Discussion Paper No. 908

ONE-LEADER AND MULTIPLE-FOLLOWER STACKELBERG GAMES WITH PRIVATE INFORMATION

Tomoya Nakamura

July 2014

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

One-Leader and Multiple-Follower Stackelberg Games with Private Information

Tomoya Nakamura*

Osaka University, ISER

July 16, 2014

Abstract

This study analyzes one-leader and multiple-follower Stackelberg games with private information regarding demand uncertainty. In the equilibrium of the Stackelberg games, a leader's private information becomes public information among followers. This study demonstrates that the strategic relationship between the leader and each follower is determined by the weight on public information regarding a follower's estimation of demand uncertainty. If the weight is sufficiently low (high), then the relationship is a strategic substitute (complement), and the leader has a first-mover (dis)advantage, respectively. In the case of strategic complementarity, the leader can exit from a market. The threshold is determined by the intensity of Cournot competition among the followers.

Keywords: Stackelberg games; Cournot games; First-mover and second-mover advantages; Public and private information

JEL classification: C72, D82, and L13

*E-mail address: nakamura@iser.osaka-u.ac.jp

1 INTRODUCTION

In strategic situations with state uncertainty, each agent decides his action by using his available information. If the agent can know the information that others possess, then he can anticipate their actions. By definition, public information is held by all agents. Therefore, public information is a focal point for the actions of others. Recent literature on information economics, following Morris and Shin (2002), includes discussions on the interrelationships between strategic behavior and public information. Angeletos and Pavan (2007) and Ui and Yoshizawa (2013) provide characterizations of those interrelationship using general Bayesian games.[1] These studies consider simultaneous-move games with uncertainty. We call simultaneous-move games *horizontal* competitions.[2] Furthermore, the studies assume existence of exogenous public information

In contrast, this study analyzes sequential-move games with uncertainty. We call sequential-move games *vertical* competitions. Specifically, we consider one-leader and multiple-follower Stackelberg competition with demand uncertainty. In the vertical competition, if the leader intends to gain a profit from the advantage of a first move, then his action is exposed to all followers. Moreover, by followers' observation of the leader's actions, followers can know the leader's private information. Consequently, the leader's private information endogenously becomes public information among followers. Furthermore, in horizontal competition, the public information is a focal point for the actions of followers, as described in the recent literature on information economics. Hence, our model can extend the literature to analyze the interrelationships between strategic behavior and public information, for both horizontal and vertical competitions, which endogenously generate public information.

[1]Other studies in the literature include Angeletos and Pavan (2004), Arato and Nakamura (2011, 2013), Cornand and Heinemann (2008), Dewan and Myatt (2008, 2012), Hellwig (2002), James and Lawler (2011, 2012a,b), Morris and Shin (2007), Svensson (2006), and Myatt and Wallace (2014).

[2]Angeletos and Pavan (2007) and Ui and Yoshizawa (2013) consider Cournot games as an industrial organization application. In another branch of the literature, Vives (1984, 1988, 2008, 2011) and Myatt and Wallace (2013) analyze large Cournot games.

It is well-known that, under deterministic demand, the leader has a first-mover advantage because he/she can commit a quantity of supply. However, under demand uncertainty, a follower can have second-mover advantage. A follower can decide the quantity of supply after observing the leader's private information, which is inferred from the leader's equilibrium supply. Hence, a follower can estimate unknown demand more correctly using his own private information as well as that of the first mover. This suggests that the follower has an information advantage. As a result, a second-mover's information advantage can dominate a first-mover commitment advantage. We demonstrate that the strategic relationship in vertical competition is determined by the weight on public information regarding followers' estimations of uncertainty. If the weight is sufficiently high (low), then the relationship is a strategic substitute (complement), and the leader has a first-mover (dis)advantage because the commitment (information) advantage dominates the information (commitment) advantage, respectively.[3]

On the other hand, horizontal competition is a strategic substitute regardless of the weight on public information because of the fundamental structure of Cournot competition. As Cournot competition among followers becomes intense, the output of each follower decreases and the total output of followers increases, similar to deterministic Cournot competitions. However, vertical strategic relationships change the degree of output reduction of followers because the action of each follower is affected by strategic relationships with other followers as well as the leader. In the case of vertically strategic substitutability (complementarity), followers strongly (weakly) decrease their output; and the total output of followers increases weakly (strongly). Particularly, in the case of vertically strategic complementarity, the leader can exit from a market if the weight on public information, or the intensity of competition among followers, is sufficiently high. Furthermore, we analyze total industry profits, using a benchmark case where all firms move simultaneously. Given the intensity of competition among followers, total

[3]This result is closely related to Gal-Or (1987). She shows similar results in one-leader and one-follower Stackelberg games with demand uncertainty in a segmented market. Furthermore, there exists the vertically extended research of Gal-Or (1987). Shinkai (2000) and Cumbul (2014) analyze sequential n-times-move Stackelberg games.

industry profits are greater (less) than the benchmark if the vertical relationship is a strategic substitute (complement). Additionally, total industry profits are maximized when the leader exits from a market.

Finally, we briefly discuss two points. The first point is the endogenous timing of a leader's entry. When the leader has a first-mover advantage in the case of vertically strategic substitution, the leader naturally moves first. In contrast, when the leader has first mover disadvantage in the case of vertically strategic complementarity, he/she gives up the right of first mover and simultaneously makes a decision with followers. Then, the leader can achieve the same level of profit as the followers, who never have second-mover advantages. The second point of discussion is the concavity of the function regarding total industry profits. If there exists one follower, then that follower uses public information only to estimate demand uncertainty. However, if the number of followers increases, then each follower uses public information to forecast not only the uncertainty but also the actions of others. In the case of a low number of followers, the value of public information as a focal point is low, and the value of reducing uncertainty is relatively high. As the number of followers increases, the value of a focal point becomes relatively higher. As a result, the form of the function changes form concave to convex as the number of followers increases.

This paper is organized as follows. Section 2 describes the model, and Section 3 derives equilibrium. In Section 4, we analyze the properties of equilibrium. In Section 5, we discuss items such as the endogenous timing of entry and the value of public information for industry. Finally, in Section 6, conclusions are provided.

2 THE MODEL

Demands and payoffs A market consists of $n+1$ firms indexed by $i \in \{0, 1, \dots, n\}$. Firm i chooses the quantity of production $q_i \geq 0$. The inverse demand function is given by

$$p = a - \bar{\theta} + u - Q, \qquad a > \bar{\theta} > 0, \tag{1}$$

where p is the market price, $Q \equiv \sum_{i=0}^{n} q_i$ is the aggregate production, and u is a random variable with mean $\bar{\theta} > 0$ and variance $1/\gamma$, $\gamma > 0$. No firm can directly observe the realized value of the prior random variable u. Payoff function of firm i is defined as

$$\pi_i(q, x) \equiv p \cdot q_i = (a - \bar{\theta} + u - Q)q_i. \tag{2}$$

Information structure Assume that one of the firms acquires a chance to move first. The firm is denoted by $i = 0$ without loss of generality. Firm 0 receives private information x_0 on u. Then, x_0 satisfies that $E(x_0|u) = u$ and $Var(x_0|u) = 1/\alpha$, $\alpha > 0$. We further assume that the other firms $i \neq 0$ produce the goods after observing the output of firm 0. They also observe the private signal x_i on u. Here, x_i satisfies $E(x_i|u) = u$ and $Var(x_i|u) = 1/\beta$, $\beta > 0$.

We restrict our attention to the posterior expectation of u and the conditional expectation for x_i given $x_{j\neq i}$ that satisfy linearity. Some combinations of prior and posterior distributions, for example, the combination of Gamma-Poisson, Beta-Binomial, and Normal-Normal distributions, satisfy following linearity.[4]

Assumption 1.

$$E(u|x_0, x_i) = E(x_{j\neq i}|x_0, x_i) = \frac{\alpha x_0 + \beta x_i + \gamma\bar{\theta}}{\Delta}, \tag{3}$$

[4]The first two combinations satisfy non-negativity. If we assume $a > \gamma^{-1/2}$ in Normal-Normal distributions, then $a - \bar{\theta} + u$ is positive with a probability more than 0.997, that is, nearly 1. More detailed discussions are found in DeGroot (1970), Gal-Or (1987), Shinkai (2000), and Cumbul (2014).

$$E(u|x_0) = E(x_{i\neq 0}|x_0) = \frac{\alpha x_0 + \gamma\bar{\theta}}{\alpha+\gamma}, \tag{4}$$

$$E(u|x_{i\neq 0}) = E(x_0|x_i) = \frac{\beta x_i + \gamma\bar{\theta}}{\beta+\gamma}, \tag{5}$$

where $\Delta \equiv \alpha + \beta + \gamma$.

Strategies We denote pure strategy space by $\mathbb{R}^+$ and support of a private signal by X_i. Firm 0 chooses its quantity of supply depending on its private information x_0. Its strategy can be denoted by $q_0 = H(x_0)$, where $H : X_0 \to \mathbb{R}^+$. Firm $i \neq 0$ chooses its quantity of output depending on its private information x_i and the leader's realized output q_0. Their strategy can be written as $q_i = G(x_i, q_0)$, where $G : X_i \times \mathbb{R}^+ \to \mathbb{R}^+$.

Importantly, $H(\cdot)$ and $G(\cdot,\cdot)$ may have many types of functional forms in equilibrium as discussed in Gal-Or (1987), Shinkai (2000), and Cumbul (2014).[5] However, we assume that $H(\cdot)$ and $G(\cdot,\cdot)$ are affine transportations because of the following reasons. First, in equilibrium, second movers can always infer a first mover's private information by the inverse function of $H(\cdot)$, if $H(\cdot)$ is a monotone function. Second, our information structure satisfies linearity. Third, the payoff function of each firm is quadratic in its action. Formally, we derive the equilibrium strategy profile that satisfies following equations: $\forall x_0 \in X_0$,

$$q_0^* = H(x_0) = \arg\max_{q_0\in\mathbb{R}^+} E[\pi_0(q_0, G(x_{i\neq 0}, q_0), u)|x_0] = A_0 + A_1 x_0 \geq 0, \tag{6}$$

$\forall x_{i\neq 0} \in X_i$ and $\forall H(x_0) = q_0 \in \mathbb{R}^+$,

$$\begin{aligned} q_{i\neq 0}^* = G(x_i, q_0) &= \arg\max_{q_i\in\mathbb{R}^+} E[\pi_i(q_0, q_i, G(x_{j\neq 0,i}, q_0), u)|x_i, q_0] \\ &= B_0 + B_1 x_i + B_2 q_0 \geq 0, \end{aligned} \tag{7}$$

where $A_0, A_1, B_0, B_1, B_2 \in \mathbb{R}$.

[5] See page 283 in Gal-Or (1987).

Timing of the game At $t = 0$, nature draws the unknown demand u and each firm receives x_i on u. At stage $t = 1$, firm 0 as a Stackelberg leader chooses q_0 given x_0. At $t = 2$, firms $i \neq 0$ as Stackelberg followers and as Cournot competitors decide q_i given x_i and q_0.

3 DERIVATION OF THE EQUILIBRIUM

Our plan to derive the equilibrium is as follows. First, we ignore the non-negativity constraint regarding q_i, and we solve the model by backward induction. Next, we check whether the derived equilibrium strategy satisfies the non-negativity constraint.

3.1 Without a non-negativity constraint

Second movers At stage 2, each follower chooses its production, given $x_{i\neq 0}$ and q_0. The objective function is $E\pi_i = E[(a - \bar{\theta} + u - Q)q_i|x_i, q_0]$, for any $i \neq 0$. The first-order conditions are:

$$\frac{\partial E\pi_i}{\partial q_i} = 0 \;\Leftrightarrow\; 2q_i^* = a - \bar{\theta} + E\left[u|x_i, q_0\right] - q_0 - E\left[\sum_{j\neq i,0} q_j \middle| x_i, q_0\right]. \tag{8}$$

In the equilibrium, from (6), second movers can correctly infer x_0 from q_0: $x_0 = (q_0^* - A_0)/A_1$. Hence, from (3), (6) and (7), we have

$$\begin{aligned} E\left[\sum_{j\neq i,0} q_j \middle| x_i, q_0\right] = (n-1)\Bigg[&B_0 + B_1\left(\frac{\gamma}{\Delta}\bar{\theta} - \frac{A_0}{A_1}\frac{\alpha}{\Delta}\right) \\ &+ B_1\frac{\beta}{\Delta}x_i + \left(\frac{B_1}{A_1}\frac{\alpha}{\Delta} + B_2\right)q_0\Bigg]. \end{aligned} \tag{9}$$

Substituting (3) and (9) into (8), we have

$$2q_i^* = a - \bar{\theta}\left(1 - \frac{\gamma}{\Delta}\right) - \frac{A_0}{A_1}\frac{\alpha}{\Delta} - (n-1)\left[B_0 + B_1\left(\frac{\gamma}{\Delta}\bar{\theta} - \frac{A_0}{A_1}\frac{\alpha}{\Delta}\right)\right]$$

$$+ [1-(n-1)B_1]\frac{\beta}{\Delta}x_i + \left\{\frac{1}{A_1}\frac{\alpha}{\Delta} - 1 - (n-1)\left(\frac{B_1}{A_1}\frac{\alpha}{\Delta} + B_2\right)\right\}q_0. \tag{10}$$

Comparing the coefficients of (10) with that of (7), we have

$$2B_0 = a - \bar{\theta}\left(1 - \frac{\gamma}{\Delta}\right) - \frac{A_0}{A_1}\frac{\alpha}{\Delta} + (n-1)\left(\frac{A_0 B_1}{A_1}\frac{\alpha}{\Delta} - B_1\frac{\gamma}{\Delta}\bar{\theta} - B_0\right), \tag{11}$$

$$2B_1 = [1-(n-1)B_1]\frac{\beta}{\Delta}, \tag{12}$$

$$2B_2 = \frac{1}{A_1}\frac{\alpha}{\Delta} - 1 - (n-1)\left(\frac{B_1}{A_1}\frac{\alpha}{\Delta} + B_2\right). \tag{13}$$

First mover At $t=1$, firm 0 chooses its supply depending on x_0. The objective function is $\pi_0 = E\left[\left(a - \bar{\theta} + u - Q\right)q_0 | x_0\right]$. The first-order condition is

$$\frac{\partial E\pi_0}{\partial q_0} = a - \bar{\theta} + E(u|x_0) - E\left(\sum_{j\neq 0} q_j \middle| x_0\right) - 2q_0^* - q_0^*\sum_{i\neq 0}\frac{\partial q_i}{\partial q_0} = 0. \tag{14}$$

Using (4), (6) and (7), we have

$$a - \frac{\alpha}{\alpha+\gamma}\bar{\theta} - 2A_0 - n\left(2A_0B_2 + B_0 + B_1\frac{\gamma}{\alpha+\gamma}\bar{\theta}\right) - \left[2A_1 - \frac{\alpha}{\alpha+\gamma} + n\left(2A_1B_2 + B_1\frac{\alpha}{\alpha+\gamma}\right)\right]x_0 = 0. \tag{15}$$

(15) should satisfy for any realization of x_0. Hence,

$$a - \frac{\alpha}{\alpha+\gamma}\bar{\theta} - 2A_0 - n\left(2A_0B_2 + B_0 + B_1\frac{\gamma}{\alpha+\gamma}\bar{\theta}\right) = 0, \tag{16}$$

$$2A_1 - \frac{\alpha}{\alpha+\gamma} + n\left(2A_1B_2 + B_1\frac{\alpha}{\alpha+\gamma}\right) = 0. \tag{17}$$

From (3), we define $\rho \equiv \frac{\alpha+\gamma}{\Delta}$ that represents weight on public information regarding followers' estimation of u. Then, solving the system of five equations (11), (12), (13), (16), and (17), we obtain the following result:

$$A_0 = \frac{X(\rho,n)}{2Y(\rho,n)}\left(a - \frac{\alpha}{\alpha+\gamma}\bar{\theta}\right), \quad A_1 = \frac{X(\rho,n)}{2Y(\rho,n)}\left(\frac{\alpha}{\alpha+\gamma}\right),$$
$$B_0 = \frac{Y_n(\rho,n)}{Y(\rho,n)}(a-\bar{\theta}), \quad B_1 = \frac{Y_n(\rho,n)}{Y(\rho,n)}, \quad B_2 = -\frac{X_n(\rho,n)}{X(\rho,n)}, \tag{18}$$

where

$$X(\rho,n) \equiv (1-3\rho)n + 1 + \rho, \quad Y(\rho,n) \equiv (1-\rho)n + 1 + \rho > 0.$$

Non-negativity constraint We can easily check $q^*_{i\neq 0} > 0$. On the other hand, q^*_0 is not always non-negative. Here, q^*_0 can be written as follows.

$$q^*_0 = \frac{X(\rho,n)}{2Y(\rho,n)}\left(a - \frac{\alpha}{\alpha+\gamma}\bar{\theta} + \frac{\alpha}{\alpha+\gamma}x_0\right)$$

Furthermore, $Y(\rho,n)$ and the value in the blanket is strictly positive because we assume $a > \bar{\theta} > 0$ and (almost all) the support of x_0 is positive. Therefore, $X(\rho,n)$ determines the sign of q^*_0. Then, we have the following non-negativity condition.

Proposition 1. *If $X(\rho,n) > 0$, then $q^*_0 > 0$. If $X(\rho,n) \leq 0$, then $q^*_0 = 0$.*

From $X(\rho,n) > 0$, we can define two thresholds regarding ρ and n:

$$\rho < \bar{\rho} \equiv \frac{n+1}{3n-1} \in (1/3, 1], \quad n < \bar{n} \equiv \begin{cases} \frac{\rho+1}{3\rho-1}, & \text{if } \rho > 1/3, \\ \infty, & \text{if } \rho \leq 1/3. \end{cases} \tag{19}$$

Observation 1 in Gal-Or (1987) shows that the leader always chooses a positive output. Similarly, in our model, if $n = 1$, then $X(\rho,n) > 0$ always holds and the leader always chooses a positive output. However, departing from her observation, if $n \geq 2$, then $\bar{\rho}$ is strictly smaller than 1. This suggests that, in contrast to Gal-Or (1987), horizontal competition by followers can drive out the leader from the market if $\rho \in [\bar{\rho}, 1)$.

3.2 With a non-negativity constraint

If $X(\rho, n) \leq 0$, the leader chooses $q_0^* = 0$. Then, we assume that the followers play a Cournot competition without firm 0. Then, an observation of firm $i \neq 0$ is only x_i. Therefore, we assume linear equilibrium such that $q_{i\neq 0}^c = G^c(x_i) = B_0^c + B_1^c x_i$, $\forall x_i \in X_i$. The payoff of followers is $\pi_i = E[(a - \bar{\theta} + u - \sum_{j\neq 0} q_j)q_i|x_i]$, and the first order condition is $q_i = \frac{1}{2}[a - \bar{\theta} + E\left(u|x_i\right) - E(\sum_{j\neq 0,i} q_j^c|x_i)]$, where $E(\sum_{j\neq 0,i} q_j^c|x_i) = (n-1)[B_0^c + B_1^c(\frac{\beta}{\beta+\gamma}x_i + \frac{\gamma}{\beta+\gamma}\bar{\theta})]$. Hence, q_i^c can be rewritten as

$$2q_i^c = a - \frac{\beta}{\beta+\gamma}\bar{\theta} - (n-1)\left(B_0^c + B_1^c\frac{\gamma}{\beta+\gamma}\bar{\theta}\right) + [1-(n-1)B_1^c]\frac{\beta}{\beta+\gamma}x_i. \tag{20}$$

Using a method of undetermined coefficient, we have

$$B_0^c = \frac{a}{n+1} - \frac{\beta\bar{\theta}}{(1+n)\beta + 2\gamma}, \quad B_1^c = \frac{\beta}{(1+n)\beta+2\gamma}.$$

Then, $Eq_0^c = 0$ and $Eq_{i\neq 0}^c = \frac{a}{n+1}$.

Summing up the results, we have following proposition.

Proposition 2. *The unique pure strategy equilibrium of the game is*

1. *If $X(\rho, n) > 0$, then*

$$q_0^* = \frac{X(\rho,n)}{2Y(\rho,n)}\left[a - \frac{\alpha}{\alpha+\gamma}(\bar{\theta} + x_0)\right], \tag{21}$$

$$q_{i\neq 0}^* = \frac{\partial \ln Y(\rho,n)}{\partial n}(a - \bar{\theta} + x_i) - \frac{\partial \ln X(\rho,n)}{\partial n}q_0. \tag{22}$$

2. *If $X(\rho, n) \leq 0$, then $q_0^c = 0$ and*

$$q_{i\neq 0}^c = \frac{a}{1+n} - \frac{\beta}{2\gamma + (1+n)\beta}(\bar{\theta} - x_i).$$

We also have ex ante expected production:

Corollary 1. *(i) If $X(\rho, n) > 0$, then*

$$Eq_0^* = \frac{a}{2}\left[\frac{X(\rho,n)}{Y(\rho,n)}\right], \qquad Eq_i^* = \frac{a}{2}\left[\frac{2Y_n(\rho,n) - X_n(\rho,n)}{Y(\rho,n)}\right],$$

$$E\sum_{i\neq 0} q_i^* = n\frac{a}{2}\left[\frac{2Y_n(\rho,n) - X_n(\rho,n)}{Y(\rho,n)}\right], \qquad E\sum_i q_i^* = \frac{a}{2}\left[\frac{Z(\rho,n)}{Y(\rho,n)}\right],$$

where $Z(\rho, n) = 2(1-\rho)n + 1 + \rho$. (ii) If $X(\rho, n) \leq 0$, then $Eq_0^c = 0$ and $Eq_{i\neq 0}^c = \frac{a}{n+1}$.

3.3 Benchmark case

For future reference as a benchmark, we derive the equilibrium that all firms play Cournot competition. Hence, the strategies of firms are dependent on their private signals and there is no public signal except for prior belief. The proof of the following Lemma is similar to Proposition 1.

Lemma 1. *The linear equilibrium in which all firms simultaneously move is $q_0^{ac} = A_0^{ac} + A_1^{ac} x_0$ and $q_{i\neq 0}^{ac} = B_0^{ac} + B_1^{ac} x_i$, where*

$$A_0^{ac} = \frac{a}{n+2} - \frac{\alpha(\beta+2\gamma)\bar{\theta}}{4\gamma^2 + \gamma[4\alpha + 2\beta(n+1)] + \alpha\beta(n+2)},$$
$$A_1^{ac} = \frac{\alpha(\beta+2\gamma)}{4\gamma^2 + \gamma[4\alpha + 2\beta(n+1)] + \alpha\beta(n+2)},$$
$$B_0^{ac} = \frac{a}{n+2} - \frac{\beta(\alpha+2\gamma)\bar{\theta}}{4\gamma^2 + \gamma[4\alpha + 2\beta(n+1)] + \alpha\beta(n+2)},$$
$$B_1^{ac} = \frac{\beta(\alpha+2\gamma)}{4\gamma^2 + \gamma[4\alpha + 2\beta(n+1)] + \alpha\beta(n+2)},$$

and ex ante expected production and profits are

$$Eq_0^{ac} = Eq_{i\neq 0}^{ac} = \frac{a}{n+2}, \quad E\pi_0^{ac} = E\pi_{i\neq 0}^{ac} = \frac{a^2}{(n+2)^2}. \tag{23}$$

4 EQUILIBRIUM ANALYSIS

In this section, we analyze properties of the equilibrium. First, we characterize the strategic relationship between the leader and followers. Second, we show the results of comparative statistics regarding ex ante expected production. Third, total industry profit is analyzed.

4.1 Strategic relationship in vertical competition

Note that B_2 represents slope of followers' reaction function to leader's action.

$$B_2 = -\frac{\partial \ln X(\rho, n)}{\partial n} = -\frac{X_n(\rho, n)}{X(\rho, n)} = \frac{-(1-3\rho)}{(1-3\rho)n + 1 + \rho}$$

In the case of $X(\rho, n) > 0$, the strategic relationship between the leader and each follower is determined by the sign of $X_n(\rho, n) = 1 - 3\rho$.

Proposition 3. *If $\rho > 1/3$, $\rho = 1/3$ and $\rho < 1/3$, then followers' reaction functions are upward, constant and downward sloping; thereby, the strategic relationship is complementary, neutral, and substitutive, respectively.*

This result is closely related to Gal-Or (1985, 1987). Gal-Or (1985) shows that, in the deterministic Stackelberg duopoly model, the first mover has an (dis)advantage if the reaction functions of the players are downwards (upwards) sloping. The slope in her model is determined by deterministic parameters. On the other hand, Gal-Or (1987) and our model show that, in the model of Stackelberg games with demand uncertainty, the slope of reaction functions can be upward sloping because of the conditions of uncertainties. Gal-Or (1987) assumes that the markets faced by each firm are partially segmented. She shows that, if the markets are *sufficiently segmented*, then the reaction function of follower can be upward sloping. On the other hand, our model assumes a *perfectly integrated* market. The results of Gal-Or (1987) and our study differ because of the generality of assumptions regarding signals.

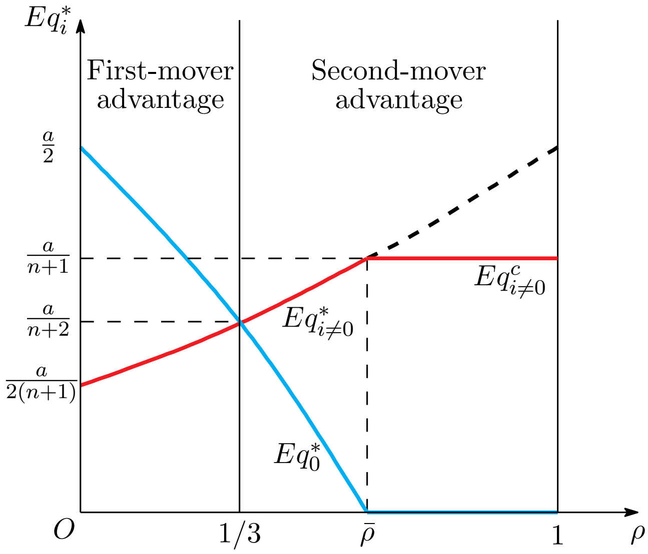

Figure 1: Effect of weight on public signals

4.2 Comparative statics

Proposition 4 (Comparative statics)**.** *(i) If $X(\rho, n) > 0$, then*

$$\frac{\partial}{\partial \rho} Eq_0^* \leq 0, \quad \frac{\partial}{\partial n} Eq_0^* < 0, \quad \frac{\partial}{\partial \rho} Eq_{i\neq 0}^* \geq 0, \quad \frac{\partial}{\partial n} Eq_{i\neq 0}^* < 0,$$

$$\frac{\partial}{\partial \rho} \sum_{i\neq 0} Eq_i^* \geq 0, \quad \frac{\partial}{\partial n} \sum_{i\neq 0} Eq_i^* > 0, \quad \frac{\partial}{\partial \rho} \sum_{i} Eq_i^* \leq 0, \quad \frac{\partial}{\partial n} \sum_{i} Eq_i^* > 0.$$

(ii) If $X(\rho, n) \leq 0$, then

$$\frac{\partial}{\partial \rho} Eq_{i\neq 0}^c = 0, \quad \frac{\partial}{\partial n} Eq_{i\neq 0}^c < 0, \quad \frac{\partial}{\partial n} E \sum_{i\neq 0} q_i^c > 0.$$

Weight on public information Figure 1 shows the results of comparative statics regarding the weight on public signals given the intensity of followers' horizontal competition. When $\rho = 0$, the leader produces more than the followers: $q_0^* > q_i^*$. As ρ increases, the leader's production decreases, and each follower's production increases: $\partial Eq_0^*/\partial \rho < 0$ and $\partial Eq_i^*/\partial \rho > 0$. If $\rho = 1/3$, then the leader's output corresponds with the follower's output. This suggests that,

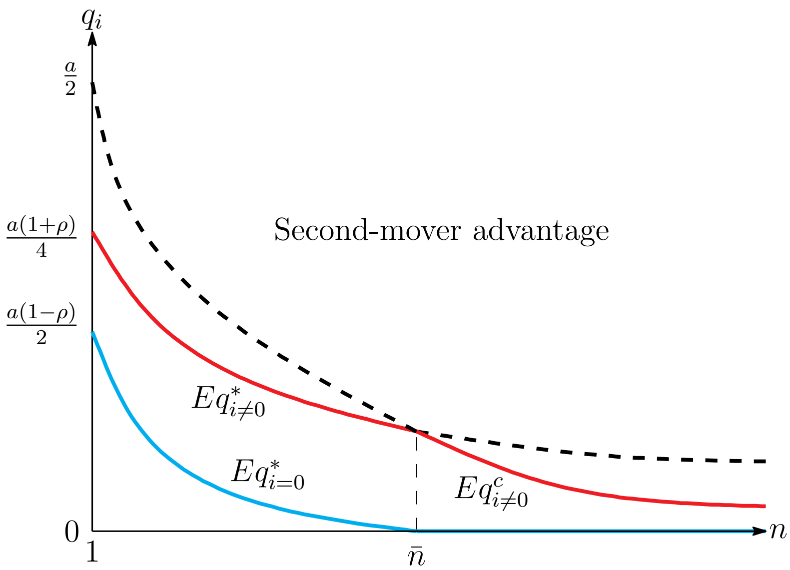

Figure 2: Effect of intensity of horizontal competition: $\rho > 1/3$

if $\rho < 1/3$ ($\rho > 1/3$), the leader has a first-mover (dis)advantage. Intuition is related to the tradeoff between commitment advantage and information advantage. If a leader who has a low (high) precision of private signal reveals a low (high) precision of private information, then that leader's commitment advantage dominates (is dominated by) the followers' information advantage. Finally, if $\rho \geq \bar{\rho} \in (1/3, 1]$, the leader refrains from production; then, $\bar{\rho}$ decreases with n. This indicates that the more followers that exist, the leader stops production at smaller $\bar{\rho}$. This effect can also be examined by $\partial^2 Eq_0^*/\partial\rho\partial n < 0$ and $\partial^2 Eq_{i\neq 0}^*/\partial\rho\partial n > 0$. If n increases, slopes of Eq_0 and $Eq_{i\neq 0}$ become steeper.

Intensity of horizontal competition All firms reduce output with respect to n that can be regarded as intensity of horizontal competition: $\partial Eq_0^*/\partial n < 0$ and $\partial Eq_{i\neq 0}/\partial n < 0$. However, the effect is changed by ρ. If $\rho \leq 1/3$, that is, the region of a first-mover advantage, then the output of firm 0 is no less than that of the follower's. Hence, firm 0 always chooses non-negative output because followers produce non-negative output. If $\rho > 1/3$, then the output of firm 0 is less than that of follower's. That is, if horizontal competition is sufficiently

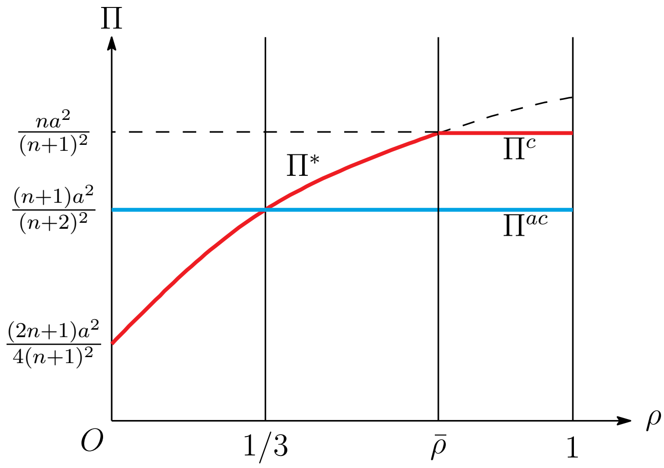

Figure 3: Total industry profit

intense ($n \geq \bar{n} \equiv (\rho+1)/(3\rho-1)$), then firm 0 stops production and followers play a Cournot competition without firm 0. Therefore, a follower's production is kinked at $\bar{n}$ (see Figure 2). The reason why firm 0 stops production is that, unless an individual follower reduces output, the total output of followers increases with respect to n that can be regarded as the intensity of followers' horizontal competition: $\partial \sum_{i\neq 0} Eq_i/\partial n > 0$. Consequently, unless firm 0 has a first-mover commitment advantage, the intensity of followers' competition excludes firm 0 from the market.

4.3 Total industry profit

From Proposition 1, we can easily derive ex ante expected profits.

Corollary 2 (Ex ante expected profits)**.**

$$E\pi_0 = \begin{cases} \frac{a^2(1+\rho)X(\rho,n)}{4Y(\rho,n)^2}, & \textit{if } \rho < \bar{\rho} \\ 0, & \textit{if } \rho \geq \bar{\rho} \end{cases}, \quad E\pi_{i\neq 0} = \begin{cases} \frac{a^2(1+\rho)^2}{4Y(\rho,n)^2}, & \textit{if } \rho < \bar{\rho} \\ \frac{a^2}{(n+1)^2}, & \textit{if } \rho \geq \bar{\rho} \end{cases}.$$

A total industry profit is defined as follows:

$$\Pi^* \equiv \sum_{i=0}^{n} E\pi_i = \frac{a^2(1+\rho)Z(\rho,n)}{4Y(\rho,n)^2}, \qquad \text{if } \rho < \bar{\rho}.$$
$$\Pi^c \equiv \sum_{i=1}^{n} E\pi_i^c = n\frac{a^2}{(n+1)^2}, \qquad \text{if } \rho \geq \bar{\rho}.$$

We treat total profits of Cournot competition by all firms as the benchmark.

$$\Pi^{ac} = \sum_{i=0}^{n} E\pi_i^{ac} = (n+1)\frac{a^2}{(n+2)^2}$$

Figure 3 shows total industry profits. If $\rho < \bar{\rho}$, then Π^* increases with ρ. The reason is that the increase of $\sum_{i\neq 0} Eq_i^*$ is greater than the decrease of Eq_0^* as ρ increases; $\partial \sum_{i\neq 0} Eq_i^*/\partial\rho > \partial Eq_0^*/\partial\rho$. Then, the increase in the total profit of followers is greater than the decrease in the leader's profit. When $\rho = 1/3$, Π^* corresponds to Π^{ac}. This is because $B_2 = 0$ suggests that followers ignore a leader's production, and this corresponds to the situation where all firms play Cournot competition. If $\rho \geq \bar{\rho}$, then the leader chooses $q_0 = 0$; thereby, Π^* corresponds to Π^c. Furthermore, Π^* is maximized in this case. Because the leader exited from the market, competition reduces in intensity. Hence, the profit of each follower increases.

5 Discussions

Endogenous timing of production A large body of literature discusses the endogenous timing of entry, for example, as in Hamilton and Slutsky (1990), Normann (2002), and Hoffmann and Rota-Graziosi (2012). We briefly discuss the issue of timing of a leader's entry. In the case of a first-mover advantage ($\rho < \bar{\rho}$), the leader naturally moves first. However, in the case of a first-mover disadvantage, the leader should not move first. If the leader moves simultaneously with the followers, then the profit of the leader is higher than that in the case of first-mover disadvantage from (23). The profit of the leader is the same as that of the followers. Figure 4

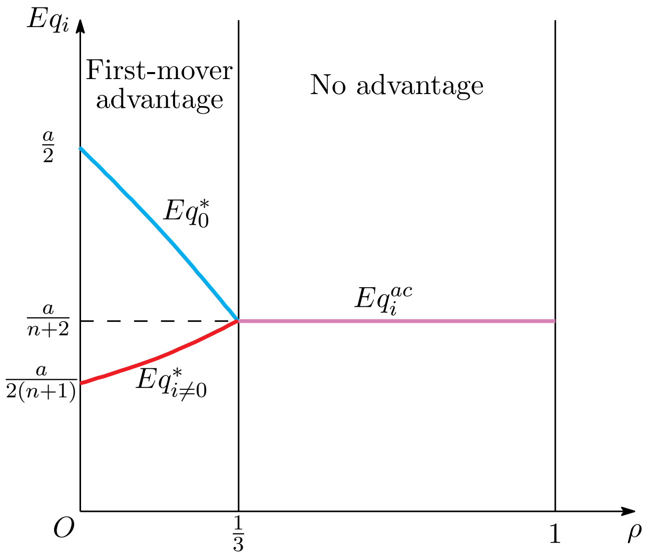

Figure 4: Effect of weight on public signal

shows the result.

Industry value of public signal We restrict our attention to $\rho < \bar{\rho}$. Then,

$$\frac{\partial \Pi^*}{\partial \rho} = \frac{a^2 n^2 (1-\rho)}{Y(\rho, n)^3} \geq 0, \quad \text{and} \quad \frac{\partial^2 \Pi^*}{\partial \rho^2} = \frac{a^2 n^2 \Phi(\rho, n)}{Y(\rho, n)^4},$$

where $\Phi(\rho, n) \equiv (1-n)\rho - 2 + n$. These results show that the industry value of the public signal is increasing, but that concavity of Π^* is determined by $\Phi(\rho, n)$. Figure 5 graphically depicts the results. When $n \leq 2$ ($n \geq 3$), Φ is negative (positive), that is, the industry profit is concavely (convexly) increases with ρ. The intuition is as follows. If $n = 1$, a public signal cannot work as a focal point because the follower need not to coordinate other followers. However, if $n \geq 2$, a public signal starts to work as a focal point. Furthermore, the coordination becomes difficult with increasing number of followers. Simultaneously, the value of public signal as a focal points becomes higher. Consequently, the form of Π^* changes from concave to convex as the number of followers increases. This result suggests that, if we endogenize information acquisition in future research, we might discover solutions that are determined by the precision of signals.

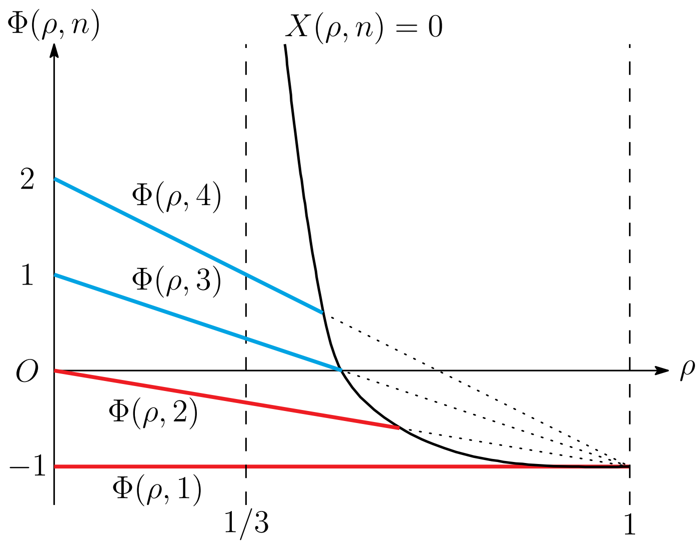

Figure 5: Concavity of industry value of public signal

6 Conclusion and future researches

In this study, we analyzed one-leader and multiple-follower Stackelberg games with demand uncertainty, and obtained the following results. First, the strategic relationship in vertical competition is determined by the weight on common signals regarding follower's estimation of uncertainty. If the weight is sufficiently high (low), then the relationship is a strategic substitute (complement), and the leader has a first-mover (dis)advantage. Second, in contrast to Gal-Or (1987), a first mover can exit from the market if the intensity of horizontal competition is sufficiently high, or if the weight on common signals is sufficiently high. Third, total industrial profit is maximized when the leader exits from the market. These results connect two branches of the literatures. One branch is the classical literature on industrial organization. One of its main interests is first-mover (dis)advantages in vertical competition. Another branch is the recent literature on information economics. One of its main interests is the interaction between strategic behavior and public information in horizontal competition. The main contribution of this study is providing a simultaneous analysis of these two branches of scholarship.

There are still some open questions. First, we assume the precision of exogenous signals.

Introducing the cost of acquiring signals, we could endogenize the equilibrium precision of signals as in Hellwig and Veldkamp (2009), Myatt and Wallace (2012), Colombo et al. (2013), Ui (2013), and Arato et al. (2014). Then, the discussion in Section 5 is pertinent. Second, we assume the number of followers exogenously. Assuming the cost of entering a market, we could endogenize the number of followers. Third, it might be useful to consider the implications of analyzing the social surplus as in the study of Vives (1984, 1988, 2008, 2011), which addresses the large Cournot model with strictly convex production costs. While our paper chose to emphasize strategic situations, these three points merit attention in future research.

References

Angeletos, G.-M., Pavan, A., 2007. Efficient Use of Information and Social Value of Information. Econometrica 75(4), 1103–1142.

Angeletos, G.-M., Pavan, A., 2004. Transparency of Information and Coordination in Economies with Investment Complementarities. American Economic Review 94, 91–98.

Arato, H., Nakamura, T., 2010. The Benefit of Mixing Private Noise into Public Information in Beauty Contest Games. B.E. Journal of Theoretical Economics 11(1). doi: 10.2202/1935–1704.1694

Arato, H., Nakamura, T., 2013. Endogenous Alleviation of Overreaction Problem by Aggregate Information Announcement. Japanese Economic Review 64, 319–336.

Arato, H., Hori, T., Nakamura, T., 2014. Endogenous Information Acquisition and the Partial Announcement Policy. ISER Discussion papers 892, Osaka University.

Colombo, L., Femminis, G., Pavan, A., forthcoming. Information Acquisition and Welfare. Review of Economic Studies. doi: 10.1093/restud/rdu015

Cornand, C., Heinemann F., 2008. Optimal Degree of Public Information Dissemination. Economic Journal 118, 718–742.

Cumbul, E., 2014. Stackelberg versus Cournot Oligopoly with Private Information. Working Paper.

DeGroot, M. H., 1970. Optimal Statistical Decisions. McGraw-Hill, New York.

Dewan, T., Myatt D. P., 2008. The Qualities of Leadership: Direction, Communication, and Obfuscation. American Political Science Review 102, 351–368.

Dewan, T., Myatt D. P., 2012. On the Rhetorical Strategies of Leaders: Speaking Clearly, Standing Back, and Stepping Down. Journal of Theoretical Politics 24, 431–460.

Gal-Or, E., 1985. First Mover and Second Mover Advantages. International Economic Review 26(3), 649–653.

Gal-Or, E., 1987. First Mover Disadvantages with Private Information. Review of Economic Studies 54(2), 279–292.

Hellwig, C., 2002. Public Announcements, Adjustment Delays and the Business Cycle. Working Paper, UCLA.

Hellwig, C., Veldkamp, L., 2009. Knowing What Others Know: Coordination Motives in Information Acquisition. Review of Economic Studies 76(1), 223–251.

Hamilton, J. H., Slutsky, S. M., 1990. Endogenous timing in duopoly games: Stackelberg or Cournot equilibria. Games Economic Behavior 2, 29–46.

Hoffmann, M., Rota-Graziosi G., 2012. Endogenous Timing in General Rent-Seeking and Conflict Models. Games and Economic Behavior 75(1), 168–184.

James, J., Lawler, P., 2011. Optimal Policy Intervention and the Social Value of Public Information. American Economic Review 101, 1561–1574.

James, J., Lawler, P., 2012a. Heterogeneous Information Quality; Strategic Complementarities and Optimal Policy Design. Journal of Economic Behavior and Organization 83, 342–352.

James, J., Lawler, P., 2012b. Strategic Complementarity, Stabilization Policy, and the Optimal Degree of Publicity. Journal of Money, Credit and Banking 44, 551–572.

Myatt, D. P., Wallace, C., 2012. Endogenous Information Acquisition in Coordination Games. Review of Economic Studies 79(1), 340–374.

Myatt, D. P., Wallace, C., 2013. Cournot Competition and the Social Value of Information. Working Paper.

Myatt, D. P., Wallace, C., forthcoming. Central Bank Communication Design in a Lucas-Phelps Economy. Journal of Monetary Economics.

Morris, S., Shin H. S., 2002. Social Value of Public Information. American Economic Review 92(5), 1521–1534.

Morris, S., Shin, H. S., 2007. Optimal Communication. Journal of the European Economic Association 5, 594–602.

Normann, H.-T., 2002. Endogenous Timing with Incomplete Information and with Observable Delay. Games and Economic Behavior 39(2), 282–291.

Rasmusen, E., Yoon Y-R., 2012. First Versus Second Mover Advantage with Information Asymmetry about the Profitability of New Markets. Journal of Industrial Economics 60(3), 374–405.

Shinkai, T., 2000. Second Mover Disadvantages in a Three-player Stackelberg Game with Private Information. Journal of Economic Theory 90(2), 293–304.

Svensson, L. E., 2006. Social Value of Public Information: Comment: Morris and Shin (2002) Is Actually Pro-transparency, Not Con. American Economic Review 96, 453–455.

Ui, T., 2013. Welfare Effect of Information Acquisition Costs. Working Paper, Hitotsubashi University.

Ui, T., Yoshizawa, Y., 2013. Characterizing the Social Value of Information. Working Paper, Hitotsubashi University.

Vives, X., 1984. Duopoly Information Equilibrium: Cournot and Bertrand. Journal of Economic Theory 34(1), 71–94.

Vives, X., 1988. Aggregation of Information in Large Cournot Markets, Econometrica 56(4), 851–876.

Vives, X., 2008. Information and Learning in Markets: The Impact of Market Microstructure. Princeton University Press, Princeton, NJ.

Vives, X., 2011. Strategic Supply Function Competition with Private Information. Econometrica 79(6), 1919–1966.