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FROM DUTY TO RIGHT:
THE ROLE OF PUBLIC EDUCATION
IN THE TRANSITION TO
AGING SOCIETIES

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From Duty to Right: The Role of Public Education in the Transition to Aging Societies

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Abstract

This paper argues that currently advanced, aging economies experienced a qualitative change in the role of public education during the process of industrialization. In the early phases of the Industrial Revolution, public education was regarded as a duty that regulated child labor and thereby discouraged childbirth. As these economies developed and the population aged, younger generations came to view public education as a right, whereas older generations desirous of other public services became more politically influential. The eventual policy bias in favor of the elderly placed a heavier education burden on the young, inducing them to have fewer children. This vicious cycle between population aging and the undersupply of public education may have decelerated the growth of advanced economies in the last few decades.

Keywords: Compulsory Education; Generational Conflict; Fertility; Growth.

JEL Classification: D70; H50, J10; J20; O40.

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1 Introduction

This study hypothesizes that the origin of population aging, pervasive in developed economies, dates back to the educational reforms during the era of the Industrial Revolution. We argue that the qualitative effect on fertility of education policy changed with the individual stances on public education. In the early phases of the Industrial Revolution, investment in human capital was not desirable for low-income households needing child labor. Compulsory education thus worked as a 'duty' that regulated child labor and thereby discouraged childbirth. By contrast, in the later phases of the Industrial Revolution, younger generations came to regard education investment as desirable. That is, public education was desired as a 'right'. The undersupply of public education in the postwar period placed the financial burden of child education on households, and thereby discouraged childbirth.

A question to be raised is how public schooling arose and which factors have affected subsequent education policies. To tackle this question, the present article highlights demographic factors as the determinants of social spending, for currently advanced economies over the modern period. Until the early 19th century, population expansion gradually raised the demand for a public education system by promoting skill- (age-) biased technological progress. In the postwar period, on the other hand, older generations, who have increased their population share and political representation, have sought national resources for their own well-being.[1]

In fact, one of the postwar aging societies, Japan, has improved the relative well-being of the elderly in some respects.[2] Figure 1 plots the trends in population aging (the dotted line) and the allocation of public spending between the young and the old (the solid line). Noting that old-age and education expenditures are both in per capita terms, one finds that the public service allocation

[1] In the U.S., for example, the American Association of Retired Persons (AARP) and the National Council on Aging (NCOA) increased their political power in the 1960s (Longman, 1987, p. 234). According to a recent article of *The Economist* (2005), the AARP has by far the largest members (35 millions) among the politically active groups in the country.

[2] There is some evidence for other countries. Pampel (1994) conducts econometric analysis by using the 1959–1986 annual data for 18 advanced industrial democracies. The result shows that, in the absence of class-based corporatism and strong leftist parties, a large aged population tends to exacerbate age inequality in public spending in favor of the elderly. Preston (1984) shows that the aged American society reduced poverty among the elderly by the expansion of social security benefits in contrast to raising poverty among children. Poterba (1997, 1998), using the U.S. panel data over the 1960–1990 period, finds that an increase in the fraction of the elderly in a jurisdiction is associated with a significant reduction in per-child education spending. For earlier periods, the regression results of Lindert (1994), based on the 1880–1930 panel of 21 (mostly currently advanced) countries, indicate that aged countries tend to have large public spending on social transfers including pension and health subsidies, and low primary- and secondary-school enrollment rates.

is shifting over the period in favor of the elderly. Such a trend would be attributed in large part to increased political pressures from retiring workers and pensioners.

These arguments assert that demographic changes and public policies have interacted with each other in the process of industrialization. In particular, developed economies may experience a vicious cycle between social aging and the undersupply of public education, possibly leading to a population decline and a slowdown in economic growth.[3] This spiral can therefore be one of the potential forces for the postwar convergence across advanced economies. The main objective of this paper is to develop a theoretical framework to understand the socioeconomic structures of such phenomena and to consider its policy implications.

The growth theory developed features five properties deserving special attention. First, parents face a trade-off between the quantity and quality of children, as shown in standard models of fertility and education such as Becker et al. (1990) and Galor and Weil (2000).[4] Second, public and private education are substitutes for each other in terms of skill acquisition. Those two properties together generate a potential link between education policy and fertility: in response to the undersupply of public education, for instance, parents may spend more on private education by having fewer children. Third, public education compulsorily takes away the time children have for paid work. Hence, their forgone wages equal the opportunity costs of child education. Fourth, technological progress is assumed to be skill- (age-) biased. This improves the productivity of adult workers over that of children, and thus makes sending children to school more beneficial than having them work.[5] Finally, older generations prefer social services that directly and immediately benefit themselves, such as pensions and social security, over public education. This leads to age-cohort differences in the willingness to enlarged education policy.[6]

[3] The undersupply of public education does not necessarily mean a decline in public expenditures for education per pupil. Indeed, it means that the provision of public education becomes insufficient for the given level of economic development.

[4] This paper extends the unified growth theory developed by Galor and Weil (2000), who explore the transition from Malthusian stagnation to modern growth through the demographic transition. In contrast to their representative-agent model, this article takes a political economy approach in analyzing the demographic trend since the early Industrial Revolution. The extended model also allows for the other four properties discussed.

[5] A similar mechanism is proposed by Hazan and Berdugo (2002). However, their single-agent model makes no distinction between public education and private education.

[6] Using the American National Election Survey for 1988, Vinovskis (1993, pp. 202–209) finds that additional federal assistance for public schools is supported by 77.1 % of interviewees aged through 18–29, and only by 46.9 % of those aged 70 and above. Furthermore, his multiple classification analysis based on the same data reveals that, after controlling for the effects of other factors (such as sex, race, education, family income and so on), there remains the tendency of the elderly to oppose federal funding for public schools.

This paper also establishes a more solid foundation for the recent and growing literature on demographic change and macroeconomics. It appears that no established theory has examined the interrelationship between public education, private education and fertility, despite its importance for the assessment of education policy. As discussed below, while there are three related seminal theories, these can be interpreted as partial theories from the perspective of this research, in that none encompasses the rise and fall of education policy over the long term.

First, Galor and Moav (2006) demonstrate the mechanism of the birth of public schooling on the course of industrialization. Unlike this research, however, their growth theory analyzes the monotonic evolution of education policy, excluding the possibility of its aggravation in mature development stages. Their model also assumes away demographic changes so as to highlight the profit rates of capitalists as a key determinant in their policy preferences. The current study takes the opposite approach by omitting physical capital to highlight the demographic factors. Furthermore, in their scenario, educational reform occurs without any political conflict—in sharp contrast to this analysis, which emphasizes compulsory education as a duty for the poor.[7]

Second, Doepke and Zilibotti (2005) explore the mechanism of child labor legislation. By noting the interaction between the number of children per family and policy preferences, they show the possibility of multiple equilibria such that child labor may or may not be abolished. Whereas their attention is directed at underdeveloped stages, the present study builds on a longer-term perspective, and encompasses more development stages where population aging provokes political confrontation between generations.

Third and finally, Holtz-Eakin et al. (2004) develop a growth model where the fraction of the elderly population is a prime determinant of government spending on education.[8] They demonstrate that a fertility decline alters the policy preferences of the median voter in favor of the elderly. However, unlike the present article, their analysis leaves unexplored an opposing causality between population aging and education policy. This is because their model abstracts from the family strategies of child rearing by assuming exogenous fertility and no private education.

The remainder of the paper is organized as follows. Section 2 presents historical evidence

[7] This difference is largely owing to the fact that the present article highlights the role of compulsory education as a child labor regulation. As evidenced in Section 2, early factory legislation required education for child workers while limiting their work hours.

[8] Similar demographic approaches are employed in Kemnitz (1999), Pecchenino and Utendorf (1999) and Gradstein and Kaganovich (2004). All of these, however, assume exogenous population growth.

supporting the central argument of the paper. Section 3 describes the basic structure of the model and then solves the optimization problems. This section also derives the political equilibrium that determines the allocation of public spending between generations. Section 4, as the main part of the paper, conducts a dynamic analysis to demonstrate the evolution of the economy. Further, it indicates the possibility for a slowdown in economic growth in the developed stage. Section 5 concludes the discussion and addresses some possible modifications and extensions to the research. The description of the data used and the proofs of the technical results are provided in the Appendix.

2 Historical Evidence

This section presents historical and empirical evidence supporting the central thesis of the paper. The focus here is placed on the experiences of advanced economies—Japan, the United States, and Western Europe—over the last two centuries. Consistent with the theory developed below, it shows that industrialization forces triggered the decisive shift of individuals' attitudes toward child labor and education.

2.1 Public Education as a Duty

The notion of compulsory education as a duty has been embraced by the poor, who live from hand to mouth and need the earnings of their children. In the early stages of their industrial revolutions, these countries employed children, and their respective child labor regulations and educational reforms were not embraced by the lower class. As a result of compromise, early educational reforms were designed to provide special classes for working children. Consistent with this observation, the growth theory developed here encompasses the coexistence of child labor and public schooling.

2.1.1 Japan

It is widely recognized that the Japanese industrial revolution took place after the Meiji Restoration in 1868. Ohkawa and Rosovsky (1965, p. 66; 1978, p. 142) argue that modern economic growth began in the mid-1880s. Similarly, Rostow (1978, p. 425) dates the years 1885–1905 as the period of the take-off.

Japan's modern educational reform was launched with the proclamation of the Educational System Order in 1872, which aimed for egalitarian education by establishing school districts throughout the country.[9] The law made four-year elementary education compulsory, along with the charging of school fees (Taira, 1978, p. 196). An educational system in the early stages of development such as this placed a heavy financial burden on the lower classes at that time. As described by Taira (ibid.), "To poor farmers, compulsory education appeared as an encouragement to the children to loaf in school when they could be helping on the farm. In some poorer parts of Japan there were a number of riots against compulsory education, in which hundreds of school buildings were destroyed."

In fact, statistical records indicate that not many children were sent to primary school in the early years of industrialization. The classroom attendance rate was approximately 20.44% in 1873 and 31.24% in 1890, and exceeded 50% for the first time in 1900 (Japan National Commission for UNESCO, 1966, p. 64).[10] Poor classroom attendance might have even been observed in not only day schools, but also night schools, which allowed children to work during the day. A public survey reports that among 4043 pupils registered in primary night schools (run by the city of Tokyo) in 1927, about 74% dropped out before graduation (Ishii, 1992, p. 165).

The low attendance rates were a result of the economic dependence of poor households on child labor. According to the Annual Report of Factory Inspection, even in 1926 nearly 20% of workers in textile mills were aged 16 or less (Ayusawa, 1966, pp. 186–188). Saito (1996, pp. 87–89) argues that, while child employment in paid work was not as extensive as in Britain, Japanese children were mostly expected to support their parents in the fields or around the house.

In order to encourage education for child workers, some forms of half time education were introduced in the late 19th century. In 1875, the city of Tokyo permitted night classes opened by private elementary schools, and in 1876 permitted night classes by public elementary schools (Ishii, 1992, p. 25). In 1894, the Ministry of Education ordered local governments to develop special curriculums for children who could not attend regular classes, and subsequently night classes, Sunday schools, and special classes for children involved in the care of babies were opened by state

[9] See the webpage of the Ministry of Education:
`http://www.mext.go.jp/b_menu/hakusho/html/hpbz198103/index.html`
where the history of Japan's modern educational system is documented comprehensively. According to Dore (1964, p. 176), although there were many small schools for reading, writing and arithmetic before the Meiji Restoration, they were typically run privately with no subsidies from the government.

[10] While the commission also reports the rate of school attendance, they believe that the rate of classroom attendance provides more accurate information about actual attendance: the former is defined as "an official ratio of the number of children attending school to the total number of the school-age children," whereas the latter is "the ratio of average daily attendance to the total number of the school-age children (ibid., p. 64–65)."

schools and sometimes by factories (Saito, 1996, pp. 83–85).[11]

2.1.2 The United States

As is well known, there were regional differences in the timing of industrialization in the United States. The economic take-off of New England occurred in the period 1815–1850, whereas that of the American North in 1843–1870 (Rostow, 1978, p. 392). In the north-eastern region, the share of child workers employed in manufacturing was 23.1% in 1820 (Goldin and Sokoloff, 1982, p. 748). It is estimated that in 1832 children amounted to about 40% of factory workers in New England (Weiner, 1991, p. 142). The proportion of child labor would have been much higher in agriculture, a dominant sector in the early 19th century.[12] At the national level, about one-sixth of children between the ages 10 to 15 were gainfully employed even in 1880 (Sanderson, 1974, p. 297; Weiner, 1991, p. 145). Given these figures, one can imagine that earlier years would have seen higher ratios of children in the labor force.

These records indicate that one of the principal obstacles to universal education was how to provide education for those working children. Hence, early educational reforms were presumably a compromise between the need for child labor and that for child education. In the late 1830s, industrial states passed laws which required factory children to go to school three months every year (Church, 1976, p. 59). In 1836, Massachusetts legislated that (at least) three months of school attendance was required, in advance, for the employment of children under the age of 15 in manufacturing (Weiner, 1991, pp. 142–143). Under the Massachusetts' labor-attendance law in 1842, it was the duty of the local school committees to secure the school attendance of children, while all factory children under the age of 12 were prohibited from working more than 10 hours a day (Ensign, 1969, p. 49).

2.1.3 Britain

Child labor played an important role even in early 19th century Britain, one of the most advanced economies at that time. In 1816, workers under the age of 18 accounted for 51.2% of workers

[11] A careful investigation by Osada (1995) identifies the locations of about 300 childminders' schools that were established in 36 prefectures between 1876 and 1930 (pp. 211–215), and the typical student in the subsample he analyzed were girls between the ages of 11 and 14 (pp. 325–334).

[12] In 1820, the proportion of workers employed in agriculture was 73% in New England, and 74% in the Middle Atlantic (Goldin and Sokoloff, 1982, p. 748).

in the British cotton industry, and 60.2% in the Scottish flax industry (Nardinelli, 1990, p. 109). The Census of England and Wales records that at least 36.6% of boys aged 10–14 were working in 1851 (Cunningham, 1996, p. 42). The earnings of children were an important income source of the working class. In the period from 1817 to 1839, records show that the children of mining, factory, and outworking workers earned, respectively, 23.9%, 28.2%, and 27.5% of household income (Horrell and Humphries 1995, p. 491).

These figures indicate parents' hesitation in sending their children to school at that time. For instance, the records of the Mitcham National School show that during the years 1830–1939, the average length of stay of pupils was 34.3 months, and 57% of boys left school to start work (Madoc-Jones, 1977, pp. 45, 47). Even in 1889, the London School Board issued as many as 96,450 initial notices to parents failing to send their children to school (Rubinstein, 1977, p. 245).[13] Furthermore, there is a view that the London poor, estimated to be 30.7% of the population in 1891, opposed education for their children, in part because those poor needed the earnings of their children (ibid., pp. 235–236). It is conjectured from these figures that the first half of the 19th century saw widespread, intense opposition to compulsory education.[14]

In these circumstances, early factory legislation compromised between regulating child labor and promoting child education. The Factories Act of 1833 imposed two-hour schooling on six days a week on working children aged 9 to 11 in the main textile industries (Silver 1977, p. 141). The Factory Act of 1844 enacted the half-time system, which restricted the work time of children to half a day, either before or after the dinner hour (Hutchins and Harrison, 1926, pp. 76, 85). As stated by Silver (1977, p. 141), "The half-time system, foreshadowed in the early Factory Acts and effective from 1845, began as a strategy for combating excessive child labour and became, in the 1850s and 1860s, an educational theory."

2.1.4 France

As in Britain's case, the progress of French educational reform was gradual. The education law in 1833 made primary education universally available (Weissbach, 1989, p. 3). Nevertheless, that was not fully made use of by the poorer classes, who needed child labor for their daily life. In 1840,

[13] Hurt (1979, p. 155) introduces two court cases, from *The Times* in November 1875, in which the London School Board was involved. They show that compulsory school attendance was a big burden on the poor.

[14] See Hopkins (1994, p. 142) for supporting episodes in the mid-19th century Black Country.

more than a quarter of the children aged 6 to 12 in France did not attend school in part because their labor was needed at home (ibid.).[15]

Evidence shows that children were working for their families out of necessity, at least before the mid-19th century. According to a study published in 1840, it cost 960 francs a year for a four-member family to live in Melhouse, whereas a laborer or dyer in the local cotton industry could earn at most 450 francs a year (Heywood, 1988, p. 108). As for handloom weavers in Rouen, the annual cost of living, 912 francs, was greater than the standard parental income, 861 francs (ibid., pp. 108–109). In the mid-1840s, several School Inspectors found that school attendance in rural areas declined when bread became expensive (ibid., p. 88).

Given such economic importance of children for households, it is not surprising that France began its educational reform while accepting child employment. Evening classes were held by the municipality in Mulhouse in the early 1830s, and in Lille in the early 1840s, although they were not very successful (ibid., pp. 244–245). The child labor law passed in 1841 provided a certain level of education for children aged 12 or below, while regulating the minimum age of employment to 8 (Ogg and Sharp, 1929, p. 382). Faced with working class opposition, this first child labor law was ineffective, and the child labor law passed in 1874 was more effectively enforced by a special inspectorate (Nardinelli, 1990, pp. 126–127, 137).

2.1.5 Germany

In Prussia, the first school edict was issued as the General-Landschul-Geglement of 1763, with the aim of providing compulsory education for all children aged 5 to 13 (Melton, 1988, p. 174). However, such an early attempt was unsuccessful. The four schools in Berlin, Kloster Berg, Stettin and Königsberg "produced far too few graduates to meet the goals of the reform (ibid., p. 175–176)." One of the reasons for the failure was the need for child labor for the poor. Indeed, summer school was impossible to run because rural families, especially in summer, needed their children to be working (ibid., p. 176).

Similarly, 40 out of about 100 petty schools in Bremen were closed between 1788 and 1810, during which children found new work opportunities in tobacco processing (Cunningham 1995, p. 103). This indicates that a large proportion of households in Bremen gave priority to child labor

[15] In 1840, 756,464 boys were in communal primary schools in winter, whereas the number attending declined to 463,464 in summer, possibly a season when their labor was especially needed at home (ibid.).

over child education at that time, and probably, in earlier periods as well. During the period 1800–1846, children under the age of 14 accounted for between 10 and 20% of factory workers (Lee, 1978, p. 466). In particular, until the 1850s children were of great assistance in textile factories, where their hands were suitable for picking up the threads (ibid.).

Under these circumstances, early educational reform encountered difficulties in its enforcement. The Prussian law of 1839 obliged children under the age of 16 to attend school five hours a day, while limiting their work time to 10 hours (Ogg and Sharp, 1929, p. 389). However, the local authorities (including police, teachers and clergymen) were unwilling and unable to enforce the law and, accordingly, the schooling time was cut to three hours a day in 1853 with limited enforcement (ibid., p. 390). One can imagine from these events that the local authorities had sympathy for poor children who were working for their families. Similar results were observed in other German states such as Bavaria, Baden, Württemberg and Hesse: they failed to restrict child labor and to spread public schooling (ibid.).

2.2 Public Education as a Right

More recently, compulsory education is widely perceived to be essential for securing the minimum living standard in advanced countries. In fact, children in OECD countries attend on average 11.9 years of schooling (OECD, 2006, p. 41), exceeding the compulsory years in most cases.[16] This fact, along with the evidence given in Section 2.1, suggests that the notion of compulsory education for their citizens had shifted from a duty to a right.[17]

Such a shift in individual attitudes toward education became widespread around the turn of the 20th century, although the timing varied across countries. In Japan, the aforementioned classroom attendance rate at primary level rose to 89.63% in 1913 (Japan National Commission for UNESCO, 1966, p. 64). This implies that most school-age children remained at primary schools after being enrolled. In the United States, 32.3% of children aged 14 to 17 were enrolled in secondary schools in 1920, a sharp increase from 6.7% in 1890 (Church, 1976, p. 289). In *Secondary Education for All,* published in 1922, the British Labor Party suggests the need for post-elementary education for

[16] See the webpage of the Japanese Ministry of Education:
`http://www.mext.go.jp/b_menu/shingi/chukyo/chukyo3/gijiroku/001/04052101/009/005.htm`

[17] This point has been emphasized by many scholars. See Shibuya (1988, p. 167) for Japan's case, and Weiner (1991) for a more comprehensive analysis.

working-class children (Hopkins, 1994, p. 321).[18]

This hypothesis is supported by Nardinelli (1990), who investigates the historical evidence for Britain, France, Germany, Japan, and the U.S. He argues that "As working-class incomes rose, parents kept children out of the labor force until later and later ages. The production of well-educated children became a realistic and desirable alternative for the working-class family in the late nineteenth and twentieth centuries. The desire of parents to improve the quality of their children would have been sufficient, in the absence of child labor laws, to remove children from factories in the long run (p. 149)." The minor role of child labor laws on school levels is also found by Landes and Solmon's (1972) regression analysis for the U.S. states in the late 19th century. Also, in Prussian textile factories, the number of child workers dropped by about 30% for the period 1846–1853, when child labor legislation was not effective (Lee, 1978, p. 467).

While several socioeconomic forces were at work in the transformation, two forces are emphasized as of particular importance in this paper: the rise in the net return on education investment, and the improvement in living standards. The first force motivated parents to educate their children if they could afford it.[19] By 1880, Félix Pécaut, apostle of progressive education in France, reported that families began to notice the advantage of obtaining a school certificate for finding jobs, inducing parents to permit longer school attendance by their children (Weber, 1976, pp. 73, 328). Likewise, high school education in the United States began to be regarded as a means to secure white-collar occupations emerging at the end of the 19th century (Church, 1976, p. 290). In Britain at that time, secondary and university education were increasing their importance in achieving or maintaining social status (Glass and Grebenik 1965, p. 117). On the cost side, technological progress reduced the forgone wages of children attending school. For example, in the aforementioned German textile factories, the significant role of child labor was replaced by improved machines in the late 1840s (Lee, 1978, p. 467).

The second force, improved living standards, did more than reduce child labor: it altered parents' myopic views on child rearing, and thereby induced them to care about the future well-being of their children. In the United States, the aforementioned spread of secondary education

[18] In 1950, the share of secondary school pupils in the 10–19 age group reached 33.7% in England and Wales (Flora et al., 1983, pp. 626).

[19] The underlying assumption here is that imperfect credit markets prevent households from borrowing school fees. This credit-market imperfection appears to be plausible in under- and semi-developed stages.

was in part because of rapid, widespread growth in per capita income after 1900, which would have allowed more families to keep sending their children to school without relying on their forgone wages (Church, 1976, pp. 289–290). Similarly in France, fewer children were in the labor force in the second half of the 19th century, during which real wages kept rising (Heywood, 1988, p. 109; Nardinelli, 1990, p. 146). In Britain, the average real wage over the period 1900–1909 increased to almost twice the level of the period 1850–1859 (Polland, 1978, p. 171). The improved living standard was also confirmed by looking at the number of prosecutions. By 1910, it dropped to 36,823 from the peak level of 92,274 in 1883 and, furthermore, truancy was the second most common case in roughly the same period (Hurt, 1979, p. 203).[20] Finally, in Japan, real GDP per capita more than doubled between 1890–1990 (Maddison, 2001, p. 206). Taira (1971, p. 374) points out that accelerated Japanese growth in this period lightened the burden on households while the rate of classroom attendance increased by about 20%.

3 The Model

Consider a closed overlapping-generations economy operating over an infinite discrete-time horizon. In perfectly competitive environments, producers generate a single homogeneous final good by employing individuals of all generations. Growth in output per worker is driven by education investment and population growth.

In every period, the government levies an income tax on working adults in order to provide education for children and an old-age related service. The allocation of tax revenues between the two services is determined by the adult and old generations, whose political power varies with the age composition of the population. Compulsory schooling works as a child labor regulation: this affects households' child-rearing strategies.

3.1 Producers

The final good is produced in two independent sectors, A and B. In sector A, competitive firms employ adult workers as a single factor of production. Letting Y_t^A be the aggregate level of output

[20] Although not mentioned, this is probably the record for England and Wales because these two countries are focused on throughout the book.

produced by this sector in period t (≥ 0), the production function is written as

$$Y_t^A = A_t H_t,$$

where $A_t > 0$ is the level of technology in period t and H_t is the aggregate amount of efficiency labor employed in period t. For simplicity, the price of the final good is normalized to unity. Standard profit maximization reveals that the wage rate per efficiency labor in period t, denoted as w_t, equals the marginal labor productivity, A_t.

On the other hand, sector B employs children and old individuals by using a less advanced technology. Letting Y_t^B be the aggregate level of sector B's output in period t, the production function is

$$Y_t^B = \bar{w} L_t,$$

where $\bar{w} > 0$ is the stationary level of technology and L_t is the aggregate amount of raw labor employed in period t. As a result of profit maximization, $\bar{w}$ equals the competitive wage rate per raw labor in sector B.

3.2 Households

A new generation, consisting of a continuum of individuals, is born at the beginning of every period and lives for three periods. This means that there are three generations in society at each point in time. All individuals have perfect foresight and behave as price takers. Adult individuals make three choices in child rearing: how many children to bear, how much education to give these children, and how much their children will work.[21] The political system is controlled by three interest groups: the adult rich, the adult poor, and the old. While adult individuals differ in their skill levels, old individuals are homogeneous in this respect.

3.2.1 The Environment

Consider the life stream of an individual who is born in period $t-1$ to be a member of group i ($= R, P$). The individual has one unit of time in each of the three life periods. In the first period (childhood), the individual works and/or receives education, while consuming a time fraction of

[21] The omission of capital markets would not affect households' child rearing strategies. Considering the limited access to capital markets in the early years of the Industrial Revolution, presumably most households at that time were unable to borrow education costs.

her/his single parent. The work time of the child depends on the levels of private and public education. There is no decision made by the child.

In the second period (adulthood or parenthood), the individual is obliged to serve τ units of time for the government, where $\tau \in (0,1)$ is exogenously given and stationary over time. Thus the time left to the individual is $1-\tau \equiv T$, which is allocated between working and child rearing. The individual works in sector A by supplying efficiency labor, which is the product of her/his work time and skill level h_t^i. Meanwhile, the individual raises n_t^i units of children by spending a fixed time fraction $\delta > 0$ per child, and possibly further invests e_t^i units of time per child for skill acquisition. Each of the children supplies l_t^i units of labor in sector B to earn $\bar{w} l_t^i$ for her/his parent. These wage incomes are spent for their consumption in this period, c_t^i. It follows that the budget constraint of the parent is

$$c_t^i \leq z_t^i [T - (\delta + e_t^i - \omega_t^i) n_t^i], \tag{1}$$

where $z_t^i \equiv w_t h_t^i$ denotes parental potential income, and $\omega_t^i \equiv \bar{w} l_t^i / z_t^i$ is the child labor income in terms of the parent's work time. It is assumed that ω_t^i is not high enough to fully cover the noneducation cost of child rearing; i.e., $\omega_t^i < \delta \ \forall t$.[22] Human capital h_t^i depreciates completely by the end of adulthood.

In the third period (elderhood), the individual becomes unskilled, and earns $\bar{w}$ by inelastically supplying one unit of labor in Sector B. The obtained income is spent for her/his own consumption c_{t+1}^3, and there is no intergenerational transfers. The individual is exempted from taxation and receives an old-aged related public service for free of charge. It follows that the budget constraint of the old individual is $c_{t+1}^3 \leq \bar{w}$.

Preferences in adulthood are defined over the level of consumption and the quantity of her/his children. Preferences in elderhood are defined over three economic factors: her/his consumption, the received amount of public service, and the average quality of her/his children. The lifetime utility function of an adult individual i (an adult member of group i) in period t is given by

$$u_t^i = (1-\alpha)\ln c_t^i + \alpha \ln n_t^i + \rho_t^i [(1-\beta)\ln \min(\gamma c_{t+1}^3, x_{t+1}) + \beta \ln h_{t+1}^i], \tag{2}$$

where $\alpha \in (0,1)$; $\beta \in (0,1)$; $\gamma > 0$, and ρ_t^i denotes the income-dependent discount rate taking a

[22] This assumption negates child labor income as an incentive for childbirth, and excludes the feasibility of an infinite number of children to be born. See Footnote 25.

discrete form

$$\rho_t^i = \rho(z_t^i) \equiv \begin{cases} 0 & \text{for } z_t^i < z^*; \\ \rho > 0 & \text{for } z_t^i \geq z^*. \end{cases} \tag{3}$$

A plausible interpretation of (3) is that low-income adults live from hand to mouth, and do not care about the future.[23] Following Galor and Weil (2000), it is assumed for simplicity that the weight on the quantity of children equals the weight on the quality of children; i.e., $\alpha = \rho\beta$.

3.2.2 The Production of Child Labor and Human Capital

Let e_t^G denote the time the government spends for the education of each child in period t. Then, the labor supply of a child i (a child of group i) in period t, denoted as l_t^i, is given by a continuous function

$$l_t^i = l(e_t^i + e_t^G) \begin{cases} > 0 & \text{if } e_t^i + e_t^G < \hat{e}; \\ = 0 & \text{if } e_t^i + e_t^G \geq \hat{e}, \end{cases} \tag{4}$$

where $\hat{e} > 0$.[24] Namely, the work time is left to the child if and only if less than $\hat{e}$ units of time is invested in her/his education. Furthermore, the function satisfies $l(0) = 1$, $l'(e) < 0$ and $l''(e) \leq 0 \ \forall e \in [0, \hat{e})$. Thus child labor supply is inversely related to the level of education, and no education investment allows the child one unit of labor supply. In this sense, compulsory schooling plays the role of a child labor regulation.

It follows that the labor income of a child i in period t is, in terms of her/his parent's work time,

$$\omega_t^i = \frac{\bar{w} l(e_t^i + e_t^G)}{z_t^i} \equiv \omega(e_t^i, e_t^G, z_t^i). \tag{5}$$

In light of (1), this shows that the forgone wages of children are the only costs of public education incurred by households. Thus, in terms of cost, public education is superior to private education.

The skill level of an adult individual i in period $t+1$ is determined by the production function

$$h_{t+1}^i = a^i h(e_t^i, e_t^G), \tag{6}$$

where a^i ($a^R > a^P = 1$) denotes the stationary learning ability of group i. This formulation means that only teaching time enhances the human capital children will acquire. The function $h(\cdot)$ is

[23] If instead $\rho_t^i = \rho > 0 \ \forall z_t^i \geq 0$, parents might invest in private education while having their children work. As will become apparent, (3) is useful to exclude this possibility and simplifies the dynamic analysis without changing the central argument of the paper.

[24] $e_t^i + e_t^G$ is the total time spent for the education of a child i in period t, rather than the time the child spends for education. For this reason, the critical level $\hat{e}$ does not have to be the total time the child has (i.e. one unit of time).

continuous and is defined on $\mathbb{R}^2_+$, featuring three properties. First, a noneducated adult of dynasty P obtains one unit of human capital; that is, $h(0,0) = 1$. Second, the function is increasing and strictly concave with respect to each argument. Third, and as mentioned in the introduction, private and public education are substitutes. These properties are expressed as $h_j(\cdot) > 0$, $h_{jj}(\cdot) < 0$ and $h_{12}(\cdot) < 0$ for $(e^i_t, e^G_t) \geq 0$ and $j = 1, 2$.

3.2.3 Optimization

By taking market prices and government policies as given, an adult individual i in period t makes a decision of childbirth and private education so as to maximize her/his utility (2). Since this decision has no effect on the wage income $\bar{w}$ and thus consumption when old, the optimal choice is

$$\{n^i_t, e^i_t\} = \arg\max (1-\alpha)\ln[T - (\delta + e^i_t - \omega(e^i_t, e^G_t, z^i_t))n^i_t] + \alpha \ln n^i_t + \rho(z^i_t)\beta \ln h(e^i_t, e^G_t), \qquad (7)$$

subject to the budget constraint (1) and $(n^i_t, e^i_t) \geq 0$; $z^i_t > \bar{w}/\delta$.[25]

The objective function is strictly concave with respect to n^i_t. The first-order condition for n^i_t reveals that the net cost of child rearing is

$$(\delta + e^i_t - \omega^i_t)n^i_t = \alpha T, \qquad (8)$$

implying that $n^i_t > 0$. The relationship between n^i_t and e^i_t in (8) indicates that the individual faces a trade-off between the quantity and the quality of children. Equally important, an increase in the relative wage of child labor, ω^i_t, induces her/him to have more children.

Optimization with respect to e^i_t is straightforward for individuals who cannot afford to care about the future: in light of (3), $\rho(z^i_t) = 0$ and thus $e^i_t = 0$ if $z^i_t < z^*$. If $z^i_t \geq z^*$ and thus $\rho(z^i_t) = \rho$, on the other hand, optimization is more complicated because, as shown below, multiple local optima may exist in this case.

The situation in the second case above is shown by Figure 2. On the lower panel is the (partially indirect) utility function obtained from substituting (8) into (7).[26] The solid curve there indicates the utility levels based on a given pair $e^G_t \geq 0$ and $z^{\min} \geq z^*$. Note that the curve is kinked at $e^i_t = \hat{e} - e^G_t$, where no work time is left to a child i in period t. In fact, there may be two locally

[25] The restriction on z^i_t is necessary to assure the existence of the optimal pair (n^i_t, e^i_t). If this condition is violated, n^i_t can be infinitely large by choosing $e^i_t = 0$, for example.

[26] The slope of the indirect utility function is depends on the signs of the functions $D^l(e^i_t, e^G_t, z^i_t)$ and $D^h(e^i_t, e^G_t)$ in the upper panel. See Appendix 1 for details.

optimal levels of private education, respectively, below and above the kink point: the lower level allows children to work, whereas the higher level does not.[27]

In what follows, it is assumed that the marginal productivity of private education, $h_1(e_t^i, e_t^G)$, is large enough to satisfy

$$h_1(\hat{e}, \hat{e})(\delta + \hat{e}) > h(\hat{e}, \hat{e}). \tag{A1}$$

Lemma 1 *If (A1) is satisfied and $z_t^i \geq z^*$, a locally optimal level of private education is given by a continuous single-valued function $\phi(e_t^G)$ such that $\phi(e_t^G) + e_t^G > \hat{e}$ and $\phi'(e_t^G) \leq 0$ $\forall e_t^G \geq 0$, with strict inequality if $\phi(e_t^G) > 0$.*

Proof. See Appendix 1. □

According to the first property of $\phi(\cdot)$ above, choosing the level $\phi(e_t^G)$ for private education leaves no time for children to work. The second property means that parents not using child labor aim to substitute private education for reduced public education.

The global optimality of the level $\phi(e_t^G)$ depends on the relative productivity of child workers to their parents. A rise in parental potential income makes child labor less productive, and thereby reduces the opportunity cost of schooling. This transition is illustrated by Figure 2. As z^i increases to z^{high} and further, the utility curve in the lower panel shifts downward only on the interval $[0, \hat{e})$, where child labor is used, and ultimately the level $\phi(e_t^G)$ becomes globally optimal. This result is formally given by Lemma 2 below.

Lemma 2 *If (A1) is satisfied and $z_t^i \geq z^*$, there exists a level of potential income, $\hat{z}$, above which $\phi(e_t^G)$ is the globally optimal level of private education for any $e_t^G \geq 0$.*

Proof. See Appendix 1. □

Lemmas 1–2 are summarized as follows: Parents do not rely on child labor and invest in private education, provided that their potential incomes are greater than $\hat{z}$ as well as than z^*. In order to simplify the dynamic analysis below, it is assumed that parents do not use child labor if they are wealthy enough to care about the future skills of their children (i.e. $\rho_t^i = \rho$). That is to say,

$$\hat{z} < z^*. \tag{A2}$$

[27] There is the possibility of a corner solution. For example, suppose that $e_t^i = e_t^G = 0$. Substituting (8) into (7) yields $\alpha \ln \alpha + (1-\alpha)\ln(1-\alpha) + \ln T - \alpha \ln(\delta - \bar{w}/z_t^i)$. In this case, utility goes to infinity as $\bar{w}/z_t^i$ approaches δ. On the other hand, if $e_t^i > e_t^G = 0$, then (8) shows that n_t^i and thus utility remain finite when $\bar{w}/z_t^i$ approaches δ. Hence a corner solution $e_t^i = 0$ may exist depending on the values of z_t^i and e_t^G.

This condition assures that no parents choose a level of private education on the range $(0, \hat{e})$, where children have time to work. In other words, it excludes the case where parents privately educate their children while sending them to work.

It follows that the optimal decision on private education is summarized as

$$e_t^i = e(e_t^G, z_t^i) \equiv \begin{cases} 0 & \text{if } \bar{w}/\delta < z_t^i < z^*; \\ \phi(e_t^G) \geq 0 & \text{if } z_t^i \geq z^*, \end{cases} \tag{9}$$

where $\phi(e_t^G) + e_t^G > \hat{e}$ and $\phi'(e_t^G) \leq 0 \; \forall e_t^G \geq 0$, with equality only if $\phi(e_t^G) = 0$. Substituting (9) into (8) and noting (5) yield that

$$n_t^i = n(e_t^G, z_t^i) \equiv \begin{cases} \frac{\alpha T}{\delta - \omega(0, e_t^G, z_t^i)} > 1 & \text{if } \bar{w}/\delta < z_t^i < z^*; \\ \frac{\alpha T}{\delta + \phi(e_t^G)} > 0 & \text{if } z_t^i \geq z^*, \end{cases} \tag{10}$$

where $\alpha T \geq \delta$ by assumption.[28] Note that the choice of childbirth hinges on education policy as well as the level of parental potential income, and their effects change qualitatively depending on the three cases below.

First, if $e_t^G < \hat{e}$ and $\bar{w}/\delta < z_t^i < z^*$, an adult individual i in period t has her/his children work instead of educating them privately. Hence, an increase in e_t^G works as a child labor regulation, and an increase in the parent's potential income z_t^i reduces the relative wage of children ω_t^i. Both of them discourage childbirth, meaning that $n_1(e_t^G, z_t^i) < 0$ and $n_2(e_t^G, z_t^i) < 0$.

Second, if $e_t^G > \hat{e}$ and $\bar{w}/\delta < z_t^i < z^*$, the parent does not invest in private education, while the government regulates child labor completely. Under these environments, an increase in either e_t^G or z_t^i has no effect on the relative wage of children, no effect on the decision of private education, and thus no effect on the incentive of childbirth. That is, $n_1(e_t^G, z_t^i) = n_2(e_t^G, z_t^i) = 0$.

Third and finally, if $z_t^i \geq z^*$, the parent invests in private education with no use of child labor, regardless of education policy. An increase in e_t^G induces the parent to reduce e_t^i and to increase n_t^i, because of the substitutability between public and private education, and through the quantity–quality trade-off in child rearing. Furthermore, as in the second case, no child labor income implies that n_t^i is independent of z_t^i. It is now clear that $n_1(e_t^G, z_t^i) \geq 0$ and $n_2(e_t^G, z_t^i) = 0$ in this case.

[28] This assumption requires the weight parameter on childbirth, α, to be sufficiently larger than the noneducation cost per child, δ. Without this condition, even a family relying on child labor may not have more than one child—a case not supported by historical evidence.

3.3 The Government

In period t, the government imposes a tax, in the form of time, on N_t units of adult workers. Each of them is obliged to serve $\tau \in (0, 1)$ units of time for public services. Thus the government obtains τN_t units of time in period t, allocating them between education for children and the old-age related service. These two public services are free of charge for all beneficiaries. All adult individuals have the same ability in the production of the public services.

Let $n_t \equiv N_{t+1}/N_t$ denote the average number of children per adult in period t. Each public school class consists of n_t units of students and one teacher; that is to say, each adult individual equally teaches n_t units of children at the same time.[29] Thus τ is the maximum spending on public education per child, and the budget constraint facing the government is $e_t^G \in [0, \tau]$.

Let x_t be the level of public service each old individual receives in period t. For simplicity, x_t is assumed to be the time allocated for this service; that is

$$x_t = \frac{(\tau - e_t^G)N_t}{N_{t-1}} = (\tau - e_t^G)n_{t-1} \equiv x(e_t^G, n_{t-1}), \tag{11}$$

where $n_{t-1} \equiv N_t/N_{t-1}$ denotes the child/adult ratio in period $t-1$ (or the adult/old ratio in period t). The effects on x_t of education policy and of fertility is straightforward: A rise in e_t^G shifts the government spending from the elderly to the young, leading to a reduction in x_t. A rise in n_{t-1} increases the number of taxpayers per old individual, and thereby increases x_t.

3.4 The Political System

In every period, adult individuals, together with the old, decide the budget allocated to young generations. The decision on childbirth is made in advance by perfectly forecasting the political decision.[30] As will become apparent, the three interest groups, the adult rich, the adult poor, and the old, have different political powers.

First, consider the level of public education desirable for an adult individual i in period t, denoted as e_t^{Gi}. As implied by (11), education policy in this period has no direct effect on the old-age related service x_{t+1}. It thus follows from (2) that

$$e_t^{Gi} = \underset{0 \le e_t^G \le \tau}{\arg\max} (1-\alpha)\ln[T - (\delta + e_t^i - \omega(e_t^i, e_t^G, z_t^i))n_t^i] + \rho(z_t^i)\beta \ln h(e_t^i, e_t^G), \tag{12}$$

[29] More generally, one may assume that εn_t units of children are enrolled in each class, where $\varepsilon > 0$ is a constant.

[30] As will be disccussed in Footnote 32, the timing of childbirth changes the reason for poor families to oppose educational reform.

subject to $e_t^i \geq 0$, $n_t^i > 0$ and $z_t^i > \bar{w}/\delta$.

Next, consider the level of public education desirable for the elderly in period t, e_t^{G3}. Substituting the budget constraint $c_t^3 = \bar{w}$ and (11) into (2), one finds that

$$e_t^{G3} = \arg\max_{0 \leq e_t^G \leq \tau} \min[\gamma\bar{w}, x(e_t^G, n_{t-1})],$$

where $x(e_t^G, n_{t-1}) = (\tau - e_t^G)n_{t-1}$. Note that there exists a unique value $\tilde{e}_t$ such that $\gamma\bar{w} = x(\tilde{e}_t, n_{t-1})$, and the qualitative effect of education policy changes at this critical level. Increasing e_t^G up to $\tilde{e}_t$ has no effect on the elderly, whereas the effect becomes negative on the range above $\tilde{e}_t$.

The elderly, when indifferent, are assumed to accept the policy proposed by younger generations. Then the desirable level of public education for old individuals in period t, denoted as e_t^{G3}, is

$$e_t^{G3} = \begin{cases} e_t^{G2} & \text{if } e_t^{G2} \leq \tilde{e}_t; \\ \max(\tilde{e}_t, 0) & \text{if } e_t^{G2} > \tilde{e}_t, \end{cases} \tag{13}$$

where

$$\tilde{e}_t = \tau - \frac{\gamma\bar{w}}{n_{t-1}} \equiv \tilde{e}(n_{t-1}). \tag{14}$$

Note that $\tilde{e}(n_{t-1}) < \tau$ and $\tilde{e}'(n_{t-1}) > 0$ $\forall n_{t-1} > 0$. The first property means that the maximum spending on public education, τ, is large enough to provoke opposition by the elderly. This is straightforward because such an extreme policy leaves no budget for the old-age related service x_t. The second property implies that the elderly can accept higher levels of public education as the relative number of taxpayers increases.

It is now ready to discuss how education policy is determined in the presence of potential conflict. Based on the evidence in Section 2, the political system considered here incorporates two ideas. First, all interest groups have some political influence regardless of their voting status.[31] Second, the political power of adult individuals in period t increases with the adult/old population ratio in period t, n_{t-1}. More specifically, the equilibrium level of public education is a weighted average such that

$$e_t^G = \lambda(n_{t-1})[\theta e_t^{GR} + (1-\theta)e_t^{GP}] + [1 - \lambda(n_{t-1})]e_t^{G3}, \tag{15}$$

where $\theta \in (0,1)$ and $\lambda(0) = 0$, $\lim_{n\to\infty} \lambda(n) = 1$ and $\lambda'(n_{t-1}) > 0$ $\forall n_{t-1} \geq 0$.

[31] Flora et al. (1983) show that suffrage was limited in Western Europe in the first half of the 19th century. Nevertheless, as evidenced in Section 2.1, administrators at that time encountered practical difficulties in enforcing school attendance.

3.5 Population Growth and Technological Change

Let N^i_{t+1} denote the number of adult members i in period $t+1$. Then, noting that there is no within-group heterogeneity, one finds that

$$N^i_{t+1} = \int_{s=0}^{N^i_t} n^i_t ds = n^i_t N^i_t.$$

The average fertility rate in period t, n_t, is defined as

$$n_t \equiv \frac{N_{t+1}}{N_t} = \frac{N^R_{t+1} + N^P_{t+1}}{N_t} = q_t n^R_t + (1-q_t) n^P_t, \tag{16}$$

where $q_t \equiv N^R_t / N_t$ is the share of group R in the adult generation in period t. Note that n_t is the child/adult ratio in period t, and also the adult/old ratio in period $t+1$.

Then it follows from (8) that the aggregate level of efficiency labor employed by Sector A in period $t+1$ is

$$\begin{aligned} H_{t+1} &= \sum_{i=R,P} \left\{ [T - (\delta + e^i_{t+1} - \omega^i_{t+1}) n^i_{t+1}] h^i_{t+1} N^i_{t+1} \right\} \\ &= (1-\alpha) T N_t [q_t n^R_t h^R_{t+1} + (1-q_t) n^P_t h^P_{t+1}]. \end{aligned} \tag{17}$$

Suppose that H_{t+1} is the single determinant of technological progress between periods t and $t+1$. That is

$$g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t} = g(H_{t+1}), \tag{18}$$

where $g(\cdot)$ is a single-valued continuous function such that $g(0) = 0$ and $g'(H) > 0 \ \forall H \geq 0$. This formulation implies that there is no depreciation in the level of technology (or knowledge) and A_t is nondecreasing over time. Equally important, it incorporates the well-known idea that larger populations tend to create more ideas (cf. Kuznets [1960]; Kremer [1993]; Galor and Weil [2000]). In the presence of such a scale effect, education investment is not necessary for economic growth: as long as a population exists, the productivity of adult workers improves over time. By contrast, there is no scale effect and no technological progress in sector B. This type of age-biased technological progress decreases the relative productivity of child labor over time.

4 The Evolution of the Economy

This section demonstrates that the dynamic model presented above generates the development process consistent with the historical experience in Section 2. The economy goes through three stages of development through which individuals' views toward public education change. The resulting progress in educational reform is followed by a steady decline in the birthrate.

Stage I is the premature development stage without skill investment. Because of low living standards, adult workers are not inclined to educate their children at the sacrifice of child labor income. As a result, neither private nor public investment in education occurs, and population expansion is the single driving force of technological progress. It is shown that a small degree of the scale effect leads to the acceleration of population growth.

Stage II is marked by the emergence of public education. Skill-biased technological progress gradually raises the relative productivity of skilled workers and makes them wealthy enough to care about the future performance of their children. These environmental changes induce them to abandon child labor, triggering the educational reform movement. However, they encounter opposition from the less skilled poor, who still need child labor income for consumption. These conflicting policy stances limit the provision of public education and, accordingly, its growth-enhancing effects. On other hand, the legislation of compulsory schooling discourages childbirth for the poor by regulating the work time of their children.

Stage III encompasses population aging and generational conflict over public policies. In this developed stage, all adult individuals are wealthy enough to care about the future skills of their children, and the relative productivities of child workers are sufficiently low. Now even the less skilled adults come to regard the receipt of public education as a right and there is no political conflict within the adult generation. Accordingly, educational reform makes further progress. Moreover, the onset of universal investment in education, both by households and the government, decelerates population growth.

The rise of education policy, however, is followed by a drop in later stages. As society ages, old generations demanding other public services become more politically influential. The rising pressures of the old cause a policy bias in their favor, and thereby lay a financial burden on young households. To make up for the undersupply of public education (as a right), they spend more on private education by having fewer children. This chain reaction generates a vicious cycle between social aging and the deterioration of public education, and this spiral may cause population decline and a slowdown in economic growth in the long run.

4.1 Stage I: Economic Growth Driven by Population Expansion

Stage I is the initial development stage that occurs before period t^*, when z_t^R exceeds z^* for the first time (namely, $z_t^R < z^* \ \forall t < t^*$ and $z_t^R \geq z^*$ for $t = t^*$). In this stage, no parents are wealthy enough to care about the future of their children. Consequently, neither private nor public education investment occurs, and technological progress is driven solely by population expansion.

4.1.1 Education, Child Labor, and Fertility

Sector A in period 0 consists of adult workers who have only innate skills. Since their potential incomes are $z_0^R = a^R A_0$ and $z_0^P = A_0$, there is between-group inequality. As shown by (6) and (9), a rise in z_t^i has a nonnegative effect on private investment in child education, e_t^i, and then on the acquired skill level, $h_{t+1}^i = a^i h(e_t^i, e_t^G)$. Therefore, the persistent ability gap between adult workers results in

$$z_t^R > z_t^P \qquad \forall t \geq 0. \tag{19}$$

Thus it follows from (3) that $z_t^P < z_t^R < z^*$ and $\rho_t^R = \rho_t^P = 0 \ \forall t \in [0, t^*)$. Hence in Stage I, adult individuals do not gain utility from the future skills of their children. In this situation, as shown by (9), no one invests in private education regardless of education policy; i.e., $e_t^i = e(e_t^G, z_t^i) = 0 \ \forall e_t^G \geq 0$.

Using these results for (12), the desirable education policy for the parental generation in Stage I is

$$e_t^{Gi} = \underset{0 \leq e_t^G \leq \tau}{\arg\max} \{1 - [\delta - \omega(0, e_t^G, z_t^i)] n_t^i\} = 0.$$

In other words, parents do not desire compulsory education as sending their children to school takes away the opportunity for them to work. Educational reform would merely diminish their child labor income ω_t^i, consumption c_t^i, and thus utility u_t^i, because their choice on childbirth is made in advance. These predicted adverse effects prompt the adult generation to oppose the introduction of public education.[32] Then it follows from (13) and (15) that there is no generational conflict and $e_t^G = 0 \ \forall t \in [0, t^*)$.

[32] The reason for their opposition changes depending on the timing of childbirth. If parents have children after the education policy was determined, n_t^P is adjusted so that (8) holds. Then it follows that consumption is $c_t^P = (1-\alpha) T z_t^P$ regardless of education policy. In this case, it is the negative effect on childbirth, rather than on consumption, that induces the opposition of group P against educational reform.

As a result of no schooling, all children spend their entire time on working in sector B. They grow up to have only innate skills, $h_t^i = a^i h(0,0) = a^i$, and potential incomes $z_t^i = a^i A_t$. Thus in Stage I, technological progress is the single force pushing up the level of z_t^i, and A_t grows toward z^*/a^R.

Then it follows from (5) and (10) that households' birthrates in Stage I are

$$n_t^P = \frac{\alpha T}{\delta - \bar{w}/A_t} > n_t^R = \frac{\alpha T}{\delta - \bar{w}/(a^R A_t)} > 1, \tag{20}$$

where $\bar{w}/\delta < A_t < z^*/a^R$. Note that parents of group P are more fertile than those of group R, as child labor is more valuable for the formers than the latters. Equally important, n_t^i decreases with A_t because technological progress in Sector A reduces the relative productivity of children, working in Sector B.

4.1.2 The Dynamical System

As shown above, n_t^R and n_t^P depend only on A_t whereas h_{t+1}^R and h_{t+1}^P are constant for $t \in [0, t^*)$. Thus the resulting level of efficiency labor, H_{t+1} in (17), depends on A_t, N_t and q_t.

Using these results for (16) and (18), the evolution of the economy in Stage I, where $A_t < z^*/a^R$, is governed by the first-order, three-dimensional autonomous system:

$$\begin{aligned} A_{t+1} &= [1 + g(H_{t+1})]A_t \equiv G^I(A_t, N_t, q_t)A_t; \\ N_{t+1} &= n_t N_t \equiv n^I(A_t, q_t)N_t; \\ q_{t+1} &= \frac{n_t^R}{n_t} q_t \equiv m^I(A_t, q_t)q_t, \end{aligned} \tag{21}$$

where the initial set (A_0, N_0, q_0) satisfies[33]

$$\frac{\bar{w}}{\delta} < A_0 < \frac{z^*}{a^R}; \qquad N_0 > 0; \qquad 0 < q_0 < q^{\max}. \tag{A3}$$

The three state variables of the dynamical system evolve in the following ways. First, the growth rate of technology g_{t+1} has a positive lower bound, and thus the level of technology A_t grows without converging to a certain level.[34] Second, $n_t > 1$ and thus the total population N_t keeps expanding. Third, $n_t^R < n_t$ and thus the share of group R in the adult generation, q_t, strictly monotonically declines over time.

[33] The upper limit $q^{\max} \in (0, 1]$ is defined by the proof of Equation (31) in Appendix 1.

[34] As follows from (17) and (20), H_{t+1} in (21) has a positive lower bound such that $H_{t+1} > (1-\alpha)TN_{t+1} > (1-\alpha)TN_0$.

Accordingly, the parental potential income $z_t^R = a^R A_t$ grows strictly monotonically over time, ultimately exceeding the critical level z^*. This means that there is no steady state in Stage I and the economy inevitably departs from there. As for the average fertility rate, n_t may or may not decrease over time because of two opposing forces at work: On the one hand, technological progress discourages childbirth by making child labor less productive compared to adult labor. On the other hand, the more fertile group, the adult poor, increases its share over time (note that $n^I(A_t, q_t)$ is decreasing in q_t). The lemma below suggests that the average fertility rate increases when slow technological progress makes the former negative force negligible.

Lemma 3 *Under (A1)–(A3), $n_t > n_{t-1}$ for $t \in (0, t^*)$ if A_t is sufficiently small.*

Proof. See Appendix 1. □

The possibility of population growth acceleration in this initial stage of industrialization appears to be historically plausible. According to Maddison (2001, p. 242), the average growth rate of population reached a peak during the period 1820–1870 in the U.S., 1870–1913 in Germany and the U.K., and 1913–1950 in Japan.

4.2 Stage II: Educational Reform with Class Conflict

The economy develops in Stage II on the time interval $[t^*, t')$, where t' is the critical period in which z_t^P exceeds z^* for the first time (i.e., $z_t^P < z^*$ $\forall t < t'$ and $z_t^P \geq z^*$ for $t = t'$). In this stage, members of group R are wealthy enough to care about the future skills of their children, whereas this is not the case for group P. Furthermore, the relative productivity of child workers is low enough for group R to abandon child labor. In these circumstances, educational reform is promoted by the rich class, who face the opposition of the poor. Compulsory schooling works as a child labor regulation that discourages childbirth in group P.

4.2.1 Education and Child Labor

In the initial period of Stage II, $z_{t^*}^R = a^R A_{t^*} \geq z^*$. Since the level of technology A_t is, according to (18), nondecreasing over time, it follows that

$$z_t^R = A_t h_t^R \geq z^*; \qquad \rho_t^R = \rho; \qquad \forall t \geq t^*. \tag{22}$$

As for group P, on the other hand, $z_t^P < z^*$ and thus $\rho_t^P = 0$ in Stage II.

Under these circumstances, (9) reveals that individuals' decisions of private education are $e_t^R = \phi(e_t^G)$ and $e_t^P = 0$ $\forall e_t^G \geq 0$. In fact, since $\phi(e_t^G) + e_t^G > \hat{e}$ $\forall e_t^G \geq 0$, (4) shows that $l_t^R = 0$ $\forall e_t^G \geq 0$. In other words, whether the government regulates child labor or not, adult members of group R do not send their children to work. By contrast, those of group P are still myopic and rely on child labor income, which depends on the level of compulsory education e_t^G.

Using these results for (12), the desirable policy for adult individuals in Stage II are

$$\begin{aligned} e_t^{GR} &= \underset{0 \leq e_t^G \leq \tau}{\arg\max}\ h(e_t^R, e_t^G) = \tau; \\ e_t^{GP} &= \underset{0 \leq e_t^G \leq \tau}{\arg\max} \{1 - [\delta - \omega(0, e_t^G, z_t^P)] n_t^P\} = 0. \end{aligned}$$

These results indicate the existence of between-group conflict (class conflict) over education policy: Unlike group P, keeping children at school is no longer a burden for group R. The shift in their policy preferences is caused by z_t^R exceeding z^* and $\hat{z}$. Given such a high potential income, parents of group R take into account the future skills of their children (i.e. $\rho_t^R = \rho$). The parents also find that sending their children to work, rather than to school, does not compensate for the future loss.

The two adult groups facing such class conflict make a compromise at $\theta\tau$, according to (15). In light of the historical evidence, suppose that the compromise level $\theta\tau$ does not lead to a drastic reform. In particular, $\theta\tau$ is not large enough to completely regulate child labor, and not enough to provoke the opposition by the elderly. That is to say,[35]

$$\theta\tau < \min(\hat{e}, \tilde{e}_t) \qquad \forall t \geq 0. \tag{A4}$$

It thus follows from (15) that the resulting education policy is

$$e_t^G = \theta\tau < \hat{e} \qquad \forall t \in [t^*, t'), \tag{23}$$

This outcome is interpreted as the half-time system mentioned in Section 2.1, in that children of group P engage in both work and compulsory education.

4.2.2 Fertility

Substituting the results obtained above into (10), households' birthrates in Stage II are

$$n_t^R = \frac{\alpha T}{\delta + \phi(\theta\tau)} \gtreqless 1; \qquad n_t^P = \frac{\alpha T}{\delta - \omega(0, \theta\tau, z_t^P)} > 1, \tag{24}$$

[35] In light of (10) and (14), $\theta\tau < \tilde{e}_t$ $\forall t \geq 0$ if $\theta\tau < \tau - \gamma\bar{w}/n^{\min}$, where $n^{\min} \equiv \alpha T/[\delta + \phi(0)]$ is a lower bound such that $n^{\min} \leq n_t^i$ $\forall t \geq 0$.

where $z_t^P = A_t h(0, \theta\tau)$. This shows that n_t^R remains constant over Stage II, whereas the higher rate n_t^P depends on the level of technology, A_t.

Note that introducing compulsory education promotes group R's childbearing by supporting their child education; i.e., $n(\theta\tau, z_t^R) > n(0, z_t^R)$. In this sense, public education works as a childcare service, rather than as a child labor regulation, for the rich class. By contrast, it discourages group P's childbearing by decreasing their child labor income; i.e., $n(\theta\tau, z_t^P) < n(0, z_t^P)$. In light of (16), their overall effect on average fertility depends on the relative share between the two groups. If q_t is sufficiently small, the former positive effect on group R is negligible and

$$q_t n(\theta\tau, z_t^R) + (1 - q_t) n(\theta\tau, z_t^P) < q_t n(0, z_t^R) + (1 - q_t) n(0, z_t^P).$$

Now one obtains the proposition below.

Proposition 1 *If* (A1) *is satisfied and q_t is sufficiently small, the provision of compulsory education in period $t \in [t^*, t')$ decreases the average fertility rate in the same period.*

Thus, compulsory education is likely to delay population growth in the developing stage where child labor is needed by the masses. The initial value q_0 is important for this result to hold because, as will become apparent, q_t strictly monotonically declines over Stages I–II.

4.2.3 The Dynamical System

As shown above, n_t^P depends only on A_t whereas n_t^R, h_{t+1}^R and h_{t+1}^P are all constant for $t \in [t^*, t')$. Thus it follows from (17) that the resulting level of efficiency labor H_{t+1} depends on A_t, N_t and q_t.

Using these results for (16) and (18), the evolution of the economy in Stage II, where $z^*/a^R < A_t < z^*/h(0, \theta\tau)$, is governed by the first-order, three-dimensional autonomous system:

$$\begin{aligned} A_{t+1} &= [1 + g(H_{t+1})] A_t \equiv G^{II}(A_t, N_t, q_t) A_t; \\ N_{t+1} &= n_t N_t \equiv n^{II}(A_t, q_t) N_t; \\ q_{t+1} &= \frac{n_t^R}{n_t} q_t \equiv m^{II}(A_t, q_t) q_t, \end{aligned} \qquad (25)$$

where the initial condition $(A_{t^*}, N_{t^*}, q_{t^*}) \gg 0$ is determined by (21).

The dynamical system above characterizes the evolution of the three state variables as follows. First, as in Stage I, the growth rate of technology g_{t+1} has a positive lower bound, and thus A_t

grows without converging to a certain level.[36] Second, unlike in Stage I, the total population N_t may not grow monotonically because, in light of (24), $n_t \gtreqless 1$. As long as the share of group R is sufficiently small, however, n_t is greater than unity and thus the total population keeps growing. Third, $n_t^R < n_t$ and thus $q_{t+1} < q_t$; namely, the share of group R in the parental generation declines over time.

Under these circumstances, both z_t^R and z_t^P strictly monotonically grow over time and z_t^P ultimately reaches the critical level z^* ($z_{t'}^P = A_{t'}h(0, \theta\tau) \geq z^*$). This assures the economy's departure from Stage II. Furthermore, as in Stage I, the time trend of average fertility is generally ambiguous because of two opposing effects. While the birthrate of group P declines over time, group R, which is less fertile, reduces its share. As asserted below, the former downward pressure becomes dominant and decelerates population growth when the share of group R is sufficiently small.

Lemma 4 *Under (A3), $n_{t+1} < n_t$ for $t \in [t^*, t')$ if q_t is sufficiently small.*

Proof. See Appendix 1. □

4.3 Stage III: Social Aging and the Rise of Generational Conflict

Stage III, which begins in period t', is symbolized by the rise and fall of education policy. In this stage, all adult individuals are wealthy enough to care about the future performance of their children. In addition, the relative wages of child workers are sufficiently low for even less skilled parents to abandon child labor. As a result, adult generations unanimously support extensive education policy. Meanwhile, population aging gradually enhances the political power of the elderly, who demand the old-age related public service. The resulting undersupply of public education lays an education burden on young families, and thereby discourages childbirth. These results well fit the postwar evidence of social aging paralleled by generational inequality in public services.

4.3.1 Private Education, Child Labor, and Education Policy

Consider a period in which $z_t^P \geq z^*$. Noting (19), this implies that $z_t^R > z^*$. In this circumstance, (3) and (9) show that $\rho_t^R = \rho_t^P = \rho$ and $e_t^R = e_t^P = \phi(e_t^G)\ \forall e_t^G \geq 0$. Since $\phi(e_t^G) + e_t^G > \hat{e}\ \forall e_t^G \geq 0$, any parents no longer send their children to work *regardless of* the degree of child labor regulation

[36] As follows from (17) and (24), H_{t+1} in (25) has a positive lower bound such that $H_{t+1} > (1-\alpha)TN_{t+1}^P > (1-\alpha)TN_{t^*}^P$.

(i.e., $l_t^R = l_t^P = 0$ for any $e_t^G \geq 0$). In this sense public schooling does not play a role of child labor regulation.

Therefore, it follows from (12) that the policy stance of an adult individual i in this period is

$$e_t^{Gi} = \underset{0 \leq e_t^G \leq \tau}{\arg\max}\ h(e_t^i, e_t^G) = \tau.$$

This shows that public education is beneficial and viewed as a right for all adult individuals, and no political conflict arises among them. The shift in the policy preferences of group P is caused by the improvement in parental potential income. First, z_t^P exceeding z^* allows parents of the group to care about the future performance of their children. Second, z_t^P exceeding $\hat{z}$ reduces the relative labor productivity of their children.

The adult generation's positive attitude to public education triggers generational conflict, because τ exceeds the upper limit for the elderly, $\tilde{e}_t$ in (14). In this case, as follows from (13) and (A4), the policy stance of the elderly is $e_t^{G3} = \tilde{e}_t > \theta\tau$. These results and (15) reveal that the resulting education policy is

$$e_t^G = \lambda_t \tau + (1 - \lambda_t)\tilde{e}_t > \theta\tau. \tag{26}$$

It follows that the parental potential income of group P in the subsequent period is

$$z_{t+1}^P = A_{t+1} h(\phi(e_t^G), e_t^G) > A_{t'} h(0, \theta\tau) = z_{t'}^P \geq z^*,$$

where $A_{t+1} \geq A_t \geq A_{t'}$. Therefore, $z_{t+1}^P > z^*$ if $z_t^P \geq z^*$. Because of the persistent inequality shown by (19), it is now clear that

$$z_t^R > z_t^P \geq z^*; \qquad \rho_t^R = \rho_t^P = \rho, \qquad \forall t \geq t'. \tag{27}$$

Thus the results obtained above hold for any $t \geq t'$. In particular, note that the level of public education in Stage III is always higher than in Stages I–II, where $e_t^G \leq \theta\tau$.

4.3.2 The Evolution of Fertility and Education Policy

As follows from (10), (16) and (27), the average fertility rate in Stage III is

$$n_t^R = n_t^P = n_t = \frac{\alpha T}{\delta + \phi(e_t^G)}, \tag{28}$$

where $\phi(e_t^G)$ is decreasing in e_t^G. The birthrate of group P in Stage III is lower than in the previous stages, as the group no longer relies on child labor.

Now consider how e_t^G is determined in period t, in which $e_t^{GR} = e_t^{GP} = \tau$ and the adult/old ratio $n_{t-1} > 0$ is exogenously given. Since (13) reveals that $e_t^{G3} = \max[0, \tilde{e}(n_{t-1})]$ in this case, the education policy based on the political system (15) is given by a single-valued function such that

$$e^G(n_{t-1}) = \lambda(n_{t-1})\tau + [1 - \lambda(n_{t-1})] \max[0, \tilde{e}(n_{t-1})], \tag{29}$$

where $0 < e^G(n_{t-1}) < \tau$ and $e^G(n_{t-1})$ is strictly monotonically increasing in $n_{t-1} > 0$. Note that a decline in n_{t-1} enhances the relative political power of the old generation in period t, and accordingly reduces e_t^G.

Substituting (29) into (28) reveals that the evolution of the average fertility in Stage III is governed by a first-order, autonomous system

$$n_t = \frac{\alpha T}{\delta + \phi(e^G(n_{t-1}))} \equiv n^{III}(n_{t-1}), \tag{30}$$

where $n^{III}(n_{t-1})$ is monotonically increasing in $n_{t-1} > 0$ and the initial value $n_{t'-1}$ is given by (25).[37]

In addition to the positive correlation between n_t and n_{t-1}, the dynamical system (30) features two important properties. First, as shown by Appendix 1,

$$n_{t'} = n^{III}(n_{t'-1}) < n_{t'-1}. \tag{31}$$

This fertility decline reflects the switch of group P from the reliance on child labor to the education of children. Second, because $0 < e^G(n_{t-1}) < \tau \ \forall n_{t-1} > 0$, the function $n^{III}(n_{t-1})$ is bounded in such a way that

$$\frac{\alpha T}{\delta + \phi(0)} < n^{III}(n_{t-1}) < \frac{\alpha T}{\delta + \phi(\tau)} \qquad \forall n_{t-1} > 0. \tag{32}$$

These properties are depicted by Figure 3. The initial value of Stage III, $n_{t'} = n^{III}(n_{t'-1})$, is below the 45 degree line. Equally important, the function is monotonically increasing and has a positive lower bound. These ensure the existence of a nontrivial steady-state equilibrium $n_t = n^{III}(n_t) > 0$, towards which n_t monotonically declines over Stage III.[38] Since the corresponding path of e_t^G is characterized by (26) and (29), one obtains the following proposition.

[37] As shown by Lemma 1, $\phi'(e_t^G) \leq 0 \ \forall e_t^G \geq 0$, with strict inequality if $\phi(e_t^G) > 0$. Therefore, $n^{III}(n_{t-1})$ is strictly monotonically increasing in $n_{t-1} > 0$ if $\phi(\tau) > 0$.

[38] The properties of $n^{III}(n_{t-1})$ shown above do not ensure the uniqueness of the steady-state equilibrium. In the case of multiple equilibria, n_t converges to the largest steady-state level on the interval $(0, n_{t'-1})$.

Proposition 2 (Population Aging and Political Bias) *Under* $(A1)$–$(A4)$,

(a) $n_t \leq n_{t-1}$ *and* $\theta\tau < e_{t+1}^G \leq e_t^G$ $\forall t \geq t'$;

(b) n_t *converges to a nontrivial steady-state equilibrium where* $n_t = n^{III}(n_t) > 0$ *and* $e_{t+1}^G = e_t^G$.

Proposition 2 asserts a vicious cycle between population aging and a deterioration in education policy. As the old/adult ratio increases, the old generation becomes more politically influential in the allocation of public resources. The resulting budget cut in public education raises education costs on young families, and thereby discourages childbirth. The level of public education in Stage III is nevertheless higher than in Stages I–II, where $e_t^G \leq \theta\tau$.

There are a few remarks on this result. First, as mentioned in Footnote 3, the proposition does not mean a nominal decline in aggregate spending on public education. e_t^G should be viewed as the real education expenditure in terms of the time of adult workers. Second, private education is stimulated by a reduction in e_t^G because, in this stage, child education is beneficial for the adult generation. Such a substitutional relationship between private and public education does not exist in underdeveloped stages, where households rely on child labor. Third and finally, the steady-state fertility rate, $n_t = n^{III}(n_t)$, may be smaller than unity, depending on the structural parameters of the economy.[39]

The sequence $\{n_{t-1}\}_{t=t'}^{\infty}$ determines the population dynamics given by

$$N_{t+1} = n^{III}(n_{t-1})N_t, \tag{33}$$

where the initial condition $N_{t'}$ is obtained from (25). Population keeps either shrinking or expanding in the long run, depending on the steady-state fertility rate.

4.3.3 Technological Progress

The results above demonstrate that n_t^R, n_t^P, h_{t+1}^R and h_{t+1}^P depend only on e_t^G in period $t \geq t'$. Furthermore, because $n_t^R = n_t^P$ the group ratio is stationary over Stage III; i.e., $q_{t+1} = q_{t'} \equiv \bar{q}$ $\forall t \geq t'$, where $q_{t'}$ is given by (25). It then follows from (17) that the aggregate level of efficiency

[39] Since $\phi(e_t^G) > \hat{e} - e_t^G$ in (28),

$$n_t < \frac{\alpha T}{\delta + \hat{e} - e_t^G} \qquad \forall t \geq t',$$

where $e_t^G \in (\theta\tau, \tau)$ as shown by (26). This shows that $n_t < 1$ if the difference $\hat{e} - \tau$ is sufficiently large, for example.

labor H_{t+1} becomes

$$H_{t+1} = (1-\alpha)TN_t n_t[\bar{q}h_{t+1}^R + (1-\bar{q})h_{t+1}^P] \equiv H^{III}(e_t^G, N_t), \tag{34}$$

where $H_1^{III}(e_t^G, N_t) > 0$ and $H_2^{III}(e_t^G, N_t) > 0$ $\forall (e_t^G, N_t) \gg 0$. The first property is because a rise in e_t^G increases $n_t^i h_{t+1}^i$, and the second property reflects the scale effect.[40]

Substituting (34) into (18), the evolution of technology in Stage III is

$$A_{t+1} = [1 + g(H_{t+1})]A_t \equiv G^{III}(e_t^G, N_t)A_t, \tag{35}$$

where the initial value $A_{t'}$ is obtained from (25), and the sequence $\{e_t^G, N_t\}_{t=t'}^{\infty}$ is fully determined by (29), (30) and (33). Since $e_t^G > \theta\tau$ for all $t \geq t'$, the growth rate of technology in this stage is higher than in the previous stages, as long as population keeps growing. However, such rapid growth may not be sustainable in the long run, depending on the demographic trend.

Proposition 3 (Economic Growth Slowdown) *If* (A1)–(A4) *are satisfied and* $n_t = n^{III}(n_t) < 1$, *the growth rate of technology,* g_t, *converges toward zero in the long run.*

Proof. Proposition 2 and (33) reveal that under the conditions above, the adult population N_t declines towards zero over Stage III. Since (35) shows that $g(0) = 0$ if $N_t = 0$, g_t converges toward zero in the long run. □

The ultimate population extinction implied by this proposition does not appear to be quite implausible, considering the current fertility rates of industrial economies.[41] Nevertheless, the long-run result needs to be interpreted carefully since it is based on the political system (26). Rather than offering an accurate prediction, the proposition alerts the possibility of economic growth slowdown in the presence of political bias: If public policies excessively reflect the opinions of the elderly, aging economies will underinvest in young generations, who will be the main labor force in the future.

In order to avoid such a miserable consequence, the government needs to be impartial between generations. By allocating a fair budget to public education, the government is able to lighten

[40] Substituting $\rho(z_t^i) = \rho$, $e_t^i = \phi(e_t^G)$ and (28) into (7) yields the indirect utility function of (e_t^G, z_t^i). Then the envelope theorem reveals that the indirect utility function is strictly increasing in e_t^G. Moreover, as implied by (8), consumption c_t^i is constant for any e_t^G. Thus recalling $\alpha = \rho\beta$, one finds that $n_t^i h_{t+1}^i$ increases with e_t^G.

[41] For example, a French demographer Bourgeois-Pichat estimates that the population of European countries would die out within 300 years if their fertility rates were and remained the West German level (Johnson et al., 1989).

the households' burden on education, $\phi(e_t^G)$, and thereby encourage childbirth. Equally important, these effects can be augmented by improving the efficiency of education policy.[42] Therefore, with a high level of quantity and quality, public education works as a substitute for private education, and as an effective means of preventing population decline.

5 Concluding Remarks

This research has unveiled the crucial role played by public education in the transition to aging societies since the early Industrial Revolution. In the early phases of Industrial Revolution, skill investment is undesirable for the masses, and compulsory education worked as a child labor regulation that discouraged childbirth. On the other hand, in the later phases of the Industrial Revolution, public education began to be viewed as a desirable service that assists households' child rearing. Nevertheless, it appears that public education may have been undersupplied in the postwar aging countries.

The theory developed in this paper demonstrates that advanced economies may have poor growth performance through a vicious cycle between population aging and the deterioration of education policy. In order to make an escape from this cycle and sustain national pension systems, policymakers should aim to prevent the allocation of national resources from being biased to a particular age cohort. Furthermore, they need to carefully examine whether the budgets allocated to public education are being used efficiently. Without their dedicated efforts, public education would not be an effective substitute for private education, and thus fail to encourage childbirth.

The theoretical foundation behind these arguments is that education policy stimulates individuals' incentives for childbirth as well as for skill investment. By taking this two-sided effect into account, one finds that education policy may be more productive than other policies, such as child benefits, for the purpose of enhancing human capital at the national level. Future research should be directed at investigating this potential superiority of public education.

[42] To see this, consider public education investment in efficiency units, denoted as $\sigma_t e_t^G$, where σ_t measures how efficiently e_t^G is invested: for instance, it depends on the allocation of e_t^G between facilities and teachers. In this case the level of private education in Stage III is $e_t^i = \phi(\sigma_t e_t^G)$.

Appendix 1 Technical Discussions

The Local Optimality Conditions for Private Education

This section discusses the case that $z_t^i \geq z^*$ and thus $\rho(z_t^i) = \rho > 0$ in (3). Substituting (8) into the objective function (7) yields the (partially) indirect utility function that depends on e_t^i, e_t^G and z_t^i. The sign of the first derivative of this function with respect to e_t^i is determined by the sign of

$$D(e_t^i, e_t^G, z_t^i) \equiv \begin{cases} D^l(e_t^i, e_t^G, z_t^i) & \text{if } e_t^i + e_t^G < \hat{e}; \\ D^h(e_t^i, e_t^G) & \text{if } e_t^i + e_t^G > \hat{e}, \end{cases}$$

where

$$\begin{aligned} D^l(e_t^i, e_t^G, z_t^i) &\equiv h_1(e_t^i, e_t^G)[\delta + e_t^i - \omega(e_t^i, e_t^G, z_t^i)] - [1 - \omega_1(e_t^i, e_t^G, z_t^i)]h(e_t^i, e_t^G); \\ D^h(e_t^i, e_t^G) &\equiv h_1(e_t^i, e_t^G)(\delta + e_t^i) - h(e_t^i, e_t^G). \end{aligned}$$

For analytical convenience, the domain of $D^l(e_t^i, e_t^G, z_t^i)$ is defined as

$$\Omega^l \equiv \{(e_t^i, e_t^G, z_t^i) \in \mathbb{R}_+^3 : e_t^i + e_t^G \leq \hat{e} \text{ and } z_t^i \geq z^*\}.$$

Noting the properties of $\omega(\cdot)$ in (5) and $h(\cdot)$ in (6), one obtains

$$\begin{aligned} &D_1^l(e_t^i, e_t^G, z_t^i) < 0; \\ &\lim\nolimits_{z_t^i \to \infty} D^l(e_t^i, e_t^G, z_t^i) = D^h(e_t^i, e_t^G) > D^l(e_t^i, e_t^G, z_t^i), \end{aligned} \tag{36}$$

for $(e_t^i, e_t^G, z_t^i) \in \Omega^l$. On the other hand, $D^h(e_t^i, e_t^G)$ is defined on $\mathbb{R}_+^2$ and has properties such that, $\forall (e_t^i, e_t^G) \geq 0$,

$$D_1^h(e_t^i, e_t^G) < 0; \qquad D_2^h(e_t^i, e_t^G) < 0; \qquad \lim_{e_t^i \to \infty} D^h(e_t^i, e_t^G) < 0. \tag{37}$$

In order to prove the last property in (37), note that

$$\frac{\partial}{\partial e_t^i} \left[\frac{h(e_t^i, e_t^G)}{h_1(e_t^i, e_t^G)} \right] > 1.$$

Thus it follows that

$$\lim_{e_t^i \to \infty} \left[(\delta + e_t^i) - \frac{h(e_t^i, e_t^G)}{h_1(e_t^i, e_t^G)} \right] < 0.$$

The properties with respect to e_t^i are depicted by the upper panel of Figure 2. The two solid curves indicate $D^l(e_t^i, e_t^G, z_t^i)$ and $D^h(e_t^i, e_t^G)$, respectively, for a given e_t^G and $z_t^i = z^{\text{low}}$. They are both negatively-sloped and are disconnected at $e_t^i = \hat{e} - e_t^G$, where the wage function $\omega(\cdot)$ is kinked. The corresponding utility level is shown by the solid curve in the lower panel. The curve is continuous and kinked at $e_t^i = \hat{e} - e_t^G$. Furthermore, its slope is positive where $D(e_t^i, e_t^G, z^i) > 0$, and is negative where $D(e_t^i, e_t^G, z^i) < 0$. These properties establish the following lemma.

Lemma 5 *For an adult individual with $z_t^i \geq z^*$, a level of private education, e_t^i, is at least locally optimal if (a) $D(e_t^i, e_t^G, z_t^i) = 0$ and $e_t^i > 0$ or (b) $D(e_t^i, e_t^G, z_t^i) \leq 0$ and $e_t^i = 0$.*

Proof of Lemma 1

(a) $0 \leq e_t^G \leq \hat{e}$. Since $(A1)$ is equivalent to $D^h(\hat{e}, \hat{e}) > 0$, the properties of $D^h(\cdot)$ in (37) reveal that

$$D^h(\hat{e} - e_t^G, e_t^G) > 0 \qquad \forall e_t^G \in [0, \hat{e}].$$

Using (37) again, one finds that there exists a unique value $e_t^i > \hat{e} - e_t^G$ such that $D^h(e_t^i, e_t^G) = 0$. Thus as depicted by Figure 2, the curve $D^h(e_t^i, e_t^G)$ crosses the horizontal line at a unique point above $\hat{e} - e_t^G$.

(b) $e_t^G > \hat{e}$. The results in case (a), along with (37), imply that there exists a unique $e_t^i \geq 0$ such that $D^h(e_t^i, e_t^G) \leq 0$, with equality if $e_t^i > 0$.

Cases (a)–(b) show that there exists a single-valued function $\phi(e_t^G) \geq 0$ such that

$$D^h(\phi(e_t^G), e_t^G) \leq 0; \qquad \phi(e_t^G) + e_t^G > \hat{e},$$

where the equality holds if $\phi(e_t^G) > 0$. In light of Lemma 5, $\phi(e_t^G)$ is the optimal level of private education at least on the interval where $D(e_t^i, e_t^G, z_t^i) = D^h(e_t^i, e_t^G)$.

Furthermore, applying the Implicit Function Theorem and noting (37), one obtains

$$\phi'(e_t^G) = -\frac{h_{12}(e_t^i, e_t^G)(\delta + e_t^i) - h_2(e_t^i, e_t^G)}{h_{11}(e_t^i, e_t^G)(\delta + e_t^i)} < 0,$$

where $e_t^i = \phi(e_t^G) > 0$. Lastly, $\phi'(e_t^G) \leq 0$ if $\phi(e_t^G) = 0$. $\square$

Proof of Lemma 2

(a) $0 \leq e_t^G \leq \hat{e}$. Using (36)–(37) with (A1),

$$\lim_{z_t^i \to \infty} D^l(e_t^i, e_t^G, z_t^i) = D^h(e_t^i, e_t^G) > 0 \qquad \forall e_t^i \in [0, \hat{e} - e_t^G].$$

This result is shown by Figure 2. As z_t^i increases to z^{high} and further, the curve $D^l(e_t^i, e_t^G, z_t^i)$ in the upper panel shifts upward, approaching the upper bound $D^h(e_t^i, e_t^G)$. Correspondingly, the (partially indirect) utility function in the lower panel has a strictly positive slope on $[0, \hat{e} - e_t^G)$. On the other hand, the proof of Lemma 1 shows that $\phi(e_t^G)$ is the optimal level of private education at least on the interval $(\hat{e} - e_t^G, \infty)$. Since the utility function is continuous on $[0, \infty)$ in the diagram, one finds that $\phi(e_t^G)$ is globally optimal.

(*b*) $e_t^G > \hat{e}$. The proof of Lemma 1 shows that $\phi(e_t^G)$ is the optimal level of private education on the interval $[0, \infty)$. This directly proves the global optimality of $\phi(e_t^G)$. □

Proof of Lemma 3

Substituting $q_{t+1} = n_t^R q_t / n_t$ into the expression of $n_{t+1} - n_t$ obtained from (16),

$$n_{t+1} - n_t = \left(\frac{n_{t+1}^R}{n_t} - 1 \right) q_t n_t^R + \left(\frac{n_{t+1}^P}{n_t} - 1 \right) (1 - q_t) n_t^P,$$

where $q_t \leq q_0 < 1$ for $t \in [0, t^*]$ because of (21). Furthermore, in light of (16) and (20),

$$\begin{aligned} n_t^i &= \frac{\alpha T}{\delta - \bar{w}/a^i A_t}; \\ \frac{n_t}{n_{t+1}^i} &= q_t \frac{\delta - \bar{w}/a^i A_{t+1}}{\delta - \bar{w}/a^R A_t} + (1 - q_t) \frac{\delta - \bar{w}/a^i A_{t+1}}{\delta - \bar{w}/A_t}, \end{aligned}$$

for $t \in [0, t^* - 1)$, where $a^R > a^P = 1$ and $A_{t+1} \geq A_t > \bar{w}/\delta$ because of (18) and (A3). Therefore, as A_{t+1} (and thus A_t) approaches $\bar{w}/\delta$ from above, n_t^P goes to infinity and n_{t+1}^P / n_t becomes greater than unity, while n_t^R and n_{t+1}^R / n_t remain finite. In this case $n_{t+1} - n_t$ goes to infinity. □

Proof of Lemma 4

Note that $H_{t+1} > (1-\alpha) T N_{t+1}^P$ in (17). Furthermore, (20) and (24) imply that $N_{t+1}^P = n_t^P N_t^P > N_t^P$ for $t \in [0, t')$. Then it follows from (18) that the growth rate of technology has a lower bound such that

$$g_{t+1} > g^{\min} \equiv g((1 - \alpha) T N_0^P) > 0 \qquad t \in [0, t'),$$

where $N_0^P = (1 - q_0) N_0 > 0$ under (A3). Using this result with (24) and (28) reveals that, $\forall t \in [t^*, t')$,

$$n_{t+1}^P < \frac{\alpha T}{\delta - \bar{w} l(\theta\tau) / [(1 + g^{\min}) A_t h(0, \theta\tau)]} < n_t^P,$$

where, in light of (25), A_t and n_t^P are independent of q_{t+1} and q_t ($> q_{t+1}$). On the other hand, (16) shows that n_t^P approaches n_t as q_t goes to zero. Thus, the relationship $n_{t+1}^P < n_t^P$ above implies that $n_{t+1} < n_t$ if q_t (and thus q_{t+1}) is sufficiently small in period $t \in [t^*, t')$. □

Proof of Equation (31)

As follows from (16), (24) and (32), a sufficient condition for $n_{t'} < n_{t'-1}$ is

$$\frac{\alpha T}{\delta + \phi(\tau)} \leq q_{t'-1} \frac{\alpha T}{\delta + \phi(\theta\tau)} + (1 - q_{t'-1}) \frac{\alpha T}{\delta - \omega^{\min}},$$

where $\omega^{\min} \equiv \omega(0, \theta\tau, z^*) < \omega(0, \theta\tau, z^P_{t'-1})$ because $z^P_t < z^* \ \forall t \in [0, t')$. Solving the expression above for $q_{t'-1}$ yields

$$q_{t'-1} \leq \left(1 - \frac{\delta - \omega^{\min}}{\delta + \phi(\tau)}\right) \left(1 - \frac{\delta - \omega^{\min}}{\delta + \phi(\theta\tau)}\right)^{-1} \equiv q^{\max},$$

where $\phi(\tau) \leq \phi(\theta\tau)$ and thus $q^{\max} \in (0, 1]$. Note that the condition $q_{t'-1} \leq q^{\max}$ is satisfied under (A3) because, as shown in Section 4, $q_{t+1} \leq q_t \ \forall t \geq 0$. □

Appendix 2 Data Sources

Figure 1 (Japan)

- Old-age related benefits (*Koreisha Kankei Kyufuhi*) are obtained from the National Institute of Population and Social Security Research (2007, p. 102).
- The number and the percentage of the population aged 65 and above are published by the Japanese Statistics Bureau. The data for 1973–2000 and 2001–2004, respectively, are obtained from the following websites:

  `http://www.stat.go.jp/data/jinsui/wagakuni/index.htm`

  `http://www.stat.go.jp/data/jinsui/2004np/index.htm#05k16-b`

- Public expenditures for national schools are recorded on the special account budgets (revised) of fiscal years 1973–2003 as "*Kokuritsu Daigaku*", and on the General Account budget (revised) of 2004 as "*Kokuritsu Daigaku Hojin Uneihi*".[43] The whole data are published online by the Ministry of Finance Japan. Besides, the data for 1973–2000 are compiled in the electric file "*Kokuritsu Gakko Tokubetsu Kaikei* (98)" by the Research Institute for Higher Education of Hiroshima University. Their respective URLs are:

  `http://www1.mof.go.jp/data/index.htm`

  `http://rihe.hiroshima-u.ac.jp/data_list.php?%20-%2035`

[43] The difference in the budget sources is due to the incorporation of national universities in 2004.

- The number of students enrolled in national universities and graduate schools is obtained from the file "Zaigakushasu (2)" available at the Hiroshima University's website listed above.

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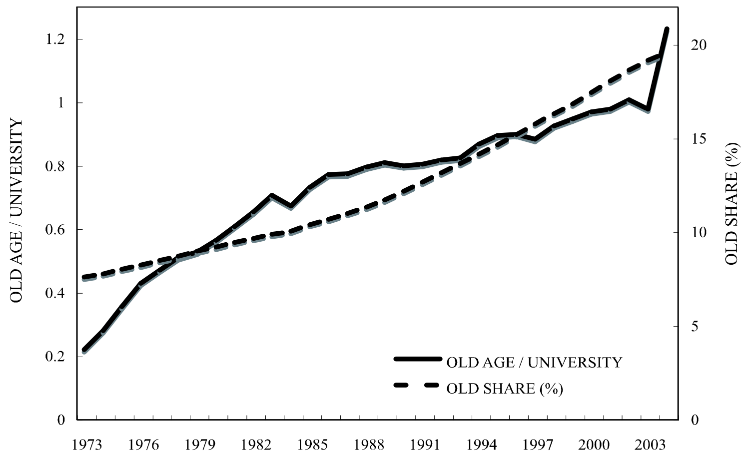

Figure 1. Population Aging and Generational Bias in Public Spending: Japan, 1973–2004

Notes: OLD AGE = old-age related benefits per person aged 65 and above. UNIVERSITY = public expenditures for national schools (mostly university) per student. OLD SHARE = the share of the population aged 65 and above.
Sources: See Appendix 2.

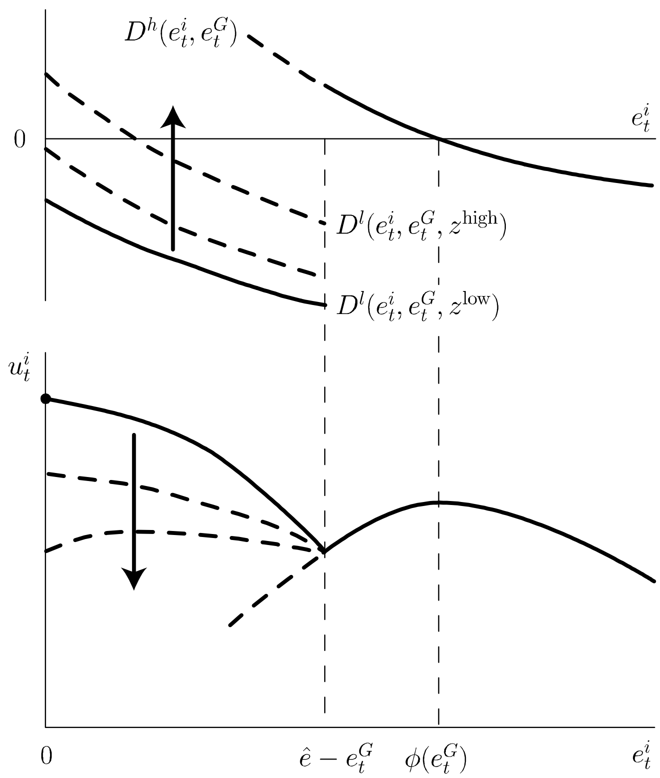

Figure 2. The Determination of Private Education for $\rho_t^i = \rho > 0$

Notes: The diagram depicts the decision of private education by an adult individual who cares about the utility in elderhood. There may exist multiple local optima, depending on the levels of public education e_t^G and parental potential income z_t^i. Given a sufficiently large amount of z_t^i, the individual spends $\phi(e_t^G) > 0$ units of time per child on education.

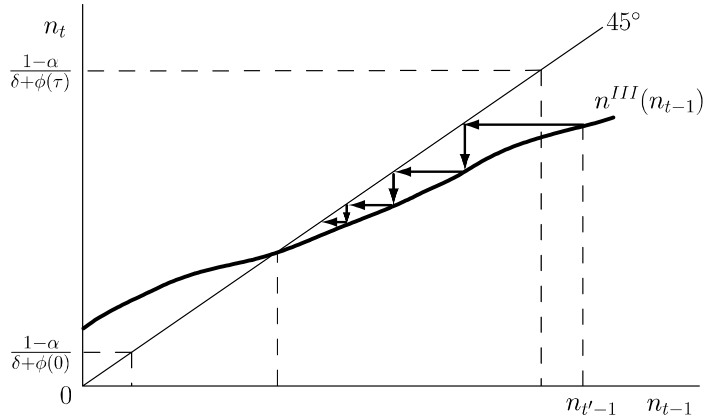

Figure 3. The Evolution of Fertility in Stage III

Notes: The diagram shows that fertility declines monotonically over Stage III, converging to a nontrivial stationary-state equilibrium. The long-run fertility rate may or may not be lower than unity (the critical level for stationary population), depending on how private education reacts to undersupplied public education.