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EDUCATION, INNOVATION, AND LONG-RUN GROWTH

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Education, Innovation, and Long-Run Growth*

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Abstract

This study augments a second-generation Schumpeterian growth model to employ human capital explicitly. We clarify the general-equilibrium interactions of subsidy policies to R&D and human capital accumulation in a unified framework. Despite a standard intuition that subsidizing these growth-enhancing activities is always mutually growth promoting, we find asymmetric effects for subsidies on R&D and those on education. Our theoretical result of asymmetric policy effects provides an important empirical caveat that empirical researchers may find false negative relationships between education subsidies and the output growth rate, if they merely rely on the standard human capital model.

JEL classification: O15; O32; O41

Keywords: Schumpeterian growth model; human capital accumulation; subsidies

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1 Introduction

A growing consensus in the literature of endogenous growth theory is that Schumpeterian growth theory (e.g. Dinopoulos and Thompson [1998], Peretto [1998], and Howitt [1999]) is more consistent with empirical facts than earlier versions of endogenous growth theory including semi-endogenous growth theory.[1] Well after the first critique against the semi-endogenous growth theory by Jones [1995], Ha and Howitt [2007] compare two growth models and show that the data is more supportive of the Schumpeterian endogenous growth theory than the semi-endogenous growth theory. Recent findings further reinforce the finding, showing that a Schumpeterian prediction that *per capita R&D input determines the output growth rate* can be seen in the U.S. (Zachariadis [2003]), in OECD countries (Madsen [2008]), and in developing countries (Ang and Madsen [2011]).[2]

Despite this data-consistent feature of the theory, there is one missing point in the Schumpeterian endogenous growth theory, which is that usually roles of human capital are not explicitly taken into account in the models. This omission exhibits a contrast with a traditional view that human capital accumulation is an important engine of the growth (Uzawa [1965], Lucas [1988], and Rebelo [1991]). Recently, Madsen [2010] examined an extended version of conventional growth accounting with data the OECD countries, showing that output growth has been predominantly driven by total factor productivity (TFP) growth. He then showed that TFP, in turn, has been driven by R&D and by educational attainment, among others. His estimates in the growth accounting exercises suggested permanent growth effects of R&D and human capital.

In order to supplement the gap, this study augments a second-generation Schumpeterian growth model to employ human capital explicitly. The main contribution of the paper is to clarify the general-equilibrium interactions of subsidy policies to these two engines of growth, R&D and human capital accumulation, in a unified framework. We see how innovation and education activities interact, and discern implications for subsidy policies. Despite a standard intuition that subsidizing growth-enhancing activities is always mutually growth promoting, we show that subsidies on the education sector and those on the innovation sector exhibit asymmetric effects. Namely, a subsidy to education has a positive impact on inno-

[1]Semi-endogenous growth theory such as Jones [1995], Kortum [1997], Segerstrom [1998], and Young [1998] predicts that "the long-run rate of TFP growth, and hence the long-run growth rate of per capita income, depends on the rate of population growth, which ultimately limits the growth rate of R&D labor, to the exclusion of all economic determinants." (p. 734, Ha and Howitt [2007])

[2]Madsen, Ang, and Banerjee [2010] also show that innovative activity was an important force in shaping the Industrial Revolution and that the British growth experience after the Malthusian trap is consistent with Schumpeterian growth theory.

vation as well as on human capital accumulation, while a subsidy to innovation may not affect, or even negatively affect, human capital accumulation.[3] Our theoretical result of asymmetric policy effects provides an important empirical caveat that empirical researchers may find false negative relationships between education subsidies and the output growth rate, if they merely rely on the standard human capital model.

There are also theoretical studies considering education and innovation at the same time. Nelson and Phelps [1966] is the first paper showing that enhancement of human capital accumulation promotes innovation. Redding [1996] and Acemoglu [1997] consider human capital accumulation and R&D in search-theoretic models with linear preferences and binary decision making, and show that multiplicity of equilibria is a consequence of the complementarities between education and innovation. Arnold [1998] constructs an endogenous growth model with these two sources of R&D and human capital accumulation. However, the effects of any subsidy policy on economic growth would vanish in the long run, since it is essentially based on a semi-endogenous growth model. Recenly, Zeng [2003] constructs a similar model to our model and investigate the effects of subsidy policies on output growth, but does not consider the asymmetric effects of these policies on engines of economic growth. Connolly and Peretto [2003] also construct an endogenous growth model close to ours to investigate interactions of fertility choices and innovation. In their model, endogenous fertility choices affect per capita output growth rate through the channel of labor inputs. More recenly, Grossmann [2007] investigates the effects of public education and R&D subsidy to innovation, and shows that R&D subsidy may reduce the R&D investment when there are congestion externalities on public education. Along the lines of these studies, we construct a second-generation Schumpeterian growth model with human capital accumulation, and clarify the general-equilibrium interactions of subsidy policies between R&D and human capital accumulation.

This paper is organized as follows. Section 2 presents the baseline model

[3] As is well known, an important theoretical result of the Schumpeterian endogenous growth theory is that subsidizing R&D activities can have positive long-run effects on the output growth rate, not an implication of semi-endogenous growth theory. Direct empirical evidence supporting the theoretical results are scarce. Using Japanese and the U.S. data sets, Sakakibara and Branstetter [2001] showed that patent reform policies did not lead to increase in either R&D spending or innovative output, but they did not work on subsidy policies. Zachariadis [2004] and Ulku [2007] showed that in OECD countries there are positive relationship between R&D intensity and the growth rate of output. However, their conclusions were not from natural experiments of exogenous subsidy policy shocks. Similarly, standard human capital models also allow for policies that enhance economic growth, but empirical evidence on effects of these policies on the output growth rate is scarce. Instead, the effects of increases in minimum education due to school reforms in European countries over 1960–1980 are well examined. See Machin, Pelkonen, and Salvanes [2008] and references therein for this topic.

where households have logarithmic preferences. Section 3 generalizes the baseline model to allow for general constant intertemporal elasticity of substitution (CIES) felicity function. We show that main results in the baseline case remain. Section 4 extends the baseline model where only human capital is required for intermediate goods production and vertical innovation needs final goods only. In the extended model, these activities use both human capital and final goods. It is shown that our main results remain. Finally, Section 5 concludes the paper.

2 The Baseline Model

The production structure of the baseline model is the same as that of Howitt [1999] except that the raw labor input in the intermediate goods production is replaced by human capital in our baseline model. We also employ an Uzawa-Lucas-Rebelo-type process for human capital accumulation. Population of the economy is assumed constant over time and normalized to one. The final goods output, which is the numéraire in the model, is used for consumption (C_t) and investments for vertical innovation (Z_{At}), horizontal innovation (Z_{Nt}), and human capital creation (Z_{ht}).

2.1 Production

The final goods sector operates under perfect competition. The technology for final goods production is given by

$$Y_t = X^{1-\alpha} \int_0^{N_t} A_{it} x_{it}^{\alpha} \mathrm{d}i, \tag{1}$$

where Y_t is the final goods output, X is a fixed resource such as land, N_t measures the varieties of intermediate goods at time t, A_{it} is a productivity parameter attached to the incumbent version of intermediate product $i \in [0, N_t)$, x_{it} is the amount of intermediate product i used in the economy, and $\alpha \in (0, 1)$ is a parameter for production technology. Since we assume that the total endowment of the fixed resource in the economy is equal to 1, we henceforth abbreviate variable X from the production function. Under perfect competition, the first-order condition for the final goods sector with respect to x_{it} is given as

$$p_{it} = \alpha A_{it} x_{it}^{\alpha-1}, \tag{2}$$

where p_{it} is the price of intermediate product i.

The intermediate goods sector operates under monopolistic competition. Here, we assume that one unit of intermediate good is made from one unit of human capital.

Profits Π in creating intermediate product i are given by

$$\Pi_{it} = p_{it}x_{it} - w_t x_{it},$$

where w_t is the real wage for human capital. With the demand function of x_{it} obtained from (2), the first-order condition with respect to x_{it} gives the amount of the ith intermediate good as

$$x_{it} = \alpha^{\frac{2}{1-\alpha}} A_{it}^{\frac{1}{1-\alpha}} w_t^{\frac{-1}{1-\alpha}}. \tag{3}$$

Then, profits of intermediate goods firms can be written as

$$\Pi_{it} = \alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_{it}^{\frac{1}{1-\alpha}} w_t^{\frac{\alpha}{\alpha-1}}. \tag{4}$$

2.2 Innovation

Here, we consider two types of innovation. The first is vertical innovation, which improves productivity in each intermediate goods sector. The second is horizontal innovation, which brings new varieties into the economy.

2.2.1 Vertical Innovation

The Poisson arrival rate of vertical innovation in each sector is defined as

$$\phi_t = \lambda_A \frac{Z_{At}}{A_t N_t} \equiv \lambda_A z_{At}, \tag{5}$$

where $\lambda_A > 0$ is the productivity parameter of vertical innovation, and Z_{At} is the amount of resource devoted to vertical innovation for each sector i. A_t captures the leading-edge productivity parameter with $A_t \equiv \max\{A_{it} \mid i \in [0, N_t]\}$. Hence, $z_{At} \equiv Z_{At}/\left(A_t N_t\right)$ is the per-sector expenditures on vertical innovation adjusted by productivity and complexity. The free-entry condition into vertical innovation is given by

$$\lambda_A z_{At} V_t = (1 - s_R)\frac{Z_{At}}{N_t}, \tag{6}$$

where V_t is the expected present value of a vertical innovation at time t from the stream of future profits. Here, $s_R \in [0, 1)$ is the general subsidy rate to R&D, which is an important policy parameter in this study.

Because at every time t, the innovation will be replaced by the next innovator with the Poisson arrival rate ϕ_t, V_t is given as

$$V_t = \int_t^\infty \exp\left[-\int_t^\tau (r_s + \lambda_A z_{At})\mathrm{d}s\right]\pi_{\tau t}\mathrm{d}\tau, \tag{7}$$

where r_s is the interest rate, and $\pi_{\tau t}$ is the profit of the incumbent on date τ for any sector with vintage technology at time t, which can be expressed as

$$\pi_{\tau t} = \alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} w_\tau^{\frac{\alpha}{\alpha-1}}.$$

Further, we can define the quality-adjusted value of vertical innovation as $v_t \equiv V_t/A_t$. Then, the free-entry condition (6) is rewritten as

$$v_t = \frac{1-s_R}{\lambda_A}. \tag{8}$$

Finally, the intensity of the quality improvement for each vertical innovation is captured by a parameter $\sigma > 0$, with which the growth rate of the leading-edge productivity is given as

$$g_{At} = \sigma \lambda_A z_{At}, \tag{9}$$

where g_{At} denotes the growth rate of the leading-edge productivity, $g_{At} = \dot{A}_t/A_t$. Hereafter, we denote the growth rate of a variable χ_t by $g_{\chi t}$. Therefore, by differentiating (8) with respect to t and using (7), (8), and (9), a little algebra provides the expression of the interest rate as

$$r_t = \lambda_A \left[(1-s_R)^{-1} \alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \omega_t^{\frac{\alpha}{\alpha-1}} - \left(1 + \frac{\sigma\alpha}{1-\alpha}\right) z_{At} \right], \tag{10}$$

where ω_t denotes the quality-adjusted wage rate ($\omega_t \equiv w_t/A_t$).

2.2.2 Horizontal Innovation

The variety of intermediate goods can be augmented by horizontal innovation. The evolution of varieties is specified as

$$\dot{N}_t = \lambda_N \frac{Z_{Nt}^{\beta} Y_t^{1-\beta}}{A_t},$$

where $\lambda_N > 0$ and $\beta \in (0,1)$ are parameters of horizontal innovation. Z_{Nt} is the amount of numéraire devoted to horizontal innovation. Knowledge spillover effects captured by the term Y_t is also included. Therefore, the growth rate of the variety of intermediate goods is given as

$$g_{Nt} = \lambda_N \frac{Z_{Nt}^{\beta} Y_t^{1-\beta}}{A_t N_t}. \tag{11}$$

Each horizontal innovation results in a new intermediate product whose productivity is randomly drawn from the distribution of existing intermediate products. Further, from the definition of the value of the leading-edge intermediate good (A_t) given by (7), and from the definition of the profits of intermediate good firm with quality A_{it} given by (4), the expected value of a horizontal innovation is derived as $\mathrm{E}[(A_{it}/A_t)^{1/(1-\alpha)}]V_t$. Hence, the free-entry condition in the horizontal innovation sector read as

$$\dot{N}_t \mathrm{E}\left[\left(\frac{A_{it}}{A_t}\right)^{\frac{1}{1-\alpha}}\right] V_t = (1 - s_R)\, Z_{Nt}. \tag{12}$$

Then, plugging the definition of $\dot{N}_t$ into the above equation, we get

$$\lambda_N \frac{Z_{Nt}^{\beta} Y_t^{1-\beta}}{A_t} \mathrm{E}\left[\left(\frac{A_{it}}{A_t}\right)^{\frac{1}{1-\alpha}}\right] V_t = (1 - s_R)\, Z_{Nt}. \tag{13}$$

From the specification of vertical innovation, the distribution of the relative productivity A_{it}/A_t converges to the time-invariant distribution function $F(q) = q^{1/\sigma}$, where $0 < q \leq 1$.[4] Hence, in the long run, we obtain

$$\mathrm{E}\left[\left(\frac{A_{it}}{A_t}\right)^{\frac{1}{1-\alpha}}\right] = \frac{1}{\Gamma},$$

where $\Gamma \equiv 1 + \sigma/(1 - \alpha) > \sigma$. Throughout this paper, we assume that the relative productivity A_{it}/A_t is drawn from the time-invariant distribution function. From (8), (13), and the definition of Γ, we get

$$\frac{Z_{Nt}}{Y_t} = \left(\frac{\lambda_N}{\lambda_A}\Gamma^{-1}\right)^{\frac{1}{1-\beta}} \equiv \zeta. \tag{14}$$

Note that $\zeta \in (0, 1)$ must hold because of a resource constraint, $Y_t > Z_{Nt}$. To guarantee that $\zeta \in (0, 1)$ holds, we impose the following assumption.

Assumption 1. *We assume that $\lambda_A \in (\lambda'_A, \infty)$, where*

$$\lambda'_A = \frac{\lambda_N}{\Gamma}.$$

Then, (11) is rewritten as

$$g_{Nt} = \lambda_N \zeta^{\beta} y_t, \tag{15}$$

where y_t is the amount of output adjusted by the quality and variety ($y_t \equiv Y_t/(A_t N_t)$).

[4]See Howitt [1999] and Segerstrom [2000] for the proof.

2.3 Household

For the baseline analysis, we assume that the felicity function of household is in log. The maximization problem of a representative household is given by

$$\max \int_0^\infty \exp[-\rho t] \log C_t \mathrm{d}t$$

where $\rho > 0$ is the subjective discount rate, and C_t is the per capita consumption. The laws of motion for financial assets and human capital in per capita terms are respectively given as

$$\dot{W}_t = r_t W_t + w_t u_t h_t - C_t - (1 - s_h) Z_{ht} - T_t, \quad (\mu_t)$$

and

$$\dot{h}_t = \lambda_h \left(\frac{Z_{ht}}{A_t} \right)^\gamma [(1 - u_t) h_t]^{1-\gamma}, \quad (\nu_t) \tag{16}$$

where W_t denotes the per capita asset, h_t is the per capita amount of human capital, $u_t \in [0, 1]$ is the ratio of human capital devoted to the intermediate goods sector, Z_{ht} is the expenditure on human capital accumulation, $\gamma \in [0, 1)$ is a parameter for human capital accumulation process, and $s_h \in [0, 1)$ is the other important policy parameter of the subsidy rate to expenditure on human capital accumulation. Co-state variables of μ and ν are attached to the respective constraints. In (16), we divide the amount of expenditure (Z_{ht}) by A_t following the idea that the higher the leading-edge quality in the economy, the more difficult may be the acquisition of new skills to handle the cutting-edge technology (Howitt [2005]). Finally, the expenditure of the government is financed by lump sum tax (T_t), and the budget of government is balanced at any time.

From the above specifications, the first-order conditions of the problem are obtained as

$$C_t^{-1} = \mu_t \tag{17}$$

$$\mu_t w_t = \nu_t \lambda_h (1 - \gamma)(1 - u_t)^{-\gamma} A_t^{-\gamma} h_t^{-\gamma} Z_{ht}^\gamma \tag{18}$$

$$\mu_t (1 - s_h) = \nu_t \lambda_h \gamma A_t^{-\gamma} h_t^{1-\gamma} Z_{ht}^{\gamma-1} (1 - u_t)^{1-\gamma} \tag{19}$$

$$\mu_t r_t = \rho \mu_t - \dot{\mu}_t \tag{20}$$

and

$$\mu_t w_t u_t + \nu_t \lambda_h (1 - \gamma)(1 - u_t)^{1-\gamma} A_t^{-\gamma} h_t^{-\gamma} Z_{ht}^\gamma = \rho \nu_t - \dot{\nu}_t. \tag{21}$$

In addition, the usual transversality conditions (TVCs) are imposed on W and h.

Combining (17) through (21), the first-order conditions are reduced to the following equations.

$$g_{ct} = r_t - \rho - g_{At} - g_{ht}, \tag{22}$$

$$g_{\omega t} = (1-\gamma)^{-1}(r_t - g_{At}) - \lambda_h \gamma^{\gamma}(1-\gamma)^{-\gamma}(1-s_h)^{-\gamma}\omega_t^{\gamma}, \tag{23}$$

$$g_{ht} = \lambda_h \gamma^{\gamma}(1-\gamma)^{-\gamma}(1-s_h)^{-\gamma}(1-u_t)\omega_t^{\gamma}, \tag{24}$$

and

$$z_{ht} = \frac{\gamma(1-u_t)}{(1-\gamma)(1-s_h)}\omega_t. \tag{25}$$

Here, z_{ht} denotes expenditure to education adjusted by the quality and human capital ($z_{ht} = Z_{ht}/(A_t h_t)$) and g_{ct} denotes the growth rate of consumption adjusted by quality and human capital ($c_t = C_t/(A_t h_t)$). Finally, we define the growth rate of human capital per variety as $l_t \equiv h_t/N_t$. Hence,

$$g_{lt} = g_{ht} - g_{Nt}. \tag{26}$$

2.4 Equilibrium

The market clearing conditions for final good and human capital market at each time t read, respectively, as

$$Y_t = C_t + Z_{ht} + Z_{At} + Z_{Nt}, \tag{27}$$

and

$$\int_0^{N_t} x_{it}\mathrm{d}i = u_t h_t. \tag{28}$$

Finally, from (1), (3), and the definition of y_t, we have

$$y_t = \alpha^{\frac{2\alpha}{1-\alpha}}\Gamma^{-1}\omega_t^{\frac{\alpha}{\alpha-1}}. \tag{29}$$

The equilibrium dynamics of the economy is defined as follows.

Definition 1 (Equilibrium Dynamics)**.** *An interior equilibrium dynamics is defined as a sequence of prices,* (w_t, r_t, p_{it})*, and allocation,* $(Z_{At}, Z_{Nt}, Z_{ht}, x_t, C_t, Y_t, u_t)$*, that satisfies (i) profit maximization:* (1), (2), (3), (10) *and* (14)*, (ii) utility maximization:* (22), (24), (25)*, and the transversality conditions with* $u \in (0,1)$*,* $g_{A*} \in (0,\infty)$ *(iii) market clearing:* (27) *and* (28)*, and (iv) dynamics:* (9), (15)*, and* (23)*, given* A_0*,* N_0*,* h_0*, and the distribution of* A_{i0}/A_0 *which is given by* $F(q) = q^{1/\sigma}$*, where* $0 < q \leq 1$.

2.5 Steady State

2.5.1 The Existence and Uniqueness of the Steady State

Hereafter, we focus on the interior steady state of the equilibrium dynamics, where $g_{ct} = g_{\omega t} = g_{lt} = 0$ for all t. We add subscript $*$ to any variable whenever it is constant in the steady state.

To begin with, it follows from (29) and from $g_{\omega *} = 0$ that the steady-state growth rate of y is equal to zero. Hence, the definition of y gives the economic growth rate in the steady state as

$$g_{Y*} = g_{A*} + g_{N*}. \tag{30}$$

Since in the steady state it holds that $g_{lt} = 0$, (26) implies that $g_{N*} = g_{h*}$. Hence, (30) can be rewritten as the sum of the growth rate of leading-edge quality and that of human capital

$$g_{Y*} = g_{A*} + g_{h*}. \tag{31}$$

Equation (31) provides a structural basis of growth regressions ever since the work of Mankiw, Romer, and Weil [1992]: while the arguments in production functions (technology, labor, physical capital, and human capital) are counted in an ad-hoc manner in the growth empirics literature, our model provides market structures in which all of these production components are contributing to the economic growth.

Regarding the growth rate of human capital, firstly we have from (22) that

$$r_* = \rho + g_{A*} + g_{h*}. \tag{32}$$

This relation is considered the steady-state version of the Euler equation.

From (23), we have

$$r_* = \xi \left(\frac{\omega_*}{1 - s_h} \right) + g_{A*}, \tag{33}$$

where $\xi(\chi) = \lambda_h \gamma^\gamma (1-\gamma)^{1-\gamma} \chi^\gamma$. Note that $\xi'(\chi) > 0$ for all $\chi > 0$ and $\lim_{\chi \to \infty} \xi(\chi) = \infty$.

Combining (32) and (33), we have

$$g_{h*} = \xi \left(\frac{\omega_*}{1 - s_h} \right) - \rho \equiv g_h \left(\frac{\omega_*}{1 - s_h} \right). \tag{34}$$

Since (34) is derived only from household's optimal conditions, it can be referred to as the *supply-side* growth rate of human capital accumulation.

Secondly, from (29) and (15), we have

$$g_{N*} = \lambda_A \zeta \alpha^{\frac{2\alpha}{1-\alpha}} \omega_*^{\frac{\alpha}{\alpha-1}} \equiv g_N(\omega_*), \tag{35}$$

where $g_N'(\chi) < 0$ for all $\chi > 0$. Since the above equation is derived only from firm's optimal conditions, it can be referred to as the *demand-side* growth rate of human capital accumulation.

Combining (34) and (35) determines the steady-state growth rate of human capital g_{h*} and the quality-adjusted wage rate ω_*:

$$g_h\left(\frac{\omega_*}{1-s_h}\right) = g_N(\omega_*). \tag{H}$$

Since the left-hand side of the above equation is increasing in ω and the right-hand side is decreasing, it determines the unique steady-state value of ω.

Next, we calculate the steady-state values of u and g_{A*}. Combining (24) and (34), we have

$$u_* = \gamma + (1-\gamma)\frac{\rho}{\xi\left(\frac{\omega_*}{1-s_h}\right)}. \tag{U}$$

Substituting (9), (10), and (35) into (32) yields

$$g_{A*} = \frac{\sigma}{\Gamma}\left\{\lambda_A\left[\frac{\alpha(1-\alpha)}{1-s_R} - \zeta\right]\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}} - \rho\right\}. \tag{A}$$

We have then the following proposition.

Proposition 1 (Steady State)**.** *In the steady state of the equilibrium dynamics, ω_*, u_*, g_{h*}, and g_{A*} are given by* (H), (U)*, and* (A).

Finally, we give the following proposition showing the existence and uniqueness of the steady state.

Proposition 2 (The Existence and Uniqueness of the Steady State)**.** *There exist $\underline{\lambda}_A \in (0,\infty)$ and $\bar{\rho} \in (0,\infty)$ such that, for any $\lambda_A \in (\underline{\lambda}_A,\infty)$ and $\rho \in (0,\bar{\rho})$, there is the unique steady state of the interior equilibrium dynamics.*

Proof. See Appendix A. □

In the remaining of this paper, we assume the interior steady state.

2.5.2 Subsidy policies in the baseline model

We provide subsidy policy implications for the baseline model as Theorem 1 and Theorem 2. Despite a standard intuition that subsidizing these growth-enhancing activities is always mutually growth promoting, we find asymmetric effects for subsidies on R&D and those on education.

Theorem 1. *The subsidy to human capital accumulation has positive effects on the growth rates of the leading-edge technology and human capital.*

$$\frac{\mathrm{d}g_{A*}}{\mathrm{d}s_h} > 0 \tag{36}$$

and

$$\frac{\mathrm{d}g_{h*}}{\mathrm{d}s_h} > 0. \tag{37}$$

Hence, we have

$$\frac{\mathrm{d}g_{Y*}}{\mathrm{d}s_h} > 0.$$

Proof. Differentiating both the sides of (H), we obtain

$$\frac{\mathrm{d}\omega_*}{\mathrm{d}s_h} = -\frac{g_h'\left(\frac{\omega_*}{1-s_h}\right)\omega_*}{(1-s_h)\left(g_h'\left(\frac{\omega_*}{1-s_h}\right) - (1-s_h)\,g_N'(\omega_*)\right)} < 0.$$

Therefore, noting that the left-hand side of (H) is equal to g_h and the right-hand side is decreasing in ω, we obtain (37). Similarly, since from (A), z_{A*} is decreasing in ω, it gives (36). The proof of the last statement in the theorem is straightforward. □

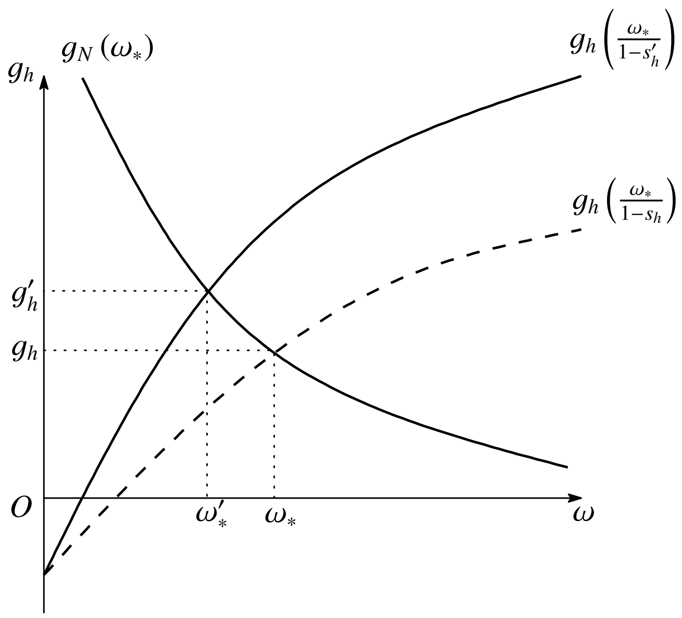

Figure 1: Effects of subsidy to human capital accumulation $(s_h' > s_h)$

Intuitively, the reason for the indirect positive impact of subsidy to human capital accumulation process on the R&D sector is explained as follows. The subsidy to human capital accumulation raises the growth rate of the supply-side human capital through (34), which reduces the adjusted wage rate to maintain the equality of (H). When the adjusted wage rate of human capital declines, the adjusted profit and the expected value of firms (7) increase, motivating the vertical and horizontal innovations. Hence, boosting human capital accumulation augments the growth rate of products quality, which in turn accelerates the growth of per capita output. If we consider only the relationship between the growth rate of output and R&D investment, as in standard Schumpeterian growth models, we miss the growth-enhancing effects of the subsidy policies coming by way of human capital accumulation process.

Figure 1 depicts the effect of subsidy to human capital accumulation on ω_*. In the figure, the g_N curve depicts the right-hand side of (H). Two g_h curves respectively plot the left-hand side of (H) corresponding to the subsidy rates of human capital accumulation, s_h and s_h'. Since g_h curve is increasing in s_h, and s_h' is assumed to be greater than s_h, the curve corresponding to s_h' lies above that corresponding to s_h. Hence, the figure shows that the increase in the subsidy rate on human capital raises the growth rate of human capital accumulation g_{h*}.

Theorem 2. *The subsidy to R&D has a positive effect on the growth rate of the leading-edge technology, but does not have any effect on the growth rate of human capital.*

$$\frac{\mathrm{d}g_{A*}}{\mathrm{d}s_R} > 0 \qquad \textit{and} \qquad \frac{\mathrm{d}g_{h*}}{\mathrm{d}s_R} = 0.$$

Therefore, we have

$$\frac{\mathrm{d}g_{Y*}}{\mathrm{d}s_R} > 0.$$

Proof. The differentiation of (A) with respect to s_R gives

$$\frac{\mathrm{d}g_{A*}}{\mathrm{d}s_R} = \sigma\Gamma\lambda_A(1-s_R)^{-2}\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}}. \tag{38}$$

Here, it should be noted that ω_* is determined by (H), which is independent of s_R. Since the right-hand side of (38) is positive, we obtain the theorem. The proof of the last statement in the theorem is straightforward. □

The reason that subsidy to R&D investment has *no effect* on human capital accumulation is as follows. In the steady state, an increase in the subsidy rate to R&D investment raises both the real interest and productivity growth rates by the same amount, since we assume the logarithmic preferences. These increases have two effects on human capital accumulation process. The first is the effect induced by the increase in the interest rate, shifting human capital resource to production. The other is the effect induced by the increase in the productivity growth rate, shifting human capital resource to education, since the increase of productivity growth rate implies the increase of the wage rate in the future. Since in the baseline case we assume the logarithmic preferences, these two effects offset each other, leaving u_* and ω_* unchanged. Hence, subsidy to R&D investment has no effect on the growth rate of human capital.

Before closing this section, we add the following proposition.

Proposition 3. *The steady state of the equilibrium dynamics is locally saddle stable.*

Proof. See Appendix B. □

Not likely to previous studies such as Redding [1996] and Acemoglu [1997], where multiple steady states result as a consequence of the complementarities between human capital and R&D, our model has the unique and locally stable steady growth path which allows for subsidy policies implications along the path.

3 CIES Preferences

In this section, we address CIES preferences instead of log felicity. We show that the previous results of asymmetric policy effects remain for the generalized case. Our theoretical result of asymmetric policy effects provides an important empirical caveat. We specify that utility function is given as

$$\int_0^\infty \exp[-\rho t]\frac{\eta}{\eta-1}\left(C_t^{1-\frac{1}{\eta}}-1\right)\mathrm{d}t,$$

where η denotes the intertemporal elasticity of substitution. In the steady state, the Euler equation is modified from (32) to

$$r_* = \rho + \frac{g_{A*}+g_{h*}}{\eta}. \tag{39}$$

The above equation shows that, given the interest rate and the growth rate of human capital, a higher degree of intertemporal elasticity of substitution η yields a higher technological progress.

Combining (33) and (39), we get

$$g_{h*} = \eta\left(\xi\left(\frac{\omega_*}{1-s_h}\right)-\rho\right)+(\eta-1)g_{A*}. \tag{40}$$

Equation (40) is the modified version of *supply-side* growth rate of human capital accumulation. Notice that g_{h*} in this case depends on g_{A*}, not likely to the case of log felicity in (34).

Next, from (9), (10), and (39), we obtain the modified version of the growth rate of technology as

$$g_{A*} = \frac{\eta\sigma/\Gamma}{\eta - (\eta - 1)\,\sigma/\Gamma}\left[\frac{\lambda_A}{1 - s_R}\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}} - \rho - \frac{g_{h*}}{\eta}\right]. \tag{41}$$

Substituting (35) into the above equation, we see that

$$g_{A*} = \frac{\eta\sigma/\Gamma}{\eta - (\eta - 1)\,\sigma/\Gamma}\left\{\lambda_A\left[\frac{\alpha(1-\alpha)}{1 - s_R} - \frac{\zeta}{\eta}\right]\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}} - \rho\right\}. \tag{A1}$$

From (24), (35), (40), and (41) we have

$$g_h\left(\omega_*, s_h, s_R\right) = g_{N*}\left(\omega_*\right) \tag{H1}$$

and u^* is determined as

$$\begin{aligned} u^* =& 1 - (1-\gamma)\left[\eta - (\eta - 1)\frac{\sigma}{\Gamma}\right] \\ &+ \frac{1-\gamma}{\xi\left(\frac{\omega_*}{1-s_h}\right)}\left[\eta\rho - (\eta - 1)\frac{\sigma}{\Gamma}\frac{\lambda_A}{1 - s_R}\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}}\right]. \end{aligned} \tag{U1}$$

Here,

$$\begin{aligned} g_h\left(\omega_*, s_h, s_R\right) \equiv& \left[\eta - (\eta - 1)\frac{\sigma}{\Gamma}\right]\xi\left(\frac{\omega_*}{1 - s_h}\right) \\ &+ (\eta - 1)\frac{\sigma}{\Gamma}\frac{\lambda_A}{1 - s_R}\alpha(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}} - \eta\rho. \end{aligned} \tag{42}$$

Therefore, we put the following proposition for the interior steady state of the economy with CIES preferences.

Proposition 4. *In the interior steady state with CIES preferences, ω_*, u_*, g_{h*}, and g_{A*} are given by* (A1)*,* (H1)*,* (U1)*.*

Note that the existence of the interior steady state is guaranteed from the continuity around $\eta = 1$ under the assumptions in Proposition 2. We then provide the following proposition.[5]

[5]The proof of the proposition and theorems in this section is given in Appendix C.

Proposition 5 (Uniqueness of the Steady State)**.** *If there is an interior steady state, it is unique.*

We provide subsidy policy implications for this generalized version of the economy as Theorem 3 and Theorem 4. As in the log felicity case, we find asymmetric effects for subsidies on R&D and those on education. In this case, however, subsidy to R&D investment can affect human capital accumulation process in addition to R&D sector. The novelty here is that the indirect effect of subsidy to R&D can slow down human capital accumulation, depending on the magnitude of η.

Theorem 3 (Effects of Subsidy to Human Capital Accumulation)**.** *In the interior steady state, we have*

$$\frac{\mathrm{d}g_{A*}}{\mathrm{d}s_h} > 0, \qquad \frac{\mathrm{d}g_{h*}}{\mathrm{d}s_h} > 0, \qquad \textit{and} \qquad \frac{\mathrm{d}g_{Y*}}{\mathrm{d}s_h} > 0.$$

Theorem 4 (Effects of Subsidy to R&D Investment)**.** *In the interior steady state, we have*

$$\frac{\mathrm{d}g_{A*}}{\mathrm{d}s_R} > 0 \qquad \textit{and} \qquad \frac{\mathrm{d}g_{h*}}{\mathrm{d}s_R} \lesseqgtr 0 \qquad \textit{for} \quad \eta \lesseqgtr 1.$$

And

$$\frac{\mathrm{d}g_{Y*}}{\mathrm{d}s_R} > 0.$$

Theorem 3 is a simple replication of Theorem 1. We obtain Theorem 4 since in this case the two opposite effects of subsidy to R&D investment on the human capital accumulation process do not offset each other. An increase of subsidy to R&D investment raises the real interest rate, r_*, through (10). The increase in r_* raises the productivity growth rate, g_{A*}, by the smaller (larger) amount than the increase of r_* through (39) if η is smaller (larger) than one, given the growth rate of human capital, g_{h*} fixed. This implies that, given ω_*, the growth rate of the marginal return to raise human capital investment, which is given by the right-hand side of (33), falls behind (goes beyond) the growth rate of the marginal return to raise savings, which is given by the left-hand side of (33), causing the decrease (increase) of g_{h*}. The figures 2 and 3 indicate the effects of subsidy to R&D investment in the cases $\eta < 1$ and $\eta > 1$, respectively.

Our theoretical result of asymmetric policy effects provides an important empirical caveat. For a thought experiment, now suppose that there is an economy where the government has launched unexpected subsidies both on the education sector and on the innovation sector. Assume that deep parameters in the economy are given such that a subsidy to the innovation sector negatively affects human capital accumulation, and assume also that the subsidy to the R&D sector is so great,

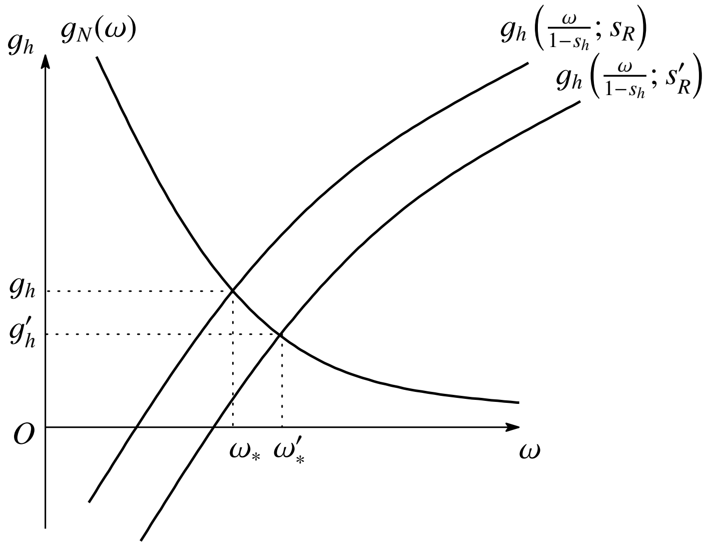

Figure 2: Effects of subsidy to R&D investment in the case $\eta < 1$ $(s_R' > s_R)$

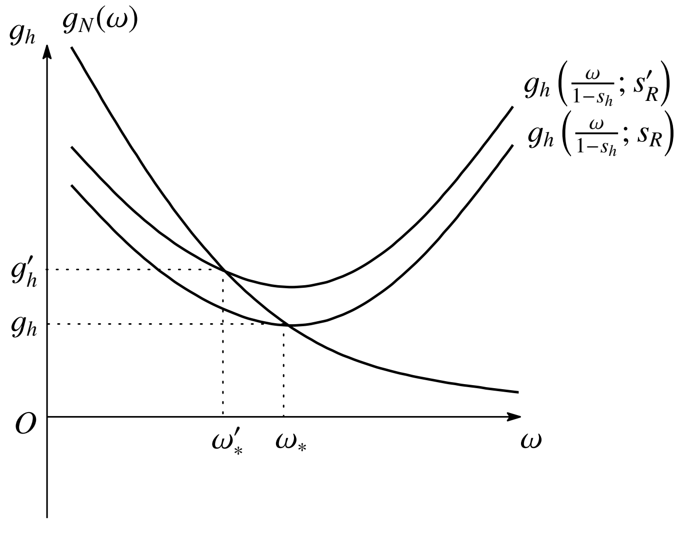

Figure 3: Effects of subsidy to R&D investment in the case $\eta > 1$ $(s_R' > s_R)$

compared to that for education sector, that the net effect of these two subsidies is negative on the growth rate of human capital. On one hand, when researchers focus on conventional Schumpeterian growth models, they will find that the positive policy shock on the R&D sector promotes the output growth rate as it is expected, while the effect is under-estimated because of the omission of the channel of human capital accumulation. This is a typical omission variable problem. On the other hand, when researchers focus on conventional human capital models, empirical researchers find false negative relationships between education subsidies and the output growth rate: after a positive policy shock on the education sector, the growth rate in human capital is *decreased* while the growth rate in the output is increased. Hence, we provide a caveat for future empirical research examining the relationship between subsidy policies and economic growth.

4 Extension

4.1 Vertical Innovation with Final Good and Human Capital Inputs

In the baseline model, we examined the case where vertical innovation required final good input only. Following Eicher and Turnovsky [1999], here we extend the baseline model to incorporate human capital as inputs for vertical innovation. We show that our previous results do not depend on the assumption that vertical innovation needs final good input only.

Instead of (5), this section defines the arrival rate of vertical innovation as

$$\phi_t = \lambda_A \left(\frac{Z_{At}}{A_t N_t}\right)^{\theta} \left(\frac{H_{At}}{N_t}\right)^{1-\theta}.$$

Here, Z_{At} and H_{At} denote the inputs of final good and human capital to develop the leading-edge technology at time t, respectively. In this case, equations (H) and (A) are respectively replaced by

$$g_h\left(\frac{\omega_*}{1-s_h}\right) = \Lambda(\omega_*)\,\zeta\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}}. \tag{H2}$$

and

$$g_{A*} = \frac{\sigma}{\Gamma}\left\{\Lambda(\omega_*)\left[\frac{\alpha(1-\alpha)}{1-s_R}-\zeta\right]\alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}}-\rho\right\}, \tag{A2}$$

where

$$\Lambda(\omega) = \lambda_A\left(\frac{\theta}{1-\theta}\right)^{1-\theta}\omega^{\theta-1}. \tag{43}$$

Equation (H2) uniquely determines the steady state values of ω_* and g_{h*}, since $\Lambda(\omega)$ is decreasing in ω.

Since (H2) is independent of s_R, g_{h*} is not affected by the change of s_R. Also, we see that a rise in s_h reduces ω_* through (H2), increasing the growth rate of the leading-edge productivity g_{A*} through (A2). Hence, our extension here does not change previous results in the baseline model.

4.2 Production of the Intermediate Goods with Final Good and Human Capital Inputs

The other extension from the baseline model is about production of the intermediate goods that now require final good input in addition to human capital inputs. We specify the production function of the intermediate goods as

$$x_{it} = \left(\frac{Z_{Iit}}{A_t}\right)^{\delta} H_{Iit}^{1-\delta}.$$

Here, Z_{Iit} and H_{Iit} denote the firm i's final good and human capital inputs at time t, respectively. In this case, (H) and (A) can be respectively rewritten as

$$g_h\left(\frac{\omega_*}{1-s_h}\right) = \lambda\zeta\alpha^{\frac{2\alpha}{1-\alpha}}\delta^{\frac{\alpha\delta}{1-\alpha}}(1-\delta)^{\frac{\alpha(1-\delta)}{1-\alpha}}\omega_*^{\frac{\alpha(1-\delta)}{\alpha-1}} \tag{H3}$$

and

$$g_{A*} = \frac{\sigma}{\Gamma}\left\{\lambda\left[\frac{\alpha(1-\alpha)}{1-s_R}-\zeta\right]\alpha^{\frac{2\alpha}{1-\alpha}}\delta^{\frac{\alpha\delta}{1-\alpha}}(1-\delta)^{\frac{\alpha(1-\delta)}{1-\alpha}}\omega_*^{\frac{\alpha(1-\delta)}{\alpha-1}}-\rho\right\}. \tag{A3}$$

Similarly to the first extension, (H3) uniquely determines ω_* and is not affected by the change of the R&D subsidy rate s_R. We can see that the increase in s_R does not affect g_{h*}, while the increase in s_h raises g_{A*}. Hence, previous results in the baseline model remain.

5 Conclusion

In this paper, we augmented Schumpeterian endogenous growth model to discern general-equilibrium effects of subsidy policies to R&D and human capital accumulation. We addressed a variant of Schumpeterian endogenous growth model after recent empirical findings of Zachariadis [2003], Ha and Howitt [2007]), Madsen [2008]), and Ang and Madsen [2011]. New findings in Madsen [2010] of permanent growth effects of human capital have provided direct motivations for this study, since in the Schumpeterian endogenous growth theory usually roles of human capital are not explicitly taken into account in the models.

We showed that, despite a standard intuition that subsidizing these growth-enhancing activities is always mutually growth promoting, asymmetric effects for subsidies on R&D and those on education result. Our theoretical result of asymmetric policy effects provides an important empirical caveat that empirical researchers may find false negative relationships between education subsidies and the output growth rate. Namely, when the indirect negative effect of R&D subsidies on human capital accumulation dominates the direct positive effect of education subsidies on human capital accumulation, empirical researchers may see that after a positive policy shock on the education sector, the growth rate in human capital is *decreased* while the growth rate in the output is increased. We suggest that empirical researchers should employ a model where technology improvements by R&D and human capital accumulation are considered in a unified framework in order not to miss the general equilibrium effects from one sector to the other.

Appendix

A Proof of Proposition 2

We show that the steady growth path uniquely exists. We first give the following lemma.

Lemma 1. *The values of ω_*, u_*, and g_{h*} given by* (H) *and* (U) *are uniquely determined as $\omega_* \in (0, \infty)$, $u_* \in (0, 1)$, and $g_{h*} \in (0, \infty)$.*

Proof. $g_{h*} > 0$ is straightforward from (H), since $g_N(\omega) > 0$ for all $\omega > 0$.

The existence of ω_* is also confirmed by (H) as we have that

$$\lim_{\omega \to 0} g_h\left(\frac{\omega}{1 - s_h}\right) < \lim_{\omega \to 0} g_N(\omega) \qquad \text{and} \qquad \lim_{\omega \to \infty} g_h\left(\frac{\omega}{1 - s_h}\right) > \lim_{\omega \to \infty} g_N(\omega).$$

The uniqueness of ω_* follows from the fact that the left-hand side of (H) is strictly increasing and that the right-hand side of the same equation is strictly decreasing.

Finally, we prove $u_* \in (0, 1)$. It is obvious from (U) that $u_* > 0$. In order to show that $u_* < 1$, here we define $\underline{\omega}$ such that

$$g_h\left(\frac{\underline{\omega}}{1 - s_h}\right) = 0,$$

or equivalently

$$\underline{\omega} \equiv (1 - s_h)\,\xi^{-1}(\rho) > 0.$$

Since g_h is increasing in ω, (H) shows that $\omega_* > \underline{\omega}$. At the same time, since (U) implies that u_* is decreasing in ω_*, we have

$$1 = \gamma + (1-\gamma)\frac{\rho}{\xi\left(\frac{\underline{\omega}}{1-s_h}\right)} > \gamma + (1-\gamma)\frac{\rho}{\xi\left(\frac{\omega_*}{1-s_h}\right)} = u_*.$$

□

Next, we prove Proposition 2.

Proof of Proposition 2. By Lemma 1, we see that ω_*, u_*, and g_{h*} uniquely exist with $\omega_* \in (0,\infty)$, $u_* \in (0,1)$, and $g_{h*} \in (0,\infty)$.

In order to show the existence of g_{A*} in the range of $(0,\infty)$, take λ''_A which satisfies the condition that

$$\frac{\alpha(1-\alpha)}{1-s_R} = \zeta,$$

from which we can explicitly obtain λ''_A as

$$\lambda''_A = \left[\frac{1-s_R}{\alpha(1-\alpha)}\right]^{1-\beta}\lambda_N\Gamma.$$

Let $\underline{\lambda}_A$ be $\max\left[\lambda'_A, \lambda''_A\right]$ and take any $\lambda_A \in (\underline{\lambda}_A, \infty)$. Then, for a sufficiently small value of $\rho > 0$, we obtain $g_{A*} > 0$ from (A). Define that $\bar{\rho} = \sup_{g_{A*}>0}\rho$. Then, since ω_* is increasing in ρ from (34) and (H), (A) shows that g_{A*} takes a positive value for any $\rho \in (0,\bar{\rho})$, which completes the proof. □

B Local Stability of the Steady State

Local saddle-stability of the steady growth path is proved as follows.

From (3) and the definitions of ω_t and l_t, (28) is rewritten as

$$u_t = \alpha^{\frac{2}{1-\alpha}}\Gamma^{-1}\omega_t^{\frac{-1}{1-\alpha}}l_t^{-1}. \tag{44}$$

From (14), (27), and (29), we get

$$z_{At} = (1-\zeta)\alpha^{\frac{2\alpha}{1-\alpha}}\Gamma^{-1}\omega_t^{\frac{-\alpha}{\alpha-1}} - (c_t + z_{ht})l_t. \tag{45}$$

Given the value of ω_* and u_* determined by (H) and by (U), (44) gives the steady-state value of l_* as

$$l_* = \alpha^{\frac{2}{1-\alpha}}\Gamma\omega_*^{\frac{1}{\alpha-1}}u_*^{-1}. \tag{46}$$

With the steady-state values of ω_*, u_*, and l_*, substituting (25) into (45) we have

$$c_* = (1-\zeta)\alpha^{\frac{2\alpha}{1-\alpha}}\Gamma\omega_*^{\frac{\alpha}{\alpha-1}} l_*^{-1} + \frac{\gamma(1-u_*)}{(1-\gamma)(1-s_h)}\omega_* - z_{A*} l_*^{-1}. \tag{47}$$

The system of equations for the equilibrium dynamics of the model consisting of (22), (23), and (26). Together with supplemental conditions of (9), (10), (15), (24), (25), (29), (44), and (45), we linearize the system around the steady state as

$$\begin{pmatrix} \frac{\dot{c}}{c} \\ \frac{\dot{\omega}}{\omega} \\ \frac{\dot{l}}{l} \end{pmatrix} \approx J \begin{pmatrix} c - c_* \\ \omega - \omega_* \\ l - l_* \end{pmatrix},$$

where J is the Jacobi matrix of

$$J = \begin{pmatrix} \frac{\partial r}{\partial c} - \sigma\lambda_A \frac{\partial z_A}{\partial c} & \frac{\partial r}{\partial \omega} - \sigma\lambda_A \frac{\partial z_A}{\partial \omega} - \frac{\partial g_h}{\partial \omega} & \frac{\partial r}{\partial l} - \sigma\lambda_A \frac{\partial z_A}{\partial l} - \frac{\partial g_h}{\partial l} \\ \frac{\partial r}{\partial c} - \sigma\lambda_A \frac{\partial z_A}{\partial c} & \frac{\partial r}{\partial \omega} - \sigma\lambda_A \frac{\partial z_A}{\partial \omega} - \gamma(1-\gamma)\Psi\omega_*^{\gamma-1} & \frac{\partial r}{\partial l} - \sigma\lambda_A \frac{\partial z_A}{\partial l} \\ 0 & \frac{\partial g_h}{\partial \omega} - \lambda_N \zeta^\beta \frac{\partial y}{\partial \omega} & \frac{\partial g_h}{\partial l} \end{pmatrix},$$

with $\Psi \equiv \lambda_h \gamma^\gamma (1-\gamma)^{-\gamma}(1-s_h)^{-\gamma} > 0$. After elementary row operations, J can be reduced to

$$J' = \begin{pmatrix} \frac{\partial r}{\partial c} - \sigma\lambda_A \frac{\partial z_A}{\partial c} & \frac{\partial r}{\partial \omega} - \sigma\lambda_A \frac{\partial z_A}{\partial \omega} - \frac{\partial g_h}{\partial \omega} & \frac{\partial r}{\partial l} - \sigma\lambda_A \frac{\partial z_A}{\partial l} - \frac{\partial g_h}{\partial l} \\ 0 & \frac{\partial g_h}{\partial \omega} - \gamma(1-\gamma)\Psi\omega_*^{\gamma-1} & \frac{\partial g_h}{\partial l} \\ 0 & \gamma(1-\gamma)\Psi\omega_*^{\gamma-1} - \lambda_N \zeta^\beta \frac{\partial y}{\partial \omega} & 0 \end{pmatrix}.$$

In order to show the local saddle-stability of the equilibrium dynamics, it is sufficient to prove that the determinant of J' is negative and the trace is positive. The determinant of $\det J'$ is obtained as

$$\det J' = -\left(\frac{\partial r}{\partial c} - \lambda_A \sigma \frac{\partial z_A}{\partial c}\right)\left[\gamma(1-\gamma)\Psi\omega_*^{\gamma-1} - \lambda_N \zeta^\beta \frac{\partial y}{\partial \omega}\right]\frac{\partial g_h}{\partial l},$$

where

$$\frac{\partial r}{\partial c} = \lambda_A\left(1 + \sigma + \frac{\alpha\sigma}{1-\alpha}\right) l_* > 0$$
$$\frac{\partial z_A}{\partial c} = -l_* < 0$$
$$\frac{\partial y}{\partial \omega} = \frac{\alpha}{\alpha-1}\alpha^{\frac{2\alpha}{1-\alpha}}\Gamma\omega_*^{\frac{1}{\alpha-1}} < 0,$$

and

$$\frac{\partial g_h}{\partial l} = \alpha^{\frac{2}{1-\alpha}}\Psi\Gamma\omega_*^{\gamma-\frac{1}{1-\alpha}} l_*^{-2} > 0.$$

The signs of partial derivatives in the above expressions hold that the determinant is negative.

The trace of J', in turn, can be obtained as

$$\mathrm{trace}J' = \frac{\partial r}{\partial c} - \lambda_A \sigma \frac{\partial z_A}{\partial c} + \frac{\partial g_h}{\partial \omega} - \gamma(1-\gamma)\Psi\omega_*^{\gamma-1} ,$$

where

$$\begin{aligned}\frac{\partial g_h}{\partial \omega} - \gamma(1-\gamma)\Psi\omega_*^{\gamma-1} &= \gamma\Psi\omega_*^{\gamma-1} - (\gamma - \frac{1}{1-\alpha})\alpha^{\frac{2}{1-\alpha}}\Psi\Gamma\omega_*^{\gamma-\frac{1}{1-\alpha}-1}l_*^{-1} - \gamma(1-\gamma)\Psi\omega_*^{\gamma-1} \\ &= \gamma^2\Psi\omega_*^{\gamma-1} - (\gamma - \frac{1}{1-\alpha})\alpha^{\frac{2}{1-\alpha}}\Psi\Gamma\omega_*^{\gamma-\frac{1}{1-\alpha}-1}l_*^{-1}.\end{aligned}$$

It is easy to show that this expression is always positive, since $\alpha \in (0, 1)$ and since $\gamma \in (0, 1)$. Hence, the trace of J' is positive. These results show that the steady state of the equilibrium dynamics is locally saddle stable. □

C Proof of Proposition and Theorems in the Case of CIES Preferences

To begin with, using (35) and (42), we introduce an *excess supply function* for human capital accumulation as

$$\begin{aligned}E(\omega, s_h, s_R) &\equiv g_h(\omega, s_h, s_R) - g_N(\omega) \\ &= \left[\eta - (\eta-1)\frac{\sigma}{\Gamma}\right]\xi\left(\frac{\omega}{1-s_h}\right) + \left[(\eta-1)\frac{\sigma}{\Gamma}\frac{\alpha(1-\alpha)}{1-s_R} - \zeta\right]\lambda_A\alpha^{\frac{2\alpha}{1-\alpha}}\omega^{\frac{\alpha}{\alpha-1}} - \eta\rho. \quad (48)\end{aligned}$$

In the steady state, it should be satisfied that

$$E(\omega_*, s_h, s_R) = 0. \quad (49)$$

We then have the following lemma.

Lemma 2. *If $\eta \in (0, \zeta(1-s_R)/[\alpha(1-\alpha)])$, we have*

$$\lim_{\omega\to 0} E(\omega, s_h, s_R) = -\infty \qquad \textit{and} \qquad \lim_{\omega\to\infty} E(\omega, s_h, s_R) = +\infty,$$

and

$$\frac{\partial E}{\partial \omega}(\omega, s_h, s_R) > 0 \qquad \textit{for all} \quad \omega \in (0, \infty).$$

Proof. Since it holds that $\zeta(1-s_R)/[\alpha(1-\alpha)] < (1-s_R)\zeta\Gamma/[\sigma\alpha(1-\alpha)] + 1$ and that $\eta < \zeta(1-s_R)/[\alpha(1-\alpha)]$, the second term of (48) diverges to $-\infty$ as $\omega \to 0$ and converges to 0 as $\omega \to \infty$. Therefore, since the first term of (48) is positive, (48) converges to 0 as $\omega \to 0$ and diverges to $+\infty$ as $\omega \to \infty$.

The last argument is true since

$$\frac{\partial E}{\partial \omega}(\omega, s_h, s_R) = \left[\eta - (\eta-1)\frac{\sigma}{\Gamma}\right]\xi'\left(\frac{\omega}{1-s_h}\right)\frac{1}{1-s_h} + \left[\frac{\zeta(1-s_R)}{\alpha(1-\alpha)} - (\eta-1)\frac{\sigma}{\Gamma}\right]\frac{\lambda_A}{1-s_R}\alpha^{\frac{2}{1-\alpha}}\omega^{\frac{1}{\alpha-1}} > 0.$$

□

Proof of Proposition 5. It is straightforward from Lemma 2. □

Proof of Theorem 3. By applying the implicit function theorem to (49), we have

$$\frac{\mathrm{d}\omega_*}{\mathrm{d}s_h} = -\frac{\frac{\partial E}{\partial s_h}(\omega_*, s_h, s_R)}{\frac{\partial E}{\partial \omega}(\omega_*, s_h, s_R)}, \tag{50}$$

where

$$\frac{\partial E}{\partial s_h}(\omega_*, s_h, s_R) = \left[\eta - (\eta-1)\frac{\sigma}{\Gamma}\right]\xi'\left(\frac{\omega_*}{1-s_h}\right)\frac{\omega_*}{(1-s_h)^2} > 0.$$

Notice that (50) is positive by Lemma 2. Therefore, it follows from (H1) that

$$\frac{\mathrm{d}g_{h*}}{\mathrm{d}s_h} = g_N'(\omega_*)\frac{\mathrm{d}\omega_*}{\mathrm{d}s_h} > 0.$$

Differentiating (A1) with respect to s_h, we have

$$\frac{\mathrm{d}g_{A*}}{\mathrm{d}s_h} = -\frac{\eta\sigma/\Gamma}{\eta-(\eta-1)\sigma/\Gamma}\left[\frac{\alpha(1-\alpha)}{1-s_R} - \frac{\zeta}{\eta}\right]\lambda_A\alpha^{\frac{2\alpha}{1-\alpha}}\frac{\alpha}{1-\alpha}\omega^{\frac{1}{\alpha-1}}\frac{\mathrm{d}\omega_*}{\mathrm{d}s_h} > 0. \tag{51}$$

This holds, since $g_{A*} > 0$ by assumptions which requires a parametric restriction from (A1) that

$$\frac{\alpha(1-\alpha)}{1-s_R} < \frac{\zeta}{\eta}.$$

We naturally obtain $\mathrm{d}g_{Y*}/\mathrm{d}s_R > 0$. □

Proof of Theorem 4. From (49), we have

$$\frac{\mathrm{d}\omega_*}{\mathrm{d}s_R} = -\frac{\frac{\partial E}{\partial s_R}(\omega_*, s_h, s_R)}{\frac{\partial E}{\partial \omega}(\omega_*, s_h, s_R)}, \tag{52}$$

where

$$\frac{\partial E}{\partial s_R}(\omega_*, s_h, s_R) = (\eta - 1)\frac{\sigma}{\Gamma}\frac{\alpha(1-\alpha)}{(1-s_R)^2}\lambda_A \alpha^{\frac{2\alpha}{1-\alpha}}\omega_*^{\frac{\alpha}{\alpha-1}}.$$

We show the case of $\eta < 1$ because of the space constraint. It is easy to verify the case of $\eta > 1$. Since the numerator of (50) is negative and the denominator is positive by Lemma 2, we have $\mathrm{d}\omega_*/\mathrm{d}s_R > 0$. Hence, from (H1) we have

$$\frac{\mathrm{d}g_{h*}}{\mathrm{d}s_R} = g_N'(\omega_*)\frac{\mathrm{d}\omega_*}{\mathrm{d}s_R} < 0. \tag{53}$$

We also obtain from (40) that

$$\frac{\mathrm{d}g_{A*}}{\mathrm{d}s_R} = \frac{\frac{\mathrm{d}g_{h*}}{\mathrm{d}s_R} - \eta\xi'\left(\frac{\omega_*}{1-s_h}\right)\frac{1}{1-s_h}\frac{\mathrm{d}\omega_*}{\mathrm{d}s_R}}{\eta - 1}. \tag{54}$$

Since $\mathrm{d}g_{h*}/\mathrm{d}s_R < 0$ and $\mathrm{d}\omega_*/\mathrm{d}s_R > 0$, the above equation is found to be positive.

Finally, with $\mathrm{d}\omega_*/\mathrm{d}s_R > 0$, from (31), (53), and (54), we have

$$\frac{\mathrm{d}g_{Y*}}{\mathrm{d}s_R} = \frac{\eta}{\eta - 1}\left(g_N'(\omega_*) - \xi'\left(\frac{\omega_*}{1-s_R}\right)\frac{1}{1-s_R}\right)\frac{\mathrm{d}\omega_*}{\mathrm{d}s_R} > 0.$$

□

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