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PRE-NEGOTIATION COMMITMENT AND INTERNALIZATION IN PUBLIC GOOD PROVISION THROUGH BILATERAL NEGOTIATIONS

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Pre-negotiation commitment and internalization in public good provision through bilateral negotiations*

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Abstract

We investigate public good provision through bilateral negotiations between a public-good supplier and the beneficiaries of the good. We find that although a pre-negotiation commitment on the production level of the public good by the supplier enhances the internalization of beneficiaries' preferences, the good is not necessarily provided efficiently. We show that with the commitment, the supplier produces the public good at an efficient level in equilibrium if and only if its bargaining power is sufficiently weak. In addition, the public good may be produced excessively as a result of the commitment when the supplier's bargaining power is sufficiently strong.

Keywords: Public good; Simultaneous bilateral bargaining; Supplier bargaining power; Nash bargaining solution.

JEL Classification: C78, D42, D62, H41, H44.

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1 Introduction

We examine the role of commitments prior to negotiations in the provision of public goods. In bargaining theory, many studies have examined how a pre-commitment affects negotiation outcomes.[1] However, few studies have investigated this topic in public goods theory. Here, we examine how a pre-commitment on the production level of a public good affects the provision of the good, as negotiated between the supplier and the beneficiaries of the good.

In our case, the supplier has the technology to produce the public good, but does not benefit from the public good itself. On the other hand, the beneficiaries enjoy the public good, but cannot produce it on their own. Hence, the provision of the public good via a negotiation, including a monetary transfer, is beneficial for the supplier and beneficiaries. This situation is worth analyzing in the context of environmental economics because it includes a bargaining problem over pollution reduction: a private firm in an upstream area produces a good by polluting a river, and residents in the downstream area suffer from this pollution. While the firm must pay costs to control the pollution level, the residents benefit from the pollution reduction. For mutual benefit, the polluter and pollutees need to negotiate a pollution reduction in exchange of compensation. The pollution reduction can be considered a public good for the residents because they benefit non-exclusively from it, irrespective of whether they pay compensation for it. In summary, our study examines two important topics in public economics: public good provision and environmental externalities.

With regard to environmental externalities between polluters and pollutees, as in the example above, the effectiveness of the negotiation between them to achieve Pareto efficiency has been investigated numerous times since the seminal work of Coase (1960). However, many subsequent papers report a limitation of the Coase theorem (or collective negotiation) in various environments, including two-player models (e.g., Porter, 1988; Huber and Wirl, 1998; Robson and Skaperdas, 2008; MacKenzie and Ohndorf, 2013, 2016),

[1] See, for example, Muthoo (1999, Ch. 8).

multiple-player models (Dixit and Olson, 2000; Ellingsen and Paltseva, 2012, 2016), and unilateral actions before negotiation (e.g., Hoel, 1991; Konrad and Thum, 2014). In addition, although the efficiency in public good provision has been investigated theoretically and experimentally (e.g. Ray and Vohra, 1997, 2001; Dixit and Olson, 2000; Thoron et al., 2009; McEvoy, 2010), some studies, such as Ray and Vohra (1997, 2001) and Dixit and Olson (2000), show that negotiations over the provision of a public good rarely overcome the well-known free-rider problem. From the viewpoint of these lines of research, it is natural to evaluate whether some tactics prior to negotiations facilitate allocative efficiency.

To address our question, we build a bargaining model under complete information, based on *simultaneous bilateral negotiations*. In this model, there is a supplier of a public good and n consumers of the good. The supplier negotiates with each consumer, simultaneously and bilaterally. Thus, the supplier has n simultaneous bilateral bargaining sessions. The supplier and each consumer in the session anticipate the outcome of the other bilateral bargaining sessions because all sessions are simultaneous. The anticipation in one session of the results of the other sessions affects the surplus of the session. In equilibrium, each session correctly anticipates the outcome of the other sessions. We assume that, through a *Nash bargaining solution*, each session shares the correctly anticipated surplus. The supplier and a consumer in each session share the surplus in proportion to their relative levels of *bargaining power*, which are given exogenously.[2]

From the viewpoint of real-world situations, it is reasonable to examine the provision of a public good through bilateral negotiations. In particular, in resolving environmental externalities, polluters and pollutees often negotiate individually, rather than collectively. According to Depres et al. (2008), who study the case of Vittel, a drinking water bottler in France, the company negotiated and contracted with farmers to solve the problem in which farmers' practices were significantly polluting Vittel's water catchment area. Seem-

[2]In the context of industrial organization, several studies employ the simultaneous bilateral bargaining model (Chipty and Snyder, 1999; Raskovich, 2001, 2003; Marshall and Merlo, 2004; Matsushima and Shinohara, 2014; Collard-Wexler et al., 2017).

ingly, the Vittel case is different from ours because Vittel is the only pollutee, and there are numerous polluters (farmers). However, Depres et al. (2008) report some important aspects, which seem to be widely applicable to negotiations in many environmental problems: a bilateral negotiation is preferable to a collective negotiation for the pollutee and for some of the polluters. In the case of Vittel, collective bargaining would most likely have increased the farmers' (polluters') bargaining power. Hence, negotiating with each farmer individually provided the company with a stronger relative bargaining position. On the other hand, some farmers who trusted their bargaining skills preferred individual negotiations to exploit the advantages of bargaining.[3]

First, we examine a case in which the supplier does not commit to a level of the public good prior to negotiations with consumers. In this case, the supplier and each consumer negotiate the joint production level of the public good, as well as transfers from the consumer to the supplier. We prove that the realized public good level through the negotiation is far from the efficient level (Proposition 1), owing to a kind of free-rider problem on public goods. When the supplier and consumers negotiate bilaterally, there is an externality between the bilateral negotiations. As a result of this externality, a consumer in bilateral bargaining can free ride a public good that is provided by compensation from other consumers.

Second, we examine a case in which the supplier commits to the level of the public good prior to the negotiations. In this case, the level of the public good is separated from the negotiation, and the supplier and each consumer negotiate the transfers only. Formally, we model this case as follows: in the first stage, the supplier commits to the level of the public good; in the second stage, observing the level, each consumer bilaterally and simultaneously negotiates with the supplier over his/her monetary compensation

[3] Note that some aspect of the Vittel case might be captured by our situation. According to Depres et al. (2008, p. 423),

> the quality of Vittel water had several positive spillovers (employment at the Vittel company, as well as in related activities, such as thermalism and tourism).

Those spillovers were achieved by the farmers contracted by Vittel protecting the water quality. On the other hand, non-contracted farmers were able to enjoy the spillovers, even if they did not engage in protecting the water quality. (Indeed, some farmers did not enter into a contract with Vittel.)

for the provision of the public good. We prove that under mild conditions, there is a threshold value of the supplier's bargaining power below which the public good is produced efficiently in equilibrium (see Theorem 1). This result shows that the public good is produced efficiently if and only if the supplier's bargaining power is "sufficiently weak." We evaluate to the extent to which the pre-commitment leads to the efficient provision of the public good under specific functional forms (see Section 6). We show that as the number of consumers increases, the efficient provision of the public good is less likely to be observed in equilibrium. However, even if the number of consumers is very large, the efficient provision of the public good in equilibrium may be observed with a certain high likelihood (see Proposition 3, and the discussion thereafter). In addition, we explore what level of the public good is provided when the supplier's bargaining power is beyond the threshold. Here, we provide a numerical example in which the supplier with "sufficiently strong" bargaining power produces excessive amounts of the public good. That is, the pre-commitment may lead to an excessive provision of the public good (see Section 6.2).

We note several interesting points with regard to our results. While the pre-commitment on the production level of the public good facilitates the internalization of consumers' preferences, the public good is not necessarily provided efficiently. Then, as shown in Theorem 1 and Section 6.2, when the supplier can pre-commit to the level of the public good, it has an incentive to set the level "sufficiently high" so that it receives transfers from all consumers.[4] In doing so, the supplier takes into account consumers' preferences for the public good. In this sense, the internalization of the consumers' preferences succeeds through the pre-commitment. However, depending on the level of per-capita transfers from the consumers, which is governed by the supplier's bargaining power, the supplier may set the public good level above the efficient level prior to the negotiations. Therefore, although the internalization of the preferences are completed via the pre-commitment, the supplier may not provide the public good efficiently (see also Remark 2).

The remainder of this paper is organized as follows. Section 2 reviews the related

[4]Refer to the discussion immediately after Theorem 1.

literature. Section 3 presents the preliminaries, and Section 4 analyzes the case of no pre-commitment. Section 5 analyzes the case of a pre-commitment. Section 6 presents an analysis under parametric functions, and Section 7 discusses the extension of the basic model. Section 8 concludes the paper.

2 Literature review

We review several studies on the effectiveness of pre-play actions before (i) public good provision (e.g., Hoel, 1991; Varian, 1994; Ray and Vohra, 1997, 2001; Dixit and Olson, 2000; Konrad and Thum, 2014) or (ii) Coasian bargaining (e.g., MacKenzie and Ohndorf, 2013; Ellingsen and Paltseva, 2016), which are important factors in our study.

As in our study, Ray and Vohra (1997, 2001) and Dixit and Olson (2000) examine public good provision through bargaining, under complete information. Ray and Vohra (1997, 2001) introduce two models in which a coalition with several agents can force each of the agents to provide a negotiated level of a public good. Dixit and Olson (2000) introduce a two-stage model in which agents determine whether to participate in a Coasian bargaining process, which is itself efficient, given the number of participants determined in the first stage. However, in these studies, the internalization of beneficiaries' preferences fails, and the provision of public goods is inefficient, except in specific cases.[5] Compared with these studies, our Proposition 1 shares these negative results. Fortunately, in our model, the pre-commitment on the level of the public good can improve the efficiency of providing the good. In addition, Proposition 3 contributes to the literature of the group-size effects of public good provision, following the seminal work of Olson (1965) (see the discussion in Section 6.1).[6] In a different context, incorporating incomplete information on agents' costs when providing public goods into the model of Hoel (1991), Konrad and Thum (2014) show that a unilateral pre-negotiation commitment by an agent on the

[5] Ellingsen and Paltseva (2012) show that even when providing *excludable* public goods, players do not necessarily participate in the provision of the goods when they are allowed to negotiate access to the goods, ex post.

[6] For a comprehensive survey on this topic, see Pecorino (2015).

level of the public good reduces the likelihood of achieving efficient outcomes, which is a negative result.[7] In contrast to their result, our study shows certain positive aspects of the pre-commitment, although our model of public good provision differs to theirs.[8]

Several researchers have examined how the restriction of feasible bargaining outcomes affects the efficiency of negotiations. MacKenzie and Ohndorf (2013) show that the efficiency of two-person Coasian bargaining can be enhanced by a restriction imposed by an outside authority on agents' property rights, which diminishes the feasible set of pollution abatement levels. Recently, Ellingsen and Paltseva (2016) analyzed the validity of the Coase theorem with N players by introducing a pre-play stage, in which players voluntarily negotiate contracts with regard to how to play the game and how much money is transferred among them. They show significantly different results between the two-player case and the N-player case. They prove that an efficient outcome is achievable in the two-player case, but not in the general N-player case, owing to strategic nonparticipation in the negotiation stage.[9] Our results are similar, in that the negotiation outcome is efficient if there are two players (a supplier and one consumer), and never efficient otherwise (see Proposition 1). Our result implies that restrictions on the strategies employed prior to negotiations, via the commitment of a player, helps to negotiate a better outcome. Similarly, Ellingsen and Paltseva (2016) show that an appropriate property right, which is related to the restriction of players' strategies, achieves an efficient allocation via contracting among players.

[7] Hoffmann et al. (2015) test Konrad and Thum's (2014) result experimentally.

[8] In the context of a voluntary contribution to public goods (e.g., Bergstrom et al., 1986), Varian (1994) considers a game of sequential contribution games, in which the player moving first can credibly commit to its contribution. He shows that such a unilateral commitment does not increase the total level of a public good.

[9] It is also shown in the context of a common agency game that the contract that achieves efficiency suffers from strategic nonparticipation. Bernheim and Whinston (1986) and Laussel and Le Breton (2001) show that, in common agency games, the efficient provision of a public good is achieved through transfer schemes that are completely contingent on the level of the good. Furusawa and Konishi (2011) show that this efficiency in equilibrium is vulnerable to voluntary participation behavior in the contracting process.

3 The model

Consider an economy with a *pure* public good and a private good (money), with a supplier of the public good and $n \geq 2$ consumers of the public good.[10] Only the supplier has the technology to provide the public good, but without any compensation from the consumers, it has no incentive to provide it. On the other hand, consumers enjoy the public good, but cannot produce the public good by themselves. Hence, each consumer needs to delegate the provision of the public good to the supplier in exchange for monetary compensation. How much money is compensated from each consumer to the supplier is decided through bargaining.

Formally, the set of players is denoted by $\{s, 1, \ldots, n\}$, where s is the supplier and i ($i = 1, \ldots, n$) is a consumer. Let N be the set of consumers. The level of the public good is typically denoted by $g \geq 0$, and consumer i's transfer to the supplier is denoted by T_i. The cost function of the public good is denoted by $c : \mathbb{R}_+ \to \mathbb{R}_+$, such that $c(0) = 0$, and c is an increasing, convex (sometimes, weakly convex), and twice continuously differentiable function. When the supplier provides g units of the public good and receives payment T_i from each $i \in N$, its payoff is $\sum_{i \in N} T_i - c(g)$ (a profit). Each consumer i receives payoff $v(g) - T_i$, where $v : \mathbb{R}_+ \to \mathbb{R}_+$ is a benefit function from the public good, such that $v(0) = 0$ and v is an increasing, concave (sometimes, weakly concave), and twice continuously differentiable function. Note that in this model, each consumer has the same benefit function, v.

Definition 1 As a reference level for the public good, we define

$$g(m) \in \arg\max_{g \geq 0} \; mv(g) - c(g),$$

for each $m \in \{1, \ldots, n\}$.

Furthermore, we impose the following conditions on $v(g)$ and $c(g)$.

[10] See Remark 1 for the case of $n = 1$.

Assumption 1 We impose the following assumption on $c(\cdot)$ and $v(\cdot)$:

(1.1) $\lim_{g\to 0} v'(g) = \infty$, $\lim_{g\to\infty} v'(g) = 0$, $\lim_{g\to 0} c'(g) = 0$, and $\lim_{g\to\infty} c'(g) = \infty$.

(1.2) For all $g > 0$, $c'(g)/c(g) > v'(g)/v(g)$.

Condition (1.1) guarantees interior solutions of the model: that is, $g(m) > 0$ for each $m \in \{1, \ldots, n\}$.[11] Condition (1.2) describes the relation between the ratio of the marginal benefit of the public good to the gross benefit and the ratio of the marginal cost to the overall cost. Condition (1.2) implies that $c(g)/v(g)$ is increasing in g, that is, the cost per value is increasing in the level of the public good, which sounds reasonable.[12]

We model a bargaining process between the supplier and consumers in Sections 4 and 5, based on simultaneous and bilateral bargaining, which is often used in the context of buyer–supplier relationships (e.g., Horn and Wolinsky, 1988; Marshall and Merlo, 2004).[13] This is based on observations of real-world negotiations concerning environmental externalities, as discussed in the Introduction. The question we address is how the commitment on the production level of the public good prior to negotiations affects the efficiency of the provision of the good. To answer this question, we consider a case in which the supplier does not pre-commit the level of the public good (Section 4), and then a case in which the supplier unilaterally commits to the level of the public good prior to negotiations (Section 5).

4 No pre-commitment on the level of the public good

In this section, the supplier does not commit to the production level of the public good prior to negotiations. Hence, the supplier negotiates the production level and the transfers

[11] Note that our main results hold whenever $g(m)$ is interior for each $m \in \{1, \ldots, n\}$. Thus, we can obtain these results if we assume $\lim_{g\to 0} v'(g) = \infty$, $\lim_{g\to\infty} v'(g) = 0$, and $c'(g)$ is finite, instead of assuming Condition (1.1).

[12] The conditions in Assumption 1 are satisfied by reasonable parametric benefit and cost functions (see also Proposition 2). By l'Hôpital's rule and (1.1), we obtain

$$\lim_{g\to 0} \frac{c(g)}{v(g)} = 0. \tag{1.3}$$

[13] See also the references in Milliou and Petrakis (2007).

with consumers. We provide a two-stage model of negotiation, based on simultaneous and bilateral bargaining. In the first stage, the supplier and each consumer $i \in N$ bilaterally negotiate the joint production level of the public good $g_i \geq 0$ and the transfer to the supplier from consumer i $T_i(\geq 0)$. Hence, in this stage, the supplier faces n independent bilateral negotiations. We assume that every bilateral negotiation is *simultaneous* and *Nash bargaining*. We further assume that the outcome of each negotiation is given by the *(asymmetric) Nash bargaining solution*, in the belief that the bargaining outcomes with the other parties are determined in the same way. Hence, the joint surplus of one bilateral bargaining process is maximized through the choice of the public good level, and the maximized joint surplus is divided between the consumer and the supplier in the proportion of $1 - \beta$ to β, where $\beta \in [0, 1]$ represents the supplier's *bargaining power*. We assume that, in equilibrium, the negotiators in each bilateral negotiation have consistent beliefs about the other negotiations: the outcome of each negotiation is predicted correctly by the others. Let $(g_j, T_j)_{j\in N}$ be the equilibrium outcome of the simultaneous bilateral bargaining. The second stage is an approval stage by the supplier, in which the supplier decides whether to execute the first-stage outcome $(g_j, T_j)_{j\in N}$. If the supplier approves $(g_j, T_j)_{j\in N}$, it provides $\sum_{j\in N} g_j$ units of the public good in exchange for transfers $\sum_{j\in N} T_j$, and the supplier's and consumer i's payoffs are $\sum_{j\in N} T_j - c(\sum_{j\in N} g_j)$ and $v(\sum_{j\in N} g_j) - T_i$, respectively. If the supplier disapproves, it annuls $(g_j, T_j)_{j\in N}$, and the payoff to every player is zero. We establish this stage to guarantee the supplier's participation constraint. In our situation, the supplier reaps no benefit from producing the public good in the absence of transfers from consumers, although it has the technology to do so. Hence, it is natural for the supplier to check whether it will benefit from a negotiation.[14]

The method of simultaneously applying the Nash bargaining solution follows Chipty and Snyder (1999) and Raskovich (2003). However, note that in their models, free riding is impossible. See, for example, the condition "$v_i(0, q_{-i}) = 0$" in Raskovich (2003, p. 410,

[14] We do not explicitly model consumers' approval of $(g_j, T_j)_{j\in N}$, because the participation constraint always holds for all consumers since the Nash bargaining solution guarantees them non-negative payoffs. Thus, consumers will approve the second-stage bargaining outcome.

12th line from the bottom).

A non-cooperative foundation for the asymmetric Nash bargaining solution has been presented by several studies. As Binmore et al. (1986) show, the subgame perfect equilibrium outcome in Rubinstein's bargaining model, with alternating offers and a risk of breakdown, approximates the Nash bargaining solution. In addition, they discuss the relationship between the Nash bargaining solution and asymmetric bargaining power. Hence, we can approximately interpret that in the first-stage game of our model, each pair of the supplier and a consumer play the alternating offer bargaining game with a risk of breakdown, anticipating the other sessions' outcomes. Therefore, by backward induction, our solution is consistent with a subgame perfect Nash equilibrium.

We solve this model in a backward manner. In the second stage, given $(g_j, T_j)_{j \in N}$, the supplier approves it if $\sum_{j \in N} T_i > c(\sum_{j \in N} g_j)$, is indifferent between approval and disapproval if $\sum_{j \in N} T_j = c(\sum_{j \in N} g_j)$, and disapproves otherwise. Henceforth, we assume that when $\sum_{j \in N} T_j = c(\sum_{j \in N} g_j)$, the supplier disapproves.[15]

In the first stage, the supplier negotiates with each consumer bilaterally and simultaneously. In the equilibrium of this stage, the bargaining participants (the supplier and a consumer) correctly anticipate the other bargaining outcomes and conduct a Nash bargaining process. That is, for each $i \in N$, the supplier and consumer i negotiate over (g_i, T_i) to maximize the Nash product function, given $(g_j, T_j)_{j \neq i}$.

Players' surpluses in each bilateral negotiation take different forms, depending on the outcomes of the other bilateral negotiations.

First, consider bilateral bargaining between the supplier and consumer i, given $(g_j, T_j)_{j \neq i}$ such that $\sum_{j \neq i} T_j \leq c(\sum_{j \neq i} g_j)$. Then, based on the second-stage equilibrium, if this bargaining breaks down, the supplier does not approve $(g_j, T_j)_{j \neq i}$ in the next stage. Hence, the payoffs to the supplier and consumer i are zero. If this bargaining reaches an agreement (g_i, T_i) and the supplier approves $(g_j, T_j)_{j \in N}$ in the next stage, then the supplier and consumer i receive the payoffs $T_i + \sum_{j \neq i} T_j - c(g_i + \sum_{j \neq i} g_j)$ and $v(g_i + \sum_{j \neq i} g_j) - T_i$,

[15]The assumption does not change the subsequent result.

respectively. Therefore, the supplier and consumer i decide (g_i, T_i) to maximize the Nash product function $\beta \ln(T_i + \sum_{j\neq i} T_j - c(g_i + \sum_{j\neq i} g_j)) + (1-\beta)\ln(v(g_i + \sum_{j\neq i} g_j) - T_i)$. Thus, given $(g_j, T_j)_{j\neq i}$, the bargaining between the supplier and consumer i achieves (g_i^*, T_i^*) such that

$$\{g_i^*\} = \arg\max_{g_i \geq 0} \; v(g_i + \sum_{j\neq i} g_j) - c(g_i + \sum_{j\neq i} g_j) \quad \text{and} \tag{1}$$

$$T_i^* = v(g_i^* + \sum_{j\neq i} g_j) - (1-\beta)\left(v(g_i^* + \sum_{j\neq i} g_j) - c(g_i^* + \sum_{j\neq i} g_j) + \sum_{j\neq i} T_j \right). \tag{2}$$

From (2),

$$T_i^* + \sum_{j\neq i} T_j - c(g_i^* + \sum_{j\neq i} g_j) = \beta \left[v(g_i^* + \sum_{j\neq i} g_j) - c(g_i^* + \sum_{j\neq i} g_j) + \sum_{j\neq i} T_j \right].$$

Thus, $T_i^* + \sum_{j\neq i} T_j > c(g_i^* + \sum_{j\neq i} g_j)$ if and only if

$$v(g_i^* + \sum_{j\neq i} g_j) - c(g_i^* + \sum_{j\neq i} g_j) + \sum_{j\neq i} T_j > 0. \tag{3}$$

The left-hand side of (3) is the joint surplus of the supplier and consumer i. From (3), the supplier approves $((g_i^*, T_i^*), (g_j, T_j)_{j\neq i})$ if and only if the joint surplus is positive. Note that if the supplier approves $((g_i^*, T_i^*), (g_j, T_j)_{j\neq i})$, consumer i's payoff $v(g_i^* + \sum_{j\neq i} g_j) - T_i^* = (1-\beta)(v(g_i^* + \sum_{j\neq i} g_j) - c(g_i^* + \sum_{j\neq i} g_j) + \sum_{j\neq i} T_j)$ is non-negative. In contrast, if the joint surplus is negative (i.e., (3) does not hold), the bargaining does not reach an agreement; that is, $(g_i^*, T_i^*) = (0, 0)$.

Second, consider the case of $\sum_{j\neq i} T_j > c(\sum_{j\neq i} g_j)$. Based on the second-stage equilibrium, even if this bilateral bargaining breaks down, the supplier approves the outcome. Hence, in the case of disagreement, the supplier and consumer i receive $\sum_{j\neq i} T_j - c(\sum_{j\neq i} g_j)$ and $v(\sum_{j\neq i} g_j)$, respectively. If this bargaining reaches agreement (g_i, T_i) and the supplier approves $(g_j, T_j)_{j\in N}$ in the next stage, then the supplier and consumer i receive $T_i + \sum_{j\neq i} T_j - c(g_i + \sum_{j\neq i} g_j)$ and $v(g_i + \sum_{j\neq i} g_j) - T_i$, respectively. As in the previous case, (g_i^*, T_i^*) maximizes the Nash product function $\beta \ln(T_i - c(g_i + \sum_{j \neq i} g_j) + c(\sum_{j \neq i} g_j)) + (1-\beta)\ln(v(g_i + \sum_{j \neq i} g_j) - T_i - v(\sum_{j \neq i} g_j))$: that is,

$$\{g_i^*\} = \arg\max_{g_i \geq 0} \; v(g_i + \sum_{j \neq i} g_j) - c(g_i + \sum_{j \neq i} g_j) \quad \text{and} \tag{4}$$

$$T_i^* = v(g_i^* + \sum_{j \neq i} g_j) - v(\sum_{j \neq i} g_j)$$
$$- (1-\beta)\left(v(g_i^* + \sum_{j \neq i} g_j) - c(g_i^* + \sum_{j \neq i} g_j) - \left(v(\sum_{j \neq i} g_j) - c(\sum_{j \neq i} g_j)\right)\right). \tag{5}$$

The maximized joint surplus of this negotiation is $v(g_i^* + \sum_{j \neq i} g_j) - c(g_i^* + \sum_{j \neq i} g_j) - (v(\sum_{j \neq i} g_j) - c(\sum_{j \neq i} g_j))$, which is non-negative, from (4). From (5),

$$T_i^* + \sum_{j \neq i} T_j - c(\sum_{j \in N} g_j) = \beta\left(v(g_i^* + \sum_{j \neq i} g_j) - c(g_i^* + \sum_{j \neq i} g_j) - \left(v(\sum_{j \neq i} g_j) - c(\sum_{j \neq i} g_j)\right)\right)$$
$$+ \sum_{j \neq i} T_i - c(\sum_{j \neq i} g_j) > 0,$$

implying that the supplier's payoff is positive and so it always approves $((g_i^*, T_i^*), (g_j, T_j)_{j \neq i})$.

From (1) and (4), in every bargaining in which the supplier and a consumer reach an agreement, the supplier and consumer i maximize $v(g_i + \sum_{j \neq i} g_j^*) - c(g_i + \sum_{j \neq i} g_j^*)$, given $(g_j^*)_{j \neq i}$ through the choice of g_i. Thus, if the tuple of the public goods at equilibrium is denoted by $(g_j^*)_{j \in N}$, then

$$\{g_i^*\} = \arg\max_{g_i \geq 0} \; v(g_i + \sum_{j \neq i} g_j^*) - c(g_i + \sum_{j \neq i} g_j^*) \text{ for each } i \in N.$$

By this property of the equilibrium, we obtain the next proposition.

Proposition 1 *In any equilibrium, the equilibrium level of the public good $\sum_{j \in N} g_j^*$ is exactly $g(1)$.*

Proof. Suppose that in an equilibrium, $(g_j^*, T_j^*)_{j \in N}$ is the first-stage outcome, which the supplier approves in the second stage. Recall that by Assumption 1, $g(1)$ is the unique maximizer of $v(g) - c(g)$ and $g(1) > 0$. Therefore, $g_i^* + \sum_{j \neq i} g_j^* = g(1)$ for each $i \in N$. ■

There always exists an asymmetric equilibrium such that a consumer is the one and only contributor, and the others free ride; that is, $(g_j, T_j)_{j \in N}$, such that $(g_i, T_i) = (g(1), \beta v(g(1)) + (1-\beta)c(g(1)))$ for some $i \in N$, and $(g_j, T_j) = (0, 0)$ for each $j \in N \backslash \{i\}$. In addition to this asymmetric equilibrium, if β is sufficiently low, then there exists a symmetric equilibrium at which $g_i^* = g(1)/n$, for each $i \in N$.[16] Note that the asymmetric equilibrium always exists in any case, but the symmetric equilibrium does not always exist. Proposition 1 shows that at any equilibrium, the level of the public good is unique.

By Proposition 1, in each bilateral negotiation, the total surplus $nv(g) - c(g)$ is never maximized, which induces the inefficient provision of the public good (recall $g(1) < g(n)$). This is because no bilateral negotiation internalizes consumers' benefits from other bilateral negotiations. Proposition 1 resembles the well-known result of under-provision of the public good through voluntary contribution (e.g., Bergstrom et al., 1986) in that no player takes other players' benefits into account when deciding on a contribution to a public good. In our model, owing to externalities among bilateral negotiations, some consumers may be able to enjoy the public good at no cost. In the asymmetric equilibrium, the level of the public good provided by some consumer is $g(1)$, which is sufficiently high so as not to generate a positive surplus from negotiations between the supplier and the other consumers. Since there is no gain from negotiating, some consumers send no transfers and become free riders.

5 Pre-commitment on the production level of the public good

In this section, the supplier commits to the level of the public good prior to negotiations, that is, the supplier decides the level of the public good before it negotiates with each consumer on transfers. The modified model in this section is as follows: in the first stage,

[16] The analysis is available upon request.

the supplier decides the commitment level of the public good $g \geq 0$. In this stage, the supplier does not provide the public good at this level or incur the cost. The public good is provided after the supplier's approval in the third stage. In the second stage, the supplier and each consumer $i \in N$ bilaterally and simultaneously negotiate over the division of the joint surplus, thus, determining $T_i \geq 0$. The equilibrium outcome is determined in a similar way to the model in the previous section. In the third stage, the supplier decides whether to approve a project $(g, (T_j)_{j \in N})$, which is a tuple of the level of the public good and the transfers. If the supplier approves, then it provides g units of the public good and receives T_i from each consumer i. As a result, the supplier's payoff is $\sum_{i \in N} T_i - c(g)$ and each consumer i's payoff is $v(g) - T_i$. Otherwise, no public good is provided and no money is transferred. Then, the supplier's and each consumer's payoffs are zero.

5.1 The third stage

The analysis of this stage is the same as before. Given $(g, (T_i)_{i \in N})$, the supplier approves it if and only if $\sum_{i \in N} T_i > c(g)$.

5.2 The second stage

In the second stage, who contributes to the project and how much money each contributor transfers to the supplier are determined. Who contributes to the project depends on who is pivotal to the project, as defined below.

Definition 2 Let $g \geq 0$ be a level of the public good. Let $i \in N$. Let T_j be a transfer from consumer $j \in N \backslash \{i\}$. Given $(T_j)_{j \neq i}$, consumer i is *pivotal* to the approval of a project $(g, (T_j)_{j \in N})$ if $\sum_{j \neq i} T_j \leq c(g) < v(g) + \sum_{j \neq i} T_j$.[17]

The pivotal consumers are defined based on the third-stage equilibrium. We consider bargaining between the supplier and consumer $i \in N$ when i is pivotal to the approval of a project. If this bargaining breaks down, then the supplier disapproves in the third

[17] The definition of a pivotal consumer is the same as that of Raskovich (2003).

stage, and the supplier's and consumer i's payoffs are zero. If an agreement is reached in the bargaining, then the supplier's payoff is $\sum_{j\in N} T_j - c(g)$ and consumer i's payoff is $v(g) - T_i$. Then, the joint surplus of this bargaining is

$$v(g) - T_i + T_i + \sum_{j\neq i} T_j - c(g) = v(g) + \sum_{j\neq i} T_j - c(g) > 0.$$

In Nash bargaining, this joint surplus is divided between the supplier and consumer i in the proportion of β and $1-\beta$. Hence,

$$T_i = v(g) - (1-\beta)\left(v(g) + \sum_{j\neq i} T_j - c(g)\right) = \beta v(g) + (1-\beta)c(g) - (1-\beta)\sum_{j\neq i} T_j. \quad (6)$$

From (6), we confirm that $\sum_{j\in N} T_j > c(g)$. Thus, the transfer from the pivotal consumer is necessary for the supplier to provide the public good.

Second, we consider the case in which consumer i is not pivotal to the project g, that is, the case in which either (i) $v(g) + \sum_{j\neq i} T_j \leq c(g)$ or (ii) $\sum_{j\neq i} T_j > c(g)$ is satisfied. In case (i), the joint surplus of the bargaining is zero because the supplier disapproves. Case (ii) means that the supplier approves the project at the third stage, irrespective of whether the bargaining with consumer i succeeds. Hence, the supplier's net surplus from this bargaining is $T_i + \sum_{j\neq i} T_j - c(g) - (\sum_{j\neq i} T_j - c(g)) = T_i$ and the consumer i's is $v(g) - T_i - v(g) = -T_i$; the joint surplus of this bargaining is zero. In any case, the joint surplus of the bargaining with the non-pivotal consumer i is zero, which implies $T_i = 0$; thus, consumer i free rides the public good.

Finally, we examine how many consumers become pivotal and how much money the pivotal consumers transfer to the supplier. Suppose that m pivotal consumers $(1 \leq m \leq n)$ exist. Let $M \subseteq N$ be the set of pivotal consumers. Since condition (6) holds for all m consumers, solving the system of equations yields $(T_j^m)_{j\in M}$ such that for each $j \in M$,

$$T_j^m = \frac{\beta v(g) + (1-\beta)c(g)}{\beta + (1-\beta)m} \ (> 0 \text{ if } g > 0). \quad (7)$$

The payoff to the supplier is

$$\pi_S^m(g) \equiv \sum_{j \in M} T_j^m - c(g) = \frac{\beta(mv(g) - c(g))}{\beta + (1-\beta)m}, \tag{8}$$

the payoff to the pivotal consumer $i \in M$ is

$$\pi_i^m(g) \equiv v(g) - T_i^m = \frac{(1-\beta)(mv(g) - c(g))}{\beta + (1-\beta)m},$$

and the payoff to the non-pivotal consumer $i \in N \backslash M$ is $v(g)$ (the superscript "m" of T_j^m and π_S^m refers to the number of pivotal consumers). Henceforth, we call pivotal consumers *contributors* and non-pivotal consumers *free riders*.

The sum of the joint surplus of the bargaining sessions with m pivotal consumers is $mv(g) - c(g)$, and the supplier's share of the surplus and the pivotal consumer's share are $\beta/(\beta + (1-\beta)m)$ and $(1-\beta)/(\beta + (1-\beta)m)$, respectively. Since $\beta + (1-\beta)m$ is the sum of the bargaining powers over the supplier and m pivotal consumers, the surplus is distributed in proportion to the bargaining power.[18]

Lemma 1 shows that the equilibrium number of contributors is determined according to the level of the public good in the first stage.

Lemma 1

(1.a) For any given $g \geq 0$, such that $nv(g) \leq c(g)$, the equilibrium number of contributors at g is zero.

(1.b) For any given $g > 0$, such that $nv(g) > c(g)$, m is the equilibrium number of contributors at g if and only if

$$\frac{c(g)}{v(g)} < m \leq \frac{c(g)}{\beta v(g)} + 1. \tag{9}$$

At least one integer m exists that satisfies (9).

Proof. (1.a) If g satisfies $nv(g) \leq c(g)$, there is no surplus in any bilateral bargaining.

[18] Note that $(T_j^m)_{j \in M}$ is supported by other bargaining models. See Section 7.1.

Hence, no consumer pays a positive fee; the number of contributors is zero.

(1.b) m is the number of contributors if and only if $\sum_{j\in M\setminus\{i\}} T_j^m \leq c(g) < v(g) + \sum_{j\in M\setminus\{i\}} T_j^m$ for each $i \in M$. From (7), these inequalities hold if and only if $(m-1)\beta v(g) \leq c(g) < mv(g)$, implying (9). Clearly, an integer m exists that satisfies $c(g)/v(g) < m \leq (c(g)/v(g)) + 1$. Hence, at least one integer m exists that satisfies (9) because $\beta \leq 1$. ■

Note that if $nv(g) > c(g)$, then the pivotal condition restricts the number of contributors, which means that the supplier does not necessarily receive positive transfers from all consumers. We summarize the second-stage equilibria as follows:

The second-stage equilibria. After the level of the public good is decided at g in the first stage, the equilibrium outcome of the second-stage subgame is $(M, (T_j^m)_{j\in N})$, where M is the set of contributors, the equilibrium number of contributors $m = |M|$ is determined according to Lemma 1, and the equilibrium transfer is

$$T_j^m = \begin{cases} \dfrac{\beta v(g) + (1-\beta)c(g)}{\beta + (1-\beta)m} & \text{if } j \in M, \\ 0 & \text{if } j \in N\setminus M. \end{cases} \tag{10}$$

Note that for some level of the public good in the first stage, there may be multiple second-stage equilibria that support different equilibrium numbers of contributors, because (9) may include multiple integers. For the backward induction analysis, we focus on the equilibrium that maximizes the payoff to the supplier among the second-stage equilibria in every second-stage subgame.

To investigate which second-stage equilibrium maximizes the supplier's payoff, we start by restating (9), which is based on the level of the public good. Define $\bar{g}^m$ for each $m \in \{1, \ldots, n\}$, and $\underline{g}^m$ for each $m \in \{2, \ldots, n\}$, such that

$$m = \frac{c(\bar{g}^m)}{v(\bar{g}^m)} \quad \text{and} \quad m = \frac{c(\underline{g}^m)}{\beta v(\underline{g}^m)} + 1. \tag{11}$$

Note that $c(g)/v(g)$ is increasing in g, by (1.2) in Assumption 1. Thus, by (1.2) in Assumption 1 and (1.3), $\bar{g}^m$ can be defined uniquely for each $m \in \{1, \ldots, n\}$, and $\underline{g}^m$ can be defined uniquely for each $m \in \{2, \ldots, n\}$. We adopt the convention that $\underline{g}^1 \equiv 0$, by (1.3).

Lemma 2

(2.a) For any given $g > 0$, such that $nv(g) > c(g)$, m is the equilibrium number of contributors at g if and only if

$$\underline{g}^m \leq g < \bar{g}^m. \tag{9'}$$

(2.b) For each $m \in \{2, \ldots, n\}$, $\underline{g}^m < \bar{g}^{m-1}$ if $\beta < 1$, and $\underline{g}^m = \bar{g}^{m-1}$ if $\beta = 1$.

Proof. (2.a) is the restatement of (1.b) in Lemma 1. We now show (2.b). By definition, $c(\bar{g}^{m-1})/v(\bar{g}^{m-1}) = m-1$ and $c(\underline{g}^m)/(\beta v(\underline{g}^m)) = m-1$. Under these conditions, we obtain $c(\underline{g}^m)/v(\underline{g}^m) = \beta(m-1) \leq m-1 = c(\bar{g}^{m-1})/v(\bar{g}^{m-1})$. Since $c(g)/v(g)$ is increasing in g, we obtain $\underline{g}^m \leq \bar{g}^{m-1}$. ■

Lemma 2 shows that $\underline{g}^m$ and $\bar{g}^m$ make the lower and upper bounds of the public good level, where m is the equilibrium number of contributors in the second stage. In addition, Lemma 2 shows the condition under which the second-stage subgame has multiple equilibria that support different numbers of contributors. By (2.b), for each $m \in \{2, \ldots, n\}$, $\underline{g}^{m-1} \leq g < \bar{g}^{m-1}$ (the range of $m-1$ contributors) and $\underline{g}^m \leq g < \bar{g}^m$ (the range of m contributors) overlap if and only if $\beta < 1$. Thus, in the case of $\beta < 1$, if the supplier chooses g between $\underline{g}^m$ and $\bar{g}^{m-1}$ in the first stage, multiple second-stage equilibria exist that support the existence of $m - 1$ contributors and that of m contributors in the subsequent second stage.

When there are multiple numbers of contributors in the equilibria of a second-stage subgame, we focus on the equilibrium that maximizes the payoff to the supplier from the set of second-stage equilibria. Lemma 3 is helpful to clarify which second-stage equilibrium maximizes the supplier's payoff.

Lemma 3 For each $g > 0$ and each $m \in \{1, \ldots, n-1\}$,

$$\pi_S^m(g) \leq \pi_S^{m+1}(g), \tag{12}$$

which is satisfied with equality if and only if $\beta = 0$.

Proof. From (8), we obtain

$$\pi_S^{m+1}(g) - \pi_S^m(g) = \frac{\beta(\beta v(g) + (1-\beta)c(g))}{(\beta + (1-\beta)(m+1))(\beta + (1-\beta)m)} \geq 0,$$

which is satisfied with equality if and only if $\beta = 0$. ■

Lemma 3 shows that for each $g > 0$, $\pi_S^m(g)$ is non-decreasing in m. By Lemma 3, in the second stage, immediately after the supplier chooses g such that $nv(g) > c(g)$, *the second-stage equilibrium that maximizes the supplier's payoff within the set of the second-stage equilibria is the equilibrium at which the number of contributors is the maximal integer among m that satisfies* (9).

The refined second-stage equilibrium. For each $g \geq 0$, denote the equilibrium number of contributors by $m(g) \in \{0, \ldots, n\}$. After the level of the public good g is decided in the first stage, the equilibrium outcome of the second-stage subgame is $(M, (T_j^{m(g)})_{j \in N})$, where M is the set of contributors, the equilibrium number of contributors $m(g) = |M|$ is

$$m(g) = \begin{cases} 0 & \text{if } g = \underline{g}^1 \ (= 0), \\ 1 & \text{if } g \in (\underline{g}^1, \underline{g}^2), \\ k & \text{if } g \in [\underline{g}^k, \underline{g}^{k+1}) \quad (k \in \{2, \ldots, n-1\}), \\ n & \text{if } g \in [\underline{g}^n, \bar{g}^n), \\ 0 & \text{if } g \geq \bar{g}^n, \end{cases} \tag{13}$$

and the equilibrium transfer $(T_j^{m(g)})_{j \in N}$ conforms to (10).

We learn from (13) that the equilibrium number of contributors increases as the level of the public good increases. Hence, the supplier sets the level of the public good "sufficiently high" if and only if it receives transfers from all consumers.

5.3 The first stage

As a preparation for Theorem 1, we introduce some notation. Using (11), we define $\beta(g,m) \in \mathbb{R}_+$ for each $g \geq 0$ and $m \in (1,n]$, such that

$$\beta(g,m) \equiv \frac{c(g)}{v(g)} \cdot \frac{1}{m-1}.$$

For each g and m, $\beta(g,m)$ is the value of bargaining power at which g is the lower bound over which m contributors exist: that is, $g = \underline{g}^m$ if $\beta = \beta(g,m)$. By this,

$$\beta\left(g(m),m\right) = \frac{c(g(m))}{v(g(m))} \cdot \frac{1}{m-1} \quad \text{and} \quad \beta\left(g(m),m+1\right) = \frac{c(g(m))}{v(g(m))} \cdot \frac{1}{m}. \tag{14}$$

Thus, $g(m) = \underline{g}^m$ if $\beta = \beta\left(g(m),m\right)$ and $g(m) = \underline{g}^{m+1}$ if $\beta = \beta\left(g(m),m+1\right)$.

Imposing an additional condition of the monotonicity of π_S^m (Condition 1 below), we establish a necessary and sufficient condition under which the supplier produces the public good efficiently.

Condition 1 $\pi_S^m(\underline{g}^m) < \pi_S^n(\underline{g}^n)$ for each $m \in \{1,\ldots,n-1\}$.

Theorem 1 *Under Assumption 1 and Condition 1, there is an equilibrium at which the supplier produces the efficient level of the public good ($g(n)$) if and only if β satisfies $0 \leq \beta \leq \beta\left(g(n),n\right)$.*

The proof of Theorem 1 is provided in the Appendix. As Theorem 1 shows, $\beta(g(n),n)$ marks the threshold value of the supplier's bargaining power, below which the supplier produces the public good efficiently. The supplier provides the public good efficiently at an equilibrium if and only if its bargaining power is "sufficiently weak," under several assumptions and conditions. At this equilibrium, the supplier receives transfers from all consumers, as we see in the proof of Theorem 1, and its payoff is “sufficiently close” to zero because β is sufficiently small.

Condition 1 can be interpreted as a condition with regard to the change rate of a joint surplus of the supplier and contributors when the number of contributors increases. After some calculation, we obtain

$$\pi_S^n(\underline{g}^n) - \pi_S^m(\underline{g}^m) > 0 \text{ if and only if } \frac{nv(\underline{g}^n) - c(\underline{g}^n)}{mv(\underline{g}^m) - c(\underline{g}^m)} > \frac{\beta + (1-\beta)n}{\beta + (1-\beta)m},$$

where $nv(\underline{g}^n) - c(\underline{g}^n)$ is the surplus of the supplier and of the contributors when all consumers are contributors, and $mv(\underline{g}^m) - c(\underline{g}^m)$ is that when m consumers are contributors. Those surpluses are evaluated at minimum levels of the public good when the contributors are m and n. Since $(\beta + (1-\beta)n)/(\beta + (1-\beta)m) \geq 1$, Condition 1 requires that the surplus increases at a greater rate than a certain number when the number of the contributors increases from m to n. Remarks with regard to Condition 1 are in order. First, this condition is not necessary to prove the sufficiency of Theorem 1. Thus, even without Condition 1, if the supplier's bargaining power is sufficiently weak, then it provides the public good at the efficient level. Second, Condition 1 is satisfied in many applications because it is satisfied by reasonable parametric benefit and cost functions, as we see in Proposition 2.

Intuitively, we can confirm from Lemma 3 that for any level of the public good, the more consumers from which the supplier receives transfers, the higher is the supplier's payoff; hence, the supplier desires to receive transfers from all consumers. Since the public good is pure, the supplier must set the level of the public good “sufficiently high” in order to receive transfers from all consumers; if the level of the public good is low, so that the cost of the public good is compensated by the transfers from fewer than n consumers, then free riders exist. However, there is another problem: the supplier may choose an excessively higher level of the public good. If the supplier's bargaining power is strong, the transfers from contributors are large. Then, there may be a case in which the cost of $g(n)$ is compensated by fewer than n consumers. In this case, even if the supplier chooses

$g(n)$, there are free riders. Then, the supplier sets the level of the public good over $g(n)$ to eliminate the free riders because it wants to receive transfers from all consumers. This leads to excessive provision of the public good.[19] Therefore, the supplier provides the public good efficiently if and only if its bargaining power is sufficiently weak, such that the cost of $g(n)$ is compensated by all consumers, but not by fewer than n consumers.

Remark 1 We have not examined the case of $n = 1$. In this case of the models in Sections 4 and 5, the public good is obviously always produced efficiently for any value of the supplier's bargaining power in equilibrium. This is very different from the result in the case of $n \geq 2$. The difference between the cases of $n \geq 2$ and $n = 1$ is the existence of externalities between bilateral negotiations. Thus, we conclude that the externalities between bilateral bargaining sessions cause the Pareto inefficiency.

6 Analysis under parametric functions

6.1 Number of consumers and likelihood of allocative efficiency

We consider an example in which $v(g) = \theta g^{\alpha}$ and $c(g) = \gamma g^{\delta}$, where α, θ, δ, and γ are positive constants such that $0 < \alpha < 1 < \delta$. Note that $v(g)$ and $c(g)$ satisfy Assumption 1. From (11) and (14), we obtain

$$\underline{g}^m = \left(\frac{\beta\theta(m-1)}{\gamma}\right)^{\frac{1}{\delta-\alpha}}, \quad \bar{g}^m = \left(\frac{m\theta}{\gamma}\right)^{\frac{1}{\delta-\alpha}}, \quad g(m) = \left(\frac{m\alpha\theta}{\delta\gamma}\right)^{\frac{1}{\delta-\alpha}}, \tag{15}$$
$$\beta(g,m) = \frac{\gamma}{\theta(m-1)}g^{\delta-\alpha}, \quad \beta(g(m),m) = \frac{\alpha}{\delta}\left(\frac{1}{m-1}+1\right), \quad \text{and} \quad \beta(g(m),m+1) = \frac{\alpha}{\delta}.$$

[19] In Section 6.2, we provide an example in which the public good is provided excessively in equilibrium.

Moreover, we obtain that

$$\begin{aligned}\pi_S^m(\underline{g}^m) &= \frac{\beta}{\beta+(1-\beta)m}\left(\frac{\beta\theta(m-1)}{\gamma}\right)^{\frac{\alpha}{\delta-\alpha}}\left(m\theta-\gamma\cdot\frac{\beta\theta(m-1)}{\gamma}\right)\\ &= \left(\frac{(\beta\theta)^{\delta}}{\gamma^{\alpha}}\right)^{\frac{1}{\delta-\alpha}}(m-1)^{\frac{\alpha}{\delta-\alpha}}\,.\end{aligned}$$

Since $\alpha/(\delta-\alpha)>0$ and $m\geq 1$, $\pi_S^m(\underline{g}^m)$ is increasing in m. Thus, Condition 1 holds.

Proposition 2 *Assumption 1 and Condition 1 hold if* $v(g)=\theta g^{\alpha}$ *and* $c(g)=\gamma g^{\delta}$*, where* α*,* θ*,* δ*, and* γ *are positive constants, such that* $0<\alpha<1<\delta$*.*

By Proposition 2, Theorem 1 applies to this example.

Corollary 1 Suppose that $v(g)=\theta g^{\alpha}$ and $c(g)=\gamma g^{\delta}$, where α, θ, δ, and γ are positive constants such that $0<\alpha<1<\delta$. The supplier provides the public good efficiently in equilibrium if and only if $\beta\in[0,\beta(g(n),n)]$, where $\beta(g(n),n)=\alpha n/(\delta(n-1))$.

Proposition 3 follows directly from $\beta(g(n),n)$ being decreasing in n.

Proposition 3 *The set of the supplier's bargaining powers in which the supplier provides the public good efficiently at an equilibrium,* $[0,\beta(g(n),n)]$*, shrinks and converges to* $[0,\alpha/\delta]$ *as* n *becomes large, if* $v(g)=\theta g^{\alpha}$ *and* $c(g)=\gamma g^{\delta}$*, where* α*,* θ*,* δ*, and* γ *are positive constants such that* $0<\alpha<1<\delta$*.*

From Proposition 3, if we treat $\beta(g(n),n)$, the upper bound of the set in Proposition 3, as the measure of the likelihood for the efficient provision of public good, we conclude that the equilibrium likelihood of the efficient provision of the public good becomes lower as the number of consumers increases. However, note that this likelihood does not necessarily vanish, even if the number of consumers approaches infinity (α/δ is close to one if α and δ are close to one). This implies that there is a possibility that a sufficiently weak supplier provides the public good efficiently, even if it negotiates with many consumers.

A comparison of our results with those of Dixit and Olson (2000) might be important, because whether beneficiaries are *pivotal* to public good provision plays a role in both sets of results. Under the two-stage game of public good provision, Dixit and Olson (2000) examine mixed-strategy Nash equilibria, and show that each beneficiary's equilibrium probability of participation diminishes to zero as the number of the players increases. Thus, the likelihood of the efficient provision of the public good is extremely low when the number of beneficiaries is large.[20] Intuitively, in their game, the probability of being *pivotal* to the public good provision affects the participation probability of each beneficiary.[21]

Note that in our analysis, the probability of being pivotal is irrelevant. As discussed in Section 5, in our model, the supplier has an incentive to set the level of the public good such that all consumers are pivotal to the provision of the public good. This is the same, even when the number of consumers is very large. Hence, Proposition 3 is derived from a different reason to that of Dixit and Olson (2000). Although our model and the measurement of the likelihood of efficiency are different to those of Dixit and Olson (2000), we share a similar implication: as the number of consumers (beneficiaries) increases, the likelihood of achieving efficiency in equilibrium decreases.

6.2 Equilibrium level of the public good when the supplier's bargaining power is sufficiently strong

Theorem 1 proves that if $\beta > \beta(g(n), n)$, there is no equilibrium that supports the efficient provision of the public good. We discuss which of underprovision and overprovision of the public good causes inefficiency in public good provision.

Consider an example in which $v(g) = \sqrt{g}$ and $c(g) = g$.[22] Then,

$$g(m) = \left(\frac{m}{2}\right)^2, \ \underline{g}^m = (\beta(m-1))^2, \ \beta(g(m), m) = \frac{m}{2(m-1)}, \text{ and } \beta(g(m), m+1) = \frac{1}{2}.$$

Note that $\beta(g(n), n) = n/(2(n-1))$. In the Appendix, we show that there is an equilibrium

[20] A similar implication can be obtained from the results of the voluntary participation game for a public good mechanism (e.g., Saijo and Yamato, 1999; Shinohara, 2009; Furusawa and Konishi, 2011; Konishi and Shinohara, 2014).

[21] The meaning of pivotal here is the same as that in our analysis: without a pivotal beneficiary, the public good is not provided.

[22] The linear cost function violates Assumption (1.1). However, note that Theorem 1 holds whenever solutions of the model are interior. See footnote 11.

at which $\underline{g}^n$ units of the public good are supplied. Since $g(n) < \underline{g}^n$, the supplier produces the public good over the efficient level at this equilibrium. Recall that in the paragraphs after Theorem 1, we mentioned that the supplier has an incentive to increase the level of the public good to receive transfers from all consumers. This example presents a case in which this incentive leads to the overprovision of the public good in equilibrium when the supplier's bargaining power is strong.

Finally, we note that the possibility cannot be denied that the supplier with strong bargaining power underprovides the public good under other benefit and cost functions.[23] Thus, the inefficiency in public good provision is due to the overprovision of the public good in some cases and the underprovision of the public good in other cases.

Remark 2 We discuss the relation between the pre-commitment and the internalization of consumers' preferences as a result of simultaneous bilateral bargaining. The supplier receives transfers from all consumers if its bargaining power is sufficiently weak (see the proof of Theorem 1). It may also receive transfers from all consumers if its bargaining power is strong, as shown in the example in this section. In general, when the supplier receives transfers from all consumers, consumers' benefits from the public good are embedded in the supplier's payoff (see (8)). In this sense, the internalization succeeds when the supplier can pre-commit to the production level of the public good. This property may induce the supplier to provide the public good efficiently. However, the supplier with sufficiently strong bargaining power provides the public good inefficiently. Thus, while the pre-commitment enhances the internalization of consumers' preferences through the bargaining, this does not necessarily lead to the efficient provision of a public good. Whether the opportunity of the pre-commitment successfully yields efficient provision depends on the supplier's bargaining power.

[23] The formal analysis of the case in which the supplier has strong bargaining power is available upon request.

7 Extensions

7.1 Other bargaining models

We briefly discuss how the main result changes if we consider other kinds of bargaining in the second stage of the model in Section 5.[24]

Sequential bargaining We consider a model in which the supplier bilaterally, but sequentially negotiates with each consumer.[25] Formally, we replace the second-stage bargaining game of the model in Section 5 with the following sequential bilateral bargaining game: the supplier first negotiates with consumer 1, then negotiates with consumer 2 after the bilateral bargaining with consumer 1, ..., and finally, bilaterally negotiates with consumer n after the bilateral bargaining sessions with the other consumers. Each bilateral bargaining session is assumed to be Nash bargaining. The supplier has only one bilateral negotiation with each of the n consumers. We can show that the equilibrium transfer attained through this sequential bargaining is the same as that in (10). Hence, a result similar to Theorem 1 can be obtained under sequential bilateral bargaining.

Multilateral bargaining based on cooperative games Some solutions of the cooperative games support the outcome through simultaneous bilateral bargaining in the model in Section 5, while others do not. We consider a cooperative game $(\{s\} \cup N, w^g)$, where $w^g : 2^{\{s\} \cup N} \to \mathbb{R}_+$ is the characteristic function, such that for any nonempty subset $\mathcal{C} \subseteq N$, $w^g(\{\emptyset\}) = w^g(\{s\}) = w^g(\mathcal{C}) = 0$; $w^g(\{s\} \cup \mathcal{C}) = \max\{|\mathcal{C}|v(g) - c(g), 0\}$, where $|\mathcal{C}|$ represents the cardinality of $\mathcal{C}$. First, we confirm that for each level of the public good in the first stage $g \geq 0$, the payoffs attained at the simultaneous bilateral bargaining are supported as one of the core elements. Thus, if we focus on this core element, then we obtain the same result as that in Theorem 1.

Second, we consider other solutions of the cooperative game. Since several famous

[24] The formal analysis in this section is available upon request.

[25] A sequential bargaining protocol based on the Nash bargaining solution is analyzed by Valencia-Toledo and Vidal-Puga (2017).

solutions, such as the Shapley value, kernel, and nucleolus, satisfy the *equal-treatment property* (ETP), we focus on the solution with the ETP.[26] Given the solution, it is always supported in an equilibrium that the supplier chooses $g(n)$ in the first stage, which is different from the main result of this study. Which solution is appropriate in the analysis of pure public good provision seems to depend on real-world situations, which we would like to analyze. As discussed in the Introduction, in some cases of environmental externalities, negotiations are conducted bilaterally. Those cases should not be captured by solutions based on multilateral negotiations.

7.2 Heterogeneous consumers

We prove that it is partially correct that the supplier produces the public good inefficiently if its bargaining power is sufficiently strong, even when consumers are heterogeneous. We consider the case in which there are two consumers ($N = \{1, 2\}$) and they have different benefit functions and bargaining power to the supplier. The payoff to consumer $i \in N$ is $v_i(g) - T_i$, where $v_i : \mathbb{R}_+ \to \mathbb{R}_+$ is consumer i's benefit function from the public good that satisfies $v_i(0) = 0$, $v_i' > 0$, $v_i'' \leq 0$, and is twice continuously differentiable. Let $\beta_i \in [0, 1]$ be the supplier's bargaining power with consumer $i \in N$. The cost function $c(g)$ satisfies $c(0) = 0$, $c' > 0$, $c'' > 0$, and is twice continuously differentiable. The timing of the game is the same as that for the basic model. In this extended model, the Pareto-efficient allocation is not achieved in equilibrium if the supplier's bargaining power over *some* (not *every*) consumer is sufficiently strong. In this sense, the main result in the case of identical consumers remains partially true.

8 Concluding remarks

We examine public good provision through simultaneous bilateral negotiations between a supplier of a public good and the consumers of the good. Our question is whether a pre-negotiation commitment for the public good plays an effective role in achieving the

[26]See Peleg and Sudhölter (2007) for the definitions of those solutions.

efficient provision of the public good. First, we show that under simultaneous bilateral negotiations, the efficient provision of the public good is never accomplished without the pre-commitment (Proposition 1). Second, we show that if the supplier has an opportunity to pre-commit to the production level of the public good, then under some conditions, the supplier produces the public good efficiently in equilibrium if and only if its bargaining power is sufficiently weak (Theorem 1). In addition, we investigate the likelihood of the efficient public good provision in equilibrium under reasonable parametric benefit and cost functions, and show that the equilibrium likelihood of the efficient provision of the public good diminishes as the number of consumers increases. However, in some case, the equilibrium likelihood is sufficiently high, even if the number of consumers is very large (Section 6.1). A supplier with strong bargaining power, and who obtains a large surplus, provides the public good inefficiently. In some cases, this inefficiency stems from the excessive provision of the public good (Section 6.2).

Our contribution is to clarify the relation between the effectiveness of the pre-commitment and the supplier's strength in bargaining. This provides some positive and negative aspects of the commitment prior to negotiations: while the pre-commitment on the level of the public good plays a role in enhancing the internalization of beneficiaries' preferences, the public good is not necessarily provided efficiently, even under this enhanced internalization. Our results seem to have some implication to real-world cases of the resolution of environmental externalities. In real-world bargaining, the bargaining power would be determined by the relative positions of a polluter and pollutees. If the polluter is a big company, then it may naturally be stronger than the pollutees. In this case, the polluter abates pollution with a greater pre-commitment than without it, but the reduction level of the pollution with pre-commitment may not be an efficient level. Note that the increase in the production level of the public good through commitment is not necessarily desirable for the whole economy.

Appendix

Preliminaries

Before proving Theorem 1, we provide some preliminaries.

By $\{g(m)\} = \arg\max_{g \geq 0}\ mv(g) - c(g)$, we obtain several observations, as follows.

1. By (8), if the number of contributors is fixed at m, then $\pi_S^m(g)$ is maximized at $g = g(m)$. This implies that $\pi_S^m(g)$ is increasing in $g \in [0, g(m))$ and decreasing in $g \in [g(m), \infty)$.

2. $g(m) < g(m+1)$ for each $m \in \{1, \ldots, n-1\}$.

3. $g(n)$ is the (Pareto) efficient level of the public good because $g(n)$ maximizes $nv(g) - c(g)$.

Corollary 2 shows that $\pi_S^m(g(m))$ is increasing in m.

Corollary 2 For each $m \in \{1, \ldots, n-1\}$, $\pi_S^m(g(m)) < \pi_S^{m+1}(g(m+1))$.

Proof. By Lemma 3, $\pi_S^m(g(m)) \leq \pi_S^{m+1}(g(m))$. From the above-mentioned first and second observations, $\pi_S^{m+1}(g(m)) < \pi_S^{m+1}(g(m+1))$. ■

Lemma 4 shows the relative relationship between $\beta(g(m), m)$ and $\beta(g(m), m+1)$.

Lemma 4

(4.a) $\lim_{m \to 1} \beta\left(g(m), m\right) = \infty$ and $\lim_{m \to 1} \beta\left(g(m), m+1\right) = \dfrac{c(g(1))}{v(g(1))}$.

(4.b) For each $m > 1$, $\beta\left(g(m), m+1\right) < \beta\left(g(m), m\right)$.

Proof. (4.a) By (14), as $m \to 1$,

$$\beta\left(g(m), m\right) = \frac{c(g(m))}{v(g(m))(m-1)} \to \infty \quad \text{and} \quad \beta\left(g(m), m+1\right) = \frac{c(g(m))}{v(g(m))m} \to \frac{c(g(1))}{v(g(1))}.$$

(4.b) follows immediately from (14). ■

Note that while $g(m)$ is dependent on m, but independent of β, $\underline{g}^m$ and $\underline{g}^{m+1}$ depend on m and β. Lemma 5 shows how $g(m)$ and $\underline{g}^m$ (or $\underline{g}^{m+1}$) are related according to the values of m and β.

Lemma 5 Let $\beta \in (0,1]$ and $m \in (1,n]$. Then,

$$\underline{g}^{m+1} \leq g(m) \text{ if } \beta \leq \beta\left(g(m), m+1\right), \tag{16}$$

$$\underline{g}^{m} \leq g(m) < \underline{g}^{m+1} \text{ if } \beta\left(g(m), m+1\right) < \beta \leq \beta\left(g(m), m\right), \text{ and} \tag{17}$$

$$g(m) < \underline{g}^{m} \text{ if } \beta > \beta\left(g(m), m\right). \tag{18}$$

Proof. $\underline{g}^m$ satisfies

$$\beta = \frac{c(\underline{g}^m)}{v(\underline{g}^m)(m-1)}.$$

From (14), if $\beta \leq \beta\left(g(m), m\right)$, then

$$\frac{c(\underline{g}^m)}{v(\underline{g}^m)(m-1)} = \beta \leq \beta\left(g(m), m\right) = \frac{c(g(m))}{v(g(m))(m-1)}.$$

We obtain $\underline{g}^m \leq g(m)$ because $c(g)/v(g)$ is increasing in g. Conversely, if $\underline{g}^m \leq g(m)$, then

$$\frac{c(\underline{g}^m)}{v(\underline{g}^m)(m-1)} \leq \frac{c(g(m))}{v(g(m))(m-1)}$$

because $c(g)/v(g)$ is increasing in g. Then, by (14), $\beta \leq \beta\left(g(m), m\right)$. Thus, $\beta \leq \beta\left(g(m), m\right)$ if and only if $\underline{g}^m \leq g(m)$. Similarly, we obtain $\beta \leq \beta\left(g(m), m+1\right)$ if and only if $\underline{g}^{m+1} \leq g(m)$. Finally, we obtain (16)–(18). ■

Lemma 6 shows what level of the public good g maximizes $\pi_S^m(g)$ under the constraint $g \in [\underline{g}^m, \underline{g}^{m+1})$, for each m and β.

Lemma 6 Let $m \in \{1, \ldots, n\}$ and $\beta \in (0,1]$.

(6.a) If $\beta \leq \beta\left(g(m), m+1\right)$, then $\underline{g}^{m+1} \leq g(m)$ (see (16)); hence, $\pi_S^m(g)$ is increasing within the interval $[\underline{g}^m, \underline{g}^{m+1})$. Thus, there is no maximizer when restricting to this interval.

(6.b) If $\beta\left(g(m), m+1\right) < \beta \leq \beta\left(g(m), m\right)$, then $g(m) \in [\underline{g}^m, \underline{g}^{m+1})$ (see (17)); hence, within the interval $[\underline{g}^m, \underline{g}^{m+1})$, $\pi_S^m(g)$ is maximized at $g = g(m)$.

(6.c) If $\beta\left(g(m), m\right) < \beta$, then $g(m) < \underline{g}^m$ (see (18)); hence, within the interval $[\underline{g}^m, \underline{g}^{m+1})$, $\pi_S^m(g)$ is maximized at $g = \underline{g}^m$.

Proof of Theorem 1

($\Leftarrow$) Suppose that $0 \leq \beta \leq \beta\left(g(n), n\right)$. If $\beta = 0$, the supplier's payoff is zero, irrespective of its choice of g. Thus, $g(n)$ is one of the optimal levels for the supplier.

Hereafter, we restrict our focus to the case in which $0 < \beta \leq \beta\left(g(n), n\right)$. Given that the number of contributors is $m(g)$ in (13), we investigate the public good level that maximizes the supplier's payoff. From $0 < \beta \leq \beta\left(g(n), n\right)$ and (17), we obtain

$$\underline{g}^n \leq g(n) < \overline{g}^n. \tag{19}$$

We partition the set $\{1, \ldots, n-1\}$ (the number of contributors less than n) into three subsets as follows:

$$\begin{aligned}
\mathcal{N}_1 &\equiv \left\{m \in \{1, \ldots, n-1\} |\ \underline{g}^{m+1} \leq g(m)\right\}, \\
\mathcal{N}_2 &\equiv \left\{m \in \{1, \ldots, n-1\} |\ \underline{g}^{m} \leq g(m) < \underline{g}^{m+1}\right\}, \text{ and} \\
\mathcal{N}_3 &\equiv \left\{m \in \{1, \ldots, n-1\} |\ g(m) < \underline{g}^{m}\right\}.
\end{aligned}$$

We examine which number of the contributors maximizes the supplier's payoff for each of the three subsets. First, consider the numbers of contributors in $\mathcal{N}_1$. Let $m \in \mathcal{N}_1$. Note that m is the number of contributors if and only if $g \in [\underline{g}^m, \underline{g}^{m+1})$. Because $\underline{g}^{m+1} \leq g(m)$, $\pi_S^m(g)$ is increasing in $[\underline{g}^m, \underline{g}^{m+1})$. Since $\pi_S^m(g)$ is continuous at every g, we can define

$\pi_S^m(\underline{g}^{m+1})$ and

$$\lim_{g \uparrow \underline{g}^{m+1}} \pi_S^m(g) = \pi_S^m(\underline{g}^{m+1}) = \frac{\beta\left(mv(\underline{g}^{m+1}) - c(\underline{g}^{m+1})\right)}{\beta + (1-\beta)m}.$$

Thus, $\sup_{g \in [\underline{g}^m, \underline{g}^{m+1})} \pi_S^m(g) = \pi_S^m(\underline{g}^{m+1})$. By Lemma 3, $\pi_S^m(\underline{g}^{m+1}) < \pi_S^n(\underline{g}^{m+1})$. Because $\pi_S^n(g)$ is increasing if $g < g(n)$ and $\underline{g}^{m+1} \leq \underline{g}^n \leq g(n)$, we further obtain $\pi_S^n(\underline{g}^{m+1}) \leq \pi_S^n(g(n))$. Therefore,

$$\pi_S^m(\underline{g}^{m+1}) < \pi_S^n(g(n)) \text{ for each } m \in \mathcal{N}_1. \tag{20}$$

Second, we consider the numbers of contributors in $\mathcal{N}_2$. Let $m \in \mathcal{N}_2$. Since $\underline{g}^m \leq g(m) < \underline{g}^{m+1}$, $\pi_S^m(g)$ is maximized at $g = g(m)$. Then, by Corollary 2,

$$\pi_S^m(g(m)) < \pi_S^n(g(n)) \text{ for each } m \in \mathcal{N}_2. \tag{21}$$

Third, consider the numbers of contributors in $\mathcal{N}_3$. Let $m \in \mathcal{N}_3$. Given that the number of contributors is m, the supplier's payoff $\pi_S^m(g)$ is maximized at $g = \underline{g}^m$ because $g(m) < \underline{g}^m$. By Lemma 3, $\pi_S^m(\underline{g}^m) < \pi_S^n(\underline{g}^m)$. Since $\pi_S^n(g)$ is increasing if $g < g(n)$ and $\underline{g}^m < \underline{g}^n \leq g(n)$, we further obtain $\pi_S^n(\underline{g}^m) \leq \pi_S^n(g(n))$. Therefore,

$$\pi_S^m(\underline{g}^m) < \pi_S^n(g(n)) \text{ for each } m \in \mathcal{N}_3. \tag{22}$$

Finally, we prove that the supplier's payoff is maximized at $g = g(n)$. Since $g(n) \in [\underline{g}^n, \overline{g}^n)$ from (19), then given that the number of contributors is n, the suppliers' payoff is maximized at $g = g(n)$. Thus, (20) and (22) imply that the supplier's payoff when the number of contributors belongs to $\mathcal{N}_1 \cup \mathcal{N}_3$ is less than $\pi_S^n(g(n))$. (21) implies that the supplier's payoff when the number of contributors belongs to $\mathcal{N}_2$ is less than $\pi_S^n(g(n))$. In conclusion, the supplier chooses $g(n)$ in equilibrium.

($\Rightarrow$) To the contrary, suppose that $\beta > \beta\,(g(n), n)$ and an equilibrium exists at which the supplier chooses $g(n)$. Suppose that on the path of this equilibrium, the supplier chooses $g(n)$ in the first stage, the number of contributors is $\mu \in \{1, \ldots, n\}$ in the second stage, and the project is approved in the third stage. The supplier obtains $\pi_S^{\mu}(g(n))$ in this equilibrium.

Note that the supplier chooses a level of the public good in the set $[\underline{g}^{\mu}, \overline{g}^{\mu}]$ when μ is the number of contributors. At the same time, by (18), we obtain $g(n) < \underline{g}^n$ from $\beta > \beta(g(n), n)$. Thus, in this equilibrium, the number of contributors μ is less than n: $\mu < n$.

We need to consider two possibilities: $\underline{g}^{\mu} < g(n)$ and $\underline{g}^{\mu} = g(n)$. Suppose first that $\underline{g}^{\mu} < g(n)$. We obtain $g(\mu) < g(n)$ because $\mu < n$. Then, because $\pi_S^{\mu}(g)$ is decreasing in g if $g > g(\mu)$, if the supplier sets a level of the public good a bit lower than $g(n)$ in the interval $[\underline{g}^{\mu}, \overline{g}^{\mu}]$, the supplier's payoff increases (the supplier can choose such a level because of $\underline{g}^{\mu} < g(n)$). Second, suppose that $\underline{g}^{\mu} = g(n)$. By Condition 1 and $\mu < n$, $\pi_S^{\mu}(\underline{g}^{\mu}) = \pi_S^{\mu}(g(n)) < \pi_S^{n}(\underline{g}^{n})$. In any case, the supplier does not choose $g(n)$ in the equilibrium, which is a contradiction. ■

Remark 3 Proof of the necessity above does not depend on the equilibrium selection in the second and third stages presented in Sections 5.1 and 5.2. In addition to the equilibrium presented in Section 5.1, in the third stage, there is an equilibrium at which the supplier chooses execution if and only if $\sum_{j \in N} T_j \geq c(g)$. Depending on which third-stage equilibrium we consider, the condition under which μ consumers transfer to the supplier is $g(n) \in [\underline{g}^{\mu}, \overline{g}^{\mu})$ or $g(n) \in (\underline{g}^{\mu}, \overline{g}^{\mu}]$; that is, in any equilibrium, if μ consumers transfer to the supplier, then $g(n) \in [\underline{g}^{\mu}, \overline{g}^{\mu}]$. In the proof, we consider this interval.

Analysis in Section 6.2

Suppose that the second and third-stage equilibria are the same as those in Sections 5.1 and 5.2. We now prove that the supplier's payoff is maximized when it chooses $\underline{g}^n$. Suppose that $\beta > \beta(g(n), n) = 1/(2(n-1)) + 1/2$.

Note that $\beta(g(m), m) = 1/(2(m-1)) + 1/2$ is decreasing in m. Since $\beta(g(m), m+1) = 1/2 < \beta(g(m), m) = 1/(2(m-1)) + 1/2$, it follows that $g(m) < \underline{g}^{m+1}$ for each m (see

Lemma 5). We obtain that $\beta(g(m), m) = \beta$ at $m \equiv 2\beta/(2\beta - 1)$. Let $\tilde{m} \equiv 2\beta/(2\beta - 1)$. Then, $\underline{g}^m \leq g(m)$ if $m \leq \tilde{m}$ and $g(m) \leq \underline{g}^m$ if $m \geq \tilde{m}$ (see Lemma 5). In summary, $\underline{g}^m \leq g(m) < \underline{g}^{m+1}$ if $m \leq \tilde{m}$ and $g(m) \leq \underline{g}^m < \underline{g}^{m+1}$ if $m \geq \tilde{m}$.

We also obtain

$$\pi_S^m(g(m)) = \frac{\beta m^2}{4(1 + (1-\beta)(m-1))} \quad \text{and} \quad \pi_S^m(\underline{g}^m) = \beta^2(m-1).$$

Since $\pi_S^m(g(m))$ is increasing in m, we obtain $\pi_S^m(g(m)) < \pi_S^{\tilde{m}}(g(\tilde{m}))$ for each $m < \tilde{m}$. By (6.b) of Lemma 6, in $[0, \underline{g}^{\tilde{m}})$, the supplier's payoff is maximized at $g = g(\tilde{m})$, and the maximized payoff is

$$\pi_S^{\tilde{m}}(g(\tilde{m})) = \frac{\beta^2}{2\beta - 1}.$$

Since $\pi_S^m(\underline{g}^m)$ is increasing in m, we obtain $\pi_S^m(\underline{g}^m) < \pi_S^n(\underline{g}^n)$ for each $m \in [\tilde{m}, n)$. From (6.c) of Lemma 6, in $[\underline{g}^{\tilde{m}}, \infty)$, it is maximized at $g = \underline{g}^n$ and the maximized payoff is $\pi_S^n(\underline{g}^n) = \beta^2(n-1)$. The difference between those payoffs is

$$\pi_S^n(\underline{g}^n) - \pi_S^{\tilde{m}}(g(\tilde{m})) = \frac{\beta^2((2n-1)\beta - n)}{2\beta - 1} > 0 \text{ since } \beta > \frac{n}{2(n-1)}.$$

Since $\pi_S^m(g(m))$ is increasing in m, we also obtain $\pi_S^n(\underline{g}^n) - \pi_S^m(g(m)) > 0$ for all integers m, such that $m \leq \tilde{m}$. Therefore, in this numerical example, the supplier oversupplies the public good when its bargaining power is sufficiently strong.

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Online Appendix for
"Pre-negotiation commitment and internalization in public good provision through bilateral negotiations"

Noriaki Matsushima and Ryusuke Shinohara

Symmetric equilibria in Section 4

We briefly discuss the possibility of the (non)existence of symmetric equilibria in the model in Section 4. We focus on an equilibrium in which $g_i^* = g(1)/n$ for each $i \in N$, and every consumer $i \in N$ is pivotal: $\sum_{j \neq i} T_j^* \leq c(\sum_{j \neq i} g_j^*)$, for each $i \in N$. In this case, for each $i \in N$,

$$T_i^* = v(\sum_{j \in N} g_j^*) - (1-\beta)\left(v(\sum_{j \in N} g_j^*) - c(\sum_{j \in N} g_j^*) + \sum_{j \neq i} T_j^* \right).$$

Solving these simultaneous equations yields

$$T_i^* = \frac{\beta v(\sum_{j \in N} g_j^*) + (1-\beta) c(\sum_{j \in N} g_j^*)}{\beta + (1-\beta)n} \text{ for each } i \in N.$$

Using this transfer value, $\sum_{i \in N} g_i^* = g(1)$, and $c(\sum_{j \neq i} g_j^*) = (n-1)g(1)/n$, we obtain that $\sum_{j \neq i} T_j^* \leq c(\sum_{j \neq i} g_j^*)$ if and only if

$$\beta \leq \frac{c(g(1))(1-n) + nc\left(\frac{n-1}{n} g(1)\right)}{(n-1)\left[v(g(1)) - (n-1)\left(c(g(1)) - c\left(\frac{n-1}{n} g(1)\right)\right)\right]}.$$

Note that the right-hand side of the previous inequalities is not necessarily positive. If it is positive, then the symmetric equilibrium exists only if the supplier's bargaining power is sufficiently small. Otherwise, the equilibrium does not exists for any β.

Equilibrium level of the public good when β is sufficiently high

We focus on the analysis of the first stage, given the second- and third-stage equilibrium in Subsections 5.1 and 5.2. Assuming Condition 1, we examine the case in which the supplier determines the level of the public good in the first stage.

Suppose that $\beta > \beta\,(g(n), n)$. Then, by Lemma 5, $g(n) < \underline{g}^n$. Similarly to the proof of Theorem 1, we make a partition of the number of contributors less than n, such that

$$\mathcal{N}_1 = \{m \in \{1, \ldots, n-1\} |\ \underline{g}^{m+1} \leq g(m)\},$$
$$\mathcal{N}_2 = \{m \in \{1, \ldots, n-1\} |\ \underline{g}^{m} \leq g(m) < \underline{g}^{m+1}\}, \text{ and}$$
$$\mathcal{N}_3 = \{m \in \{1, \ldots, n-1\} |\ g(m) < \underline{g}^{m}\}.$$

By the same calculation in the proof of Theorem 1,[27] we obtain that for each $m \in \mathcal{N}_1$ and each $g \in [\underline{g}^m, \underline{g}^{m+1})$,

$$\pi_S^m(g) < \pi_S^m(\underline{g}^{m+1}) < \pi_S^{m+1}(\underline{g}^{m+1}) \leq \pi_S^n(\underline{g}^n).$$

For each $m \in \mathcal{N}_2$ and each $g \in [\underline{g}^m, \underline{g}^{m+1})$,

$$\pi_S^m(g) \leq \pi_S^m(g(m)) \leq \pi_S^{m^*}(g(m^*)),$$

in which m^* is the maximal integer in $\mathcal{N}_2$. For each $m \in \mathcal{N}_3$ and each $g \in [\underline{g}^m, \underline{g}^{m+1})$,

$$\pi_S^m(g) \leq \pi_S^m(\underline{g}^m) < \pi_S^n(\underline{g}^n).$$

In summary, for each $m \in \mathcal{N}_1 \cup \mathcal{N}_3$ and each $g \in [\underline{g}^m, \underline{g}^{m+1})$,

$$\pi_S^m(g) < \pi_S^m(\underline{g}^n).$$

For each $m \in \mathcal{N}_2$ and each $g \in [\underline{g}^m, \underline{g}^{m+1})$,

$$\pi_S^m(g) \leq \pi_S^{m^*}(g(m^*)).$$

[27] See also Lemma 6.

We obtain that $g(m^*) < g(n)$ by $m^* < n$ and that $g(n) < \underline{g}^n$. However, under our general setting, we cannot determine the sign of $\pi_S^{m^*}(g(m^*)) - \pi_S^n(\underline{g}^n)$. Therefore, the equilibrium level of the public good is $g(m^*)$ if $\pi_S^{m^*}(g(m^*)) > \pi_S^n(\underline{g}^n)$ and is $\underline{g}^n$ otherwise. In conclusion, in any subcase, the supplier chooses an inefficient level of the public goods, that is, a level higher or lower than the efficient level.

Simultaneous bilateral bargaining and the core in Section 7.1

We show that the payoffs attained with simultaneous bilateral bargaining belong to the core of some appropriate cooperative game.

Let $\mathcal{N} = \{s\} \cup N$, where s is the supplier and $N = \{1, \ldots, n\}$ is the set of consumers, with $n \geq 2$. For each $g > 0$, such that $nv(g) > c(g)$, define the characteristic function $w^g : 2^{\mathcal{N}} \to \mathbb{R}_+$ as follows: for any nonempty subset $\mathcal{C} \subseteq N$, $w^g(\{\emptyset\}) = w^g(\{s\}) = w^g(\mathcal{C}) = 0$; $w^g(\{s\} \cup \mathcal{C}) = \max\{|\mathcal{C}|v(g) - c(g), 0\}$.[28] By this definition, the cooperation of the supplier and consumers is necessary for a positive surplus. If $\mathcal{C}$ is sufficiently large, $|\mathcal{C}|v(g) > c(g)$ holds; hence, the supplier provides g (decided in the first stage) and the consumers in $\mathcal{C}$ pay the fee to the supplier. *The supplier is assumed to commit the level of the public good decided in the first stage. In the negotiation in coalitions, only the surplus division is considered.* A payoff profile $u = (u_i)_{i \in N}$ belongs to the *core* of $(\mathcal{N}, w^g)$ if $\sum_{i \in C} u_i \geq w^g(C)$ for each $C \subseteq \mathcal{N}$ and $\sum_{i \in \mathcal{N}} u_i = w^g(\mathcal{N})$.

Let $\beta \in [0, 1]$ be the supplier's bargaining power. Let $g > 0$ be a level of the public good such that $nv(g) - c(g) > 0$. Let m be a number of pivotal consumers. Note then that $mv(g) > c(g)$ holds by the pivotal condition. Let $M \subseteq N$ be a set of contributors. Let u^s, u^c, and u^f denote the payoffs to the supplier, the contributor, and the free rider

[28] $|\mathcal{C}|$ represents the cardinality of $\mathcal{C}$.

through the simultaneous bilateral bargaining in the second stage, respectively. Then,

$$\begin{aligned} u^s &= \pi_S^m(g) = \frac{\beta(mv(g) - c(g))}{\beta + (1-\beta)m} \geq 0, \\ u^c &= v(g) - T_i^m = \frac{(1-\beta)(mv(g) - c(g))}{\beta + (1-\beta)m} \geq 0, \text{ and} \\ u^f &= v(g) > 0. \end{aligned}$$

Proposition A1 A payoff profile $(u_i)_{i \in \mathcal{N}}$, such that $u_s = u^s$, $u_i = u^c$ for any $i \in M$, and $u_i = u^f$ for any $i \in N \backslash M$ belongs to the core of $(\mathcal{N}, w^g)$.

Proof. Trivially, $\sum_{i \in C} u_i \geq w^g(C)$ for each $C \subseteq \mathcal{N}$, such that $w^g(C) = 0$. We need to consider the coalition $\{s\} \cup \mathcal{C}$, such that $\mathcal{C} \subseteq N$ and $w^g(\{s\} \cup \mathcal{C}) > 0$; that is, $|\mathcal{C}|v(g) > c(g)$. Let $\mathcal{C}^c \subseteq \mathcal{C}$ be the set of contributors in $\mathcal{C}$ and let $\mathcal{C}^f \subseteq \mathcal{C}$ be the set of free riders in $\mathcal{C}$. We obtain

$$\sum_{i \in \{s\} \cup \mathcal{C}} u_i = \frac{\beta + (1-\beta)|\mathcal{C}^c|}{\beta + (1-\beta)m}(mv(g) - c(g)) + |\mathcal{C}^f|v(g).$$

Then, we obtain

$$\frac{\beta + (1-\beta)|\mathcal{C}^c|}{\beta + (1-\beta)m}(mv(g) - c(g)) - (|\mathcal{C}^c|v(g) - c(g)) = \frac{(m - |\mathcal{C}^c|)}{\beta + (1-\beta)m}(\beta v(g) + (1-\beta)c(g)) \geq 0.$$

Thus, $\sum_{i \in \{s\} \cup \mathcal{C}} u_i \geq |\mathcal{C}^c|v(g) - c(g) + |\mathcal{C}^f|v(g) = w^g(\{s\} \cup \mathcal{C})$. In conclusion, $(u_i)_{i \in \mathcal{N}}$ belongs to the core of $(\mathcal{N}, w^g)$. ■

Remark A1 We can prove that the core of $(\mathcal{N}, w^g)$ coincides with the set of payoffs $\{(u_i)_{i \in \mathcal{N}} | \ u_i \geq 0 \text{ for each } i \in \mathcal{N} \text{ and } \sum_{i \in \mathcal{N}} u_i = nv(g) - c(g)\}$.

Sequential bilateral bargaining in Section 7.1

We replace the second-stage bargaining of the model in Section 5 with the following *sequential* bilateral bargaining model. The supplier first negotiates with consumer 1, second negotiates with consumer 2 after the bilateral bargaining with consumer 1, ..., and finally, negotiates with consumer n bilaterally. The supplier has only one bilateral negotiation with each consumer.

We focus on the third-stage equilibrium at which for each project $(g, (T_j)_{j \in N})$, the supplier executes the project if and only if $\sum_{j \in N} T_j > c(g)$.

Given this third-stage equilibrium, we solve the second-stage sequential bargaining by the backward application of the Nash bargaining solution. Given the outcome $(g, (T_j)_{j \neq n})$, the supplier negotiates with consumer n.

- If $\sum_{j \in N \setminus \{n\}} T_j > c(g)$ or if $\sum_{j \in N \setminus \{n\}} T_j \leq c(g)$ and $v(g) \leq c(g) - \sum_{j \in N \setminus \{n\}} T_j$, then the surplus of the bargaining session of the supplier and consumer n is zero; hence, $T_n = 0$.

- If $\sum_{j \in N \setminus \{n\}} T_j \leq c(g)$ and $v(g) > c(g) - \sum_{j \in N \setminus \{n\}} T_j$, the surplus of this bargaining is positive and consumer n pays to the supplier $T_n = v(g) - (1-\beta)(v(g) + \sum_{j \in N \setminus \{n\}} T_j - c(g)) = \beta v(g) - (1-\beta)(\sum_{j \in N \setminus \{n\}} T_j - c(g))$.

Given this outcome of the Nash bargaining solution, we next investigate bilateral bargaining with consumer $n-1$.

We investigate the bilateral bargaining with consumer $k \in \{1, \ldots, n-1\}$, assuming that we have investigated the bilateral bargaining with consumer n to consumer $k+1$, by the backward application of the Nash bargaining solution. Let $(T_j)_{j=k+1}^{n}$ be the Nash bargaining outcome of the bargaining with consumers $k+1$ to n. Before reaching the bargaining with consumer k, the supplier negotiates with consumer 1 to $k-1$. Let $(T_j)_{j=1}^{k-1}$ be the outcome of the bargaining before the bargaining with consumer k. Given $(T_j)_{j=1}^{k-1}$, the supplier and consumer k negotiate, anticipating that $(T_j)_{j=k+1}^{n}$ is obtained in the subsequent negotiations.

- If $\sum_{j=1}^{k-1} T_j + \sum_{j=k+1}^{n} T_j > c(g)$ or if $\sum_{j=1}^{k-1} T_j + \sum_{j=k+1}^{n} T_j \leq c(g)$ and $v(g) \leq c(g) - (\sum_{j=1}^{k-1} T_j + \sum_{j=k+1}^{n} T_j)$, then the surplus of the bargaining session of the supplier and consumer k is zero; hence, $T_k = 0$.

- If $\sum_{j=1}^{k-1} T_j + \sum_{j=k+1}^{n} T_j \leq c(g)$ and $v(g) > c(g) - \sum_{j=1}^{k-1} T_j - \sum_{j=k+1}^{n} T_j$, the surplus of this bargaining is positive and consumer k pays to the supplier $T_k = v(g) - (1-\beta)(v(g) + \sum_{j \neq k} T_j - c(g)) = \beta v(g) - (1-\beta)(\sum_{j \neq k} T_j - c(g))$.

Given this Nash bargaining outcome, we return to the bilateral bargaining with consumer $k-1$. Repeating a similar procedure until reaching the bargaining with consumer 1, we finally obtain the following results. For each $k \in N$, $T_k = \beta v(g) - (1-\beta)(\sum_{j \neq k} T_j - c(g))$ if

$$\sum_{j \in N \setminus \{k\}} T_j \leq c(g) \text{ and } v(g) > c(g) - \sum_{j \in N \setminus \{k\}} T_j \tag{23}$$

and $T_k = 0$ if

$$\sum_{j \in N \setminus \{k\}} T_j > c(g) \quad \text{or} \quad \left[\sum_{j \in N \setminus \{k\}} T_j \leq c(g) \text{ and } v(g) \leq c(g) - \sum_{j \in N \setminus \{k\}} T_j \right]. \tag{24}$$

Conditions (23) and (24) are equivalent to the pivotal condition and the non-pivotal condition, respectively. For each $k \in N$, if (23), k is pivotal to the supplier's approval of a project and $T_k = \beta v(g) - (1-\beta)(\sum_{j \neq k} T_j - c(g))$; if (24), then k is non-pivotal and $T_k = 0$. This is the same as the outcome of the simultaneous bilateral bargaining. Therefore, even under the sequential bilateral negotiations, Theorem 1 still holds.

Multilateral bargaining based on cooperative games in Section 7.1

Since several famous solutions, such as the Shapley value, kernel, and nucleolus, satisfy the *equal treatment property* (ETP), we focus on the solution with the ETP. Since the consumers have identical benefit functions in the basic model, all distinct consumers i and j are *interchangeable* in the sense that, for each coalition $D \subseteq \{s\} \cup N$ that contains neither i nor j, $w^g(D \cup \{i\}) - w^g(D) = w^g(D \cup \{j\}) - w^g(D)$. If the solution satisfies the ETP, then it assigns the same payoff to all interchangeable players (i.e., all consumers in our model). Formally, let $(u_i)_{i \in \{s\} \cup N} \in \mathbb{R}_+^{n+1}$ be payoffs that are assigned by the solution with the ETP and that satisfy $u_s + \sum_{i \in N} u_i = w^g(\{s\} \cup N)$. Then, a surplus-sharing ratio exists that is common to all consumers, $r_c \in [0, 1/n]$, such that $u_i = r_c w^g(\{s\} \cup N)$ for each $i \in N$ and $u_s = (1 - nr_c)w^g(\{s\} \cup N)$. If g is the level of the public good in the first stage, then the supplier's payoff, $\pi_S^{\mathrm{ETP}}(g)$, and consumer i's, $\pi_i^{\mathrm{ETP}}(g)$, are $\pi_S^{\mathrm{ETP}}(g) = \max\{(1 - nr_c)(nv(g) - c(g)), 0\}$ and $\pi_i^{\mathrm{ETP}}(g) = \max\{r_c(nv(g) - c(g)), 0\}$, respectively. Given the solution, it is always supported at an equilibrium that the supplier chooses $g(n)$ in the first stage, Therefore, under the solution satisfying the ETP, the public good is always provided efficiently, which is different from the main result of this study. This difference comes from the property that all consumers obtain the same payoff under the solution with the ETP. This means that they all transfer the same amount of money to the supplier and they cannot free ride. On the other hand, the simultaneous bilateral bargaining allows asymmetry of consumers' payoffs and free riding.

What solution is appropriate in the analysis of pure public good provision seems to depend on real-world situations which we would like to analyze. As discussed in Introduction, in some cases with regard to environmental externalities, negotiations are conducted bilaterally. Those cases should not be captured by solutions based on multilateral negotiations. Theoretically, it would be a problem whether the ETP solution is appropriate for the analysis of public good provision. Under the ETP solution, consumers cannot free ride a public good because they pay the same compensation for the provision of a pure public good. However, by the non-excludability of the public good, we may naturally consider that the consumer has an option to free ride. We can easily confirm that, with this option, each consumer is made better off by free riding the public good provided by the others, even under the ETP solution.[29]

[29]Suppose that $(n-1)v(g) > c(g)$ and $i \in N$ is a consumer whose payoff is $u_i^{\mathrm{ETP}} \equiv r_c(nv(g) - c(g))$ under the solution with the ETP. Since $(n-1)v(g) > c(g)$, then $w^g(\{s\} \cup N \backslash \{i\}) > 0$, which means that the coalition $\{s\} \cup N \backslash \{i\}$ produces g units of the public good even if consumer i opts out from $\{s\} \cup N$. Thus, since the public good is pure, consumer i can enjoy the free-riding payoff $v(g)$ even if he/she opts out of N. The free-riding payoff $v(g)$ clearly outperforms u_i^{ETP} because $r_c \leq 1/n$ and $u_i^{\mathrm{ETP}} \leq v(g) - (c(g)/n) < v(g)$.

Heterogeneous consumers in Section 7.2

The analysis is based on the assumption that $v_1(g) < v_2(g)$ for all $g > 0$ and $\beta_1 \leq \beta_2$. Under this assumption, for any level of the public good, consumer 2's benefit from the public good is greater than consumer 1's; the supplier's bargaining power with consumer 2 is not lower than that with consumer 1.

The analysis of the third stage is the same as that of the case of identical consumers. Based on the third-stage equilibrium, consumer $i \in N$ is said to be *pivotal* to (g, T_j) $(i \neq j)$ if $v_i(g) + T_j > c(g) \geq T_j$.

In the second stage, if consumer i is non-pivotal to (g, T_j), then $T_i = 0$. If consumer i is pivotal, then

$$T_i = v_i(g) - (1 - \beta_i)(v_i(g) + T_j - c(g)) = \beta_i v_i(g) - (1 - \beta_i)(T_j - c(g)). \tag{25}$$

If consumer i is pivotal and consumer j is not, then $T_i = \beta_i v_i(g) + (1-\beta_i)c(g)$ and $T_j = 0$. The supplier's payoff is $\pi_S^{\{i\}}(g) \equiv \beta_i(v_i(g) - c(g))$ (The upper-script letter $\{i\}$ means that the supplier receives transfers from consumer i). Consumer i's payoff is $(1-\beta_i)(v_i(g)-c(g))$ and consumer j's payoff is $v_j(g)$.

If consumers 1 and 2 are pivotal, then (25) is satisfied for each $i \in N$; hence,

$$T_i = \frac{\beta_i v_i(g) - (1 - \beta_i)\beta_j(v_j(g) - c(g))}{\beta_i + \beta_j - \beta_i\beta_j}, \tag{26}$$

where $j \in N \backslash \{i\}$.[30] The supplier's payoff is

$$\pi_S^N(g) \equiv T_1 + T_2 - c(g) = \frac{\beta_1\beta_2}{\beta_1 + \beta_2 - \beta_1\beta_2}(v_1(g) + v_2(g) - c(g)),$$

in which the upper-script letter N means that the supplier receives transfers from consumers 1 and 2.

Consumer i is pivotal to (g, T_j) if and only if $v_i(g) + T_j > c(g) \geq T_j$ $(j \in N\backslash\{i\})$. By

[30] We assume in the subsequent analysis that $T_i > 0$ for each $i \in N$ when both consumers are pivotal.

(26), $v_i(g) + T_j > c(g)$ if and only if $v_i(g) + v_j(g) > c(g)$. From (26), $c(g) \geq T_j$ if and only if

$$0 \geq \frac{1}{\beta_i + \beta_j - \beta_i\beta_j}(\beta_j(v_j(g) - c(g)) - (1 - \beta_j)\beta_i v_i(g))$$

if and only if

$$v_j(g) - \frac{(1 - \beta_j)\beta_i}{\beta_j} v_i(g) \leq c(g).$$

Therefore, consumer i is pivotal to (g, T_j) if and only if g satisfies

$$v_j(g) - \frac{(1 - \beta_j)\beta_i}{\beta_j} v_i(g) \leq c(g) < v_i(g) + v_j(g) \quad (j \in N\backslash\{i\}). \tag{27}$$

Since $v_1(g) < v_2(g)$ for each $g > 0$ and $\beta_1 \leq \beta_2$,

$$v_1(g) - \frac{(1 - \beta_1)\beta_2}{\beta_1} v_2(g) < v_2(g) - \frac{(1 - \beta_2)\beta_1}{\beta_2} v_1(g).$$

Thus, from (27), consumers 1 and 2 are pivotal if and only if g satisfies

$$v_2(g) - \frac{(1 - \beta_2)\beta_1}{\beta_2} v_1(g) \leq c(g) < v_1(g) + v_2(g). \tag{28}$$

Clearly, (28) does not hold at $g = 0$.

To simplify the discussion on (28), we further assume that for each $i \in N$, $v_i(g) = \lambda_i g$, where $0 < \lambda_1 < \lambda_2$, and $c(g) = g^2/2$. Under those functions, consumers 1 and 2 are pivotal if and only if g satisfies

$$2\left(\lambda_2 - \frac{(1 - \beta_2)\beta_1}{\beta_2}\lambda_1\right) \leq g < 2(\lambda_1 + \lambda_2). \tag{29}$$

Let $g(N) \equiv \arg\max_{g \geq 0} \sum_{j \in N} v_j(g) - c(g)$ and $g(\{i\}) \equiv \arg\max_{g \geq 0} v_i(g) - c(g)$ for each $i \in N$. Then, $g(N) = \lambda_1 + \lambda_2$ and $g(\{i\}) = \lambda_i$ for each $i \in N$. Note that whether $g(N) = \lambda_1 + \lambda_2$ satisfies (29) depends on the values of λ_i and β_i $(i \in N)$.

In Claim 1, we show that the efficient provision of the public good is achieved at an equilibrium of the game only if consumers 1 and 2 are both pivotal to the efficient provision.

Claim 1 There is an equilibrium of the game that supports the provision of $g(N)$ units of the public good only if

$$g(N) \geq 2\left(\lambda_2 - \frac{(1-\beta_2)\beta_1}{\beta_2}\lambda_1\right). \tag{30}$$

Proof. Suppose, to the contrary, that (30) does not hold, but an equilibrium exists at which $g(N)$ units of the public good are provided.

Since (30) does not hold, it is impossible that both consumers 1 and 2 are pivotal to the provision of $g(N)$ units of the public good. Hence, one of the consumers is pivotal to the provision. For each $g > 0$, if consumer 1 is solely pivotal to the provision of g units of the public good, then $0 < g \leq 2\lambda_1$.[31] This, together with $2\lambda_1 < g(N)$, implies that $v_1(g(N)) - c(g(N)) < 0$; hence, consumer 1 cannot be solely pivotal to the provision of $g(N)$ units of the public good. Therefore, if $g(N)$ is chosen in the first stage, then consumer 2 is solely pivotal to the provision of it.

Consider $\tilde{g}$, such that $\max\{2\lambda_1, \lambda_2\} < \tilde{g} < g(N)$. Note that at $\tilde{g}$, consumer 1 cannot be solely pivotal because $2\lambda_1 < \tilde{g}$, and consumer 2 can be pivotal because $\tilde{g} < 2\lambda_2$. The supplier's payoff when it chooses $\tilde{g}$ is $\pi_S^{\{2\}}(\tilde{g}) = \beta_2(v_2(\tilde{g}) - c(\tilde{g}))$ and its payoff when it chooses $g(N)$ is $\pi_S^{\{2\}}(g(N)) = \beta_2(v_2(\lambda_1 + \lambda_2) - c(\lambda_1 + \lambda_2))$. Since $\lambda_2 < \tilde{g} < g(N)$ and $v_2(g) - c(g)$ is maximized at $g = g(\{2\}) = \lambda_2$, we obtain $\pi_S^{\{2\}}(g(N)) < \pi_S^{\{2\}}(\tilde{g})$, which contradicts that $g(N)$ units of the public good are provided in equilibrium. ■

[31] Here, $2\lambda_1$ is the maximal level of the public good that guarantees $v_1(g) - c(g) \geq 0$.