EXCHANGE-RATE ADJUSTMENT
AND
MACROECONOMIC INTERDEPENDENCE
BETWEEN STAGNANT AND
FULLY EMPLOYED COUNTRIES

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Exchange-rate adjustment and macroeconomic interdependence between stagnant and fully employed countries*

by

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Abstract

This paper presents a two-country two-commodity dynamic model with free international asset trade in which one country achieves full employment and the other suffers long-run unemployment. Own and spill-over effects of changes in policy, technological and preference parameters that emerge through exchange-rate adjustment are examined. Parameter changes that worsen the stagnant country’s current account depreciate the home currency, expand home employment and improve the foreign terms of trade, making both countries better off. The stagnant country’s foreign aid to the fully employed country also yields the same beneficial effects.

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1. Introduction

In a closed economy, an increase in productivity obviously expands national income if full employment prevails. In a two-country setting it benefits not only the home country but also the foreign country because it lowers the relative price of the home commodity and improves the foreign country’s terms of trade. If the two countries suffer long-run economic stagnation due to aggregate demand deficiency, however, the result may be different. By extending the persistent stagnation model of Ono (1994, 2001) to a two-country framework, Ono (2006, 2013) showed that in the presence of aggregate demand deficiency an increase in the home country’s productivity excessively improves the current account and leads the home currency to appreciate so much that home employment and consumption decrease. The home currency appreciation in turn causes the foreign country’s employment and national income to increase. Moreover, changes in policy, technical or preference parameters that stimulate home aggregate demand generally worsen the home current account and lead the home currency to depreciate, which reduces foreign employment and makes the foreign country worse off. Thus, an international asymmetry of business activities naturally arises. This is quite different from the standard result that holds under full employment in both countries.

There is another important case: one country faces persistent deficiency of aggregate demand while the other country realizes full employment. It may in particular be the case between a developed country that faces persistent stagnation, e.g. Japan, and an emerging country that has large demand and enjoys a boom, e.g. China. This paper treats this case and examines which of the two results mentioned above is true.

This analysis also applies to the effect of foreign aid. It is naturally believed that foreign aid makes the donor country worse off and the recipient country better off. Due to this belief the foreign aid budget is usually cut when a donor country faces persistent stagnation, as Japan did in the ‘Lost Decades’. However, this belief is true only if full employment prevails in both
countries. If both countries face stagnation, the donor country is better off while the recipient country is worse off as a result of exchange-rate adjustment (see Ono, 2013).¹

The pros and cons of foreign aid in the asymmetric case were discussed in some important policy decisions. The Marshall Plan in 1947 was an example. George C. Marshall, the US Secretary of State at that time, proposed an aid of $20 billion to European countries that had significantly lost supply capacities in World War II and faced a serious shortage of supply. He insisted that it benefited not only European consumers by enabling them to import US commodities but also US producers and workers by creating a market.² In the context of the North-South problem, the Independent Commission on International Development Issues, a panel lead by former German Chancellor Willy Brandt in the early 1980s (the Brandt commission, 1980, 1983), argued that foreign aid from the North to the South would benefit the donors as well as the recipients through not only stabilizing political/security situations but also creating import demand and expanding employment in the North.³ This paper examines the validity of those statements and finds that foreign aid makes both the recipient country and the donor country better off if the former achieves full employment and the latter has persistent deficiency of aggregate demand.

It should be noted that the stagnation considered in the present analysis is not a temporary one but a persistent one that arises as a steady-state phenomenon. Most of the recent literature on macroeconomic fluctuations in an open-economy setting focuses on the analysis of

¹ In the literature such a controversial case has been discussed as a transfer paradox. The paradox arises through not an expansion of employment but a change in the terms of trade with full employment. It arises only in the case of Walrasian instability, multiple equilibria or some distortions in a two-country case. See Bhagwati et al (1983, 1985) for a general analysis of the transfer paradox with distortions in static two-country and three-country frameworks. Polemarchakis (1983) extended it to an n-country economy. The paradoxical case in this paper arises through a change in employment.

² He stated: “The Marshall Plan, it should be noted, benefited the American economy as well. The money would be used to buy goods from the United States, and they had to be shipped across the Atlantic on American merchant vessels.” See Congressional Record, 30 June 1947.

³ This issue was treated by Ono (2007) but he used CES utility functions. This paper extends that analysis to the case of general homothetic utility.
perturbations due to various policy and technology shocks in the neighborhood of the full-employment steady state. They are new open economy macroeconomic models (see an extensive survey of Lane, 2001) and extensions of DSGE models (e.g., Smets and Wouters, 2003, 2007; Christiano et al., 2005) to a small open economy setting (e.g. Adolfson et al., 2007, 2008). It may however be inadequate to apply these approaches to such long-run stagnation as Japan’s Lost Decades and maybe EU countries’ Great Recession triggered by the financial crisis of 2008. In the recent IMF annual conference, for example, Summers (2013) criticized too much reliance on the DSGE approach in solving economic crises and emphasized the need for researchers to work on long-run recessions rather than short-run business fluctuations. This paper follows this line of thought and considers a long-run stagnation model.

2. The model

There are two countries, the home and foreign countries. The home production sector specializes in commodity 1 and the foreign one in commodity 2. Both of them use only labor to produce the commodities with constant productivity.

The household sectors of the two countries have the same utility function of the two commodities. The function is homothetic and hence for a given level of consumption expenditure $c$ (or $c^*$) the utility can be summarized as

$$u(c) \equiv \hat{u}(c_1, c_2), \quad u(c^*) \equiv \hat{u}(c_1^*, c_2^*)$$

where $c_i$ (or $c_i^*$) is the consumption of commodity $i$ ($i = 1, 2$) that satisfies

$$p_1(\omega)c_1 = \delta(\omega)c, \quad p_2(\omega)c_2 = [1 - \delta(\omega)]c,$$

$$p_1(\omega)c_1^* = \delta(\omega)c^*, \quad p_2(\omega)c_2^* = [1 - \delta(\omega)]c^*,$$

$$\delta'(\omega) > 0. \quad (1)$$

In the above expressions $\omega$ is the relative price and $p_i(\omega)$ is the real price of commodity $i$...
\((i = 1, 2)\):

\[
p_1(\omega) = \frac{P_1}{p} = \frac{P_1^*}{p^*}, \quad p_2(\omega) = \frac{P_2}{p} = \frac{P_2^*}{p^*} = \omega p_1(\omega),
\]

\[
P_1^* = \frac{P_1}{\epsilon}, \quad P_2 = \epsilon P_2^*.
\]

\(P\) and \(P^*\) are the two countries’ general price indices, \(P_i\) and \(P_i^*\) are the home and foreign nominal prices of commodity \(i\) measured in each currency, and \(\epsilon\) is the nominal exchange rate, which satisfies

\[
P = \epsilon P^*.
\]

Because the time derivative of this equation gives

\[
\pi = \frac{\dot{\epsilon}}{\epsilon} + \pi^*,
\]

where \(\pi\) and \(\pi^*\) are inflation rates, from the non-arbitrage condition between home and foreign assets whose nominal interest rates are respectively \(R\) and \(R^*\):

\[
R = \frac{\dot{\epsilon}}{\epsilon} + R^*,
\]

one obtains

\[
R - \pi = r = R^* - \pi^*,
\]

i.e, the real interest rate \(r\) is internationally the same.

Because the home and foreign general price indices \(P\) and \(P^*\) satisfy

\[
\frac{dp}{p} = \delta(\omega) \frac{dp_1}{p_1} + (1 - \delta(\omega)) \frac{dp_2}{p_2}, \quad \frac{dp^*}{p^*} = \delta(\omega) \frac{dp_1^*}{p_1^*} + (1 - \delta(\omega)) \frac{dp_2^*}{p_2^*},
\]

as shown by Deaton and Muellbauer (1980, p.175), from (2) one finds

\[
0 = \delta(\omega) \frac{p_1'(\omega)}{p_1(\omega)} + (1 - \delta(\omega)) \frac{p_2'(\omega)}{p_2(\omega)},
\]

\[
1 + \frac{\omega p_1'}{p_1} = \frac{\omega p_2'}{p_2}.
\]

These two equations yield

\[
\delta = 1 + \frac{\omega p_1'}{p_1} = \frac{\omega p_2'}{p_2}.
\]
The home and foreign representative households have the same subjective discount rate \( \rho \) and the same liquidity preference \( \nu(\cdot) \). They maximize each utility functional:

\[
\int_0^\infty [u(c) + \nu(m)] \exp(-\rho t) \, dt, \quad \int_0^\infty [u(c^*) + \nu(m^*)] \exp(-\rho t) \, dt,
\]

subject to each flow budget equation and asset constraint:

\[
\dot{a} = ra + wx - c - Rm - z, \quad \dot{a}^* = ra^* + w^*x^* - c^* - R^*m^* - z^*, \quad a = m + b, \quad a^* = m^* + b^*,
\]

(5)

where \( w \) (or \( w^* \)) is the real wage, \( z \) (or \( z^* \)) is the lump-sum tax, and \( x \) (or \( x^* \)) is the employment. Real total assets \( a \) (or \( a^* \)) consist of real money balances \( m \) (or \( m^* \)) and foreign asset \( b \) (or \( b^* \)). The firm value is zero under the linear technology. Real rate of interest \( r \) is internationally the same, as shown by (3). The two countries’ labor endowments are normalized to 1 and may not be fully employed. Therefore, each country’s actual employment \( x \) (or \( x^* \)) implies each employment rate.

From the Hamiltonian function of each household’s optimization behavior:

\[
H = u(c) + \nu(m) + \lambda(ra + wx - c - Rm - z),
\]

\[
H^* = u(c^*) + \nu(m^*) + \lambda^*(ra^* + w^*x^* - c^* - R^*m^* - z^*),
\]

one obtains the first-order optimal conditions:

\[
\lambda = u'(c), \quad \lambda R = \nu'(m), \quad \frac{\lambda}{\lambda} = \rho - r,
\]

\[
\lambda^* = u'(c^*), \quad \lambda^* R^* = \nu'(m^*), \quad \frac{\lambda^*}{\lambda^*} = \rho - r.
\]

(6)

From the Ramsey equations in (6), one finds

\[
\lambda^* = \kappa \lambda, \quad \kappa = \text{constant over time.}
\]

(7)

From (3), (6) and (7), world total consumption \( C \) satisfies

\[
C (= c + c^*) = u''^{-1}(\lambda) + u''^{-1}(\kappa \lambda) \to \lambda = \lambda(C, \kappa),
\]

---

4 Apparently, by replacing \( u(c) \) by \( \hat{u}(c_1, c_2) \) one obtains the intratemporal and intertemporal optimal conditions given by (1) and (6) all at once.
\[ \left( \lambda \frac{\dot{c}}{c} \right) \frac{\dot{c}}{c} = \rho + \pi - \frac{\nu'(m)}{u'(c)}, \]  

(8)

and \( \lambda \) satisfies

\[ \frac{\lambda c}{\lambda} = -1/\left( \frac{c + \nu'}{\eta} \right) < 0, \quad \frac{\kappa \lambda}{\lambda} = -\left( \frac{\nu'}{\eta^*} \right) < 0, \]  

(9)

where

\[ \eta = -\frac{u''(c)c}{u'(c)}, \quad \eta^* = -\frac{u''(c^*)c^*}{u'(c^*)}. \]

The home (or foreign) government imposes lump-sum tax \( z \) (or \( z^* \)) and purchases commodity \( i \) (\( i = 1, 2 \)) by the amount of \( g_i \) (or \( g_i^* \)). Therefore,

\[ z = p_1(\omega)g_1 + p_2(\omega)g_2, \quad z^* = p_1(\omega)g_1^* + p_2(\omega)g_2^*. \]  

(10)

From the home and foreign demand functions presented by (1), the world demand for commodity 1 and that for commodity 2 are \( \delta(\omega)C/p_1(\omega) \) and \( [1 - \delta(\omega)]C/p_2(\omega) \), respectively. Commodity prices perfectly adjust in the international competitive market so that

\[ \frac{\delta(\omega)}{p_1(\omega)} C + g_1 + g_1^* = \theta_1 x, \]

\[ \frac{1 - \delta(\omega)}{p_2(\omega)} C + g_2 + g_2^* = \theta_2 x^*. \]  

(11)

where \( \theta_1 \) is the home productivity and \( \theta_2^* \) is the foreign productivity. Real balances \( m \) and \( m^* \) satisfy

\[ m = \frac{M}{p}, \quad m^* = \frac{M^*}{p^*}, \]  

(12)

where \( M \) and \( M^* \) are nominal money supplies and are assumed to be constant over time, for simplicity, but may increase in a once-and-for-all manner. Because the above equations yield

\[ \dot{m} = -\pi m, \quad \dot{m}^* = -\pi^* m^*, \]

and \( z \) and \( z^* \) satisfy (10), the flow budget equations in (5) reduce to

\[ \dot{b} = rb + p_1(\omega)\theta_1 x - c - [p_1(\omega)g_1 + p_2(\omega)g_2], \]

\[ \dot{b}^* = rb^* + p_2(\omega)\theta_2^* x^* - c^* - [p_1(\omega)g_1^* + p_2(\omega)g_2^*]. \]  

(13)

Foreign assets \( b \) and \( b^* \) always satisfy
\[ b + b^* = 0. \]  \hspace{1cm} (14)

3. The condition for the asymmetric steady state to arise

This section presents the condition for the asymmetric case where the home country faces persistent unemployment while the foreign country achieves full employment to appear. If the economy is in steady state, \( c \) and \( c^* \) are constant and then from (6) and (12),

\[ \rho = \frac{v'(M/P)}{u'(c)}, \quad \rho = \frac{v'(M^*/P^*)}{u'(c^*)}. \]  \hspace{1cm} (15)

Current accounts \( \dot{b} \) and \( \dot{b}^* \) given in (13) are zero. Therefore, if both countries achieve full employment, i.e.,

\[ x = 1, \quad x^* = 1, \]

from (11), (13) and (14), \( c, c^* \) and \( \omega \) satisfy

\[ c = c_f \equiv \rho b + p_1(\omega^f)\theta_1 - [p_1(\omega^f)g_1 + p_2(\omega^f)g_2], \]
\[ c^* = c^*_f \equiv -\rho b + p_2(\omega^f)\theta_2^* - [p_1(\omega^f)g_1^* + p_2(\omega^f)g_2^*], \]

\[ \frac{\omega^f}{1-\delta(\omega^f)} = \frac{\theta_1}{\theta_2^*} \frac{g_1 - g_1^*}{g_2 - g_2^*}. \]  \hspace{1cm} (16)

Substituting these \( c \) and \( c^* \) into (15) gives the steady state levels of \( P \) and \( P^* \).

However, in the presence of a liquidity trap the above-mentioned steady state may not exist. In the present setting a liquidity trap emerges if the marginal utility of money \( v'(m) \) has a positive lower bound \( \beta \), \(^5\)

\[ \lim_{m \to \infty} v'(m) = \beta > 0, \]

because the first two equations in (6) gives the home money demand function:

\(^5\) Ono (1994, 2001) assumes this property in a closed-economy setting and proves that the dynamic equilibrium path uniquely exists and converges to a steady state with persistent unemployment. The validity of this property is empirically shown by Ono, Ogawa and Yoshida (2003) using both a parametric and a non-parametric approach.
\[ R = \frac{v'(m)}{u'(c)} (> \frac{\beta}{u'(c)} \text{ for any } m), \]

where \( R \) stays strictly positive as \( m \) increases, implying a liquidity trap. In this case, if and only if the home consumption under full employment \( c_f \) is so large as to satisfy

\[ \rho < \frac{\beta}{u'(c_f)} \left( \left\langle \frac{v'(M/P)}{u'(c_f)} \right\rangle \text{ for any } P \right), \]

there is no \( P \) that satisfies the first equation in (15), implying that there is neither a steady state that accommodates full employment in the home country nor an equilibrium path that achieves full employment. Given that the left- and right-hand sides of (17) respectively represent the time preference rate and the liquidity premium, (17) shows the case where the marginal desire for holding money dominates that for consumption if this household consumes enough to realize full employment. The same argument applies to the foreign country.

If the home country has no steady state with full employment and the foreign country achieves full employment, (17) is valid in the home country while foreign consumption \( c^* \) given in (16) satisfies

\[ \rho > \frac{\beta}{u'(c^*)} \]

so that there is \( P^* \) that makes the second equation of (15) valid. From (16), (17) and the above inequality, the asymmetric case emerges when

\[ \rho < \frac{\beta}{u'[^{(\rho b + p_1(\omega f)g_1 + p_2(\omega f)g_2)]}}, \]

\[ \rho > \frac{\beta}{u'[-(\rho b + p_2(\omega f)g_2^* + p_2(\omega f)g_2^*)]}, \]

or equivalently,

\[ \rho b > \min(u'^{-1}\left(\frac{\rho}{\beta}\right) - p_1(\omega f)(\theta_1 - g_1) + p_2(\omega f)g_2, \]

\[ -u'^{-1}\left(\frac{\rho}{\beta}\right) - p_1(\omega f)g_1^* + p_2(\omega f)(\theta_2^* - g_2^*)) \]

(18)

Ono (2013) deals with the case where the first inequality is valid but the second one is opposite: neither country has a full-employment steady state.
This case holds when home productivity $\theta_1$ is high while foreign productivity $\theta_2^*$ is low, when home government purchases $g_1$ and $g_2$ are small while foreign government purchases $g_1^*$ and $g_2^*$ are large, and when the home country owns huge foreign assets (i.e., $b$ is large). In the following analysis we treat this case.

4. Dynamics and local stability

In the asymmetric case there is no equilibrium path with full employment. In order to accommodate the possibility of persistent unemployment, sluggish wage adjustments must be introduced. Recent dominant settings of wage adjustments are the New Classical, the New Keynesian, and the hybrid Phillips curves. They well fit to analyze short-run fluctuations around the full-employment steady state, but does not to examine persistent stagnation because they are set up so that the inflation-deflation rate cumulatively expands as long as market disequilibrium exists. Thus, the possibility of unemployment in a steady state is intrinsically eliminated and thus under (17) no equilibrium path exists. In order for the unemployment steady state to be possible, a dynamic extension of Akerlof’s fair wage model (1982), presented by Ono and Ishida (2013), is adopted in the following analysis.

In each country there are three kinds of workers, employed, unemployed and newly hired ones. Employed workers randomly separate from the current job at the Poison rate $\alpha$ so that employment $x$ follows

$$
\dot{x} = -\alpha x + \chi,
$$

(19)

---


8 Ono and Ishida (2013) extended the fair-wage hypothesis a la Akerlof (1982) and Akerlof and Yellen (1990) to a dynamic setting and proposed a microeconomic foundation of wage adjustment that converges to the conventional Walrasian one. That adjustment mechanism intrinsically eliminates neither unemployment nor full employment in the steady state.
where $\chi$ is the number of workers that are newly hired. While workers are employed, they form fair wage $W_F$ in mind by referring to their past wages, their fellow workers’ fair wages (which equal their own fair wages) and the unemployment situation of the society. More precisely, they first consider the rightful wage $v$, which is the wage that they believe fair if everybody is employed. Therefore, $v(t - \Delta t)$, meaning the rightful wage that is ex post conceived at time $t - \Delta t$, is calculated so that the current fair wage $W_F(t - \Delta t)$ equals the average of $v(t - \Delta t)$ and the zero income of the unemployed. Because the number of the employed is $x(t - \Delta t)$ and that of the unemployed is $1 - x(t - \Delta t)$, it satisfies

$$v(t - \Delta t)x(t - \Delta t) + 0 \times [1 - x(t - \Delta t)] = W_F(t - \Delta t).$$

(20)

Newly hired workers, in contrast, do not have any preconception about the fair wage and simply follow the employed workers’ conceptions. Therefore, when the employed workers calculate the fair wage $W_F(t)$ at time $t$, the total number of workers that they care is $1 - \chi(t)\Delta t$ because the number of new comers is $\chi(t)\Delta t$. The rightful wage that they have in mind is the one that was ex post conceived at time $t - \Delta t$, which is $v(t - \Delta t)$ in (20), and the number of the employed workers is $x(t - \Delta t)(1 - \alpha \Delta t)$. Thus, the fair wage $W_F(t)$ is formed to be:

$$W_F(t) = \frac{v(t - \Delta t)x(t - \Delta t)(1 - \alpha \Delta t)}{1 - \chi(t)\Delta t}.$$

From (20) and the above equation, one obtains

$$\frac{W_F(t) - W_F(t - \Delta t)}{\Delta t} = \chi(t)W_F(t) - \alpha W_F(t - \Delta t).$$

Reducing $\Delta t$ to zero results in

$$\frac{\dot{W}_F}{W_F} = \chi - \alpha.$$ 

(21)

In the presence of unemployment, the firm will set wage $W$ equal to fair wage $W_F$ because $W_F$ is the lowest wage under which the employees properly work. In the home country the commodity price $P_1$ adjusts to $W_F/\theta_1$ since there is no commodity supply if $P_1 < W_F/\theta_1$ and
excess commodity supply if $P_1 > W_F / \theta_1$. Under full employment, in contrast, the firm tries to pick out workers from rival firms to expand the market share by increasing $W$ from $W_F$ so long as the marginal profits are positive. Therefore, $W$ is higher than the fair wage $W_F$ that follows (21), and is equalized to $\theta_1 P_1$. The same argument is valid in the foreign country.

Note that each commodity price, $P_1$ in the home country and $P_2^*$ in the foreign country, follows the movement of the fair wage $W_F$ when there is unemployment and $W$ declines, and that $W$ follows the movement of each commodity price regardless of $W_F$ when full employment maintains and $W$ rises. Thus, anyway one has

$$w = \frac{W}{P_1} = \theta_1, \quad w^* = \frac{W^*}{P_2^*} = \theta_2^*.$$  \hspace{1cm} (22)

In the following the asymmetric case represented by (18) is considered and then:

$$x < 1, \quad x^* = 1.$$  

Therefore, $W = W_F$ that follows (21) in the home country while $W^*$ and $P_2^*$ always adjust so that full employment maintains in the foreign country. Having such price and wage adjustments in mind, from (2), (4), the second equation of (11) in which $x^* = 1$, (21) and (22), one obtains

$$\frac{\dot{W}}{W} = \frac{\dot{P}_1}{P_1} = \pi - (1 - \delta) \frac{\dot{\omega}}{\omega} = \chi - \alpha.$$

$$\frac{1 - \delta(\omega)}{\delta(1 - \delta)} \frac{\dot{C}}{C} + g_2 + g_2^* = \theta_2^*.$$  \hspace{1cm} (23)

Using (4), the time derivative of the first equation in (11) and that of the second equation in (23), one finds

$$\frac{\dot{\omega}}{\omega} = \left( \frac{1 - \delta}{\delta(1 - \delta) + \delta^2 \omega} \right) \frac{\dot{C}}{C},$$

$$\dot{x} = \left( \frac{C}{\theta_1 P_1} \right) \frac{\dot{C}}{C}.$$  \hspace{1cm} (24)

Equation (19), the first equation of (23) and the two equations of (24) give

$$\pi = \alpha(x - 1) + \left( \frac{C}{\theta_1 P_1} + \frac{(1 - \delta)^2}{\delta(1 - \delta) + \omega \delta^2} \right) \frac{\dot{C}}{C}.$$
Note that the Poison rate of job separation $\alpha$ represents the price adjustment speed around the steady state in which $C$ is constant and that $1/\alpha$ denotes the average duration of employment.

Substituting $x$ obtained from the first equation of (11) into the above expression of $\pi$ and applying the result to $\pi$ in (8) leads to

$$
\left( \frac{C}{p_1 \theta_1} + \frac{(1-\delta)^2}{\delta(1-\delta)+\omega \delta'} \frac{\lambda C}{\lambda} \right) \frac{C}{C} = \frac{v'(m)}{\lambda(C,\kappa)} - \rho - \alpha \left( \frac{\delta(\omega)}{p_1(\omega) \theta_1} C + \frac{g_1 + g_1^*}{\theta_1} - 1 \right),
$$

where $\omega$ is a function of only $C$ derived from the second equation of (23). From (1) and (9), the coefficient of $\dot{C}/C$ in (25) is positive. In the neighborhood of the steady state, deflation continues and $v'(m)$ sticks to $\beta$, and hence the dynamic equation given by (25) reduces to

$$
\left( \frac{C}{p_1 \theta_1} + \frac{(1-\delta)^2}{\delta(1-\delta)+\omega \delta'} \frac{\lambda C}{\lambda} \right) \frac{C}{C} = \frac{\beta}{\lambda(C,\kappa)} - \rho - \alpha \left( \frac{\delta(\omega)}{p_1(\omega) \theta_1} C + \frac{g_1 + g_1^*}{\theta_1} - 1 \right) \equiv \Delta(C,\kappa).
$$

If $C$ takes the full employment level $C_f$ that leads the home country to full employment, the first equation of (11) turns to be

$$
\delta(\omega) C_f + g_1 + g_1^* = p_1(\omega) \theta_1.
$$

Therefore, from (6), (8) and (17),

$$\rho < \frac{\beta}{u'(C_f)} = \frac{\beta}{\lambda(C_f,\kappa)},$$

and $\Delta(C,\kappa)$ defined by (26) satisfies

$$\Delta(C_f,\kappa) > 0.$$ 

Therefore, in order for the steady-state level of $C$ to exist in the range of $(0, C_f)$, it must be valid that

$$\Delta(0,\kappa) < 0,$$

Because (6), (7) and (8) imply that $c = 0$ if $C = 0$ for a given $\kappa$, $\lambda(0,\kappa) = u'(0) = \infty$. Therefore, from (26), in the neighborhood where $g_1 + g_1^* = 0$ the above condition is equivalent to

$$\rho - \alpha > 0.$$  

Figure 1 illustrates this case – i.e., $R_m$ is located above $\rho$ and (27) is valid.
In this case $\Delta(C, \kappa)$ must be positively inclines as $C$ increases around the steady state, which is represented by $E$ in the figure. From (4), (9), the second equation of (23), and (26), this property is represented as follows:

$$
\Delta_C(C, \kappa) = \left(\frac{\beta}{\lambda(C, \kappa)}\right)\left[\frac{c+\sigma}{\eta} - \frac{\alpha}{p_1\theta_1}\right] > 0.
$$

This property guarantees the uniqueness of equilibrium path in the dynamics of (26) because $C$ is jumppable. Therefore, $C$ jumps to the level that satisfies

$$
\Delta(C, \kappa) = 0,
$$

and stays, once $\kappa$ is given.

Moreover, from (6) and (7), $\kappa$ must equal $u'(c^*)/u'(c)$ where $c$ and $c^*$ make $\dot{b} (= -\dot{b}^*)$ in (13) equal zero because otherwise $\dot{b}$ and $-\dot{b}^*$ continue to either expand or decline and the non-Ponzi game condition is violated. Therefore,

$$
u'(c^*) = \kappa u'(c),
$$

$$
c = C - c^*,
$$

$$
c^* = -\rho b + p_2(\omega)\theta_2^* - [p_1(\omega)g_1^* + p_2(\omega)g_2^*],
$$

(28)

where $\omega$ satisfies the second equation of (23). This $\kappa$ depends on only $C$ besides exogenous parameters. Substituting it into (26) rewrites $\Delta(C, \kappa)$ to

$$
\Phi(C; \alpha, \beta, b, \theta_1, \theta_2^*, g_1, g_2, g_1^*, g_2^*) \equiv \frac{\beta}{\lambda(C, \kappa)} - \rho - \alpha \left(\frac{\delta(\omega)}{p_1(\omega)\theta_1}C + \frac{g_1+g_1^*}{\theta_1} - 1\right),
$$

(29)

where $\omega$ satisfies the second equation of (23). Making $\Phi$ equal zero gives the complete solution of the steady-state level of $C$. Substituting this $C$ to the second equation of (23) gives $\omega$, and hence from (28) $c^*$ and $c (= C - c^*)$ obtain.

Let us finally prove that this steady state indeed exists in the neighborhood where

$$
g_1 = g_1^* = 0, \quad g_2 = g_2^* = 0.
$$

(30)

From (4), the second equation of (23) and the second and third equations of (28), one obtains

$$
\frac{dc}{dc} = \frac{\delta^2(1-\delta) + \omega\delta'}{\delta(1-\delta) + \omega\delta'} > 0,
$$

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i.e., the steady-state level of $c$ is larger as $C$ increases. If $C$ is small enough to make $c$ equal zero, $C = c^*$ and $\lambda = u'(0) = \infty$. Thus, under (27) $\emptyset$ given by (29) satisfies

$$\emptyset(c^*; ...) = -\rho - \alpha \left( \frac{\delta(\omega)c^*}{\rho_1(\omega)\theta_1} - 1 \right) < 0.$$ 

If $C$ is so large as to make $c$ equal the full-employment level $c_f$, $C = C_f$ and then from the first equation of (11) in which $x = 1$, the third term of the right-hand side of (29) is zero. Thus, from (17) $\emptyset$ given by (29) satisfies

$$\emptyset(C_f; ...) = \frac{\rho}{u'(c_f)} - \rho > 0.$$ 

These two properties guarantees the existence of $C$ that makes $\emptyset$ equal zero and $c$ locate within $(0, c_f)$. It also implies that around the steady state level of $C$

$$\emptyset_C(C; ...) > 0. \quad (31)$$

### 5. Own and spillover effects of policy and parameter changes

This section analyzes the effects on $C$, $c$ and $c^*$ of changes in various policy, technological and preference parameters. As shown in the previous section, after those parameters change, the new steady state is immediately reached. Therefore, one can obtain the effects on $c$ and $c^*$ of changes in the parameters by ignoring the transitional process and simply calculating the effects on their steady-state levels. In order for such adjustment to emerge, it is assumed that the world economy is initially in the asymmetric steady state and that the foreign monetary authority, if necessary, increases the money stock in a once-and-for-all manner to avoid even short-run unemployment. The home monetary authority does not change the money stock because the home country is in the unemployment steady state, where a monetary expansion has no effect.
From (4), (9), (29) to which \( \omega \) in the second equation of (23) and \( \kappa \) in (28) are applied, and (31), in the neighborhood of (30) one obtains

\[
\Phi_C dC = - \phi_{\alpha} d\alpha - \phi_{\beta} d\beta - \phi_{g_1} dg_1 - \phi_{g_2} dg_2 - \phi_{\theta_1} d\theta_1 - \phi_{g_1^*} dg_1^* - \phi_{g_2^*} d(g_2^* - g_2^*).
\]

\[
\Phi_C = \left( \frac{\eta}{c} \right) \left( \frac{\beta}{\lambda} \right) \frac{d^2(1-\delta) + \omega \delta'}{\delta(1-\delta) + \omega \delta'} - \frac{\alpha}{p_1 \theta_1} > 0,
\]

\[
\phi_\alpha = 1 - x > 0, \quad \phi_\beta = \frac{1}{\lambda} > 0, \quad \phi_\beta = \rho \left( \frac{\eta}{c} \right) \left( \frac{\beta}{\lambda} \right) > 0,
\]

\[
\phi_{g_1} = -\frac{\alpha}{\theta_1} < 0, \quad \phi_{g_2} = -\left( \frac{\eta}{c} \right) \left( \frac{\beta}{\lambda} \right) \frac{p_2 \delta(1-\delta)}{\delta(1-\delta) + \omega \delta'} - \frac{\alpha \omega}{\theta_1} < 0, \quad \phi_{\theta_1} = \frac{\alpha x}{\theta_1} > 0,
\]

\[
\phi_{g_1^*} = \left( \frac{\eta}{c} \right) \left( \frac{\beta}{\lambda} \right) p_1 - \frac{\alpha}{\theta_1} > 0, \quad \phi_{g_2^*} = -p_2 \left[ \left( \frac{\eta}{c} \right) \left( \frac{\beta}{\lambda} \right) \frac{\omega \delta'}{\delta(1-\delta) + \omega \delta'} - \frac{\alpha}{p_1 \theta_1} \right],
\]

and

\[
\Phi_C dC = -\frac{\rho \alpha}{p_1 \theta_1} db - \frac{\delta^2(1-\delta) + \omega \delta'}{\delta(1-\delta) + \omega \delta'} \left[ \phi_\alpha d\alpha + \phi_\beta d\beta + \phi_{g_1} dg_1 + \phi_{\theta_1} d\theta_1 \right]
\]

\[
+ \left( \frac{\alpha}{p_1 \theta_1} \right) \frac{\delta(1-\delta)^2 + \omega \delta'}{\delta(1-\delta) + \omega \delta'} p_2 dg_2 - \left( \frac{\alpha}{\theta_1} \right) \frac{\delta(1-\delta)^2}{\delta(1-\delta) + \omega \delta'} dg_1^*
\]

\[
- \left( \frac{\alpha}{p_1 \theta_1} \right) \frac{\delta^2(1-\delta)}{\delta(1-\delta) + \omega \delta'} p_2 d(g_2^* - g_2^*),
\]

\[
\Phi_C dC^* = -\rho \left( \frac{\eta}{c} \right) \left( \frac{\beta}{\lambda} \right) \frac{\delta(1-\delta)^2}{\delta(1-\delta) + \omega \delta'} \left[ \phi_\alpha d\alpha + \phi_\beta d\beta + \phi_{g_1} dg_1 + \phi_{\theta_1} d\theta_1 \right]
\]

\[
+ \left( \frac{\delta(1-\delta)}{\delta(1-\delta) + \omega \delta'} \right) \left( \frac{\eta}{c} \right) \left( \frac{\beta}{\lambda} \right) \frac{\delta(1-\delta)^2}{\delta(1-\delta) + \omega \delta'} p_2 dg_2 - \left[ \frac{\delta(1-\delta)}{\delta(1-\delta) + \omega \delta'} \phi_{g_1^*} dg_1^* \right]
\]

\[
+ \left[ \frac{\eta}{c} \left( \frac{\beta}{\lambda} \right) \frac{\omega \delta'}{\delta(1-\delta) + \omega \delta'} - \left( \frac{\alpha}{p_1 \theta_1} \right) \frac{\delta(1-\delta)^2 + \omega \delta'}{\delta(1-\delta) + \omega \delta'} \right] p_2 d(g_2^* - g_2^*).
\]

Therefore,

\[
\alpha \downarrow, \beta \downarrow, b \downarrow, g_1 \uparrow, g_2 \uparrow, \theta_1 \downarrow, g_1^* \downarrow \Rightarrow C \uparrow, c \uparrow, c^* \uparrow,
\]

\[
\theta_2^* - g_2^* \uparrow \Rightarrow c \downarrow, c^* \downarrow, C \downarrow.
\]

(34)

From the second equation of (23),

\[
d\omega = \omega_C dC + \omega_{g_2^* - g_2^*} d(g_2^* - g_2^*),
\]
\[
\left(\frac{c}{\omega}\right) \omega c = \frac{1-\delta}{\delta(1-\delta)+\omega \delta^2} > 0, \quad \left(\frac{c}{\omega}\right) \omega \theta_2^*-g_2^* = -\frac{p_2}{\delta(1-\delta)+\omega \delta^2} < 0.
\]

From (32) and the above equations, the effect on \(\omega\) of a change in \(\theta_2^*-g_2^*\) is
\[
\left(\frac{c}{\omega}\right) \frac{d\omega}{d(\theta_2^*-g_2^*)} = -\left(\frac{\delta p_2}{\delta(1-\delta)+\omega \delta^2}\right) \left[\left(\frac{c}{\omega}\right) \left(\frac{\beta}{\lambda}\right) - \frac{\alpha}{p_1 \theta_1}\right] < 0.
\]

These properties and (33) give
\[
\alpha \downarrow, \beta \downarrow, b \downarrow, g_1 \uparrow, g_2 \uparrow, \theta_1 \downarrow, g_1^* \downarrow, \theta_2^* - g_2^* \downarrow \Rightarrow \omega \uparrow. \quad (35)
\]

Let us discuss the implication of the main results summarized in (34) and (35). When the home country suffers persistent unemployment, a decrease in the Poison rate of job separation (or the price adjustment speed) \(\alpha\) alleviates deflation in the home country. Therefore, it makes more advantageous for households to consume than to hold money and home consumption is stimulated. Consequently, the current account worsens and the home currency depreciates, which improves the competitiveness of home firms and expands employment, further alleviating deflation. The home-currency depreciation in turn improves the terms of trade for the foreign country and makes it better off in the case where full employment prevails in the foreign country.

A decrease in the liquidity preference parameter \(\beta\) yields the same effects on the two countries because it stimulates home consumption and moderates deflation. An expansion in home government purchases on the home commodity \(g_1\) creates new home employment and reduces the deflationary gap. Therefore, the same positive effects emerge. An improvement in home productivity \(\theta_1\), in contrast, widens the deflationary gap and hence the opposite effects arise in both countries. Home employment shrinks so much as to dominate the initial improvement in \(\theta_1\) and the foreign terms of trade worsens, which makes both countries worse off.

A transfer from the home country to the foreign country worsens the home foreign-asset position and thereby decreases the home current account. It causes the home currency to
depreciate and increases home production, employment and consumption. Consequently, the home country is better off. The foreign country is also better off because it not only receives the transfer but also benefits from the improvement in the terms of trade. This result may be consistent with that of the Marshall Plan and the statement of the Brandt commission, both mentioned in the introduction.

The above argument implies that a parameter or policy change that worsens the home current account generally depreciates the home currency, stimulates home production and employment, mitigates deflation and hence stimulates consumption. An expansion in home government purchases on the foreign commodity $g_2$ is another example. It worsens the home current account, causes the home currency to depreciate, and yields the positive effects on the two countries.

An increase in foreign government purchases on the foreign commodity raises the relative price of the foreign commodity, which expands home employment and stimulates home consumption. The rise in the foreign commodity price benefits the foreign country. However, it is caused by an increase in foreign government purchases on the foreign commodity, which harms the foreign country. Therefore, the welfare effect on the foreign households is ambiguous. The effect of an increase in $\theta_2^*$ is just the opposite to that of an increase in foreign government purchases on the foreign commodity.

6. Conclusions

A two-country economy in which the home country suffers persistent unemployment due to aggregate demand deficiency while the foreign country realizes full employment is considered. The stagnant country often attempts to improve production efficiency, expecting that such an effort would expand the world market share of home commodities and increase
national income. However, it improves the current account and thereby appreciates the home currency. In the presence of aggregate demand deficiency the appreciation of the home currency is so high that employment eventually decreases and national income and consumption decline, making the home country worse off. It also worsens the foreign terms of trade and harms the foreign country if the foreign country achieves full employment.

In contrast, the home country’s changes in policy and preference parameters that worsen the home current account cause the home currency to depreciate and thereby expand employment and consumption. It also improves the foreign terms of trade and makes the foreign country better off. Typical examples are expansions of home government purchases on home and foreign commodities.

Foreign aid may be another important example. Foreign aid expenditures tend to be cut when a donor country suffers long-run stagnation. However, an expansion in foreign aid in fact makes the country better off because it worsens the home current account, depreciates the home currency, stimulates business, increases home employment, decreases deflation, and urges people to consume more. Moreover, if the recipient country is fully employed, it is also better off because it not only receives foreign aid but also benefits from the improvement in the terms of trade.
References


Cambridge University Press.


Figure 1: Unemployment steady state

\[ R = \frac{\beta}{\lambda(C, \kappa)} \]

\[ R = \rho + \alpha \left( \frac{\delta(\omega)}{p_1(\omega)\theta_1} C - 1 \right) \]