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DYNAMIC LEGISLATIVE BARGAINING

Hülya Eraslan
Kirill S. Evdokimov
Jan Zápal

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The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

Dynamic Legislative Bargaining *

Hülya Eraslan, Kirill S. Evdokimov and Jan Zápal

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Abstract

This article surveys the theoretical literature on legislative bargaining with endogenous status-quo. These are the legislative bargaining situations in which in each period a new policy is decided and the policy implemented in the event of no agreement is endogenously determined by the outcome of bargaining in the previous period. After describing a general framework, we discus bargaining over redistributive policies, bargaining over spatial policies, existence issues, efficiency issues and open questions.

JEL codes: A3, C7, D7

Keywords: legislative bargaining, multilateral bargaining, spatial bargaining, redistributive bargaining, endogenous status-quo, dynamic political economy

1 Introduction

In this paper, we survey the literature on legislative bargaining with endogenous status-quo. Since the term "endogenous status-quo" is used to express different things by different authors, we start by proposing a typology to distinguish "endogenous status-quo" from "evolving status-quo."[1] By bargaining with endogenous status-quo, we mean dynamic bargaining situations where *in each period a new policy is decided* and the policy implemented in the event of no agreement is endogenously determined by the outcome of bargaining in the previous period. By bargaining with evolving status-quo, we mean dynamic bargaining situations in which *a single policy is decided* during the entire bargaining time horizon, the status-quo policy in each round of bargaining depends on the history up to that bargaining round, and at the final bargaining round, in the event of no agreement, the status-quo at that round is implemented.

As in Eraslan and Evdokimov (2019), by "legislative bargaining" we mean multilateral bargaining situations in which agreement requires less than unanimous consent

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[1] Anesi and Seidmann (2014, Section 6.1) discusses some of the literature using these two concepts without introducing new terminology to distinguish between them.

and agreement on a proposal binds all parties. They survey the literature on *legislative bargaining with exogenous status-quo* in which players decide on a policy through voting between an exogenously given status-quo and a proposal offered by a proposer. This protocol is repeated in each period until an agreement is reached. Here we survey the literature on *legislative bargaining with endogenous status-quo*. In these models the protocol just described is the stage game in a dynamic model in which, in each period, a new policy is decided and the status-quo is endogenously determined by the outcome of the bargaining in the previous period.

Baron (1996) is typically credited as the first paper to start the literature on legislative bargaining with endogenous status-quo. Two important early precursors are Ingberman (1985) and Epple and Riordan (1987) who study bargaining over spatial and redistributive policy respectively. The key difference between these two papers and the subsequent work is their focus on deterministic proposer recognitions, and in the case of the latter paper, on equilibria supported by history-dependent punishments.

We start by introducing a general framework that incorporates two main strands of the literature we discuss in detail: bargaining over redistributive policy and bargaining over spatial policy. After discussing the equilibrium existence issues, we use this framework to study the many interesting questions the literature on legislative bargaining with endogenous status-quo has raised: How does the endogeneity of the status-quo affect the incentives of players? What is the equilibrium dynamics of policies? Do the policies converge? If so, to which policy? If not, what is their long-run behavior? What is the appropriate efficiency concept? Are equilibrium policies (Pareto) efficient? We end our survey by briefly discussing some avenues for future research.

The general framework we study incorporates related political economy models with bilateral bargaining, bargaining with evolving status-quo, random proposal models, costly policy change, models in which either a dictator decides policy or median chooses policy.[2] We do not review these papers due to space limitations. We also ignore other related literature on elections,[3] dynamic linkages via macro variables[4] or power structures,[5] and coalitional bargaining.[6]

[2]With apologies for inevitable omissions, see, for example, Bernheim, Rangel, and Rayo (2006); Anesi and Seidmann (2014) on bargaining with evolving status-quo, Roberts (2007); Penn (2009); Hortala-Vallve (2011); Acharya and Ortner (2017) on random proposal models, Gersbach and Tejada (2018); Dziuda and Loeper (2019); Eraslan and Piazza (2019); Gersbach, Muller, and Tejada (2019); Gersbach, Jackson, and Tejada (2020) on costly policy change, Tabellini and Alesina (1990); Gieczewski (2017) on models in which median chooses policy.

[3]See, for example, Alesina and Tabellini (1990); Baron, Diermeier, and Fong (2012); Cho (2014); Forand (2014); Callander and Raiha (2017); Nunnari and Zápal (2017); Baron (2018); Duggan and Forand (2018a,b); Gersbach, Jackson, Muller, and Tejada (2018).

[4]See, for example, Persson and Svensson (1989); Krusell and Ríos-Rull (1996, 1999); Leblanc, Snyder, and Tripathi (2000); Hassler, Rodríguez Mora, Storesletten, and Zilibotti (2003a); Hassler, Storesletten, and Zilibotti (2003b); Battaglini and Coate (2007, 2008, 2016); Klein, Krusell, and Ríos-Rull (2008); Azzimonti (2011); Battaglini, Nunnari, and Palfrey (2012, 2014, forthcoming); Azzimonti, Battaglini, and Coate (2016).

[5]See, for example, Jack and Lagunoff (2006); Lagunoff (2009); Acemoğlu, Egorov, and Sonin (2010, 2012, 2018); Bai and Lagunoff (2011); Diermeier, Prato, and Vlaicu (2015, 2016); Gibilisco (2015); Jeon (2015); Karakas (2016, 2017).

[6]See, for example, Konishi and Ray (2003); Gomes (2005); Gomes and Jehiel (2005); Hyndman and Ray (2007); Acemoğlu et al. (2010, 2012). We refer the reader to Ray and Vohra (2015) for an excellent survey of this literature.

2 General framework

In a typical model of dynamic legislative bargaining, legislators in set $N = \{1, \dots, n\}$ must choose a policy x_t in each period $t = 0, 1, \dots, T$, where T can be finite or infinite. The policy space is X. When $X \subseteq \mathbb{R}$, bargaining is over a one-dimensional spatial policy. When X is the $(n-1)$-dimensional simplex, bargaining is over redistributive policy.

The game in each period t proceeds as follows. First, period t starts with a publicly observed state denoted by s_t. The state s_t is a vector whose components include x_{t-1} but it can include other (potentially stochastic) variables as well. One of the players $\kappa(s_t)$ is the proposer and she makes a proposal $x \in X(s_t)$ where $X(s_t) \subseteq X$ is the set of feasible policies in state s_t. All players vote to either accept or reject x. If the set of players who accept x is in $\mathcal{W}(s_t)$, which is the collection of winning coalitions in state s_t, then the policy x_t implemented at time t is x, i.e., $x_t = x$. Otherwise, the policy x_t implemented at time t is the status-quo. We let $\zeta(s_t)$ to denote the status-quo in period t when the state is s_t, where ζ is some known function.

Stage utility of player i from policy x in any period when the state is s is $u_i(x, s)$. Player i discounts the future at a rate δ_i, thus the utility of player i from a sequence of policies $(x_t)_{t=0}^T$ and states $(s_t)_{t=0}^T$ is given by

$$(1 - \delta_i) \sum_{t=0}^{T} \delta_i^t u_i(x_t, s_t).$$

The papers we survey fit the framework outlined. For example, Baron (1996) and Kalandrakis (2004) study, respectively, spatial and distributive settings with simple majority and random proposer recognitions. Under our notation, they assume that s_t includes the policy x_{t-1} from the previous period and identity of the proposer in the current period, and, for any state s_t, $X(s_t) = X$, $\mathcal{W}(s_t) = \{C \subseteq N : |C| \geq \frac{n+1}{2}\}$, and $\zeta(s_t) = x_{t-1}$. The framework naturally incorporates more complex models with state-dependent stage utilities (as in Riboni and Ruge-Murcia, 2008; Dziuda and Loeper, 2016; Bowen, Chen, Eraslan, and Zápal, 2017), models in which the status-quo in period t equals the $(t-1)$-period policy with some noise, i.e., $\zeta(s_t) = x_{t-1} + \varepsilon$ (as in Duggan and Kalandrakis, 2012), models in which players redistribute pie with size that stochastically changes over time (as in the stochastic bargaining models in the spirit of Merlo and Wilson, 1995), or models with state-dependent winning coalitions and policy spaces (as in Chen and Eraslan, 2017).

Histories, strategies and subgame-perfect equilibrium for the class of games above can be defined in the standard way. The papers we survey restrict attention to (stationary) Markov strategies and equilibria.[7] A Markov (behavioral) *proposal* strategy of player i specifies a distribution of proposals $\pi_i(s, t)$ over $X(s)$ for each period t and each state s in which i is the proposer. A Markov (behavioral) *voting* strategy of player i specifies the probability $\alpha_i(x, s, t)$ that player i accepts proposal x in period t when the state is s. Infinite horizon models further restrict attention to stationary strategies that do not allow dependence on t. Moreover, the strategies are usually allowed to depend only on certain (payoff relevant) components of the state, e.g., the previous-period policy or preference

[7] There is also a strand of the literature that analyzes subgame perfect equilibria in dynamic political economy models. See, for example, Dixit, Grossman, and Gül (2000); Acemoğlu, Golosov, and Tsyvinski (2008, 2011); Aguiar and Amador (2011); Halac and Yared (2014).

shocks, but not allowed to depend on others, e.g., the identity of the proposer.[8]

A Markov strategy of player i is denoted $\sigma_i = (\pi_i, \alpha_i)$ and a profile of Markov strategies is denoted $\sigma = (\sigma_i)_{i \in N}$. A profile of strategies σ induces a dynamic utility $V^\sigma(x, s, t)$ from policy x accepted in period t when the state is s, which satisfies the following recursive relation

$$V_i^\sigma(x, s, t) = (1 - \delta_i)u_i(x, s) + \delta_i \int V_i^\sigma(x', s', t+1) dP^\sigma(x', s'|x, s, t) \tag{1}$$

where $P^\sigma(x', s'|x, s, t)$ is the probability measure over $(t+1)$-period policies and states induced by profile σ and policy x in period t when the state is s. Denote by $\alpha(x, s, t)$ the probability that a proposal x is accepted in period t when the state is s. Formally, it satisfies

$$\alpha(x, s, t) = \sum_{C \in \mathcal{W}(s)} \prod_{i \in C} \alpha_i(x, s, t) \prod_{i \in N \setminus C} (1 - \alpha_i(x, s, t)). \tag{2}$$

A profile of strategies σ constitutes a Markov Perfect equilibrium (MPE) if

i) for each i, s in which i is the proposer, t and p in the support of $\pi_i(s, t)$,

$$p \in \arg\max_{x \in X(s)} \alpha(x, s, t)V_i^\sigma(x, s, t) + (1 - \alpha(x, s, t))V_i^\sigma(\zeta(s), s, t),$$

ii) for each i, x, s and t, if $V_i^\sigma(x, s, t) > V_i^\sigma(\zeta(s), s, t)$, then $\alpha_i(x, s, t) = 1$, and if $V_i^\sigma(x, s, t) < V_i^\sigma(\zeta(s), s, t)$, then $\alpha_i(x, s, t) = 0$.

A profile of strategies σ is a stationary MPE (SMPE) if it is an MPE and π_i and α_i are independent of t for each player i.

The first condition in the definition of MPE requires that the proposal strategies are optimal while the second condition requires that the voting strategies are optimal. As is standard in the literature, the definition of MPE uses the one-stage deviation principle, and hence implicitly assumes that it holds.[9] The voting strategies in the definition assume that players vote *as if* pivotal. This rules out implausible equilibria that support arbitrary outcomes because no voter is pivotal. Some work further focuses on voting strategies with indifferent players voting for the proposal. This implies that any proposal is either accepted or rejected with probability one, and hence allows one to focus on proposal strategies that generate proposals that are always accepted.[10]

3 Existence of equilibria

Existence of SMPE in dynamic legislative bargaining models is an open issue, outside special cases represented by finite-horizon models with finite policy spaces, for which MPE existence follows by standard backward induction arguments. The literature studying discounted stochastic games, of which the dynamic legislative bargaining models are a

[8]See Maskin and Tirole (2001) for a rigorous definition of Markov equilibria and payoff relevant histories.

[9]Discounting and bounded stage utilities suffice for the one-stage deviation principle to apply (see Fudenberg and Tirole, 1991, Theorem 4.2).

[10]If the voting strategies are pure and $\zeta(s) \in X(s)$ for each state s, it is without loss of generality to assume that proposers choose policies *only* from the set of policies that would be accepted. This is because proposing a policy that would be rejected is identical to proposing the default outcome $\zeta(s)$.

special case, includes numerous conditions for SMPE existence (see He and Sun, 2017, for a recent contribution) but also examples of SMPE non-existence (see Levy, 2013; Levy and McLennan, 2015). Moreover, none of the known conditions for SMPE existence applies to legislative bargaining models (see Duggan and Kalandrakis, 2007, for further discussion of the issues involved).

One possible approach to proving SMPE existence in an infinite-horizon stationary dynamic legislative bargaining model is to work with a one-shot auxiliary game in which the payoff of player i from policy x_t in state s_t combines her stage payoff $(1-\delta_i)u_i(x_t, s_t)$ with the expected dynamic utility $\delta_i U_i(x_t, s_t)$. The expected dynamic utility of a player represents her payoff from playing the dynamic legislative bargaining game starting from period $t+1$ before the state s_{t+1} is drawn, knowing that the distribution of s_{t+1} depends on the state s_t and policy x_t at t. That is, it corresponds to the second term on the right hand side of (1).[11] Fixing a profile $(U_i)_{i\in N}$ in the one-shot auxiliary game, one derives the Nash equilibrium payoffs of this game and integrates these payoffs, as in (1), into a new profile $(\tilde{U}_i)_{i\in N}$. Any fixed point of the implied mapping Γ from $(U_i)_{i\in N}$ to $(\tilde{U}_i)_{i\in N}$ corresponds to an SMPE of the original game.

The map Γ is well-defined once one assumes that $(U_i)_{i\in N}$ are continuous in both of its arguments, which ensures that the proposer i's problem of choosing x_t to maximize $(1-\delta_i)u_i(x_t, s_t)+\delta_i U_i(x_t, s_t)$ has a solution. However, continuity of $\tilde{U}_i$ in its first argument is not guaranteed even when $(U_i)_{i\in N}$ are continuous, which implies that no fixed-point theorem can be used to conclude that Γ has a fixed point.[12] $\tilde{U}_i(x_{t-1}, s_{t-1})$ might not be continuous in x_{t-1} because a small change in x_{t-1} in general leads to a small change in the state s_t that parametrizes the proposer's problem in the auxiliary game, and even a small change in this state might induce a large change in the policy that solves the proposer's problem and hence in the value of being a responder.

The difficulties described typically arise in models where $\zeta(s_t) = x_{t-1}$ for all states s_t induced by the policy x_{t-1}, that is, in models where the status-quo in period t equals the policy in period $t-1$. One approach that overcomes these difficulties is then to work with models where x_{t-1} induces a distribution over $\zeta(s_t)$ that changes smoothly when x_{t-1} changes. This is how Duggan and Kalandrakis (2012) prove a very general SMPE existence result for dynamic legislative bargaining models; they require shocks to status-quo transitions and players' preferences. Similarly, Duggan (2017a) assumes smooth transitions from a continuous policy space to a countable state space. An alternative strategy is to assume finite policy space (Anesi, 2010; Diermeier and Fong, 2012), which automatically guarantees existence of a solution to the proposer's problem.

Because SMPE is not guaranteed to exist in a general dynamic legislative bargaining model, most papers in the literature construct an equilibrium and study its properties. In the next two sections we display such constructions in two canonical settings, distributive and spatial.

4 Distributive policy

In this section, we review the results for the case of distributive policies. This type of model with infinite policy sequences raises many interesting questions regarding equilib-

[11] Expected dynamic utilities are sometimes called continuation values.

[12] This is because all fixed-point theorems require functions or correspondences between identical spaces. Constructing the fixed point in the strategy space, not in the expected dynamic utility space, would face similar problems.

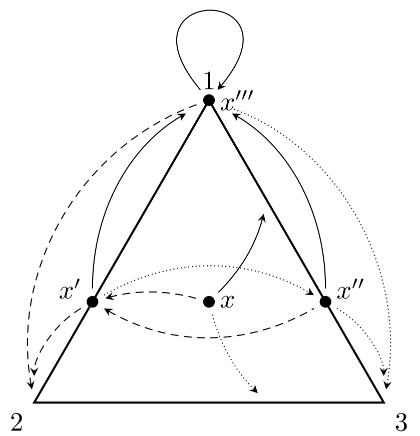
rium strategies and outcomes in addition to those questions raised in the introduction. Specifically, what is the structure of winning coalitions? For example, does the size principle of Riker (1962)[13] hold? Is the entire surplus shared in every period in equilibrium or is there waste? More generally, is there inefficiency in equilibrium? Most existing papers use constructive arguments to find SMPE to answer these questions.

As we will see, even for a given set of primitives this model has a great variety of equilibria, some of which differ drastically in their answer to the above questions. To emphasize this point, we will focus on two equilibrium constructions that have been influential in the literature: Kalandrakis (2004) which is an early paper that features an intuitive construction, and Anesi and Seidmann (2015) which is a very thorough investigation of a more general model.[14]

Kalandrakis (2004) is the first to describe an SMPE in a dynamic legislative bargaining game in which three players bargain over redistribution of a surplus of size 1. The policy space is the two-dimensional unit simplex, that is, $X = \{x \in \mathbb{R}^3_+ : \sum_{i=1}^3 x_i = 1\}$. Under our earlier notation, s_t includes the policy x_{t-1} from the previous period, and, for any state s_t, $X(s_t) = X$, $\mathcal{W}(s_t) = \{C \subseteq N : |C| \geq 2\}$, and $\zeta(s_t) = x_{t-1}$. Player i's utility from policy x is $u_i(x) = x_i$ and players discount the future at a common rate δ.

We describe the equilibrium constructed by Kalandrakis (2004) with the help of Figure 1. The figure depicts an equilateral triangle with each vertex designated for one of the players. Each edge has length $\frac{2}{\sqrt{3}}$, and therefore the distances between an arbitrary point inside the triangle and the edges sum to unity.[15] For example, x represents the equal allocation $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, x' and x'' represent allocations in which player 3 and player 2 respectively receive zero share, and x''' represent the unequal allocation $(1, 0, 0)$ favorable to player 1.

Figure 1: Equilibrium dynamics in the distributive model



The figure represents the equilibrium dynamics. For four different status-quo policies x, x', x'' and x''', the arrows point to the policy proposed by player 1 (solid line), player

[13]Riker's size principle states that in n-person zero-sum games with perfect information, when side payments are permitted, only minimum winning coalitions can occur.

[14]The earliest paper we know of is Epple and Riordan (1987) which characterizes subgame perfect equilibria in a three-player game to show that a wide range of outcomes can be sustained in equilibrium using punishment strategies.

[15]This is known as Viviani's theorem.

2 (dashed line) and player 3 (dotted line).[16] The germane feature of the equilibrium the figure illustrates is the dynamics that moves policies from strictly inside the triangle to its edges and from the edges to its vertices. That is, the equilibrium dynamics moves over allocations that differ in the number of players who receive zero share: starting with no, then with one and then with two players receiving zero share. Eventually, the equilibrium reaches a state in which the entire resource is claimed by the randomly chosen proposer. The literature has dubbed this a *rotating dictator* equilibrium.

The equilibrium dynamics arises due to the the nature of winning coalitions. We explain the intuition with the help of Figure 2. All four figures show indifference curves of the equilibrium dynamic utility. The left column is for player 1 and the right column is for player 3 (higher utility levels closer to the player's vertex). The top row is for $\delta = 0$ and the bottom row is for $\delta = \frac{6}{10}$.

Start with the simple case when $\delta = 0$, so that the model we study is an infinitely-repeated one-shot dictator game with three players and simple majority. Consider player 1's proposer problem assuming she seeks the approval of player 3, given some status-quo utility level of player 3. Using top row of Figure 2, the optimal proposal of player 1 will be a policy at the north-eastern edge of the policy space that gives player 3 the same utility as the status-quo, allocates zero share to player 2 and allocates the entire remaining resources to player 1. When, in addition, the status-quo is located at the north-western edge of the policy space, the status-quo utility of player 3 is zero and player 1 will propose a policy in which she obtains the entire pie. That is, the equilibrium dynamics moves policies from within the policy space to its edges and from the edges to the vertices. Inspection of the bottom row of Figure 2 shows that similar forces operate when $\delta = \frac{6}{10}$, although now the equilibrium is considerably more complex and its construction significantly more involved.[17]

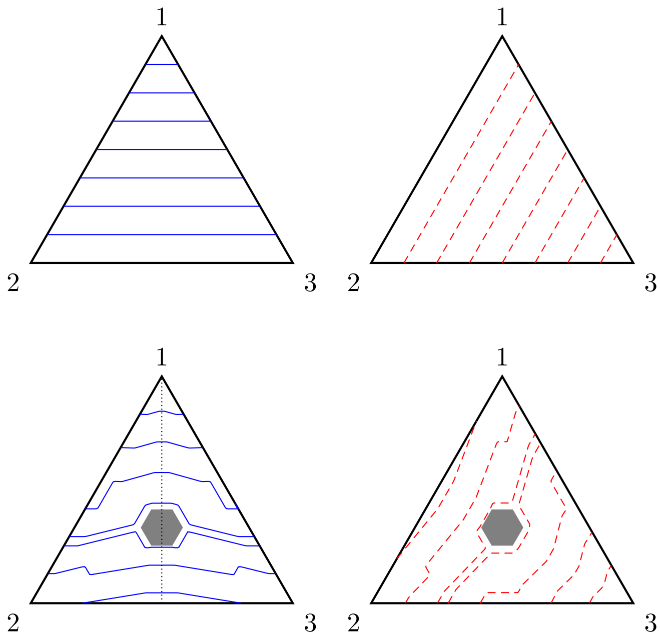
The rotating dictator equilibrium constructed by Kalandrakis (2004) has several interesting properties. First, players prefer policies that maintain their share of the resource but make the other players' shares unequal. For example, in the left bottom part of Figure 2, starting on the dotted line and moving horizontally towards the edges of the policy space typically increases player 1's dynamic utility. This preference arises endogenously; the more unequal players 2 and 3 are, the more accommodating will the disadvantaged of the two players be as a responder and hence the stronger the bargaining position of player 1 if recognized as a proposer in the next period.

Second, the strategies supporting the rotating dictator equilibrium are continuous in the status-quo. This implies that the equilibrium dynamic utilities are continuous in the status-quo. However, as Kalandrakis (2004) shows, the dynamic utilities lack quasi-concavity and might be constant over some parts of the policy space (the shaded areas in the bottom row of Figure 2). This implies that the equilibrium acceptance sets, which contain the policies that would be accepted given some status-quo, in general are not convex and that the acceptance correspondence, which maps status-quos to acceptance sets, is not lower hemicontinuous. These results highlight the difficulties in working with

[16]The equilibrium shown is symmetric, which allows one to work out the entire equilibrium policy dynamics starting from x. In particular, given a status-quo at the vertex, all players allocate the entire pie to themselves. And given a status-quo at the edge close to one of the vertices, both players with non-zero status-quo share allocate the entire pie to themselves and the player with zero status-quo share allocates zero share to the player with the largest status-quo share.

[17]The construction in Kalandrakis (2004) involves mixed proposal strategies for status-quos that allocate similar shares to two or more players. His discussion of the construction shows that this mixing is required for his construction (page 303).

Figure 2: Indifference curves of the dynamic utility in the rotating dictator equilibrium



the dynamic legislative bargaining games because the endogenous objects (e.g., dynamic utilities and acceptance sets) are not guaranteed to be well-behaved.

Finally, Kalandrakis (2004) notes that the equilibrium he constructs is not unique, although all the additional equilibria he finds are payoff equivalent to the equilibrium he studies in detail.[18] The subsequent work finds other equilibria and culminates in Anesi and Seidmann (2015) who study a significant generalization of the model from Kalandrakis (2004) and show that large number of outcomes can be supported by SMPE when players become arbitrarily patient.[19] Their model features general number of players, policy space $X = \{x \in \mathbb{R}^n_+ : \sum_{i \in N} x_i \leq 1\}$ that allows waste, any quota voting rule between simple majority and unanimity, and heterogeneous recognition probabilities, discount factors and nonlinear utilities.

The argument in Anesi and Seidmann (2015) is constructive and we describe its main features for n odd and a threshold voting rule with quota $q \in [\frac{n+1}{2}, n)$. Thus, $\mathcal{W}(s_t) = \{C \subseteq N : |C| \geq q\}$. The key ingredient of their construction is a concept called *simple solutions*. A simple solution consists of n policies. For each player i, each of these policies allocates either a bad share b_i or a good share $g_i > b_i$ to i, with policy x^i being a good policy and at least one of the other policies being a bad policy for i. In addition, each of these policies is a good policy for at least q players. Formally, a

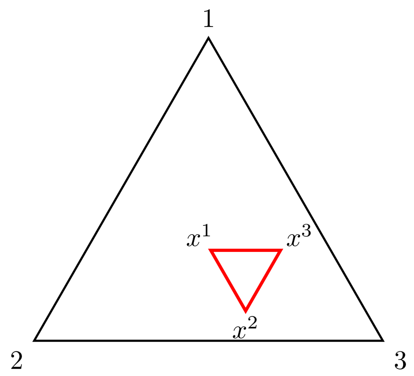
[18] Kalandrakis (2010) extends the rotating dictator equilibrium of Kalandrakis (2004) to an odd number of at least five players with heterogeneous recognition probabilities and mildly concave utilities.

[19] Bowen and Zahran (2012) construct a *compromise* equilibrium in which more than a minimum winning coalition is allocated a positive share in each period. Their construction relies on risk-sharing incentives provided by concave utilities. By additionally allowing waste, Richter (2014) constructs an *equal division* equilibrium. Baron and Bowen (2015) independently uses a construction similar to Anesi and Seidmann (2015). Our discussion focuses on the latter since its construction is also applicable to spatial bargaining (Anesi and Duggan, 2018). Baron (2019) simplifies the construction in Baron and Bowen (2015) and studies equilibrium coalitions and proposal power.

simple solution $S = (x^j)_{j \in N}$ consist of n policies such that (i) $x^j \in X$ for all j, (ii) for any i, $x_i^j \in \{b_i, g_i\}$ for all j, where $b_i < g_i$, with $x_i^i = g_i$ and $x_i^j = b_i$ for some j, and (iii) $|\{i \in N : x_i^j = g_i\}| \geq q$ for all j. Given a simple solution S, let $B_i = \{x \in S : x_i = b_i\}$ and $G_i = \{x \in S : x_i = g_i\}$ be the set of bad and good policies in S for i.

Given a simple solution S, let σ be the following strategy profile. Player i recognized to propose given status-quo x_{t-1} proposes x^i if $x_{t-1} \notin S$ and proposes x_{t-1} when $x_{t-1} \in S$. If player i is a responder given status-quo x_{t-1} and proposal $x \neq x_{t-1}$, he votes accept if and only if either (i) $x_{t-1} \in B_i$, or (ii) $x_{t-1} \notin S$ and $x \in G_i$, or (iii) $x_{t-1}, x \notin S$ and $u_i(x) \geq u_i(x_{t-1})$. Notice the policy dynamics induced by σ: starting from any status-quo $x_{t-1} \notin S$, x_t is a random variable that takes the value x^j with probability equal to the probability of recognition of player j, while it takes the value x_{t-1} starting from a status-quo $x_{t-1} \in S$. That is, starting from any status-quo, the equilibrium policy reaches S in at most one period and stays constant thereafter. Anesi and Seidmann (2015) show that the profile σ constitutes an SMPE for sufficiently patient players. Their argument is surprisingly straightforward due to the simplicity of S and σ and because patient players care mostly about the limit of policy dynamics induced by σ.

Figure 3: A simple solution in the distributive model



Studying simple solutions thus provides information about the equilibrium outcomes in the dynamic distributive legislative bargaining model. Figure 3 draws part of the three-player policy space such that $\{x \in \mathbb{R}^3_+ : \sum_{i=1}^3 x_i = c\}$, where $c \in (0, 1]$, along with a simple solution $S = (x^1, x^2, x^3)$ that corresponds to vertices of an inverted equilateral triangle inscribed into the policy space. With patient players, there exists an SMPE with equilibrium dynamics converging on S in at most one period from any initial status-quo. However, a continuum of simple solutions exists and hence large variety of outcomes are compatible with SMPE. For example, vertices of any inverted equilateral triangle inscribed into the policy space in Figure 3 correspond to a simple solution, irrespective of the size of the inscribed triangle and irrespective of c used to draw the policy space. This implies that any policy in $\{x \in \mathbb{R}^3_+ : \sum_{i=1}^3 x_i \leq 1\}$ can be supported as an equilibrium outcome including policies with strictly positive share allocated to all three players or with waste. The only policies that cannot be supported in equilibrium are the policies that assign zero share to two or three players.[20]

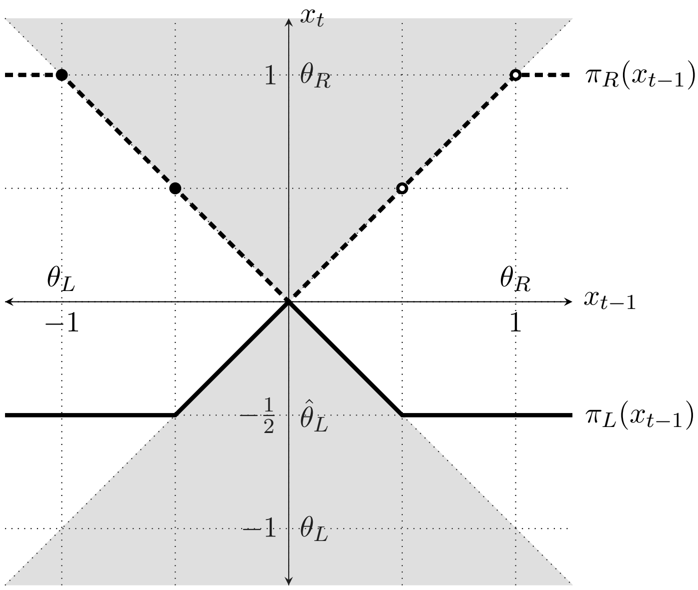
[20]Outside the three-player example, the only policies ruled out by equilibrium are those in which $n - q + 1$ or more players receive zero share.

5 Spatial policy

The spatial dynamic legislative bargaining literature starts with Baron (1996) who analyzes a model in which players choose the scale of collective goods, and each player i has a strictly concave utility with bliss-point θ_i. The policy space is the positive real line, that is, $X = \mathbb{R}_+$. The state s_t includes the policy x_{t-1} from the previous period, and, for any state s_t, $X(s_t) = X$, $\mathcal{W}(s_t) = \{C \subseteq N : |C| \geq \frac{n+1}{2}\}$, and $\zeta(s_t) = x_{t-1}$. Players discount the future at a common rate δ. Kalandrakis (2016) and Zápal (2016) analyze models similar to Baron (1996) focusing on the case of three players. This is the version of the model we study here: three players with quadratic stage utilities, bliss-points $\theta_L = -1$, $\theta_M = 0$ and $\theta_R = 1$ and equal recognition probabilities choose policies from $\mathbb{R}$.[21]

As shown by Zápal (2016), this model admits an SMPE. An equilibrium consists of pure proposal strategies, mapping status-quo into the proposed policy, and pure voting strategies, specifying voting behavior given any status-quo proposed-policy pair. We first discuss the asymmetric equilibrium shown in Figure 4 and later turn to other (symmetric and asymmetric) equilibria the game admits.

Figure 4: Asymmetric pure strategy equilibrium in the spatial model



In Figure 4, for relevant subset of the policy space, the white area is the acceptance correspondence induced by the equilibrium voting strategies while the π_L and π_R functions are the equilibrium proposal strategies of players L and R. We do not show the equilibrium proposal strategy of the median player M; for any status-quo, she proposes her bliss-point $\theta_M = 0$. (For example, given status-quo $x_{t-1} = -1$, a proposal is accepted if and only if it falls into $[-1, 1]$, player L proposes $-\frac{1}{2}$, player M proposes 0 and player R proposes 1.)

The shape of the acceptance correspondence is identical to the one that would arise with a myopic median player: the median player accepts any proposal that is weakly closer to her bliss-point $\theta_M = 0$ than the status-quo. To see this, first note that the

[21] Baron (1996) assumes existence of a 'dynamic median' voter, which is guaranteed to exist when stage utilities are quadratic (Banks and Duggan, 2006; Duggan, 2014).

shape of the acceptance correspondence is driven by the dynamic preferences of player M. This follows because under quadratic utilities the decisiveness of player M in voting over deterministic alternatives extends into the decisiveness of player M in voting over stochastic policy paths (Banks and Duggan, 2006; Duggan, 2014). Second, the median player accepts any proposal that is closer to her bliss-point θ_M since both her stage utility u_M and her expected dynamic utility are both single-peaked at θ_M. The latter is determined by the entire profile of strategies and cannot be assumed to be single-peaked. Nevertheless, the intuition behind its single-peakedness is not hard to grasp: the proposal strategies of all the players are such that (i) given any status-quo, any proposal made is accepted, and (ii) as the status-quo moves away from θ_M, the equilibrium proposals move away from θ_M, and hence the expected dynamic utility of the median player decreases.

The shape of the proposal strategies of players L and R are also related to those that would arise with myopic players as in Romer and Rosenthal (1979): for any status-quo, player R proposes a policy that is as close as possible to his bliss-point $\theta_R = 1$ out of those the median player accepts, that is, out of those policies that are weakly closer to θ_M than the status-quo. In this asymmetric equilibrium, player L behaves similarly, except that she is not using her bliss-point $\theta_L = -1$ and uses her 'strategic' bliss-point $\hat{\theta}_L = -\frac{1}{2}$ instead. Because $\hat{\theta}_L = -\frac{1}{2}$ is closer to $\theta_M = 0$, that is, it is a more moderate than $\theta_L = -1$, we call this behavior *moderation*.

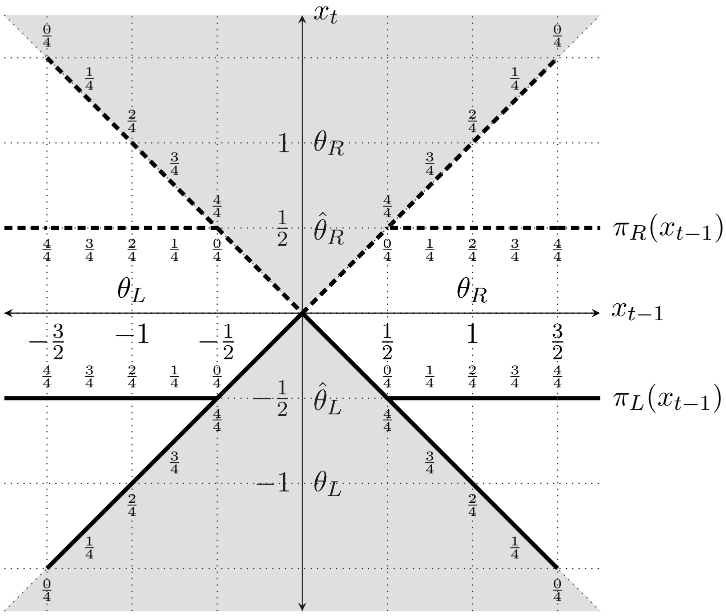
Why does player L moderate? Her incentive to do so is strategic and is driven by the endogenous status-quo. Consider status-quo $x_{t-1} = -1$. We claim player L proposes $\hat{\theta}_L = -\frac{1}{2}$ in equilibrium. To see this, consider a one-stage deviation to propose $\theta_L = -1$, which is the policy that maximizes L's static utility. Both of these policies are accepted given the status-quo. Player L proposes $\hat{\theta}_L = -\frac{1}{2}$ instead of $\theta_L = -1$ in order to constrain the proposed policy of player R if this player is recognized in the next period: player R's policy will be 1 if player L proposes $\theta_L = -1$ now, while player R's policy will be $\frac{1}{2}$ if player L proposes $\hat{\theta}_L = -\frac{1}{2}$ now (Figure 4 highlights these policies by dots in its left half). At the same time, player L is not constraining herself if she is recognized next period: player L's policy will be $-\frac{1}{2}$ irrespective of whether she proposes θ_L or $\hat{\theta}_L$ now. That is, player L moderates in equilibrium using the acceptance set of the median player M and the endogenous status-quo to constrain the future policies of her opponent player R.

Why does *not* player R moderate? Player R has the same incentive to constrain the policies of player L. Consider status-quo $x_{t-1} = 1$. We claim player R proposes $\theta_R = 1$ in equilibrium. To see this, consider a one-stage deviation to propose $x_t = \frac{1}{2}$. Both of these policies are accepted given the status-quo. However, the only effect of moderating and proposing $x_t = \frac{1}{2}$ rather than $\theta_R = 1$ is for player R to constrain herself if she is recognized next period: player R's policy will be $\frac{1}{2}$ if she proposes $x_t = \frac{1}{2}$ now and will be 1 if she proposes $\theta_R = 1$ (Figure 4 highlights these policies by circles in its right half). In order to constrain the policy of player L, player R would have to moderate to some proposal below $\frac{1}{2}$, which is too costly for her in terms of the foregone static utility. That is, the incentive to moderate is a *strategic substitute*: because player L moderates, she is effectively constraining herself and player R has no incentive to moderate. Because player R does not moderate, her strategic bliss-point coincides with her bliss-point.

Equilibrium moderation and its strategic substitute nature are two key insights of the model. But the model allows us to make several additional observations. First, in order to simplify the exposition we have chosen $\delta = \frac{3}{4}$, identical recognition probabilities and particular values for the bliss-points such that the strategic bliss-point of player L is $\hat{\theta}_L = -\frac{1}{2}$. For general values of these parameters, Zápal (2016) shows that the strategic bliss-point of a non-median player i is $\theta_i(1-2\delta r_{-i})$, where r_{-i} is the recognition probability of player $\{L, R\} \setminus \{i\}$. That is, the strategic bliss-point is a point where two forces offset each other. The first force is standard: policies are pushed towards players' bliss-points. The second force is strategic: players moderate in order to constrain their opponents. The second force intensifies and the strategic bliss-point of a player becomes more moderate when the player becomes more patient and when the probability of recognition of her opponent increases. In fact, when $1 - 2\delta r_{-i} < 0$, player i's incentive to moderate is strong enough for this player to propose $\theta_M = 0$ for any status-quo in equilibrium, that is, to propose in the identical way as the median player M does.

Second, because the game underlying Figure 4 is symmetric, it is intuitive that a mirror equilibrium exists in which player L does not moderate and uses a proposal strategy with her bliss-point $\theta_L = 1$, while player R moderates and uses a proposal strategy with strategic bliss-point $\hat{\theta}_R = \frac{1}{2}$. This multiplicity is driven by the strategic substitute nature of moderation: when player L moderates player R has no incentive to do so, while when player L does not moderate player R has incentive to do so. However, results in Zápal (2020) suggest that this multiplicity is non-generic and restricted to symmetric games. Namely, he shows that, generically, at most one equilibrium exists in a class of equilibria in which the proposal strategy of each player is pinned down by a single strategic bliss-point (as in Figure 4).

Figure 5: Symmetric mixed strategy equilibrium in the spatial model



Third, Kalandrakis (2016) shows that the game underlying Figure 4 admits a symmetric mixed strategy SMPE. Figure 5 shows key features of this equilibrium. For status-quos outside $X_m = \left[-\frac{3}{2}, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \frac{3}{2}\right]$, both players L and R use strategies similar to those from Figure 4, *except* that both players use strategies with moderate strategic bliss-points $\hat{\theta}_L = -\frac{1}{2}$ and $\hat{\theta}_R = \frac{1}{2}$ respectively. For any status-quo $x_{t-1} \in X_m$, player R mixes over two proposals $|x_{t-1}|$ and $\hat{\theta}_R = \frac{1}{2}$ putting probability $|x_{t-1}| - \frac{1}{2}$ on the latter. Player L mixes in the similar way over $-|x_{t-1}|$ and $\hat{\theta}_L = -\frac{1}{2}$. (Figure 5 shows the mixing probabilities for $x_{t-1} \in X_m$.) This equilibrium displays two interesting features: first, it is symmetric and does not inherit the strategic substitute nature of moderation of the pure strategy equilibrium from above, and, second, policies outside the set of statically Pareto efficient policies $[-1, 1]$ are proposed and accepted with strictly positive probability for a non-negligible set of status-quos.

The game underlying Figures 4 and 5 features three symmetric players and admits multiple SMPE. Going beyond this setup opens new questions, some of which has been tackled by existing work. First, Zápal (2016) studies a game with three players, without restricting their bliss-points and recognition probabilities. He proves an SMPE existence result and shows that the equilibria he describes are generically unique, but only in a restrictive class of equilibria with proposal strategies pinned down by strategic bliss-points. The equilibria he describes all involve moderation that is a strategic substitute, that is, in equilibrium only one of the non-median players moderates. Moreover, when the patience and the recognition probabilities of the players are the same, it is the player with less extreme bliss-point who moderates.

Second, do all equilibria involve moderation driven by strategic incentives? In fact, Dziuda and Loeper (2016, 2018) highlight that bargaining with an endogenous status-quo leads to strategic polarization in a bilateral bargaining under unanimity. The logic behind their *polarizing effect* and the *moderating effect* just discussed is the same: with an endogenous status-quo a policy determines both the current and future policies. Consider player L and two policies $x < x'$ both in $[\theta_L, \theta_M]$. In the equilibrium described in Figure 4, both x and x' are revised when player R proposes, but x is revised to a policy that is worse for L than the policy x' is revised to. Although player L prefers x to x', policy x brings about worse policies than x', and hence L's preference for x becomes moderate or even reverses as a result of an endogenous status-quo. Dziuda and Loeper (2016) study equilibria in which policies are unlikely to be revised, because acceptance of a policy requires unanimity. In these equilibria, player L prefers x to x' and x brings about better policies than x', and hence L's preference for x becomes stronger as a result of an endogenous status-quo. An endogenous status-quo thus induces moderation over policies that are revised when they become status-quo and polarization over policies that remain unchanged. The contrast between Dziuda and Loeper (2016, 2018) and Zápal (2016) highlights the importance of voting rule on polarization.

Third, Kalandrakis (2016) and Zápal (2016) restrict attention to three players. Whether equilibria exist when there are more than three players, and whether the equilibria, if they exists, involve moderation that is a strategic substitute, are open questions. Zápal (2020) provides affirmative but partial answer to these questions. The key to his analysis is an algorithm that derives strategic bliss-points for games with more than three players. The algorithm is based on the insight that the incentive of players to moderate is driven by their opponents, players with bliss-points on the other side of the median's bliss-point, who moderate to a smaller extent. The reason Zápal (2020) does not provide a complete answer is because his equilibrium construction fails for some games.[22]

Fourth, Baron (1996) and subsequently Kalandrakis (2016) and Zápal (2016) analyze simple majority voting. The existence and structure of equilibria in spatial model with supermajority, unanimity or more general voting rules remain an open question.

Fifth, little is known about existence and properties of equilibria with general utility functions or multi-dimensional policy spaces. The key complication associated with gen-

[22]His construction fails in games with players who have bliss-points far from the median and strong incentives to moderate. The resulting large interval between the bliss-point and the strategic bliss-point is challenging for the equilibrium construction because it might give rise to profitable deviations.

eral utility functions is that the median player might cease to be decisive in voting over lotteries over policy paths. Partial progress has been made with multi-dimensional policy spaces; Zápal (2014) uses similar construction as in Zápal (2016, 2020) to construct equilibria with moderation that is strategic substitute, while Anesi and Duggan (2018) show generic indeterminacy of Markovian equilibria in games with patient players and policy spaces with sufficient number of dimensions. Moreover, Baron (2018), using similar techniques as Anesi and Seidmann (2015) and Baron and Bowen (2015), constructs equilibria in a two-dimensional model with three parties with bliss-points located at vertices of an equilateral triangle. Baron and Herron (2003) study a similar model using numerical methods.

Finally, our understanding of dynamic bargaining models with changing preferences is very limited, despite the obvious appeal of such extension. Several papers advanced in this direction but only partially (Riboni and Ruge-Murcia, 2008 assume fixed proposer; Duggan and Kalandrakis, 2011 compute equilibria numerically; Dziuda and Loeper, 2016, 2018 have only two policies in the policy space; Austen-Smith, Dziuda, Harstad, and Loeper, 2019 have three policies; Bowen et al., 2017; Buisseret and Bernhardt, 2017, 2018 focus on two-period models). Similarly limited is our understanding of models with a stochastic drift in policies (Callander and Krehbiel, 2014; Callander and Martin, 2017), or models with endogenously changing policy spaces (Chen and Eraslan, 2017).

6 Efficiency

Are the equilibrium outcomes of the dynamic legislative bargaining models efficient or not? Answering this question requires a framework for thinking about (in)efficiency of political outcomes. One possibility is to follow Besley and Coate (1998) who draw a parallel with general equilibrium theory: one defines feasible allocations and uses players' preferences to decide, without reference to the process that determines which allocations arise, whether an allocation is Pareto efficient or not, and, only then, looks at the allocations that arise as a result of an economic or political process.

Under our notation, assuming momentarily that, in any period t, the state in period t is x_{t-1} and that the stage utilities and policy spaces are state independent, an allocation $\mathbf{x} = (x_t)_{t=0}^T$ is a sequence of policies and is feasible if $\mathbf{x} \in X^T$. A feasible allocation $\mathbf{x}$ is *dynamically Pareto efficient* if there is no feasible allocation $\mathbf{y}$ such that $(1-\delta_i)\sum_{t=0}^T \delta_i^t u_i(y_t) \geq (1-\delta_i)\sum_{t=0}^T \delta_i^t u_i(x_t)$ for all $i \in N$, with at least one inequality strict. When states include other variables, let ω_t be those other variables in t, i.e., $s_t = (x_{t-1}, \omega_t)$, and let $\boldsymbol{\omega} = (\omega_t)_{t=0}^T \in \Omega$. An allocation now is a contingent sequence of policies $(\mathbf{x}^{\boldsymbol{\omega}})_{\boldsymbol{\omega}\in\Omega} = ((x_t^{\boldsymbol{\omega}})_{t=0}^T)_{\boldsymbol{\omega}\in\Omega}$ and is feasible if $x_t^{\boldsymbol{\omega}} \in X(x_{t-1}^{\boldsymbol{\omega}}, \omega_t)$ for all t and $\boldsymbol{\omega}$.[23] A feasible allocation $(\mathbf{x}^{\boldsymbol{\omega}})_{\boldsymbol{\omega}\in\Omega}$ is *ex-ante dynamically Pareto efficient* if there is no feasible allocation $(\mathbf{y}^{\boldsymbol{\omega}})_{\boldsymbol{\omega}\in\Omega}$ such that $(1-\delta_i)\mathbb{E}_{\boldsymbol{\omega}}[\sum_{t=0}^T \delta_i^t u_i(y_t^{\boldsymbol{\omega}}, (y_{t-1}^{\boldsymbol{\omega}}, \omega_t))] \geq (1-\delta_i)\mathbb{E}_{\boldsymbol{\omega}}[\sum_{t=0}^T \delta_i^t u_i(x_t^{\boldsymbol{\omega}}, (x_{t-1}^{\boldsymbol{\omega}}, \omega_t))]$ for all $i \in N$, with at least one inequality strict. For given $\boldsymbol{\omega}$, feasible $\mathbf{x}^{\boldsymbol{\omega}}$ is *ex-post dynamically Pareto efficient in* $\boldsymbol{\omega}$ if there is no feasible $\mathbf{y}^{\boldsymbol{\omega}}$ such that $(1-\delta_i)\sum_{t=0}^T \delta_i^t u_i(y_t^{\boldsymbol{\omega}}, (y_{t-1}^{\boldsymbol{\omega}}, \omega_t)) \geq (1-\delta_i)\sum_{t=0}^T \delta_i^t u_i(x_t^{\boldsymbol{\omega}}, (x_{t-1}^{\boldsymbol{\omega}}, \omega_t))$ for all $i \in N$, with at least one inequality strict. Looking at a single period t, $x_t \in X(s_t)$ is

[23] The variable $\boldsymbol{\omega}$ is what Mas-Colell, Whinston, and Green (1995, chapter 19) call a state and should be understood as a 'complete description of a possible outcome of uncertainty' (page 688). Our contingent policy sequences are analogous to (state) contingent commodities, one of the central concepts in general equilibrium under uncertainty.

statically Pareto efficient in s_t if there is no $y_t \in X(s_t)$ such that $u_i(y_t, s_t) \geq u_i(x_t, s_t)$ for all $i \in N$, with at least one inequality strict.

In the distributive setting with policy space $\{x \in \mathbb{R}^n_+ : \sum_{i\in N} x_i \leq 1\}$, linear preferences and homogeneous discount factors, i.e., $u_i(x) = x_i$ and $\delta_i = \delta$ for all i, $(\mathbf{x}^{\boldsymbol{\omega}})_{\boldsymbol{\omega}\in\Omega}$ is ex-ante dynamically Pareto efficient if and only if $\sum_{i\in N} x^{\boldsymbol{\omega}}_{t,i} = 1$ for all t and $\boldsymbol{\omega}$, that is, if $(\mathbf{x}^{\boldsymbol{\omega}})_{\boldsymbol{\omega}\in\Omega}$ consists of policies without waste. This implies that the rotating dictator equilibrium of Kalandrakis (2004) is ex-ante dynamically Pareto efficient. This also implies that the equilibria constructed by Anesi and Seidmann (2015), in case of linear preferences and homogeneous discount factors, are ex-ante dynamically Pareto efficient if and only if based on simple solutions that consist of policies without waste.

General characterization of Pareto efficient policy sequences, outside special cases like the one just discussed, is an open question.[24] Nevertheless, we make the following general observations. First, in the distributive setting, heterogenous discount factors require efficient policy sequences to allocate larger shares to more patient players in later periods. Failure of equilibria to generate this patter is one of the sources of inefficiency in the distributive setting of Anesi and Seidmann (2015).

Second, with risk averse players, when states include variables that do not affect players' preferences and the policy space, the efficient policy sequences do not vary with these variables. That is, if $(\mathbf{x}^{\boldsymbol{\omega}})_{\boldsymbol{\omega}\in\Omega}$ is ex-ante dynamically Pareto efficient and if $\boldsymbol{\omega}$ and $\boldsymbol{\omega}'$ differ only in the preference and policy space irrelevant variables, then $\mathbf{x}^{\boldsymbol{\omega}} = \mathbf{x}^{\boldsymbol{\omega}'}$. Standard example of variables included in states that are preference and policy space irrelevant are proposer identities. Dynamic ex-ante Pareto efficiency then requires that equilibrium policies do not depend on proposer identities, which they often do. The resulting inefficiency stemming from sensitivity of equilibrium policies to proposer identities is one of the sources of inefficiency in Anesi and Seidmann (2015) and is what Bowen et al. (2017) call political risk. The converse arises when states include preference or policy space relevant variables. Ex-ante dynamic Pareto efficiency then requires that equilibrium policies track the preference or policy space relevant variables. However, in most spatial models the equilibrium policy in the current period is often the status-quo because change to any other policy is unacceptable to a sufficient number of players. The resulting inefficiency stemming from insensitivity of equilibrium policies to preference or policy space relevant variables is what Bowen et al. (2017) call gridlock.[25] However, gridlock can also improve efficiency as Piguillem and Riboni (2013, 2015) show. They study environments where government would benefit from ability to commit to future levels of taxation. In the absence of commitment, endogenous status-quo and the resulting gridlock represents a form of (endogenous) commitment that improves efficiency.

Third, it is not hard to see that if $(\mathbf{x}^{\boldsymbol{\omega}})_{\boldsymbol{\omega}\in\Omega}$ is dynamically ex-ante Pareto efficient, then, for each $\boldsymbol{\omega}$, the policy sequence $\mathbf{x}^{\boldsymbol{\omega}}$ is dynamically ex-post Pareto efficient in $\boldsymbol{\omega}$, which in turn implies that $x^{\boldsymbol{\omega}}_t$ is statically Pareto efficient in $(x^{\boldsymbol{\omega}}_{t-1}, \omega_t)$ for each t.[26] This means that the static Pareto inefficiencies noticed by Riboni and Ruge-Murcia (2008), Dziuda and Loeper (2016, 2018) and Kalandrakis (2016) imply dynamic ex-post and ex-

[24] See Anesi and Seidmann (2015) for the discussion of the issues involved. Standard approach to characterization of Pareto efficient allocations is via an optimization problem in which utility of one player is maximized subject to a lower bound on the utilities of the other players. See Mas-Colell et al. (1995, chapter 16) and Duggan (2017b) for details of this approach.

[25] Bowen et al. (2017) link political risk and gridlock to dynamic ex-post Pareto inefficiency. Because ex-post inefficiency implies ex-ante inefficiency, as we discuss in the next paragraph, political risk and gridlock are linked to ex-ante inefficiency as well.

[26] See Mas-Colell et al. (1995, chapter 19.D) for sketch of the argument.

ante Pareto inefficiencies. The sources of inefficiency in Riboni and Ruge-Murcia (2008) and Dziuda and Loeper (2016, 2018) are similar. These papers analyze models in which players' preferences are subject to stochastic shocks. In some periods the shocks are such that players agree, i.e., have identical stage utilities, while in other periods the shocks are such that players disagree, i.e., have different stage utilities. An important insight of these papers is that players might fail to implement policies that maximizes their common stage utility even in the periods when they agree. This is because with endogenous status-quo, the implemented policy acquires two roles: it constitutes a policy and hence enters stage utility of the players, but it also becomes the status-quo for the next period and hence determines players' future bargaining position. And agreeing to implement the policy that maximizes players' stage utility might compromise future bargaining position for some of them.

In light of Riboni and Ruge-Murcia (2008) and Dziuda and Loeper (2016, 2018), one might think that endogenous status-quo leads to inefficient policies. Bowen, Chen, and Eraslan (2014) show that this is not true. In their model two parties bargain repeatedly over the division of a fixed budget into public good and private transfers. Contrasting equilibria with exogenous and endogenous status-quo level of public goods, they show that having endogenous status-quo increases the equilibrium level of public goods and improves efficiency. The effect arises because higher status-quo level of public goods provides players with higher status-quo utility and hence protects them from expropriation when not recognized as proposers. These contrasting results show that the link between endogenous status-quo and efficiency is not fully understood, despite the partial progress made by Bowen et al. (2017) who suggest that efficiency might be achieved by combining exogenous and endogenous status-quo. This can be attained, for example, by having public goods with endogenous status-quo along with public goods with exogenous status-quo. Alternatively, a generalized endogenous status-quo in the form of sunset provisions might lead to efficiency.

Ultimately, we are interested in efficiency of political institutions. The models we write capture key features of these institutions. Because any given model can give rise to multiple outcomes, either due to multiplicity of equilibria or due to equilibrium mixing, one has to take a stand on whether to require one or all possible outcomes to be Pareto efficient.

7 Open questions

We end the survey with a list of what we deem to be open questions, beyond the ones already mentioned in Sections 5 and 6. First, Anesi and Duggan (2018) show that the large multiplicity of SMPE observed by Anesi and Seidmann (2015) for the distributive model extends to other environments. Their result requires infinite horizon, continuous policy space, patient players, absence of veto players, and strictly positive recognition probabilities and hence does not apply to a number of papers.[27] Nevertheless, their work highlights that the literature we survey lacks an equilibrium refinements that would restore the predictive power of the models it studies. Eraslan and Piazza (2019) propose

[27] For example, Anesi (2010); Diermeier and Fong (2011); Dziuda and Loeper (2016, 2018); Anesi and Duggan (2017); Diermeier, Egorov, and Sonin (2017) study models with finite policy spaces, Buisseret and Bernhardt (2017, 2018); Chen and Eraslan (2017) focus on finite horizon, Duggan, Kalandrakis, and Manjunath (2008); Diermeier et al. (2017); Nunnari (2018) include veto players, and Riboni and Ruge-Murcia (2008); Riboni (2010); Diermeier and Fong (2012) feature fixed proposers.

refining MPE to ensure that the property chosen to refine MPE is satisfied in (i) a single player version of the model, and (ii) a finite horizon version of the model. Specifically, they restrict attention to the class of equilibria in which equilibrium strategies satisfy a shape restriction that is also satisfied by the optimal policy of a dictator. Remarkably, the refinement allows them to fully characterize equilibria and establish its uniqueness when it is unique. Interestingly, the identical shape refinement allows Gersbach et al. (2018) to establish a unique equilibrium as well. They also provide a micro-foundation of this refinement using a finite horizon version of their model.

Second, although the surveyed literature includes a number of applications, further, especially economically relevant ones, should provide novel insights.[28] In particular, the dynamic nature of the surveyed models seems to be well suited to the study of debt (see Piguillem and Riboni, 2018; Bouton, Lizzeri, and Persico, forthcoming, for headways), dynamic issues of taxation (see Piguillem and Riboni, 2013; Ma, 2014, for existing work), and should be embedded in realistic macro models (see Grechyna, 2017; Azzimonti, Karpuska, and Mihalache, 2020, for early examples).

Third, a study of incomplete information is missing, as in the literature with exogenous status-quo. This is an obvious gap that should be addressed. Possible developments in this spirit include dynamic bargaining over pie with size that stochastically changes over time (as in Merlo and Wilson, 1995), or dynamic bargaining with privately-observed type-dependent preferences (see Anesi, 2018, for an initial contribution).

Finally, very little effort has been devoted to testing the predictions of the surveyed models. The limited existing evidence comes from laboratory experiments (Battaglini and Palfrey, 2012; Battaglini et al., 2012, forthcoming; Baron, Bowen, and Nunnari, 2017; Nunnari, 2018; Agranov, Cotton, and Tergiman, 2020, see also the chapter on laboratory experiments in this volume by Marina Agranov) and we are not aware of any evidence using observational data.

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