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BRAND PROLIFERATION IS USELESS TO DETER ENTRY

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Abstract

This paper considers an incumbent firm that is faced with a potential entrant in a vertically differentiated market. It demonstrates that an incumbent firm cannot prevent entry through product proliferation because of a commitment problem. The incumbent always makes one product only, and it degrades the quality to deter entry of a low-quality firm if entry is not blockaded. Hence the social welfare decreases as the entrant becomes more dangerous.

Keywords: entry deterrence, vertical differentiation, brand proliferation, commitment.

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1. Introduction

This paper considers an incumbent firm that is faced with a potential entrant in a vertically differentiated market. It shows that to make two or more kinds of products and fill up the market is *not* an effective measure of entry deterrence. Suppose an incumbent firm makes some kinds of products and entry occurs near to one of them. Keeping all products enables the incumbent to discriminate among its customers, but it induces tough competition with the entrant. Withdrawal of the products near to the entrant, on the other hand, relaxes competition and increases the incumbent's profit from the remaining products. Thus the incumbent withdraws competing products and consequently entry occurs in equilibrium.

Ironically, choosing two or more kinds of products makes entry *easier*: it warrants an entrant a large market because the incumbent is forced to withdraw all products near to the entrant. Therefore, faced with an entrant, the incumbent always chooses one kind of product in equilibrium. [1] It selects the good of the highest quality when fixed cost is large and entry is blockaded. It chooses a product of middle quality to prevent entry of a low-quality firm when fixed cost decreases. Finally, when fixed cost is too small to deter entry by one product, it produces the highest quality to secure its profit.

The above result indicates that the social welfare *increases* in fixed cost when the incumbent deters entry. It cuts down the quality of the product as fixed cost decreases, and each customer obtains less utility from consuming it. Therefore both of the incumbent's profit and consumer's surplus are reduced.

There are a considerable number of studies about product-line selection in a differentiated market. We can classify them into three types. The studies of the first type assume that two firms choose their product varieties simultaneously. Gal-Or (1983) and Wernerfelt (1986) investigate the optimal product line when firms compete in quantity. Brander and Eaton (1984) analyze the optimal product choice when each firm chooses two varieties. Martinez-Giralt and Neven (1988) consider the horizontal differentiation model of d'Aspremont, Gabszewicz, and Thisse (1979) (Hotelling (1929)'s model with quadratic transportation cost: we call it 'Horizontal model' hereafter) and show that firms choose only one product each even if they can select any number. Martinez-Giralt (1989) obtains the same result in the vertically differentiated Hotelling model with quadratic transportation cost. [2] Champsaur and Rochet (1989) and Cremer and Thisse (1991) prove that the vertical differentiation model of Mussa and Rosen (1978) is mathematically equivalent to Horizontal model if marginal cost is a quadratic function of the quality. Therefore two firms choose one product each in the model of Mussa and Rosen (1978). However, the studies of this type leave out of account that an incumbent firm can choose its product line in advance of an entrant.

The papers of the second type assume that firms move sequentially. Schmalensee (1978) argues that an incumbent firm can successfully prevent entry by producing enough varieties. Bonanno (1987) uses Horizontal model and shows that an incumbent firm can stop entry by changing location of its products. Constantatos and Perrakis (1997) obtains a similar result in the vertical differentiation model of Gabszewicz and Thisse (1979). Nevertheless, these papers have two shortcomings: they cannot explain why monopoly is rare in reality, and they do not check whether or not the incumbent's strategy is credible.

The third type explicitly deals with the commitment problem of an incumbent firm after entry occurs. Judd (1985) shows with a two-goods model that, if exit cost is small, the incumbent cannot stop entry by choosing both goods because it has an ex post incentive to withdraw the product entry occurs. Ashiya (1998) proves that choosing two or more products does not help the incumbent deter entry in Horizontal model. This paper demonstrates robustness of their results: when exit cost is small, brand proliferation is useless for an incumbent to prevent entry in a vertically differentiated market.

The paper is organized as follows. Section 2 describes the model. Section 3 shows that the incumbent withdraws products near to the entrant. Section 4 investigates the optimal product of the incumbent given fixed cost. Section 5 analyzes the social welfare, and Section 6 considers the extended model where three firms move sequentially. Section 7 concludes this paper. The Appendix contains the formal proofs.

2. The model

We use the vertical differentiation model of Gabszewicz and Thisse (1979): A denotes an incumbent and B denotes an entrant. [3] The timing of each firm's action, which is common knowledge, is as follows. At date 1, A chooses a set of products $Q_A = \{q_1, \cdots, q_n\}$ ($q_1 < \cdots < q_n$) from a technologically feasible range of qualities $(1, Q]$. We assume $Q = 6$ for tractability. [4] At date 2, B observes Q_A and chooses $Q_B = \{q_B, q_{B2}, \cdots q_{Bm}\}$ ($q_B < \cdots < q_{Bm}$). Each firm sinks a fixed cost F per product if it enters the market. If firm i does not enter the market, $Q_i = \phi$.

At date 3, each firm observes (Q_A, Q_B) and simultaneously selects a set of products to withdraw. Since a firm can get out of the market at will in reality, each firm can withdraw its product(s) with no additional cost (It cannot recover the fixed cost). [5] Let $\hat{Q}_i$ be the set of products firm i does not withdraw ($\hat{Q}_i \subseteq Q_i$). If $\hat{Q}_i = \{q_k\}$, we write $\hat{Q}_i = q_k$.

At date 4, each firm observes $(\hat{Q}_A, \hat{Q}_B)$, and simultaneously selects prices of its products. Each pays variable costs and earns sales revenue. Each firm makes goods at constant marginal cost, which is assumed to be zero regardless of the quality.

Consumers are identical in tastes but differing in income. Their incomes are uniformly distributed on the segment $[1, h]$. Shaked and Sutton (1982) show that only two firms of the highest and the second highest quality can earn positive gross profit if $2 < h < 4$. Only one firm of the highest quality can earn positive profit if $h < 2$. Thus we assume $h = 3$ (Consequently, consumers are distributed with density 0.5). Each consumer purchases one unit of the good for which her indirect utility is maximized, or buys nothing if it is better. The utility of a consumer of income $y \in [1, 3]$ who bought q_i at price P_i is

$$U(q_i; y) = q_i (y - P_i).$$

If she bought nothing, her utility is

$$U(0; y) = y.$$ [6]

The equilibrium concept we adopt is a weak refinement of subgame-perfect Nash equilibrium that assumes no weakly dominated strategy is played in equilibrium. [7] Let $\Pi_i(\hat{Q}_A, \hat{Q}_B)$ be the equilibrium profit of firm i gross of fixed cost. Entry occurs when Π_i is larger than the fixed cost.

3. The equilibrium profits

Gabszewicz and Thisse (1979) calculate the equilibrium prices when A and B choose one product each. We derive B's oprimal product using it. Lemma 1 shows that B will locate its product far apart from A to mitigate competition. (The proofs of all Lemmata and propositions can be found in the Appendix).

Lemma 1.

(a) Suppose $q_1 < 6$. Then $\Pi_B(q_1, 6) > \Pi_B(q_1, q_B)$ for any $q_B \in [q_1, 6)$.

(b) Suppose $q_1 \leq 3$. Then $\Pi_B(q_1, 0.25(q_1 + 3)) \geq \Pi_B(q_1, q_B)$ for any $q_B \leq q_1$.

(c) Suppose $q_1 > 3$. Define

$$q_B^*(q_1) \equiv \frac{-1 - q_1 + (q_1 - 1)\sqrt{1 + q_1}}{q_1 - 3}.$$

Then $\Pi_B(q_1, q_B^*(q_1)) \geq \Pi_B(q_1, q_B)$ for any $q_B \leq q_1$.

Lemma 2 considers the case that A chooses n (≥ 2) goods and B chooses higher quality than A. Since we assume consumers' incomes (i.e. willingness to pay for quality) are similar, everyone prefers 'an expensive but high-quality good' to 'a cheap but low-quality good'. Accordingly A's products have no sales in equilibrium except the highest quality. Then it is better for A to withdraw products near to B and relax competition.

Lemma 2.

Suppose A chooses n (≥ 2) goods ($q_1 < \cdots < q_n$) and B chooses $q_B > q_n$. Define

$$q_M \equiv \max\{q_i | \Pi_A(q_i, q_B) \geq \Pi_A(q_j, q_B) \quad \forall j\}.$$

Then A withdraws at least all products larger than q_M, and B earns $\Pi_B(q_M, q_B)$ or more in this subgame.

Lemma 3 considers the case that B chooses a low quality. If A keeps its all products, it can separate the market and operate discrimination. The gain from it is small, however, because consumers' tastes are similar in our model. Hence A withdraws the products near to B in order to avoid competition. Lemma 2 and Lemma 3 show that choosing two or more products is useless for entry deterrence.

Lemma 3.

(a1) Suppose $q_{n-1} \le q_B < q_n$ and $q_B \le 0.25(3q_{n-1} + q_n)$. Then A keeps only q_n and B earns $\Pi_B(q_n, q_B)$ in the unique equilibrium of this subgame.

(a2) Suppose $q_{n-1} \le q_B < q_n$ and $q_B > 0.25(3q_{n-1} + q_n)$. Then B earns $\Pi_B(q_n, q_B)$ in any equilibrium of this subgame.

(b) Suppose $q_{k-1} \le q_B < q_k < \cdots < q_n$ and $2 \le q_B \le 0.25(3 + q_n)$. Then A keeps only q_n and B earns $\Pi_B(q_n, q_B)$ in the unique equilibrium of this subgame.

4. The optimal product of the incumbent

The last section proved that an incumbent firm faces the commitment problem if it has two or more products. It cannot commit to keep them after entry occurs since withdrawal of competing products relaxes competition and raises profits from the remaining products.

This section shows that the incumbent never chooses more than one product in equilibrium. Figure 1 indicates the optimal product of the incumbent as a function of fixed cost. When fixed cost is large enough to blockade entry, A chooses the product of the highest quality (The proofs can be found in the Appendix).

Proposition 1 (Blockaded entry).

Suppose $F \ge \Pi_B(6, q_B^*(6))$. Then A chooses $Q_A = 6$ and B does not enter the market in the unique equilibrium.

Proposition 2 investigates the case that A cannot deter entry by choosing the highest quality. It shows that, in order to prevent entry of a low-quality firm, A degrades the quality of its product as fixed cost decreases.

Lemma 4.

Define q_1^* such that

$$\Pi_B(q_1^*, 6) = \Pi_B(q_1^*, q_B^*(q_1^*)).$$

Then A can deter entry by choosing one product if and only if $F \geq \Pi_B(q_1^*, 6)$.

Proposition 2 (Deterred entry).

Suppose $\Pi_B(q_1^*, 6) \leq F < \Pi_B(6, q_B^*(6))$. Define $\overline{q}_1$ such that

$$\Pi_B(\overline{q}_1, q_B^*(\overline{q}_1)) = F.$$

Then A chooses $Q_A = \overline{q}_1$ and B does not enter the market in the unique equilibrium.

When fixed cost is too small to deter entry, A chooses the highest quality to secure its profit.

Proposition 3 (Allowed entry).

Suppose $F < \Pi_B(q_1^*, 6)$. Then A chooses $Q_A = 6$ and B chooses $Q_B = q_B^*(6)$ in the unique equilibrium.

The combination of Proposition 1, 2, and 3 yields Theorem 1: the incumbent always chooses one product because it has an ex post incentive to withdraw all but one product after entry occurs. Calculation shows that A would choose two products if there were no entrant and $F < \Pi_A(\{\sqrt{6}, 6\}, \phi) - \Pi_A(6, \phi)$. Therefore, contrary to the argument of Schmalensee (1978), A *stops* proliferating its brand when there is an entrant.

Theorem 1.

A chooses one product in equilibrium regardless of F.

Corollary of Theorem 1.

The number of products A chooses when faced with an entrant is equal to or smaller than that in the absence of an entrant.

5. Welfare analysis

Let us define the social welfare, W, as the sum of consumer's surplus and each firm's net profit. This section investigates how the social welfare changes as fixed cost decreases. See Figure 2.

When fixed cost is so large that entry is blockaded, the incumbent firm always chooses the same quality and price. Thus the social welfare increases by the same amount as the decrease of fixed cost.

When fixed cost becomes small and entry is not blockaded, the incumbent degrades the quality as fixed cost decreases. Consumer's surplus decreases since each buyer obtains lower utility and calculation shows that the market served by the incumbent does not change. Therefore, if the incumbent deters entry, the social welfare *decreases* as fixed cost decreases (i.e. as the entry threat is strengthened).

At last, when fixed cost is so small that the incumbent cannot deter entry, each firm chooses the fixed product ($Q_A = 6$ and $Q_B = q_B^*(6)$). Consequently the social welfare increases by the same amount as the decrease of fixed cost. The social welfare under duopoly is larger than that under monopoly because competition drives the prices down and more people buy the top quality good.

Proposition 4.

Define W as the sum of consumer's surplus and each firm's net profit. Then

(a) $\frac{dW}{dF} = -1$ if $F \geq \Pi_B(6, q_B^*(6))$;

(b) $\frac{dW}{dF} > 0$ if $\Pi_B(q_1^*, 6) \leq F < \Pi_B(6, q_B^*(6))$;

(c) $\frac{dW}{dF} = -2$ if $F < \Pi_B(q_1^*, 6)$;

(d) W under duopoly is always larger than that under monopoly.

6. Extensions

This section extends the model and assumes that the third firm, C, moves after firm B. Then A and B change the qualities of their products as the fixed cost decreases. When fixed cost is large, entry of C is blockaded: A and B choose $Q_A = 6$ and $Q_B = q_B^*(6)$. Otherwise A continues to choose the top quality in order to charge a high price, and B is forced to upgrade its product in order to deter entry. Since we assume that income dispersion is small, the third firm cannot enter the market for any positive fixed cost. [8]

Proposition 5.

Suppose the third firm, C, moves after firm B. Let $\Pi_i(q_A, q_B, q_C)$ be the profit function of firm i. Then

(a) C never enters the market in equilibrium.

(b) $(q_A, q_B) = (6, q_B^*(6))$ in the unique equilibrium if

$$\Pi_C(6, q_B^*(6), 0.6(4 + q_B^*(6))) \le F < \Pi_B(q_1^*, 6, \phi).$$

(c) $(q_A, q_B) = (6, \overline{q}_B)$ where $\Pi_C(6, \overline{q}_B, 0.6(4 + \overline{q}_B)) = F$ in the unique equilibrium if

$$\Pi_C(6, 2.25, 3.75) \le F < \Pi_C(6, q_B^*(6), 0.6(4 + q_B^*(6))).$$

(d) $(q_A, q_B) = (6, \overline{q}_B)$ is an equilibrium if $F < \Pi_C(6, 2.25, 3.75)$.

7. Conclusions

When entry occurs, it may be profitable for an incumbent firm to withdraw products near to the entrant and relax competition. We have explicitly dealt with this commitment problem, and have proved that choosing two or more kinds of products cannot deter entry in a vertically differentiated market.

We have also shown that the entry threat causes the incumbent *not* to fill up the market by brand proliferation: the incumbent firm always chooses one good in equilibrium. If entry is not blockaded, the incumbent chooses a product of lower quality as fixed cost decreases. Therefore the social welfare is decreasing in entry threats when the incumbent deters entry.

Notes

1. If there were no entrant and fixed cost were small, the incumbent would choose two or more products to screen its customers.
2. It assumes that consumers are located on $[0,1]$ and products are located on $[1,\infty)$.
3. Section 6 considers the model with three firms.
4. Other studies assume narrower ranges than ours. For example, Constantatos and Perrakis (1997) consider the cases of $Q \in [1.3, 5]$. Our argument can be easily extended when Q takes other values.
5. Exit cost is the cost arising only because of the act of exit. One example is printing cost of a new catalogue (from which withdrawn products are deleted). Note that irreversible investment in product-specific capital is sunk cost and is not exit cost.
6. This model differs from Horizontal model in that all consumers choose the product of the highest quality when prices of all products are equal to their marginal costs.
7. Consider the subgame where both of the incumbent and the entrant enter the market. If exit cost is zero, it is also a subgame-perfect Nash equilibrium after this history that the entrant exits and the incumbent keeps its products at date 3. We need further refinement to exclude this rather unrealistic equilibrium.
8. If consumers' incomes are uniformly distributed on $[1,h]$ and $h > 4$, firm C is viable and $q_C < q_B < q_A = 6$ in equilibrium for sufficiently small fixed cost.

Appendix: Formal proofs of lemmata.

Proof of Lemma 1.

From Gabszewicz and Thisse (1979),

$$\Pi_B(q_1, q_B) = \frac{(3q_B - 2q_1 - 1)^2}{8q_B(q_B - q_1)} \text{ if } q_B \geq 4q_1 - 3;$$

$$= \frac{25(q_B - q_1)}{18q_B} \text{ if } q_1 \leq q_B \leq 4q_1 - 3;$$

$$= \frac{(q_1 - q_B)}{18q_B} \text{ if } 0.25(q_1 + 3) \leq q_B \leq q_1; \text{ and}$$

$$= \frac{(q_B - 1)(q_1 - 2q_B + 1)}{4q_B(q_1 - q_B)} \equiv \pi_1 \text{ if } q_B \leq 0.25(q_1 + 3)$$

(Note that the density of consumers is 0.5 in our model). Then

$$\frac{\partial \pi_1}{\partial q_B} = \frac{q_1^2 + q_1 - 2q_1 q_B - q_1 q_B^2 + 3q_B^2 - 2q_B}{4q_B^2 (q_1 - q_B)^2}$$

and it is positive at $q_B = 0.25(q_1 + 3)$ if and only if $q_1 < 3$. Thus it is straightforward to show (a), (b), and (c). Q.E.D.

Proof of Lemma 2.

B always keeps q_B (and charges $P_B > 0$) because it is the weakly dominant strategy. Define $q_Z \equiv \max\{q_i | q_i \in \hat{Q}_A\}$. Then $P_i = 0$ for any $i \neq \{Z, B\}$ and

$$\Pi_A(\hat{Q}_A, q_B) = \Pi_A(q_Z, q_B) = \begin{cases} \dfrac{(q_B - q_Z)}{18q_Z} & \text{if } q_Z \geq 0.25(q_B + 3) \\ \dfrac{(q_Z - 1)(1 + q_B - 2q_Z)}{4q_Z(q_B - q_Z)} & \text{otherwise} \end{cases}$$

in equilibrium. Thus *A* withdraws any q_i that satisfies $\Pi_A(q_i, q_B) < \Pi_A(q_j, q_B)$ for some $j < i$. Consequently $q_Z \leq q_M$ must hold in equilibrium. Since $\Pi_B(\hat{Q}_A, q_B)$ $(= \Pi_B(q_Z, q_B))$ is decreasing in q_Z, *B* earns $\Pi_B(q_M, q_B)$ or more. Q.E.D.

Proof of Lemma 3 (a1).

Suppose A keeps some products smaller than q_B. Define

$$q_X \equiv \max\left\{q_i \middle| q_i \in \hat{Q}_A \text{ and } q_i \le q_B\right\}.$$

Then $P_B = \dfrac{q_B - q_X}{q_B}$, $P_n = \dfrac{3q_n - 2q_B - q_X}{2q_n}$, and $P_i = 0$ for any $i \ne \{n, B\}$ in equilibrium.

Hence $\Pi_A\left(\hat{Q}_A, q_B\right) = \dfrac{(3q_n - 2q_B - q_X)^2}{8q_n(q_n - q_B)}$. Since Lemma 1 shows that

$$\Pi_A(q_n, q_B) = \begin{cases} \dfrac{25(q_n - q_B)}{18q_n} & \text{if} \quad \mathrm{q_B} \ge 0.25(q_n + 3) \\ \dfrac{(3q_n - 2q_B - 1)^2}{8q_n(q_n - q_B)} & \text{otherwise} \end{cases}.$$

A has an incentive to withdraw all products except q_n. Q.E.D.

Proof of Lemma 3 (a2)

When $q_B > 0.25(3q_{n-1} + q_n)$ and A keeps q_n, B chooses $P_B = \dfrac{q_n - q_B}{3q_B}$ and earns $\Pi_B(q_n, q_B)$ even if A keeps some products smaller than q_B. Q.E.D.

Proof of Lemma 3 (b)

If A keeps some $q_i \le q_B$, $P_i = 0$ in equilibrium and it causes negative effect on P_B. Therefore A withdraws any $q_i \le q_B$ in equilibrium.

Suppose A keeps q_k and q_{k+1}. Then in equilibrium

$$P_B = 0.5q_B^{-1}Z^{-1}\left(2q_k^3 - q_k q_B(q_k + q_{k+1}) + q_B^2(q_{k+1} - q_k)\right),$$

$$P_k = Z^{-1}(3q_k + 2q_{k+1})(q_k - q_B),$$

$$P_{k+1} = 0.5Z^{-1}\left(2q_k(q_k + 4q_{k+1}) + q_B(q_{k+1} - 11q_k)\right),$$

and $\Pi_A(\{q_k, q_{k+1}\}, q_B) = 0.125\, Z^{-2}V$

where $Z \equiv q_k(q_k + 2q_{k+1}) + q_B(q_{k+1} - q_k)$

and $V \equiv 12q_k^4 + q_k^3(40q_{k+1} - 42q_B) + 2q_k^2\left(24q_{k+1}^2 - 43q_{k+1}q_B + 15q_B^2\right)$

$$+ 7q_k q_{k+1} q_B(4q_{k+1} - 5q_B) + 5q_{k+1}^2 q_B^2.$$

Define $\Delta\Pi_{21} \equiv \Pi_A(\{q_k, q_{k+1}\}, q_B) - \Pi_A(q_{k+1}, q_B)$. Then $\Delta\Pi_{21}$ is maximized at $q_B = 2$ because $\frac{\partial^2}{\partial q_B^2}\Delta\Pi_{21} > 0$ and $\frac{\partial}{\partial q_B}\Delta\Pi_{21} < 0$ at $q_B = 2.25$. Given $q_B = 2$, $\Delta\Pi_{21} < 0$ at $q_k = q_B$, $\Delta\Pi_{21} = 0$ at $q_k = q_{k+1}$, and $\frac{\partial^2}{\partial q_k^2}\Delta\Pi_{21} > 0$. Consequently $\Delta\Pi_{21} < 0$ is always satisfied and *A* withdraws q_k in equilibrium.

Next suppose *A* keeps *n* products. Then $\Pi_A(\{q_k, \cdots, q_n\}, q_B) - \Pi_A(q_n, q_B)$ is decreasing in q_n for given q_B. The reason is that a change in q_n has a direct effect on P_B in the latter case, while it has an indirect effect on P_B in the former case. Hence

$$
\begin{aligned}
&\Pi_A(\{q_k, \cdots, q_{n-1}, q_n\}, q_B) - \Pi_A(q_n, q_B) \\
&< \Pi_A(\{q_k, \cdots, q_{n-1}, q_{n-1}\}, q_B) - \Pi_A(q_{n-1}, q_B) \\
&= \Pi_A(\{q_k, \cdots, q_{n-1}\}, q_B) - \Pi_A(q_{n-1}, q_B) \\
&< \Pi_A(\{q_k, \cdots, q_{n-2}\}, q_B) - \Pi_A(q_{n-2}, q_B) \\
&< \cdots < \Pi_A(\{q_k, q_{k+1}\}, q_B) - \Pi_A(q_{k+1}, q_B) = \Delta\Pi_{21} < 0
\end{aligned}
$$

for given q_B. Namely *A* withdraws $\{q_k, \cdots, q_{n-1}\}$ in equilibrium. Q.E.D.

Proof of Lemma 4.

Since $\Pi_B(3,6) > \Pi_B(3,1.5)$, $q_1^* > 3$ must hold. Then Π_B is maximized at either $q_B = 6$ or $q_B = q_B^*(q_1)$ from Lemma 1. Hence choosing q_1^* deters entry if $F \geq \Pi_B(q_1^*, 6)$. If $F < \Pi_B(q_1^*, 6)$, *B* can enter either $q_B = 6$ or $q_B = q_B^*(q_1)$. Q.E.D.

Proof of Proposition 1.

If $Q_A = 6$, *B* does not enter the market because Lemma 1 and Lemma 2 show that it can earn $\Pi_B(6, q_B^*(6)) - F$ at most. Then *A* earns $\Pi_A(6, \phi) - F$. If $Q_A = q_1 < 6$, Π_A decreases because

$$\Pi_A(q_1, \phi) = 1.125(1 - q_1^{-1}).$$

If *A* produces the second product,

$$\Pi_A(\{q_1, q_2\}, \phi) = \frac{4.5q_1(q_2 - 1)}{q_2 + 3q_1q_2 - q_1 + q_1^2}$$

and calculation shows that it can earn at most

$$\max_{Q_A} \Pi_A(\{q_1, q_2\}, \phi) - 2F = \Pi_A(\{\sqrt{6}, 6\}, \phi) - 2F$$

$$(\leq \Pi_A(\{\sqrt{6}, 6\}, \phi) - \Pi_B(6, q_B^*(6)) - F < \Pi_A(6, \phi) - F).$$

Therefore A never chooses two or more products. Q.E.D.

Proof of Proposition 2.

If $Q_A = \overline{q}_1$, B does not enter the market and A earns $\Pi_A(\overline{q}_1, \phi) - F$. We shall prove Π_A decreases when A deviates from this. First suppose that A chooses one product other than $\overline{q}_1$. If $Q_A = q_1 < \overline{q}_1$, Π_A decreases because $\Pi_A(q_1, \phi)$ is increasing in q_1. If $Q_A = q_1 > \overline{q}_1$, B enters $q_B^*(q_1)$ because $\Pi_B(q_1, q_B^*(q_1)) > F$ and Lemma 2 shows that choosing other product(s) decreases Π_B. Then A earns $\Pi_A(q_1, q_B^*(q_1)) - F$ ($\leq \Pi_A(6, q_B^*(6)) - F$), and numerical calculation shows that $\Pi_A(6, q_B^*(6)) < \Pi_A(\overline{q}_1, \phi)$.

Next suppose that A chooses $n \geq 2$ products and $q_i > q_B^*(6)$ if and only if $i \geq k$, and that B does not enter the market. Lemma 2 shows that, in order to deter entry of B, q_k and q_{k-1} must satisfy

$$\Pi_B(q_k, 6) = \frac{25(6 - q_k)}{108} \leq F(\leq \Pi_B(6, q_B^*(6)))$$

and $\Pi_A(q_{k-1}, 6) < \Pi_A(q_k, 6)$.

Define $\overline{q}_k$ such that $\Pi_B(\overline{q}_k, 6) = \Pi_B(6, q_B^*(6))$, and define $\overline{q}_{k-1}$ such that $\overline{q}_{k-1} \leq q_B^*(6)$ and $\Pi_A(\overline{q}_{k-1}, 6) = \Pi_A(\overline{q}_k, 6)$. Then, since calculation shows that

$$\Pi_A(q, \phi) < \Pi_B(q_1^*, 6)(\leq F) \text{ for any } q \leq \overline{q}_{k-1},$$

A does not have an incentive to choose products smaller than $q_B^*(6)$. Hence $q_1 \geq \overline{q}_k$ if A deters entry of B. On condition that $q_1 \geq \overline{q}_k$,

$$\max \Pi_A(\{q_1, \cdots, q_{n-1}, q_n\}, \phi) - \max \Pi_A(\{q_1, \cdots, q_{n-1}\}, \phi)$$
$$< \max \Pi_A(\{q_1, \cdots, q_{n-2}, q_{n-1}\}, \phi) - \max \Pi_A(\{q_1, \cdots, q_{n-2}\}, \phi),$$

and calculation shows that

$$\max \Pi_A(\{q_1, q_2, q_3\}, \phi) = \Pi_A\left(\left\{\overline{q}_k, \sqrt{6\overline{q}_k}, 6\right\}, \phi\right)$$

$$< \Pi_A(\{\overline{q}_k, 6\}, \phi) + \Pi_B(q_1^*, 6) \leq \Pi_A(\{\overline{q}_k, 6\}, \phi) + F.$$

Namely, choosing more than two products decreases net profit. Thus A can earn $\Pi_A(\{\overline{q}_k, 6\}, \phi) - 2F$ ($\leq \Pi_A(\{\overline{q}_k, 6\}, \phi) - \Pi_B(q_1^*, 6) - F$) at most. Since calculation shows that $\Pi_A(\{\overline{q}_k, 6\}, \phi) - \Pi_B(q_1^*, 6) < \Pi_A(\overline{q}_1, \phi)$, Π_A decreases by deviation.

Finally suppose that A chooses $n \geq 2$ products and $q_i > q_B^*(6)$ if and only if $i \geq k$, and that B enters the market. We will determine an upper bound of A's payoff. Since $\Pi_A(\hat{Q}_A, \{q_B, \cdots, q_{Bm}\}) \leq \Pi_A(\hat{Q}_A, q_B)$, we assume B chooses one product. If $q_k < \overline{q}_k$, B enters $q_B = 6$. Then

$$\Pi_A(\hat{Q}_A, 6) \leq \max\{\Pi_A(q_{k-1}, 6), \Pi_A(q_k, 6)\} \text{ (from Lemma 2)}$$

$$< \Pi_A(6, q_B^*(6)) < \Pi_A(\overline{q}_1, \phi) \text{ (from Lemma 1).}$$

If $q_k \geq \overline{q}_k$, B chooses $q_B^*(q_n)$ and calculation shows that

$$2 < q_B^*(\overline{q}_k) \leq q_B^*(q_n) < 0.25(q_n + 3).$$

Then Lemma 3 (b) shows that A withdraws all products except q_n, and it earns $\Pi_A(q_n, q_B^*(q_n)) - nF$ ($< \Pi_A(\overline{q}_1, \phi) - F$). Accordingly Π_A decreases in this case. Q.E.D.

Proof of Proposition 3.

If $Q_A = 6$, B chooses only one product because Lemma 2 shows that only the highest quality among $\hat{Q}_B$ has positive sales in equilibrium. Thus B chooses $q_B^*(6)$ to maximize its profit, and A earns $\Pi_A(6, q_B^*(6)) - F$. We shall prove Π_A decreases when A deviates from this.

Suppose A chooses $Q_A = q_1 < 6$. Then B enters either $q_B = 6$ or $q_B = q_B^*(q_1)$. A earns either $\Pi_A(q_1, 6) - F$ or $\Pi_A(q_1, q_B^*(q_1)) - F$, both of which is smaller than $\Pi_A(6, q_B^*(6)) - F$.

Next suppose that A chooses $n \geq 2$ products and $q_i > q_B^*(6)$ if and only if $i \geq k$.

Since we consider an upper bound of A's payoff, assume that B chooses one product. If $q_k < q_1^*$, B enters $q_B = 6$. Then

$$\Pi_A\left(\hat{Q}_A, 6\right) \leq \max\{\Pi_A(q_{k-1}, 6), \Pi_A(q_k, 6)\} \text{ (from Lemma 2)}$$

$$< \Pi_A\left(6, q_B^*(6)\right) \text{ (from Lemma 1).}$$

If $q_k \geq q_1^*$, B chooses $q_B^*(q_n)$ ($\geq q_B^*(q_1^*) > 2$) and A withdraws all products except q_n from Lemma 3 (b). Then it earns $\Pi_A(q_n, q_B^*(q_n)) - nF$ ($< \Pi_A(6, q_B^*(6)) - F$). Q.E.D.

Proof of Proposition 4.

A consumer of income y is willing to pay up to $y\left(1 - q_i^{-1}\right)$ for good q_i. When A chooses q_1 and B does not enter, A offers $P_1 = 1.5\left(1 - q_1^{-1}\right)$ in equilibrium and

$$W = 0.5\int_{1.5}^{3} y\left(1 - q_1^{-1}\right)dy - F$$

$$= \frac{27(q_1 - 1)}{16q_1} - F.$$

If $F \geq \Pi_B\left(6, q_B^*(6)\right)$, Proposition 1 shows that $q_1 = 6$ and $W = \frac{45}{32} - F$.

If $\Pi_B\left(q_1^*, 6\right) \leq F < \Pi_B\left(6, q_B^*(6)\right)$, A offers $\bar{q}_1$ that satisfies

$$F = \frac{(\bar{q}_1 - 1)(5r + \bar{q}_1 r - 4\bar{q}_1 - 4)}{4(\bar{q}_1 - r - 1)(\bar{q}_1 r - \bar{q}_1 - r - 1)} \text{ where } r \equiv \sqrt{\bar{q}_1 + 1}.$$

Then

$$W(\bar{q}_1) = \frac{27(\bar{q}_1 - 1)}{16\bar{q}_1} - \frac{(\bar{q}_1 - 1)(5r + \bar{q}_1 r - 4\bar{q}_1 - 4)}{4(\bar{q}_1 - r - 1)(\bar{q}_1 r - \bar{q}_1 - r - 1)}$$

and calculation shows that $\frac{dW}{d\bar{q}_1} > 0$ and $\frac{d\bar{q}_1}{dF} > 0$. Therefore $\frac{dW}{dF} > 0$ in this case.

If $F < \Pi_B\left(q_1^*, 6\right)$, then $(Q_A, Q_B) = \left(6, q_B^*(6)\right)$, $P_1 = \frac{17 - 2q_B^*}{12}$, and $P_B = \frac{q_B^* - 1}{q_B^*}$ in equilibrium. Consumers whose income are less than $\frac{19 - 4q_B^*}{12 - 2q_B^*} \equiv b$ buy q_B, and other consumers buy q_1. Thus

$$W = 0.5\int_1^b y\left(1 - q_B^{-1}\right)dy + 0.5\int_b^3 y\left(1 - q_1^{-1}\right)dy - 2F .$$

Calculation shows that

$$0.5\int_1^b y\left(1 - q_B^{-1}\right)dy + 0.5\int_b^3 y\left(1 - q_1^{-1}\right)dy - 2\Pi_B\left(q_1^*, 6\right) > \frac{45}{32} - \Pi_B\left(6, q_B^*(6)\right).$$

Therefore W under duopoly is always larger than W under monopoly. Q.E.D.

Proof of Proposition 5.

If $q_C < \min\{q_A, q_B\}$, $P_C = 0$ in equilibrium. Hence we assume $q_B < q_C < q_A$. When $q_C \le 0.2(2q_A + 3q_B)$,

$$\Pi_C(q_A, q_B, q_C) = \frac{9(q_A - q_B)(q_A - q_C)(q_C - q_B)}{2q_C(4q_A - 3q_B - q_C)^2} \text{ and } \frac{\partial \Pi_C}{\partial q_C} > 0 .$$

When $q_C \ge 0.2(2q_A + 3q_B)$,

$$\Pi_C(q_A, q_B, q_C) = \frac{q_A - q_C}{18q_C} \text{ and } \frac{\partial \Pi_C}{\partial q_C} < 0 .$$

Therefore Π_C is maximized at $q_C = 0.2(2q_A + 3q_B)$ for $q_C \in [q_B, q_A]$, and entry is blockaded if $\Pi_C\left(6, q_B^*(6), 0.6\left(4 + q_B^*(6)\right)\right) < F$.

Otherwise B must choose closer product to A. The reason is it cannot earn positive gross profit when C enters higher quality than its product. Define

$$K \equiv \frac{\partial}{\partial q_B}\Pi_A(q_A, q_B, \phi) \Big/ \frac{\partial}{\partial q_B}\Pi_C\left(q_A, q_B, 0.2(2q_A + 3q_B)\right)$$
$$- \frac{\partial}{\partial q_A}\Pi_A(q_A, q_B, \phi) \Big/ \frac{\partial}{\partial q_A}\Pi_C\left(q_A, q_B, 0.2(2q_A + 3q_B)\right).$$

A continues to choose $q_A = 6$ if $K < 0$ for $q_A = 6$. Calculation shows that

$$K = -\frac{3(3q_A - 2q_B - 1)(2q_A + 3q_B)^2}{10q_A^2 q_B (q_A - q_B)} < 0 \text{ for } q_B \le 0.25(q_A + 3)$$

and $K = 0$ for $q_B > 0.25(q_A + 3)$.

Thus A chooses $q_A = 6$ and B chooses $\bar{q}_B$ such that $\Pi_C\left(6, \bar{q}_B, 0.6(4 + \bar{q}_B)\right) = F$ in the unique equilibrium for $\Pi_C(6, 2.25, 3.75) \le F$.

If $F < \Pi_C(6, 2.25, 3.75)$, q_B must be larger than 2.25 to prevent entry of C. Then

A chooses q_A such that $\Pi_B(q_A, 6, \phi) \leq F$ since $K = 0$ for any q_A. *B* chooses q_B such that $q_B < q_A$ and $\Pi_C(q_A, q_B, 0.2(2q_A + 3q_B)) = F$. $(q_A, q_B) = (6, \overline{q}_B)$ is an example of such equilibrium. Q.E.D.

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Figure 1: The optimal product of the incumbent

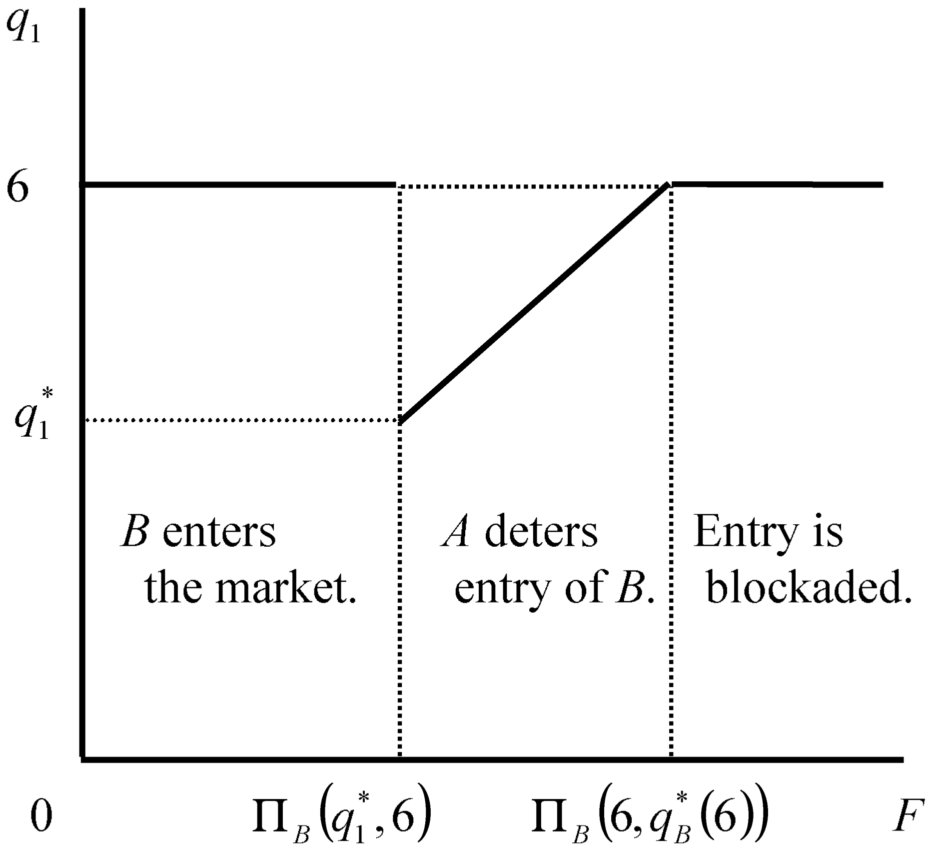

Figure 2: The social welfare