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CAN MORE INFORMATION FACILITATE COMMUNICATION?

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Abstract

In this paper we analyze a cheap talk model with a partially informed receiver. In clear contrast to the previous literature, we find that there is a case where the receiver's prior knowledge enhances the amount of information conveyed via cheap talk. The point of departure is our explicit focus on the "dual role" of the sender's message in this context: when the receiver has imperfect private information of her own, the sender's message provides information about the true state as well as about the reliability of the receiver's private information. This feature gives rise to the asymmetric response of the receiver's action, where the receiver reacts less to the truthful message and more to the misrepresented one, which is essential in disciplining the sender to be more truthful.

Keywords: Cheap talk, Informed receiver.

JEL Classification Number: D23; D82.

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1 Introduction

This paper analyzes a cheap talk model with a partially informed receiver. More precisely, we extend the canonical model of Crawford and Sobel [3] (hereafter, CS) to a setting where the receiver is also endowed with some private information of her own, on top of the sender's message which can be observed subsequently. Within this framework, we ask how the receiver's prior knowledge affects the strategic nature of communication and, in particular, the amount of information conveyed by the sender via cheap talk messages.

Recently, several papers have explored this problem, i.e., how the nature of communication alters when the receiver becomes more informed in a broad sense (Chen [2], Lai [7], Moreno de Barreda [4]).[1] While they differ in their ways to add the receiver's private information to the model, all of these studies by and large show that the more informed the receiver is, the less information she can extract from the sender. A similar conclusion is also obtained in models with multiple senders (Austen-Smith [1], Morgan and Stocken [8], Galeotti et al. [5]).[2] For instance, Morgan and Stocken [8] examine information aggregation in polls and show that truth telling is impossible when the size of a poll is sufficiently large.

The main logic behind this result is fairly simple, if we carefully dissect why any information can be conveyed via cheap talk messages. To see this, consider the standard setup of CS where the state of nature is denoted by $t \in [0, 1]$. The receiver's bliss point is t whereas the sender's is $t + b$, $b > 0$, meaning that the sender always prefers a larger action than the receiver, and hence has an incentive to exaggerate his message. The key insight of CS is that even in this case, the receiver can still extract some information from the sender by dividing the state space into intervals. These intervals endogenously create the cost of exaggeration because if the sender exaggerates and sends a message in the next interval, the resultant action could move further to the right (towards one) and away from his bliss point. In other words, what makes this strategy work is the sensitivity of the receiver's action to the sender's (misrepresented) message. Since the receiver naturally becomes less sensitive to the sender's message when she has private information of her own, the sender's incentive to exaggerate his information is magnified and, as a consequence, the quality of communication deteriorates.

[1] In a model with the discrete state space, Ishida and Shimizu [6] also show that the receiver's prior information becomes an impediment to efficient communication.

[2] Since the model with an informed receiver does not specify the source of her private information, it inherently has a close connection with the model with multiple senders.

So, does this mean that the impact of the receiver's prior knowledge on the quality of communication is *invariably* negative? While this conclusion, and the reasoning behind it, appear fairly robust, it is still hard to believe that this single mechanism is all there is to this problem: intuition certainly suggests that there are also situations where it is more difficult to lie to a better informed receiver, so that the receiver's prior knowledgable facilitates, rather than impedes, communication. If this is the case, i.e., there is a route through which more information facilitates communication, it means that the existing literature overlooks some critical link between the receiver's information on one hand and the quality of communication on the other. The aim of this paper is to find this "missing piece" in this class of problems, if any, which hopefully gives us a clearer insight on the role of information in strategic communication.

The point of departure from the existing literature is our explicit focus on the "dual role" of the sender's message: when the receiver is partially informed, the sender's message can provide information not only about the true state but also about the reliability of the receiver's private information. The latter aspect, which has been largely neglected in the literature, is the driving force of our model. To see how this works, note that the receiver knows that her private information is less reliable than the sender's.[3] If the sender's message is consonant with what she privately knows, she thinks that her private information is more likely to be correct and, consequently, places more weight on it. As the receiver relies more on her own information, her action necessarily becomes *less* sensitive to the sender's message. If the message is not consonant with her private information, on the other hand, she loses her confidence in her private information and her action becomes *more* sensitive to the message. Since the sender's message is more likely to be consonant with the receiver's private information when the sender truthfully reveals his information, the receiver reacts less to the truthful message and more to the misrepresented one. As we will clarify in more detail later, this asymmetric response of the receiver is essential in disciplining the sender to be more truthful and hence facilitating communication between them.

We think that adding private information to the receiver's side is a natural extension of the existing literature and poses an intriguing question in itself, as it is not *a priori* clear whether the quality of communication improves or deteriorates when the receiver has more precise information. The extension also yields more practical implications when how much information to collect on her own is the receiver's endogenous choice. If the quality of communication diminishes as the receiver becomes more informed, then information acquisition and communication

[3]As in CS and most cheap talk models, we assume that the sender knows the true state with precision.

are substitutes, and the incentive to collect information certainly diminishes when the sender's information is expected to be reliable. In contrast, if the quality of communication enhances as the receiver becomes more informed, information acquisition and communication are complements, and the incentive to collect information intensifies. Our analysis shows that information acquisition and communication can be complements, rather than substitutes as the previous literature indicates, depending on the information structure on the receiver's side.

The paper is organized as follows. Section 2 outlines the model which is an extension of CS. Section 3 characterizes equilibria of the model and discusses their implications. Finally, section 4 offers some concluding remarks. All the proofs are relegated to Appendix.

2 The Model

We consider an extended version of CS's uniform-quadratic model. There are two players, the sender (male) and the receiver (female), and the model goes as follows.

1. Nature randomly draws the state of nature $t \in [0,1]$ from the uniform distribution. The state is the sender's private information.

2. The sender sends a message $m \in [0,1]$ to the receiver.

3. The receiver observes a private signal $r \in [0,1]$ which is drawn according to the following probability:

$$P(r \in A|t) = q\mathbb{I}(t \in A) + (1-q)\lambda(A),$$

   where $\mathbb{I}$ is the indicator function and λ is Lebesgue measure. In other words, the signal reflects the true state, i.e., $r = t$, with probability q while it is randomly drawn from the uniform distribution on $[0,1]$ with probability $1-q$.

4. Upon observing m and r, the receiver chooses an action $a \in [0,1]$.

The payoff for the receiver is

$$U^R(t,a) = -(t-a)^2,$$

whereas that for the sender is

$$U^S(t,a) = -(t+b-a)^2.$$

We call b the bias and assume $b \in (0, 0.5)$.

The only difference from the original CS model is that we allow for the possibility that the receiver observes a possibly informative signal of the state of nature. The signal is either perfectly informative (with probability q) or noisy (with the remaining probability), but the receiver cannot tell whether any given signal is informative or noisy. What is critical in this specification is the way noise is introduced into the receiver's signal: in the current setting, there is a positive probability that the observed signal is a complete noise containing no useful information.[4] This feature is essential in giving rise to the dual role of the sender's message as we detail below. We interpret q as the accuracy of the signal where the model is equivalent to the original CS model when $q = 0$.

3 Analysis

3.1 The equilibrium concept

Throughout the analysis, we focus on the class of *monotone partition equilibria*, which is a subset of perfect Bayesian equilibria, defined as below.[5]

Definition 1 Let μ_t denote the type t sender's strategy (i.e., a probability distribution over the message space $[0, 1]$). *A monotone partition strategy (MPS)* is the sender's strategy where there exists a partition of $[0, 1]$, $\{T_i\}_{i \in I}$ (I is some index set) such that

- T_i is a non-empty interval for any $i \in I$,
- $\mu_t = \mu_{t'}$ for any $i \in I$ and any $t, t' \in T_i$, and
- *Supp* $\mu_t \cap$ *Supp* $\mu_{t'} = \emptyset$ for any distinctive $i, i' \in I$ and any $t \in T_i$, $t' \in T_{i'}$.

A monotone partition equilibrium (MPE) is a perfect Bayesian equilibrium where the sender's strategy is an MPS.

[4]The assumption that the signal contains no information with some probability is made only for analytical simplicity and not essential. Our results holds in a qualitative sense as long as there is some positive probability that the signal is sufficiently weakly correlated with the true state.

[5]The need for this focus arises from a special feature of cheap talk models with an informed receiver. When the receiver is endowed with some information of her own, the sender typically induces lotteries over actions, not actions themselves. This feature produces some complicated equilibria that never exist in the original CS model: for example, Chen [2] shows that there exist a non-monotone equilibrium in a version of cheap talk models with an informed receiver. We rule out this possibility given the question we set out to solve, although it is certainly intriguing as a theoretical possibility. Note that CS shows that in the case of $q = 0$, there exists only monotone partition equilibria.

We can show that any MPE consists of finite intervals.

Proposition 1 For any MPE, I is a finite set.

Based on this proposition, we denote any MPE partition by $\{T_n\}_{n=1,\ldots,N}$ where N is some natural number and $\sup T_n = \inf T_{n+1}$ for $n = 1, \ldots, N-1$. We call T_n the nth interval. Furthermore, we identify an MPS with a partition $\{T_n\}_{n=1,\ldots,N}$ as a vector $\mathbf{t} = (t_0, \ldots, t_N)$ defined as

$$t_n \overset{d}{=} \begin{cases} 0 & \text{for } n = 0, \\ \sup T_n & \text{for } n = 1, \ldots, N. \end{cases}$$

We call such t_n a threshold. Note that $0 = t_0 < t_1 < \cdots < t_N = 1$. Let $\tau_n = t_n - t_{n-1}$ denote the length of the nth interval which, as usual, is taken as a measure of the quality of communication: the shorter each interval is, the more information is conveyed via cheap talk in equilibrium.

3.2 The receiver's problem

In any MPS, the receiver has two sources of information: her own signal r and the sender's message m which indicates in which interval the true state is lying. Given this, the receiver's equilibrium strategy is pure and we denote it by $\alpha(m, r)$. Furthermore, the actions induced on any equilibrium path are determined as follows:

$$\alpha(m, r) = \begin{cases} \frac{q}{q+(1-q)\tau_n} r + \frac{(1-q)\tau_n}{q+(1-q)\tau_n} \frac{t_n + t_{n-1}}{2} & \text{if } \exists n \ \forall t \in T_n \ \mu_t(m) > 0 \text{ and } r \in T_n, \\ \frac{t_n + t_{n-1}}{2} & \text{if } \exists n \ \forall t \in T_n \ \mu_t(m) > 0 \text{ and } r \notin T_n. \end{cases}$$

Note that the receiver uses her own information only when the sender's message falls into the same interval. On the other hand, the receiver sees her private information as a noise and disregards it altogether if the message does not agree with her private signal, given that the sender plays the equilibrium (truth-telling) strategy. This is the critical feature of the current model: when the message is "close" to the receiver's signal, she places more confidence in her signal; when it is "further away", she relies less on it and more on the sender's message. As we will see later, this asymmetric response is what disciplines the sender to be more truthful. The current model provides a setup which captures this dual role of the sender's message in a relatively tractable manner.

Remark 1 This argument implies that a subtle difference in the information structure could result in a large qualitative change in equilibrium outcomes. To elaborate more on the difference from the existing literature, consider an alternative information structure in which the receiver can observe whether the state is low $[0, 0.5]$ or high $(0.5, 1]$, as assumed in Lai [7].[6] Now suppose that there exists an equilibrium with two intervals, $\{[0, t_1], (t_1, 1]\}$, and moreover that $t \leq t_1 < 0.5$ so that the receiver observes a "low" signal. In this environment, if the sender deviates and claims that the state is in $(t_1, 1]$, the receiver thinks that the state must lie in $(t_1, 0.5]$ and chooses $a = \frac{2t_1+1}{4}$.[7] Since the uninformed receiver would think that the state must lie in $(t_1, 1]$ and choose $a = \frac{t_1+1}{2}$, the presence of the receiver's prior information makes her less sensitive to the misrepresented message, which magnifies the incentive to exaggerate.

3.3 The equilibrium conditions

Given the receiver's strategy, we can now identify the conditions for an MPE with a partition $\mathbf{t}$ (an MPE with $\mathbf{t}$ for short) by checking the sender's incentives. In particular, what we need to see is that given some partition $\mathbf{t}$ and the true state t, the sender has no incentive to deviate by sending a "nearby" message. To this end, define $\Delta(t; t_{n-1}, t_n, t_{n+1})$ as follows:

$$\Delta(t; t_{n-1}, t_n, t_{n+1}) = -\int_0^1 [t + b - \alpha(m_{n+1}, r)]^2 \, P(dr \mid t) + \int_0^1 [t + b - \alpha(m_n, r)]^2 \, P(dr \mid t),$$

where m_n is any message sent (with positive probability) when the true state lies in the nth interval. Then, a necessary condition for the equilibrium with N intervals is that

$$\Delta(t; t_{n-1}, t_n, t_{n+1}) \begin{cases} \leq 0 & \text{for } t \in (t_{n-1}, t_n) \\ \geq 0 & \text{for } t \in (t_n, t_{n+1}) \end{cases}$$

holds for any $n = 1, \ldots, N-1$. Moreover, a sufficient condition is that

$$\Delta(t; t_{n-1}, t_n, t_{n+1}) \begin{cases} \leq 0 & \text{for } t < t_n \\ \geq 0 & \text{for } t > t_n \end{cases}$$

holds for any $n = 1, \ldots, N-1$.

In the original CS model, the length of each partition must satisfy certain conditions in equilibrium: in the linear-quadratic specification, each interval must be exactly $4b$ longer than the last. The following result establishes the conditions along this line in our extended setup.

[6] Strictly speaking, in Lai [7], the threshold, which is exogenously set at 0.5 in this example, is also only private known and drawn from the uniform distribution.

[7] In contrast, if the sender reveals truthfully, the receiver ignores her own signal and chooses $a = \frac{t_1}{2}$. We do not focus much on this side because there is no qualitative difference in the way the receiver updates her belief. Even though the receiver ignores her own signal, this is to some extent a figment of the simplified information structure as assumed here. For instance, if the receiver's signal space is partitioned into three intervals, very low $[0, \varepsilon)$, low $[\varepsilon, 0.5)$ and high $[0.5, 1]$, and the true state lies in $[\varepsilon, 0.5)$, she uses both pieces of her information and chooses $a = \frac{\varepsilon+t_1}{2}$ on the equilibrium path.

Proposition 2 Define

$$
\begin{aligned}
&G^{+}(\tau_n, \tau_{n+1}; q)\\
&\quad \overset{d}{=} (\tau_{n+1} + \tau_n)(\tau_{n+1} - \tau_n - 4b)\\
&\qquad + \frac{q^2(1-q)\tau_{n+1}^3}{3\,(q+(1-q)\tau_{n+1})^2} + \frac{2q^2(1-q)\tau_n^3}{3\,(q+(1-q)\tau_n)^2} + \frac{q^2\tau_n(\tau_n+4b)}{q+(1-q)\tau_n},\\
&G^{-}(\tau_n, \tau_{n+1}; q)\\
&\quad \overset{d}{=} (\tau_{n+1} + \tau_n)(\tau_{n+1} - \tau_n - 4b)\\
&\qquad - \frac{2q^2(1-q)\tau_{n+1}^3}{3\,(q+(1-q)\tau_{n+1})^2} - \frac{q^2(1-q)\tau_n^3}{3\,(q+(1-q)\tau_n)^2} - \frac{q^2\tau_{n+1}(\tau_{n+1}-4b)}{q+(1-q)\tau_{n+1}}.
\end{aligned}
$$

(i) A necessary and sufficient condition for an MPE with MPS $\{T_n\}_{n=1,\ldots,N}$ is $G^{+}(\tau_n, \tau_{n+1}; q) \geq 0$ and $G^{-}(\tau_n, \tau_{n+1}; q) \leq 0$ for $n = 1, \ldots, N-1$.

(ii) For any $\tau_n \in (0,1)$ and $q \in [0,1)$, there exists $\underline{\tau}(\tau_n, q)$ and $\overline{\tau}(\tau_n, q)$ such that $G^{+}(\tau_n, \tau_{n+1}; q) \geq 0$ and $G^{-}(\tau_n, \tau_{n+1}; q) \leq 0$ if and only if $\underline{\tau}(\tau_n, q) \leq \tau_{n+1} \leq \overline{\tau}(\tau_n, q)$.

(iii) For any $\tau_n \in (0,1)$ and $q \in [0,1)$, $G^{+}(\tau_n, \underline{\tau}(\tau_n, q); q) = 0$ and $G^{-}(\tau_n, \overline{\tau}(\tau_n, q); q) = 0$.

(iv) For any $\tau_n \in (0,1)$ and $q \in [0,1)$, $2b < \underline{\tau}(\tau_n, q) \leq \tau_n + 4b \leq \overline{\tau}(\tau_n, q)$ and

$$
\begin{aligned}
q = 0 \ &\Leftrightarrow\ \underline{\tau}(\tau_n, q) = \tau_n + 4b = \overline{\tau}(\tau_n, q),\\
q > 0 \ &\Leftrightarrow\ \underline{\tau}(\tau_n, q) < \tau_n + 4b < \overline{\tau}(\tau_n, q).
\end{aligned}
$$

The proposition shows that the equilibrium conditions are less stringent when $q > 0$ in that the length of each partition only needs to be in some range. With more breathing room, we can construct "more informative" equilibria. To see the intuition behind this result, consider an equilibrium with two intervals, $\{[0, t_1], (t_1, 1]\}$. When $q = 0$, it is straightforward to compute $\alpha(m_0, r) = t_1/2$ and $\alpha(m_1, r) = (t_1 + 1)/2$ regardless of r. At $t = t_1$, the sender must be indifferent between the two messages, i.e., his bliss point must be at the midpoint of $\alpha(m_0, r)$ and $\alpha(m_1, r)$. Let b' denote the bias which satisfies

$$
t_1 + b' = \frac{1}{2}\left(\frac{t_1}{2} + \frac{t_1+1}{2}\right) \ \Leftrightarrow b' = \frac{1-2t_1}{4}.
$$

It is well known that the second interval must be exactly $4b$ longer than the first in this linear-quadratic specification.

We now let q increase above zero and see how that changes the sender's incentives. There are two effects at work, which we call the information effect and the risk effect for expositional clarity, depending on whether the receiver observes a correct signal or not. These effects are absent when the receiver is uninformed ($q = 0$), and mark a key departure from the original CS model.

First, suppose that the true state is $t = t_1$, and also that the receiver happens to observe the true state, i.e., $r = t_1$ (though she does not know it for sure). If the sender reveals truthfully, i.e., $m = m_0$, the receiver's private signal is consonant with the sender's message, and the receiver combines the two pieces of evidence to determine her action: the resultant action is hence necessarily gravitated towards the true state t_1 away from what the message indicates, i.e., $t_1/2$. If the sender chose to deviate and send $m = m_1$, on the other hand, the receiver's reaction would totally be different, now that the sender's message is dissonant with the receiver's signal. Under the presumption that the sender plays the equilibrium strategy, the receiver must think that her signal is a noise and places zero weight in Bayesian updating: the resultant action hence stays at $(t_1+1)/2$ regardless of q. We refer to this as the information effect of the receiver's prior knowledge, which works to discipline the sender to be more truthful.

Second, suppose that the receiver observes $r \neq t$, in which case she (mistakenly) uses her private signal with some positive probability. Given that $\tau_1 > \tau_0$, $r \in \tau_1$ is the more dominant case. In this case, if the sender follows the equilibrium strategy, the receiver now ignores her signal and deterministically chooses an action regardless of r; if he deviates, the receiver combines the wrong signal with the message and hence produces a stochastic action. The stochastic nature of the latter case introduces irrelevant noise into the receiver's action, thereby reducing the expected payoff. We refer to this as the risk effect which also works to discipline the sender to be more truthful.[8]

With these two effects and the consequent asymmetric response, $t = t_1$ is no longer on the border. It is now strictly better for the sender with $t = t_1$ to send $m = m_0$, meaning that the threshold can be "pushed further to the right." Even with the same number of intervals, we can construct a more informative equilibrium by having more equally divided intervals. We will

[8]Obviously, applying the same logic, the risk effect works in a way to make the deviation more attractive when the receiver observes a wrong signal but that happens to fall into the right interval. In a typical MPE, however, this is less likely to occur and its effect is usually dominated.

formalize the above discussion in the next subsection.

3.4 Can more information facilitate communication?

We are now ready to address our main question of how a change in q affects the nature of communication. We first establish that an increase in q *per se* enhances the payoffs of both players. To this end, define $V^i(\mathbf{t}; q)$, $i = S, R$, as player i's ex ante expected payoff. Then, we can obtain the following result.

Proposition 3 $\dfrac{\partial V^i}{\partial q} > 0$ for $i = S, R$.

This result is somewhat straightforward as it simply states that the players benefit from having access to more information. What is more interesting is whether an increase in q can afford a more efficient way of communication, or a more efficient configuration of the equilibrium partition. To this end, it is convenient to associate each MPE explicitly with the information accuracy q and denote it by an MPE-q. Given this, we define the following notions.

Definition 2 We say that:

(i) A partition $\mathbf{t}$ is *more efficient* than $\mathbf{t}'$ at q if $V^i(\mathbf{t}, q) > V^i(\mathbf{t}', q)$ for $i = S, R$.

(ii) *More information facilitates communication* at (q, q'), $q > q'$, if for any MPE-q' with $\mathbf{t}'$, there exists an MPE-q with a partition $\mathbf{t}$ which is more efficient than $\mathbf{t}'$ at q'.

Given this, we can obtain the following result which is derived directly from Proposition 2.

Proposition 4 For any MPE-0 with $\mathbf{t}$ and any $q > 0$, $\mathbf{t}$ also constitutes an MPE-q. Furthermore, more information facilitates communication at $(q, 0)$ for any $q > 0$.

Remark 2 While we show the above result by comparing among the MPEs with the same number of intervals, we can also easily construct an example where an increase in q results in an increase in the maximum number of intervals. For instance, suppose that $b = 0.25$, in which case the unique equilibrium is babbling when $q = 0$. This conclusion does not hold, however, when $q > 0$. Since $G^+(0, 1) > 0$ and $G^-(0, 1) < 0$, for a sufficiently small $t_1 > 0$, $(0, t_1, 1)$ also satisfies $G^+(\tau_n, \tau_{n+1}) > 0$ and $G^-(\tau_n, \tau_{n+1}) < 0$ due to the continuity of G^+ and G^-. In other words, there always exists an MPE-q, $q > 0$, with two intervals when $b = 0.25$.

Proposition 4 establishes that more information facilitates communication at $(q, 0)$ for any $q > 0$, but does not show that more information *monotonically* facilitates communication. The next question is thus whether this condition holds for any arbitrary pair (q, q') for $q > q' > 0$. To do so, we need to identify the most efficient MPE-q (an MPE with the most efficient partition). If $\tau_n < \tau_{n+1}$ holds for $n = 1, \dots N-1$, the most efficient equilibrium requires $\tau_n = \underline{\tau}(\tau_n, q)$ for $n = 1, \dots, N-1$ (Lemma 6 in the Appendix). Unfortunately, in our setting, $\tau_n < \tau_{n+1}$ is violated in some MSE. See the following example:

Example 1 When $b < \frac{1}{8}$ and q is sufficiently large, there exists an MPE with two intervals where $\tau_1 \geq \tau_2$. This is verified from the fact that $G^+(0.5, 0.5; q) > 0$ holds for a sufficiently large q.

However, it is verified that a partition of this kind never constitutes an MPE whenever q is sufficiently small (Lemma 7 in the Appendix). Moreover, it is also verified that $\underline{\tau}$ is strictly decreasing in q whenever q is sufficiently small (Lemma 8 in the Appendix). From these facts, we conclude that more information facilitates communication whenever we focus on the situation where the accuracy is sufficiently low.

Proposition 5 There exists a $\overline{q} > 0$ such that for any $q \in (0, \overline{q}]$ and $q' < q$, more information facilitates communication at (q, q').

As a final note, we would like to present some counterexamples to show that the effect of the receiver's prior knowledge does not monotonically improve the quality of communication. Figure 1 focuses on MPEs which admit two intervals, and depicts the length of the first interval of the most efficient MPE for different values of b. Since these lengths are below 0.5, they also directly represent the efficiency level. The figure clearly shows that for $b = 0.225$ and $b = 0.25$, more information results in a less efficient partition when q is sufficiently close to one. The reason for this is that as q increases, the receiver's signal becomes more accurate and more likely to fall into the right interval. An increase in q thus makes the risk effect less relevant, which works as the first-order effect. Since the risk effect is more salient when the bias is large, this first-order effect eventually dominates and lowers the quality of communication as q approaches one.

It is important to note, however, that our focus is generally on the case where q is relatively small because there is little point in communicating when the receiver already knows the true state with sufficient precision. We can thus argue that in a class of situations where communication is relevant and beneficial, the effect of the receiver's prior knowledge on the quality of communication is largely positive, if the underlying information structure has a feature that gives rise to the dual role of the sender's message.

4 Conclusion

In this paper we analyze a cheap talk model with a partially informed receiver. In clear contrast to the previous literature, we find that there is a case where the receiver's prior knowledge enhances the amount of information conveyed via cheap talk messages. This contrasting result is mainly due to the structure of the receiver's private information we assume. While we do not intend to insist that the information structure of the current form is a necessary feature in this class of problems, it provides an insight that the information structure matters for the impact of the receiver's information on the quality of communication. In future, it seems worthwhile to explore more on this point, as to when and under what conditions the receiver's information facilitates communication.

Appendix

Proof of Proposition 1:

The proof closely follows that of Proposition 1 of Moreno de Barreda [4] and directly stems from the following lemma.

Lemma 1 The width of an interval T_i in Difinition 1 is longer than or equal to $2b$ unless $\inf T_i = 0$.

Proof:

Suppose to the contrary that there exists an interval T_i such that

$$0 < \inf T_i \leq \sup T_i < \inf T_i + 2b.$$

We divide the situation into the following two cases: $\{\inf T_i\} \notin T_i$ or $\{\inf T_i\} \in T_i$.

We first consider the case of $\{\inf T_i\} \notin T_i$. In this case the point $\inf T_i$ belongs to another interval, say T_j, and

$$\inf T_j \leq \sup T_j = \inf T_i < \sup T_i$$

holds. Pick any two distinctive messages m_i and m_j such that

$$\mu_t(m_i) > 0 \quad \text{iff } t \in T_i,$$
$$\mu_t(m_j) > 0 \quad \text{iff } t \in T_j.$$

Then,

$$\sup T_i > \alpha(m_i, r) > \inf T_i \geq \alpha(m_j, r)$$

holds for any $r \in [0, 1]$. This implies that

$$|\inf T_i + b - \alpha(m_i, r)| < b \leq |\inf T_i + b - \alpha(m_j, r)|$$

holds for any $r \in [0, 1]$. However, this eliminates the sender's incentive to send a message m_j at $t = \inf T_i$. This is a contradiction.

Next, we consider the case of $\{\inf T_i\} \in T_i$. In this case there exists an interval T_j such that

$$\inf T_j < \sup T_j = \inf T_i \leq \sup T_j.$$

Pick any two distinctive messages m_i and m_j such that

$$\mu_t(m_i) > 0 \quad \text{iff } t \in T_i,$$
$$\mu_t(m_j) > 0 \quad \text{iff } t \in T_j.$$

Then,

$$\sup T_i \geq \alpha(m_i, r) \geq \inf T_i > \alpha(m_j, r)$$

holds for any $r \in [0, 1]$. This implies that

$$|\inf T_i + b - \alpha(m_i, r)| < b < |\inf T_i + b - \alpha(m_j, r)|$$

holds for any $r \in [0, 1]$. Then, there exists a sufficiently small $\epsilon > 0$ such that $\{\inf T_i - \epsilon\} \in T_j$ and

$$|\inf T_i - \epsilon + b - \alpha(m_i, r)| < |\inf T_i - \epsilon + b - \alpha(m_j, r)|$$

holds. However, this eliminates the sender's incentive to send a message m_j at $t = \inf T_i - \epsilon$. This is a contradiction. ∎

Proof of Proposition 2:

For $t \in T_n$,

$$
\begin{aligned}
&\Delta(t; t_{n-1}, t_n, t_{n+1}) = \\
&\quad - \left(t + b - \frac{t_{n+1} + t_n}{2} \right)^2 \\
&\quad - \frac{q(1-q)}{q + (1-q)\tau_{n+1}} \int_{r \in T_{n+1}} \left(r - \frac{t_{n+1} + t_n}{2} \right) \\
&\quad\quad \times \left\{ \frac{q}{q + (1-q)\tau_{n+1}} r - 2t - 2b + \left[2 - \frac{q}{q + (1-q)\tau_{n+1}} \right] \frac{t_{n+1} + t_n}{2} \right\} dr \\
&\quad + \left(t + b - \frac{t_n + t_{n-1}}{2} \right)^2 \\
&\quad + \frac{q^2}{q + (1-q)\tau_n} \left(t - \frac{t_n + t_{n-1}}{2} \right) \left\{ - \left[2 - \frac{q}{q + (1-q)\tau_n} \right] \left(t - \frac{t_n + t_{n-1}}{2} \right) - 2b \right\} \\
&\quad + \frac{q(1-q)}{q + (1-q)\tau_n} \int_{r \in T_n} \left(r - \frac{t_n + t_{n-1}}{2} \right) \\
&\quad\quad \times \left\{ \frac{q}{q + (1-q)\tau_n} r - 2t - 2b + \left[2 - \frac{q}{q + (1-q)\tau_n} \right] \frac{t_n + t_{n-1}}{2} \right\} dr.
\end{aligned}
$$

$$
\frac{\partial \Delta}{\partial t} = \tau_{n+1} + \tau_n - \frac{2q^2}{q + (1-q)\tau_n} \left[\left(2 - \frac{q}{q + (1-q)\tau_n} \right) \left(t - \frac{t_n + t_{n-1}}{2} \right) + b \right].
$$

Then, if $\lim_{t \uparrow t_n} \frac{\partial \Delta}{\partial t} \geq 0$, or equivalently

$$
\frac{2q^2 b}{q + (1-q)\tau_n} \leq \tau_{n+1} + \tau_n \left\{ 1 - \frac{q^2}{q + (1-q)\tau_n} \left[2 - \frac{q}{q + (1-q)\tau_n} \right] \right\} \tag{1}
$$

holds, then $\frac{\partial \Delta}{\partial t} \geq 0$ for $t \in T_n$. Therefore, $\Delta(t; t_{n-1}, t_n, t_{n+1}) \leq 0$ for $t \in T_n$ if and only if $\lim_{t \uparrow t_n} \Delta(t; t_{n-1}, t_n, t_{n+1}) \leq 0$. This is written as

$$
G^+(\tau_n, \tau_{n+1}) \geq 0. \tag{2}
$$

Clearly, (2) is also a necessary condition. Here, we obtain the following results by direct calculation:

Lemma 2 For any $\tau_n \in [0,1]$ and τ_{n+1},

$$\begin{aligned}
&\frac{\partial^2 G^+(\tau_n,\tau_{n+1})}{\partial \tau_{n+1}^2} > 0,\\
&\frac{\partial G^+(\tau_n,0)}{\partial \tau_{n+1}} < 0,\\
&\frac{\partial G^+(\tau_n,2b)}{\partial \tau_{n+1}} \geq 0,\\
&G^+(\tau_n,0) < 0,\\
&G^+(\tau_n,2b) < 0,\\
&G^+(\tau_n,\tau_n+4b) \geq 0, \quad \text{"="} \text{ holds iff } q=0.
\end{aligned}$$

This lemma implies that for any $\tau_n \in [0,1]$ and $\tau_{n+1} \geq 0$, (2) holds if and only if $\tau_{n+1} \geq \underline{\tau}(\tau_n,q)$ where $\underline{\tau}(\tau_n,q)$ is defined as the unique solution $\tilde{\tau} \in [0,\infty)$ of $G^+(\tau_n,\tilde{\tau}) = 0$. Moreover, $\underline{\tau}(\tau_n,q) \in (2b,\tau_n+4b]$ and $\underline{\tau}(\tau_n,q) = \tau_n+4b$ if and only if $q=0$.

Since $\tau_{n+1} \geq 2b$ implies that (1) holds for any $\tau_n \geq 0$, (1) is redundant for any $\tau_n \in [0,1]$ and $\tau_{n+1} \geq \underline{\tau}(\tau_n,q)$.

For $t \in T_{n+1}$,

$$\begin{aligned}
&\Delta(t;t_{n-1},t_n,t_{n+1}) = \\
&\quad -\left(t+b-\frac{t_{n+1}+t_n}{2}\right)^2 \\
&\quad -\frac{q^2}{q+(1-q)\tau_{n+1}}\left(t-\frac{t_{n+1}+t_n}{2}\right)\left\{-\left[2-\frac{q}{q+(1-q)\tau_{n+1}}\right]\left(t-\frac{t_{n+1}+t_n}{2}\right)-2b\right\} \\
&\quad -\frac{q(1-q)}{q+(1-q)\tau_{n+1}}\int_{r\in T_{n+1}}\left(r-\frac{t_{n+1}+t_n}{2}\right) \\
&\qquad \times\left\{\frac{q}{q+(1-q)\tau_{n+1}}r-2t-2b+\left[2-\frac{q}{q+(1-q)\tau_{n+1}}\right]\frac{t_{n+1}+t_n}{2}\right\}dr \\
&\quad +\left(t+b-\frac{t_n+t_{n-1}}{2}\right)^2 \\
&\quad +\frac{q(1-q)}{q+(1-q)\tau_n}\int_{r\in T_n}\left(r-\frac{t_n+t_{n-1}}{2}\right) \\
&\qquad \times\left\{\frac{q}{q+(1-q)\tau_n}r-2t-2b+\left[2-\frac{q}{q+(1-q)\tau_n}\right]\frac{t_n+t_{n-1}}{2}\right\}dr.
\end{aligned}$$

$$\frac{\partial\Delta}{\partial t} = \tau_{n+1}+\tau_n+\frac{2q^2}{q+(1-q)\tau_{n+1}}\left[\left(2-\frac{q}{q+(1-q)\tau_{n+1}}\right)\left(t-\frac{t_{n+1}+\tau_n}{2}\right)+b\right] > 0.$$

Then, $\Delta(t; t_{n-1}, t_n, t_{n+1}) \geq 0$ for $t \in T_{n+1}$ if and only if $\lim_{t \downarrow t_n} \Delta(t; t_{n-1}, t_n, t_{n+1}) \geq 0$. This is written as

$$G^{-}(\tau_n, \tau_{n+1}) \leq 0. \tag{3}$$

Clearly, (3) is also a necessary condition. Here, we obtain the following results by direct calculations:

Lemma 3 For any $\tau_n \in [0,1]$ and $\tau_{n+1} \geq 0$,[9]

$$\begin{aligned}
&\frac{\partial^3 G^{-}(\tau_n, \tau_{n+1})}{\partial \tau_{n+1}^3} \geq 0, \\
&\frac{\partial^2 G^{-}(\tau_n, 2b)}{\partial \tau_{n+1}^2} > 0, \\
&\frac{\partial G^{-}(\tau_n, 2b)}{\partial \tau_{n+1}} \leq 0, \\
&\lim_{\tau_{n+1} \to \infty} \frac{\partial G^{-}(\tau_n, \tau_{n+1})}{\partial \tau_{n+1}} > 0, \\
&G^{-}(\tau_n, 2b) < 0, \\
&G^{-}(\tau_n, \tau_n + 4b) \leq 0, \quad \text{"="} \text{ holds iff } q = 0.
\end{aligned}$$

This lemma implies that for any $\tau_n \in [0,1]$ and $\tau_{n+1} \geq 2b$, (3) holds if and only if $\tau_{n+1} \leq \overline{\tau}(\tau_n, q)$ where $\overline{\tau}(\tau_n, q)$ is defined as the unique solution $\tilde{\tau} \in [2b, \infty)$ of $G^{-}(\tau_n, \tilde{\tau}) = 0$. Moreover, $\overline{\tau}(\tau_n, q) \geq \tau_n + 4b$ and the equality holds if and only if $q = 0$.

[9]The second inequality holds under the assumption that $b < 0.5$.

For $t \in T/(T_n \cup T_{n+1})$,

$$
\begin{aligned}
&\Delta(t; t_{n-1}, t_n, t_{n+1}) = \\
&\quad -\left(t + b - \frac{t_{n+1} + t_n}{2}\right)^2 \\
&\quad - \frac{q(1-q)}{q + (1-q)\tau_{n+1}} \int_{r \in T_{n+1}} \left(r - \frac{t_{n+1} + t_n}{2}\right) \\
&\quad\quad \times \left\{ \frac{q}{q + (1-q)\tau_{n+1}} r - 2t - 2b + \left[2 - \frac{q}{q + (1-q)\tau_{n+1}}\right] \frac{t_{n+1} + t_n}{2} \right\} dr \\
&\quad + \left(t + b - \frac{t_n + t_{n-1}}{2}\right)^2 \\
&\quad + \frac{q(1-q)}{q + (1-q)\tau_n} \int_{r \in T_n} \left(r - \frac{t_n + t_{n-1}}{2}\right) \\
&\quad\quad \times \left\{ \frac{q}{q + (1-q)\tau_n} r - 2t - 2b + \left[2 - \frac{q}{q + (1-q)\tau_n}\right] \frac{t_n + t_{n-1}}{2} \right\} dr.
\end{aligned}
$$

Note that

$$\frac{\partial \Delta}{\partial t} = \tau_{n+1} + \tau_n > 0.$$

Then, $\Delta(t; t_{n-1}, t_n, t_{n+1}) \leq 0$ for $t \in T_{n'}$ and $n' < n$ if and only if $\lim_{t \uparrow t_{n'}} \Delta(t; t_{n-1}, t_n, t_{n+1}) \leq 0$. The latter holds if and only if $\lim_{t \uparrow t_{n-1}} \Delta(t; t_{n-1}, t_n, t_{n+1}) \leq 0$ since $\lim_{t \uparrow t_{n'}} \Delta(t; t_{n-1}, t_n, t_{n+1}) = \lim_{t \downarrow t_{n'}} \Delta(t; t_{n-1}, t_n, t_{n+1})$ for $n' < n-1$. It is verified that (3) assures this.

Similarly, $\Delta(t; t_{n-1}, t_n, t_{n+1}) \geq 0$ for $t \in T_{n'}$ and $n' > n+1$ if and only if $\lim_{t \downarrow t_{n'}} \Delta(t; t_{n-1}, t_n, t_{n+1}) \geq 0$. The latter holds if and only if $\lim_{t \downarrow t_{n+1}} \Delta(t; t_{n-1}, t_n, t_{n+1}) \geq 0$ since $\lim_{t \downarrow t_{n'}} \Delta(t; t_{n-1}, t_n, t_{n+1}) = \lim_{t \uparrow t_{n'}} \Delta(t; t_{n-1}, t_n, t_{n+1})$ for $n' > n+1$. It is verified that (2) assures this. ∎

Proof of Proposition 3:

This proposition is obtained from the following lemma:

Lemma 4 $V^S(\mathbf{t}; q) = -\sum_{n=1}^{N} \frac{(1-q)(q+\tau_n)\tau_n^3}{12\,(q + (1-q)\tau_n)} - b^2$ and $V^R(\mathbf{t}; q) = -\sum_{n=1}^{N} \frac{(1-q)(q+\tau_n)\tau_n^3}{12\,(q + (1-q)\tau_n)}$

This lemma is in turn obtained by direct calculation. ∎

Proof of Proposition 4:

This proposition is obtained from Proposition 2 and the following lemma:

Lemma 5 Given $\mathbf{t} = (t_0, \dots t_N)$, $\mathbf{t}' = (t'_0, \dots t'_N)$, and $\hat{n}$ such that

- $t_{\hat{n}} > t'_{\hat{n}}$,
- $t_n = t'_n$ for $n \neq \hat{n}$, and
- $\tau_{\hat{n}} < \tau_{\hat{n}+1}$.

Then, $\mathbf{t}$ is more efficient than $\mathbf{t}'$ under any q.

Proof:

We denote

$$M(\tau, q) \stackrel{d}{=} -\frac{(q+\tau)\tau^3}{q+(1-q)\tau}.$$

Then, it suffices to show that

$$M(\tau_n, q) + M(\tau_{n+1}, q) - M(\tau'_n, q) - M(\tau'_{n+1}, q) > 0.$$

It is verified that

$$\frac{\partial}{\partial \tau}\left(\frac{\partial M}{\partial q}\right) < 0,$$
$$\frac{\partial^2}{\partial \tau^2}\left(\frac{\partial M}{\partial q}\right) < 0.$$

These imply that

$$\frac{\partial}{\partial q}\left[M(\tau_n, q) + M(\tau_{n+1}, q) - M(\tau'_n, q) - M(\tau'_{n+1}, q)\right] > 0.$$

Combining this with the fact that

$$M(\tau_n, 0) + M(\tau_{n+1}, 0) - M(\tau'_n, 0) - M(\tau'_{n+1}, 0) > 0,$$

we obtain the lemma. ∎

Proof of Proposition 5:

The proof is based on the following series of lemmas.

Lemma 6 Given b and $q > 0$. If $\mathbf{t}$ is the most efficient MPE and $\tau_n < \tau_{n+1}$ for $n = 1, \dots, N-1$, then $\tau_{n+1} = \underline{\tau}(\tau_n, q)$ for $n = 1, \dots, N-1$.

Proof:

Suppose the contrary. Define $\hat{n}$ such that

$$\hat{n} \stackrel{d}{=} \min\left\{n | \tau_{n+1} > \underline{\tau}(\tau_n, q)\right\}.$$

Then, $\hat{n} = 1$ or

$$\tau_{\hat{n}} = \underline{\tau}(\tau_{\hat{n}-1}, q) < \overline{\tau}(\tau_{\hat{n}-1}, q).$$

Thus, we can find another MPE $\mathbf{t}'$ where $t'_{\hat{n}}$ is slightly higher than $t_{\hat{n}}$, $t'_n = t_n$ for $n \neq \hat{n}$, and the presupposition of Lemma 5 holds at $n = \hat{n}$. Then the lemma is obtained from Lemma 5. ∎

Lemma 7 There exists $\hat{q} > 0$ such that $\underline{\tau}(\tau, q) > \tau$ for any $\tau \in (0, 1)$ and $q \in [0, \hat{q}]$.

Proof:

By Proposition 2, $\underline{\tau}(\tau, 0) = \tau + 4b > \tau$ for any $\tau \in (0, 1)$. Then, there must exist $\hat{q} > 0$ such that $\hat{\tau}(\tau, q) > \tau$ for any $\tau \in (0, 1)$ and $q \in [0, \hat{q}]$. ∎

Lemma 8 There exists $\check{q} > 0$ such that $\dfrac{\partial \underline{\tau}}{\partial q} < 0$ for any $\tau \in (0, 1)$ and $q \in (0, \check{q}]$.

Proof:

Since $\frac{\partial G^+}{\partial \tau_{n+1}} > 0$ for $\tau_{n+1} > 2b$, $\frac{\partial G^+}{\partial \tau_{n+1}}|_{\tau_{n+1}=\underline{\tau}(\tau_n)} > 0$ by Lemma 2.

On the other hand,

$$\frac{1}{q}\frac{\partial G^+}{\partial q} =$$
$$\frac{\left(-q^2 + (1-q)(2-q)\tau_{n+1}\right)\tau_{n+1}^3}{3\left(q + (1-q)\tau_{n+1}\right)^3} + \frac{2\left(-q^2 + (1-q)(2-q)\tau_n\right)\tau_n^3}{3\left(q + (1-q)\tau_n\right)^3} + \frac{\left(q + (2-q)\tau_n\right)\tau_n(\tau_n + 4b)}{\left(q + (1-q)\tau_n\right)^2}.$$

Then, Lemma 2 implies

$$\begin{aligned}
\frac{1}{q}\frac{\partial G^+}{\partial q}|_{q=0,\tau_{n+1}=\underline{\tau}(\tau_n)} &= \frac{2}{3}\underline{\tau}(\tau_n) + \frac{4}{3}\tau_n + 2(\tau_n + 4b) \\
&> \frac{2}{3}2b + 8b \\
&= \frac{28}{3}b.
\end{aligned}$$

Therefore, there exists $\check{q}$ such that $\frac{\partial G^+}{\partial q}|_{\tau_{n+1}=\underline{\tau}(\tau_n)} > 0$ for any $q \in (0, \check{q}]$. ∎

We now return to the proof of Proposition 5. Letting $\overline{q} \stackrel{d}{=} \min\{\hat{q}, \check{q}\}$, the proposition then follows from Lemmas 5, 6, 7, and 8.

∎

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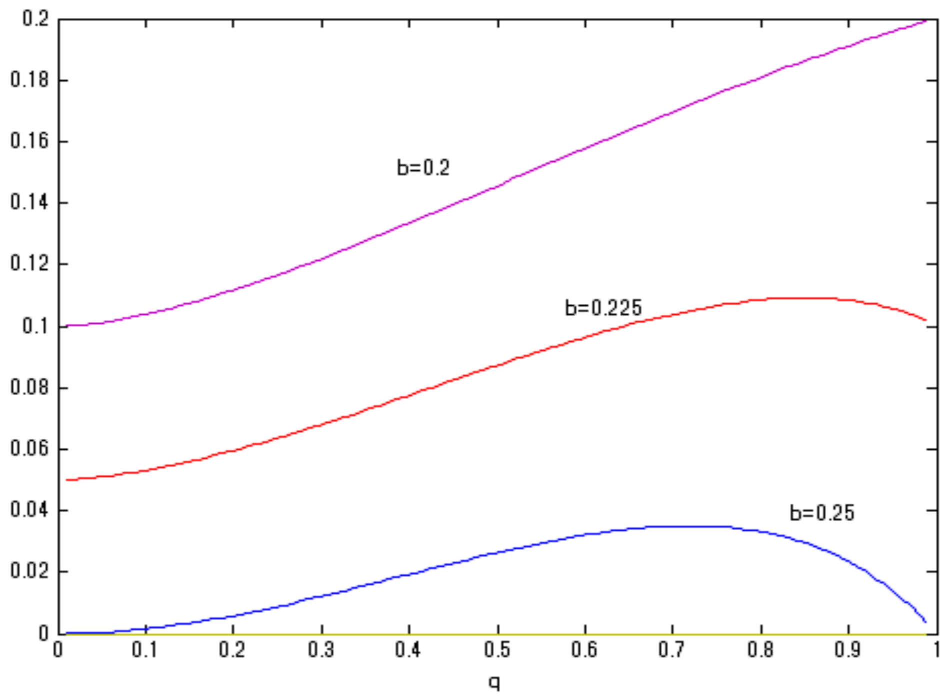
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b=0.25
q
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0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 1: