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**A COORDINATION GAME MODEL OF
CHARITABLE GIVING AND
SEED MONEY EFFECT**

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A Coordination Game Model of Charitable Giving and Seed Money Effect*

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Abstract: If potential donors for a charity project possess the warm-glow properties in their preferences, we can represent their behavior with a coordination game. Accordingly, we construct a simultaneous incomplete information game model of charitable giving based on a simple global coordination game. We demonstrate that merely by the effect of seed money to shift the threshold requirement of the donations for project success downwards, the proportion of donors and the total amount of donations strictly and continuously increase with the amount of seed money. This result is partially compatible with the field experimental evidence in List and Lucking-Reiley [List, J. A., Lucking-Reiley, D., 2002. The Effects of Seed Money and Refunds on Charitable Giving: Experimental Evidence from a University Capital Campaign. *Journal of Political Economy* 110 (1), 215-233].

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1 Introduction

1.1 Charitable Giving and Coordination Game

Charitable giving is a topic that has attracted the attention of many public economists for a long time and has been studied in various aspects.¹ This paper presents a new view to this topic that charitable behavior is well represented as a coordination game. Further, we claim that if the number of potential donors is large and they have little information about other potential donors' valuations for the charity project, the global game, which is a recently developed coordination game with incomplete information, is quite suitable to represent the situation and brings several insights to the study of charitable giving. In this first subsection of Introduction, we explain the reason for this.

In recent years, it has been revealed that when considering charitable behavior, especially in large economies, the warm-glow property of preference is important. A preference with warm-glow property is typically represented by a utility function that possesses a three-dimensional domain: the private consumption, the total amount of donations from all contributors to a charity, and her own contribution to a charity. This contrasts with the classical preference represented by a utility function with only a two-dimensional domain: the private consumption, and the total amount of donations from all contributors to a charity. Warm-glow is the property that increases a donor's utility as a result of her own contribution.² This property has been developed theoretically by Andreoni (1988, 1989, 1990), and supported empirically by studies such as Palfrey and Prisbrey (1996, 1997) and Ribar and Wilhelm (2002).³ In particular, Andreoni (1988) and Ribar and Wilhelm (2002) suggest that in a large economy, the warm-glow property is crucial to collect adequate amounts of charitable contributions.

A simple simultaneous game of a capital campaign for charity confirms their finding as follows. A capital campaign constitutes a funding drive for a charity that incurs a large fixed cost to ensure project success.⁴ A campaign to collect funds to build a hospital is a case in point. If the fixed cost for the project success is very large and there are many potential donors, each contribution from each donor has almost no effect on project success. Assume a potential donor in a large economy has two choices *Donate* and *Not Donate*, and possesses a simple classical preference that depends on her own private consumption and whether the project is a success or a failure. The latter condition is equivalent to whether the total amount of donations exceeds the fixed cost or not. Assume that even if the project fails, the

¹See Andreoni (2006b) for a recent survey on the studies of charitable giving.

²Warm-glow is also called impure-altruism or joy-of-giving.

³Andreoni (2006b) contains a summary and discussions on the warm-glow property.

⁴We owe this definition of a capital campaign to Andreoni (1998).

donations are nonrefundable. Also assume that if a sufficiently large number of potential donors choose *Donate*, the project succeeds. Then her payoff is represented as follows.

	Success	Failure	
Donate	Happy	Unhappy	(1)
Not Donate	Very Happy	Neutral	

Given that a potential donor cares only for her own private consumption and project success and she has no effect on project success, she is better off choosing *Not Donate* if the project succeeds. Of course, *Not Donate* also yields greater happiness if the project fails. From the Payoff Table (1), *Not Donate* is the strictly dominant strategy and each potential donor chooses this action. It is a prisoner's dilemma game and represents the classical free-riding problem.

However, if people possess the warm-glow property in their preferences, what is the payoff table in the same capital campaign game for a large number of potential donors? It is natural to consider that the payoff table is given as follows.

	Success	Failure	
Donate	Very Happy	Unhappy	(2)
Not Donate	Happy	Neutral	

A contribution makes a donor with a warm-glow property happier than when not contributing if the project is a success.⁵ Then it has two pure strategy Nash equilibria; that is, either everybody chooses *Donate*, or everybody chooses *Not Donate*. This is a typical coordination game.

The coordination game is one of the most classical problems in game theory and has significantly improved in recent years due to Carlsson and van Damme (1993). They consider a simultaneous incomplete information coordination game where each agent has little knowledge about the type distribution, and they show the existence of a unique equilibrium with few assumptions concerning the strategies of agents. Even in the Bayesian coordination game where the type distribution is common knowledge, a unique equilibrium is difficult to obtain. Consequently, their work has attracted much attention. Their game model, known as the global game, has become popular with game theorists and has been extended in a number of different directions. Thanks to Morris and Shin (1998, 2003), it has now been extended to encompass a coordination problem in a large economy.⁶

⁵Ribar and Wilhelm (2002) find that in a large economy, the warm-glow property has a much stronger effect than the altruistic property concerned only with project success or failure in this example. Following this, the payoff to choose *Not Donate* with project success is even represented by *Neutral*.

⁶Well-known applications of the global game include debt pricing (Morris and Shin, 2004) and bank runs (Goldstein and Pauzner, 2005). Both are typical coordination problems.

If there are a sufficiently large number of potential donors with the warm-glow property of preference, donation behavior for a charity project becomes a coordination game. In the situation of simultaneous donations, the incompleteness of information on the valuation of each potential donor for the charity project is also appropriate. These are the reasons why we consider that a global game is suitable for modeling charitable behavior. We construct a simultaneous incomplete information game model of charitable giving based on a simple global coordination game, and we characterize a unique equilibrium of the game. By means of comparative static analysis, we show that the model is compatible with the empirical studies of charitable behavior, especially the field experimental evidence in List and Lucking-Reiley (2002). We discuss this in the following subsection.

Finally, in this subsection, we note that this model has the same structure as the voluntary provision of an excludable threshold public good. If a contribution sufficient to exceed a threshold is necessary to produce a public good, and if the benefit for each contributor from the public good depends on how much she pays, then this has the same representation as the capital campaign to donors with the strong warm-glow property.⁷ Hence, the results in this paper also apply to the problem of producing an excludable threshold public good under incomplete information.

1.2 Seed Money

Seed money is a preliminary fund for charity publicly announced at the time when the project is announced. Leaders directly solicited to donate before the public announcement sometimes provide this seed money. Even before Andreoni (1998) introduced this problem to public economists, it was well known among charity fund-raisers that if seed money is granted, the donations from general contributors increase.

The field experimental evidence in List and Lucking-Reiley (2002) supports this knowledge among charity fund-raisers.⁸ List and Lucking-Reiley (2002) divide 3,000 potential donors into six groups. For three groups, each with 500 potential donors, they solicit the donation of a laboratory for environmental studies to purchase a computer at a fixed cost of \$3,000 with different levels of seed money.⁹ To the first group, they announce that the seed contribution is \$300, to the second group, \$1000, and to the third group, \$2,000. Table 1 provides the result. As shown, List and Lucking-Reiley (2002) clearly demonstrate that the total amount of contributions,

⁷To represent the excludability of the public good in the Payoff Table (2), we need only change the payoff of choosing *Not Donate* with project success to *Neutral*.

⁸Other empirical studies on seed money include List and Rondeau (2003) and Potters *et al.* (2005).

⁹The other three groups of potential donors are assigned to an experiment concerning refunds.

the proportion of donors who actually contribute, and the per capita amount of contributions strictly and continuously increase according to the amount of seed money.

[Table 1 about here.]

By drawing on a global coordination game, our model demonstrates that the proportion of actual donors and the total amount of donations strictly and continuously increase with the amount of seed money. The result is compatible with List and Lucking-Reiley's (2002) field experimental work, and it supports the view that charitable giving actually involves a coordination game. The key to deriving this result is *the threshold shift effect* of seed money. This is the effect of seed money shifting the minimal requirement of donations for project success from the cost of the project itself to the amount of the cost minus the seed money. Under the global game setting, each potential donor forecasts the distribution of other potential donors' valuations for the project based on her own valuation. Once the threshold for project success is given, she predicts the proportion of actual contributors and calculates her expected payoffs in both cases of donating and not donating. When seed money shifts the threshold downwards, for some proportion of potential donors, the expected payoff by donating becomes greater than when not donating. This makes both the proportion of actual donors and the total amount of donations increase. This behavior can be observed only under global game setting of incomplete information. In an alternative Bayesian game where the distribution of potential donors' valuations is common knowledge, they often together decide to donate or not to donate in an equilibrium. This is inconsistent with the above empirical data by List and Lucking-Reiley (2002).

While this paper is the first to model charitable giving as a coordination game and to capture the threshold shift by seed money, there is a considerable theoretical literature on the effect of seed money. Andreoni (1998) considers the seed money as a contribution from leaders, and he shows that the voluntary contribution from leaders cancels the zero-contribution equilibrium and leaves only the equilibrium with a positive amount of contributions. His discussion bases on a complete information game suitable to charitable giving among a small number of potential donors, while we construct a game with a large number of potential donors under incomplete information. Vesterlund (2003) and Andreoni (2006a) direct their attention to *the quality signal effect* of seed money. They explain that seed money donated by leaders indicates the high quality of the charity and therefore increases contributions from remaining donors. If there is a quality signal effect from seed money, it is easy to derive the strict increase in donations

corresponding to the amount of seed money.¹⁰ In contrast, our model shows that even without the quality signal effect, we can derive the positive effect of seed money. Indeed, the threshold shift effect and the quality signal effect are not contradictory but rather are complementary in increasing total contributions. Our study is meaningful even if we take the quality signal effect of seed money into account.¹¹

1.3 The Structure of the Paper

The structure of the paper is as follows. Section 2 examines the basic model of charitable giving without seed money. In Subsection 2.1, we construct a model based on a simple global coordination game. We derive the equilibrium in Subsection 2.2, and examine the conditions for the efficiency and success of a project in Subsection 2.3. In Section 3, we consider the effect of seed money. In Subsection 3.1, we introduce seed money with the threshold shift effect, and we observe the impact on the behavior of potential donors. In Subsection 3.2, we calculate the minimal seed money needed to achieve success. Section 4 discusses remaining inconsistencies with List and Lucking-Reiley (2002) and their conceivable solutions. Section 5 concludes. All proofs of propositions and remarks are in the Appendix.

2 Basic Model and Results

In this section, we present a model of charitable giving that provides a foundation for future comparative static analysis. The effect of seed money is not yet considered. We present the detailed game structure in Subsection 2.1, derive a unique equilibrium in Subsection 2.2, and provide remarks on the efficiency and success of the project in Subsection 2.3.

2.1 Preliminaries

There is a charity project and $n \in \mathbb{N}$ potential donors (agents). The project has a threshold level of cost $C \in \mathbb{R}_{++}$, and if the total provision from agents exceeds the threshold C , then the quality of the project is significantly improved. Imagine, for example, a charitable campaign to promote child

¹⁰An interesting aspect of Vesterlund (2003) and Andreoni (2006a) is that the leaders donating the seed money and the amount of seed money donated are determined endogenously.

¹¹There are several other studies on the effect of seed money. Romano and Yildirim (2001) show that in a two stage game of public good provision, the announcement of the contribution in the first stage sometimes brings about larger total contributions because of the warm-glow property. Bac and Bag (2003) consider a similar question about when it is beneficial for a charity fund-raiser to reveal the number of potential donors who are actually willing to donate. Bag and Roy (2008) explain the effect of seed money by the uncertainty regarding other donors' valuations of the project.

health care in a rural area of a developing country. If the charity collects sufficient donations to build a hospital, the quality of the project increases substantially. Refer to the project as a *success* (in the ex-post sense) if the donations collected from agents are sufficient to cover C , and a *failure* (in the ex-post sense) if the total donations are less than C .

We consider cases where the number of agents are sufficiently large.¹² We adopt the assumption of a continuum of agents. To simplify the analysis, we normalize the length of the continuum of agents to 1. Let $c = \frac{C}{n}$. Under the normalized setting, c plays a role as the threshold cost instead of C . Thus we also refer to c as the threshold cost.

The charity announces the threshold cost c and solicits agents for donation. Through the announcement, each agent receives a signal of how worthy the project is in the case of both project success and failure. Based on this signal, each agent makes a decision about whether to donate. To simplify the analysis, we assume that her decision is merely a choice from her action set $\{Donate, Not\ Donate\}$ and that the amount of donation on the basis of her signal is given exogenously.¹³

Let $v_i \in \mathbb{R}$ be the signal of an agent i 's degree of warm-glow for this project. $v_i > 0$ means that that if i decides to donate and the project succeeds, she is delighted because of her warm-glow property. $v_i \leq 0$ means that she thinks that the project is not worthwhile and even if it succeeds, she does not gain anything from her own contribution.

Assume that an agent i with v_i donates $x(v_i)$ when she makes a decision *Donate*, where $x : \mathbb{R} \rightarrow \mathbb{R}_{++}$ is the same among all agents, weakly increasing, continuous and there exists an upper bound $\bar{x} \in \mathbb{R}_{++}$ such that for all $v_i \in \mathbb{R}$, $x(v_i) \leq \bar{x}$. This upper bound \bar{x} can be interpreted as the constraint of their incomes. Also assume simply that v_i works as the coefficient of $x(v_i)$ in her payoff when the project succeeds.

Any agent is assumed to take zero payoff if she does not donate, regardless of whether the project is a success or a failure. Because of the warm-glow property discussed in Introduction, when the project is a success, the payoff not to donate is smaller than that of donating. Thus we adopt the assumption of a zero payoff without donation for simplicity.

The table below summarize the payoffs.

¹²In each field experiment in List and Lucking-Reiley (2002), the number of potential donors (people who received solicitation letters) was 500.

¹³It is widely adopted as a simplification technique to limit the elements of each agent's action set to binary in the literature of global games. See, for example, Morris and Shin (2003). Moreover, even in classical models of discrete public good provision, such as Palfrey and Rosenthal (1984) and Gradstein (1994), the elements of the action set are binary limited to reduce complexity.

	Success	Failure
Donate	$v_i \cdot x(v_i)$	$-x(v_i)$
Not Donate	0	0

(3)

Based on this ex-post payoff table, each agent is assumed to calculate her own von Neumann-Morgenstern expected interim payoff, and to decide whether to donate.¹⁴

Let us introduce the signal distribution structure and information setting for agents. Assume that the signal for each agent v_i follows a normal distribution with mean θ and standard deviation σ independently and identically. Given that the density function for a normal distribution approximates a wide class of symmetric density functions with a peak, this assumption is adequate.¹⁵ Let $F(\cdot; \theta)$ denote the normal cumulative distribution function with mean θ and standard deviation σ , and let $f(\cdot; \theta)$ denote its probability density function.

Following Morris and Shin (2003), we assume that it is common knowledge among agents that each signal v_i for an agent i is independently, identically, and normally distributed with standard deviation σ . However, each agent i has no information about the mean θ . She has to predict the mean θ based on her realized signal v_i . We assume that her prior belief for θ is uniformly distributed over the real line \mathbb{R} . This prior belief is improper as the total probability mass is infinite; however, the posterior belief through Bayesian estimation is well defined.¹⁶ She believes, based on her signal v_i , that θ follows the normal distribution with mean v_i and standard deviation σ . This is to say, from her point of view, $F(\cdot; v_i)$ is the cumulative distribution function of the actual mean θ , and $f(\cdot; v_i)$ is its probability density function. It is an interesting aspect of this improper uniform prior belief that the same function is available to represent both the distribution of real signals and the distribution of the predictable mean based on the belief of a single agent.

¹⁴We can extend the model to consider the case where an agent who decides to donate takes a refund if the project is in failure. For example, let $r \in (0, 1]$ and assume $-r \cdot x(v_i)$ as i 's payoff when she decides to donate and the project is in failure. r represents the rate of her losses including those incurred by waist of time and negative emotional influence. Even with this change in payoff, the structure of the following investigation and the consequences are unchanged. A detailed explanation is given in the Supplementary Note attached to the discussion paper version of this study (Hatsumi, 2011). However, note that in the field experiments in List and Lucking-Reiley (2002), the refund policy did not affect the proportion of actual contributors.

¹⁵In the field experiments in List and Lucking-Reiley (2002), all potential donors met two criteria: (i) the household's annual income was above \$70,000, and (ii) the household was known to have previously given to a charity. In this case, the standard deviation σ is relatively small.

¹⁶See Morris and Shin (2003) for a detailed explanation. They discuss the improper uniform prior belief with respect to the philosophy of Laplace.

In comparison with a Bayesian game wherein the signal distribution is common knowledge among agents, the assumption that the posterior belief of signal distribution for each agent is slightly different from those of others is quite natural. This is because of the following reasons. First, in a large economy where a continuum can approximate the total agents, if the distribution of signals is common knowledge, it almost represents a complete information game. This appears unrealistic. Second, especially when agents are relatively homogeneous (*i.e.*, σ is small) as in the field experiments in List and Lucking-Reiley (2002), it is reasonable that each agent predicts the signal distribution on the basis of her own signal.

A strategy for an agent i is a function mapping her receiving signal v_i to her action set $\{Donate, Not\ Donate\}$. If both actions occasion the equivalent expected payoff for i , we assume that she selects *Not Donate*. This assumption is just to simplify the description, and the opposite assumption is, of course, available.

At this point, we define a Bayesian type of incomplete information game. Let G denote this game. In the next subsection, we consider the equilibrium in this game.

2.2 The Equilibrium

We are interested in Bayesian Nash equilibria of the game defined in the previous subsection. In this subsection, we characterize a unique Bayesian Nash equilibrium under the assumption that all strategies of agents have symmetry and switching properties. A strategy is symmetric if all agents follow the same strategy. A strategy is a switching strategy around some cutoff point $k \in \mathbb{R}$ if there exists a unique cutoff point k such that if an agent's receiving signal v_i is above k , she donates; otherwise, she does not donate. A symmetric switching strategy $s : \mathbb{R} \rightarrow \{Donate, Not\ Donate\}$ is formally written as follows.

$$s(v_i) = \begin{cases} Donate & \text{if } v_i > k \\ Not\ Donate & \text{if } v_i \leq k \end{cases}$$

First, let us introduce two functions that play important roles in the following investigation.

$$\int_k^\infty f(v; \theta) dv \tag{4}$$

$$\int_k^\infty x(v) f(v; \theta) dv \tag{5}$$

(4) represents the proportion of agents whose signal is larger than k when the mean signal is θ . This function is useful no matter what strategies are adopted. (5) represents the total donation when the mean of the signals is θ and any agent receiving a signal larger than the cutoff point k decides

to donate. (5) is meaningful only when any agent adopts the switching strategy.

At the beginning of the investigation, notice the remark below.

Remark 1. For any agent i with $v_i \leq 0$, *Not Donate* is the strictly dominant action.

This remark is straightforward from the payoff tables of the previous section. Hereafter, we focus on the actions of agents with signals larger than 0. Note that the interim payoff of an agent i with a positive signal $v_i > 0$ is represented by

$$\Pr(\text{Success})(v_i \cdot x(v_i)) + (1 - \Pr(\text{Success}))(-x(v_i)). \quad (6)$$

Thus, agent i decides to donate if

$$\begin{aligned} (6) &> 0 \\ \iff \Pr(\text{Success}) &> \frac{1}{v_i + 1}. \end{aligned}$$

Let a function $p : \mathbb{R}_{++} \rightarrow (0, 1)$ be such that $p(v_i) = \frac{1}{v_i + 1}$. $p(v_i)$ represents the threshold probability for i with $v_i > 0$ in the sense that if the probability of project success is larger than $p(v_i)$, *Donate* leads to a higher expected payoff than *Not Donate*. On the other hand, if the probability of project success is less than $p(v_i)$, *Not Donate* guarantees a higher expected payoff than *Donate*. Then $p(\cdot)$ has the following properties obviously.

Remark 2. $p(\cdot)$ is strictly decreasing and continuous in $v_i > 0$.

[Figure 1 about here.]

Let $\hat{\theta} : \mathbb{R}_{++} \rightarrow \mathbb{R}$ be a function such that for $v_i > 0$,

$$\int_{\hat{\theta}(v_i)}^{\infty} f(v; v_i) dv = p(v_i).^{17}$$

See Figure 1 for an illustration. Remember that an agent i with a signal v_i believes the mean of normally distributed signals to be v_i itself. Thus the equation represents that i considers that the probability of the mean of signal distribution being larger than $\hat{\theta}(v_i)$ is $p(v_i)$ (*i.e.*, i considers $\Pr(\theta \geq \hat{\theta}(v_i)) = p(v_i)$). Since $p(\cdot)$ is strictly decreasing in $v_i > 0$ by Remark 2, we have the remark below.

¹⁷In the explicit form, $\hat{\theta}(v_i) = F^{-1}(1 - p(v_i); v_i)$, where $F^{-1}(\cdot; v_i)$ is the inverse function of $F(\cdot; v_i)$. This expression is rarely used in the following investigation.

Remark 3. (i) $\hat{\theta}(v_i)$ is strictly increasing and continuous in $v_i > 0$. (ii) For $v'_i > v_i > 0$, $\hat{\theta}(v'_i) - \hat{\theta}(v_i) > v'_i - v_i$. (iii) $\lim_{v \rightarrow +\infty} \hat{\theta}(v) = +\infty$. (iv) $\lim_{v \rightarrow 0} \hat{\theta}(v) = -\infty$.

We introduce two more functions and discuss their properties.

$$\int_k^\infty x(v)f(v; \hat{\theta}(v_i))dv \quad (7)$$

(7) is a special case of (5), and the mean of signals is $\hat{\theta}(v_i)$. Since $\hat{\theta}(v_i)$ is strictly increasing in $v_i > 0$, the following remark is straightforward.

Remark 4. (7) is strictly increasing and continuous in $v_i > 0$ and strictly decreasing and continuous in $k > 0$.

$$\int_{v_i}^\infty x(v)f(v; \hat{\theta}(v_i))dv \quad (8)$$

(8) is a special case of (7). The mean of the signals is $\hat{\theta}(v_i)$ and the cutoff point $k = v_i$. Remark 3 induces the next remark.

Remark 5. (8) is strictly increasing and continuous in $v_i > 0$, and as $v_i \rightarrow 0$, (8) $\rightarrow 0$.

Now, we characterize a cutoff point and the equilibrium in a game G . Given a threshold cost c , let $k^* \in \mathbb{R}_{++}$ be such that

$$\int_{k^*}^\infty x(v)f(v; \hat{\theta}(k^*))dv = c.$$

Note that by Remark 5, k^* is uniquely determined.

Proposition 1. *Let a symmetric switching strategy $s^*(\cdot)$ be such that*

$$s^*(v_i) = \begin{cases} \text{Donate} & \text{if } v_i > k^* \\ \text{Not Donate} & \text{if } v_i \leq k^*. \end{cases}$$

Then, in a game G , $s^(\cdot)$ is a unique symmetric switching strategy constituting a Bayesian Nash equilibrium.*

Note that from the definition of k^* , the actual mean of the signal distribution θ has no effect in determining the equilibrium cutoff point k^* . In contrast, $\hat{\theta}(\cdot)$ is quite important in determining k^* . The example below illustrates this proposition.

Example 1. Let a threshold level of cost $c = 6$.¹⁸ Let the standard deviation of the signal distribution $\sigma = 3$. Let for all $v_i \in \mathbb{R}$, $x(v_i) = 10$. Then, we calculate $\hat{\theta}(k^*) = 2.26$ and $k^* = 1.50$, and have Table 2.

¹⁸In each field experiment in List and Lucking-Reiley (2002), the threshold level of project cost for success is \$3,000 and the number of potential donors is 500 (*i.e.*, $C = 3,000$ and $n = 500$). In this case, $c = C/n = 6$.

[Table 2 about here.]

Note that the proportion of actual contributors is represented by

$$\int_{k^*}^{\infty} f(v; \theta) dv \quad (9)$$

and the total donation is represented by

$$\int_{k^*}^{\infty} x(v) f(v; \theta) dv. \quad (10)$$

The project is a success if (10) is larger than c .

As in Example 1, the equilibrium cutoff point k^* is strictly larger than 0. This is because of a particular assumption of the global game that each agent possesses a different belief about the signal distribution. Under a Bayesian game model where the signal distribution and the strategies of other agents constitute common knowledge, all agents with signals larger than 0 decide together whether to donate or not depending on the actual mean of the signals θ in an equilibrium. Under the same setting as Example 1, this alternative Bayesian game analysis provides this in an equilibrium; if $\theta > 2.26$, agents with signals larger than 0 together decide to donate, and if $\theta \leq 2.26$, any agent does not donate and the total amount of donation is equal to zero. This prediction is incompatible with the empirical research such as List and Lucking-Reiley (2002). It suggests the effectiveness of global game analysis in the study of charitable giving.¹⁹

2.3 Conditions for Efficiency and Success

In this subsection, we first consider the condition that achieves the ex-post Pareto efficiency of the donation for agents. Next, we consider the condition for the success of the project.

We refer to a project as *efficient* if in the equilibrium, the actions of agents derive ex-post Pareto efficiency. Formally, this is represented when $\int_0^{\infty} x(v) f(v; \theta) dv \geq c$, any agent i with signal $v_i > 0$ chooses *Donate*, and any agent j with signal $v_j \leq 0$ chooses *Not Donate*, otherwise, all agents select *Not Donate*. This is achievable in an equilibrium of the alternative Bayesian Game discussed in the last paragraph of the previous subsection. However, it is never obtained under the more realistic global game setting.

¹⁹In the literature of global games, the strategy profile surviving the iterated elimination of interim strictly dominated strategies is widely used as the equilibrium concept. See, for example, Morris and Shin (2003). However, in the present model, to derive the same equilibrium to Proposition 1 under this stronger equilibrium concept, we need an additional outlandish assumption that any agent with sufficiently large signal has the strictly dominant action to donate. A detailed explanation is given in the Supplementary Note attached to the discussion paper version of this study (Hatsumi, 2011).

Proposition 2. *Any project is inefficient.*

The intuition for this proposition is quite simple. Even though the project is a failure, some agents with signals larger than k^* choose *Donate*. This implies that the project is not efficient. If the project is a success, an agent i with $0 < v_i < k^*$ chooses *Not Donate*, which brings about inefficiency. Therefore, there is no condition that occasions efficiency.

As already mentioned in the previous subsection, a project is a success if

$$\int_{k^*}^{\infty} x(v)f(v;\theta)dv \geq c. \quad (11)$$

This may be the most important matter for a charity fund-raiser. The proposition below states the condition for project success.

Proposition 3. *A project is a success if and only if $\theta \geq \hat{\theta}(k^*)$.*

Since $\int_{k^*}^{\infty} x(v)f(v;\hat{\theta}(k^*))dv = c$, $\theta \geq \hat{\theta}(k^*)$ induces $\int_{k^*}^{\infty} x(v)f(v;\theta)dv \geq c$ and the project succeeds. If $\theta < \hat{\theta}(k^*)$, then $\int_{k^*}^{\infty} x(v)f(v;\hat{\theta}(k^*))dv = c > \int_{k^*}^{\infty} x(v)f(v;\theta)dv$. This induces project failure.

In the next section, we introduce seed money. The main purpose for introducing seed money is to obtain the success of a project. We show how to calculate the minimal amount of seed money for success.

3 Seed Money

We exogenously introduce seed money. We see the threshold shift effect of the seed money on the donation behavior of agents in Subsection 3.1. We also see how to obtain the success of the project by announcing seed money in Subsection 3.2.

3.1 The Threshold Shift Effect

Let $L \in [0, C)$ denote the amount of seed money. $L = 0$ represents the state where no seed money is granted. If seed money L is granted, the threshold level for project success shifts from C to $C - L$. Assume that information about the project never changes except concerning the existence and amount of seed money. We consider that all other structures of the game except for the shift of the threshold for project success remain as before.

Let ℓ denote the normalized version of seed money; *i.e.*, $\ell = \frac{L}{n}$. Consequently, the normalized version of the shifted threshold is $c - \ell$.

Let $G(\ell)$ denote a Bayesian game with the same structure as that in Subsection 2.1 with the shift of threshold level from c to $c - \ell$. Then we derive a unique equilibrium strategy $s^*(\cdot; \ell)$ in a game $G(\ell)$ as a corollary of Proposition 1.

Corollary 1. *In a game $G(\ell)$, a unique symmetric switching strategy $s^*(\cdot; \ell)$ constituting a Bayesian Nash equilibrium is such that*

$$s^*(v_i; \ell) = \begin{cases} \text{Donate} & \text{if } v_i > k^*(\ell) \\ \text{Not Donate} & \text{if } v_i \leq k^*(\ell), \end{cases}$$

where $k^*(\ell) \in \mathbb{R}_{++}$ is such that

$$\int_{k^*(\ell)}^{\infty} x(v) f(v; \hat{\theta}(k^*(\ell))) dv = c - \ell.$$

According to Corollary 1, we have the remark below.

Remark 6. $k^*(\ell)$ is strictly decreasing and continuous in ℓ , and $\lim_{\ell \rightarrow c} k^*(\ell) = 0$.

From Remark 6, we derive a proposition below that is quite compatible with the field experiments in List and Lucking-Reiley (2002).

Proposition 4. *If a granted amount of seed money ℓ strictly increases, then both the proportion of agents who decide to donate and the total amount of donations strictly increase.*

The example below illustrates this proposition.

Example 2. Similarly to Example 1, let a threshold level of cost $c = 6$, a standard deviation of signal distribution $\sigma = 3$, and for all $v_i \in \mathbb{R}$, $x(v_i) = 10$. Let the actual mean of signal distribution $\theta = 0$. Consequently, we have the results in Table 3.

[Table 3 about here.]

The mechanism behind Proposition 4 is quite simple and only depends on Remark 6. If $k^*(\ell)$ decreases, it is obvious that the number of agents with signals larger than the cutoff point $k^*(\ell)$ strictly increases, and then the proportion of agents who decide to donate and the total amount of donations also strictly increase. As discussed in the last part of Subsection 2.2, the emergence of the cutoff point $k^*(\ell)$ strictly larger than 0 is due to the global game assumption that the actual signal distribution is not common knowledge but is noisily observed and guessed by each agent i on the basis of her own signal v_i . This realistic setting of the global game is the key to the derivation of Proposition 4.

3.2 Minimal Seed Money for Success

The main purpose for introducing seed money is to obtain the efficiency or success of a project that is not achieved in its absence. Unfortunately, by Proposition 2, efficiency is never obtained by introducing any amount of seed money. Hence, in this subsection, we only focus on the minimal amount of seed money to success.

From the viewpoint of a charity fund-raiser, the attainment of project success is crucial. Even though we do not explicitly consider the cost of introducing seed money, charity fund-raisers actually require a considerable effort to collect seed money and are better off if the amount of seed money needed for success is as small as possible.

The project is not a success without seed money if $\theta < \hat{\theta}(k^*(0))$. In this case, in order to achieve success, we have the following proposition.

Proposition 5. *When $\theta < \hat{\theta}(k^*(0))$, the minimal amount of seed money for success ℓ^* is such that $\hat{\theta}(k^*(\ell^*)) = \theta$.*

Note that from Remarks 3 and 6, $\hat{\theta}(k^*(\ell))$ is continuously and strictly decreasing in $\ell \in [0, \bar{\ell}]$ and $\hat{\theta}(k^*(\ell)) \rightarrow -\infty$ as $\ell \rightarrow c$. Thus ℓ^* is uniquely determined. In the case of Example 2, $\ell^* = 1.91$.

To calculate the minimal seed money for success, a charity fund-raiser needs to know the actual mean of the distribution θ . Indeed, the collection of the actual signal distribution is quite difficult for a charity fund-raiser. It involves a considerable cost for preliminary surveys on the potential donors to approach the actual mean θ , and it is very beneficial.

4 Discussion

In the previous section, we establish Proposition 4, which is quite compatible with the empirical result in List and Lucking-Reiley (2002). In this section, we discuss the still remaining inconsistency of our model with List and Lucking-Reiley (2002). In our model, the per-capita amount of donations weakly decreases along with the increase of seed money, whereas the per-capita amount of donations strictly increases corresponding to the increase of seed money in List and Lucking-Reiley (2002). We discuss the source of this inconsistency and propose some ideas for its resolution.

The mechanism behind this inconsistency is quite simple. We assume that the amount of donation $x(v_i)$ of agent i with signal v_i is exogenously given and never changes to the amount of seed money. From Remark 6, if the amount of seed money increases, the threshold signal $k^*(\ell)$ decreases. Therefore, an agent who changes her action from *Not Donate* to *Donate* because of the increase in seed money has a smaller signal than agents who continue to choose to *Donate*. Then, since the amount of donation $x(v_i)$

weakly increases in her signal v_i , the per-capita amount of donation should weakly decrease in the amount of seed money.²⁰

To remove this inconsistency in the extended version of our model, we need to increase $x(v_i)$ according to the increase of seed money. Two approaches are considerable for this purpose.

The first approach is to render $x(v_i)$ endogenously determined as an optimal value corresponding to the probability of success. This maintains the assumption that the effect of seed money is only the threshold shift effect. As the probability of success depends on the amount of donation obtained from other agents, this optimization problem is highly complicated. At present, we are unsure about whether this approach makes sense.

The second approach entails the introduction of the quality signal effect of seed money. The increase of $x(v_i)$ corresponding to the increase in seed money can be considered as the consequence of the quality signal effect. For example, consider the simple introduction of the quality signal effect as follows. Let θ be the mean of each agent's signal, similarly to Section 2. Let $\theta : [0, c) \rightarrow \mathbb{R}$ be a strictly increasing and continuous function such that $\theta(0) = \theta$. Assume that when seed money ℓ is granted, the mean of signal θ is shifted to $\theta(\ell)$. Moreover, assume that the standard deviation σ remains the same. We interpret this shift of θ to $\theta(\ell)$ as indicating that an agent i receiving a signal v_i when no seed money is granted receives a signal $v_i(\ell) = v_i + (\theta(\ell) - \theta)$ when seed money ℓ is granted. It is obvious that in this model, the inconsistency regarding the per-capita amount of donation is resolved.²¹

The second explanation also suggests the possibility of constructing a model in which the threshold shift effect and the quality signal effect are complementary. Which effect actually works remains an interesting question in both the theoretical and empirical work.

5 Concluding Remarks

In this paper, we construct a global coordination game model of charitable giving. We show that by merely considering the threshold effect of seed money, the proportion of agents who decide to donate and the total amount of donations strictly and continuously increase according to the increase in seed money.

We construct a simultaneous model for comparison with the empirical work in List and Lucking-Reiley (2002). However, the actual capital campaigns of charities usually allow agents to donate in a certain period, and

²⁰The assumption that the function $x(\cdot)$ is weakly increasing is crucial for the uniqueness of the equilibrium. See the proof of Remark 5.

²¹It may be more sophisticated in the global game framework to represent the quality signal of seed money by the *public signal*, which is a signal that all agents can observe in the same way. See Morris and Shin (2003) for the details.

possibly represented by dynamic games. Even in dynamic models, our view that charitable donation is indeed a coordination game plays an important role.²² We hope that this paper encourages further study of donation behavior and thereby assists future charity projects.

Appendix

In the Appendix, we provide the proofs of the Propositions and Remarks. We omit the proofs of Remarks 1, 2 and 4 since they are straightforward.

Proof of Remark 3. Remember that $\int_{\hat{\theta}(v_i)}^{\infty} f(v; v_i) dv = p(v_i)$ for $v_i > 0$. Since $p(\cdot)$ is strictly decreasing in $v_i > 0$ by Remark 2, for $v'_i > v_i > 0$,

$$\int_{\hat{\theta}(v_i)}^{\infty} f(v; v_i) dv = p(v_i) > p(v'_i) = \int_{\hat{\theta}(v'_i)}^{\infty} f(v; v'_i) dv.$$

The continuity of $p(\cdot)$ implies (i) $\hat{\theta}(v_i)$ is strictly increasing and continuous in $v_i > 0$.

Note that if $p(v_i) > p(v'_i)$, $\hat{\theta}(v'_i) - \hat{\theta}(v_i) > v'_i - v_i$. (See Figure 2 for an illustration.) This implies (ii) for $v'_i > v_i > 0$, $\hat{\theta}(v'_i) - \hat{\theta}(v_i) > v'_i - v_i$.

$\lim_{v \rightarrow +\infty} p(v) = 0$, $\lim_{v \rightarrow 0} p(v) = 1$, and $\int_{\hat{\theta}(v_i)}^{\infty} f(v; v_i) dv = p(v_i)$ implies (iii) $\lim_{v \rightarrow +\infty} \hat{\theta}(v) = +\infty$ and (iv) $\lim_{v \rightarrow 0} \hat{\theta}(v) = -\infty$. \square

[Figure 2 about here.]

Proof of Remark 5. Continuity of (8) is obvious. Let $v' > v > 0$. Since $\hat{\theta}(v'_i) - \hat{\theta}(v_i) > v'_i - v_i$ by Remark 3, $\int_{v_i}^{\infty} f(v; \hat{\theta}(v_i)) dv < \int_{v'_i}^{\infty} f(v; \hat{\theta}(v'_i)) dv$. Since $x(\cdot)$ is a weakly increasing function, it implies that

$$\int_{v_i}^{\infty} x(v) f(v; \hat{\theta}(v_i)) dv < \int_{v'_i}^{\infty} x(v) f(v; \hat{\theta}(v'_i)) dv.$$

Thus (8) is strictly increasing in $v > 0$. When $v_i \rightarrow 0$, by Remark 3, $\hat{\theta}(v_i) \rightarrow -\infty$. Since $x(v)$ has an upper bound \bar{x} , (8) $\rightarrow 0$ as $v_i \rightarrow 0$. \square

²²Famous dynamic models of investments in projects include Admati and Perry (1991) and Marx and Matthews (2000). Dynamic models of investment in projects in the global game framework include, for example, Heidhues and Melissas (2006), Dasgupta (2007), and Barbieri and Mattozzi (2009).

Proof of Proposition 1. First, we show that $s^*(\cdot)$ is a symmetric switching strategy constituting a Bayesian Nash equilibrium. Suppose that all agents except i follow the strategy $s^*(\cdot)$. By the payoff matrix of this model, if $v_i \leq 0$, it is obvious that she has no incentive to donate. We focus on the case of $v_i > 0$.

Note that for any $\theta > \hat{\theta}(k^*)$,

$$\int_{k^*}^{\infty} x(v)f(v; \theta)dv > \int_{k^*}^{\infty} x(v)f(v; \hat{\theta}(k^*))dv = c, \quad (12)$$

and for any $\theta' \leq \hat{\theta}(k^*)$,

$$\int_{k^*}^{\infty} x(v)f(v; \theta')dv \leq \int_{k^*}^{\infty} x(v)f(v; \hat{\theta}(k^*))dv = c. \quad (13)$$

This means that under the assumption of that all agents except i follow the strategy $s^*(\cdot)$, if the actual mean of the signals is larger than $\hat{\theta}(k^*)$, the project is a success, and if it is smaller than $\hat{\theta}(k^*)$, the project is a failure. Also note that for any $v_i > k^*$,

$$\int_{\hat{\theta}(k^*)}^{\infty} f(v; v_i)dv > \int_{\hat{\theta}(k^*)}^{\infty} f(v; k^*)dv = p(k^*), \quad (14)$$

and for any $v'_i \leq k^*$,

$$\int_{\hat{\theta}(k^*)}^{\infty} f(v; v'_i)dx \leq \int_{\hat{\theta}(k^*)}^{\infty} f(v; k^*)dv = p(k^*). \quad (15)$$

This means that i with a signal $v_i > k^*$ forecasts $\Pr(\theta > \hat{\theta}(k^*)) > p(k^*)$, and that with a signal $v'_i \leq k^*$ forecasts $\Pr(\theta \leq \hat{\theta}(k^*)) \leq p(k^*)$.

Thus, (12) and (14) together imply that an agent i with signal $v_i > k^*$ has an incentive to donate, and (13) and (15) together imply that an agent i with signal $v'_i \leq k^*$ has no incentive to donate.

Therefore, $s^*(\cdot)$ surely constitutes an Bayesian Nash equilibrium.

Next, we show the uniqueness of $s^*(\cdot)$ under the assumption that strategies have symmetric and switching properties.

First, suppose, on the contrary, that a symmetric switching strategy $s(\cdot)$ with cutoff point $k > k^*$ constitutes a Bayesian Nash equilibrium. In this case, by Remark 5, $\int_k^{\infty} x(v)f(v; \hat{\theta}(k))dv > c$. Thus, by that $\hat{\theta}(\cdot)$ is continuous and Remark 4, for a small $\epsilon > 0$,

$$\int_k^{\infty} x(v)f(v; \hat{\theta}(k - \epsilon))dv > c. \quad (16)$$

This means that if the realized mean of signals $\theta \geq \hat{\theta}(k - \epsilon)$ and any agent with a signal larger than k donates, the project is a success. Note that

$$\int_{\hat{\theta}(k-\epsilon)}^{\infty} f(v; k - \frac{\epsilon}{2})dv > \int_{\hat{\theta}(k-\epsilon)}^{\infty} f(v; k - \epsilon)dv \quad (17)$$

$$\begin{aligned}
&= p(k - \epsilon) && \text{(by the definition of } \hat{\theta}(\cdot)\text{)} \\
&> p(k - \frac{\epsilon}{2}). && \text{(by Remark 2)}
\end{aligned}$$

This means that an agent with signal $k - \frac{\epsilon}{2}$ predicts $\Pr(\theta \geq k - \epsilon) > p(k - \frac{\epsilon}{2})$. Therefore, (16) and (17) together imply that an agent with signal $v_i = k - \frac{\epsilon}{2}$ has an incentive to donate. This is a contradiction.

Second, suppose, on the contrary, that a symmetric switching strategy $s(\cdot)$ with a cutoff point $k < k^*$ constitutes a Bayesian Nash equilibrium.

In this case, by Remark 5, $\int_k^\infty x(v)f(v; \hat{\theta}(k))dv < c$. Thus, by Remark 4, for a small $\epsilon > 0$,

$$\int_k^\infty x(v)f(v; \hat{\theta}(k + \epsilon))dv < c. \quad (18)$$

This means that if the realized mean of the signals $\theta \leq \hat{\theta}(k + \epsilon)$ and any agent with a signal smaller than k does not donate, the project is a failure. Note that

$$\begin{aligned}
\int_{\hat{\theta}(k+\epsilon)}^\infty f(v; k + \frac{\epsilon}{2})dv &< \int_{\hat{\theta}(k+\epsilon)}^\infty f(v; k + \epsilon)dv && (19) \\
&= p(k + \epsilon) && \text{(by the definition of } \hat{\theta}(\cdot)\text{)} \\
&< p(k + \frac{\epsilon}{2}). && \text{(by Remark 2)}
\end{aligned}$$

This means that an agent with signal $k + \frac{\epsilon}{2}$ predicts $\Pr(\theta \geq k + \epsilon) < p(k + \frac{\epsilon}{2})$. Therefore, (18) and (19) together imply that an agent with signal $k + \frac{\epsilon}{2}$ has no incentive to donate. This is a contradiction.

By the above two cases, we have that $s^*(\cdot)$ with cutoff point k^* is a unique symmetric switching strategy for a Bayesian Nash equilibrium. \square

Proof of Proposition 2. The proof is given in the paragraph after this proposition. \square

Proof of Proposition 3. The proof is given in the paragraph after this proposition. \square

Proof of Remark 6. Let a small $\epsilon > 0$. Then, from Corollary 1, we have

$$\int_{k^*(\ell)}^\infty x(v)f(v; \hat{\theta}(k^*(\ell)))dv = c - \ell \quad (20)$$

and

$$\int_{k^*(\ell+\epsilon)}^\infty x(v)f(v; \hat{\theta}(k^*(\ell+\epsilon)))dv = c - (\ell + \epsilon). \quad (21)$$

By subtracting both sides of (21) from those of (20), we have

$$\int_{k^*(\ell)}^\infty x(v)f(v; \hat{\theta}(k^*(\ell)))dv - \int_{k^*(\ell+\epsilon)}^\infty x(v)f(v; \hat{\theta}(k^*(\ell+\epsilon)))dv = \epsilon > 0. \quad (22)$$

Remark 5 and (22) imply $k^*(\ell) > k^*(\ell + \epsilon)$. Hence, $k^*(\ell)$ is strictly decreasing and continuous in ℓ . \square

Proof of Proposition 4. Note that the proportion of agents who decide to donate is represented by $\int_{k^*(\ell)}^{\infty} f(v; \theta) dv$ and the total amount of donations is given by $\int_{k^*(\ell)}^{\infty} x(v) f(v; \theta) dv$. Since both are strictly decreasing in $k^*(\ell)$, Remark 6 induces this proposition. \square

Proof of Proposition 5. Let $\ell \geq \ell^*$. Remark 6 implies that $k^*(\ell) \leq k^*(\ell^*)$. Thus we have $\hat{\theta}(k^*(\ell)) \leq \hat{\theta}(k^*(\ell^*)) = \theta$. Since Corollary 1 states that

$$\int_{k^*(\ell)}^{\infty} x(v) f(v; \hat{\theta}(k^*(\ell))) dv = c - \ell,$$

$\hat{\theta}(k^*(\ell)) \leq \theta$ implies that

$$\int_{k^*(\ell)}^{\infty} x(v) f(v; \theta) dv \geq c - \ell.$$

This occasions project success. Let $\ell' < \ell^*$. Then the symmetric reasoning brings $\int_{k^*(\ell')}^{\infty} x(v) f(v; \theta) dv < c - \ell'$. This induces project failure. \square

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Table 1: a Result of the Field Experiment by List and Lucking-Reiley (2002)

A. Experimental Design			
Number of solicitations mailed	500	500	500
Seed money (%)	10%	33%	67%
Seed money (\$)	\$300	\$1,000	\$2,000
B. Result			
Number of contributions	17	33	42
Participation rate	3.40%	6.60%	8.40%
Total contributions	\$202	\$805	\$1,485
Mean amount given	\$11.88	\$24.39	\$35.36
Standard error of mean amount	\$2.27	\$2.50	\$2.26

Source: Table 1 of List and Lucking-Reiley (2002)

Table 2: the Results of Example 1

θ	proportion of donors	total donation	success/failure
5	0.88	8.78	s
2.5	0.63	6.31	s
0	0.31	3.09	f
-2.5	0.09	0.91	f

Table 3: the Results of Example 2

ℓ	k^*	$\hat{\theta}(k^*)$	proportion of donors	total donation	success/failure
0	1.50	2.26	0.31	3.09	f
0.5	1.22	1.60	0.34	3.42	f
1	1.00	1.00	0.37	3.69	f
1.5	0.82	0.44	0.39	3.99	f
2	0.67	-0.09	0.41	4.12	s

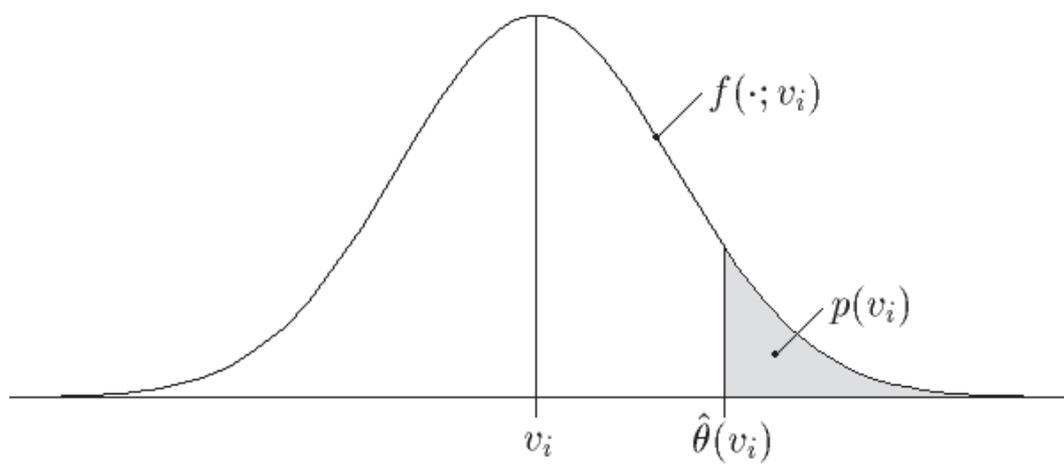


Figure 1: The relationship between v_i , $p(v_i)$, and $\hat{\theta}(v_i)$

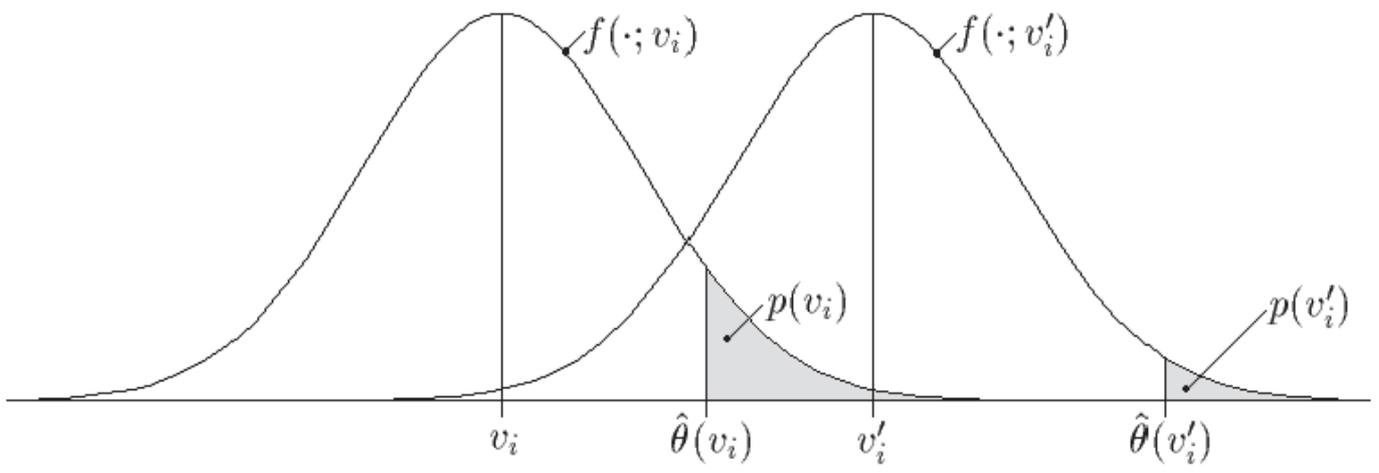


Figure 2: Illustration for the proof of Remark 3

Supplementary Note for “A Coordination Game Model of Charitable Giving and Seed Money Effect”

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In this supplementary note, (I) we first show that an alternative payoff structure that captures a refund policy instead of that in Payoff Table (3) is available with the same consequences. (II) Next, we show that with additional two assumptions, the equilibrium in Proposition 1 can be derived as a unique strategy profile surviving the iterated elimination of interim strictly dominated strategies.

(I) Assume that the payoff table is

	Success	Failure	
Donate	$v_i \cdot x(v_i)$	$-r \cdot x(v_i)$	(23)
Not Donate	0	0	

where $r \in (0, 1]$. r represents loss ratio when a refund is allowed. $r > 0$ means that even if 100% refund is possible, disutility occurs because of waist of time or negative emotional influence.

To derive the equivalent result to Proposition 1, it is sufficient to show the equivalent result to Remark 2 because this is the key to the uniqueness of the equilibrium.

Given Payoff Table (23), an agent i with a positive signal $v_i > 0$ chooses to donate if

$$\begin{aligned} & \Pr(\text{Success})(v_i \cdot x(v_i)) + (1 - \Pr(\text{Success}))(-r \cdot x(v_i)) > 0 \\ \iff & \Pr(\text{Success}) > \frac{r}{v_i + r}. \end{aligned}$$

Let a function $\bar{p} : \mathbb{R}_+ \rightarrow \mathbb{R}_{++}$ be such that $\bar{p}(v_i) = \frac{r}{v_i + r}$. Then obviously the following remark holds.

Remark 2. $\bar{p}(\cdot)$ is strictly decreasing and continuous in $v_i \geq 0$.

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Because of Remark 2', for this alternative payoff table, we can derive the equivalent results to Remarks 3 to 5 and Proposition 1 by using the same proofs.

(II) Next, we show that the strategy $s^*(\cdot)$ defined in Proposition 1 constitutes a unique strategy profile surviving the iterated elimination of interim strictly dominated strategies. Instead of the assumption that symmetry and switching properties of agents' strategies are common knowledge, we need two additional assumptions.

We require a similar property to Remark 1 that for any agent i with a sufficiently large signal v_i , *Donate* is the strictly dominant action.

Remember that we assume the number of agents to be large and approximated by a continuum of agents with length 1. Therefore, an agent regards her contribution itself as zero even if she donates a positive amount of money. However, an agent with so large a signal that her own contribution covers the whole project cost C (*i.e.*, an agent i with signal $v_i > 0$ such that $x(v_i) > C$) may not consider her contribution to be zero. Note that it is not the normalized cost c but rather the original C . As her actual contribution covers the whole cost C , she has no reason to fear project failure.

Assumption 1 below represents the above discussion. Let $V \in \mathbb{R}_{++}$ be such that $x(V) = C$.

Assumption 1. For any agent i with $v_i > V$, *Donate* is the strictly dominant action.²³

We add one more assumption related to Assumption 1.

Assumption 2. $\int_V^\infty x(v)f(v;\hat{\theta}(V))dv > c$

Assumption 2 states that if the mean of signals is $\hat{\theta}(V)$ and agents with signals more than V donate, the total donation from agents with signals larger than V exceeds c and the project is a success. As already discussed when we introduced Assumption 1, the signal V is so large that $x(V) = C$. Given that $\hat{\theta}(V)$ is also large in accordance with V , Assumption 2 is supported.

Then we have the following proposition.

Proposition 6. *In a game G with Assumptions 1 and 2, $s^*(\cdot)$ defined in Proposition 1 is a unique strategy constituting the strategy profile surviving the iterated elimination of interim strictly dominated strategies.*

Proof of Proposition 6. Proposition 2.1 of Morris and Shin (2003) provides the equilibrium of a general global game where the (ex-post) payoff of an agent depends on her action, the proportion of other agents' strategies,

²³It is important that an agent with a large enough signal has the strictly dominant action to donate. The value V itself is not essential.

and her own signal. Even though in our game G , the payoff of an agent depends on her action, the total amount of donation, and her own signal, our Proposition 6 is essentially the same as their Proposition 2.1. We owe Morris and Shin (2003) and originally Carlsson and van Damme (1993) their proof techniques.

First, we construct a sequence of recursive values $\bar{v}^0, \bar{v}^1, \bar{v}^2, \dots$ such that for agents with signals larger than each value, *Not Donate* is the interim strictly dominated action.

By Assumption 1, for any agent i with $v_i > V$, *Donate* is the strictly dominant action, and *Not Donate* is strictly dominated. Let $\bar{v}^0 = V$.

Let $t \in \mathbb{N} \cup \{0\}$. Assume as an induction hypothesis that for any agent i with $v_i > \bar{v}^t$, *Not Donate* is the interim strictly dominated action, and they choose to *Donate*. Let \bar{v}^{t+1} be such that

$$\int_{\bar{v}^t}^{\infty} x(v)f(v; \hat{\theta}(\bar{v}^{t+1}))dv = c. \quad (24)$$

Suppose, at first, that all the other agents with signals smaller than \bar{v}^t choose *Not Donate*. Then, (24) represents that if the mean signal is $\hat{\theta}(\bar{v}^{t+1})$, the total amount of donation equals c . Thus if the mean signal is larger than $\hat{\theta}(\bar{v}^{t+1})$, the project is a success.

Note that for an agent i with $v_i > \bar{v}^{t+1}$, $\Pr(\theta \geq \hat{\theta}(\bar{v}^{t+1})) > \Pr(\theta \geq \hat{\theta}(v_i)) = p(v_i)$ since $\hat{\theta}(v_i) > \hat{\theta}(\bar{v}^{t+1})$. This means that from i 's point of view, the probability of success is larger than $p(v_i)$, and *Donate* is more profitable than *Not Donate*.

Notice that if we drop the supposition that all other agents with signals smaller than \bar{v}^t choose *Not Donate*, the probability of success for i with $v_i > \bar{v}^{t+1}$ is higher than that with the supposition. Thus, for i with $v_i > \bar{v}^{t+1}$, *Not Donate* is the interim strictly dominated action.

Next, we show that $k^* < \bar{v}^{t+1} < \bar{v}^t \leq V$.

First, notice $k^* < V = \bar{v}^0$ by Assumption 2 and Remark 5.

Assume as an induction hypothesis that $k^* < \bar{v}^t$. Then $\int_{k^*}^{\infty} x(v)f(v; \hat{\theta}(k^*))dv = c$ by the definition of k^* and Remark 5 imply that

$$\int_{\bar{v}^t}^{\infty} x(v)f(v; \hat{\theta}(\bar{v}^t))dv > c. \quad (25)$$

(24), (25) and Remark 4 imply that $\bar{v}^t > \bar{v}^{t+1}$. Similarly, since $k^* < \bar{v}^t$, $\int_{k^*}^{\infty} x(v)f(v; \hat{\theta}(k^*))dv = c$, (24) and Remark 4 imply $\bar{v}^{t+1} > k^*$. Hence $k^* < \bar{v}^{t+1} < \bar{v}^t \leq V$.

By recursive means, we have that the sequence $\bar{v}^0, \bar{v}^1, \bar{v}^2, \dots$ is weakly decreasing and has a lower bound k^* . Thus it has the limit value. Let \bar{v} be the limit value. (*i.e.*, $\lim_{t \rightarrow \infty} \bar{v}^t = \bar{v}$.) Since (7) is continuous in both k and v_i by Remark 4, we have that $\int_{\bar{v}}^{\infty} x(v)f(v; \hat{\theta}(\bar{v}))dv = c$. This implies $\bar{v} = k^*$.

(B) Similarly, we can construct a sequence of recursive values $\underline{v}^0, \underline{v}^1, \underline{v}^2, \dots$ such that for agents with signals smaller than each value, *Donate* is the interim strictly dominated action.

Let $\underline{v}^0 = 0$. Given $t \in \mathbb{N} \cup \{0\}$, let \underline{v}^{t+1} be such that

$$\int_{\underline{v}^t}^{\infty} x(v) f(v; \hat{\theta}(\underline{v}^{t+1})) dv = c.$$

Then, Remark 1 and the symmetric reasoning of (A) guarantee that for agents with signals smaller than each value, *Donate* is the interim strictly dominated action.

Similarly to (A), we have that $0 \leq \underline{v}^t < \underline{v}^{t+1} < k^*$ for any $t \in \mathbb{N} \cup \{0\}$, and $\lim_{t \rightarrow +\infty} \underline{v}^t = k^*$.

By discussions (A) and (B), we have the statement of Proposition 6. \square