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ONLY THE FINAL OUTCOME MATTERS: PERSISTENT EFFECTS OF EFFORTS IN DYNAMIC MORAL HAZARD

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Abstract

In dynamic principal-agent relationships, it is sometimes observed that the agent's reward depends only on the final outcome. For example, a student's grade in a course quite often depends only on the final exam score, where the performance in the problem sets and the mid-term exam is ignored. The present paper shows that such an arrangement can be optimal if the agent's effort in each period has strong persistent effects. It is shown that the optimality of such a simple payment scheme crucially depends on the first order stochastic dominance of the final outcome under various effort sequences.

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1 Introduction

In long-term principal-agent relationships, the principal write payment schedules that can potentially depend on period-by-period performance (that is related to the level of efforts), in order to provide proper incentives. In the light of the celebrated *Sufficient Statistic Theorem* (Hölmstrom [3]), one may expect that using the detailed history of past performances that is informative of the agent's efforts is optimal for the principal in writing payment schedules.

However, we often observe various incentive schemes which are not dependent on a part of performances, although those performances would provide certain information about the agent's effort levels. Especially, examples of incentive scheme that depends *only* on the *final* performance are abundant.

For instance, in many undergraduate courses, the instructors' grading policy mainly focuses on scores in the final exam, although week-by-week homework may reflect students' effort levels in detail. Other examples include university admissions in Japan that are completely dependent on entrance exams, where high-school records are hardly taken into consideration in admission process. Private tutors in Japan for entrance exams are often compensated with special bonus if the student has achieved the final objective, while usual tutorial fees are fixed, and do not depend on the students' period-by-period performance.

The present paper shows that such an arrangement can be optimal if the agent's effort in each period has strong persistent effects. If agent's effort in each period has strong persistent effect on the probability distribution of outcomes in later periods, the payment contract which depends *only* on the final outcome can provide the agent with sufficient incentive to work harder in every period. Therefore, all outcomes except for the final-period one are ignored in the optimal long-term contract, although those outcomes would provide detailed information about agent's effort levels in preceding periods.

Theorems 1–3 of the paper provide sufficient conditions for such *simple* contracts to be optimal in various models of dynamic moral hazard circumstance in which the cost of effort is the same in all periods. The common feature of our sufficient conditions can be simply summarized as follows: The probability distribution of the final outcome when the agent shirks only in the final period *first-order stochastically dominates* (*FOS-dominates* or *FOSD*, hereafter) the distribution when the agent shirks in any other periods in such a way that *the expected number of shirking is one.* To grasp the idea behind this condition intuitively, consider the two-period model in which the agent's first-period action also affects the probability distribution of the second-period outcome. Let (a, a') denote the action profile in which the first element (second element) indicates the agent's first period action (second period action, respectively), and let $\bar{a}$ ($\underline{a}$) denote the high effort (the shirk, respectively). Then the sufficient condition has the following two requirements (Theorem 1).

(i) The probability distribution of the second-period outcome when the agent shirks only in the second period $(\bar{a}, \underline{a})$ *FOS-dominates* the distribution when the agent shirks only in the first period $(\underline{a}, \bar{a})$.

(ii) The probability distribution of the second-period outcome when the agent shirks only in the second period $(\bar{a}, \underline{a})$ *FOS-dominates* the half-by-half randomization of (a) the distribution when the agent shirks in *both* periods $(\underline{a}, \underline{a})$ and (b) the distribution when the agent *never* shirks in any periods $(\bar{a}, \bar{a})$.

Requirement (i) ensures that shirking in the first period $(\underline{a}, \bar{a})$ is *always* worse off to the agent than shirking in the second period $(\bar{a}, \underline{a})$. Due to the FOSD, the agent can obtain larger expected payoff *from wages* in $(\bar{a}, \underline{a})$ than in $(\underline{a}, \bar{a})$[1], and as the number of efforts is the same in both action profiles, the agent obtains larger *overall* expected payoff if he takes $(\bar{a}, \underline{a})$ than $(\underline{a}, \bar{a})$. Thus, in designing the optimal contract, the principal need not take into account the possibility that the agent may shirk in the first period $(\underline{a}, \bar{a})$. Requirement (ii) ensures that shirking in both periods $(\underline{a}, \underline{a})$ is worse off to the agent than shirking in the second period $(\bar{a}, \underline{a})$. As the number of efforts is different between the two alternatives, the FOSD condition should be arranged in such a way that the expected number of efforts is set to be the same. In requirement (ii), this is achieved by setting the expected number of efforts of both sides to be one $(1 = 0.5 \times 2 + 0.5 \times 0)$.[2] Thus, in designing the optimal contract, the principal need not take into account the possibility that the agent may shirk in both periods $(\underline{a}, \underline{a})$.

Under (i) and (ii), the principal need not take into consideration any possibilities that the agent shirks in the first period whatsoever $(\underline{a}, \cdot)$. Therefore, the principal's interest is concentrated on incentivizing the agent's second-period effort only, which induces the *simple* optimal contract that depends *only* on the final outcome. It is noteworthy that requirement (ii) together with (i) can be summarized as follows: The probability distribution of the final outcome when the agent shirks only in the final period *FOS-dominates* the distribution when the agent shirks in any other periods in such a way that *the expected number of shirking is one.* Such arguments of the role of FOSD and the expected number of efforts also apply to T-period models,

[1] As will be presented formally in Section 3, we assume that the distribution of outcomes has the *monotone likelihood ratio property.* Therefore, in the optimal contract, the wage scheme is an increasing function of outcomes, which enables us to make comparison between expected payoffs *from wages* by means of FOSD.

[2] The reader may wonder why $(\bar{a}, \bar{a})$, which is irrelevant in the comparison between $(\bar{a}, \underline{a})$ and $(\underline{a}, \underline{a})$, appears in requirement (ii). This is because the incentive compatibility constraint between $(\bar{a}, \bar{a})$ and $(\bar{a}, \underline{a})$ is binding (indifferent to the agent) in the optimum. See Section 3 for the detail.

and sufficient conditions are provided in similar manners (Theorems 2–3).

Strong persistent effects of efforts as characterized by the FOSDs is the main source of our result. Historical dependence of this sort can be often seen in real economic environments. For example, if an effort has a time-lag effect to the next period as well as the direct effect to the current period, then the probability of success in period 2 will be influenced by the effort level in period 1. If the production technology bears irreversibility, then the model becomes history dependent in a similar manner.[3]

A brief review of the related literature is as follows. The result of the paper (Theorems 1–3) is in contrast with the ones in *repeated* moral hazard literature that payments in the optimal long-term contract should be dependent on the whole history of past performances (Lambert [5], Rogerson [11], Malcomson and Spinnewyn [8] and Chiappori et al. [1]). In those literature, it is assumed that there are no exogenous links between one period and the next, and the complementarity between incentives as discussed in preceding paragraphs cannot emerge.

Holmström [3] shows that any signal that is informative of the agent's efforts should be used to condition the agent's compensation scheme when there are no exogenous links between actions the agent might take (*The Sufficient Statistic Theorem*). In our model, all outcomes except for the final-period one should be ignored in the optimal contract, although those outcomes are informative of the agent's efforts in the preceding periods. Our result is in contrast to Holmström's in that some part of "informative" signals can be ignored in the optimal contract if there are exogenous links between the agent's actions.

After completing the earlier version of this paper, we became aware of an independent work by Kwon [4], who investigates a dynamic moral hazard model and derives a similar result to ours in a sense that the optimal contract should be *simple* if the probability distributions satisfy "non-increasing

[3] These examples are examined in detail in Section 4.

marginal returns" assumption. Kwon deals with a simplified model in which there are only two performance levels ("success" and "failure") and the effect of efforts in every period is symmetric. The present paper deals with more general environment in which there are N performance levels and the effect of efforts in each period can be asymmetric, and reveals that conditions provided with *first-order stochastic dominance* is sufficient for the simple contract result, which is a weaker condition than Kwon's "non-increasing marginal returns" assumption.[4]

The remainder of the paper is organized as follows. Section 2 describes the basic model of 2-period dynamic moral hazard. Section 3 provides the main result of the paper. It is shown that the optimal long-term contract is dependent only on the final outcome and a sufficient condition for the result is presented (Theorem 1). Section 4 provides some examples of environments in which the sufficient condition is satisfied. In Section 5, we extend the basic model to T-period, and present sufficient conditions for the optimal contract to be simple as in Theorem 1. Section 6 contains some concluding remarks.

2 The Basic Model

We study a simple dynamic moral hazard model with "history dependence." The relationship between a principal (she) and an agent (he) lasts for two periods ($t = 1, 2$).

In each period, the agent chooses his action a^t from the action space $A = \{\underline{a}, \bar{a}\}$. These actions are kept unobservable to the principal. We may find it convenient to interpret those actions as effort levels, and say that he works hard (respectively, shirks) when he chooses $\bar{a}$ (respectively, $\underline{a}$).

In period t, after the agent has chosen his action a^t, the outcome $x^t \in$

[4]In Section 5, it is shown that "non-increasing marginal returns" is a special class of the sufficient condition provided with *FOSD*. Empirical evidence from health insurance is also presented in Kwon [4], which is consistent with the derived optimal contract.

$\{x_1, \cdots, x_N\}$ realizes according to probabilities that depend on the history of agent's actions; that is, the distribution of x^1 depends on a^1, whereas that of x^2 depends on the pair (a^1, a^2). These outcomes are immediately observed by both parties (and assumed to be verifiable to third parties, such as a court). We may regard these outcomes as performances, and identify each of them with the corresponding revenue to the principal.

We assume that x^1 and x^2 are independently distributed[5]; hereafter, we will write the distributions as follows:

$$p_i^1(a^1) = \Pr\left[x^1 = x_i \mid a^1\right] \qquad (i = 1, \cdots, N),$$
$$p_i^2(a^1, a^2) = \Pr\left[x^2 = x_i \mid (a^1, a^2)\right] \qquad (i = 1, \cdots, N).$$

Throughout the paper, we assume that the distributions are of full supports:

$$p_i^1(a^1) > 0 \qquad \text{for all } (i, a^1) \in \{1, \cdots, N\} \times A,$$
$$p_i^2(a^1, a^2) > 0 \qquad \text{for all } (i, a^1, a^2) \in \{1, \cdots, N\} \times A^2.$$

At the beginning of the game (i.e., before $t = 1$), the principal and the agent sign a contract in the manner described in detail below.

First, the principal offers a long-term contract $\mathbf{w} = (\mathbf{w}^1, \mathbf{w}^2)$, where $\mathbf{w}^1 = (w^1(x^1))_{x^1 \in X}$ and $\mathbf{w}^2 = (w^2(x^1, x^2))_{(x^1, x^2) \in X^2}$ are payment schedules for periods 1 and 2, respectively, under outcome realizations (x^1, x^2). Such a contract stipulates $N + N^2$ possible payments, depending on the realizations of outcomes. Next, the agent decides whether to accept or refuse the contract offered by the principal. If the agent refuses the offered contract, both parties receive their reservation utilities, and the game comes to an end. If the agent accepts the contract, the game enters into the two times moral

[5]This assumption says that the realized value of x^1 does not influence the distribution of x^2, so that the former yields no information on the current likelihood of any particular production levels in period 2. "History dependence" discussed in this paper treats the case where x^2 is affected by a^1, but not by the realization of x^1.

hazard repetition discussed above.

We assume that the principal can commit to the long-term contract that she has offered before $t = 1$ and so, once the contract is accepted by the agent, the principal cannot change the payment schedule $\mathbf{w}$ and must make the payment each period according to the history of outcome realizations up to the date. We also assume that the agent can commit to his participation to the game and so, once he accepts the contract, he cannot exit in the midst of the game and must participate in it until the end of period 2.

In each period, the agent attains a payoff of $u(w) - c(a)$, where u is strictly increasing and strictly concave (the agent is risk-averse) and $c(\underline{a}) < c(\bar{a})$ (harder work makes more cost). We normalize this as $c(\underline{a}) = 0$ and $c(\bar{a}) = C$.

Given a long-term contract $\mathbf{w}$, the agent's strategy consists of two parts: one is the action he takes in the first period, a^1, and the other is the action schedule for the second period $a^2 = (a_i^2)_{i=1}^N$, each of which specifies the action he will take in period 2 under the outcome realization of x^1 in period 1.[6] Let $U_i(a^1, a_i^2, \mathbf{w}^2)$ denote the expected utility in period 2 for the agent when he took a^1 and the outcome was x_i in the first period:

$$U_i(a^1, a_i^2, \mathbf{w}^2) = \sum_{j=1}^{N} p_i^2(a^1, a_i^2) u(w^2(x_i, x_j)) - c(a_i^2).$$

Using this notation, the intertemporal expected utility for the agent $U(a^1; a^2; \mathbf{w})$ under the agent's strategy $(a^1; a^2)$ can be written as

$$U(a^1; a_1^2, \cdots, a_N^2; \mathbf{w}) = \sum_{i=1}^{N} p_i^1(a^1) \left[u(w^1(x_i)) + U_i(a^1, a_i^2, \mathbf{w}^2) \right] - c(a^1).^7$$

[6] Accordingly, we allow the agent to change his action in period 2 after he observes the outcome realization in period 1, which is one of the standard assumptions in the literature. Once we cease this assumption and assume that the agent had to commit to a pair of actions (a^1, a^2) *ex ante*, then the model reduces to a one-shot multitask incentive problem. We shall take the sequentiality assumption to focus on the dynamics of the model, but note that the main result of the paper (Theorems 1–3) also apply to the one-shot multitask model.

[7] We assume that both the Principal and the Agent have the common discount factor

The optimization problem for the principal when she wishes to implement an action profile (a^1, a^2) can now be written as:

$$\min_{\mathbf{w}} \sum_{i=1}^{N} p_i^1(a^1) \left[w^1(x_i) + \sum_{j=1}^{N} p_j^2(a^1, a_i^2) w^2(x_i, x_j) \right], \tag{P}$$

subject to

$$U(a^1, a^2, \mathbf{w}) \geq U(a', a'', \mathbf{w}), \quad a' \neq a^1, \quad \forall a'' \in A^N, \tag{IC1}$$

$$U_i(a^1, a_i^2, \mathbf{w}^2) \geq U_i(a^1, a', \mathbf{w}^2), \quad a' \neq a_i^2, \quad i = 1, \cdots, N, \tag{IC2}$$

$$U(a^1, a^2, \mathbf{w}) \geq 2\bar{u}, \tag{PC}$$

where $\bar{u}$ denotes the reservation utility for the agent.

At this point, we should emphasize how the optimization problem (P) differs from the one for *repeated* moral hazard models. When the model is just a repetition of two moral hazard stages, the action taken in period 1, a^1, does not affect the probability distribution of outcomes in period 2 so that $U_i(a', a_i^2, \mathbf{w}^2) = U_i(a'', a_i^2, \mathbf{w}^2)$ for any $a' \neq a''$. This would reduce the incentive constraints for the first period (IC1) to

$$U(a^1, a^2, \mathbf{w}) \geq U(a', a^2, \mathbf{w}), \quad (a' \neq a^1), \tag{IC1$^{\text{ind}}$}$$

under which we must only take into account the deviation strategies from a^1 to the other a', with a^2 fixed. For the *dynamic* model which we investigate in the paper, this would not be sufficient: we must take into account all possibilities of deviation the agent might make during the two periods, as it is no longer assured that he will always take a^2 regardless of the action he takes in period 1, even if (IC2) is satisfied for the a^1.

of 1. If the common discount factor were less than 1 (but positive) and the outcome space consists of three elements or more, we cannot attain plausible sufficient conditions as in Assumption 1, which can be described only with the nature of $(p_i^1(\cdot))$ and $(p_i^2(\cdot, \cdot))$. An independent related paper by Mukoyama and Sahin [8] shows in case of $N = 2$ that an extension of Assumption 1 is a sufficient condition for $w^1(x^1)$ to be constant in a similar model in which both players have a common discount factor less than 1.

3 Simple Contract

In this section, we show that the optimal long-term contract is dependent only on the second-period outcome if the probability distribution of the second period outcome satisfies certain conditions as briefly discussed in Introduction. The result (Theorem 1) lies in contrast to that in the *repeated* moral hazard literature where the optimal long-term contract would always be dependent on the whole history of past outcomes.

The following assumption gives the sufficient condition for such simple contracts. We may regard this assumption as "strong persistent effects" in the sense that the action chosen in period 1 has a stronger influence on the outcome in period 2 than the action chosen in period 2.

Assumption 1. $p_i^2(a^1, a^2)$ satisfies the following three conditions:

(i) $p_i^2(a^1, \bar{a})/p_u^2(a^1, \underline{a})$ is increasing in i for all a^1. (MLRC)

(ii) $\sum_{i=1}^{I} p_i^2(\underline{a}, \bar{a}) \geq \sum_{i=1}^{I} p_i^2(\bar{a}, \underline{a})$ for all $I \in \{1, \cdots, N\}$.

(iii) $\frac{1}{2}\sum_{i=1}^{I} (p_i^2(\underline{a}, \underline{a}) + p_i^2(\bar{a}, \bar{a})) \geq \sum_{i=1}^{I} p_i^2(\bar{a}, \underline{a})$ for all $I \in \{1, \cdots, N\}$.

In Assumption 1, (ii) says that the action profile $(\bar{a}, \underline{a})$ stochastically dominates the action profile $(\underline{a}, \bar{a})$ in the distribution of x^2, while (iii) says that $(\bar{a}, \underline{a})$ stochastically dominates the half-by-half randomization between $(\underline{a}, \underline{a})$ and $(\bar{a}, \bar{a})$. We should note that neither (ii) nor (iii) in Assumption 1 can be satisfied in *repeated* moral hazard models.

Theorem 1. *Suppose that the probability distribution of second period outcome satisfies Assumption 1. Then the optimal long-term contract* $\mathbf{w}$ *which implements* $a^1 = \bar{a}$ *and* $a^2 = (\bar{a}, \ldots, \bar{a})$ *is such that*

(a) $w^1(x^1)$ *is a constant for all* x^1*,*

(b) $w^2(x^1, x^2)$ *is independent of* x^1*, and is increasing in* x^2*.*

We should note here that Assumption 1 is not only a sufficient condition for the simple contract result, but also *almost* necessary condition in the sense that if the simple contract is optimal for *any* increasing and concave functions $u(\cdot)$ then the probability distribution necessarily satisfies Assumption 1.

Proof. The proof proceeds in two steps. In the first step, we solve a "relaxed" optimization problem as follows:

$$\min_{\mathbf{w}} \sum_{i=1}^{N} p_i^1(a^1) \left[w^1(x_i) + \sum_{j=1}^{N} p_j^2(a^1, a_i^2) w^2(x_i, x_j) \right], \tag{P'}$$

subject to

$$U_i(a^1, a_i^2, \mathbf{w}^2) \geq U_i(a^1, a', \mathbf{w}^2), \quad a' \neq a_i^2, \quad i = 1, \cdots, N, \tag{IC2}$$

$$U(a^1, a^2, \mathbf{w}) \geq 2\bar{u}, \tag{PC}$$

and show that the solution satisfies the properties (a) and (b). In the second step, we verify that (any) contract satisfying properties (a) and (b) is always compatible with the constraint (IC1). By these two steps, we can conclude that the solution to the "original" optimization problem (P) satisfies properties (a) and (b).

1. The first-order condition for $w^1(x_i)$ in the "relaxed" problem (P′) is

$$\frac{1}{u'(w^1(x_i))} = \nu \quad \text{for all } x_i,$$

where ν is the Lagrange multiplier with respect to (PC). Thus, $w^1(x_i)$ is a constant for all x_i.

The first-order condition for $w^2(x_i, x_j)$ is

$$\frac{1}{u'(w^2(x_i, x_j))} = \frac{\mu_i}{p_i(\bar{a})} \left[1 - \frac{p_j^2(\bar{a}, \underline{a})}{p_j^2(\bar{a}, \bar{a})} \right] + \nu,$$

where μ_i is the Lagrange multiplier with respect to (IC2) for the corresponding i. Here, $w^2(x_i, x_j)$ is independent of i (otherwise the principal could be strictly better off by offering the certainty equivalence $\tilde{w}'_j$ such that $u(\tilde{w}'_j) = \sum_i p^1_i(\bar{a})u(w^2(x_i, x_j))$, without affecting the remaining constraints (IC2) and (PC)). Hence, the ratio $\mu_i/p_i(\bar{a})$ is a constant for all i.

If $\mu_i = 0$, then $w^2(x_i, x_j)$ would be a constant for all j, which violates (IC2) for i. Hence, $\mu_i > 0$ should be satisfied for all i, which means that (IC2) is binding in the optimum. Therefore, from Assumption 1 (i) and the concavity of $u(\cdot)$, $w^2(x_i, x_j)$ must be increasing in j.

2. Firstly, we check that (IC1) is satisfied for two deviation strategies $(a^1; a^2) = (\underline{a}; \bar{a}, \cdots, \bar{a})$ and $(a^1; a^2) = (\underline{a}; \underline{a}, \cdots, \underline{a})$ under the optimal contract derived in 1. Here, we write $w^1(x_i) = w^1$ and $w^2(x_i, x_j) = w^2_j$ as the contract is not dependent on x_i.

As shown in 1., (IC2) is binding at the optimum; therefore,

$$C = \sum_{j=1}^{N} p^2_j(\bar{a}, \bar{a})u(w^2_j) - \sum_{j=1}^{N} p^2_j(\bar{a}, \underline{a})u(w^2_j) \tag{1}$$

(IC1) to hold against deviation strategy $(a^1; a^2) = (\underline{a}; \bar{a}, \cdots, \bar{a})$ is equivalent to

$$\sum_{j=1}^{N} p^2_j(\bar{a}, \bar{a})u(w^2_j) - 2C \geq \sum_{j=1}^{N} p^2_j(\underline{a}, \bar{a})u(w^2_j) - C,$$

which, by substituting (1), yields

$$\sum_{j=1}^{N} p^2_j(\bar{a}, \underline{a})u(w^2_j) \geq \sum_{j=1}^{N} p^2_j(\underline{a}, \bar{a})u(w^2_j).$$

Since $u(w^2_j)$ in increasing in j, a sufficient condition for this inequality to hold is that $(\bar{a}, \underline{a})$ stochastically dominates $(\underline{a}, \bar{a})$ in the probability distribution of x^2: Assumption 1 (ii).

(IC1) to hold against deviation strategy $(a^1; a^2) = (\underline{a}; \underline{a}, \cdots, \underline{a})$ is equivalent to

$$\sum_{j=1}^{N} p_j^2(\bar{a},\bar{a})u(w_j^2) - 2C \geq \sum_{j=1}^{N} p_j^2(\underline{a},\underline{a})u(w_j^2),$$

which, by substituting (1), yields

$$2\sum_{j=1}^{N} p_j^2(\bar{a},\underline{a})u(w_j^2) \geq \sum_{j=1}^{N} p_j^2(\bar{a},\bar{a})u(w_j^2) + \sum_{j=1}^{N} p_j^2(\underline{a},\underline{a})u(w_j^2).$$

Likewise a sufficient condition for this inequality to hold is that $(\bar{a},\underline{a})$ stochastically dominates the half-by-half randomization between $(\bar{a},\bar{a})$ and $(\underline{a},\underline{a})$: Assumption 1 (iii).

Finally we check that (IC1) is satisfied for any deviation strategies $(a^1;a^2) = (\underline{a};a_1^2,\cdots,a_N^2)$. Suppose the agent is to take $a_i^2 = \bar{a}$ if $i \in \bar{I} \subset \{1,\cdots,N\}$ and $a_i^2 = \underline{a}$ if $i \in \underline{I} = \{1,\cdots,N\} \setminus \bar{I}$. The intertemporal payoff to the agent following this deviation strategy would satisfy

$$\begin{aligned}
& u(w^1) + \sum_{i\in\bar{I}} p_i^1(\underline{a}) \left[\sum_{j=1}^{N} p_j^2(\underline{a},\bar{a})u(w_j^2) - C\right] + \sum_{i\in\underline{I}} p_i^1(\underline{a}) \left[\sum_{j=1}^{N} p_j^2(\underline{a},\underline{a})u(w_j^2)\right] \\
& \leq u(w^1) + \max\left\{\sum_{j=1}^{N} p_j^2(\underline{a},\bar{a})u(w_j^2) - C,\ \sum_{j=1}^{N} p_j^2(\underline{a},\underline{a})u(w_j^2)\right\} \\
& = \max\left\{U(\underline{a};\bar{a},\cdots,\bar{a};\mathbf{w}),\ U(\underline{a};\underline{a},\cdots,\underline{a};\mathbf{w})\right\} \\
& \leq U(\bar{a};\bar{a},\cdots,\bar{a};\mathbf{w}),
\end{aligned}$$

where the last inequality comes from the previous result that (IC1) is satisfied both for $(a^1;a^2) = (\underline{a};\bar{a},\cdots,\bar{a})$ and for $(a^1;a^2) = (\underline{a};\underline{a},\cdots,\underline{a})$. Hence, (IC1) is satisfied for any deviation strategy $(a^1;a^2) = (\underline{a};a_1^2,\cdots,a_N^2)$. □

The intuition behind the proof is as follows. For the principal who is willing to induce the agent to exert the positive effort $\bar{a}$ in period 2, it is necessary to make the second-period payment $w^2(x_i,x_j)$ dependent on the second-period outcome x_j as this is the only source of incentive power available. However, such a payment schedule would induce the agent to work hard in period 1 since the distribution of second-period outcomes is affected not only by a^2 but also by a^1. Moreover, this gives the agent an incentive enough to work hard in period 1 under Assumption 1: Assumption 1 (ii) ensures that the agent can always obtain larger *gross* expected payoff from wages by action profile $(\bar{a}, \underline{a})$ than that by $(\underline{a}, \bar{a})$ due to the FOSD, and as the cost of effort, C, is the same in both periods, the agent obtains larger *net* expected payoff as well. Thus, if the contract is to induce working hard in the second period, it automatically provides the agent with incentive to work hard in the first period. Assumption 1 (iii), on the other hand, ensures that the agent would not deviate to shirking in both periods (i.e., to $(\underline{a}, \underline{a})$). Half-by-half randomization of the two probability distributions, $(\bar{a}, \bar{a})$ and $(\underline{a}, \underline{a})$, gives the agent's *gross* expected payoff by taking $(\underline{a}, \underline{a})$ in accordance with the benefit of effort cost reduction normalized to C (one-time shirk). Thus, if the contract is to induce working hard in the second period, it automatically makes the agent worse off if he shirks in both periods, $(\underline{a}, \underline{a})$.

To summarize, if the probability distribution of the second-period outcome when the agent shirks only in the second period $(\bar{a}, \underline{a})$ *FOS-dominates* the distribution when the agent shirks in any other periods in such a way that *the expected number of shirking is one*, providing incentive to work hard in the second period becomes enough to induce the agent to make high efforts in both periods. As we will see in Section 5, such arguments of *FOSD* and *one-time shirk* play central roles in T-period models as well and the sufficient conditions for simple contracts are provided in similar manners.

4 Examples

In this section, we give a few examples in which $p_i^2(\cdot, \cdot)$ satisfies Assumption 1. These examples incorporate "strong persistent effects" in the sense that the action chosen in period 1 has a stronger influence on the outcome of period

2 than the action chosen in period 2. Under such circumstances, the optimal long-term contract is simple by which we mean that the payment schedule would be dependent only upon the second-period outcome.

In the following examples, we suppose $N = 2$ ("success" and "failure") and let $\pi_t(\cdot)$ denote the probability of "success" in period t; that is, $\pi_t(\cdot) = p_2^t(\cdot)$ and $1 - \pi_t(\cdot) = p_1^t(\cdot)$.

Example 1 (Time lag)**.** There is a time lag between the effort and its effect.

If the agent works hard in period t, it not only increases the probability of success in the same period by α but also increases the probability of success in the following period by β. We assume $0 < \alpha < \beta$ in which we can regard β as a "full effect" of the effort and α as a "partial effect" of the effort. Let $\underline{\pi}$ denote the probability of success when the agent has never taken any positive efforts. Then, we can write $\pi_t(\cdot)$ as follows:

$$\begin{aligned} \pi_1(\underline{a}) &= \underline{\pi}, & \pi_1(\bar{a}) &= \underline{\pi} + \alpha, \\ \pi_2(\underline{a}, \underline{a}) &= \underline{\pi}, & \pi_2(\underline{a}, \bar{a}) &= \underline{\pi} + \alpha, \\ \pi_2(\bar{a}, \underline{a}) &= \underline{\pi} + \beta, & \pi_2(\bar{a}, \bar{a}) &= \underline{\pi} + \alpha + \beta. \end{aligned}$$

Assumption 1 (ii) is satisfied as $\pi_2(\bar{a}, \underline{a}) > \pi_2(\underline{a}, \bar{a})$; this is the "time-lag effect" since the positive effort $\bar{a}$ taken in period 1 has greater influence β than it has if taken in period 2 (α). Assumption 1 (iii) is also satisfied as $\pi_2(\bar{a}, \underline{a}) > \frac{1}{2}\left[\pi_2(\bar{a}, \bar{a}) + \pi_2(\underline{a}, \underline{a})\right]$.

Example 2 (Irreversibility)**.** The agent has to make a positive effort every period to maintain the highest probability of success $\bar{\pi}$. If he shirks, the probability of success declines by γ and this will never be recovered, even if the agent makes a positive effort in the following period:

$$\begin{aligned} \pi_1(\underline{a}) &= \bar{\pi} - \gamma, & \pi_1(\bar{a}) &= \bar{\pi}, \\ \pi_2(\underline{a}, \underline{a}) &= \bar{\pi} - 2\gamma, & \pi_2(\underline{a}, \bar{a}) &= \bar{\pi} - \gamma \\ \pi_2(\bar{a}, \underline{a}) &= \bar{\pi} - \gamma, & \pi_2(\bar{a}, \bar{a}) &= \bar{\pi}. \end{aligned}$$

It is clear that the distribution satisfies Assumption 1 (ii) and (iii) with equalities.

5 Extensions

In this section, we extend the basic model to T-period setup, and show that similar results as in Theorem 1 can be obtained. As in Section 2, we let a^t and x^t denote agent's action and outcome in each period $t = 1, \cdots, T$, respectively. Distribution of each outcome x^t is dependent on the whole past history of actions $\mathbf{a}^t = (a^1, \cdots, a^t)$, and we write them as follows:

$$p_{it}(\mathbf{a}^t) = \Pr\left[x^t = x_i \mid \mathbf{a}^t\right], \quad i = 1, \cdots, N.$$

For simplicity, we assume throughout this section that the agent decides his whole action profile $\mathbf{a}^T = (a^1, \cdots, a^T)$ in the beginning of period 1 and he never changes this profile after observing any outcomes in each period.[8] We split the agent's action space $A^T = A \times \cdots \times A$ into partition $(\mathcal{A}_0, \ldots, \mathcal{A}_T)$, where

$$\mathcal{A}_k = \left\{(a^1, \ldots, a^T) \mid \# \left\{t \mid a^t = \underline{a}\right\} = k\right\}, \quad k = 0, \ldots, T.$$

That is, $\mathcal{A}_k$ is the set of action profile $\mathbf{a}^T$ in which there are k low efforts $\underline{a}$ (and hence, $T - k$ high efforts $\bar{a}$).[9] For instance, if $T = 3$, then above notation gives us $\mathcal{A}_0 = \{(\bar{a}, \bar{a}, \bar{a})\}$, $\mathcal{A}_1 = \{(\bar{a}, \bar{a}, \underline{a}), (\bar{a}, \underline{a}, \bar{a}), (\underline{a}, \bar{a}, \bar{a})\}$, etc.

[8] In the basic model of Section 2, it is assumed that the agent can make his second period action a^2 after observing the first period outcome x^1; therefore, the action profile consists of $N+1$ components $(a^1; a_1^2, \ldots, a_N^2)$, where a_i^2 denotes the second period action when the first period outcome is x_i. For T-period model in this section, we can also think of possibility that the agent's actions depend on past outcomes (in which case the action profile consists of $(N^T - 1)/(N-1)$ components), but such a consideration does not change the result in Theorems 2–3. See the Appendix for more on this point.

[9] It is obvious that $(\mathcal{A}_0, \ldots, \mathcal{A}_T)$ satisfies $\mathcal{A}_k \cap \mathcal{A}_l = \emptyset$ for any k, l, and $\bigcup_k \mathcal{A}_k = A$. Hence $(\mathcal{A}_0, \cdots, \mathcal{A}_T)$ is a partition of A.

Let $V(\mathbf{a}, \mathbf{w})$ denote the agent's *gross* expected payoff from payment schedule $\mathbf{w}$ when he takes action profile $\mathbf{a}$. Then the agent's *net* expected payoff can be written as

$$V(\mathbf{a}, \mathbf{w}) - C \cdot m(\mathbf{a}),$$

where C is the cost of high effort $\bar{a}$ (as in Section 2) and $m(\mathbf{a})$ is the number of high efforts in action profile $\mathbf{a}$. Then the incentive compatibility constraint in N-period model can be simplified as follows: for $k = 1, \ldots, T$,

$$V(\bar{a}, \ldots, \bar{a}, \mathbf{w}) - C \cdot k \geq V(\mathbf{a}', \mathbf{w}), \quad \text{for all } \mathbf{a}' \in \mathcal{A}_k. \tag{IC$_k$}$$

Here we offer two models of N-period *dynamic* moral hazard.

Extention 1 (Summary outcome in the final period)

Suppose that the agent is to make mutually independent outcomes in each period, but in the final period $t = T$, there will be a "summary" outcome that is dependent on the whole past actions $\mathbf{a}^t = (a_1, \cdots, a_T)$. To be specific,

$$\begin{aligned} p_{it}(a^t) &= \Pr\left[x^t = x_i \mid a^t\right], \quad t = 1, \cdots, T-1, \\ p_{jt}(\mathbf{a}^T) &= \Pr\left[x^T = x_j \mid \mathbf{a}^T\right]. \end{aligned}$$

In this setup, t-period action a^t is assumed to affect the current outcome x^t as well as the final outcome x^T. We can think of this situation as the relationship between week-by-week homeworks and the final exam. Each week students are assigned homework concerning the topic they have just studied, but in the end, students must challenge the final exam concerning the entire topic they learned in the semester.

Theorem 2. *Suppose that $p_{jT}(\mathbf{a}^T)$ satisfies the following two conditions:*

1. $p_{jT}(\mathbf{a}^{T-1}, \bar{a})/p_{jT}(\mathbf{a}^{T-1}, \underline{a})$ *is increasing in j for all $\mathbf{a}^{T-1}$ (MLRC),*

2. *For all $k = 1, \ldots, T$ and all $\mathbf{a}' \in \mathcal{A}_k$, the inequality*

$$\frac{\sum_{j=1}^{J} [(k-1)p_{jT}(\bar{a}, \ldots, \bar{a}, \bar{a}) + p_{jT}(\mathbf{a}')]}{k} \geq \sum_{j=1}^{J} p_{jT}(\bar{a}, \ldots, \bar{a}, \underline{a}) \qquad (2)$$

holds for all $J \in \{1, \cdots, N\}$.

Then payments in the optimal long-term contract is dependent only on the final outcome x^T.

First note that conditon 2. is the generalization of conditions (ii) and (iii) in Assumption 1. Substituting $k = 1$ into equation (2) gives

$$\sum_{j=1}^{J} p_{jT}(\mathbf{a}') \geq \sum_{j=1}^{J} p_{jT}(\bar{a}, \ldots, \bar{a}, \underline{a}) \quad \text{for all } \mathbf{a}' \in \mathcal{A}_1 \text{ and } J,$$

which states that the distribution of final outcome x^T when the agent takes action profile $(\bar{a}, \ldots, \bar{a}, \underline{a})$ *first-order stochastically dominates* that of any action profile $\mathbf{a}'$ in which the agent takes low effort only in one period; this is exactly an extension of condition (ii) in Assumption 1. Similarly, substituting $k = 2$ into inequality (2) provides an extention of condition (iii) in Assumption 1. In general, inequality (2) can be seen as the following condition: shirking in the final period (right-hand side) first-order stochastically dominates any shirkings *whose expected number is exactly one* (left-hand side).

Proof. As in the proof of Theorem 1, we can show that the optimal long-term contract is independent of outcome history $\mathbf{x}^{T-1}$ up to the period $T-1$, if all incentive constraints but

$$V(\bar{a}, \ldots, \bar{a}, \bar{a}, \mathbf{w}) - C \geq V(\bar{a}, \ldots, \bar{a}, \underline{a}, \mathbf{w}) \qquad (3)$$

are not binding. In the following we show that the derived contract, which is dependent only on the final outcome x^T, automatically satisfies all the incentive compatibility constraints.

As the derived contract $\mathbf{w}^*$ satisfies (3) with equality, we have

$$C = V(\bar{a}, \dots, \bar{a}, \bar{a}, \mathbf{w}^*) - V(\bar{a}, \dots, \bar{a}, \underline{a}, \mathbf{w}^*). \tag{4}$$

For each $k = 1, \dots, T$, substituting (4) into (IC_k) yields

$$k \cdot V(\bar{a}, \dots, \bar{a}, \underline{a}, \mathbf{w}^*) \geq (k-1) \cdot V(\bar{a}, \dots, \bar{a}, \bar{a}, \mathbf{w}^*) + V(\mathbf{a}', \mathbf{w}^*).$$

This inequality is guaranteed to hold by (2). □

Extention 2 (Human Capital Investment)

Suppose that distributions of outcome in each period are dependent not on the detail of past actions, but on the number of high efforts that the agent has taken up to the date. To be specific, we let $p_i(k)$ denote the probability distribution when the agent has taken k high efforts:

$$p_i(k) = \Pr\left[x^t = x_i \mid \#\left\{t \mid a^t = \bar{a}\right\} = k\right], \quad k = 0, \dots, T.$$

We can think of this situation as agent's human capital investment or learning-by-doing effect of agent's effort.

Theorem 3. *Suppose that distributions $p_i(k)$, $k = 0, \dots, T$, satisfy the following two conditions:*

1. *$p_i(T)/p_i(T-1)$ is increasing in i,*

2. *For all $k = 2, \cdots, T$, the inequality*

$$\frac{\sum_{j=1}^{J}\left[(k-1)p_j(T) + p_j(T-k)\right]}{k} \geq \sum_{j=1}^{J} p_j(T-1)$$

   holds for all $J \in \{1, \dots, N\}$.

Then the optimal long-term contract is dependent only on the final outcome x^T.

We should note that substituting $k = 2$ into condition 2 yields the counterpart of condition (iii) in Assumption 1.[10] As in Extension 1, condition 2 can be seen as the following condition: shirking in the final period (right-hand side) first-order stochastically dominates any shirkings *whose expected number is one* (left-hand side).

It is also important to note that condition 2 is a reasonable assumption since it is a weaker condition of *non-increasing marginal returns* to investment: for all $k = 1, \ldots, T-1$,

$$\sum_{j=1}^{J} [p_j(k+1) - p_j(k)] \geq \sum_{j=1}^{J} [p_j(k) - p_j(k-1)], \quad J = 1, \ldots, N. \tag{5}$$

This inequality states that the marginal "benefit" in the probability distribution by one additional effort is decreasing in k. To see that the "non-increasing marginal returns" implies condition 2, replace k with $T - k + l$ and multiply the inequality (5) by l:

$$l \sum_{j=1}^{J} [p_j(T-k+l+1) - p_j(T-k+l)] \geq l \sum_{j=1}^{J} [p_j(T-k+l) - p_j(T-k+l-1)],$$

then by summing up both sides for $l = 1, \ldots, k-1$, we have

$$(k-1) \sum_{j=1}^{J} p_j(T) + \sum_{j=1}^{J} p_j(T-k) \geq k \sum_{j=1}^{J} p_j(T-1),$$

which is equivalent to condition 2.[11]

[10] Note that the counterpart of condition (ii) in Assumption 1 becomes an identity in this extention since two action profiles $(\bar{a}, \ldots, \bar{a}, \underline{a})$ and $(\bar{a}, \ldots, \underline{a}, \bar{a})$ yield the same distribution $p_i(T-1)$ in the final period T.

[11] An independent related paper Kwon [4] investigates a similar model with binary outcomes ($N = 2$) and shows that the optimal long-term contract is dependent only on

Proof. As in the proof of Theorem 1, we can show that the optimal long-term contract is independent of outcome history $\mathbf{x}^{T-1}$ up to the period $T-1$, if all incentive constraints but

$$V(\bar{a},\ldots,\bar{a},\bar{a},\mathbf{w}) - C \geq V(\bar{a},\ldots,\bar{a},\underline{a},\mathbf{w}) \tag{6}$$

are not binding. In the following we show that the derived contract, which is dependent only on the final outcome x^T, automatically satisfies all the incentive compatibility constraints.

As the derived contract $\mathbf{w}^*$ satisfies (6) with equality, we have

$$C = V(\bar{a},\ldots,\bar{a},\bar{a},\mathbf{w}^*) - V(\bar{a},\ldots,\bar{a},\underline{a},\mathbf{w}^*). \tag{7}$$

For each $k = 1,\ldots,T$, substituting (7) into (IC_k) yields

$$k \cdot V(\bar{a},\ldots,\bar{a},\underline{a},\mathbf{w}^*) \geq (k-1)\cdot V(\bar{a},\ldots,\bar{a},\bar{a},\mathbf{w}^*) + V(\mathbf{a}',\mathbf{w}^*).$$

This inequality is guaranteed to hold by the condition 2. □

6 Concluding Remarks

This paper has examined the role of history dependence in a dynamic moral hazard model. It is shown that, under certain conditions on the probability distributions of outcomes, the optimal long-term contract is such that the payment schedules are not contingent upon the realization of past outcomes. This finding lies in striking contrast to the results in *repeated* moral hazard models where the optimal long-term contracts would generally be dependent

the final outcome under "non-increasing marginal returns" assumption. Our result (Theorem 3) in N-outcome model is more general, and the weaker sufficient condition is provided with *FOSD* relationships. Kwon also presents empirical analysis using personnel records in a health insurance company, and the findings are consistent with the main feature of derived optimal contract.

on the whole history of past outcomes.

We see a variety of circumstances in reality where the effort has persistent effects, and the result of the paper that the payments do not fully reflect the realization of past outcomes under such circumstances is persuasive. However, the assumption of full commitment may be too strong in some of these economic contexts. In the study of moral hazard problems, renegotiation-proof contracts have been investigated by Fudenberg and Tirole [2], Ma [6, 7], and Park [10]. In this respect, the study of dynamic moral hazard would call for further research on renegotiation.

This paper has favored the simplest models to focus upon the role of history dependence. In particular, we have assumed two actions and independent distributions over periods in the paper. Generalizations of this model also deserve further investigation.

Appendix

In this Appendix, we provide some mathematical arguments for footnote 8 in Section 5.

Suppose that the agent is to make actions dependent on past outcomes. We can think of such agent's strategy as a sequence of "behavior strategy" such as

$$\boldsymbol{\alpha} = (\alpha^1, \alpha^2(x^1), \alpha^3(x^1, x^2), \ldots, \alpha^T(x^1, \ldots, x^{T-1})),$$

where each $\alpha^t : \{1, \ldots, N\}^{t-1} \to A$ is a mapping from the history of past outcomes (up to period $t-1$) to the action in period t.

The problem in footnote 8 is whether the agent can improve his payoff by taking such history-dependent strategy $\boldsymbol{\alpha}$ (rather than history-independent strategy $\mathbf{a} = (a^1, a^2, \ldots, a^T)$).

Theorem 4. *If the contract is* simple*, then the agent cannot improve by taking history-dependent strategy* $\boldsymbol{\alpha}$.

Proof. The expected payoff to the agent taking such strategy $\boldsymbol{\alpha}$ can be written as

$$\sum_{\mathbf{a}\in A^T}\left\{\left(\sum_{(x^1,\ldots,x^{T-1})\in I(\mathbf{a},\boldsymbol{\alpha})}\prod_{t=1}^{T-1}\Pr[x^t \mid \alpha^t(x^1,\ldots,x^{t-1})]\right)\right.$$
$$\left.\times\left(\sum_{t=1}^{T-1}u(w^t)+\sum_{j=1}^{N}p_j^T(\mathbf{a})u(w_j^T)-C\cdot m(\mathbf{a})\right)\right\} \tag{8}$$

where

$$I(\mathbf{a},\boldsymbol{\alpha})=\left\{(x^1,\ldots,x^{T-1}) \mid \alpha^t(x^1,\ldots,x^{t-1})=a^t \text{ for } t=1,\ldots,T\right\},$$

that is, $I(\mathbf{a},\boldsymbol{\alpha})$ is the set of history of outcomes that action profile $\mathbf{a}$ will be played under strategy $\boldsymbol{\alpha}$ with positive probability.

Since every each history $(x^1,\ldots,x^{T-1})$ generates exactly one action profile $\mathbf{a}$ given $\boldsymbol{\alpha}$, the first parentheses in (8) can be seen as a probability distribution of $\mathbf{a}$ over A^T; that is,

$$\sum_{\mathbf{a}\in A^T}\left(\sum_{(x^1,\ldots,x^{T-1})\in I(\mathbf{a},\boldsymbol{\alpha})}\prod_{t=1}^{T-1}\Pr[x^t \mid \alpha^t(x^1,\ldots,x^{t-1})]\right)=1 \quad \text{for any } \boldsymbol{\alpha}.$$

As the expected value of random variables cannot exceed the maximum of the variables, we establish that

$$\begin{aligned}(8) &\leq \max_{\mathbf{a}\in A^T}\left(\sum_{t=1}^{T-1}u(w^t)+\sum_{j=1}^{N}p_j^T(\mathbf{a})u(w_j^T)-C\cdot m(\mathbf{a})\right)\\ &=\max_{\mathbf{a}\in A^T}\left(V(\mathbf{a},\mathbf{w})-C\cdot m(\mathbf{a})\right).\end{aligned}$$

□

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