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LOCAL HOME ENVIRONMENT EXTERNALITY IS A SOURCE OF INDETERMINACY

Shiro Kuwahara
and
Katsunori Yamada

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The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

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Shiro Kuwahara

Graduate School of Systems and Information Engineering, University of Tsukuba, Japan

Katsunori Yamada†

Institute of Social and Economic Resarch, Osaka University

Mihogaoka 6-1, Ibaraki 567-0047, Japan

Abstract

We address *the local home environment externality* conceptualized by Galor and Tsiddon (1997a; 1997b) in the two sector growth model of Lucas (1988). We show that this version of externality related to human capital accumulation process can be a source of indeterminacy.

Keywords: Local home environment externality; human capital; indeterminacy

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†Corresponding author. Tel and fax: +81-742-27-9981. E-mail address: kyamada@econ.osaka-u.ac.jp (K. Yamada)

1 Introduction

As clearly documented in Benhabib and Perli (1994), dynamic equilibrium can be indeterminate in models with externalities. Because indeterminacy are useful to explain business fluctuations, economists have tried to find out appropriate externalities. Recently, Chen and Hsu (2007) show that a version of consumption externalities, admiration, is a source of indeterminacy. In this paper, following the spirit of Chen and Hsu (2007), we present another clear-cut and plausible source of indeterminacy.

The externality we address here is related to human capital; the local home environment externality (LHEE) conceptualized by Galor and Tsiddon (1997a; 1997b). With this externality, newly born agents are affected by parents at home and obtain positive amount of human capital before they enter production process.[1] Hence, it is plausible that there is inter-generational spillover of human capital, which effects parents cannot internalize, and this version of externality is a source of indeterminacy. We show that it is the case. We also argue that LHEE has merits that it is plausible and simpler as a source of indeterminacy than a version of human capital externality documented in Benhabib and Perli (1994).

In section 2 we provide our model. Section 3 investigates the stability of the equilibrium. Section 4 concludes the paper.

2 The Model

We study the two sector growth model of Lucas (1988) with LHEE. Specifically, addressing a Cobb-Douglas production function on final output as

$$Y(t) = Ak(t)^{\beta}(u(t)h(t))^{1-\beta}, \quad A > 0, \quad \beta \in (0,1)$$

the model is given as

$$\max_{c(t),u(t)} \int_0^{\infty} \frac{c(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \tag{P}$$

subject to

$$\dot{k}(t) = Ak(t)^{\beta}(u(t)h(t))^{1-\beta} - c(t) - nk(t)$$
$$\dot{h}(t) = \delta(1-u(t))h(t) - nh(t) + \alpha h(t),$$

where c is consumption, k is physical capital, h is human capital, and u is the fraction of human capital devoted to production of final output. Note here

[1]Behrman et al. (1999) argue that the educational achievements of mothers are positively correlated with the intensity of home schooling towards their children.

that human capital per capita is diluted because of the population growth, $nh(t)$, but augmented with LHEE, $\alpha h(t)$. We assume that agents cannot internalize the effect of LHEE on human capital augmentation. $\sigma(>0)$, $\delta(>0)$, $n(>0)$, $b(>0)$, and $\alpha(>0)$ are parameters.

The problem (P) is solved by defining the current value Hamiltonian[2]

$$H \equiv \frac{c(t)^{1-\sigma}-1}{1-\sigma}+\lambda_1(t)[Ak(t)^{\beta}(u(t)h(t))^{1-\beta}-c(t)-nk(t)]+\lambda_2(t)[\delta(1-u(t))h(t)-nh(t)]$$

and deriving the optimal conditions;

$$c(t)^{-\sigma} = \lambda_1(t)$$

$$\lambda_1(t)(1-\beta)Ak(t)^{\beta}u(t)^{-\beta}h(t)^{1-\beta} = \lambda_2(t)\delta h(t)$$

$$\dot{\lambda_1}(t) = [\rho + n - \beta Ak(t)^{\beta-1}u(t)^{1-\beta}h(t)^{1-\beta}]\lambda_1(t)$$

$$\dot{\lambda_2}(t) = -\lambda_1(t)(1-\beta)Ak(t)^{\beta}u(t)^{1-\beta}h(t)^{-\beta} + [\rho + n - \delta(1-u(t))]\lambda_2(t)$$

and the usual two transversality conditions,

$$\lim_{t\to\infty} \lambda_1(t)k(t)e^{-\rho t} = 0$$

$$\lim_{t\to\infty} \lambda_2(t)h(t)e^{-\rho t} = 0,$$

where λ_1 and λ_2 represent shadow prices of physical and human capital, respectively.

We define new two variables which are stationary on the balanced growth path in order to investigate the stability of the equilibrium path; $q \equiv c/k$ and $x \equiv k/h$. Then, with a little algebra, the intensive form dynamical system consisting of $(u(t), q(t), x(t))$ can be obtained as

$$\frac{\dot{u}(t)}{u(t)} = \delta u(t) - q(t) + \frac{\delta}{\beta} - \delta - \alpha,$$

$$\frac{\dot{q}(t)}{q(t)} = \left(\frac{\beta}{\sigma} - 1\right) Ax(t)^{\beta-1}u(t)^{1-\beta} + q(t) + n - \frac{n}{\sigma} - \frac{\rho}{\sigma}$$

$$\frac{\dot{x}(t)}{x(t)} = Ax(t)^{\beta-1}u(t)^{1-\beta} + \delta u(t) - q(t) - \delta - \alpha,$$

[2]Although $\alpha h(t)$ appears in the low of motion for human capital, it does not taken into account by agents so that in the Hamiltonian $\alpha h(t)$ is not included. Alternatively, one can think that $\alpha h(t)$ is a uncontrollable constant term for agents.

together with stationary variables of

$$u^* = 1 - \frac{(\delta - \rho - n)/\sigma + n - \alpha}{\delta}$$

$$q^* = \delta u^* + \frac{\delta}{\beta} - \delta - \alpha$$

$$x^* = \left(\frac{q^* + \delta(1 - u^*) + \alpha}{A(u^*)^{1-\beta}} \right)^{\frac{1}{\beta - 1}}.$$

Inner solution conditions of $u^* \in (0, 1)$ and $q^* > 0$ require that parameters satisfy the following conditions

$$\Lambda - \delta < \alpha < \Lambda,$$

where $\Lambda \equiv (\delta - \rho - n)/\sigma + n$ can be innocuously assumed positive to ensure a positive growth rate of the economy, and

$$\Lambda < \frac{\delta}{\beta}.$$

For later use, here we put another condition on Λ as[3]

$$\Lambda - \delta > 0. \tag{C1}$$

3 Stability Analysis

The linearized dynamical system around the steady state, (u^*, q^*, x^*), can be derived as

$$\begin{pmatrix} \dot{u} \\ \dot{q} \\ \dot{x} \end{pmatrix} = \begin{pmatrix} \delta u^* & -u^* & 0 \\ \left(\frac{\beta}{\sigma} - 1\right)(1 - \beta)\frac{\Upsilon^*}{u^*} q^* & q^* & \left(\frac{\beta}{\sigma} - 1\right)(\beta - 1)\frac{\Upsilon^*}{x^*} q^* \\ \left\{(1 - \beta)\frac{\Upsilon^*}{u^*} + \delta\right\} x^* & -x^* & (\beta - 1)\Upsilon^* \end{pmatrix} \begin{pmatrix} u - u^* \\ q - q^* \\ x - x^* \end{pmatrix},$$

where $\Upsilon^* \equiv A(u^*/x^*)^{1-\beta}$. Then, we define the characteristic equation of the dynamical system as

$$\Omega(\omega) = -\omega^3 + Tr\omega^2 + B\omega + Det,$$

where ω denotes eigenvalues, Tr is the trace and Det is the determinant of the Jacobean matrix of the dynamical system. We can easily see that Det is always negative as

$$Det = -(1 - \beta)\delta \Upsilon^* q^* u^* < 0.$$

With $Det < 0$, we have the following lemma.

[3] In order to satisfy (C1), σ must be sufficiently small.

Lemma:

Indeterminacy of the dynamical system requires $Tr < 0$ and $B < 0$.

Proof:

See the proof in Chen and Hsu (2007). Q.E.D.

We then investigate if there are parameter sets to satisfy these requirements. Now we can obtain B as

$$B = (1-\beta)\frac{\delta}{\beta}(\delta u^* + q*) - \delta u^* q^*.$$

Although the sign of B is non-determinate, the following condition ensures $B < 0$ since $\delta u^* q^* > 0$ and does not converge to zero with β close to one.

$$\beta \quad \text{is enough close to} \quad 1 \tag{C2}$$

We assume (C2) to be satisfied: the economy is strongly physical capital intensive.

With respect to Tr, it is derived as

$$Tr = 2\delta u^* - \alpha,$$

which is always positive when we do not consider LHEE ($\alpha = 0$). Hence, we have $Tr < 0$ if and only if

$$\alpha < 2(\Lambda - \delta). \tag{C3}$$

Now we have the proposition as

Proposition:

Local home environment externality is a source of Indeterminacy.

Proof:

A simple algebra shows that parameter region satisfying (C1), (C2), and (C3) is not empty as shown in Figure 1. Hence, the dynamical system can be indeterminate around the steady state. Q.E.D.

To finish this section, we recap that there is no possibility of indeterminacy if we do not consider any externalities in this type of growth models (Mulligan and Sala-i-Martin; 1993).

4 Conclusion

Following the spirit of Chen and Hsu (2007), we present a plausible source of indeterminacy related to human capital accumulation. This paper contributes to the literature since the externality considered here seems plausible and has empirical supports. Also, the introduction of LHEE does not provide any analytical complexity, which will be a technical merit in explaining real world phenomena with formal models.

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Figure 1: Parameter restrictions for indeterminacy