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EDUCATION, INNOVATION, AND LONG-RUN GROWTH

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Abstract

This paper combines three prototype endogenous growth models, the models with human capital accumulation introduced by Uzawa [1965] and Lucas [1988], variety expansion by Romer [1990], and quality improvements by Aghion and Howitt [1992], in order to investigate how these three engines of growth interact. We show that a subsidy to human capital accumulation has a positive impact on R&D effort, as well as on human capital accumulation. On the other hand, a subsidy to R&D sectors does not affect human capital accumulation in our model. Moreover, we show that equilibrium dynamics is locally saddle-path stable around the steady growth path. It suggests that Schumpeterian growth models à la Howitt [1999] should share the locally saddle-path stable property. Finally, since in our model the per-capita output growth rate is endogenously determined by both technology improvements and human capital accumulation, it bridges the gap between the literature on Schumpeterian growth models and that on growth empirics.

JEL classification: O15; O32; O41
Keywords: quality improvements; variety expansion; human capital accumulation; subsidies

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1 Introduction

This study aims to investigate general-equilibrium interactions between two major engines of growth, R&D investment and human capital accumulation, by integrating three major classes of endogenous growth models: human capital accumulation models initiated by Uzawa [1965] and Lucas [1988],[1] variety-expansion models developed by Romer [1990], and quality-improvement models developed by Grossman and Helpman [1991] and Aghion and Howitt [1992]. To the best of our knowledge, this is the first study featuring an endogenous growth model with these three engines of growth. A significant merit of our model is that the per-capita output growth rate depends on both technology improvements and human capital accumulation, allowing us to obtain the following new results. On the one hand, a subsidy to R&D effort accelerates the R&D but not human capital accumulation. On the other hand, a subsidy to education positively affects not only R&D but also human capital accumulation.

Since Jones [1995] pointed out that ealier versions of endogenous growth models have an empirically unsupported property of scale effect of population on growth, two different types of endogenous growth theories, namely, the semi-endogenous growth theory (e.g. Jones [1995], Kortum [1997], and Segerstrom [1998], and Young [1998]), and the Schumpeterian endogenous growth theory (e.g. Dinopoulos and Thompson [1998], Peretto [1998], and Howitt [1999]) have become popular. The main policy implications of the former theory are that the per-capita growth rate in the long run is pinned down to a constant including an exogenous population growth rate, and that there is no room for policies to affect the long-run growth rate. On the other hand, the main implication of the latter theory is that a subsidy on R&D investment has long-run effects. Recently, Ha and Howitt [2007] compared these two growth models and found that the evidence is more supportive of the Schumpeterian endogenous growth theory than the semi-endogenous growth theory.

In this light, we augment Howitt [1999]'s Schumpeterian endogenous growth model to consider general-equilibrium interactions between R&D and human capital accumulation. Our motivation to consider human capital in the framework of a Schumpeterian endogenous growth model is straightforward: while in the usual Schumpeterian endogenous growth models human capital accumulation is disregarded, there is a strong consensus among economists that human capital is also an engine of growth. In particular, from the viewpoint of growth empirics literature since Mankiw, Romer, and Weil [1992], if we disregard human capital accumulation, then the estimators in growth regressions will be biased because the

[1] More precisely, our model incorporates a general version of two-sector model. See, for example, Mulligan and Sala-i Martin [1993], Bond, Wang, and Yip [1996], etc.

per capita output growth will be attributable to capital accumulation, technology progress, and human capital accumulation.[2]

Regarding other related literature, some studies such as Redding [1996] and Acemoglu [1997] consider education and innovation.[3] These studies, however, focus on complementarity between education and innovation, and hence, multiplicity of equilibrium as a consequence of the complementarity and welfare implications associated with the choice of equilibium are the main themes. On the other hand, our interests lies in investigating what are the subsidy policy effects in a hybrid version of endogenous growth models where the equilibrium is uniquely determined.[4]

This paper is organized as follows. In Section 2, we set up the model. The equilibrium is analyzed in Section 3. Section 4 focuses on the steady growth path and derives its main features, and Section 5 concludes the paper.

2 The Model

In this study, human capital can be used for the production of intermediate product and for investment in human capital creation. The population growth rate n is exogenously determined, and the population size is denoted as L_t. The final goods output, which is the numéraire in the model, is used for consumption (C_t) and investments in vertical R&D (Z_{At}), horizontal R&D (Z_{Nt}), and human capital creation (Z_{ht}).

2.1 Production structure

The final goods sector operates under perfect competition. The technology for final goods production is given by

$$Y_t = X^{1-\alpha} \int_0^{N_t} A_{it} x_{it}^{\alpha} \mathrm{d}i, \tag{1}$$

[2]The model below may provide a structural basis of growth regressions: while the arguments in production functions are just assumed in growth empirics, our model provides market structures in which technological improvements through R&D activities and human capital accumulation are contributing to the economic growth.

[3]Nelson and Phelps [1966] is the first literature investigating complementary relationship between education and innovation.

[4]There is another study constructing a model that integrates R&D investments and human capital accumulation. Arnold [1998] constructed a semi-endogenous growth model with these two sorces of R&D and human capital accumulation. However, he did not delve on a subsidy policy to human capital accumulation. In his model, a subsidy to R&D investment would have no long-run effects since the model is a semi-endogenous one.

where Y_t is the final goods output, X is a fixed resource such as land, N_t measures the varieties of intermediate goods at time t, A_{it} is a productivity parameter attached to the incumbent version of intermediate product i, x_i is the amount of intermediate product i used in the economy, and $\alpha \in (0,1)$ is the capital share. Since this study assumes that the total endowment of the fixed resource in the economy is equal to 1, we henceforth abbreviate variable X from the production function. Under perfect competition, the first-order condition for the final goods sector with respect to x_{it} is given as

$$p_{it} = \alpha A_{it} x_{it}^{\alpha-1}, \tag{2}$$

where p_{it} is the price of intermediate product i.

The intermediate goods sector operates under monopolistic competition. In this study, we assume that one unit of intermediate good is made from one unit of human capital. By this assumption, two sources of endogenous growth-R&D and human capital accumulation-are interacted. The profit in creating intermediate product i is given by

$$\Pi_{it} = p_{it} x_{it} - w_t x_{it},$$

where w_t is the real wage for human capital. With the demand function of x_{it} from (2), the first-order condition with respect to x_{it} gives the demand of x_{it}:

$$x_{it} = \alpha^{\frac{2}{1-\alpha}} A_{it}^{\frac{1}{1-\alpha}} w_t^{\frac{-1}{1-\alpha}}, \tag{3}$$

and the profits of an intermediate goods firm are given by

$$\Pi_{it} = \alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_{it}^{\frac{1}{1-\alpha}} w_t^{\frac{\alpha}{\alpha-1}}. \tag{4}$$

2.2 Innovations

Here, we consider two types of innovations. Vertical innovations improve productivity in each intermediate goods sector i, A_{it}, and horizontal innovations bring new varieties into the economy, N_t.

2.2.1 Vertical Innovations

The Poisson arrival rate of vertical innovations in each sector is defined as

$$\phi_t = \lambda_A \frac{Z_{At}}{A_t N_t} \equiv \lambda_A z_{At},$$

where $\lambda_A > 0$ is the productivity parameter of vertical innovations, and Z_{At} is the amount of resource devoted to vertical R&D for each sector i. Here, $z_{At} \equiv$

$Z_{At}/(A_t N_t)$ is the per sector expenditure on vertical R&D adjusted by productivity and complexity, and A_t can be regarded as the leading-edge productivity parameter, with $A_t \equiv \max\{A_{it} \mid i \in [0, N_t]\}$. Therefore, the free-entry condition is given by

$$\lambda_A z_{At} V_t = (1 - s_R)\frac{Z_{At}}{N_t}, \tag{5}$$

where V_t is the expected present value of a vertical innovation at time t from the stream of future profits, and $s_R \in [0,1)$ is the general subsidy rate to R&D. Because at every time t, the innovation will be replaced by the next innovator with the Poisson arrival rate ϕ_t, V_t is determined as

$$V_t = \int_t^{\infty} \exp\left[-\int_t^{\tau}(r_s + \lambda_A z_{At})\mathrm{d}s\right]\pi_{\tau t}\mathrm{d}\tau, \tag{6}$$

where r_s is the interest rate, and $\pi_{\tau t}$ is the profit of the incumbent on date τ for any sector with vintage technology at time t. Further, we can define the quality adjusted value of a vertical innovation as $v_t \equiv V_t/A_t$.

Finally, the intensity of the quality improvement for each vertical innovation is captured by a parameter $\sigma > 0$, with which the growth rate in the leading-edge productivity is given as

$$\frac{\dot{A}_t}{A_t} = \sigma\lambda_A z_{At}. \tag{7}$$

2.2.2 Horizontal Innovations

The variety of intermediate goods can be augmented by horizontal innovations and the evolution of varieties is specified as

$$\dot{N}_t = \lambda_N \frac{Z_{Nt}^{\beta} Y_t^{1-\beta}}{A_t},$$

where $\lambda_N > 0$ and $\beta \in (0,1)$ are the parameters, Z_{Nt} is the amount of numéraire devoted to horizontal innovations, and Y_t is included as a proxy of knowledge spill over. To guarantee the inner solution, we impose the following assumption.

Assumption 1.

$$\lambda_A > \lambda_N.$$

Each horizontal innovation results in a new intermediate product whose productivity is randomly drawn from the distribution of existing intermediate products. Further, from the definition of the value of the leading-edge intermediate good A_t given by (6) and from the definition of the profits of intermediate good firm with quality A_{it} given by (4), the expected value of a horizontal innovation is derived as $\mathrm{E}[(A_{it}/A_t)^{1/(1-\alpha)}]V_t$. Hence, from the free-entry condition in the horizontal R&D sector, we obtain the next condition.

$$\lambda_N \frac{Z_{Nt}^{\beta} Y_t^{1-\beta}}{A_t} \mathrm{E}\left[\left(\frac{A_{it}}{A_t}\right)^{\frac{1}{1-\alpha}}\right] V_t = (1-s_R) Z_{Nt}. \tag{8}$$

From the structure described, the distribution of relative productivity A_{it}/A_t converges to the time-invariant distribution function $F(q) = q^{1/\sigma}$, where $0 < q \leq 1$.[5] Hence, in the long run, we obtain

$$\mathrm{E}\left[\left(\frac{A_{it}}{A_t}\right)^{\frac{1}{1-\alpha}}\right] = \frac{1}{1+\sigma/(1-\alpha)} \equiv \Gamma^{-1} < 1.$$

2.3 Households' problem

The maximization problem of a representative household is given by

$$\max \int_0^{\infty} \exp[-(\rho - n)t] \log C_t \mathrm{d}t$$

where $\rho > n$ is the subjective discount rate, and C_t is the per capita consumption. The laws of motion for financial assets and human capital in per capita terms are respectively given as

$$\dot{W}_t = (r_t - n)W_t + w_t u_t h_t - C_t - (1-s_h)Z_{ht} - T_t, \quad (\mu_t)$$

and

$$\dot{h}_t = \left(\frac{Z_{ht}}{A_t}\right)^{\gamma} [(1-u_t)h_t]^{1-\gamma} - nh_t, \quad (\nu_t) \tag{9}$$

where W_t denotes the per capita asset, h_t is the per capita amount of human capital, $u_t \in [0,1]$ is the ratio of human capital devoted to the intermediate goods sector, Z_{ht} is the expenditure on human capital accumulation, $\gamma \in (0,1)$, $s_h \in [0,1)$ is the subsidy rate to expenditure on human capital accumulation, and μ and ν are the co-state variables attached to the respective constraints. We divide the amount of expenditure (Z_{ht}) by A_t because (1) the higher the leading-edge quality in the economy, the more difficult is the acquisition of new skills to handle the cutting-edge technology and (2) the more the economy has human capital, the more difficult it will be to obtain additonal human capital. Finally, the expenditure of the government is financed by lump sum tax (T_t), and the budget of government is balanced at any time.

[5] See Howitt [1999] and Segerstrom [2000] for the proof.

From the above specifications, the first-order conditions of the problem are obtained as

$$C_t^{-1} = \mu_t \tag{10}$$

$$\mu_t w_t = \nu_t (1-\gamma)(1-u_t)^{-\gamma} A_t^{-\gamma} h_t^{-\gamma} Z_{ht}^{\gamma} \tag{11}$$

$$\mu_t (1-s_h) = \nu_t \gamma A_t^{-\gamma} h_t^{1-\gamma} Z_{ht}^{\gamma-1} (1-u_t)^{1-\gamma} \tag{12}$$

$$\mu_t (r_t - n) = (\rho - n)\mu_t - \dot{\mu}_t \tag{13}$$

and

$$\mu_t w_t u_t + \nu_t \left[(1-\gamma)(1-u_t)^{1-\gamma} A_t^{-\gamma} h_t^{-\gamma} Z_{ht}^{\gamma} - n \right] = (\rho - n)\nu_t - \dot{\nu}_t. \tag{14}$$

In addition, the usual transversality conditions (TVCs) are imposed on W and h.

Combining (10) through (14), the first-order conditions are reduced to the following equations.

$$\frac{\dot{C}_t}{C_t} = r_t - \rho, \tag{15}$$

$$(1-\gamma)\frac{\dot{w}_t}{w_t} = r_t - \gamma^{\gamma}(1-\gamma)^{1-\gamma}(1-s_h)^{-\gamma} A_t^{-\gamma} w_t^{\gamma}, \tag{16}$$

$$\frac{\dot{h}_t}{h_t} = \gamma^{\gamma}(1-\gamma)^{-\gamma}(1-s_h)^{-\gamma}(1-u_t) A_t^{-\gamma} w_t^{\gamma} - n, \tag{17}$$

and

$$Z_{ht} = \frac{\gamma(1-u_t)}{(1-\gamma)(1-s_h)} w_t h_t. \tag{18}$$

3 Equilibrium

In this section, we derive the equilibrium path of the model. Next, as usual, it is convenient to define new variables that are constant along the steady growth path. Specifically, we define quality-adjusted human capital wage ($\omega_t \equiv w_t / A_t$), per-capita consumption adjusted by quality and human capital ($c_t \equiv C_t / (A_t h_t)$), human capital per variety ($l_t \equiv h_t L_t / N_t$), the amount of output adjusted by the quality and variety ($y_t \equiv Y_t / (A_t N_t)$), the share of expenditure on horizontal R&D from total output ($z_{Nt} = Z_{Nt} / Y_t$), and expenditure to education adjusted by quality and human capital ($z_{ht} = Z_{ht} / (A_t h_t)$).

From the free-entry condition of vertical R&D (5), we obtain

$$v_t = \frac{1 - s_R}{\lambda_A}. \tag{19}$$

By differentiating (19) with respect to t and using the definitions of V_t in (6), (7) and (19), A little algebra leads to

$$r_t = \lambda_A \left[(1-s_R)^{-1} \alpha (1-\alpha) \alpha^{\frac{2\alpha}{1-\alpha}} \omega_t^{\frac{\alpha}{\alpha-1}} - \left(1 + \frac{\sigma\alpha}{1-\alpha}\right) z_{At} \right]. \tag{C1}$$

Moreover, combining (8) and (19) and using the definition of γ, we get

$$z_{Nt} = \zeta \equiv \left(\frac{\lambda_N}{\lambda_A} \Gamma^{-1} \right)^{\frac{1}{1-\beta}}. \tag{20}$$

Here, it should be noted that ζ derived satisfies the inner solution condition that $\zeta \in (0,1)$ since $\Gamma > 1$ and $\lambda_A > \lambda_N$ by Assumption 1.

From (7) and (16), the evolution of quality-adjusted human capital wage can be written as[6]

$$\frac{\dot{\omega}_t}{\omega_t} = r_t - \gamma^\gamma (1-\gamma)^{1-\gamma} (1-s_h)^{-\gamma} \omega_t^\gamma - \sigma \lambda_A z_{At}. \tag{E1}$$

In addition, from (15) and (17), the evolution of the quality and human capital adjusted per capita consumption is derived as

$$\frac{\dot{c}_t}{c_t} = r_t - \rho + n - \sigma \lambda_A z_{At} - \gamma^\gamma (1-\gamma)^{-\gamma} (1-s_h)^{-\gamma} (1-u_t) \omega_t^\gamma. \tag{E2}$$

Finally, from the definition of l_t, (8) and (17), the following is derived.

$$\frac{\dot{l}_t}{l_t} = \gamma^\gamma (1-\gamma)^{-\gamma} (1-s_h)^{-\gamma} (1-u_t) \omega_t^\gamma - \lambda_N z_{Nt}^\beta y_t. \tag{E3}$$

In this model, the market clearing conditions for final good and human capital market at each time t are as follows.

$$Y_t = C_t L_t + Z_{ht} L_t + Z_{At} + Z_{Nt}. \tag{21}$$

and

$$\int_0^{N_t} x_{it} \mathrm{d}i = u_t h_t L_t. \tag{22}$$

From (21), we have

$$z_{At} = (1 - z_{Nt}) y_t - (c_t + z_{ht}) l_t. \tag{23}$$

By using (3) and the definitions of ω and l, (22) can be rewritten as

$$u_t = \alpha^{\frac{2}{1-\alpha}} \Gamma^{-1} \omega_t^{\frac{-1}{1-\alpha}} l_t^{-1}. \tag{C2}$$

[6]Notice here that the per sector productivity adjusted expenditure on vertical R&D (z_{At}) is also stationary along the steady growth path.

Finally, using the definition of y, (1), and (3), we have

$$y_t = \alpha^{\frac{2\alpha}{1-\alpha}} \Gamma^{-1} \omega_t^{\frac{\alpha}{\alpha-1}}, \tag{24}$$

and from (23) and (24), we get

$$z_{At} = (1 - z_{Nt}) \alpha^{\frac{2\alpha}{1-\alpha}} \Gamma^{-1} \omega_t^{\frac{-\alpha}{\alpha-1}} - (c_t + z_{ht}) l_t. \tag{C3}$$

Therefore, an *equilibrium* of the economy consists of six variables, $(c_t, \omega_t, l_t, z_{At}, u_t, r_t)$, satisfying (E1) – (E3), together with (C1) – (C3).

4 Steady Growth Path

In this section, we focus on the steady growth path, where $\hat{c}$, ω, and l are constant over time. Hereafter, we add subscript $*$ to any variable whenever it is constant in the steady growth path.

From (E1) and (E2), we have

$$u_* = \gamma^{-\gamma} (1-\gamma)^{\gamma} (1-s_h)^{\gamma} (\rho - n) \omega_*^{-\gamma}. \tag{S1}$$

Next, substituting (20), (24), and (S1) into (E3), we get

$$\gamma^{\gamma} (1-\gamma)^{-\gamma} (1-s_h)^{-\gamma} \omega_*^{\gamma} - \rho = \zeta \alpha^{\frac{2\alpha}{1-\alpha}} \omega_*^{\frac{\alpha}{\alpha-1}} - n. \tag{S2}$$

Here, it should be noted that if (S2) has a solution, it uniquely determines ω in the steady state since the left-hand side of the above equation is increasing in ω, while the right-hand side is decreasing. Similarly, substituting (20), (24), and (S1) into (C1), we obtain

$$l_* = \alpha^{\frac{2}{1-\alpha}} \Gamma^{-1} \omega_*^{\frac{1}{\alpha-1}} u_*^{-1}, \tag{S3}$$

respectively. Therefore, using (C1) and (S1) – (S3), (E1) yields

$$z_{A*} = \left(1 + \sigma + \frac{\alpha\sigma}{1-\alpha}\right)^{-1} \left\{ \left[\frac{\alpha(1-\alpha)}{1-s_R} - \frac{\zeta}{\lambda_A} \right] \alpha^{\frac{2\alpha}{1-\alpha}} \omega_*^{\frac{\alpha}{\alpha-1}} - \frac{\rho - n}{\lambda_A} \right\}. \tag{S4}$$

Moreover, from (18), (20), (23), (24), and (S1) – (S4), we have

$$c_* = (1-\zeta) \alpha^{\frac{2\alpha}{1-\alpha}} \Gamma^{-1} \omega_*^{\frac{\alpha}{\alpha-1}} l_*^{-1} + \frac{\gamma(1-u_*)}{(1-\gamma)(1-s_h)} \omega_* - z_{A*} l_*^{-1}. \tag{S5}$$

Finally, from (C1), together with (S2) and (S4), we have

$$r_* = \lambda_A \left[(1-s_R)^{-1} \alpha (1-\alpha) \alpha^{\frac{2\alpha}{1-\alpha}} \omega_*^{\frac{\alpha}{\alpha-1}} - \left(1 + \frac{\sigma\alpha}{1-\alpha}\right) z_{A*} \right]. \tag{S6}$$

Therefore, (S1) – (S6) give the candidates of the steady state values of the dynamical system concerning the equations characterizing the economy, (E1) – (E3), with (C1) – (C3). Therefore, we have the following proposition.

Proposition 1. *There exist* $\bar{\rho} \in (n,\infty)$ *and* $\underline{\lambda}_A \in (0,\infty)$ *such that the system of differential equations, (E1) – (E3) with (C1) – (C3), has a unique steady-state equilibrium, where* $u_* \in (0,1)$ *and* $z_{A*} \in (0,\infty)$*, if* $\rho \in (n,\bar{\rho})$ *and* $\lambda_A \in (\underline{\lambda}_A,\infty)$*. The set of steady-state values* $(c_*, \omega_*, l_*, z_{A*}, u_*, r_*)$ *is given by (S1) – (S6).*

Proof. Since the candidates of the steady-state values are given by (S1) – (S6), it is sufficient to show the existence of $\bar{\rho}$ and $\underline{\lambda}_A$ with $u_* \in (0,1)$ and $z_{A*} \in (0,\infty)$. From (S1), in order to satisfy $u_* \in (0,1)$, it must hold that

$$\omega_* > \underline{\omega} \equiv \frac{\gamma(\rho-n)^{\frac{1}{\gamma}}}{(1-\gamma)(1-s_h)}.$$

Therefore, it follows from the fact that the right-hand side of (S2) is increasing in ω and that the left-hand side is decreasing, and the condition for $u_* \in (0,1)$ is given by

$$\gamma^{\gamma}(1-\gamma)^{-\gamma}(1-s_h)^{-\gamma}\underline{\omega}^{\gamma} - \rho < \zeta\alpha^{\frac{2\alpha}{1-\alpha}}\underline{\omega}^{\frac{\alpha}{\alpha-1}} - n,$$

or

$$1 < \zeta\alpha^{\frac{2\alpha}{1-\alpha}}\left[\frac{\gamma(\rho-n)^{\frac{1}{\gamma}}}{(1-\gamma)(1-s_h)}\right]^{\frac{\alpha}{\alpha-1}} + (\rho-n).$$

Since the right-hand side of the above inequality diverges to $+\infty$ as $\rho \to n$, and it is obvious that $u_* > 0$ from (S1), we find that there exists a sufficiently small value $\bar{\rho} \in (n,\infty)$ such that $u_* \in (0,1)$ if $\rho \in (n,\bar{\rho})$. Finally, if follows from (S4) that

$$z_{A*} \to \left(1+\sigma+\frac{\alpha\sigma}{1-\alpha}\right)^{-1}(1-s_R)^{-1}\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}} > 0 \quad \text{as} \quad \lambda_A \to \infty.$$

Therefore, we find that there exists a sufficiently large value $\underline{\lambda}_A \in (0,\infty)$ such that $z_{A*} > 0$ if $\lambda_A \in (\underline{\lambda}_A,\infty)$. □

In this proposition, the condition that $\rho \in (n,\bar{\rho})$ guarantees a positive investment on human capital and the condition that $\lambda_A \in (\underline{\lambda}_A,\infty)$ guarantees a positive R&D expenditure. In the remaining of this paper, we assume the following.

Assumption 2. *We assume that* $\rho \in (n,\bar{\rho})$ *and* $\lambda_A \in (\underline{\lambda}_A,\infty)$*, where* $\bar{\rho}$ *and* $\underline{\lambda}_A$ *are given in Proposition 1.*

Then, we have the following proposition.

Proposition 2. *The growth rate in the steady state is given by*

$$g \equiv \frac{\dot{Y}_t}{Y_t} = g_A + g_h + n,$$

where g_A and g_h are the respective growth rates of A_t and h_t in the steady state, given as

$$g_A \equiv \frac{\dot{A}_t}{A_t} = \lambda_A \sigma z_{A*} \tag{25}$$

and

$$g_h \equiv \frac{\dot{h}_t}{h_t} = \left[\frac{\gamma\omega_*}{(1-\gamma)(1-s_h)}\right]^\gamma - \rho. \tag{26}$$

Proof. From the definition of y and the fact that y is constant over time in a steady state equilibrium, the growth rate can be written as

$$g = g_A + g_N,$$

where g_N is the growth rate of the variety of intermediate goods: $g_N \equiv \dot{N}_t/N_t$. From (7) and (S4), the growth rate of the leading-edge quality of technology, g_A, is given by (25) in a steady state. Moreover, from the definition of l and the fact that l is constant over time in a steady state, we obtain

$$g_N = g_h + n.$$

We obtain (26) by substituting (S1) into (17), which completes the proof. □

As is discussed in the introduction, proposition 2 bridges the gap between the literature concerning Schumpeterian growth models and that concerning growth empirics. With our specification, the growth rate of output depends on both technological improvements and human capital accumulation. Our model provides a structural basis for including human capital as a determinant of long-run growth by using a Schumpeterian growth model. Hence, this study can be taken as providing a micro-foundation for reduced form analyses in growth empirics literature. It should also be noted that this is the first study that demonstrates complete general equilibrium policy implications by taking into account the endogenous determinations of both technology improvements and human capital accumulation.

Next, we provide the policy implications of the model as theorems.

Theorem 1. *The subsidy to R&D has a positive effect on the growth rate of the leading-edge technology, but does not have any effect on the growth rate of human capital.*

$$\frac{\partial g_A}{\partial s_R} > 0$$

and

$$\frac{\partial g_h}{\partial s_R} = 0.$$

Proof. The differentiation of (S4) with respect to s_R gives

$$\frac{\mathrm{d}z_{A*}}{\mathrm{d}s_R} = \left(1+\sigma+\frac{\alpha\sigma}{1-\alpha}\right)^{-1}(1-s_R)^{-2}\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}}. \tag{27}$$

Here, it should be noted that ω_* is determined by (S2), independent of s_R. Since the right-hand side of (27) is positive, we obtain the theorem. □

Therefore, our modification does not provide significant new insights in terms of the effect of R&D subsidy.[7]

On the other hand, the following theorem provides a new insight into the subsidy to human capital accumulation.

Theorem 2. *The subsidy to human capital accumulation has positive effects on the growth rates of the leading-edge technology and human capital.*

$$\frac{\partial g_A}{\partial s_h} > 0 \tag{28}$$

and

$$\frac{\partial g_h}{\partial s_h} > 0. \tag{29}$$

Proof. See Appendix A. □

Figure 1 represents the effect of subsidy to human capital accumulation on ω_*. In Figure 1, the RHS curve depicts the right-hand side of (S2). Note that it is decreasing in ω_*. Two LHS curves in Figure 1 respectively depict the left-hand side of (S2) corresponding to the subsidy rates of human capital accumulation, s_h and s_h', where $s_h' > s_h$. It should be noted that the left-hand side of (S2) is equal to g_h and is increasing in ω. Moreover, since it is increasing in s_h, the LHS curve shifts up as the subsidy rate on human capital accumulation increases from s_h to s_h'. Therefore, Figure 1 shows that an increase in s_h raises the growth rate of human capital. Moreover, since it reduces the adjusted wage rate ω_* and z_{A*} are decreasing in ω from (S4), the increase in the subsidy rate on human capital also raises the growth rate of the leading-edge productivity g_A.

Intuitively, the reason for the indirect positive impact of subsidy to human capital accumulation on productivity growth is explained as follows. Since human capital accumulation is labor augmenting, an increase in the growth rate of human capital reduces the adjusted wage rate. This raises the adjusted profit and the expected value of firms (6), motivating the enhancement of horizontal and

[7] The positive effects of subsidy to R&D are clarified in standard Schumpeterian endogenous growth models.

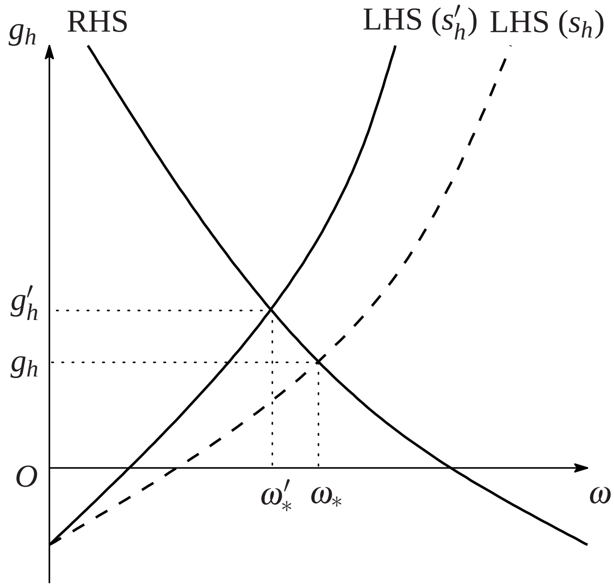

Figure 1: Effect of subsidy to human capital accumulation on ω ($s_h' > s_h$)

vertical innovations. This indirect effect on the subsidy of human capital accumulation, given by Theorem 2, provides a new insight into the subsidy policy, which is the main result of this study. Hence, human capital accumulation augments the growth rate of quality, which in turn accelerates the growth of per capita output. Further, if we consider only the relationship between the growth rate of output and R&D investment, as in standard Schumpeterian growth models, it misses the effect coming through human capital accumulation (and subsidy). The omission of this general equilibrium effect was a drawback in standard Schumpeterian growth models and is remediated with our specification.

Before completing this section, we add the following proposition.

Proposition 3. *Suppose that $\lambda_A \in (\underline{\lambda}_A, \infty)$ and $\rho \in (n, \bar{\rho})$, where $\underline{\lambda}_A$ and $\bar{\rho}$ is given in Proposition 1. Then, the system of differential equation characterizing the dynamics of the economy, (E1) – (E3), together with (C1) – (C3), is locally saddle-path stable around the steady state.*

Proof. See Appendix B. □

Proposition 3 states that equilibrium dynamics is locally saddle-path stable around the steady growth path in our model. Since our model is the human-capital augmented version of the model by Howitt [1999] and he does not consider the stability property, the result suggests that the equilibrium path in Schumpeterian endogenous growth models such as Howitt [1999] and its variant version of Segerstrom [2000] should be saddle-path stable since the human capital accumulation in our model has no externalities which can be source of indeterminacy of equilibrium dynamics.[8]

5 Conclusion

In this paper, we augmented Howitt [1999]'s Schumpeterian endogenous growth model to consider general-equilibrium interactions between R&D and human capital accumulation. We showed that on one hand, a subsidy to R&D investment has a positive impact on R&D investment but not on human capital accumulation, but on the other hand, a subsidy to human capital accumulation has a positive impact not only on R&D efforts but also on human capital accumulation. Because in our model, the per-capita output growth rate depends on both technology improvements and human capital accumulation, the model bridges the gap between the literature on Schumpeterian growth model and that on growth empirics.

Appendix

A Proof of Theorem 2

Differentiating both the sides of (S2), we obtain

$$\frac{\mathrm{d}\omega_*}{\mathrm{d}s_h} = -\frac{\gamma^{\gamma+1}(1-\gamma)^{-\gamma}(1-s_h)^{-\gamma-1}\omega_*^{\gamma}}{\gamma^{\gamma+1}(1-\gamma)^{-\gamma}(1-s_h)^{-\gamma}\omega_*^{\gamma-1} + \zeta\alpha^{\frac{2\alpha}{1-\alpha}}\frac{\alpha}{1-\alpha}\omega_*^{\frac{1}{\alpha-1}}} < 0.$$

Therefore, noting that the left-hand side of (S2) is equal to g_h and the right-hand side is decreasing in ω, we obtain (29). Similarly, since from (S4), z_{A*} is decreasing in ω, it gives (28).

[8] Dinopoulos and Thompson [1998] construct a different version of Schumpeterian endogenous growth model with vertical and horizontal product differentiation and numerically show that the equilibrium is globally saddle-path stable within plausible values of parameters.

B Local Stability of Equilibrium Dynamics around the Steady State

To prove equilibrium dynamics is locally saddle-path stable around the steady state, it is sufficient to show that the dynamical system of (c_t, ω_t, l_t) given by (E1) – (E3) is locally saddle-path stable around the steady state since the set of the remaining three variables (z_{At}, u_t, r_t) with (C1) – (C3) is fully determined by the value of (c_t, ω_t, l_t). By linearizing the system of differential equations, (E1) – (E3), we obtain

$$\begin{pmatrix} \frac{\partial \dot{c}}{\partial c} \\ \frac{\partial \dot{\omega}}{\partial \omega} \\ \frac{\partial \dot{l}}{\partial l} \end{pmatrix} = J \begin{pmatrix} c - c_* \\ \omega - \omega_* \\ l - l_* \end{pmatrix},$$

where J is the Jaccobi matrix:

$$J = \begin{pmatrix} \frac{\partial r}{\partial c} - \sigma\lambda_A \frac{\partial z_A}{\partial c} & \frac{\partial r}{\partial \omega} - \sigma\lambda_A \frac{\partial z_A}{\partial \omega} - \frac{\partial g_h}{\partial \omega} & \frac{\partial r}{\partial l} - \sigma\lambda_A \frac{\partial z_A}{\partial l} - \frac{\partial g_h}{\partial l} \\ \frac{\partial r}{\partial c} - \sigma\lambda_A \frac{\partial z_A}{\partial c} & \frac{\partial r}{\partial \omega} - \sigma\lambda_A \frac{\partial z_A}{\partial \omega} - \gamma(1-\gamma)B\omega_*^{\gamma-1} & \frac{\partial r}{\partial l} - \sigma\lambda_A \frac{\partial z_A}{\partial l} \\ 0 & \frac{\partial g_h}{\partial \omega} - \lambda_N \zeta^\beta \frac{\partial y}{\partial \omega} & \frac{\partial g_h}{\partial l} \end{pmatrix},$$

where $B \equiv \gamma^\gamma (1-\gamma)^{-\gamma}(1-s_h)^{-\gamma} > 0$. After elementary row operations, J can be reduced to

$$J' = \begin{pmatrix} \frac{\partial r}{\partial c} - \sigma\lambda_A \frac{\partial z_A}{\partial c} & \frac{\partial r}{\partial \omega} - \sigma\lambda_A \frac{\partial z_A}{\partial \omega} - \frac{\partial g_h}{\partial \omega} & \frac{\partial r}{\partial l} - \sigma\lambda_A \frac{\partial z_A}{\partial l} - \frac{\partial g_h}{\partial l} \\ 0 & \frac{\partial g_h}{\partial \omega} - \gamma(1-\gamma)B\omega_*^{\gamma-1} & \frac{\partial g_h}{\partial l} \\ 0 & \gamma(1-\gamma)B\omega_*^{\gamma-1} - \lambda_N \zeta^\beta \frac{\partial y}{\partial \omega} & 0 \end{pmatrix}.$$

To show that the dynamics of the economy is locally saddle-path stable around the steady state, it is sufficient to prove that the determinant of J' is negative and the trace is positive. Calculating the determinant of $\det J'$, we obtain

$$\det J' = -\left(\frac{\partial r}{\partial c} - \lambda_A \sigma \frac{\partial z_A}{\partial c}\right)\left[\gamma(1-\gamma)B\omega_*^{\gamma-1} - \lambda_N \zeta^\beta \frac{\partial y}{\partial \omega}\right]\frac{\partial g_h}{\partial l},$$

where

$$\frac{\partial r}{\partial c} = \lambda_A \left(1 + \sigma + \frac{\alpha\sigma}{1-\alpha}\right) l_* > 0$$

$$\frac{\partial z_A}{\partial c} = -l_* < 0$$

$$\frac{\partial y}{\partial \omega} = \frac{\alpha}{\alpha - 1}\alpha^{\frac{2\alpha}{1-\alpha}}\Gamma^{-1}\omega_*^{\frac{1}{\alpha-1}} < 0,$$

and

$$\frac{\partial g_h}{\partial l} = \alpha^{\frac{2}{1-\alpha}} B\Gamma^{-1} \omega_*^{\gamma-\frac{1}{1-\alpha}} l_*^{-2} > 0.$$

Therefore, the signs of these expressions hold that the determinant is negative.

Moreover, calculating the trace of J', we obtain

$$\mathrm{trace}J' = \frac{\partial r}{\partial c} - \lambda_A \sigma \frac{\partial z_A}{\partial c} + \frac{\partial g_h}{\partial \omega} - \gamma(1-\gamma)B\omega_*^{\gamma-1},$$

where

$$\begin{aligned}\frac{\partial g_h}{\partial \omega} - \gamma(1-\gamma)B\omega_*^{\gamma-1} &= \gamma B\omega_*^{\gamma-1} - (\gamma - \frac{1}{1-\alpha})\alpha^{\frac{2}{1-\alpha}} B\Gamma^{-1}\omega_*^{\gamma-\frac{1}{1-\alpha}-1} l_*^{-1} - \gamma(1-\gamma)B\omega_*^{\gamma-1} \\ &= \gamma^2 B\omega_*^{\gamma-1} - (\gamma - \frac{1}{1-\alpha})\alpha^{\frac{2}{1-\alpha}} B\Gamma^{-1}\omega_*^{\gamma-\frac{1}{1-\alpha}-1} l_*^{-1}.\end{aligned}$$

■

It is easy to show that this expression is always positive since $\alpha \in (0,1)$ and $\gamma \in (0,1)$. Hence, all in all, we find that the trace of J' is positive. Therefore, these results show that the steady-state equilibrium is locally saddle-path stable.

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