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MULTILAYERED TOURNAMENTS

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Abstract

In many facets of life, we often face competition with a multilayered structure in which different levels of competition take place simultaneously. In this paper, we propose a new class of tournament models, called multilayered tournaments, to capture this type of competitive environment. Among other things, we find that: (i) an increase in individual incentives, holding the level of team incentives fixed, can lower total effort as it induces inefficient allocation of effort; (ii) the optimal level of individual incentives depends on and is complementary to the level of team incentives. The analysis illuminates the essential role of economic subgroups, such as firms, in achieving some degree of cooperation in an inherently competitive environment, and provides an explanation for why high-powered incentives are more common in market arrangements than within firms.

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1 Introduction

In many facets of life, we often face competition with a multilayered structure in which within-group internal competition and across-group external competition take place simultaneously. A leading example of this situation is the competition among workers, where they compete against each other within their respective firms for promotions while those firms in turn compete against each other for market shares. Similarly, politicians compete against each other for candidacy within their political parties which in turn compete against each other in elections. Professional athletes compete with teammates within their teams which in turn compete with each other in their respective leagues. Academic researchers compete against each other for publication slots or research funds within their disciplines which in turn compete against each other for students or public recognition. Multilayered competition is fairly common and ubiquitous in society: after all, human beings have engaged in various types of competition within their respective communities, such as tribes, villages, states and countries, for the pursuit of internal resources, while those communities have simultaneously competed against each other for the pursuit of external resources.

Many of the instances raised above have a common simple structure which we characterize by the following three features. First, the nature of competition is relative, at least to some extent, where more productive ones tend to capture more of the available resources.[1] Second, even though intense internal competition undoubtedly underlies the strength of a team, team production occupies an essential part of the production process in each of these cases, so that it is essential to foster cooperation among teammates and let them help each other when necessary. Finally and most importantly, agents compete simultaneously at two distinct levels that are not clearly distinguishable in the sense that they cannot direct their resources (such as time and effort) exclusively to a particular level of competition.[2] This last feature is the defining feature of multilayered competition which sets the current analysis apart

[1]The nature of competition naturally becomes relative because the total amount of rewards is typically bounded from above, in which case one's gain necessarily becomes someone else's loss at some point. It is well recognized that many forms of internal competition, such as promotion tournaments, have a relative aspect because total prize, either pecuniary or nonpecuniary, is generally bounded from above. Likewise, many forms of external competition have this feature as well: for instance, the market share of a firm often depends on the firm's relative product quality level rather than the absolute level. A similar argument also applies to political competition in which the number of winners is exogenously fixed, both internally and externally.

[2]For some of the examples raised above, the two levels of competition literally takes place simultaneously, as in the case of the competition among workers, so that it is not feasible to make a clear distinction between them: that is, it is in principle not feasible to allocate resources to one level of competition without affecting the relative standing at the other level. In contrast, in political campaigns, the competition itself can be sequential where the internal competition (primary) precedes the external competition. Even in this case, however, if what politicians need to do is to make investment in necessary skills and/or to frame basic policies, it is hard to distinguish the two levels of competition.

from dynamic elimination tournaments (Rosen, 1990) where each round of competition is independent from others with no strategic interactions.

In this paper, we propose a new class of tournament models which intend to capture these aforementioned features of multilayered competition. We consider an environment in which homogenous agents are divided into symmetric teams and compete with each other both within and across teams simultaneously. To capture the possibility of within-group cooperation, the option of helping others a la Itoh (1991) is incorporated into this framework, so that each agent must allocate his resources between improving his own productivity (own effort) and helping teammates (helping effort). Although the analysis of multilayered competition can easily get intractable due to its complicated hierarchical interactions between different layers of competition, we devise a framework that is tractable enough to conduct various comparative statics exercises. Viewing it as a model of markets and internal organizations, the present framework then illuminates the elusive link between these two primary allocation mechanisms and provides answers to a set of questions concerning the relationship between them, such as how incentive provision within firms should depend on the extent of market competition, under what circumstances pro-competitive market policies are more desirable, why high-powered incentives are more commonly found in markets than stronger than within firms, and above all, why firm organizations are so ubiquitous in the first place.

We obtain several results and implications. First, the analysis sheds some light on the essential role of economic subgroups, such as firms, in achieving the right balance between competition and cooperation. It is in general impossible to induce cooperation which involves all agents when they cannot be rewarded jointly. Even in this situation, however, it is still possible to achieve some degree of cooperation by dividing agents into smaller subgroups and having them pit against one another. Our analysis shows that the possibility of cooperation is what makes multilayered competition both necessary and meaningful. A multilayered tournament virtually has no bite when there is no possibility of helping others, as any feasible allocation can be achieved by a conventional single-layered tournament in this case. Conversely, when the underlying production process calls for some degree of cooperation, a multilayered tournament provides a fairly effective way to induce helping effort even in an inherently competitive environment where there are necessarily winners and losers.

Second, we show that an increase in individual incentives, holding the extent of team incentives fixed, could induce inefficient substitution between the two types of effort and consequently result in lower total effort. This result points to yet another route though which the adverse effect of individual incentives arises. While there is little doubt that intense internal competition better motivates agents at the individual level, it also necessarily impedes cooperation among them. We identify conditions under which an increase in own effort induced by more intense internal competition is more than offset by a decrease in helping effort, thereby leading to a reduction in total effort. We also relate this finding to two widely cited results in the literature – sabotage in a tournament (Lazear, 1989) and the multi-task problem (Holmstrom and Milgrom, 1991) – and suggest an underlying connection between them.

Third, the analysis illuminates the general superiority of team incentives, despite the fact that their effectiveness is generally limited by the notorious freerider problem. We show that although the optimal level of individual incentives may be zero under some conditions, the optimal level of team incentives is always bounded away from zero. The key observation here is that individual incentives can induce own effort only at the expense of a decrease in helping effort. In contrast, team incentives generally yield positive effects in both directions as they are more inclusive and work at a higher level. We argue that this observation accords well with Williamson's view (1985) that high-powered incentives are more common in market arrangements than within firms.

Forth, we provide a new perspective on how individual and team incentives should be related to each other. We show that the optimal incentive scheme at one level of competition cannot be determined independently of the extent of incentives provided for the other level. We in particular show that the relationship between individual and team incentives is complementary, where a decrease in one should be accompanied by a decrease in the other. As practical implications, the result suggests that: stronger individual incentives tend to work well in environments with more intense market competition; pro-competitive policies are more likely to be justified in environments where stronger individual incentives are provided within firms.

Finally, we also conduct various comparative statics exercises on team size. A larger team obviously exacerbates the freerider problem, but could raise the incentive to help others because it may be easier or less costly to help other teammates when more of them are around. This is especially the case when helping effort takes the form of sharing information with teammates (or contributing to a public good). In the presence of such a scale effect, if the social planner can dictate the effort levels of each agent, it is always desirable to have as large a team as possible so as to fully exploit the scale effect, i.e., a grand coalition is always socially optimal. This is not incentive compatible, however, as there is clearly no incentive to help others when all of the agents are grouped into one team. The optimal team size must balance these concerns, and we derive conditions under which a larger team size is warranted.

Related literature: The paper largely falls into a class of tournament models and extends this setting by incorporating team competition. While most tournament models, starting from Lazear and Rosen (1981), focus on competition among individuals, there is a handful of works which examine competition among teams (Drago, et al., 1996; Marino and Zabojnik, 2004; Ishida 2006; Gurtler, 2008).[3] The major difference from these works is that they only consider competition among teams with no regard to competition among individuals, where the sharing rule within a team is exogenously given.[4] The scope of the current analysis is totally different since our primary focus is on the interaction between the two distinct levels of competition connected through two-dimensional effort choices.

One essential aspect of the model is the presence of team production in which each agent's contribution to total output cannot be measured accurately (Alchian and Demsetz, 1972; Holmstrom, 1981). This naturally gives rise to the freerider problem, which substantially diminishes the effectiveness of team incentives. Although the freerider problem necessarily entails efficiency loss, team production also has a bright side as it can encourage teammates to cooperate with each other for the betterment of the team. Our setup with helping effort is heavily indebted to Itoh (1991) who introduces the option of helping others to a standard moral hazard problem.[5] This paper follows this approach to capture a situation in which mutual cooperation is at the core of the production process, although its extent is confined within the boundary of a team. We then take a step further by integrating internal and external competition in order to illuminate the connection between them and derive implications from it.

There are now numerous works which analyze incentive provision within firms in the filed of organizational economics; there are also many works on various market regulations and competition policies in the field of industrial organization. With few exceptions, however, these two strands of literature have developed rather independently from each other, partly due to the technical difficulty of placing organizational and market interactions into a unified

[3]For experimental studies on team tournaments, see Nalbantian and Schotter (1997) and Sutter and Strassmair (2009).

[4]Technically, since our model builds on Tullock (1980), there is also some connection with works on rent-seeking contests. The current analysis is especially related to works on "collective contests" (Nitzan, 1991; Warneryd, 1998; Inderst et al., 2007; Munster, 2007; Nitzan and Ueda, 2011) although they differ in scope and motivation to a great extent: the main focus of this strand of literature is on the equilibrium rate of rent dissipation, i.e., the ratio of total contest expenditures to the total prize size. Along this line, Munster (2007) considers a situation where agents must make multi-dimensional effort choices as in ours. Aside from the differences in scope and motivation, the major difference is that in Munster, individual and collective appropriative activities are totaly separated from each other, so that each agent can specifically direct his resources to a particular level of competition.

[5]Che and Yoo (2001) consider a relational contracting environment and show that there exists a cooperative subgame perfect equilibrium which induces effort at a lower cost. In these works, absolute performance evaluation is feasible for contracting. Ishida (2006) extends this idea and shows that this same mechanism also works even when only relative performance measures are feasible for contracting.

framework. Still, some attempts have been made to integrate these two strands. One approach in this direction examines the effect of market competition on managerial behaviors and incentives (Hart, 1983; Hermalin, 1992; Schmidt, 1997; and Raith, 2003). Yet another approach looks at the effect of incentive provision on market outcomes through oligopolistic interactions (Fershtman and Judd, 1987). Also, see Morita (2012) for a recent attempt to integrate human resource management and market competition. The paper intends to contribute to this line of inquiry from a different perspective with particular focus on the multilayered nature of competition and to provide a framework which captures the elusive link between markets and internal organizations in an intuitive and tractable manner.

2 Model

Environment: We consider a set I of $N \geq 4$ homogeneous agents who are divided into m symmetric teams competing for prizes. We assume that N is some even (and relatively large) integer. Throughout the analysis, the number of agents within each team is denoted by $n \geq 2$ where $N = mn$. Note that the case with $n = 1$ represents the situation where each agent forms his own team, so that the nature of competition is effectively single-layered with no within-group competition.[6] For comparative statics exercises, we mainly focus on the range $\mathcal{N} := [2, \frac{N}{2}]$.

The structure of competition is multilayered in the sense that agents compete with each other both at the individual level (the internal competition) and the aggregate level (the external competition) simultaneously. Throughout the analysis, we index agents by $i = 1, 2, ..., n$ and teams by $j = 1, 2, ..., m$, and use the following notations:

- I_j: the set of agents in team j;
- I_{-ij}: the set of agents in team j other than agent i (or agent i's "teammates");
- T: the set of all teams;
- T_{-j}: the set of teams other than team j.

Note that each agent is in principle identified by a pair (i, j); in what follows, however, we abbreviate the team index and refer to each agent as agent i for brevity.

[6] Note also that the case with $n = N$ where there exists only one team (a grand coalition) which involves all the agents is strategically equivalent to the case with $n = 1$ if the reward structure is set optimally. As we will see, however, the case with $n = N$ is generally dominated by the case with $n = 1$ when the reward structure is given arbitrarily.

The current setting admits a number of possible interpretations, as mentioned in the introduction. Among those possibilities, we primarily view this as a model of markets and internal organizations although no market competition which we observe in reality may correspond physically to the external competition as specified here. Nonetheless, we here emphasize the fact that the current specification captures a situation where a team (or a firm) gains more when it does better than other competitors in a broad sense, which should be true in almost any form of market competition. The external competition in the current setting should thus be regarded as a "reduced-form" of market competition which can actually assume many different forms at a more superficial level.

Effort choice: Each agent independently and simultaneously chooses his effort pair $e_{ij} = (a_{ij}, b_{ij}) \in \mathbb{R}^2_+$, where a_{ij} is the level of own effort and b_{ij} is the effort level of helping effort. The effort cost is given by

$$c_{ij} = C(a_{ij}, b_{ij}).$$

A key departure from the previous literature is the addition of helping effort into a tournament framework. The necessity of helping each other arises largely from the fact that agents are often heterogeneous in skills or sets of information they have: in that case, they can improve the productive efficiency, e.g., by reallocating tasks to reap the benefit of skill complementarities and/or sharing critical knowledge that may be dispersedly distributed within the team. Throughout the analysis, we simply assume that helping effort is allocated evenly among the teammates and benefits each teammate equally (or alternatively focus on a class of cost functions under which it is optimal to allocate helping effort evenly among the teammates).[7] Define $h_{ij} := \frac{\sum_{i' \in I_{-ij}} b_{i'j}}{n-1}$ as the amount of helping effort agent i receives from his teammates. Agent i's total effort, denoted as x_{ij}, is then given by

$$x_{ij} = F(a_{ij}, h_{ij}),$$

We assume that these functions satisfy the usual properties.

Assumption 1 *(i) The cost function C is strictly increasing and weakly convex in each respective argument. (ii) The production function F is strictly increasing and weakly concave in each respective argument.*

For the analysis, it is often convenient to define

$$G(a, b) := \frac{F_1(a, b)}{C_1(a, b)}, \; H(a, b) := \frac{F_2(a, b)}{C_2(a, b)}.$$

[7] It is in principle possible to extend helping effort to agents in other teams. We rule out this possibility at the outset, however, as there is no incentive to do so under any monotonic incentive scheme.

Note that under the maintained assumption, G and H are strictly increasing in a and b, respectively. Moreover, we impose the following assumptions .

Assumption 2 *$\lim_{a\to 0} G = \infty$ and $\lim_{b\to 0} H = \infty$.*

Assumption 3 *$G_2 > 0$ and $H_1 > 0$.*

Assumption 2 assures the existence of interior solutions. Assumption 3 indicates that the two types of effort are complementary to each other. We make this assumption to focus our attention to the case where it is optimal to allocate resources between the two activities, rather than to devote all resources to one activity. One can easily see that a sufficient condition for them is that $F_{12} \geq 0$ and $C_{12} \leq 0$ with at least one holding with strict inequality.

Contest success function: Define r_{ij} as the probability that agent i wins the internal competition, and q_j as the probability that team j wins the external competition. We adopt the (generalized) Tullock success function to determine the winner at each respective level of competition. Letting $z_j := \sum_{i\in I_j} x_{ij}$ denote the total effort for team j, the contest success functions are specified as

$$r_{ij} = \frac{x_{ij}^{\alpha}}{\sum_{i'\in I_j} x_{i'j}^{\alpha}},\ q_j = \frac{z_j^{\alpha}}{\sum_{j'\in T} z_{j'}^{\alpha}}.$$

The Tullock specification is known to have some strategic foundations (Fullerton and McAfee, 1999; Baye and Hoppe, 2003) as well as axiomatic justifications (Skaperdas, 1996; Clark and Riis, 1998). Important features of this specification are that: (i) taking all the others' effort choices as given, the winning probability is increasing in one's effort level; (ii) taking one's effort choices as given, the winning probability is decreasing in the effort level of each of the other agents. We adopt this specification mostly for its greater tractability, among a plethora of possibilities which can capture these features.

Prize allocation: We assume that the individual winner is identifiable only in the winning team so that there are three possible contest outcomes: the winner in the winning team, the losers in the winning team, and all the others in the losing teams.[8] Given this information structure, it suffices to consider a three-level reward schedule, denoted by (W, w, l), where W is the reward for the winner in the winning team, w for the losers in the winning team, and l for all the agents in the losing teams. Define $\Delta_I := W - w$ and $\Delta_T := w - l$. Since

[8]In principle, we can also discriminate between the winner and the losers in a losing team. Here, we only consider a three-level reward scheme to reduce the number of parameters and simplify the analysis. The substance of our analysis is not affected even if we assume that the winner in a losing team earns more than the losers.

what matters for each agent is the reward dispersion, the incentive scheme can alternatively written as (Δ_I, Δ_T): for expositional purposes, we refer to Δ_I (Δ_T) as the individual (team) incentive. Let $\phi := \frac{\Delta_I}{\Delta_T}$ denote the incentive ratio. The extent of the individual incentive measures the intensity of the internal competition.

The total prize is fixed at R, which is entirely captured by the winning team and shared among its members, possibly depending on the contest outcomes, so that $R = W + (n-1)w$ and $l = 0$. We impose two assumptions on the set of feasible incentive schemes. First, there is a liquidity constraint such that each reward must be nonnegative, i.e., $W, w \geq 0$. Second, we also require that an incentive scheme be monotonic in that $W \geq w \geq l = 0$.[9] We use the prize size R as a measure of the intensity of the external competition.

3 Incentives in multilayered tournaments

3.1 Optimal effort choices

In this section, we characterize the optimal effort choices by taking the incentive scheme (Δ_I, Δ_T) and the team size n as given. Throughout the analysis, we seek for a symmetric equilibrium where all agents choose the same effort pair. When all the other agents choose (a, b), we can write

$$x_{ij} = F(a_{ij}, b), \; x_{i'j} = F(a, \tfrac{(n-2)b+b_{ij}}{n-1}), \; i' \in I_{-ij}.$$

The probability that agent i wins the internal competition conditional on his effort choice (a_{ij}, b_{ij}) is then given by

$$r_{ij} = r(a_{ij}, b_{ij}) = \frac{F(a_{ij}, b)^\alpha}{F(a_{ij}, b)^\alpha + (n-1)F(a, \frac{(n-2)b+b_{ij}}{n-1})^\alpha},$$

for $n \geq 2$. Similarly, the probability that team j wins the external competition is given by

$$q_j = q(a_{ij}, b_{ij}) = \frac{(F(a_{ij}, b) + (n-1)F(a, \frac{(n-2)b+b_{ij}}{n-1}))^\alpha}{(F(a_{ij}, b) + (n-1)F(a, \frac{(n-2)b+b_{ij}}{n-1}))^\alpha + (m-1)n^\alpha F(a, b)^\alpha}.$$

Taking (a, b) as given, the problem faced by each agent is formulated as follows:

$$\max_{a_{ij}, b_{ij}} \quad q_j r_{ij} \Delta_I + q_j \Delta_T - C(a_{ij}, b_{ij}).$$

[9] Needless to say, the optimal incentive scheme is indeed monotonic for a wide range of parameters. In this framework, however, there may actually arise a case where it is optimal to inhibit the incentive to exert own effort by offering a negative Δ_I, if it feasible to do so. We rule out this possibility because a non-monotonic incentive scheme may provide agents a detrimental incentive to "game the system," e.g., by intentionally hiding or destroying his own output. Moreover, a non-monotonic incentive scheme may not be optimal from the fairness point of view because it is highly demoralizing to punish the most productive agent.

The first-order conditions are obtained as

$$\frac{\partial q_j}{\partial a_{ij}}(r_{ij}\Delta_I + \Delta_T) + q_j \frac{\partial r_{ij}}{\partial a_{ij}}\Delta_I = C_1,$$

$$\frac{\partial q_j}{\partial b_{ij}}(r_{ij}\Delta_I + \Delta_T) + q_j \frac{\partial r_{ij}}{\partial b_{ij}}\Delta_I = C_2.$$

In a symmetric equilibrium, the first-order conditions are reduced to

$$\alpha G P_a = F(a, b), \tag{1}$$

$$\alpha H P_b = F(a, b), \tag{2}$$

where

$$P_a := \frac{(N-1)\Delta_I + (N-n)\Delta_T}{N^2}, \; P_b := \frac{-\Delta_I + (N-n)\Delta_T}{N^2}.$$

We assume that the second-order conditions are satisfied at the optimum: for instance, they are satisfied if α is sufficiently small. See Appendix B for more detail on the first- and second-order conditions.

Let $(a^*(\Delta_I, \Delta_T), b^*(\Delta_I, \Delta_T))$, or simply (a^*, b^*), denote the solution to the pair of equations, (1) and (2), and $x^* = F(a^*, b^*)$. The equilibrium effort pair (a^*, b^*) is determined largely by P_a and P_b. It is important to note that an increase in the individual incentive Δ_I could diminish the incentive to help others as it unambiguously decreases P_b. The reason for this is intuitive and fairly straightforward: more intense competition at the individual level discourages agents to allocate resources to others, as there is more to lose by helping others. This feature is the driving force of many results that follow.

3.2 On the role of multilayered tournaments

The current setup makes two key departures from standard tournament models that are widely analyzed. First and foremost, competition is multilayered where the agents compete with each other simultaneously on the two different levels. Second, the effort-choice problem in our setup is multi-dimensional in that each agent must allocate resources to two distinct activities – own effort and helping effort – that may be imperfect substitutes for each other. There is a close association between these two elements in that one ceases to have any value when the other is absent. To illustrate what a multilayered tournament can give, we consider two benchmark cases and contrast them with the current setup.

A single-layered tournament: We first consider the case of single-layered competition where each agent competes directly with all the others on the same level as usually assumed.[10] In the current setup, the case with $n = 1$ (and also $n = N$) corresponds to this standard specification. It is evident that helping effort cannot be induced in single-layered tournaments as it can only lower his chance of winning. Given this, (1) becomes

$$\alpha \frac{(N-1)(\Delta_I + \Delta_T)}{N^2} = \frac{F(a^*, 0)}{F_1} C_1,$$

for any given incentive scheme (Δ_I, Δ_T). Evidently, since Δ_I and Δ_T are perfect substitutes in this case, each agent's effort incentive depends only on $\Delta_I + \Delta_T = W$.

A multilayered tournament with no helping effort: As another benchmark, consider the case where helping effort is not a viable option for the agents, either because it is prohibitively costly or because it carries no productive value, so that $b^* = 0$. In this case, (1) becomes

$$\alpha \frac{(N-1)\Delta_I + (N-n)\Delta_T}{N^2} = \frac{F(a^*, 0)}{F_1} C_1.$$

Note that an increase in Δ_I is more effective in raising a^* than an increase in Δ_T due to the freerider problem. This suggests that zero reward should be allocated to the external competition in order to maximize the total effort.

Comparison: The two benchmark cases are instructive in illuminating the role of multilayered competition in general. To see this, we write the total effort as a function of n, denoted by $x^*(n)$, and compare these benchmarks in terms of the total effort induced.[11] We establish the following result which indicates some necessary conditions for multilayered tournaments to have any value.

Proposition 1 *Take (Δ_I, Δ_T) as given. If helping effort is not a viable option, $x^*(1) \geq x^*(n)$ for any $n \in \mathcal{N}$.*

PROOF: To prove this, it suffices to show that

$$\frac{(N-1)(\Delta_I + \Delta_T)}{N^2} \geq \frac{(N-1)\Delta_I + (N-n)\Delta_T}{N^2},$$

which holds with equality, for any $n \in \mathcal{N}$, only if $\Delta_T = 0$. ∎

[10] There are no losers in the winning team in this case, and the winner simply captures W.

[11] Define $\pi(n) := vNx^*(n) - \Delta_I - n\Delta_T$, where v is some constant, as total profit. Note that since the total prize is weakly increasing in n, $x^*(1) \geq x^*(n)$ is a sufficient condition for $\pi^*(1) \geq \pi^*(n)$ for any $n \geq 2$.

In short, an integral ingredient of multilayered competition is the possibility or the necessity of helping others: the value of multilayered competition naturally diminishes when the production process becomes more individualistic and requires less cooperation. The proposition states that multilayered competition has no bite when it is virtually impossible or meaningless to help others (the case of purely individualistic production). The factor that is especially critical is the imperfect substitutability of the two types of effort which gives rise to several interesting properties of multilayered tournaments as we will see shortly.

3.3 Efficient allocation

Properties of the efficient allocation are to a large degree independent of the shape of F. Consider a social planner who can impose any effort pair (a, b) without any cost. The social planner's problem is then defined as

$$\max_{a,b} \quad F(a, b) - C(a, b).$$

The first-order conditions with respect to a and b imply that

$$F_1 = C_1, \; F_2 = C_2.$$

The question of particular interest is whether and how this efficient allocation is implemented via an appropriately designed incentive scheme. To do so, it is easy to observe that $P_a = P_b$, which in turn implies $\Delta_I = 0$. Given this, the efficient allocation is implemented if Δ_T is set so as to satisfy

$$\alpha\frac{(N-n)\Delta_T}{N^2} = F(a^*(0, \Delta_T), b^*(0, \Delta_T)). \tag{3}$$

Let Δ_T^{ef} denote the team incentive which satisfies (3), and $R^{ef} := n\Delta_T^{\text{ef}}$ the total prize size required to achieve the (unconstrained) efficient allocation. We summarize this finding as follows (the proof abbreviated).

Proposition 2 *Suppose that the social planner can choose any R at no cost. The efficient allocation is then implemented by $(0, \Delta_T^{\text{ef}})$ where Δ_T^{ef} solves (3).*

3.4 Hidden costs of incentives and the multi-task interpretation of team

One important aspect of the model is that an increase in the individual incentive inhibits cooperative behavior among teammates. Although intense internal competition can be beneficial up to some point, as it can provide stronger incentives for each agent and induce a higher level of own effort, it is necessarily at the expense of a decrease in helping effort. This could result in inefficient substitution of the two efforts and could even lower the total effort x^* when the incentive ratio is highly unbalanced to begin with.

Proposition 3 *Suppose that the two types of effort are complementary to each other, i.e.,* $G_2 > 0$ *and* $H_1 > 0$*. Then, there exists some* $\bar{\phi} \in [0, N-n)$ *such that an increase in* Δ_I*, with* Δ_T *fixed, reduces the total effort for* $\phi \geq \bar{\phi}$ *if there exists some finite number* $L(a)$ *for any given* $a > 0$ *such that*

$$\lim_{b \to 0} \frac{F_2(F_{12}C_2 - C_{12}F_2) - F_1(F_{22}C_2 - C_{22}F_2)}{F_2^2 C_2} < L(a). \tag{4}$$

Proof: See Appendix A.

The proposition states that an increase in the individual incentive Δ_I, with Δ_T fixed, may reduce the total effort supplied by each agent, thereby shedding light on yet another mechanism through which the adverse effect of incentives arises. The condition we obtain is not something that holds only under extreme conditions, as it is satisfied by many standard specifications of F and C. To see this, when the cost is additively separable and linear in each type of effort, (4) is reduced to

$$\lim_{b \to 0} \Big(\frac{F_{12}}{F_2} - \frac{F_1 F_{22}}{F_2^2} \Big) < L(a),$$

which is satisfied by many standard production functions. See section 5 for a more specific example of the model.

When one side of the incentive scheme is exogenously fixed, the model yields implications that are closely related to two well known results in the literature. On one hand, the adverse effect of incentives is driven by the fact that an increase in Δ_I induces inefficient substitution between the two types of effort, where each agent shifts resources from more productive helping effort to less productive own effort. In this sense, the result can be seen as yet another manifestation of the multi-task problem (Holmstrom and Milgrom, 1991). On the other hand, this result also runs parallel to Lazear (1989) who points out that an increase in wage spread lowers total surplus (output net of effort cost), though not total effort, in a tournament when agents have the option of sabotaging others. Here, an increase in Δ_I leads to a reduction in helping effort, which is qualitatively equivalent to an increase in sabotage, when Δ_T is held fixed for some exogenous reasons.

The result thus illuminates an inherent connection between the two widely cited rationales for the adverse effect of incentives. The commonality of the two is that agents must allocate resources into several different activities. To induce desirable behavior along several different dimensions, the principal typically needs as many contracting measures. If there are not enough contractible variables to cover the whole set of activities, some of the incentives may need to be muted to achieve the right balance of efforts, which is precisely the point made by

Holmstrom and Milgrom (1991). In Lazear (1989), an increase in wage spread results in more sabotage (or less cooperation) because there is only one contractible measure, i.e., individual ranking, while the agents must allocate resources into two distinct activities. The analysis thus suggests a virtue of dividing agents into smaller subsets in a competitive environment where performances can be measured only in relative terms, as it is instrumental in inducing within-group cooperation by making an additional performance measure, i.e., team ranking, available for contracting.

4 Optimal provision of incentives

In this section, we assume that the total prize size is exogenously fixed at $R \in (0, R^{ef})$ and examine how a change in the underlying environment, namely the incentive scheme (Δ_I, Δ_T) and the team size n, on each agent's incentives and the aggregate outcome under this restriction.

4.1 Effort-maximizing incentive scheme

The effort-maximizing incentive scheme is the solution to the following problem:

$$\max_{\Delta_I, \Delta_T} F(a^*, b^*),$$

subject to

$$R \geq \Delta_I + n\Delta_T.$$

Our primary interest is in how the extent of internal competition, measured by Δ_I, is related to the extent of external competition, measured by R, i.e., what fraction of the total prize should be allocated to the individual winner.

Proposition 4 *Suppose that F and C are homogenous functions. For any given $R > 0$, the effort-maximizing incentive scheme can be written as*

$$\Delta_I^* = \frac{\phi^*}{\phi^* + n} R, \ \Delta_T^* = \frac{1}{\phi^* + n} R,$$

where $\phi^* \in [0, N - n)$ is some constant which is bounded from above.

Proof: See Appendix A.

A corollary of this result is that the effort-maximizing team incentive is always bounded away from zero, while the effort-maximizing individual incentive can be zero in some cases.

To see under what conditions $\Delta_I^* = 0$ holds, note that when F and C are both homogeneous, the optimality condition can be written as

$$\frac{P_a}{P_b} = \frac{H(\frac{a}{b}, 1)}{G(\frac{a}{b}, 1)}.$$

Under the maintained assumptions, there exists a unique $\underline{\psi}$ such that

$$1 = \frac{H(\underline{\psi}, 1)}{G(\underline{\psi}, 1)}.$$

This is the minimum effort ratio which can be implemented by a feasible incentive scheme: for any $\frac{a}{b} > \underline{\psi}$, there is a unique incentive ratio which can implement the effort ratio. This means that it is more likely to have $\Delta_I^* = 0$ if helping effort is relatively more productive than own effort and $\underline{\psi}$ is higher.

Our analysis is instrumental in illuminating the elusive link between the two distinct levels of competition: within-group (internal) competition and across-group (external) competition. As emphasized, the most important example of this framework is the case where agents compete for internal resources within organizations, such as firms, and for external resources across organizations. Regarding this link, we emphasize two points that are of particular importance.

General superiority of team incentives: As shown in Proposition 4, the optimal incentive ratio can be zero but is bounded from above. There are actually two sides to this statement. First, the optimal individual incentive might be zero, meaning that it is optimal to mute any within-group competition by treating all the teammates equally (low-powered individual incentives) under some conditions. Second, the optimal team incentive is always positive and bounded from below, even though its effectiveness is severely limited by the freerider problem. These two facts together point to the general superiority of team incentives in this type of setup, which is consistent with Williamson's observation that that high-powered incentives are more common in market arrangements than within firms (Williamson, 1985).

The reason why the team incentive is more effective in this context stems from the fact that it works at a higher level than the individual incentive and can hence yield a positive impact on both types of effort. This draws clear contrast to the individual incentive which can induce own effort only at the expense of a decrease in helping effort. Because of this, when the level of Δ_T is small compared to Δ_I, the agents choose to exert zero helping effort, which is never optimal given that $\lim_{b \to 0} H(a, b) = \infty$. The same argument does not apply to the case of the individual incentive because a positive team incentive, no matter how small it is, can induce a positive level of own effort even if $\Delta_I = 0$.

Complementarity between internal and external incentives: Our analysis also reveals the importance of balancing the incentives at the two ends. Proposition 4 shows that given an exogenous decrease in R, both Δ_I and Δ_T must be reduced proportionally, implying that the optimal level of incentives provided at one level of competition cannot be independent of that at the other level. We argue that the individual and team incentives are largely complementary to each other, so that a decrease in one should be accompanied by a decrease in the other in order to maintain the right balance between competition and cooperation. As more practical implications, the result suggests that: stronger individual incentives tend to work well in environments with more intense market competition; pro-competitive policies are more likely to be justified in environments where stronger individual incentives are provided within firms. The model provides a critical policy insight that there exists no set of competition policies that are universally effective because the extent of market regulations should depend on how incentives are provided within firms, which could vary substantially across industries as well as countries.

4.2 Effort-maximizing team size

It should be clear that the team size, or equivalently the number of teams, plays a critical role in almost every aspect of the model. We now turn our attention to the effect of the team size n which is another key component of the incentive scheme.[12] An increase in the team size obviously exacerbates the freerider problem and yields a detrimental effect on effort provision. Despite this, a larger team can also bring about a positive effect, as it may allow the teammates to reap the benefit of the scale effect more effectively. The basic idea is that when (i) the agents are heterogenous and complementary to each other, and (ii) the set of tasks they need to work on is highly diversified, it is intrinsically easier, or equivalently more productive, to help each teammate by a small margin than to devote all of helping effort intensively to one agent, or it is easier to find someone who an agent can help with his expertise when there are more teammates to work with. Alternatively, the cost of helping effort also decreases with n when it is a contribution to a public good, such as disclosure of indispensable production knowledge, whose benefit is non-rival among the teammates.

To capture this idea and tradeoff, we add more structure to the cost function. More specifically, we now assume that the cost function is given by

$$C(a_{ij}, b_{ij}) = c(a_{ij}, \tfrac{b_{ij}}{\beta(n)}).$$

That is, the production function is augmented with a new function $\beta(n)$ which measures the

[12] Here, we basically ignore the integer problem.

relative productivity of helping effort. For the subsequent analysis, we ignore the integer problem and treat n as if it is a continuous variable for $n \in \mathcal{N}$. We in general assume that $\beta(n)$ is weakly increasing and weakly concave in n. Let $x^*(n)$ denote the equilibrium total effort as a function of the team size. We are interested in how a change in the team size affects each agent's effort choices for $n \in \mathcal{N}$.[13]

Due to the scale effect of helping effort, the socially optimal team size, when the social planner can directly impose the effort levels, is $n = N$. This is not incentive compatible, however, because there is clearly no incentive for any agent to exert helping effort when he is in a grand coalition which involves all the agents. Because of this inability to take advantage of the scale effect, the case with $n = N$ is weakly dominated even by the least efficient case where each team consists of only one agent ($n = 1$).[14] To induce a positive level of helping effort, the team size should be bounded between $n = 2$ and $n = \frac{N}{2}$.

The optimal effort pair is now given by $a^*(\Delta_I, \Delta_T, n), b^*(\Delta_I, \Delta_T, n)$. Taking (Δ_I, Δ_T) as given, an increase in the team size raises the total effort if

$$F_1 \frac{\partial a^*}{\partial n} + F_2 \frac{\partial b^*}{\partial n} > 0.$$

Let $n^* := \text{argmax}_{n \in \mathcal{N}} x^*(n)$ denote the effort-maximizing team size in the interval $\mathcal{N}$. The effort-maximizing team size depends largely on the magnitude of the scale effect, which is captured by β', and the incentive ratio ϕ. While it is intuitive and clear that a larger scale effect always favors a larger team size, the effect of ϕ is more somewhat complicated. We state the following result concerning how the effort-maximizing team size is related to the incentive scheme in effect.

Proposition 5 $n^* > 2$ *only if*

$$\frac{N\Delta_T}{(N-1)\Delta_I + (N-n)\Delta_T} = \frac{N}{(N-1)\phi + N - n} \geq \beta'(2).$$

Under the optimal incentive scheme (Δ_I^*, Δ_T^*) *derived in Proposition 4,* $n^* > 2$ *if and only if* $\beta'(2) > 0$.

PROOF: See Appendix A.

[13] There is of course a possibility that the effort-maximizing team size is one. This is more likely to be the case if the two types of effort are more substitutive and/or the productivity of helping effort is very low. Given the spirit of this problem, however, we do not consider this possibility and simply assume that $x^*(2) > x^*(1)$.

[14] To see this, note that $b^*(N) = 0$ for $n = N$, so that the total effort is given by $\alpha \frac{(N-1)\Delta_I}{N^2} = \frac{F(a^*(N),0)}{F_1(a^*(N),0)} a^{*\gamma-1}$. See section 3.2 for the case with $n = 1$. Comparing these, it is clear that $x^*(1) \geq x^*(N)$ for any given (Δ_I, Δ_T).

First, it is clear that the total effort is monotonically decreasing in $n \in \mathcal{N}$ in the absence of the scale effect, i.e., when $\beta'(n) = 0$ for all $n \in \mathcal{N}$. When $\beta'(2) > 0$, on the other hand, a larger team size may increase the total output. To see when a larger team size is more desirable, observe that an increase in the team size renders the team incentive less effective and hence yields an effect similar to a decrease in Δ_T. Because of this, an increase in the team size tends to have a favorable effect on the total effort when the incentive ratio is relatively low: a larger team is likely to be more desirable when the individual incentive Δ_I is weak relative to the team incentive Δ_T. In contrast, when the individual incentive is relatively strong (or the incentive ratio is relatively high), a small team is generally desirable, and it may even be desirable to have no team at all.

5 An example: CES production function with linear cost

In this section, we provide a specific example of the model by employing more specific production and cost functions in order to obtain sharper predictions and implications. Specifically, we assume that the production function takes the following CES form:

$$x_{ij} = F(a_{ij}, h_{ij}) = \Big(\frac{a_{ij}^{1-\rho}}{2} + \frac{h_{ij}^{1-\rho}}{2}\Big)^{\frac{1}{1-\rho}}. \tag{5}$$

The key parameter is $\rho \in [0, \infty)$ which measures the elasticity of substitution between own effort and helping effort. Since we are interested in cases where helping effort is inherently different from own effort and cannot be easily substituted, we assume that $\rho > 1$, i.e., the two types of effort are sufficiently complementary to each other. We also assume that the cost function is given by

$$C(a_{ij}, b_{ij}) = a_{ij} + \frac{b_{ij}}{\beta}. \tag{6}$$

Under this specification, we obtain the following result.

Lemma 1 *Under the production and cost functions specified in (5) and (6), the equilibrium effort levels are given by*

$$x^* = \alpha A(P_a^\sigma + (\beta P_b)^\sigma)^{\frac{1}{\sigma}}, \; a^* = \frac{\alpha P_a^{1+\sigma}}{P_a^\sigma + (\beta P_b)^\sigma}, \; b^* = \frac{\alpha(\beta P_b)^{1+\sigma}}{P_a^\sigma + (\beta P_b)^\sigma},$$

where $\sigma := \frac{1-\rho}{\rho}$ *and* $A := 2^{-\frac{1}{1-\rho}}$.

Proof: Given the production function (5), the equilibrium conditions are computed as

$$\alpha P_a A(a^{*1-\rho} + b^{*1-\rho})^{\frac{\rho}{1-\rho}} a^{*-\rho} = A(a^{*1-\rho} + b^{*1-\rho})^{\frac{1}{1-\rho}},$$

$$\alpha\beta P_b A(a^{*1-\rho} + b^{*1-\rho})^{\frac{\rho}{1-\rho}} b^{*-\rho} = A(a^{*1-\rho} + b^{*1-\rho})^{\frac{1}{1-\rho}}.$$

With some computation, these can be written as

$$\alpha P_a = (a^{*1-\rho} + b^{*1-\rho})a^{*\rho} = \Big(\frac{x^*}{A}\Big)^{1-\rho} a^{*\rho},$$

$$\alpha\beta P_b = (a^{*1-\rho} + b^{*1-\rho})b^{*\rho} = \Big(\frac{x^*}{A}\Big)^{1-\rho} b^{*\rho},$$

from which we obtain the expressions in the lemma. ∎

The lemma shows that the total effort is given by a very simple and tractable form which allows us to conduct various comparative statics exercises and verify our preceding results. Among them, we are particularly interested in the effect of an increase in the individual incentive Δ_I on the total effort x^*. Note that in this setup, (4) can be written as

$$\lim_{b\to 0}\Big(\frac{F_{12}}{F_2} - \frac{F_1 F_{22}}{F_2^2}\Big) = \frac{\rho a^{-\rho}(1 + (\frac{a}{b})^{1-\rho})}{a^{1-\rho} + b^{1-\rho}},$$

which is bounded from above for any given $a > 0$ under the maintained assumption of $\rho > 1$, meaning that the condition in Proposition 3 is satisfied.

Proposition 6 *There exists $\bar{\phi} \in [0, N-n)$ such that an increase in Δ_I decreases x^* for $\phi > \bar{\phi}$. If $N - 1 > \beta^\sigma$, $\bar{\phi} > 0$.*

Proof: An increase in Δ_I decreases x^* if

$$\frac{A}{N}(P_a^\sigma + (\beta P_b)^\sigma)^{\frac{1-\sigma}{\sigma}}((N-1)P_a^{\sigma-1} - \beta_n^\sigma P_b^{\sigma-1}) < 0. \tag{7}$$

The left-hand side represents the marginal increase in total effort per worker. It then follows from this that an increase in Δ_I reduces the total effort if

$$(N-1)P_a^{\sigma-1} < \beta^\sigma P_b^{\sigma-1},$$

which is simplified to

$$p = \frac{(N-1)\phi + N - n}{-\phi + N - n} > \Big(\frac{N-1}{\beta_n^\sigma}\Big)^{\frac{1}{1-\sigma}}.$$

Note that p ranges from one (at $\phi = 0$) to infinity (at $\phi = N - n$). There exists an interior threshold $\bar{\phi}$ if $N - 1 > \beta^\sigma$. Otherwise, the total effort is monotonically decreasing in Δ_I. ∎

We can also obtain sharper predictions on the effect of n on the total effort x^*. To this end, we now let the cost function (6) be given by

$$C(a_{ij}, b_{ij}) = a_{ij} + \frac{b_{ij}}{\gamma(n-1)^\lambda},$$

where $\beta = \gamma(n-1)^\lambda$ and $\lambda \in [0,1]$.[15] It directly follows from Lemma 1 that the total effort is increasing in n if

$$-(n-1)(P_a^{\sigma-1} + \beta^\sigma P_b^{\sigma-1})\frac{\Delta_T}{N^2} + \lambda(\beta P_b)^\sigma > 0. \tag{8}$$

The first term indicates the loss of incentives due to the freerider problem and is always negative. The second term indicates the efficiency gain from the scale effect and is always positive by definition. The effort-maximizing team size must balance this tradeoff.

The effort-maximizing team size is relatively straightforward to obtain when the scale effect of helping effort is absent. If $\lambda = 0$, i.e., helping effort exhibits no scale effect, this condition is reduced to

$$-(P_a^{\sigma-1} + \beta^\sigma P_b^{\sigma-1}) > 0,$$

which obviously never holds. This implies that the total effort is strictly decreasing in n for $n \in \mathcal{N}$ due to the incentive to freeride. In contrast, when $\lambda > 0$, the total effort may increase with the team size if

$$\lambda(\gamma P_b)^\sigma > (n-1)(P_a^{\sigma-1} + \gamma^\sigma P_b^{\sigma-1})\frac{\Delta_T}{N^2}.$$

This condition is likely to be satisfied when the incentive ratio is small, so that the agents allocate more resources to helping effort: as $\phi \to 0$, we have

$$\frac{\lambda\gamma^\sigma(N-n)}{(n-1)\Delta_T} > 1 + \gamma^\sigma.$$

6 Conclusion

This paper proposes a tractable framework to analyze multilayered competition which we ubiquitously observe. The framework allows us to analyze the interconnection between the two levels of competition, internal (within-group) and external (across-group), and how they are and should be related to each other. Among other things, we find that: (i) an increase in individual incentives, holding the extent of team incentives fixed, can lower overall performances because it induces inefficient substitution of effort; (ii) the extent of individual

[15] Under this specification, the scale effect is absent when $\lambda = 0$ while helping effort is a pure public good whose cost is independent of the number of beneficiaries when $\lambda = 1$.

incentives should be complementary to that of team incentives. The latter finding suggests that when we devise an incentive scheme within a firm, the extent of market competition that the firm faces is an important factor to be reckoned with. We also use our framework to explain why high-powered incentives are more common in market arrangements than within firms.

As a final note, one limitation of the analysis is that we only consider symmetric cases where agents are all identical and grouped into teams of the same size. One possible extension along this line is therefore to introduce heterogeneity among agents and allow them to differ in innate productivity. The extension adds another dimension to the problem and allows us to ask how agents should be sorted into teams (positive or negative assortative matching). The other possibility is to consider the case where teams vary in size and productivity so that some teams are larger than others. Although the analysis in either direction could become much more complicated, as we rely on the imposed symmetric structure to make it tractable, it is nonetheless of some interest to analyze these extended cases.

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Appendix

Proof of Proposition 3: Differentiating the first-order conditions with respect to Δ_I yields

$$P_a\Big(G_1\frac{\partial a^*}{\partial\Delta_I}+G_2\frac{\partial b^*}{\partial\Delta_I}\Big)+\frac{(N-1)G}{N^2}=F_1\frac{\partial a^*}{\partial\Delta_I}+F_2\frac{\partial b^*}{\partial\Delta_I},$$

$$P_b\Big(H_1\frac{\partial a^*}{\partial\Delta_I}+H_2\frac{\partial b^*}{\partial\Delta_I}\Big)-\frac{H}{N^2}=F_1\frac{\partial a^*}{\partial\Delta_I}+F_2\frac{\partial b^*}{\partial\Delta_I}.$$

Solving these we obtain

$$\frac{\partial a^*}{\partial\Delta_I}=\frac{(N-1)(F_2-H_2P_b)G-(G_2P_a-F_2)H}{N^2((F_1-G_1P_a)(F_2-H_2P_b)-(G_2P_a-F_2)(H_1P_b-F_1))},$$

$$\frac{\partial b^*}{\partial\Delta_I}=\frac{(N-1)(H_1P_b-F_1)G-(F_1-G_1P_a)H}{N^2((F_1-G_1P_a)(F_2-H_2P_b)-(G_2P_a-F_2)(H_1P_b-F_1))}.$$

It follows from these that

$$\frac{\partial x^*}{\partial\Delta_I}=F_1\frac{\partial a^*}{\partial\Delta_I}+F_2\frac{\partial b^*}{\partial\Delta_I}=\frac{(N-1)(F_2H_1-F_1H_2)GP_b-(F_1G_2-F_2G_1)HP_a}{N^2((F_1-G_1P_a)(F_2-H_2P_b)-(G_2P_a-F_2)(H_1P_b-F_1))}. \quad (9)$$

We first show that the denominator of (9) is always positive, i.e., $(F_1-G_1P_a)(F_2-H_2P_b) > (G_2P_a-F_2)(H_1P_b-F_1)$. This can be written as

$$(F_2H_1-F_1H_2)P_b+(F_1G_2-F_2G_1)P_a+(G_1H_2-G_2H_1)P_aP_b>0,$$

which holds under the maintained assumption that $G_1>0$, $H_2>0$, $G_2<0$ and $H_1<0$. Given this, it now suffices to show that the numerator is negative, i.e.,

$$\lim_{P_b\to 0}\frac{(F_1G_2-F_2G_1)HP_a}{(F_2H_1-F_1H_2)GP_b}>N-1,$$

for any given $P_a > 0$. Since

$$H_1 = \frac{F_{12}C_2 - C_{12}F_2}{C_2^2},\ H_2 = \frac{F_{22}C_2 - C_{22}F_2}{C_2^2},\ \frac{P_a}{P_b} = \frac{H}{G},$$

we have

$$\lim_{P_b \to 0} \frac{(F_1 \frac{F_{12}C_1 - C_{12}F_1}{C_1^2} - F_2 \frac{F_{11}C_1 - C_{11}F_1}{C_1^2})(\frac{F_2}{C_2})^2}{(F_2 \frac{F_{12}C_2 - C_{12}F_2}{C_2^2} - F_1 \frac{F_2C_2 - C_2F_2}{C_2^2})(\frac{F_1}{C_1})^2} > N - 1,$$

which is further reduced to

$$\lim_{P_b \to 0} \frac{\frac{C_1F_{12}}{F_1} - C_{12} - \frac{F_2F_{11}C_1}{F_1^2} + \frac{F_2C_{11}}{F_1}}{\frac{C_2F_{12}}{F_2} - C_{12} - \frac{F_1F_{22}C_2}{F_2^2} + \frac{F_1C_{22}}{F_2}} = \lim_{P_b \to 0} \frac{\frac{C_1F_{12}}{F_1C_2} - \frac{C_{12}}{C_2} - \frac{F_2F_{11}C_1}{F_1^2C_2} + \frac{F_2C_{11}}{F_1C_2}}{\frac{F_{12}}{F_2} - \frac{C_{12}}{C_2} - \frac{F_1F_{22}C_2}{F_2^2} + \frac{F_1C_{22}}{F_2}} > N - 1, \quad (10)$$

Note that as $P_b \to 0$, we have $b \to 0$ and $H \to \infty$ while $a > 0$ and G is bounded from above.

We first show that the numerator diverges to infinity as $P_b \to 0$, i.e.,

$$\lim_{b \to 0} \Big(\frac{C_1F_{12}}{F_1C_2} - \frac{C_{12}}{C_2} - \frac{F_2F_{11}C_1}{F_1^2C_2} + \frac{F_2C_{11}}{F_1C_2} \Big) = \infty.$$

With some algebra, this condition can be written as

$$\lim_{b \to 0} \Big(\frac{C_1^2G_2}{F_1C_2} - \frac{HF_{11}}{GF_1} + \frac{HC_{11}}{F_1} \Big) = \infty,$$

which holds true under the maintained assumptions. Given this, (10) holds if there exists some finite number $L(a)$ for any $a > 0$ such that

$$\lim_{b \to 0} \frac{F_{12}}{F_2} - \frac{C_{12}}{C_2} - \frac{F_1F_{22}C_1}{F_2^2} + \frac{F_1C_{22}}{F_2} = \lim_{b \to 0} \frac{F_2(F_{12}C_2 - C_{12}F_2) - F_1(F_{22}C_2^2 - F_2C_2C_{22})}{F_2^2C_2} < L.$$

∎

Proof of Proposition 4: Under the restriction that $R = \Delta_I + n\Delta_T$, the optimal incentive scheme must satisfy

$$n\Big(F_1 \frac{\partial a^*}{\partial \Delta_I} + F_2 \frac{\partial b^*}{\partial \Delta_I}\Big) = F_1 \frac{\partial a^*}{\partial \Delta_T} + F_2 \frac{\partial b^*}{\partial \Delta_T}. \quad (11)$$

It is clear from this that $\Delta_T^* > 0$ under Assumption 3.

To obtain $\frac{\partial a^*}{\partial \Delta_T}$ and $\frac{\partial b^*}{\partial \Delta_T}$, we differentiate the first-order conditions with respect to Δ_T:

$$P_a\Big(G_1 \frac{\partial a^*}{\partial \Delta_T} + G_2 \frac{\partial b^*}{\partial \Delta_T}\Big) + \frac{(N-n)G}{N^2} = F_1 \frac{\partial a^*}{\partial \Delta_T} + F_2 \frac{\partial b^*}{\partial \Delta_T},$$

$$P_b\Big(H_1 \frac{\partial a^*}{\partial \Delta_T} + H_2 \frac{\partial b^*}{\partial \Delta_T}\Big) + \frac{(N-n)H}{N^2} = F_1 \frac{\partial a^*}{\partial \Delta_T} + F_2 \frac{\partial b^*}{\partial \Delta_T}.$$

Solving these, we obtain

$$\frac{\partial a^*}{\partial \Delta_T} = \frac{(N-n)((G_2P_a - F_2)H + (F_2 - H_2P_b)G)}{N^2((F_1 - G_1P_a)(F_2 - H_2P_b) - (G_2P_a - F_2)(H_1P_b - F_1))},$$

$$\frac{\partial b^*}{\partial \Delta_T} = \frac{(N-n)((H_1P_b - F_1)G + (F_1 - G_1P_a)H)}{N^2((F_1 - G_1P_a)(F_2 - H_2P_b) - (G_2P_a - F_2)(H_1P_b - F_1))}.$$

It follows from these that

$$F_1\frac{\partial a^*}{\partial \Delta_T} + F_2\frac{\partial b^*}{\partial \Delta_T} = \frac{(N-n)((F_2H_1 - F_1H_2)GP_b + (F_1G_2 - F_2G_1)HP_a)}{N^2((F_1 - G_1P_a)(F_2 - H_2P_b) - (G_2P_a - F_2)(H_1P_b - F_1))}.$$

Combined with (9), we can rewrite (11) as

$$\frac{(N-n)((F_2H_1 - F_1H_2)GP_b + (F_1G_2 - F_2G_1)HP_a}{(N-1)(F_2H_1 - F_1H_2)GP_b + (F_2G_1 - F_1G_2)HP_a} = n,$$

which is simplified to

$$(n-1)(F_2H_1 - F_1H_2)G^2 = (F_1G_2 - F_2G_1)H^2.$$

Given that F and C are homogenous functions, this condition can be rewritten as

$$n - 1 = \Omega(\tfrac{a}{b}) := \frac{(F_1(\frac{a}{b},1)G_2(\frac{a}{b},1) - F_2(\frac{a}{b},1)G_1(\frac{a}{b},1))H(\frac{a}{b},1)^2}{(F_2(\frac{a}{b},1)H_1(\frac{a}{b},1) - F_1(\frac{a}{b},1)H_2(\frac{a}{b},1))G(\frac{a}{b},1)^2}.$$

Given that there exists an interior solution to satisfy this, we have a constant optimal effort ratio ψ such that $n-1 = \Omega(\psi)$ for any R. The optimal incentive scheme must solve

$$\frac{P_a}{P_b} = \frac{(N-1)\phi + N - n}{-\phi + N - n} = \frac{H(\psi,1)}{G(\psi,1)},$$

if there exists an interior solution.

Define $\underline{\psi}$ such that

$$1 = \frac{H(\underline{\psi},1)}{G(\underline{\psi},1)}.$$

There exists an interior solution if

$$n - 1 < \Omega(\underline{\psi}).$$

If this condition is satisfied, there exists some ϕ^* such that

$$\frac{(N-1)\phi^* + N - n}{-\phi^* + N - n} = \frac{H(\psi,1)}{G(\psi,1)}.$$

The resource constraint then implies that

$$R = \phi^*\Delta_T + n\Delta_T.$$

Solving this yields the expressions in the proposition. ∎

Proof of Proposition 5: Differentiating the first-order conditions with respect to n, we obtain:

$$P_a\Big(G_1\frac{\partial a^*}{\partial n}+G_2\frac{\partial b^*}{\partial n}\Big)-\frac{\Delta_T}{N^2}G=F_1\frac{\partial a^*}{\partial n}+F_2\frac{\partial b^*}{\partial n},$$

$$P_b\Big(H_1\frac{\partial a^*}{\partial n}+H_2\frac{\partial b^*}{\partial n}\Big)-\Big(\frac{\Delta_T}{N^2}-\beta'_nP_b\Big)H=F_1\frac{\partial a^*}{\partial n}+F_2\frac{\partial b^*}{\partial n}.$$

Solving these, we obtain

$$\frac{\partial a^*}{\partial n}=\frac{(G_2P_a-F_2)(\Delta_T-\beta'P_bN)H-(F_2-\beta H_2P_b)\Delta_TG)}{N^2((F_1-G_1P_a)(F_2-\beta H_2P_b)-(G_2P_a-F_2)(\beta H_1P_b-F_1))},$$

$$\frac{\partial b^*}{\partial n}=\frac{-(\beta H_1P_b-F_1)\Delta_TG-(F_1-G_1P_a)(\Delta_T-\beta'P_bN)H}{N^2((F_1-G_1P_a)(F_2-\beta H_2P_b)-(G_2P_a-F_2)(\beta H_1P_b-F_1))}.$$

An increase in n increases the total effort if

$$F_1\frac{\partial a^*}{\partial n}+F_2\frac{\partial b^*}{\partial n}=-\frac{\beta(F_2H_1-F_1H_2)\Delta_TGP_b+(F_1G_2-F_2G_1)(\Delta_T-\beta'P_b)HP_a}{N^2((F_1-G_1P_a)(F_2-H_2P_b)-(G_2P_a-F_2)(H_1P_b-F_1))}>0. \quad (12)$$

It is evident from this that this condition holds only if

$$\Delta_T\geq\beta'P_bN \;\Leftrightarrow\; \frac{N\Delta_T}{(N-1)\Delta_I+(N-n)\Delta_T}\geq\beta'.$$

Under the optimal contract (Δ_I^*,Δ_T^*), (13) becomes

$$\beta'\frac{(F_1G_2-F_2G_1)P_b}{N^2((F_1-G_1P_a)(F_2-H_2P_b)-(G_2P_a-F_2)(H_1P_b-F_1))}>0, \quad (13)$$

which holds if and only if $\beta'(2)>0$. ∎

Appendix B: derivation of the first- and second-order conditions

B.1 The first-order conditions

The first-order conditions are given by

$$\frac{\partial q_j}{\partial a_{ij}}(r_{ij}\Delta_I+\Delta_T)+q_j\frac{\partial r_{ij}}{\partial a_{ij}}\Delta_I=C_1,$$

$$\frac{\partial q_j}{\partial b_{ij}}(r_{ij}\Delta_I+\Delta_T)+q_j\frac{\partial r_{ij}}{\partial b_{ij}}\Delta_I=C_2.$$

It is straightforward to compute

$$\frac{\partial r_{ij}}{\partial a_{ij}} = F_1 \frac{\partial r_{ij}}{\partial x_{ij}} = F_1 \frac{\alpha(n-1)x_{ij}^{\alpha-1}x^{\alpha}}{(x_{ij}^{\alpha}+(n-1)x^{\alpha})^2},\ \frac{\partial r_{ij}}{\partial b_{ij}} = \frac{F_2}{n-1}\frac{\partial r_{ij}}{\partial x} = -F_2 \frac{\alpha x_{ij}^{\alpha}x^{\alpha-1}}{(x_{ij}^{\alpha}+(n-1)x^{\alpha})^2},$$

$$\frac{\partial q_j}{\partial a_{ij}} = F_1 \frac{\partial q_j}{\partial x_{ij}} = F_1 \frac{\alpha(m-1)z_j^{\alpha-1}z^{\alpha}}{(z_j^{\alpha}+(m-1)z^{\alpha})^2},\ \frac{\partial q_j}{\partial b_{ij}} = \frac{F_2}{n-1}\frac{\partial q_j}{\partial x} = F_2 \frac{\alpha(m-1)z_j^{\alpha-1}z^{\alpha}}{(z_j^{\alpha}+(m-1)z^{\alpha})^2},$$

In a symmetric equilibrium (a^*, b^*), we have

$$\frac{\partial r_{ij}}{\partial a_{ij}} = F_1 \frac{\alpha(n-1)}{n^2 x^*},\ \frac{\partial r_{ij}}{\partial b_{ij}} = -F_2 \frac{\alpha}{n^2 x^*},\ \frac{\partial q_j}{\partial a_{ij}} = F_1 \frac{\alpha(N-n)}{N^2 x^*},\ \frac{\partial q_j}{\partial b_{ij}} = F_2 \frac{\alpha(N-n)}{N^2 x^*},$$

It follows from these that the first-order conditions can be written as

$$\alpha \frac{(N-1)\Delta_I + (N-n)\Delta_T}{N^2 x^*} F_1 = C_1,\ \alpha \frac{-\Delta_I + (N-n)\Delta_T}{N^2 x^*} F_2 = C_2.$$

B2 The second-order conditions

The second-order conditions are given by

$$\frac{\partial^2 q_j}{\partial a_{ij}^2}(r_{ij}\Delta_I + \Delta_T) + 2\frac{\partial q_j}{\partial a_{ij}}\frac{\partial r_{ij}}{\partial a_{ij}}\Delta_I + q_j \frac{\partial^2 r_{ij}}{\partial a_{ij}^2}\Delta_I < C_{11},$$

$$\frac{\partial^2 q_j}{\partial b_{ij}^2}(r_{ij}\Delta_I + \Delta_T) + 2\frac{\partial q_j}{\partial b_{ij}}\frac{\partial r_{ij}}{\partial b_{ij}}\Delta_I + q_j \frac{\partial^2 r_{ij}}{\partial b_{ij}^2}\Delta_I < C_{22}.$$

With some algebra, we obtain

$$\frac{\partial^2 r_{ij}}{\partial a_{ij}^2} = \frac{\alpha(n-1)x_{ij}^{\alpha-1}x^{\alpha}}{(x_{ij}^{\alpha}+(n-1)x^{\alpha})^2}\Big(F_{11} + F_1 \frac{(\alpha-1)(x_{ij}^{\alpha}+(n-1)x^{\alpha}) - 2\alpha x_{ij}^{\alpha}}{(x_{ij}^{\alpha}+(n-1)x^{\alpha})x_{ij}}\Big),$$

$$\frac{\partial^2 r_{ij}}{\partial b_{ij}^2} = -\frac{\alpha x_{ij}^{\alpha}x^{\alpha-1}}{(x_{ij}^{\alpha}+(n-1)x^{\alpha})^2}\Big(F_{22} + F_2 \frac{(\alpha-1)(x_{ij}^{\alpha}+(n-1)x^{\alpha}) - 2\alpha(n-1)x^{\alpha}}{(n-1)(x_{ij}^{\alpha}+(n-1)x^{\alpha})x}\Big),$$

$$\frac{\partial^2 q_j}{\partial a_{ij}^2} = \frac{\alpha(m-1)z_j^{\alpha-1}z^{\alpha}}{(z_j^{\alpha}+(m-1)z^{\alpha})^2}\Big(F_{11} + F_1 \frac{(\alpha-1)(z_j^{\alpha}+(m-1)z^{\alpha}) - 2\alpha z_j^{\alpha}}{(z_j^{\alpha}+(m-1)z^{\alpha})z_j}\Big),$$

$$\frac{\partial^2 q_j}{\partial b_{ij}^2} = \frac{\alpha(m-1)z_j^{\alpha-1}z^{\alpha}}{(z_j^{\alpha}+(m-1)z^{\alpha})^2}\Big(F_{22} + F_2 \frac{(\alpha-1)(z_j^{\alpha}+(m-1)z^{\alpha}) - 2\alpha z_j^{\alpha}}{(z_j^{\alpha}+(m-1)z^{\alpha})z_j}\Big).$$

In a symmetric equilibrium (a^*, b^*), we have

$$\frac{\partial^2 r_{ij}}{\partial a_{ij}^2} = \frac{\alpha(n-1)}{n^2 x^*}\Big(F_{11} + F_1 \frac{(\alpha-1)n - 2\alpha}{n x^*}\Big) = \frac{\partial r_{ij}}{\partial a_{ij}}\Big(\frac{F_{11}}{F_1} + \frac{(\alpha-1)n - 2\alpha}{n x^*}\Big),$$

$$\frac{\partial^2 r_{ij}}{\partial b_{ij}^2} = -\frac{\alpha}{n^2 x^*}\Big(F_{22} + F_2\frac{(\alpha-1)n - 2\alpha(n-1)}{n(n-1)x^*}\Big) = \frac{\partial r_{ij}}{\partial b_{ij}}\Big(\frac{F_{22}}{F_2} + \frac{(\alpha-1)n - 2\alpha(n-1)}{n(n-1)x^*}\Big),$$

$$\frac{\partial^2 q_j}{\partial a_{ij}^2} = \frac{\alpha(N-n)}{N^2 x^*}\Big(F_{11} + F_1\frac{(\alpha-1)N - 2\alpha n}{Nnx^*}\Big) = \frac{\partial q_j}{\partial a_{ij}}\Big(\frac{F_{11}}{F_1} + \frac{(\alpha-1)N - 2\alpha n}{Nnx^*}\Big),$$

$$\frac{\partial^2 q_j}{\partial b_{ij}^2} = \frac{\alpha(N-n)}{N^2 x^*}\Big(F_{22} + F_2\frac{(\alpha-1)N - 2\alpha n}{Nnx^*}\Big) = \frac{\partial q_j}{\partial a_{ij}}\Big(\frac{F_{22}}{F_2} + \frac{(\alpha-1)N - 2\alpha n}{Nnx^*}\Big),$$

where $x^* = F(a^*, b^*)$. It follows from these that the second-order conditions can be written as

$$\begin{aligned}&\frac{\alpha(N-n)}{N^2 x^*}\Big(F_{11} + F_1\frac{(\alpha-1)N - 2\alpha n}{Nnx^*}\Big)\Big(\frac{\Delta_I}{n} + \Delta_T\Big)\\ &+ 2(\alpha F_1)^2\frac{(n-1)(N-n)}{(Nnx^*)^2} + \frac{\alpha(n-1)}{Nnx^*}\Big(F_{11} + F_1\frac{(\alpha-1)n - 2\alpha}{nx^*}\Big)\Delta_I < C_{11}, \qquad (14)\end{aligned}$$

$$\begin{aligned}&\frac{\alpha(N-n)}{N^2 x^*}\Big(F_{22} + F_2\frac{(\alpha-1)N - 2\alpha n}{Nnx^*}\Big)\Big(\frac{\Delta_I}{n} + \Delta_T\Big)\\ &- 2(\alpha F_2)^2\frac{N-n}{(Nnx^*)^2}\Delta_I - \frac{\alpha}{Nnx^*}\Big(F_{22} + F_2\frac{(\alpha-1)n - 2\alpha(n-1)}{n(n-1)x^*}\Big)\Delta_I < C_{22}, \qquad (15)\end{aligned}$$

For any optimal solution (a^*, b^*) where $a^*, b^* > 0$, the second-order conditions are satisfied if α is sufficiently small.