problems are

\[
\begin{align*}
    &i_t^j = (1 - r)E_{t-1}^j i_t^{hs} + rE_{t-1}^j \frac{\sum_{m \in H(\{j\})} i_t^{m}}{L-1}, \quad \text{for } j \in H, \\
    &i_t^k = (1 - r)E_{t-1}^k i_t^{ds} + rE_{t-1}^k \frac{\sum_{m \in D(\{k\})} i_t^{m}}{N - L - 1}, \quad \text{for } k \in D.
\end{align*}
\]

Note that the problems of the both groups are mutually independent since every member does not pursue coordination with the others who belongs to the opponent group. Thus, I consider the following linear strategy as in the benchmark model:

\[
\begin{align*}
    &i_t^j = \sigma \left[ \gamma_a^h \varphi_t^j + (1 - \gamma_a^h) \varphi_t^c \right] + \Phi^h \left[ \gamma_c^h \psi_t^j + (1 - \gamma_c^h) \psi_t^c \right], \quad \text{for } j \in H, \\
    &i_t^k = \sigma \left[ \gamma_a^d \varphi_t^k + (1 - \gamma_a^d) \varphi_t^c \right] + \Phi^d \left[ \gamma_c^d \psi_t^k + (1 - \gamma_c^d) \psi_t^c \right], \quad \text{for } k \in D.
\end{align*}
\]

\( ^{23} \)Of course, we need to reset \( \Phi^h \) and \( \Phi^d \) to \( \frac{\lambda^h \rho_c + (1 - \rho_c) \sigma \kappa}{\lambda^h (1 - \rho_c) + \kappa^2} \) and \( \frac{\lambda^d \rho_c + (1 - \rho_c) \sigma \kappa}{\lambda^d (1 - \rho_c) + \kappa^2} \) respectively when we treats the case where the shock are serially correlated.
where $\gamma^h_u, \gamma^h_e, \gamma^d_u, \gamma^d_e$ are undetermined coefficients.

Substituting (22) and (23) into (20) and (21), by the method of undetermined coefficients, we can find $\gamma^h_u, \gamma^h_e, \gamma^d_u, \gamma^d_e$. We omit the detail of the calculation because it is long and similar to the benchmark model. The undetermined coefficients turn out to be

$$\gamma^h_u = \gamma^d_u = \frac{(1 - r)p + \gamma^h_e - \gamma^d_e - (1 - r)p + \gamma^h_e - \gamma^d_e}{p + (1 - r)p e} = \gamma_u,$$

$$\gamma^h_e = \gamma^d_e = \frac{(1 - r)p - \gamma^h_u + \gamma^d_u - (1 - r)p - \gamma^h_u + \gamma^d_u}{p + (1 - r)p e} = \gamma_e.$$

Nominal interest rate in period $t$ set by the committee is determined by (22) and (23) as follows.

$$i_t = \frac{1}{N} \left( \sum_{j=1}^{N} i^j_t + \sum_{k=1}^{N} i^k_t \right)$$

$$= \sigma \left[ u_t + \gamma_u \tilde{e}_t + (1 - \gamma_u) \mu_t \right] + \left[ q \Phi^h + (1 - q) \Phi^d \right] e_t$$

$$+ \gamma_e \left[ q \Phi^h \tilde{\eta}^h_t + (1 - q) \Phi^d \tilde{\eta}^d_t \right] + (1 - \gamma_e) \left[ q \Phi^h + (1 - q) \Phi^d \right] \nu_t,$$

(24)

where $\tilde{\eta}^h_t = \sum_{j=1}^{N} \eta^j_t$, $\tilde{\eta}^d_t = \sum_{k=1}^{N} \eta^k_t$ and $q = \frac{L}{N}$. The parameter $q$ is the share of Hawk group in the monetary policy committee and plays the most important role in this section. Note that the committee tends to response harder against the cost shock when $q$ is large and vice versa.

As in the benchmark model, macroeconomic dynamics is determined by the system (1),(2) and (24). Equilibrium output gap and inflation rate in period $t$ are linear in the (relevant) state variables in period $t$. In this case, they are $e_t, \tilde{e}_t, \tilde{\eta}^h_t, \tilde{\eta}^d_t, \mu_t$ and $\nu_t$. By the method of undetermined coefficients, we obtain equilibrium output gap and inflation rate as follows.\cite{24}

$$x_t = -\frac{1}{\sigma} \left[ q \Phi^h + (1 - q) \Phi^d \right] e_t - \gamma^h_u \tilde{e}_t - \frac{\gamma^h_e}{\sigma} \left[ q \Phi^h \tilde{\eta}^h_t + (1 - q) \Phi^d \tilde{\eta}^d_t \right]$$

$$- (1 - \gamma_u) \mu_t - \frac{1 - \gamma_e}{\sigma} \left[ q \Phi^h + (1 - q) \Phi^d \right] \nu_t,$$

(25)

$$\pi_t = \left[ 1 - \frac{\kappa}{\sigma} \left( q \Phi^h + (1 - q) \Phi^d \right) \right] e_t - \kappa \gamma^h_u \tilde{e}_t - \frac{\kappa \gamma^h_e}{\sigma} \left[ q \Phi^h \tilde{\eta}^h_t + (1 - q) \Phi^d \tilde{\eta}^d_t \right]$$

$$- \kappa (1 - \gamma_u) \mu_t - \frac{\kappa (1 - \gamma_e)}{\sigma} \left[ q \Phi^h + (1 - q) \Phi^d \right] \nu_t.$$

(26)

Equilibrium output gap and inflation rate in (25) and (26) nest those of the basic New Keynesian model as the case where information is perfect and the members’ preference are homogenous, i.e., $\lambda^h = \lambda^d$.

\cite{24}I omit the derivation because it is long but straightforward.
We can calculate asymptotic variances of output gap and inflation in the same way in Proposition 2.

**Proposition 4** In the presence of preference heterogeneity among the committee members, asymptotic variances of output gap and inflation rate are

\[
V[x] = \left[ \frac{q \Phi_h + (1-q) \Phi_d}{\sigma} \right]^2 \sigma_\psi^2 + \frac{1}{N} \left[ \gamma_u^2 \sigma_\epsilon^2 + \gamma_e^2 \left( q \left( \frac{\Phi_h}{\sigma} \right)^2 + (1-q) \left( \frac{\Phi_d}{\sigma} \right)^2 \right) \sigma_\eta^2 \right] \\
+ \left[ (1-\gamma_u)^2 \sigma_\mu^2 + (1-\gamma_e)^2 \left( \frac{q \Phi_h + (1-q) \Phi_d}{\sigma} \right)^2 \sigma_\nu^2 \right],
\]

(27)

\[
V[\pi] = \left[ 1 - \kappa \left( \frac{q \Phi_h + (1-q) \Phi_d}{\sigma} \right) \right]^2 \sigma_\psi^2 + \frac{\kappa^2}{N} \left[ \gamma_u^2 \sigma_\epsilon^2 + \gamma_e^2 \left( q \left( \frac{\Phi_h}{\sigma} \right)^2 + (1-q) \left( \frac{\Phi_d}{\sigma} \right)^2 \right) \sigma_\eta^2 \right] \\
+ \kappa^2 \left[ (1-\gamma_u)^2 \sigma_\mu^2 + (1-\gamma_e)^2 \left( \frac{q \Phi_h + (1-q) \Phi_d}{\sigma} \right)^2 \sigma_\nu^2 \right].
\]

(28)

By (3), (27) and (28), we obtain the following result immediately.

**Corollary 7** Given \( q \), asymptotic variance of output gap and inflation rate are decreasing in \( N \) and thus so is social loss. That is, the larger the monetary policy committee is, the better social welfare is unless the ratio of the size of Hawk and Dove group changes.

This is a variant of Corollary 1. Note that the substantial difference of the model in this section from that of section 2 is the existence of preference heterogeneity among the committee members, or the existence of \( q \). Since the role of the ratio \( q \) is only to determine the response of the monetary policy committee to cost shock, it does not make a difference on the role of the committee size \( N \). Thus, in this model, enlarging the committee always improves social welfare by mitigating noisiness of private signals of the committee members.

### 4.2 The Optimal Personnel Organization

In actual, the members of monetary policy committees are usually chosen from the inside and outside of the central banks subject to the legal mandates which mention the proportions explicitly. The empirical researches report preference heterogeneity between both the types and the results of the previous section tell us that this is meaningful in view of social welfare. So, how should the seats of the committees be allocated to both? The rest

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25We have to replace \( \sigma_\epsilon^2 \) by \( \frac{\sigma_\epsilon^2}{1-\rho^2} \) when we analyze the case of serial correlation.

26The abstraction of serial correlation of cost shock does not affect this point.
of this section addresses this question, i.e., the optimal personnel organization problem of the monetary policy committee. The approach is to find the ratio which minimizes the social loss function \( L \), given the values of \( \lambda^h \) and \( \lambda^d \).

### 4.2.1 Analytical Results

The analysis of the optimal ratio is slightly difficult but the most elemental results are available in the analytical way. First, I provide a characterization of the optimal ratio.

**Proposition 5** Let the optimal ratio of the size of Hawk group to the whole committee be \( q^* \). Then

\[
q^* = \arg\min_{q' \in Q} |q' - \tilde{q}|,
\]

where \( Q = \left\{ \frac{2}{N}, \ldots, \frac{N-2}{N} \right\} \) \(^{27}\) and

\[
\tilde{q} = \frac{\left[ \sigma \kappa - \Phi^d (\kappa^2 + \lambda) \right] \sigma^2_{\psi} - \left( \Phi^h + \Phi^d \right) (\kappa^2 + \lambda) \frac{\gamma^2}{\sigma^2_{\eta}} \sigma^2_{\eta} - \Phi^d (\kappa^2 + \lambda) (1 - \gamma_e)^2 \sigma^2_{\nu}}{\left( \Phi^h - \Phi^d \right) (\kappa^2 + \lambda) \left[ \sigma^2_{\psi} + (1 - \gamma_e)^2 \sigma^2_{\nu} \right]}.
\]

**Proof.**

See Appendix B.

Proposition 5 asserts that the optimal ratio of the size of Hawk group to the whole of the committee is the nearest one to some real number \( \tilde{q} \). \(^{28}\) This fact is important for the analysis on optimal ratio \( q^* \) since it provides the following useful result.

**Remark 3** The optimal ratio \( q^* \) is non-decreasing in \( \tilde{q} \).

Remark 3 suggests that comparative statics about \( \tilde{q} \) is enough to investigate the relationships between the optimal ratio \( q^* \) and the parameters. Indeed, quantitative analyses of \( q^* \) face difficulty of discreteness of \( q \) but qualitative analyses of \( q^* \) is relatively easily conducted by use of continuity of \( \tilde{q} \).

I next provide a simple but important limiting result. In the discussion on Proposition 6 below, I treat the case of \( r = 0 \). Since there is no coordination motive among the

\(^{27}\)Later, in the discussion on Proposition 6, \( Q \) is reset to \( \left\{ 0, \frac{1}{N}, \ldots, \frac{N-1}{N}, 1 \right\} \).

\(^{28}\)There is a possibility of multiple optima. Since the social loss function \( L \) is quadratic in \( q \), there are two optimal ratios when \( \tilde{q} \) corresponds the middle point of some two grid points. That is, both of two ratios \( q_- \) and \( q_+ \) (\( q_- < q_+ \)) are optimal if \( \tilde{q} = q_- + \frac{1}{N} \) (or equivalently, \( \tilde{q} = q_+ - \frac{1}{N} \)). Here, we neglect such a case.
members, I may assume that $0 \leq L \leq N$. That is, we can consider the case where the monetary policy committee is occupied by only one group. The set $Q$ in Proposition 5 is hence modified to $\{0, \frac{1}{N}, ..., \frac{N-1}{N}, 1\}$. This is an expedient setting to give a theoretical explanation to the fact that the actual monetary policy committees usually consist of heterogeneous members. However, the restriction above is not substantial and Proposition 6 below is true about the general case where $r > 0$.

Let $\Theta = \frac{\Phi}{\Phi}$. Since heterogeneity of underlying preferences affects only the responses of the members’ target to cost shock as (18) and (19) show, I interpret $\Theta$ as a measure of the effect of preference heterogeneity. This is a key element of the next proposition, which provides a necessary condition for that it is optimal to organize the monetary policy committee into a compound of hawkish and dovish persons.

**Proposition 6**

1. $\tilde{q} \geq 0$ if and only if $\lambda^d \geq \lambda^* + (\Theta - 1)(\lambda + \kappa^2)\frac{\sigma^2}{2N} \cdot \frac{\sigma^2_v}{\sigma^2_\nu}$

2. $\tilde{q} \leq 1$ if and only if $\lambda^h \leq \lambda^* - (1 - \frac{1}{\Theta})(\lambda + \kappa^2)\frac{\sigma^2}{2N} \cdot \frac{\sigma^2_v}{\sigma^2_\nu}$

**Proof.**

See Appendix C.

To gain an intuition for Proposition 6, consider the case of perfect information: $\sigma^2_\nu = \sigma^2_\eta = 0$. We obtain the following corollary immediately from Proposition 6.

**Corollary 8** Suppose that information of the committee members on cost shock is perfect, i.e., $\sigma^2_\eta = \sigma^2_\nu = 0$. Then the following statements hold.

1. $\tilde{q} \geq 0$ if and only if $\lambda^d \geq \lambda$.

2. $\tilde{q} \leq 1$ if and only if $\lambda^h \leq \lambda$.

Corollary 8 has a very simple background. Note that by (13), the optimal weight $\lambda^*$ is equal to the social preference $\lambda$ when the committee members’ information is perfect. Consider the case of $\lambda^d \leq \lambda$. All the committee members place lower or equal weights on output gap than optimum in this case. Hence, to approximate interest rate setting to optimum with respect to policy weight, it is suboptimal to appoint only the members who belong to Dove group. We can understand similarly the case where $\lambda^h \geq \lambda$.

The mechanism for the first assertion of Proposition 6 is similar to this. However, the assertion indicates that the necessary condition for that appointing a hawkish member is
valid is stricter than that of in the case of perfect information. This is because there is extra social loss due to the hawkish members’ weak interest in stabilizing the volatility of output gap from information imperfectness. So that, strong preference heterogeneity pushes up the lower bound of $\lambda_d$ for optimality of the compound committees. I interpret the second assertion of Proposition 6 in the same way.

The results of the previous section and Proposition 6 reflects the merit of appointing liberal (i.e., output-minded) central bankers, while the literature on this topic largely suggests the optimality of conservative central bankers. Proposition 6 asserts that a compound monetary policy committee is optimal unless the bias of the members’ preference is very small. It can be regarded as a justification for that the actual monetary policy committees consist of heterogeneous members with respect to preferences over the goals of monetary policy. It also answer for why the committees have replaced the single policy makers in recent years by showing the merit of preference heterogeneity. \(^{29}\)

Next, I investigate the relationships between the optimal ratio $q^*$ and the parameters. The results which can be analytically obtained are the following.

**Proposition 7**

1. The optimal ratio $q^*$ is non-decreasing in $N$ and $\sigma_\psi^2$ and non-increasing in $\lambda$.

2. Suppose that $\tilde{q} \geq 0$. Then, $q^*$ is non-increasing in $r$ if \(\frac{(1+\Theta)(1-r)}{N} \leq 2\).

3. Suppose that information of the committee members on the cost shock is perfect, i.e., $\sigma_\eta^2 = \sigma_\nu^2 = 0$. Then, $q^*$ is non-decreasing in $\kappa$ if $\lambda_h \leq \lambda \leq \lambda_d$.

**Proof.**

See Appendix D.

These are intuitively plausible in view of Corollary 4 and 6. Consider the first assertion. Since enlarging the committee makes accommodates control error of interest rate setting due to imperfect information, it lowers the value of adding output-minded members. So that, $q^*$ is non-decreasing in $N$.

Intuitively, $q^*$ seems to be non-increasing in $r$ unconditionally since the coordination behavior strengthens the volatility due to noisiness of common information. There exists, however, the possibility that an increase in $r$ lowers the dependency of the committee members on their noisy individual information and it improves social welfare. The full characterization of the relationship between $r$ and $q^*$ is somewhat complex but the sufficient condition presented in the second assertion of Proposition 7 is simple and informative. Unless the size of the committee is extremely small, noisiness of individual

\(^{29}\)Heterogeneity of members’ tastes is one of substantial features of committee decision-making.
information is considerably absorbed by aggregation. This removes the possibility mentioned above.

To gain an intuition for the third assertion of Proposition 7, at first, consider the case where a single policy maker conducts monetary policy under perfect information. Tillmann (2008) claims that too conservative central bankers are more harmful than too liberal ones. This is because the optimization condition under discretionary policy by a single policy maker

\[ x_t = -\frac{\kappa}{\lambda^c} \pi_t \]

implies that excessive conservatism (i.e. low \( \lambda^c \) relative to \( \lambda \)) leads to enormous volatility of output gap. Thus, excessive liberalism generates relatively small inefficiency to excessive conservatism. This mechanism is true in this paper. By \( \lambda^h \leq \lambda \leq \lambda^d \), the hawkish and dovish members are the too conservative and liberal central bankers respectively. To raise \( q \) means that the monetary policy committee becomes more conservative. As the equation above indicates, a rise in \( \kappa \) reduces the safety of excessive liberalism (relative to excessive conservatism) by increasing volatility of output gap. Since the benefit of appointing liberal central bankers in Proposition 2 vanishes in the case of perfect information, an increase in \( \kappa \) makes the optimal ratio \( q^* \) larger monotonically.

### 4.2.2 Numerical Results

Let us investigate other important issues by a numerical approach. First, I set the baseline parameter value. This paper does not aim to obtain quantitatively conclusive results and the qualitative results are invariant with the baseline values. As the baseline value, set \( \beta = 0.99 \), \( \sigma = 1 \), \( \lambda = 0.25 \), \( \kappa = 0.05 \). I follow Walsh (2003b) about \( \beta \), \( \lambda \) and \( \kappa \). The value of \( \sigma \) is irrelevant as seen in the analytical results above. Even if it is incorporated that \( \lambda \) is endogenously determined by particular micro foundations, it does not change the following discussion basically. Since an empirically valid values of \( r \), \( \lambda^h \) and \( \lambda^d \) are not available, fix \( r = 0.2 \), \( \lambda^h = 0.15 \) and \( \lambda^d = 0.35 \). These specifications do not make a difference in the qualitative results. I set \( \sigma^2_\psi = 1 \) since the important factor is not the absolute value of \( \sigma^2_\psi \) but the ratios of \( \sigma^2_\nu \) to \( \sigma^2_\eta \) to it. Because the values of \( \sigma^2_\psi \) affects the results significantly, I examined several cases about them. I use \( \sigma^2_\psi = 0.6 \) and \( \sigma^2_\eta = 0.8 \) as a baseline value and report the results based on them because I obtained the qualitatively same results. The size of the monetary policy committee varies among the countries. For example, the numbers of committee members of FOMC, the MPC of

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The actual sizes of the monetary policy committees are about 10. I provide a few examples later.
Bank of England and the Policy Board of Bank of Japan are 12, 9 and 9, respectively. I set \( N = 10 \) here, for it keeps \( q = \frac{q}{N} \) numerically simple.

At first, I analyze the relationship between the slope of the New Keynesian Phillips curve, \( \kappa \), and the optimal ratio, \( \hat{q} \).

\[ \text{[Figure 1 about here.]} \]

Figure 1 illustrates it. The reason why \( \hat{q} \) is hump-shaped in \( \kappa \) is as follows. An increase in \( \kappa \) has three effects on \( \hat{q} \). First, even in the case of imperfect information, the channel of the third assertion of Proposition 7 is effectual. Second, a decrease in the ratio of the response coefficients on cost shock in target rates of Hawk group and Dove group, \( \Theta \), reduces the relative benefit of appointing the output-minded person rather than inflation-minded person to stabilize output gap and eventually inflation. This is interpreted as below. The larger the trade-off of monetary policy is, the closer the attitudes about inflationary pressure of inflation-minded and output-minded group are. It reduces the merit of Dove group members for accommodating the volatility due to inefficiency of interest rate setting. Third, an increase of impact of output gap in inflation enlarges the effect of inaccurate and inefficient interest rate setting on inflation as Corollary 4 shows. The first and second dominate the third for small \( \kappa \) and vice versa for large \( \kappa \). \(^{31}\) The second and third do not exist under perfect information, which leads to the monotone result, i.e., the third assertion of Proposition 7. This result is robust with respect to other parameters’ values.

Next, let us see the relationship between the degree of information imperfectness (\( \sigma^2_\eta \) and \( \sigma^2_\nu \)) and the optimal ratio, \( q^* \).

\[ \text{[Figure 2 about here.]} \]

Figure 2 shows the result. Broadly, the optimal ratio \( q^* \) decreases in \( \sigma^2_\eta \) and \( \sigma^2_\nu \). It is intuitively plausible since the merit of increasing dovish members is generally large when the inefficiency of imperfect information is large. In detail, the optimal ratio is monotonically decreasing in \( \sigma^2_\eta \) for a fixed \( \sigma^2_\nu \) but it is not necessarily so in \( \sigma^2_\nu \) for a fixed \( \sigma^2_\eta \). The background of this asymmetric result is the asymmetric results of Corollary 5. While noisiness of the private signal always lowers the optimal weight on output gap and

\(^{31}\) In this example, the range of \( \kappa \) in which the third dominates the first and second is not so realistic in view of the empirical evidences. However, this paper focus only on the mechanism that provides the result above.
hence heightens the optimal ratio, noisiness of the common signal can make the converse
effect when it is sufficiently intense. Thus, for a given $\sigma^2_\eta$, the optimal ratio can decreases
in the intermediate values of $\sigma^2_\nu$ in Figure 2.

5 Concluding Remark

Incorporating that monetary policy is conducted by committees brings various problems
about the institution design of monetary policy to macro economists. This paper deals
with one of them: optimal personnel organization problem of monetary policy commit-
tees. Through the analysis of this problem, I find a way to model strategic situations
in the committees and its limitations. It also tell us that there is a merit of appointing
liberal central bankers when the central bank can not control the demand side of the
economy perfectly because of imperfect information on economic shocks. This paper pro-
vides a theoretical justification for that the actual monetary policy committees usually
consist of members with heterogeneous preferences.

One of the remaining problems along this paper’s direction is how large the monetary
policy committees should be. This is important for practice of monetary policy but is not
easy to solve in formal models. The key mechanism for determination of the optimal size
may be a harmful effect of common information which is augmented by adding committee
members.

There is possibility of another extension. Riboni and Ruge-Murcia (2008) and Gerlach-
Kristen (2008) reports that the outsiders of the BOE’s MPC have, on average, the asym-
metric preferences in that they are apt to avoid raising nominal interest rate harder than
lowering it. This fact is intuitive in a sense and should be incorporated into the optimal
organization problem of the monetary policy committees.
Appendices

Appendix A: Derivation of (8),(9),(10),(11)

In the benchmark model, macroeconomic dynamics of the artificial economy is given by the following system of stochastic difference equations.

\[
\begin{align*}
    x_t &= E_t x_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1}) + u_t, \quad (30) \\
    \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + e_t, \quad (31) \\
    i_t &= \sigma [u_t + \gamma_u \tilde{e}_t + (1 - \gamma_u) \mu_t] + \Phi [e_t + \gamma_e \tilde{\eta}_t + (1 - \gamma_e) \nu_t]. \quad (32)
\end{align*}
\]

Since the relevant state variables in period \( t \) are \( e_t, \tilde{e}_t, \tilde{\eta}_t, \mu_t \) and \( \nu_t \), the solution will be of the form

\[
\begin{align*}
    x_t &= A_x e_t + B_x \tilde{e}_t + C_x \tilde{\eta}_t + D_x \mu_t + E_x \nu_t, \quad (33) \\
    \pi_t &= A_\pi e_t + B_\pi \tilde{e}_t + C_\pi \tilde{\eta}_t + D_\pi \mu_t + E_\pi \nu_t, \quad (34)
\end{align*}
\]

where \( A_k, B_k, C_k, D_k \) and \( E_k (k = x, \pi) \) are undetermined coefficients. Substituting (33),(34) into (30),(32) and then (32) into (30) and comparing the coefficients of both sides,

\[
\begin{align*}
    A_x &= \rho_e A_x - \frac{\Phi - \rho_e A_\pi}{\sigma}, \quad B_x = -\gamma_u, \quad C_x = -\frac{\gamma_e \Phi}{\sigma}, \quad (35) \\
    D_x &= -(1 - \gamma_u), \quad E_x = -(1 - \gamma_e) \Phi. \quad (36)
\end{align*}
\]

Substituting (33),(34) into (31) and comparing the coefficients of both sides,

\[
\begin{align*}
    A_\pi &= \beta \rho_e A_\pi + \kappa A_x + 1, \quad B_\pi = \kappa B_x, \quad C_\pi = \kappa C_x, \quad D_\pi = \kappa D_x, \quad E_\pi = \kappa E_x. \quad (37)
\end{align*}
\]

Solving the first equations of (35) and (36), we get

\[
\begin{align*}
    A_x &= -\frac{\kappa}{\lambda^x(1 - \beta \rho_e) + \kappa^2}, \quad A_\pi = \frac{\lambda^x}{\lambda^x(1 - \beta \rho_e) + \kappa^2}.
\end{align*}
\]

Thus, we obtain equilibrium output gap and inflation rate in period \( t \) in the text.

Finally, we calculate asymptotic variances of output gap and inflation rate. By \( e_t = \rho_e e_{t-1} + \psi_t \), asymptotic variance of \( e_t \) is \( \frac{\sigma^2_e}{1 - \rho_e^2} \). Besides, since \( e_t, \tilde{e}_t, \tilde{\eta}_t, \mu_t \) and \( \nu_t \) are mutually independent, each covariance of them is zero. Noting the two facts above, we find asymptotic variance of output gap in the text. Similarly, we can calculate asymptotic variance of inflation rate in the text.
Appendix B: Proof of Proposition 5

The optimal ratio $q^*$ is defined as

$$q^* = \arg\min_{q \in \left\{ \frac{N}{N - 2}, \ldots, \frac{N}{2} \right\}} \left\{ V[\pi] + \lambda V[x] \right\}.$$ 

Note that according to (31) and (33), social loss $V[\pi] + \lambda V[x]$ is a quadratic function of $q$ with a positive coefficient of the second order term. Consider the continuation of $V[\pi] + \lambda V[x]$ with respect to $q$ and let the solution of $\frac{\partial (V[\pi] + \lambda V[x])}{\partial q} = 0$ be $\tilde{q}$. Then, $\tilde{q}$ must satisfy

$$\left\{ 2 \left[ \sigma - \kappa \left( \frac{\tilde{q} \Phi^h + (1 - \tilde{q}) \Phi^d}{\sigma} \right) \right] \left[ - \frac{\kappa}{\sigma} (\Phi^h - \Phi^d) \right] \sigma^2 \psi + \kappa^2 \gamma^2 e \frac{2}{N} (\sigma^2 - (\frac{\Phi^d}{\sigma})^2) \sigma^2 + 2 \kappa^2 (1 - \gamma_e)^2 \left( \frac{\tilde{q} \Phi^h + (1 - \tilde{q}) \Phi^d}{\sigma} \right) \left( \frac{\Phi^h - \Phi^d}{\sigma} \right) \sigma^2 \nu \right\} = 0.$$ 

Solving this with respect to $\tilde{q}$, we obtain (29). Since discrete convexity of $V[\pi] + \lambda V[x]$ with respect to $q$, the optimal ratio $q^*$ must belong to the set

$$\left\{ q_-, q_+ \left| \exists L \in \mathbb{N}, 2 \leq L \leq N - 3, \frac{L}{N} = q_- \leq \tilde{q} \leq q_+ = \frac{L + 1}{N} \right\} \cup \left\{ \frac{2}{N}, \frac{N - 2}{N} \right\}.$$ 

Moreover, since $V[\pi] + \lambda V[x]$ is quadratic in $q$ and quadratic functions are symmetric with respect to their axes, we find that $q^*$ is the nearest element of $\left\{ \frac{2}{N}, \ldots, \frac{N - 2}{N} \right\}$ to $\tilde{q}$. Q.E.D.

Appendix C: Proof of Proposition 6

Using (15), the numerator of (29) becomes

$$\left[ \sigma \kappa - \Phi^d(\kappa^2 + \lambda) \right] \sigma^2 \psi - (\Phi^h + \Phi^d)(\kappa^2 + \lambda) \frac{\gamma_e^2}{2N} \sigma^2 \eta - \Phi^d(\kappa^2 + \lambda)(1 - \gamma_e)^2 \sigma^2 \nu$$

$$= \frac{\sigma^2}{\Phi^d} \left[ \lambda^d - \lambda - (\Theta + 1)(\kappa^2 + \lambda) \frac{\gamma_e^2}{2N} \sigma^2 \eta - (\kappa^2 + \lambda)(1 - \gamma_e)^2 \sigma^2 \nu \right].$$

\(^{32}\)I omit the detail of the calculation since it is long but straightforward. It is available on request.
\[ \frac{\sigma^2}{\Phi d} \left[ \lambda^d - \left( \lambda^* + (\Theta - 1)(\kappa^2 + \lambda) \frac{\gamma^2_c}{2N} \cdot \frac{\sigma^2}{\sigma^2_{\psi}} \right) \right]. \]

This shows the first assertion. Next, \( \tilde{q} \geq 1 \) if and only if
\[
\left[ \sigma \kappa - \Phi^d(\kappa^2 + \lambda) \right] \sigma^2_{\psi} - (\Phi^h + \Phi^d)(\kappa^2 + \lambda) \frac{\gamma^2_c}{2N} \sigma^2_{\eta} - \Phi^d(\kappa^2 + \lambda)(1 - \gamma_c)^2 \sigma^2_{\nu} \geq 0.
\]

Dividing the both sides by \( \Phi^h \sigma^2_{\psi} \), we have
\[
\lambda^h + \kappa^2 - \left( 1 + \frac{1}{\Theta} \right)(\kappa^2 + \lambda) \frac{\gamma^2_c}{2N} \cdot \frac{\sigma^2}{\sigma^2_{\psi}} \geq \lambda + \kappa^2 + (1 - \gamma_c)^2(\kappa^2 + \lambda) \frac{\sigma^2}{\sigma^2_{\psi}}.
\]

which is reduced to
\[
\lambda^h \geq \lambda^* - \left( 1 - \frac{1}{\Theta} \right)(\kappa^2 + \lambda) \frac{\gamma^2_c}{2N} \cdot \frac{\sigma^2}{\sigma^2_{\psi}}.
\]

It completes the proof. \( Q.E.D. \)

**Appendix D: Proof of Proposition 7**

We can show the first assertion as follows. It is immediately obtained from (29) that \( q^* \) is non-decreasing in \( N \) and non-increasing in \( \lambda \). By (29), we have
\[
\frac{\partial \tilde{q}}{\partial \sigma^2_{\psi}} = \frac{(\Phi^d + \Phi^h)(\kappa^2 + \lambda) \sigma^2_{\psi}}{2N} + \left[ (\sigma \kappa - \Phi^d(\lambda + \kappa^2)) + \Phi^d(\kappa^2 + \lambda) \right] (1 - \gamma_c)^2 \sigma^2_{\nu}.
\]

Since \( \Phi^h > \Phi^d \) by \( \Phi^d = \frac{\sigma \kappa}{\lambda^d + \kappa^2} \), \( \Phi^h = \frac{\sigma \kappa}{\lambda^h + \kappa^2} \) and \( \lambda^d > \lambda^h \), we obtain that \( \frac{\partial \tilde{q}}{\partial \sigma^2_{\psi}} \geq 0 \).

Next, I provide a proof of the second assertion. Since I assume that \( \tilde{q} \geq 0 \) here, it is a sufficient condition for the assertion that the numerator of (29) is decreasing in \( r \). Put
\[
f(r) = (\Phi^h + \Phi^d) \gamma^2_c \sigma^2_{\eta} + \Phi^d (1 - \gamma_c)^2 \sigma^2_{\nu}.
\]

Then, after some calculations, I obtain
\[
f'(r) = -\frac{\partial \gamma_c}{\partial r} \cdot \frac{\Phi^d}{\eta^2} \left[ 2 - \frac{(1 + \Theta)(1 - r)}{N} \right].
\]

Since \( \frac{\partial \gamma_c}{\partial r} \) is negative, this proves the second assertion.
Let us prove the third assertion. By (29), under perfect information

\[ \tilde{q} = \frac{\sigma\kappa - \Phi^d(\kappa^2 + \lambda)}{(\Phi^h - \Phi^d)(\kappa^2 + \lambda)} = \frac{\lambda^d - \lambda}{(\Theta - 1)(\kappa^2 + \lambda)}, \]

where \( \Theta = \frac{\lambda^d + \kappa^2}{\lambda^h + \kappa^2} \). Note that we may regard \( \tilde{q} \) as a function of \( \kappa^2 \). Differentiating \( \tilde{q} \) with respect to \( \kappa^2 \), we have

\[ \frac{\partial \tilde{q}}{\partial \kappa^2} = -\frac{(\lambda^d - \lambda)}{(\Theta - 1)^2(\kappa^2 + \lambda)} \left[ \frac{\partial \Theta}{\partial \kappa^2} (\kappa^2 + \lambda) + (\Theta - 1) \right]. \]

By \( \frac{\partial \Theta}{\partial \kappa^2} = \frac{\lambda^h - \lambda^d}{(\lambda^h + \kappa^2)^2} \), we obtain

\[ \frac{\partial \Theta}{\partial \kappa^2} (\kappa^2 + \lambda) + (\Theta - 1) = -\frac{\lambda^d - \lambda^h}{(\lambda^h + \kappa^2)^2}. \]

This implies that \( \frac{\partial \tilde{q}}{\partial \kappa^2} \geq 0 \) as long as \( \lambda^h \leq \lambda \leq \lambda^d \). The proof is finished. \( Q.E.D. \)
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Figure 1: The relationship between $\kappa$ and $\tilde{q}$
Figure 2: The relationship between $(\sigma^2_\eta, \sigma^2_\nu)$ and $q^*$