Price-Quantity Competitions and Formation of Buyer-Seller Networks*

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1 Introduction

During 2006-2007, Intel and AMD, which are suppliers of CPU, were said to be experienced a price competition. In 2008, the price competition calmed down, and then the both companies improved their performances. Customers of such suppliers are manufacturers of computer. For example, Dell, HP, Apple, and so on. In such a market, once a manufacturer adopts a component of its product, it usually uses the component over a long period because it is difficult to change the component immediately. Therefore, for a manufacturer, the decision that which components it adopts is important. For a supplier, it is important that how many customers they can get.

We model such a market as a network formation game. Buyers choose components (i.e. sellers) they adopt. Buyers' choices are described by a network, in which a link between a buyer and a seller indicates the buyer adopts the seller's good, and they can trade. Sellers are engaged in price competition with quantity precommitment. Each seller decides a quantity and a price without knowing buyers' exact valuations.

This paper is closely related to the industrial organization literature on capacity competition. For example, by Kreps and Sheinkman (1983), the

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result that *quantity precommitment and Bertrand competition yield Cournot outcomes* is well known.[1]

Furthermore, this paper is related to the model of buyer-seller networks. In a series of papers, Kranton and Minehart (2000a, b, and 2001) analyze the formation of buyer-seller networks. Kranton and Minehart (2001) characterized the equilibrium of the English auction in buyer-seller networks. In their model, the price at which markets clear is competitive, and then, an allocation as a result of trading at the price is efficient. Corominas-Bosh (2004) studies a bargaining between buyers and sellers who are connected by a network. Besides them, the present paper focuses on a competition among sellers.

2 Model

2.1 Players and goods

Let $\mathcal{K} = \{1, \ldots, K\}$ be the set of indivisible goods. Let $\mathcal{B} = \{b_1, \ldots, b_m\}$ be the set of *buyers* who each demand only one indivisible unit of good. Let $\mathcal{S} = \{s_1, \ldots, s_n\}$ be the set of *sellers* who each s_j sell good $k_j \in \mathcal{K}$. Each buyer b_i has valuations v^i_k for good $k \in \mathcal{K}$. Let $\mathcal{V}_k = [0, 1]$ denote the set of valuations for good k. Let $\mathcal{V} = \mathcal{V}_1 \times \cdots \times \mathcal{V}_K$. Let $v^i = (v^i_1, \ldots, v^i_K)$ be a profile of buyer i's valuations. Let $v = (v^1, \ldots, v^m)$ denote a profile of buyers' valuation profiles.

2.2 Networks

Buyers and sellers can exchange only if they are linked. Let $g_{ij} \in \{0, 1\}$ denote a relationship between buyer b_i and seller s_j. If b_i and s_j are linked, then $g_{ij} = 1$, and 0 otherwise. Let G be a $m \times n$ matrix, where the (i, j)th element is g_{ij}. We denote a *network* by G. Let $\mathcal{G}$ be the set of all networks. For a given network G and a set of buyers $\mathcal{B}' \subseteq \mathcal{B}$, let $L(\mathcal{B}') = \{j \in \mathcal{S} : i \in \mathcal{B}' \text{ and } g_{ij} = 1\}$ denote the buyers' *linked set* of sellers. Similarly, for a set of sellers $\mathcal{S}' \subseteq \mathcal{S}$, let $L(\mathcal{S}') = \{i \in \mathcal{B} : j \in \mathcal{S}' \text{ and } g_{ij} = 1\}$ denote the sellers' *linked set* of buyers.

[1] Levitan and Shubik (1980), and Davidson and Deneckere (1990) also study such a market.

2.3 Strategies

Buyers and seller play the following two-stage game.

$$\begin{cases}\textbf{Stage 1.} & \text{Each buyer chooses links that connect to sellers.}\\ & \text{Each seller chooses the quantity of its product.}\\ \textbf{Stage 2.} & \text{Each Seller names the prices for the buyers.}\end{cases}$$

The buyer b_i's valuation for good k is distributed over the interval $\mathcal{V}_k = [0, 1]$ according to uniform distribution function $F\colon \mathcal{V}_k \to [0, 1]$ with associated density function f. All distributions are common knowledge. The valuations are private information for buyers. Sellers do not know the exact valuations.

In Stage 1, buyers and sellers act simultaneously. Let $\lambda_i\colon \mathcal{V} \to \{0, 1\}^n$ be *linking* of buyer b_i, denoted by

$$\lambda_i(v^i) = \big(\lambda_{i1}(v^i), \dots, \lambda_{in}(v^i)\big) \in \{0, 1\}^n,$$

where $\lambda_{ij}(v^i) = 1$ indicates that b_i can adopt seller s_j's good as input after b_i receives his valuation v^i. The network that is formed in this stage is denoted by a $m \times n$ matrix $G = [\lambda_{ij}(v^i)]$. Let $\lambda = (\lambda_1, \dots, \lambda_m)$ denote a profile of buyers' linkings, and let Λ be the set of linkings. We suppose that a link costs d for the buyer. Therefore, buyer b_i pays the cost $\sum_{j=1}^{n} \lambda_{ij}(v^i)d$ in this stage.

Each seller s_j chooses a quantity of production. Let $q_j \in \mathbb{Z}_+$ denote the quantity of s_j's production. Let $q = (q_1, \dots, q_n)$ denote a profile of sellers' production. We assume that producing one unit costs c for the seller.

In Stage 2, only sellers act. Sellers know the network and quantities that are ditermined in Stage 1. Let $p_j\colon \mathcal{G} \to \mathbb{R}^n_+$ be seller s_j's *pricing*, which is denoted by $p_j(G) = (p_{1j}(G), \dots, p_{nj}(G))$, where $p_{ij}(g)$ is the price named by seller s_j for b_i. Let $p = (p_1, \dots, p_n)$ denote a profile of all sellers' pricing.

2.4 Payoffs

We now define buyers' and sellers' payoff function. First, we define a buyer's payoff function. If buyer b_i purchases a good k_j, then the net benefit from good k_j is $v^i_{k_j} - p_{ij}$. Furthermore, buyer b_i pays a linking cost xd, where x is the number of links b_i formed. Then, b_i's payoff is $v^i_{k_j} - p_{ij} - xd$.

The *buyer's payoff function* is $V_i\colon \Lambda \times \mathbb{Z}^n_+ \times \mathbb{R}^n_{++} \times \mathcal{V} \to \mathbb{R}$, defined by

$$V_i(\lambda, q \mid v^i, p) = \int_{\mathcal{V}^{(K-1)}} \delta_{ij}(\lambda, q, p, v^{-i} \mid v^i)(v^i_{k_j} - p_{ij}) f^{K-1}(v^{-i}) dv^{-i} - \sum_{j=1}^{n} \lambda_{ij}(v^i) d,$$

where $\delta_{ij}(\lambda, q, p, p, v^{-i} \mid v^i)$ is the probability that buyer b_i buys from seller s_j. The probability δ_{ij} is defined in the next section.

The *seller's expected profit function* is $\Pi_j\colon \Lambda \times \mathbb{Z}^n_+ \times \mathbb{R}^n_{++} \to \mathbb{R}$, defined by

$$\Pi_j(\lambda, q; p) = \int_{\mathcal{V}^{(n)}} \theta_{ij}(\lambda, q, v, p) p_{ij}([\lambda_{ij}(v^i)]) f^K(v) dv - c q_j,$$

where $\theta_{ij}(\lambda, q, v, p)$ is a probability that seller s_j sells a good to buyer b_i when prices are p_{ij}. The probability θ_{ij} is defined in the next section.

This two-stage game is solved backwardly. In Stage 2, given quantities and the network, each seller names prices to maximize its expected profit. At the end of Stage 1, each seller updates information of the buyers' valuations. For example, if buyer b_i forms a link to seller s_j, then buyers consider that $v^i_{k_j} > d$. In this case, we suppose that sellers consider that b_i's valuation is uniformly ditrbuted over $(d, 1]$. Each seller maximizes its expected profit by using the newly obtained distribution function $\tilde{f}^K$. The *expected revenue* in Stage 2 is

$$\int_{\mathcal{V}^{(n)}} \theta_{ij}(\lambda, q, p, v) p_{ij}(G) \tilde{f}^K(v) dv.$$

Therefore, seller s_j names prices $p_{ij}, i = 1, \ldots, n$ such that

$$\int_{\mathcal{V}^{(n)}} \theta_{ij}(\lambda, q, p, v) p_{ij}(G) \tilde{f}^K(v) dv \geq \int_{\mathcal{V}^{(n)}} \theta_{ij}(\lambda, q, p'_i, p_{-i}, v) p'_{ij}(G) \tilde{f}^K(v) dv.$$

In Stage 1, each buyer chooses its links to maximize the expected payoff V_i subject to the expected prices that will be named in Stage 2. Each seller chooses quantities to maximize the expected payoff Π_j.

3 A small market: two buyers and two sellers

We consider a small market that both numbers of buyers and sellers are two. We suppose that sellers s_1 and s_2 produce different goods 1 and 2, respectively.

For simplicity, we suppose that costs of producing a unit and forming a link are two types: $c_L = 1/16, c_H = 1/4, d_L = 1/16$, and $d_H = 1/4$.

3.1 Sellers' pricing

In Stage 2, sellers name their prices to buyers. Since if a seller produces no unit, then the seller's expected revenue is zero, We now consider all sellers produce at least one unit. We have three cases as follows.

- **Case 1. A buyer links only one seller.**

  Let b_i and s_j be lnked, and b_i and $s_{j'}$ not be linked. Then, seller s_j consider buyer b_i's valuation as $v_j^i > d$. Thus, s_j calculates the distribution of b_i's valuation as the uniform distribution over $[d, 1]$, which is denoted by $\tilde{F}\colon [d, 1] \to [0, 1]$. If seller s_j names the price as p_{ij}, then s_j's expected revenue is $p_{ij}(1 - \tilde{F}(p_{ij}))$. Since s_j maximizes its expected revenue without considering a $s_{j'}$'s price, s_j prices $(d + 1)/2$. Thus, if $d = d_L$ or $d = d_H$, then $p_{ij} =$ or $p_{ij} =$, respectively.

- **Case 2. A buyer links two sellers.**

  Let b_i and s_1, and b_i and s_2 be linked. Then, each seller considers b_i's valuation as $v_j^i > 2d$. Sellers update the distribution of b_i's valuation as the uniform distribution over $[2d, 1]$, which is denoted by $\tilde{F}\colon [2d, 1] \to [0, 1]$. Each seller prices considering the price named by the other seller.

  If $v_1^i - p_{i1} > v_2^i - p_{i2}$, then b_i buys from s_1. Thus, the probability that b_1 buys from s_1 is

$$\begin{cases} (1-2d)^2 - \big((1-2d) - (p_{i2} - p_{i1})\big)^2/2 & \text{if } p_{i1} \le p_{i2}, \\ \big(1 - 2d - (p_{i1} - p_{i2})\big)^2/2 & \text{if } p_{i1} > p_{i2}. \end{cases}$$

Thus, seller s_1's expected revenue is

$$\begin{cases} p_{i1}\big((1-2d)^2 - \big((1-2d) - (p_{i2} - p_{i1})\big)^2\big)/2 & \text{if } p_{i1} \le p_{i2}, \\ p_{i1}\big(1 - 2d - (p_{i1} - p_{i2})\big)^2/2 & \text{if } p_{i1} > p_{i2}. \end{cases}$$

Similarly, seller s_2's expected profit is

$$\begin{cases} p_{i2}\big((1-2d)^2 - \big((1-2d)-(p_{i1}-p_{i2})\big)^2\big)/2 & \text{if } p_{i2} \le p_{i1}, \\ p_{i2}\big(1-2d-(p_{i2}-p_{i1})\big)^2/2 & \text{if } p_{i2} > p_{i1}. \end{cases}$$

In equilibrium, they name the price $(2d+1)/2$. Thus, if $d = d_L$ or $d = d_H$, then $p_{ij} =$ or $p_{ij} =$, respectively.

- **Case 3. No buyer forms any link.**

  Let no link be formed. Then, each seller consider buyer b_i's valuation as $v_j^i < d$. In this case, no trade occurs.

In the model, sellers consider that if a buyer forms two links, then they have the higer valuations than those that when they form only one link.

3.2 Buyers' linking and sellers' production

In Stage 1, buyers choose links, and sellers determine quantities. Buyers and sellers decide their actions considering the prices will be set in Stage 2. The results of the game in Period 1 are summarized as follows.

Proposition 1. There exist three-types symmetric equilibria due to the link cost and the production cost.
(i) Each buyer forms at most one link, and each seller produces one unit if a link cost is d_H and a production cost is c_H.

(ii) Each buyer forms at most one link, and each seller produces two units if a link cost is d_H and a production cost is c_L.

(iii) Each buyer forms at most two links, and each seller produces one unit if a link cost is d_L and a production cost is c_H.

The proof is in Appendix.

4 Concluding remarks

We consider that price-quantity competitons in the networked market. Even if a buyer can exchange multiple sellers, prices that the buyer faces are higher than those that when it can exchange only one buyer. The reason is that to trade with multiple sellers, a buyer form multiple links spending a large cost and, then, sellers consider that the buyer's valuations for goods are so high. Thus, in the model, price competitions do not work similar to the Bertrand competition, in which prices ditermined by sellers are set in competitive levels. Particulary, the Bertrand competition leads the price to the sufficiently low level in the sense that the price is equal to the marginal cost of production.

In our model, competitive prices are as follows. For a network G and valuations v, a price vector p is *competitive* if:
(i) if a buyer b_i and a seller s_j exchange good k_j, then $v^i_{k_j} \geq p_{ij} \geq 0$ and $p_{ij} = \min\{p_{ij'} : s_{j'} \in L(b_i)\}$ and $v^i_{k_j} - p_{ij} \geq v^i_{k_{j'}} - \min\{p_{ij''} : s_{j''} \in L(b_i)\}$,
(ii) if a buyer b_i does not buy a good, then $v^i_k \leq \min\{p_{ij} : j \in L(b_i)\}, k \in \mathcal{K}$, and
(iii) if a seller s_j does not sell a good then $p_{ij} = 0$.

Cleally, if sellers produce one unit, then prices need not satisfy the condition (ii). Furthermore, even if prices set in the game is competitive, there is no guarantee that the prices are sufficiently low. This result is derived from incomplete information.

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