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ENDOGENOUS INFORMATION ACQUISITION AND PARTIAL ANNOUNCEMENT POLICY

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Endogenous Information Acquisition and Partial Announcement Policy *

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Abstract

We extend the model of Cornand and Heinemann (2008, Economic Journal) and examine how to implement partial announcement by selling public information when the agents' action is strategic complements. In a game of information acquisition, there exist multiple equilibria and the partial announcement equilibrium is unstable if the authorities sell public information at a constant price. However, if the authorities offer an increasing pricing rule, partial announcement equilibrium is stable and implementable.

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1 Introduction

In the literature on public announcement, it is well-known that public information announcement may have a detrimental welfare effect through excess coordination of agents' actions (Morris and Shin, 2002). Extending the beauty contest model of Morris and Shin (2002), Cornand and Heinemann (2008) show that a partial-announcement policy, which means that the authorities disseminate public information to a certain fraction of agents, can alleviate the excess coordination. Cornand and Heinemann (2008) assume that the fraction of agents who receive public signal is exogenously given, and analyze the welfare effect of partial announcement by comparative statics.

For policy makers, it is an important issue how to conduct the partial-announcement policy. Cornand and Heinemann (2008) propose eight ways to exclude some fraction of the agents from acquiring information. One of the ways is to "sell data at prices that not all agents are willing to pay". In this paper, we examine how to conduct the partial-announcement policy by assuming that the authorities sell information to the agents at a certain price. As a result, in contrast to the model of Cornand and Heinemann (2008), public information acquisition is endogenously determined in our model.

We find that it is not easy for the authorities to implement partial announcement by selling data at a certain price. As shown by Hellwig and Veldkamp (2009), when agent's action is strategic complements, information acquisition is also strategic complements. Likewise, we prove that, if the authorities sell public information at a certain price, this type of strategic complementarity causes multiple equilibria, which consist of two pure strategy equilibria (full- and no-announcement equilibria), and a mixed strategy equilibrium (partial-announcement equilibrium). The partial-announcement equilibrium is unstable. Hence, unless the authorities could completely coordinate beliefs of all agents, it is difficult to realize the partial-announcement equilibrium by selling data at a certain price.

We propose a pricing rule of information in order to ensure the uniqueness and stability of the partial-announcement equilibrium. If the authorities offer a pricing rule such that a price of public information is sufficiently increasing in the number of public information users, they can coordinate the agents' expectation; hence the partial announcement is implementable.

2 The Model

We borrow the model of Cornand and Heinemann (2008) except that agents who require public information must pay a usage fee.

Payoff structure There are the authorities and a continuum of agents indexed by $i \in [0,1]$. Each agent i chooses an action $a_i \in \mathbb{R}$ to maximize following payoff,

$$u_i(a,\theta) = \underbrace{-(1-r)(a_i-\theta)^2}_{\text{Loss 1}}\underbrace{-r(L_i-\bar{L})}_{\text{Loss 2}}\underbrace{-T_i+\tau}_{\text{Usage fee}}, \qquad (1)$$

where $a \equiv \{a_i : i \in [0,1]\}$ is an action profile, $\theta \in \mathbb{R}$ is unobservable state, and $r \in (0,1)$ is a parameter that represents the degree of strategic complementarity of action. Loss 1 is standard loss. Agent

i suffers a loss from a distance between a_i and θ. Loss 2 is beauty contest loss. $L_i \equiv \int_0^1 (a_i - a_j)^2 dj$ means that agent i incurs a loss from distances between a_i and others' action a_j. Loss 2 has zero-sum structure because $\bar{L} \equiv \int_0^1 L_j dj$.

Define agents who use public information as users, others as non-users, and $P \in [0, 1]$ as the share of users. In contrast to Cornand and Heinemann (2008), P is an endogenous variable. The authorities charge a constant usage fee for public information, T, and

$$T_i \equiv \begin{cases} T, & \text{if agent } i \text{ uses public information,} \\ 0, & \text{otherwise.} \end{cases}$$

τ is lamp-sum transfer from the authorities to agents. Financial resource of τ is total fee, $\tau = PT$. From (1), agent i's optimal action is $a_i = (1-r)E_i(\theta) + rE_i(\bar{a})$, where $\bar{a} = \int_0^1 a_i di$ is an average action.

Information structure Information structure is following. Assume that all error terms are mutually independent. The state θ is uniformly distributed on $\mathbb{R}$. After nature draws θ, agent i receives a private signal $x_i = \theta + \epsilon_i$ with $\epsilon_i \sim N(0, 1/\beta)$. The authorities also receive a public signal $y = \theta + \eta$ with $\eta \sim N(0, 1/\alpha)$, and disclose it only to users. In this setting, users' and non-users' estimations of θ are $E_{iu}(\theta) \equiv E(\theta|x_i, y) = \frac{\beta x_i + \alpha y}{\beta + \alpha}$ and $E_{in}(\theta) \equiv E(\theta|x_i) = x_i$, respectively.

Timing of the game The game has two stages. At stage 1, agents decide whether to buy the public information, y, given T that is set by the authorities. At stage 2, the authorities disclose y only to the users, and all agents receive x_i and choose a_i.

3 Equilibrium

We solve the model by backward induction.

At stage 2, agents decide their actions given T and P. Because of additive separability of our payoff function, each agent's equilibrium action strategy is the same as in Cornand and Heinemann (2008).

Result 1. *The equilibrium action of non-users is* $a_{in} = x_i$*, and the equilibrium action of users is* $a_{iu} = \kappa x_i + (1-\kappa)y$*, where* $\kappa \equiv \frac{\beta(1-rP)}{\alpha+\beta(1-rP)}$.

At stage 1, each agent decides whether to use y, given the other agents' decision, hence P as given. Then, expected payoff of user iu is

$$\begin{aligned} w_{iu}(P) \equiv E[u_{iu}(a)|\theta] = E\Bigg[&-(1-r)(a_{iu}-\theta)^2 \\ -r\left\{\int_0^P (a_{iu}-a_{ju})^2 dj + \int_P^1 (a_{iu}-a_{jn})^2 dj - r\bar{L}\right\}\Bigg|\theta\Bigg] - T + \tau \\ = -\frac{(1-rP)(1-\kappa)^2}{\alpha} - \frac{r(1-P)+(1+rP)\kappa^2}{\beta} + r\bar{L} - T + \tau, \end{aligned} \tag{2}$$

and, similarly, expected payoff of non-user in is

$$w_{in}(P) \equiv -\frac{rP(1-\kappa)^2}{\alpha} - \frac{[1+r(1-P)]+rP\kappa^2}{\beta} + r\bar{L} + \tau. \tag{3}$$

Agent i's problem can be written as $\max_{p_i} p_i w_{iu}(P) + (1-p_i)w_{in}(P)$, where $p_i \in [0,1]$ is agent i's mixed strategy whether to use y. From

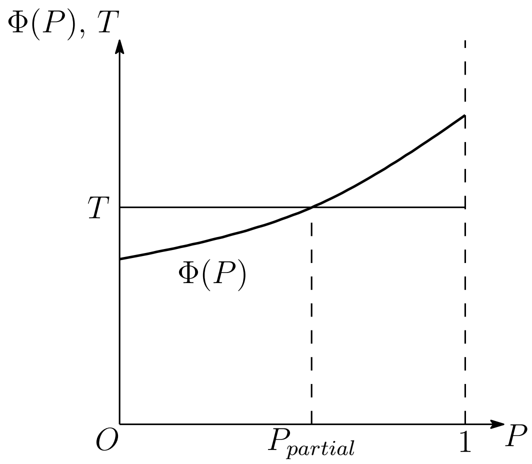

Figure 1: Benefit from acquiring public information

(2) and (3), agent i's net benefit from receiving y is $\Delta w_i(P)$:

$$\Delta w_i(P) \equiv w_{iu}(P) - w_{in}(P) = \frac{\alpha(\alpha+\beta)}{\beta\left[\alpha + (1-rP)\beta\right]^2} - T \equiv \Phi(P) - T,$$

where $\Phi(P)$ represents a gross benefit of acquiring y. If the net benefit is positive, purchasing y is optimal for agent i. If negative, refrain from buying y is optimal. If zero, the two alternatives are indifferent.

Figure 1 represents the cost and benefit of public information acquisition. The net benefit is strictly increasing in the fraction of information users because $\Phi'(P) > 0$ and T is constant. Hence, for any $T \in (\Phi(0), \Phi(1))$, there uniquely exists $P_{\text{partial}} \in (0,1)$ such that $\Phi(P_{\text{partial}}) = T$.[1] Then, for all agents, their best response function,

[1]Partial announcement does not occur when $T < \Phi(0)$ or $T > \Phi(1)$.

$R(P)$, is

$$R(P) \begin{cases} = 0 & \text{if } P < P_{\text{partial}}, \\ \in [0,1] & \text{if } P = P_{\text{partial}}, \\ = 1 & \text{if } P > P_{\text{partial}}. \end{cases} \tag{4}$$

As in Hellwig and Veldkamp (2009), $R(\cdot)$ represents that public information acquisition is strategic complements when action is strategic complements.[2] Public information is useful for inferring the other information users' action; hence, when action is strategic complements, the private value of public information becomes higher as the number of information users increases.

Multiple equilibria and (in)stability A mixed strategy profile, (p_i), is an equilibrium if, for all i, p_i is a best response for the others' strategy profile p_{-i}. From the law of large numbers, $P = R(P)$ holds in a symmetric equilibrium.

We can easily verify that the strategic complementarities about information acquisition causes multiple equilibria. Figure 2 represents the best response when $\Phi(0) < T < \Phi(1)$. $p_i = 0$ ($p_i = 1$) for all i is an equilibrium, because agent i's best response is $p_i = 0$ ($p_i = 1$) for $p_{-i} = 0$ ($p_{-i} = 1$). Moreover, $p_i = P_{partial} \in (0, 1)$ for all i, where $P_{partial}$ satisfies $\Phi(P_{partial}) = T$, is also an equilibrium because

[2] Hellwig and Veldkamp (2009) point out that, in the beauty contest situation, information acquisition is strategic complements so that multiple equilibria arise. A difference between them and ours is the proposed ways to make equiliblium unique. They propose a way to realize a unique pure strategy equilibrium. In contrast, we focus on a stability of a mixed strategy equilibrium. Hence, in section 4, we propose a way to make the mixed strategy equilibrium unique, because the mixed strategy equilibrium corresponds to partial-announcement one.

$p_i = P_{partial}$ is a best response for $p_{-i} = P_{partial}$.

Proposition 1. *Suppose that the authorities apply the constant pricing rule. Then,*

1. *If $T \in (\Phi(0), \Phi(1))$, then multiple equilibria arise as follows.*

   (a) No-announcement equilibrium: $p_i = 0$ for all i, hence $P = 0$,

   (b) Full-announcement equilibrium: $p_i = 1$ for all i, hence $P = 1$,

   (c) Partial-announcement equilibirum: $p_i = P_{partial}$ for all i, hence $P = P_{partial}$.

2. *If $T < \Phi(0)$ or $T > \Phi(1)$, there does not exist any partial-announcement equilibrium.*

Next, we define a stability of an equilibrium, following Milgrom and Roberts (1990) and Vives (1990). In what follows, we describe equilibrium by its outcome $P_l, l = 1, 2, 3$, where $P_1 = 0, P_2 = P_{partial}, P_3 = 1$, corresponds to no-announcement, partial announcement, and full-announcement, respectively. A Cournot tatonnement in our game is defined as the process $\{P(t)\}$: $P(0) \in [0, 1]$, $P(t) \in R(P(t-1))$, $t = 1, 2, \cdots$. We define the stability of equilibrium as follows.

Definition. *An equilibrium $P_l \in [0, 1]$ is stable if there exists $P(0) \neq P_l$ such that the Cournot tatonnement starting at $P(0)$ converges to P_l.*

Figure 2 represents the best-response dynamics and equilibrium stability in our information acquisition game. When $P(0) \in [0, P_{\text{partial}})$, the best-response dynamics converges to $P_1(= 0)$. When $P(0) \in (P_{\text{partial}}, 1]$, it converges to $P_3(= 1)$. Hence, the following proposition holds.

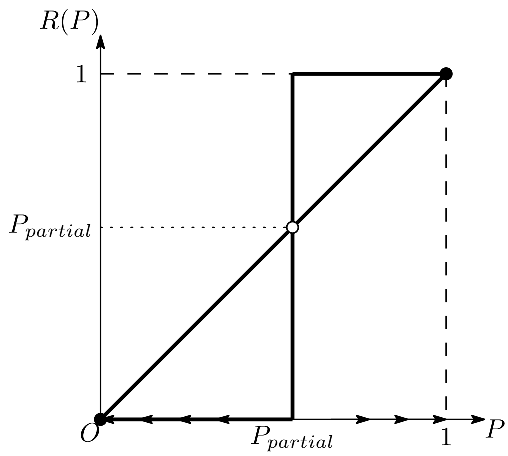

Figure 2: Best response dynamics and (in)stability of equilibrium

Proposition 2. *Suppose that the authorities apply the constant pricing rule with $T \in (\Phi(0), \Phi(1))$. Then, no-announcement and full-announcement equilibrium are stable, and partial-annoucement equilibrium is unstable.*

Such an equilibrium instability implies that coordination of the agents' expectation is essential in order to achieve the optimal degree of public information dissemination.[3]

4 A Coordination Device of Expectation

We propose a solution that the authorities guide the agents to the unique partial-announcement equilibrium. The cause of the coordina-

[3]In our model, the welfare-maximizing ratio of information users is $P^* = \min\{1, \frac{\alpha+\beta}{3r\beta}\}$, when $T^* = \frac{9\alpha}{4\beta(\alpha+\beta)}$ (or 0) if $\alpha/\beta < 3r-1$ (or $\alpha/\beta \geq 3r-1$). P^* is the same as in Cornand and Heinemann (2008). Proposition 2 implies any equilibrium in which $P_{\text{partial}}(T) \in (0, 1)$ is unstable, including the case that $P_{\text{partial}} = P^*$.

tion failure is that, owing to the strategic complementarities, $\Delta w_i(P)$ is upward sloping. To align the agents' belief, we employ another pricing rule that has strategic *substitution* effect. Assume that the fee sufficiently increases in the number of users. Formally, consider a pricing rule $T = \Psi(P)$, where $\Psi(P)$ satisfies $\Psi(P_{\text{partial}}) = T$, and

$$\Psi(P) \begin{cases} < \Phi(P) & \text{if } P > P_{\text{partial}}, \\ > \Phi(P) & \text{if } P < P_{\text{partial}}. \end{cases}$$

The strategic substitution effect of $\Phi(P)$ counteracts the strategic complementarities of information acquisition, and makes $\Delta w_i(P)$ downward sloping. Then, the agents plausibly believe that $P = P_{\text{partial}}$ is realized, because the agents' best response function is

$$R(P) \begin{cases} = 1 & \text{if } P < P_{\text{partial}}, \\ \in [0,1] & \text{if } P = P_{\text{partial}}, \\ = 0 & \text{if } P > P_{\text{partial}}, \end{cases}$$

and, hence, an equilibrium $P = P_{\text{partial}}$ uniquely exists (Figure 3). This shows that, by using the increasing pricing rule as a coordination device, the authorities can implement a partial-announcement policy.

5 Conclusion

Partial-announcement policy is a solution for alleviating over coordination problem generated by strategic complementarities in action. However, such strategic complementarities makes information acqui-

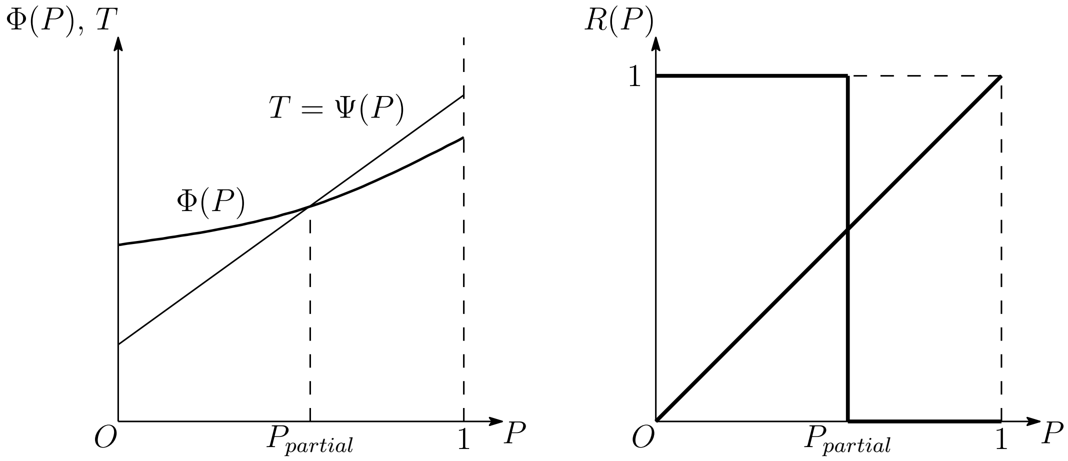

Figure 3: Increasing pricing rule and stability of equilibrium

sition also strategic complements, hence strategic complementarities themselves may disturb the implementation of partial announcement. Nevertheless, the partial announcement policy becomes implementable under some increasing pricing rules.

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