Discussion Paper No. 731

COMPLEMENTARY RELATIONSHIPS BETWEEN EDUCATION AND INNOVATION

Katsuhiko Hori
and
Katsunori Yamada

February 2009

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

Complementary Relationships between Education and Innovation*

Katsuhiko Hori† Katsunori Yamada‡

February, 2009

Abstract

This paper combines two prototype endogenous growth models: the Schumpeterian endogenous growth model developed by [Howitt(1999)] and human capital growth models developed by [Uzawa(1965)] - [Lucas(1988)]. While standard Schumpeterian growth models suggest that a subsidy to R&D has long-run effects, we show that a subsidy to human capital investment has a positive impact on R&D efforts as well as on human capital accumulation. Because in our model, the per capita output growth rate depends on both technology improvements and human capital accumulation, the model bridges the gap between the literature concerning Schumpeterian growth model and that concerning growth empirics.

JEL classification: O11; O31; O41

Keywords: Schumpeterian endogenous growth model; R&D; human capital; subsidy

*The first author would like to acknowledge the financial support provided by the Global Center of Excellence (GCOE) program entitled "Raising Market Quality · Integrated Design of Market Infrastructure" of Keio University. The second author is grateful for the research grant provided by the GCOE program entitled "Human Behavior and Socioeconomic Dynamics" of Osaka University.

†Corresponding author: Institute of Economic Research (KIER), Kyoto University. Address: Yoshida Hon-machi, Sakyo-ku, Kyoto 606-8501, Japan. E-mail address: hori@kier.kyoto-u.ac.jp

‡Institute of Social and Economic Research (ISER), Osaka University. Address: Mihogaoka 6-1, Ibaraki 567-0047, Japan. E-mail address: kyamada@econ.osaka-u.ac.jp

1 Introduction

This study aims to integrate two major strands of endogenous growth models: Schumpeterian endogenous growth models developed by [Young(1998)] and [Howitt(1999)] and human capital accumulation models developed by [Uzawa(1965)] and [Lucas(1988)]. We explain how a subsidy to human capital investment positively affects not only R&D efforts but also human capital accumulation. A significant merit of the model below is that the per capita output growth rate depends on both technology improvements and human capital accumulation. We argue that when a Schumpeterian endogenous growth model meets a human capital accumulation model, endogenous growth theory meets growth empirics.

Ever since Jones' critique ([Jones(1995)]), two different types of endogenous growth theories, namely, the Schumpeterian endogenous growth theory and the semi-endogenous growth theory, have struggled for supremacy. The main proposition of the former theory is that a subsidy on R&D investment has long-run effects, while the latter argues otherwise.

In this paper, after the recent finding by [Ha and Howitt(2007)] that the Schumpeterian endogenous growth theory accommodates to the U.S. experience, we augment Howitt's Schumpeterian endogenous growth model to consider the human capital accumulation process.[1] The motivation is straightforward: while in [Howitt(1999)], human capital accumulation is disregarded, there is a strong consensus among economists that human capital is also an engine of growth in addition to technology improvements. In particular, from the viewpoint of growth empirics literature since [Mankiw et al.(1992)Mankiw, Romer, and Weil], if we disregard human capital, then the estimators in growth regressions will be biased because the per capita output growth rate is assumed to be attributable to capital accumulation, technology progress, and human capital accumulation. The model below provides a structural basis for including human capital as a determinant of long-run growth by using a Schumpeterian growth model. In this sense, this study can be taken as providing a microfoundation for reduced form analyses in growth empirics literature, where the production function of final output is assumed.

With respect to policy implications, we obtain the following results. First, a subsidy to R&D investment accelerates the output growth rate as [Howitt(1999)]. Second, it is shown that a subsidy to human capital in-

[1] A technical feature of this study is that there are three engines of growth: horizontal R&D, vertical R&D, and human capital accumulation. To the best of our knowledge, this is the first study featuring an endogenous growth model with three engines of growth.

vestment has a direct positive effect on output growth via the promotion of human capital accumulation and has an indirect positive effect through improvements in technology.[2] To the best of our knowledge, this is the first study that demonstrates complete general equilibrium policy implications by taking into account the endogenous determinations of both technology improvements and human capital accumulation.

This paper is organized as follows. In Section 2, we set up the model. The equilibrium is analyzed in Section 3. Section 4 focuses on a steady growth path and derives chief propositions, and Section 5 concludes the paper.

2 The Model

The basic setup of our model follows [Howitt(1999)]. A significant difference between [Howitt(1999)] and our model is that we consider human capital as the sole production input for the intermediate goods sector, whereas [Howitt(1999)] assume that exogenously growing row labor force is inelastically supplied to this sector.

In this study, human capital can be used for the production of intermediate product and for investment in human capital creation. The population growth rate n is exogenously determined, and the population size is denoted as L_t. The final goods output, which is the numeraire in the model, can be used for consumption (C_t), investment in vertical R&D (Z_{At}), and investment in horizontal R&D (Z_{Nt}).

2.1 Production structure

The final goods sector is under perfect competition. The technology for final goods production is specifically given by

$$Y_t = X^{1-\alpha} \int_0^{N_t} A_{i,t} x_{i,t}^{\alpha} \mathrm{d}i, \tag{1}$$

where Y_t is the final goods output, X is a fixed resource such as land, N_t measures the varieties of intermediate goods at time t, $A_{i,t}$ is a productivity parameter attached to the incumbent version of intermediate product

[2] See [Arnold(1998)] for this point. In [Arnold(1998)], a subsidy to R&D investment has no long-run effects since the model is semi-endogenous. In addition, while a subsidy to human capital investment has positive effects according to [Arnold(1998)], he did not delve on the issue.

i, x_i is the amount of intermediate product i used in the economy, and $\alpha \in (0,1)$ is the capital share. Since this study assumes that the total endowment of the fixed resource in the economy is equal to 1, we henceforth abbreviate variable X from the production function. Under perfect competition, the first-order condition for the final goods sector with respect to $x_{i,t}$ is given as

$$\alpha A_{i,t} x_{i,t}^{\alpha-1} = p_{i,t}, \tag{2}$$

where $p_{i,t}$ is the price of intermediate product i.

The intermediate goods sector is under monopolistic competition. In this study, we assume that one unit of intermediate good is made from one unit of human capital. By this assumption, two sources of endogenous growth-R&D and human capital accumulation-are interacted. The profit in creating intermediate product i is given by

$$\Pi_{i,t} = p_{i,t} x_{i,t} - w_t x_{i,t},$$

where w_t is the real wage for human capital. With the demand function of $x_{i,t}$ from (2), the first-order condition with respect to $x_{i,t}$ is then given by

$$w_t = \alpha^2 A_{i,t} x_{i,t}^{\alpha-1}.$$

Hence, the demand of $x_{i,t}$ is determined as

$$x_{i,t} = \alpha^{\frac{2}{1-\alpha}} A_{i,t}^{\frac{1}{1-\alpha}} w_t^{\frac{-1}{1-\alpha}}, \tag{3}$$

and the profits of an intermediate goods firm are given by

$$\Pi_{it} = \alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_{i,t}^{\frac{1}{1-\alpha}} w_t^{\frac{\alpha}{\alpha-1}}. \tag{4}$$

2.2 Innovations

Following [Howitt(1999)], we consider two types of innovations. Vertical innovations improve productivity in each intermediate goods sector i, $A_{i,t}$, and horizontal innovations bring new varieties into the economy, N_t.

2.2.1 Vertical Innovations

The Poisson arrival rate of vertical innovations in each sector is defined as

$$\phi_t = \lambda_A \frac{Z_{At}}{A_t} \equiv \lambda_A z_{At},$$

where $\lambda_A > 0$ is the productivity parameter of vertical innovations, and Z_{At} is the amount of resource devoted to vertical R&D for each sector i. Here, $z_{At} \equiv Z_{At} / A_t$ is the per sector productivity adjusted expenditure on vertical R&D, and A_t can be regarded as the leading-edge productivity parameter, with $A_t \equiv \max\{A_{i,t} \mid i \in [0, N_t]\}$. From the zero-profit condition for the vertical R&D sector, the market clearing condition is given by

$$\lambda_A z_{At} V_t = (1 - s_R)\, Z_{At}, \tag{5}$$

where V_t is the expected present value of a vertical innovation at time t from the stream of future profits, and $s_R \in [0, 1)$ is the general subsidy rate to R&D. Because at every time t, the innovation will be replaced by the next innovator with the Poisson arrival rate ϕ_t, V_t is determined as

$$V_t = \int_t^{\infty} \exp[-\int_t^{\tau} (r_s + \lambda_A z_{At}) \mathrm{d}s] \pi_{\tau,t} \mathrm{d}\tau, \tag{6}$$

where r_s is the interest rate, and $\pi_{\tau,t}$ is the profit of the incumbent on date τ for any sector with vintage technology at time t.

Further, we can define the quality adjusted value of a vertical innovation as $v_t \equiv V_t / A_t$.

Finally, the intensity of the quality improvement for each vertical innovation is captured by a parameter $\sigma > 0$, with which the growth rate in the leading-edge productivity is given as

$$\frac{\dot{A}_t}{A_t} = \sigma \lambda_A z_{At}. \tag{7}$$

2.2.2 Horizontal Innovations

The variety of intermediate goods can be augmented by horizontal innovations and the evolution of varieties is specified as

$$\dot{N}_t = \lambda_N \frac{Z_{Nt}^{\beta} Y_t^{1-\beta}}{A_t},$$

where $\lambda_N > 0$ and $\beta \in (0, 1)$ are the parameters, and Z_{Nt} is the amount of numeraire devoted to horizontal innovations. To guarantee the inner solution, we impose the following assumption.

Assumption 1.

$$\lambda_A > \lambda_N.$$

Each horizontal innovation results in a new intermediate product whose productivity is randomly drawn from the distribution of existing intermediate products. Further, from the definition of the value of the leading-edge intermediate good A_t given by (6) and from the definition of the profits of intermediate good firm with quality $A_{i,t}$ given by (4), the expected value of a horizontal innovation is derived as $\mathrm{E}[(A_{i,t}/A_t)^{1/(1-\alpha)}]V_t$. Hence, from the zero-profit condition in the horizontal R&D sector, we obtain the next condition.

$$\lambda_N \frac{Z_{Nt}^{\beta} Y_t^{1-\beta}}{A_t} \mathrm{E}\left[\left(\frac{A_{i,t}}{A_t}\right)^{\frac{1}{1-\alpha}}\right] V_t = (1 - s_R)\, Z_{Nt}. \tag{8}$$

From the structure described, the distribution of relative productivity $A_{i,t}/A_t$ converges to the time-invariant distribution function $F(q) = q^{1/\sigma}$, where $0 < q \leq 1$.[3] Hence, in the long run, we obtain

$$\mathrm{E}\left[\left(\frac{A_{i,t}}{A_t}\right)^{\frac{1}{1-\alpha}}\right] = \frac{1}{1+\sigma/(1-\alpha)} \equiv \Gamma^{-1} < 1.$$

2.3 Households' problem

The maximization problem of a representative household is given by

$$\max \int_0^{\infty} \exp[-(\rho - n)t] \log C_t \mathrm{d}t$$

where $\rho > n$ is the subjective discount rate, and C_t is the per capita consumption. The laws of motion for financial assets and human capital in per capita terms are respectively given as

$$\dot{W}_t = (r_t - n)W_t + w_t u_t h_t - C_t - (1 - s_h)Z_{ht} - T_t, \quad (\mu_t)$$

and

$$\dot{h}_t = \left(\frac{Z_{ht}}{A_t}\right)^{\gamma} [(1 - u_t)h_t]^{1-\gamma} - nh_t, \quad (\nu_t) \tag{9}$$

where W_t denotes the per capita financial asset, h_t is the per capita amount of human capital, $u_t \in [0, 1]$ is the ratio of human capital devoted to the intermediate goods sector, Z_{ht} is the expenditure on human capital accumulation, $\gamma \in (0, 1)$, $s_h \in [0, 1)$ is the subsidy rate to expenditure on human capital accumulation, and μ and ν are the co-state variables attached

[3] See [Howitt(1999)] and [Segerstrom(2000)] for the proof.

to the respective constraints. We divide the amount of expenditure (Z_{ht}) by A_t because (1) the higher the leading-edge quality in the economy, the more difficult is the acquisition of new skills to handle the cutting-edge technology and (2) the more the economy has human capital, the more difficult it will be to obtain additonal human capital. Finally, the expenditure of the government is financed by lump sum tax (T_t), and the budget of government is balanced at all times.

From the above specifications, the first-order conditions of the problem are obtained as

$$C_t^{-1} = \mu_t \tag{10}$$

$$\mu_t w_t = \nu_t (1-\gamma)(1-u_t)^{-\gamma} A_t^{-\gamma} h_t^{-\gamma} Z_{ht}^{\gamma} \tag{11}$$

$$\mu_t (1-s_h) = \nu_t \gamma A_t^{-\gamma} h_t^{1-\gamma} Z_{ht}^{\gamma-1} (1-u_t)^{1-\gamma} \tag{12}$$

$$\mu_t (r_t - n) = (\rho - n)\mu_t - \dot{\mu}_t \tag{13}$$

and

$$\mu_t w_t u_t + \nu_t \left[(1-\gamma)(1-u_t)^{1-\gamma} A_t^{-\gamma} h_t^{-\gamma} Z_{ht}^{\gamma} - n \right] = (\rho - n)\nu_t - \dot{\nu}_t. \tag{14}$$

In addition, the usual total variable costs (TVCs) are imposed on a and h.

From (10) and (13), we obtain

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{15}$$

From (11) and (12), we obtain

$$Z_{ht} = \frac{\gamma(1-u_t)}{(1-\gamma)(1-s_h)} w_t h_t. \tag{16}$$

Substituting the above equation back into (11), we obtain

$$\frac{\mu_t}{\nu_t} = \gamma^{\gamma}(1-\gamma)^{1-\gamma}(1-s_h)^{-\gamma} A_t^{-\gamma} w_t^{\gamma-1}. \tag{17}$$

Therefore, substituting (16) and (17) into (14), we obtain

$$\frac{\dot{\nu}_t}{\nu_t} = \rho - \gamma^{\gamma}(1-\gamma)^{1-\gamma}(1-s_h)^{-\gamma} A_t^{-\gamma} w_t^{\gamma}. \tag{18}$$

Further, (17) implies

$$\frac{\dot{\mu}_t}{\mu_t} - \frac{\dot{\nu}_t}{\nu_t} = (\gamma - 1)\frac{\dot{w}_t}{w_t} - \gamma \frac{\dot{A}_t}{A_t}.$$

(13) and (18) together with the above condition lead to

$$(1-\gamma)\frac{\dot{w}_t}{w_t} = r_t - \gamma^\gamma (1-\gamma)^{1-\gamma}(1-s_h)^{-\gamma} A_t^{-\gamma} w_t^\gamma. \tag{19}$$

Finally, substituting (16) into (9), we obtain

$$\frac{\dot{h}_t}{h_t} = \gamma^\gamma (1-\gamma)^{-\gamma}(1-s_h)^{-\gamma}(1-u_t) A_t^{-\gamma} w_t^\gamma - n. \tag{20}$$

3 Equilibrium

In this section, we derive the equilibrium path of the model. Next, as usual, it is convenient to define new variables that are constant along the steady growth path. Specifically, we define quality adjusted human capital wage ($\omega_t \equiv w_t / A_t$), quality and human capital adjusted per capita consumption ($c_t \equiv C_t / (A_t h_t)$), and human capital per variety ($l_t \equiv h_t L_t / N_t$).

From (7) and (19), the evolution of quality adjusted human capital wage can be depicted as[4]

$$\frac{\dot{\omega}_t}{\omega_t} = r_t - \gamma^\gamma (1-\gamma)^{1-\gamma}(1-s_h)^{-\gamma}\omega_t^\gamma - \sigma\lambda_A z_{At}. \tag{S1}$$

In addition, from (15) and (20), the evolution of the quality and human capital adjusted per capita consumption is derived as

$$\frac{\dot{c}_t}{c_t} = r_t - \rho + n - \sigma\lambda_A z_{At} - \gamma^\gamma (1-\gamma)^{-\gamma}(1-s_h)^{-\gamma}(1-u_t)\omega_t^\gamma. \tag{S2}$$

Finally, from the definition of l_t, (8) and (20), the following is derived.

$$\frac{\dot{l}_t}{l_t} = \frac{\dot{h}_t}{h_t} + \frac{\dot{L}_t}{L_t} - \frac{\dot{N}_t}{N_t} = \gamma^\gamma (1-\gamma)^{-\gamma}(1-s_h)^{-\gamma}(1-u_t)\omega_t^\gamma - \lambda_N z_{Nt}^\beta y_t, \tag{S3}$$

where $y_t \equiv Y_t / (A_t N_t)$ is the quality and human capital adjusted per capita output, and $z_{Nt} = Z_{Nt} / Y_t$ is the share of expenditure on horizontal R&D from total output.

In this model, the market clearing conditions are obtained as follows. For the final goods sector, we obtain

$$Y_t = C_t L_t + Z_{ht} L_t + Z_{At} N_t + Z_{Nt}.$$

[4] Notice here that the per sector productivity adjusted expenditure on vertical R&D (z_{At}) is also stationary along the steady growth path.

Dividing the above equation by $A_t N_t$, we obtain the intensive form resource constraint as

$$y_t = c_t l_t + z_{ht} l_t + z_{At} + z_{Nt} y_t,$$

From this, we obtain

$$z_{At} = (1 - z_{Nt}) y_t - (c_t + z_{ht}) l_t. \tag{21}$$

At each time t, the human capital market clears such that

$$\int_0^{N_t} x_{i,t} \mathrm{d}i = u_t h_t L_t. \tag{22}$$

Using (3), (22) can be rewritten as

$$\int_0^{N_t} \alpha^{\frac{2}{1-\alpha}} A_{i,t}^{\frac{1}{1-\alpha}} w_t^{\frac{-1}{1-\alpha}} \mathrm{d}i = u_t h_t L_t. \tag{23}$$

With the definitions of ω and l, we divide (23) by N_t to obtain the intensive form version of (23), determining u_t as

$$u_t = \alpha^{\frac{2}{1-\alpha}} \Gamma^{-1} \omega_t^{\frac{-1}{1-\alpha}} l_t^{-1}. \tag{C1}$$

Next, using the definitions of y, (1), and (3), we obtain

$$y_t = \alpha^{\frac{2\alpha}{1-\alpha}} \Gamma^{-1} \omega_t^{\frac{\alpha}{\alpha-1}}. \tag{24}$$

Next, from (21) and (24), we obtain

$$z_{At} = (1 - z_{Nt}) \alpha^{\frac{2\alpha}{1-\alpha}} \Gamma^{-1} \omega_t^{\frac{-\alpha}{\alpha-1}} - (c_t + z_{ht}) l_t. \tag{C2}$$

From the market clearing condition of vertical R&D (5), we obtain

$$v_t = \frac{1 - s_R}{\lambda_A}, \tag{25}$$

and from the market clearing condition of horizontal R&D (8) with the definition of γ, we obtain

$$\lambda_N \Gamma^{-1} v_t = (1 - s_R) z_{Nt}^{1-\beta}.$$

Combining these two equations, we obtain

$$z_{Nt} = \zeta \equiv \left(\frac{\lambda_N}{\lambda_A} \Gamma^{-1} \right)^{\frac{1}{1-\beta}}. \tag{26}$$

Since $\Gamma > 1$ and $\lambda_A > \lambda_N$ are assumed, ζ thus derived satisfies the inner solution condition of $0 < \zeta < 1$.

Finally, from the definition of v_t, we obtain

$$\frac{\dot{v}_t}{v_t} = \frac{\dot{V}_t}{V_t} - \frac{\dot{A}_t}{A_t}.$$

In the long run, $\dot{v}_t / v_t = 0$. We can derive $\dot{V}_t / V_t$ by using the definitions of V_t in (6), (7) and (25). Hence, a little algebra leads to

$$r_t = \lambda_A \left[(1 - s_R)^{-1} \alpha (1-\alpha) \alpha^{\frac{2\alpha}{1-\alpha}} \omega_t^{\frac{\alpha}{\alpha-1}} - \left(1 + \frac{\sigma\alpha}{1-\alpha} \right) z_{At} \right]. \tag{C3}$$

The equilibrium dynamics of the model consists of three variables, (c_t, ω_t, l_t). The dynamical systems are given by (S1) – (S3), together with three instantaneous variables (z_{At}, u_t, r_t) given by (C1) – (C3).

4 Steady Growth Path

In this section, we focus on the steady growth path, where $\hat{c}$, ω, and l are constant over time. Hereafter, we add subscript $*$ to any variable whenever it is constant in the steady growth path.

From (S1) and (S2), we obtain

$$u_* = \gamma^{-\gamma} (1-\gamma)^{\gamma} (1 - s_h)^{\gamma} (\rho - n) \omega_*^{-\gamma}. \tag{27}$$

Next, substituting (24), (26), and (27) into (S3), we obtain

$$\gamma^{\gamma} (1-\gamma)^{-\gamma} (1 - s_h)^{-\gamma} \omega_*^{\gamma} - \rho = \zeta \alpha^{\frac{2\alpha}{1-\alpha}} \omega_*^{\frac{\alpha}{\alpha-1}} - n. \tag{28}$$

Since the left-hand side of the above equation is increasing in ω, while the right-hand side is decreasing, (28) uniquely determines ω in the steady state. Similarly, substituting (24), (26), and (27) into (C1), we obtain

$$l_* = \alpha^{\frac{2}{1-\alpha}} \Gamma^{-1} \omega_*^{\frac{1}{\alpha-1}} u_*^{-1}, \tag{29}$$

respectively. Therefore, substituting (C3) and (27) – (29) into (S1), we obtain

$$z_{A*} = \left(1 + \sigma + \frac{\alpha\sigma}{1-\alpha} \right)^{-1} \left\{ \left[(1 - s_R)^{-1} \alpha (1-\alpha) - \frac{\zeta}{\lambda_A} \right] \alpha^{\frac{2\alpha}{1-\alpha}} \omega_*^{\frac{\alpha}{\alpha-1}} - \frac{\rho - n}{\lambda_A} \right\}. \tag{30}$$

Finally, from (21), (16), (24), (26), and (27) – (30), we obtain

$$c_* = (1-\zeta)\alpha^{\frac{2\alpha}{1-\alpha}}\Gamma^{-1}\omega_*^{\frac{\alpha}{\alpha-1}} l_*^{-1} + \frac{\gamma(1-u_*)}{(1-\gamma)(1-s_h)}\omega_* - z_{A*} l_*^{-1}. \tag{31}$$

Therefore, (28), (29), and (31) give the candidates of the steady state values of the dynamical system concerning the equations characterizing the economy, (S1) – (S3). Therefore, we have the following proposition.

Proposition 1. *There exist $\bar{\rho} \in (n, \infty)$ and $\underline{\lambda}_A \in (0, \infty)$ such that the system of differential equations, (S1) – (S3) with (C1) – (C3), has a unique steady-state equilibrium, where $u_* \in (0,1)$ and $z_{A*} \in (0, \infty)$, if $\rho \in (n, \bar{\rho})$ and $\lambda_A \in (\underline{\lambda}_A, \infty)$. The set of steady-state values (u_*, z_{A*}, c_*) is given by (28), (29), and (31), together with (27) and (30).*

Proof. Since the candidates of the steady-state values are given by (28), (29), and (31), it is sufficient to show the existence of $\bar{\rho}$ and $\underline{\lambda}_A$. From (27), in order to satisfy $u_* \in (0,1)$, it must hold that

$$\omega_* > \underline{\omega} \equiv \frac{\gamma(\rho - n)^{\frac{1}{\gamma}}}{(1-\gamma)(1-s_h)}.$$

Therefore, it follows from the fact that the right-hand side of (28) is increasing in ω and that the left-hand side is decreasing, and the condition for $u_* \in (0,1)$ is given by

$$\gamma^{\gamma}(1-\gamma)^{-\gamma}(1-s_h)^{-\gamma}\underline{\omega}^{\gamma} - \rho < \zeta\alpha^{\frac{2\alpha}{1-\alpha}}\underline{\omega}^{\frac{\alpha}{\alpha-1}} - n,$$

or

$$1 < \zeta\alpha^{\frac{2\alpha}{1-\alpha}}\left[\frac{\gamma(\rho-n)^{\frac{1}{\gamma}}}{(1-\gamma)(1-s_h)}\right]^{\frac{\alpha}{\alpha-1}} + (\rho - n).$$

Since the right-hand side of the above inequality diverges to $+\infty$ as $\rho \to n$, and it is obvious that $u_* > 0$ from (27), we find that there exists a sufficiently small value $\bar{\rho} \in (n, \infty)$ such that $u_* \in (0,1)$ if $\rho \in (n, \bar{\rho})$. Finally, if follows from (30) that

$$z_{A*} \to \left(1 + \sigma + \frac{\alpha\sigma}{1-\alpha}\right)^{-1}(1-s_R)^{-1}\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}} > 0 \quad \text{as} \quad \lambda_A \to \infty.$$

Therefore, we find that there exists a sufficiently large value $\underline{\lambda}_A \in (0, \infty)$ such that $z_{A*} > 0$ if $\lambda_A \in (\underline{\lambda}_A, \infty)$. $\square$

In this proposition, the condition that $\rho \in (n, \bar{\rho})$ guarantees a positive investment on human capital and the condition that $\lambda_A \in (\underline{\lambda}_A, \infty)$ guarantees a positive R&D expenditure. Therefore, we have the following proposition.

Proposition 2. *Suppose that $\rho \in (n, \bar{\rho})$ and $\lambda_A \in (\underline{\lambda}_A, \infty)$, where $\bar{\rho}$ and $\underline{\lambda}_A$ are given in Proposition 1. Then, the growth rate in the steady state is given by*

$$g \equiv \frac{\dot{Y}_t}{Y_t} = g_A + g_h + n,$$

where g_A and g_h are the respective growth rates of A_t and h_t in the steady state, given as

$$g_A \equiv \frac{\dot{A}_t}{A_t} = \lambda_A \sigma z_{A*} \tag{32}$$

and

$$g_h \equiv \frac{\dot{h}_t}{h_t} = \left[\frac{\gamma \omega_*}{(1-\gamma)(1-s_h)}\right]^{\gamma} - \rho. \tag{33}$$

Proof. From the definition of y and the fact that y is constant over time in a steady state equilibrium, the growth rate can be written as

$$g = g_A + g_N,$$

where g_N is the growth rate of the variety of intermediate goods: $g_N \equiv \dot{N}_t / N_t$. From (7) and (30), the growth rate of the leading-edge quality of technology, g_A, is given by (32) in a steady state. Moreover, from the definition of l and the fact that l is constant over time in a steady state, we obtain

$$g_N = g_h + n.$$

We obtain (33) by substituting (27) into (20), which completes the proof. □

As is discussed in the introduction, proposition 2 bridges the gap between the literature concerning Schumpeterian growth models and that concerning growth empirics. With our specification, the growth rate of output depends on both technological improvements and human capital accumulation. Our model provides a structural basis for including human capital as a determinant of long-run growth by using a Schumpeterian growth model. Hence, this study can be taken as providing a microfoundation for reduced form analyses in growth empirics literature. It should also be noted that this is the first study that demonstrates complete general equilibrium policy implications by taking into account the endogenous determinations of both technology improvements and human capital accumulation.

Next, we provide the policy implications of the model as theorems.

Theorem 1. *The subsidy to R&D has a positive effect on the growth rate of the leading-edge technology, but does not have any effect on the growth rate of human capital.*

$$\frac{\partial g_A}{\partial s_R} > 0$$

and

$$\frac{\partial g_h}{\partial s_R} = 0.$$

Proof. The differentiation of (30) with respect to s_R gives

$$\frac{\mathrm{d}z_{A*}}{\mathrm{d}s_R} = \left(1 + \sigma + \frac{\alpha\sigma}{1-\alpha}\right)^{-1} (1 - s_R)^{-2}\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}}. \tag{34}$$

Here, it should be noted that ω_* is determined by (28), independent of s_R. Since the right-hand side of (34) is positive, we obtain the theorem. □

Therefore, our modification does not provide significant new insights in terms of the effect of R&D subsidy.[5]

On the contrary, the following theorem provides a new insight into the subsidy to human capital investment.

Theorem 2. *The subsidy to human capital investment has positive effects on the growth rates of the leading-edge technology and human capital.*

$$\frac{\partial g_A}{\partial s_h} > 0 \tag{35}$$

and

$$\frac{\partial g_h}{\partial s_h} > 0. \tag{36}$$

Proof. See Appendix A. □

Figure 1 represents the effect of subsidy to human capital investment on ω_*. In Figure 1, the RHS curve depicts the right-hand side of (28). Note that it is decreasing in ω_*. Two LHS curves in Figure 1 respectively depict the left-hand side of (28) corresponding to the subsidy rates of human capital investment, s_h and s_h', where $s_h' > s_h$. It should be noted that the

[5] The effects of subsidy to R&D are clarified, for example, in [Howitt(1999)].

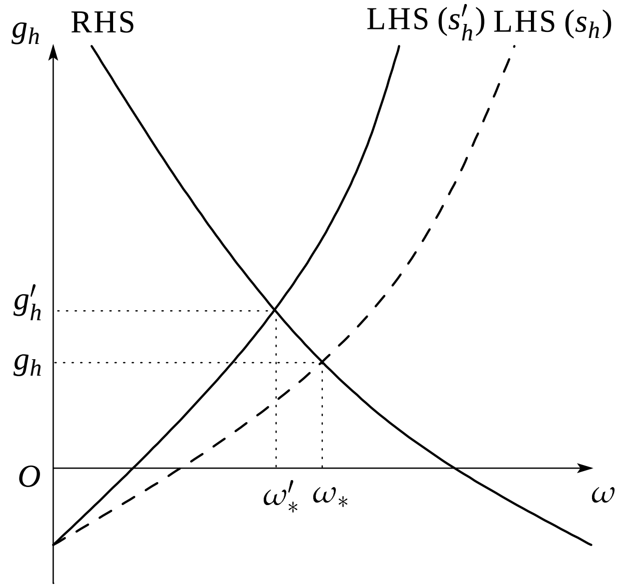

Figure 1: Effect of subsidy to human capital investment on ω

left-hand side of (28) is equal to g_h and is increasing in ω. Moreover, since it is increasing in s_h, the LHS curve shifts up as the subsidy rate on human capital investment increases from s_h to s_h'. Therefore, Figure 1 shows that an increase in s_h raises the growth rate of human capital. Moreover, since it reduces the adjusted wage rate ω_* and z_{A*} are decreasing in ω from (30), the increase in the subsidy rate on human capital also raises the growth rate of the leading-edge productivity g_A. Intuitively, the reason for the indirect positive impact of subsidy to human capital investment on productivity growth is explained as follows. Since human capital accumulation is labor augmenting, an increase in the growth rate of human capital reduces the adjusted wage rate. This raises the adjusted profit and the expected value of firms (6), motivating the enhancement of horizontal and vertical innovations. This indirect effect on the subsidy of human capital investment, given by Theorem 2, provides a new insight into the subsidy policy, which is the main result of this study.

Hence, human capital investment augments the growth rate of quality, which in turn accelerates the growth of per capita output. Further, if we consider only the relationship between the growth rate of output and R&D investment, as in [Howitt(1999)], it misses the effect coming through human capital investment (and subsidy). The omission of this general equilibrium effect was a drawback in the excellent model of [Howitt(1999)] and is remediated with our specification.

5 Conclusion

In this paper, after the recent finding by [Ha and Howitt(2007)] that the Schumpeterian endogenous growth theory accommodates to the U.S. experience, we augment Howitt's Schumpeterian endogenous growth model to consider the human capital accumulation process. We show that a subsidy to human capital investment has a positive impact not only on R&D efforts but also on human capital accumulation. Because in our model, the per capita output growth rate depends on both technology improvements and human capital accumulation, the model bridges the gap between the literature concerning Schumpeterian growth model and that concerning growth empirics.

Appendix

A Proof of Theorem 2

Differentiating both the sides of (28), we obtain

$$\frac{\mathrm{d}\omega_*}{\mathrm{d}s_h} = -\frac{\gamma^{\gamma+1}(1-\gamma)^{-\gamma}(1-s_h)^{-\gamma-1}\omega^{\gamma}}{\gamma^{\gamma+1}(1-\gamma)^{-\gamma}(1-s_h)^{-\gamma}\omega_*^{\gamma-1} + \zeta\alpha^{\frac{2\alpha}{1-\alpha}}\frac{\alpha}{1-\alpha}\omega_*^{\frac{1}{\alpha-1}}} < 0.$$

Therefore, noting that the left-hand side of (28) is equal to g_h and the right-hand side is decreasing in ω, we obtain (36). Similarly, since from (30), z_{A*} is decreasing in ω, it gives (35).

References

[Arnold(1998)] L. Arnold. Growth, Welfare, and Trade in an Integrated Model of Human-Capital Accumulation and Research. *Journal of Macroeconomics*, 20(1):81–105, 1998.

[Ha and Howitt(2007)] J. Ha and P. Howitt. Accounting for Trends in Productivity and R&D: A Schumpeterian Critique of Semi-Endogenous Growth Theory. *Journal of Money, Credit and Banking*, 39(4):733–774, 2007.

[Howitt(1999)] P. Howitt. Steady Endogenous Growth with Population and R& D Inputs Growing. *Journal of Political Economy*, 107(4):715, 1999.

[Jones(1995)] C.I. Jones. R & D-Based Models of Economic Growth. *Journal of Political Economy*, 103(4):759, 1995.

[Lucas(1988)] R. Lucas. On the Mechanisms of Economic Development. *Journal of Monetary Economics*, 22(1):3–42, 1988.

[Mankiw et al.(1992)Mankiw, Romer, and Weil] N. G. Mankiw, David Romer, and David Weil. A Contribution to the Empirics of Economic Growth. *The Quarterly Journal of Economics*, 107(2):407–37, 1992.

[Segerstrom(2000)] P. S. Segerstrom. The Long-Run Growth Effects of R&D Subsidies. *Journal of Economic Growth*, 5(3):277–305, 2000.

[Uzawa(1965)] H. Uzawa. Optimum Technical Change in an Aggregative Model of Economic Growth. *International Economic Review*, 6(1):18–31, 1965.

[Young(1998)] A. Young. Growth without Scale Effects. *Journal of Political Economy*, 106(1):41–63, 1998.