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CONSUMER SEARCH AND STOCK-OUT: A LABORATORY EXPERIMENT

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Abstract

We investigate the overall impact of stock-out on individual consumers' information search behavior through both search-theoretic and experimental approaches. As the probability of stock-out increases, search intensity decreases, while the expected number of searches may increase. Such increases in stock-out probability also deter consumers from participating in the market.

Keywords: consumer search, stock-out, search experiment, consumer behavior

JEL classification: C91, D11, D43, D83, L13

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1 Introduction

This study investigate the overall impact of stock-out on individual consumers' information search behavior through both search-theoretic and experimental approaches.

In recent decades, several studies have addressed that the stock-out is the major problem in the real world. They report the observed stock-out rates are as high as 8%.[1] Moreover, Corsten and Gruen (2003) point out that if consumers face a stock-out, then 31% of them walk out of the store, 15% delay purchase, and 9% never purchase the item (see also Emmelhainz et al. (1991); Corsten and Gruen (2004)). As such, stock-out is a significant loss of sales opportunities for firms, and the evidence above shows the importance of managing it.[2]

The possibility of stock-out affects not only sales opportunities but also consumers' information search behavior. Because consumers usually have incomplete prior information about the product (including whether the product is out-of-stock or not), they must visit a firm (or website) to gather information by incurring search costs, including time and money. If there is insufficient stock of the product at a firm, some unfortunate consumers realize that although they have incurred search costs, the product is out of stock. Consumers must incur additional search costs if they search further. Furthermore, some consumers would be reluctant to engage in the initial search because they anticipate stock-out and do not want to waste time and money. Thus, firms will lose further sales opportunities. As a result, because stock-outs harm both firms and consumers through consumers' search behavior, we must consider their overall impact.

The purposes of this paper are to present a practical theoretical model and an experimental method of examining consumers' information search behavior regarding

[1] Andersen Consulting (1996) shows that over 8.2% of items are not available in the average supermarket. Corsten and Gruen (2003) find that the average stock-out rate for 40 studies is 8.3%; Aastrup and Kotzab (2010) lament that stock-out rates seem to have an average level of 7% to 8%, despite 40 years of research.

[2] For example, Corsten and Gruen (2004) point out that a typical retailer loses about 4% of its sales from stock-out.

stock-out, as well as to provide the implications for consumer behavior and retailing. Although stock-out is an important issue, very few studies focus on the impact of the *possibility* of stock-out on consumers' information search behavior (*e.g.*, Anderson et al. (2006); Zhou et al. (2015)); instead, most studies consider consumers' reactions to *encountering* stock-out, *i.e.*, the *ex-post* effects of stock-out (*e.g.*, Emmelhainz et al. (1991); Fitzsimons (2000); Zinn and Liu (2008)). One possible reason for this focus is the absence of a tractable consumer information search model that explicitly addresses the overall impact of the possibility of stock-out, and empirical methods to confirm the results.

In our model, we consider a market with many firms that supply a product. There is a mass of consumers who wish to purchase the product, but they are initially uninformed about the actual prices and valuations of the product they want; hence, they must search for product information by incurring a search cost per firm. The product's valuations are randomly decided in each firm according to the same probability distribution, and each firm independently determines its price that is revealed to consumers who visit the firm. In addition, there exists a possibility of stock-out; a consumer may not obtain product information despite the search.[3]

First, we characterize consumers' search behavior when there is the possibility of stock-out by extending the seminal work of Wolinsky (1986) and theoretically investigate how the possibility affects consumers' information search behavior.[4][5] The intensity of the consumer's search behavior depends on her own search cost and the stock-out probability. We show that the higher the stock-out probability (and the higher the

[3] In that sense, we can also interpret the possibility of an exogenous shock (*i.e.*, a shortage owing to a disaster or store closure). Furthermore, we can interpret our model as a special form of Anderson and Renault (1999). In essence, consumers have inflexible tastes for product varieties such that they evaluate match utility of a product as zero with a certain probability.

[4] More precisely, our model is a variation of Anderson and Renault (1999), where the market is not fully covered.

[5] Because we want to focus on the effect of the possibility of stock-out on individual consumers' search behavior, we omit the analyses of firms' pricing, profits, and welfare from the main part of the study. See the discussion in Section 2 and the appendix for details.

search cost), the more likely the consumer is to terminate the search at a lower valuation. Like rising search costs, a higher stock-out probability also discourages consumers from participating in the market and leads to fewer transactions. Moreover, increased search costs always decrease the expected number of searches until each consumer terminates the search and purchases the product (therefore, the conversion rate —the number of purchases divided by the number of visitors— increases in the search costs). In contrast, the effect of an increase in the stock-out probability on the number of searches depends on the shape of the survival function of product valuation's distribution. If the survival function is log-concave (log-convex), the number of searches increases (decreases) with an increase in the stock-out probability, corresponding to the case where the probability of obtaining higher valuations is relatively low (high) and decreasing rapidly (slowly).

Second, we confirm these theoretical predictions through experimental analysis. To correct the consumer search behavior data, we conduct a search-theoretic laboratory experiment. We show that most results are consistent with theoretical predictions; hence, our model can well capture consumers' search behavior. The reservation value, which characterizes search intensity, declines as search costs or the stock-out probability increase (*i.e.*, the search intensity decreases as those parameters increase), and the expected number of searches also falls as search costs rise. As predicted by theory, we find that whether the expected number of searches increases as the stock-out probability increases depends on the distribution of valuations. Furthermore, we find that an increase in stock-out probability induces subjects with higher search costs not to participate. However, in some experimental treatments, the reservation values and the number of searches tend to be significantly smaller than the theoretical values; this issue requires further investigation.

Previous studies have shown that stock-out inherently results in a loss of sales opportunities by revealing consumers' responses to stock-out. We show that the possibility of stock-out itself may cause a loss of sales opportunities by discouraging consumers with high search costs from participating in the market. Therefore, practitioners (especially upstream suppliers) should pay particular attention to the impact of stock-out on consumer's beliefs for stock-out. On the other hand, our research suggests that the stock-out not only has a negative impact on consumers and firms but also would have a positive impact. That is, the possibility of stock-out could lead to higher conversion rates and lower total search costs by discouraging consumers from further searches. We show that it is important to know the distribution of consumers' valuations about a product to learn whether such positive effects exist. However, such a positive impact may be less than the negative impact of consumer participation decisions; hence, practitioners should be aware of the overall impact of stock-outs. We believe that our results will be useful to both practitioners and researchers and consider that the framework will prompt future in-depth discussions of stock-out.

The rest of this article proceeds as follows. In the remainder of this section, we describe related literature and our contributions to it. Section 2 describes the basic model, characterizes consumers' search behavior, analyzes how the possibility of stock-out affects consumers' search behavior, and presents some theoretical examples. Section 3 describes the experimental design and identification strategy to confirm theoretical predictions. Section 4 reports the results of the experiment and tests our predictions. Section 5 concludes and discusses the important issue.

1.1 Related literature

As stock-out is a very broad issue that can cause loss to both consumers and sellers, much research has been conducted in a wide range of fields, including marketing, management, and economics. The present study is mainly concerned with two fields: (1) consumer behavior regarding stock-out in marketing and (2) consumer search theory in economics.

In consumer search behavior on stock-out in the marketing literature, the majority of studies are empirical, focusing on consumer's *ex-post* response to the stock-out (*cf.* Campo et al. (2000)). For example, Emmelhainz et al. (1991) conduct a brief questionnaire on stock-out to see how consumers respond when they actually face the stock-out. They find that 13% of consumers delay the purchase and 14% walk out of the store.[6] (See also Campo et al. (2000); Kim and Lennon (2011); Pizzi and Scarpi (2013) for consumers' response to stock-out, and Fitzsimons (2000); Zinn and Liu (2008); Sampaio and Machline (2009) for the relationship between consumer characteristics and their reactions to stock-out). In contrast to these studies, we analyze *ex-ante* responses to stock-out, *i.e.*, we focus on how the possibility of stock-out itself affects consumers' search behavior using a search-theoretic framework. Using this framework, we reveal the overall impact of the possibility of stock-out on consumer behavior (about the search intensity, number of searches (conversion rate), and participation decisions), which has not been elucidated in previous empirical studies on *ex-post* reactions to stock-out. Thus, our study complements previous work and provides a new perspective on stock-out analysis in the marketing literature.

Our model builds on consumer search theory in economics. Since the seminal work of Stigler (1961), there have been many theoretical studies on consumer search. There are two research streams: (i) research on consumer's price search in homogeneous markets (*e.g.*, Varian (1980); Burdett and Judd (1983); Stahl (1989)) and (ii) consumer's search for both prices and suitability of products sought on the market with heterogeneous products (Wolinsky (1986); Anderson and Renault (1999); Moraga-González et al. (2017)). Our study is in the latter stream. In contrast to the previous studies, we explicitly analyze the various effects of changing the stock-out probability. Our approach provides important implications for economic welfare, especially by revealing

[6] This ratio is almost the same as that reported by Corsten and Gruen (2003). Besides, Corsten and Gruen (2004) note that 21% to 43% (depending on the product category) of consumers facing stock-out walk out of the store.

the impact of stock-out on the number of searches. We also contribute to consumer search theory by providing data on the sequential search model's robustness with various parameters for search costs and stock-out probability.

Our study also relates to the literature of search experiments, especially in economics.[7] There is a vast body of experimental studies on both consumer and labor search based on the outstanding frameworks proposed by Weitzman (1979) and Lippman and McCall (1979). Many early experimental studies tested whether a sequential search model following an optimal search rule with a reservation value could capture subjects behavior. In the seminal study of Schotter and Braunstein (1981), the authors asked subjects directly about their reservation values. They found no significant difference between the theoretical and described reservation values for risk-neutral and risk-averse subjects. Subsequent studies have also shown that such a sequential search model captures subjects behavior well (see also Hey (1987); Kogut (1990); Sonnemans (1998)). However, these studies have mainly focused on subjects' search heuristics, and few have focused on identification, particularly on the estimation of reservation values. In a more recent study, Caplin et al. (2011) developed an experimental design that can estimate the reservation value from the subjects' choice process data and statistically showed that the sequential search model with the reservation value captured the subjects' behavior well.[8] With our search-theoretic model, we extend the experimental design of Caplin et al. (2011) to estimate the reservation value under various search costs and stock-out probabilities, contributing to these studies by demonstrating the robustness and applicability of the model.

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[7] In the marketing literature, Zwick et al. (2003) conduct a search experiment with *ex-post* stock-out. Although they do not show explicit results for stock-out and consumer search behavior, some of their other results are very similar to ours.

[8] Brown et al. (2011) is also a relevant study of the sequential search in labor search experiments. "arrival rate" in the context of labor search is a somewhat similar concept to the stock-out.

2 The model

2.1 Settings

There are an infinite number of symmetric firms supplying a single product.[9] Without loss of generality, they can supply a product without any retailing costs. There is a mass of consumers with measures normalized to one per firm who have imperfect information about the actual price and the valuation for each firm's product. They sequentially gather information by incurring a search cost of s per firm.[10] We impose a free-recall assumption that each consumer can return to the stores they had visited freely, which is usual in consumer search theory. According to Perloff and Salop (1985) and the standard consumer search literature, we consider the products as differentiated between firms, and each consumer has idiosyncratic tastes for each product. The valuation of firm k's product is denoted by u^k, and we call the valuation the *match utility* of firm k. When a consumer visits firm k, her match utility u^k for the firm (product) is independently and randomly drawn from a common twice-differentiable cumulative distribution function $F(u)$ with the interval $[\underline{u}, \bar{u}]$. The distribution is commonly known. We assume that consumers have no particular preference for firms, so they visit firms in random order. Each consumer's valuation of a firm's product is independent across consumers, and her utility function is linear.

We introduce stock-out probability as follows. When a consumer visits firm k ($k = 1, 2...$), a product is available at the firm with probability q, *i.e.*, the consumer faces stock-out at the firm with probability $1 - q$. We assume that the probability of an out-of-stock event $1 - q$ is the same for all firms and consumers, but whether the product is out of stock at each firm is determined exogenously and independently for each firm and

[9] Our results hold for a finite number of firms, but the results and calculations will be complicated. Therefore, we assume an infinite number of firms for clarity.

[10] For simplicity, we mainly analyze the behavior of a representative consumer with search cost s, but we can allow search costs to vary among consumers. In Section 2.5, we consider each consumer's participation decision given her search cost s_i and the stock-out probability.

consumer.[11] The stock-out probability $1-q$ is common knowledge, but consumers do not know whether the product is actually out of stock at each firm before a search.[12][13] We also assume that once a consumer observes the product is out-of-stock at a current firm, she considers that she will never to purchase from there (so she never returns). We focus on the symmetric equilibrium such that all firms charge the same price and assume that a consumer holds a consistent belief that each firm charges the equilibrium price, even if she observes a deviating price (*i.e.*, passive belief).

The game proceeds as follows. In period 1, q is exogenously determined by nature, the upstream monopolist, or somehow. In period 2, each consumer decides whether to participate in the market given her search cost, the probability of stock-out, and her individual beliefs. In period 3, firms charge prices simultaneously. Then, consumers who are active in the market start searches in random order. We assume that firms cannot change their prices during the game.

2.2 The optimal search rule

We first characterize the optimal search rule for a consumer who decides to participate in the market. In the following analysis and the experiment, we mainly focus on the case that all consumers participate in the market (*i.e.*, the expected consumer surplus of market participation is positive). However, each rational consumer must consider whether to participate given her search cost, the stock-out probability, and her belief about the price. Therefore, we discuss the participation decision in Section 2.5. For

[11] For example, q can be interpreted as the quantities for each firm supplied by the upstream monopoly firm. The inequality $q<1$ represents a situation in which the upstream firm fails to forecast supply (or sometimes it may intentionally reduce the entire supply) and thus supply is insufficient to meet demand. In that sense, we can also interpret q as a capacity constraint on the entire market in the short term. As another example, q is interpreted as an exogenous (or unintentional) shock (*i.e.*, shortage owing to a disaster or the possibility of store closure).

[12] This assumption can be easily justified (*e.g.*, if the consumers can put a product on hold temporarily). Relaxing this assumption will strengthen the effects of stock-out on search behavior.

[13] We may use the model of Zhou et al. (2015) to perform a similar analysis of the *ex-post* stock-out problem. According to their study, an *ex-post* stock-out may induce consumers to search *more*, which is contrary to our finding.

the moment, we assume that all consumers participate in the market.

Once consumers decide to participate, they follow the optimal search rule suggested by Weitzman (1979). In each search period, each consumer compares an expected incremental benefit from an additional search with an additional search cost of s. If the expected incremental benefit from an additional search is higher than the search cost, she decides to search further. Otherwise, she stops and purchases from the current firm immediately.[14] Let x represent the current highest observed match value. Then, given x and the belief about the equilibrium price, an expected incremental benefit from an additional search in the symmetric equilibrium is given by

$$q \int_{x}^{\bar{u}} (u - x) dF(u) = q \int_{x}^{\bar{u}} [1 - F(u)] du. \tag{1}$$

Let s_i represent the search cost for consumer i.[15] Then, according to equation (1), we argue that there exists a critical value of x_i whereby the expected incremental benefit from an additional search and an individual search cost s_i for an additional search are indifferent given q:

$$h(x_i(s_i; q)) \equiv \int_{x_i(s_i;q)}^{\bar{u}} [1 - F(u)] du = \frac{s_i}{q}. \tag{2}$$

Hereafter, we refer to this value of $x_i(s_i; q)$ as a *reservation value* for consumer i given search costs of s_i and q. In the following context, we sometimes drop the index i for brevity. Because $x_i(s_i; q)$ represents the minimum amount of match utility required by consumer i, we can regard $x(s; q)$ as a measure of the *intensity* of the search for each consumer. Alternatively, we can formally define the search intensity as follows.

Definition. *Searches based on the reservation value x_1 are more intense than those with x_2 when $x_1 > x_2$ holds.*

[14] See the appendix for more formal characterization and derivation.

[15] Despite considering homogeneous consumers, we use subscript i to denote the search cost for later analysis and the experimental part.

2.3 Characteristics of the reservation value

We first investigate the characteristics of the reservation value $x_i(s_i; q)$ to characterize the consumer's search intensity. In the following analysis, we focus on a change in search costs as well as a change in the stock-out probability to highlight the common and different effects of those changes on consumer behavior. From equation (2), we immediately have

$$\frac{\partial x_i(s_i; q)}{\partial s_i} = \frac{1}{-q[1 - F(x_i(s_i))]} < 0 \ ; \ \frac{\partial x_i(s_i; q)}{\partial q} = \frac{\left(\int_{x_i(s_i)}^{\bar{u}} [1 - F(u)] du\right)^2}{s_i[1 - F(x_i(s_i))]} > 0. \tag{3}$$

These expressions yield the following first proposition.

Proposition 1. *The reservation value $x(s; q)$ is (i) decreasing in s and (ii) decreasing in $1 - q$.*

These are rather intuitive results. When consumers face a higher search cost or anticipate a high probability of stock-out, they tend to be reluctant to search further because the opportunity cost of searching is large. Therefore, consumers are more likely to accept the relatively low match utility observed in the current firm.

Then, what about p^*? We note that equation (2) is independent of p^*. As we see in Section 2.5, each consumer's participation decision is affected by p^*. However, once a consumer participates in the market, as there is no difference between firms in the symmetric equilibrium and with a consistent and passive consumer belief, the value of p^* does not matter.

Although a stock-out inherently leads to a loss of sales opportunities, Proposition 1 suggests that such a possibility may also have positive effects on both consumers and firms. That is, even if a product has such a low match utility that a consumer would not purchase it without the possibility of stock-out, she may do so if there is a possibility of stock-out in a future search. It may benefit the consumer by reducing the number of searches or benefit the firm by making sales easier. To confirm this, we next consider how the stock-out possibility affects the probability that consumers will transact with the firm.

2.4 Number of searches and conversion rate

In this subsection, we focus on the effect of a change in search cost and stock-out probability on the number of searches (or the conversion rate). In contrast to the previous subsection, we show that a change in search cost s and $1-q$ may have different effects on the number of searches.

Note that consumer i stops searching and immediately purchases from firm k if and only if $u^k - p^k \geq x_i(s_i; q) - p^*$, where p^k is the price of firm k. In a symmetric equilibrium, a product is available at each firm with probability q, so the probability of terminating a search and purchasing at the firm is given by

$$q[1 - F(x_i(s_i; q))], \tag{4}$$

which is also known as a *conversion rate*. Then, when there is an infinite number of firms, i's expected number of searches $\#_i$ is given by[16]

$$\#_i(s_i; q) \equiv \frac{1}{q[1 - F(x_i(s_i; q))]}. \tag{5}$$

Then, we have the following proposition.[17]

Proposition 2. *(i) The expected number of searches decreases in s. (ii) The expected number of searches is increasing (decreasing) in the stock-out probability $1-q$ if $1-F(u)$ is a log-concave (log-convex) function. It is independent of the stock-out probability if $1 - F(u)$ is log-linear.*

The intuition behind part (i) is simple. As an increase in search cost makes consumers reluctant to search further ($\partial x/\partial s < 0$), an increase in s indirectly increases the

[16] If there is a finite number of firms, the numerator of $\#_i$ must be modified, but it does not affect the discussion below.
[17] The proof is available in the appendix.

Table 1: Survival function and log-concavity example

Distribution	Survival function $1 - F(u)$
Uniform	log-concave
Normal	log-concave
Logistic	log-concave
Extreme	log-concave
Power Function ($\beta \geq 1$)	log-concave
Gamma ($\beta \geq 1$)	log-concave
Beta ($\alpha \geq 1,\ \beta \geq 1$)	log-concave
Exponential	log-linear (log-concave)
Pareto	log-convex
Gamma ($\beta < 1$)	log-convex
Beta ($\alpha = 2,\ \beta = 0.5$)	log-convex
Log-Normal	mixed
Cauchy	mixed

Note: α and β are the shape parameters.

probability of terminating a search but does not directly affect the expected number of searches. Hence, the number of searches always decreases in s.

On the other hand, an increase in $1 - q$ has two effects on the expected number of searches: (a) it decreases the reservation value $x(s; q)$, decreasing the expected number of searches (the indirect effect), and (b) it decreases the probability of success of the search, increasing the expected number of searches (the direct effect). Which effect is more significant depends on the shape of $1-F(u)$ (known as a *survival function*). Table 1 shows the survival functions and log-concavities of the widely used distributions (*cf.* Bagnoli and Bergstrom (2005)). From the table, we note that the survival function is log-concave in many distribution types. That is, the expected number of searches tends to increase as the stock-out probability increases. Figure 1 also shows the probability density functions of several distribution types with log-concave and log-convex survival functions, which may help the reader understand the relationship between the log-concavity of the survival function and the shape of the probability density function.

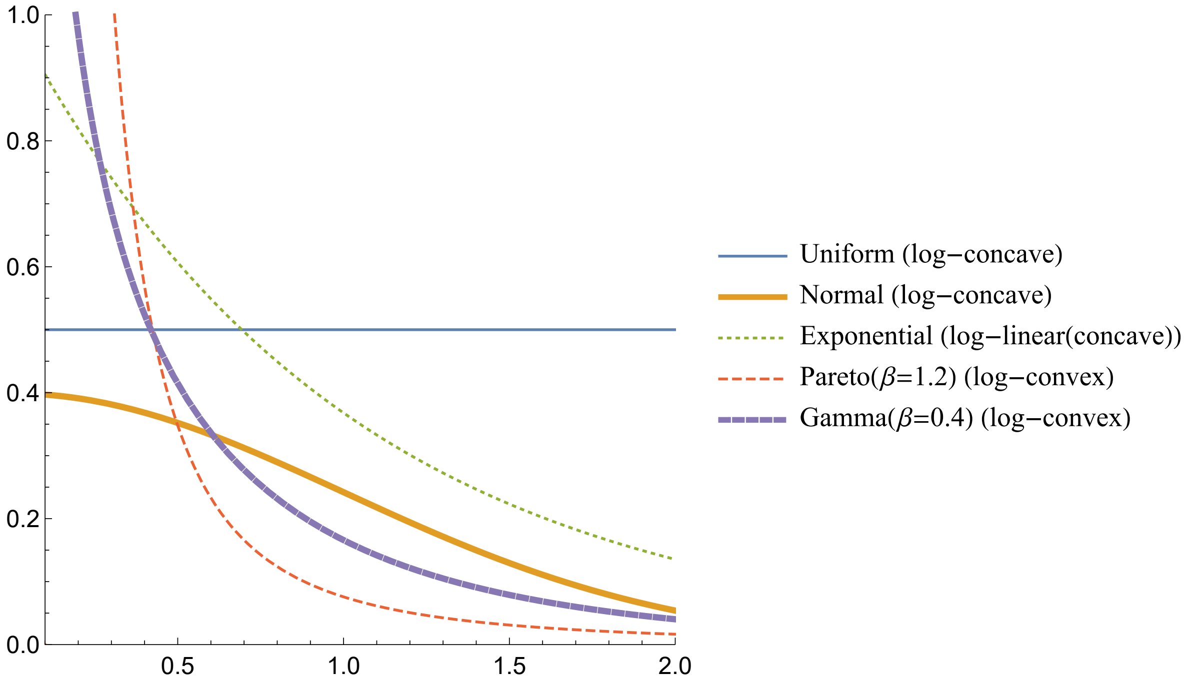

Figure 1: Probability density functions with log-concave and log-convex survival functions.

To illustrate the intuition behind Proposition 2-(ii), we first consider a case in which the number of searches decreases as the stock-out probability (*i.e.*, the survival function is log-convex). The survival function will be log-convex when the probability of obtaining high utility is relatively low and decreases rapidly (*e.g.*, Pareto distribution). In this type of distribution, consumers tend to be very reluctant to search for the next firm, so $x(s;q)$ decreases significantly when the stock-out probability increases. Because such a decrease in $x(s;q)$ also significantly decreases the number of searches, this indirect effect dominates the direct effect of increasing the number of searches in $1-q$. On the other hand, if the survival function is log-concave, the direct effect outweighs the indirect effect. Consequently, an increase in the stock-out probability leads to an increase in the number of expected searches.

As the expected conversion rate is the inverse of the expected number of searches, we can easily confirm the effect of a change of stock-out probability on the rate. That is, if the survival function is log-concave (log-convex), the expected conversion rate is decreasing (increasing) in $1-q$. For example, when the survival function is log-convex, the higher the stock-out probability, the more likely consumers are to stop searching with lower match utility, and the firm may benefit from the higher stock-out probability, thus making it easier to sell the product. Nevertheless, to consider whether a firm's profit increases in the stock-out probability, we need to consider further the impact of stock-out on the total number of transactions (the consumer's participation decisions), as discussed in the next subsection.

Proposition 2 also suggests that in analyses that reflect the consequences of consumers' search behavior, we need to be careful about assumptions concerning the distribution of consumer's valuations. For example, the distribution of consumer's valuations is frequently assumed to be a normal, log-normal, or Weibull distribution in empirical analyses of the consumers' qualitative choice behavior,[18] or for estimating consumer's willingness to pay (WTP).[19] One may also expect consumer's valuations to follow a Pareto distribution as well as income (*cf.* Piketty and Saez (2003)). The survival functions of these distributions can have different characteristics.[20] Therefore, if there is a possibility of stock-out, different distribution assumptions will also yield different results, reflecting the consequences of search behavior.

[18] According to McFadden (1973), a vast body of studies assume Weibull distribution.

[19] Most estimations of WTP assume a normal distribution, an extreme distribution, or a log-normal distribution. For example, Lusk (2003) estimates WTP for golden rice by assuming a normal distribution; Scarpa et al. (2008) assume a Gumbel distribution (type I extreme) for site choice in the Alps; and Carpio and Isengildina-Massa (2009) assume both a normal and a log-normal distribution for locally grown products.

[20] Whether the survival function of the log-normal distribution is log-concave or log-convex depends on the interval of u and, in the case of the Weibull distribution, on the shape parameter.

2.5 The participation decision

So far, we assume the consumer to be active in the market. We now turn to the consumer's participation decision problem.[21]

Each consumer should participate in the market if and only if her *ex-ante* expected surplus is nonnegative. Because consumer i only purchases from a firm where she has observed positive match utility, which should be greater than $x_i(s_i; q)$, her *ex-ante* net expected benefit from purchasing is given as

$$E[u|u \geq x_i(s_i; q)] - p^*,$$

where p^* is the symmetric equilibrium price. On the other hand, from (5), we have consumer i's expected total search costs:

$$\frac{s_i}{q[1 - F(x_i(s_i; q))]}.$$

The *ex-ante* expected surplus of consumer i from participating in the market is given by

$$E[CS(s_i, p^*, q)] = E[u|u \geq x_i(s_i; q)] - p^* - \frac{s_i}{q[1 - F(x_i(s_i; q))]}. \tag{6}$$

This expression is rewritten as

$$E[CS(s_i, p^*, q)] = x_i(s_i; q) - p^*.$$

If we set the *ex-ante* expected surplus above zero, it gives the cutoff reservation value $x(s_i; q)$ of a consumer i, who is indifferent about participating in the market, which is conditional on q:

$$x(s^*) \equiv x(s_i; q) - p^* = 0. \tag{7}$$

[21] Previous studies that considered homogeneous individual search behavior have avoided explicit discussion by considering relatively low search costs, as we have assumed an active market instead of discussing participation decisions. However, in the experiment, we needed to discuss participation decision-making explicitly because individual attributes (*e.g.*, risk preferences) can affect search costs, even if we assume a relatively low search cost.

Hence, again from (2), we obtain the critical search cost s^*, which satisfies the above equation as follows:

$$s^* = q \int_{p^*}^{\bar{u}} (u - p^*) dF(u). \tag{8}$$

This implies that the *ex-ante* expected surplus is strictly positive when $s_i < s^*$; hence, it is optimal for consumer i to participate in the market. Thus, we have the following lemma.

Lemma 1. *A consumer with search cost s_i should participate in the market if and only if*

$$s_i \leq s^*, \tag{9}$$

where s^* is implicitly defined by (8).

Furthermore, by differentiating (8) with respect to q, we have

$$\frac{\partial s^*}{\partial q} = \int_{p^*}^{\bar{u}} (u - p^*) dF(u) - q(1 - F(p^*)) \frac{\partial p^*}{\partial q}. \tag{10}$$

To summarize, we claim the following.

Proposition 3. *If p^* is independent or increasing in stock-out probability, then a decrease (increase) in the stock-out probability increases (decreases) the critical search cost s^*.*

As the effect of a change in $1 - q$ on p^* is ambiguous, we cannot obtain a clear-cut prediction the effect of a change in the stock-out probability on the participation decision.[22] However, Lemma 1 and Proposition 3 predict that an increase in stock-out probability will discourage consumers from participating in the market in many cases. Moreover, Proposition 3 shows that the higher the consumer's search cost, the more

[22] We emphasize that the independence of p^* with respect to q (or the property of decreasing in q) is a sufficient condition. By specify the distributions, one can see that s^* is usually increasing in q (*i.e.*, an increase in the stock-out probability may deter consumers from participating in the market). This is also consistent with intuition.

likely she is to decide not to participate as the stock-out probability increases. Thus, the possibility of stock-out is always undesirable for consumers and in many cases for firms as well.[23]

Note that all consumers participate in the market when s^* sufficiently exceeds the upper bound of the search cost $\bar{s}$. In this case, we (and firms) can ignore the participation issues caused by an increase in $1-q$ unless the change in s^* is sufficiently large. Otherwise, a change in $1-q$ affects participation decisions.[24]

2.6 Example

This subsection presents examples of two specified distributions with log-concave and log-linear survival functions that are widely used in consumer search theory to illustrate our results. We confirm our results in relation to these two distributions in the experiment. Furthermore, to confirm the result of Proposition 2, we also present an example with the Pareto distribution, which has a log-convex survival function.[25]

Example 1: Uniform distribution

Suppose that u is distributed uniformly as $F(u) = u$ with the interval $[0, 1]$. Then, from equation (2), the reservation value $x(s)$ is calculated as follows:

$$\int_{x(s)}^{1} (1 - F(u))du = \frac{s}{q} \Leftrightarrow x(s) = 1 - \sqrt{\frac{2s}{q}}. \tag{11}$$

Figure 2 illustrates how a change in s and $1-q$ affects $x(s)$. The reservation value decreases in search cost s and the stock-out probability $(1-q)$, which is consistent

[23] If firms can raise prices as stock-out probability increases, then a stock-out may be desirable for firms. However, based on two examples in the appendix, we suggest that the negative impact of consumer nonparticipation on firms' profits will always be significant, even if the firms can raise prices.

[24] In most of the experiments, to avoid these participation issues as much as possible, we set sufficient s^* for most subjects to participate in the search. We also chose values of s and q that make participation in the market (almost) indifferent in one of the treatments to test our claim about participation decisions and to check the impact of subject heterogeneity.

[25] The distributions with log-convex survival function (*e.g.*, Pareto distribution) show very similar features to the exponential distribution in Propositions 1 and 3; hence, we performed an additional experiment with a distribution that has log-convex survival function only to confirm Proposition 2.

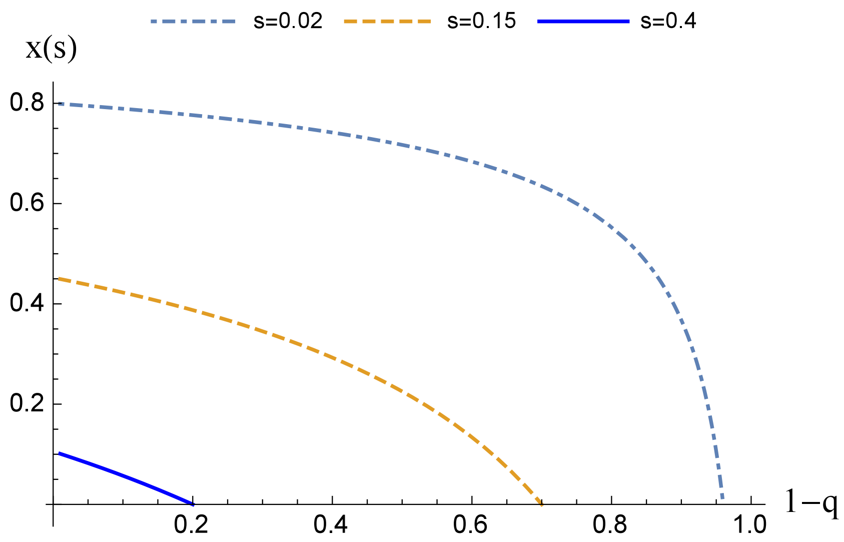

Figure 2: A graph of $x(s;q)$ and $1-q$ when u is distributed uniformly.

with Proposition 1. Moreover, we can see that the survival function of the uniform distribution is log-concave; hence, Proposition 2 means that the expected number of searches is increasing in $1-q$. From (5), the expected number of searches is given by $\sqrt{\frac{s}{q}}/\sqrt{2}s$, which is actually increasing in $1-q$. As shown in the subsection above, consumer i will not participate in the market if her reservation value $x(s_i) < 0$, because the expected consumer surplus from participating in the market is negative. This is the case where her search cost s_i is too high compared with $1-q$. From Figure 2, we can see that an increase in the stock-out probability $1-q$ causes more consumers to avoid participating in the market, and consumers with higher search costs are less likely to participate.

Example 2: Exponential distribution

Suppose that u is distributed exponentially as $F(u) = 1 - e^{-\lambda u}$ with the interval $[0, \infty)$. Then, from (2), the reservation value $x(s;q)$ is calculated as follows:

$$q \int_{x(s;q)}^{\infty} e^{-\lambda u} du = s \Leftrightarrow \frac{1}{\lambda} e^{-\lambda x(s;q)} = \frac{s}{q} \Leftrightarrow x(s;q) = -\frac{1}{\lambda} ln \left(\frac{\lambda s}{q} \right). \tag{12}$$

We can easily confirm that Proposition 1 holds. Figure 3 illustrates how a change in

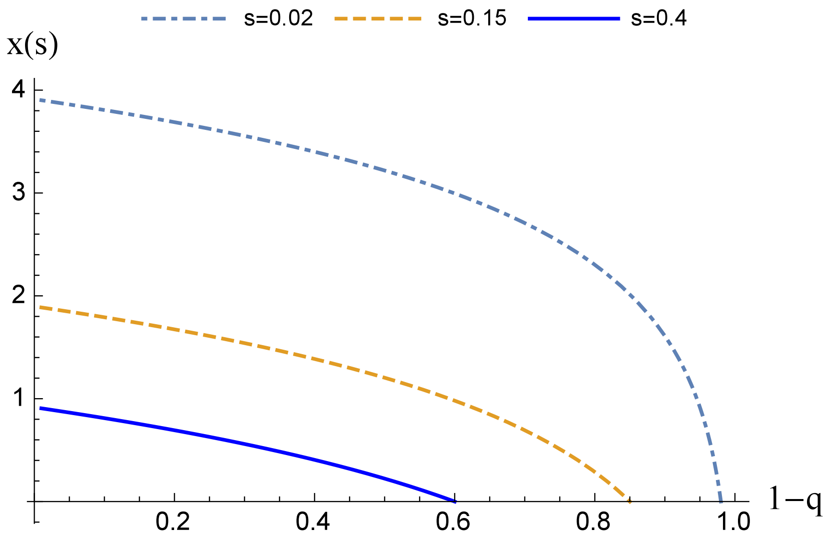

Figure 3: A graph of $x(s;q)$ and $1-q$ when u is distributed exponentially and $\lambda = 1$.

$1-q$ affects $x(s;q)$. The figure also implies that Proposition 3 holds. In addition, as the survival function of the exponential distribution is log-linear, we can see that the expected number of searches (given by $1/q(1-F(x)) = 1/s$) is decreasing in s but independent of q.

Example 3: Pareto distribution

As an example of a log-convex survival function, we consider the Pareto distribution of $F(u) = 1 - u^{-\beta}$ with the interval $[1, \infty)$. We have the reservation value $x(s;q) = \left(-\frac{(1-\beta)s}{q}\right)^{\frac{1}{1-\beta}}$, which is decreasing in both s and $1-q$ for $\beta > 1$. Figure 4 illustrates how a change in $1-q$ affects $x(s)$. Moreover, as the survival function is log-convex with the Pareto distribution, Proposition 2 means that the expected number of searches decreases in $1-q$. We can see that the expected number of searches (given by $\frac{\left(\left(-\frac{(1-\beta)s}{q}\right)^{\frac{1}{1-\beta}}\right)^{\beta}}{q}$) is indeed decreasing in both s and $1-q$.

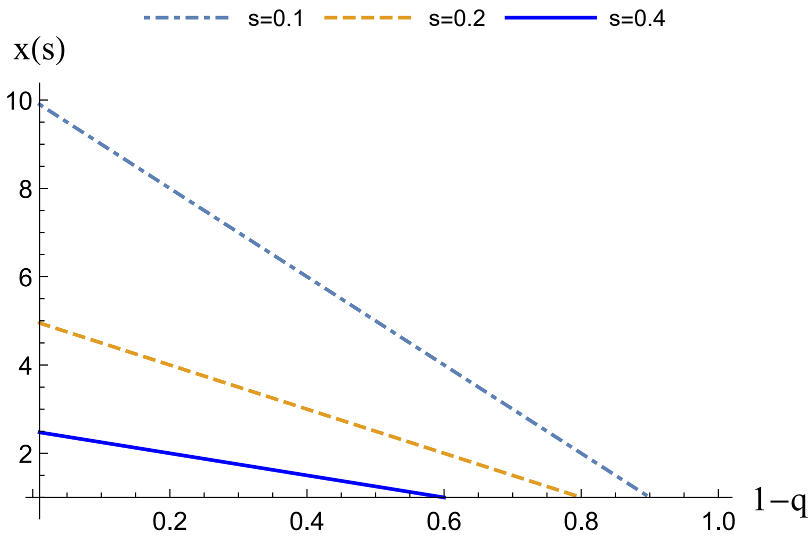

Figure 4: A graph of $x(s;q)$ and $1-q$ when match utilities are distributed according to the Pareto distribution ($\beta = 2$).

2.7 Discussion: firm's problem and welfare

So far, we have not discussed the firm's problem in depth because the focus of our attention is on the impact of stock-out on consumers rather than on the profits of firms or the industry as a whole. To address the problem, suppose like Moraga-González et al. (2017) that each consumer's search cost is drawn from twice-differentiable cumulative distribution function $G(s)$ with the interval $[\underline{s}, \bar{s}]$. In this case, a firm's optimal pricing depends on the property of $G(s)$ (see the appendix for more details). Notably, when s^* is insufficiently large, stock-out can either increase or decrease prices, depending on market parameters. According to Moraga-González et al. (2017), if the distribution of consumer search costs has an increasing likelihood ratio (ILR), the price could increase in the stock-out probability $1-q$. Intuitively, firms can charge higher prices as the stock-out probability increases if the change in $1-q$ has a significant effect on the reservation value and has little influence on inducing consumers not to participate in the market. In the appendix, we confirm in the exponential search cost case that $G(s) = 1 - e^{-s}$ (which has the ILR property), that the equilibrium price can increase in $1-q$ (we also suppose that match utility is distributed as $F(u) = u$ with the interval $[0, 1]$). We also confirm that each firm's profit does not increase the stock-out probability in this case.

What is the impact of stock-out on welfare? Because a change in p^* associated with a change in q is regarded as a transfer of surplus between consumers and firms, it does not directly affect welfare. Thus, a stock-out affects welfare through two channels: the number of transactions and the number of searches. We only consider the general case for both impacts. When a decrease in q decreases the number of participants, it would in turn decrease the number of transactions, and thus impair welfare. Furthermore, if the stock-out increases the expected number of searches, it will also harm consumer surplus. On the other hand, such an increase in the number of searches could increase the chance of obtaining a higher match utility; hence, the overall impacts on welfare are ambiguous. However, in general, an increase in stock-out probability is likely to decrease welfare by increasing total search costs. Therefore, stock-out may be undesirable for social welfare by causing a loss of transactions and waste of search costs.

3 Experimental design

To verify our theoretical predictions, we conducted a laboratory experiment at the Institute of Social Economic Research, Osaka University, Japan in February 2020.[26]

3.1 Outline

The experiment is a simple card-flipping game that simulates information search behavior of consumers described in Section 2. We conducted two search sessions, each with an assumed exponential distribution and a uniform distribution of match utility. The subject group consisted of 41 students aged 19 to 33 from various departments of Osaka University. We randomly divided the subjects between two sessions: 20 in the uniform distribution session and 21 in the exponential distribution session. Each experiment

[26] Instructions and data are available upon request.

consisted of 48 rounds, including three practice rounds, and the total number of observations was 1845. The average time required for the experiment was about 60 minutes. The average earnings were about 1260 Japanese yen (JPY), including 500 JPY as the participation fee. After all subjects had finished all rounds, one of the 45 rounds was chosen at random for each subject, and the result of that round (multiples of 100 JPY) was immediately given in cash as her earnings.[27] The experiment was computerized, and the program was created using PsychoPy.[28] A recording of spoken instructions was played.[29]

3.2 Design

In each round of the session, subjects sequentially explored a "number" (representing match utility) by flipping cards with a given search cost and the "success probability" of each card. A number from 1 to 20 was randomly and independently assigned to each card based on a uniform distribution with a mean of 10.5 in the uniform session or an exponential distribution with $\lambda = 0.15$, truncated at 20, in the exponential session.[30] Each subject was required to pay a cost of s per card to obtain information on the card number. Moreover, subjects can obtain information on the card number with the probability of q, displayed as the "Probability of success." Therefore, they still have a chance of failing to flip the card (any number on the card will be zero) with the probability of $1 - q$ even if they pay the cost. This indicates the probability of the

[27] This random incentive system (RIS) rule induces the subject to behave more cautiously and avoids subjects' hedging behavior. See Heinemann et al. (2009) for more details.

[28] PsychoPy is a software package written in the Python programming language for use in neuroscience and experimental psychology research (*cf.* Peirce (2007)). PsychoPy is available at https://www.psychopy.org/.

[29] In addition, we also conducted an experiment regarding risk, which is independent of the search experiment, to test whether subjects' risk preferences were consistent with previous studies. We used the multiple price list (MPL) method, which was developed by Holt and Laury (2002) (details, instructions, and data are available upon request). This session was also well monetary incentivezed as well as the main session. The proportion of risk-averse subjects was 59%, which is consistent with previous studies (*e.g.*, 63% in Schunk and Winter (2009)).

[30] In the exponential session, we provided a graph of the distribution and extra information about the distribution in the instructions.

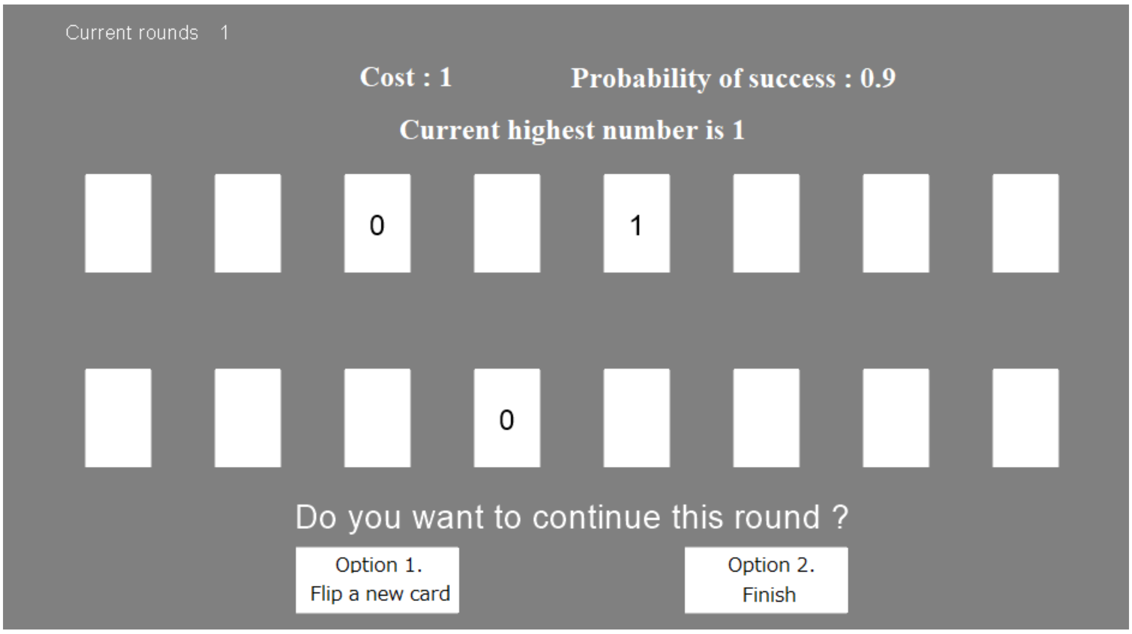

Figure 5: The main screen of the game.

product being out of stock in the store. To avoid confusion of terms, in the following context, we call $1 - q$ in the experiments the "failure probability."

Figure 5 shows the main game screen.[31] At the top of the main screen, the cost and probability of success are displayed. In addition, the highest number ever found in the round is always displayed at the top middle of the screen and is initially zero. Subjects can flip a card by clicking the "Flip a new card" button and then clicking on the card of their choice. This redundant procedure is designed to avoid mishandling and frenzied card flipping. There are 16 cards on the screen, but if subjects know the number of whole 16 cards and still wish to continue, they can obtain 16 new cards by clicking "Flip a new card." In such cases, the previous highest number is still displayed on the screen. Subjects can finish the round by clicking the "Finish" button at any time. At the end of the round, the highest number of cards minus the total search cost is recorded as the result of the round. In that sense, our experimental design is quite

[31] In the actual experiment, everything on the screen is written in Japanese.

susceptible to *recall*.[32]

For example, let search cost was 1; the subject flipped the card three times, and the numbers in order were "4," "0," and "7." This means that she failed the second observation. In this case, the highest number began at 0 and changed to 4, 4, and 7. If she clicks the "Finish" button after observing 7, the round ends immediately, and the result of this round is recorded as 4, which equals the highest number of 7 minus the total search cost of 3 (1 times 3: search cost per card multiplied by the number of card flips during the round). If this round is chosen as the earnings, she receives 900 JPY, including the participation fee. Each session included nine treatments: search costs were 1, 2, or 3, with the success probability of each card being 0.5, 0.9, or 1. Each treatment was performed five times, and the order of treatments was random for each subject. We tracked all the cards flipped, the sequence of choice processes, reaction times, the number of flips, the number of times subjects failed to flip cards, and when they finished.

3.3 Identification strategy

One of our parameters of interest is the reservation value $x(s;q)$. Hereafter, we treat both s and q as parameters, so we use (s,q) instead of $(s;q)$. Like Caplin et al. (2011), we assume that all individuals within a given choice environment have the same reservation match value $x^d(s;q)$.[33] Let $\tilde{x}^d(s,q)$ be the theoretical value of $x(s,q)$,[34] and $\hat{x}^d(s,q)$ be the estimated value from data, where $d \in \{Uni, Exp\}$ represents the type of distribution

[32] See Section 4.1 for a detailed definition and discussion of recall.

[33] We also estimated individuals' reservation values and found a similar overall trend, although the reservation values could vary among subjects. Because our interest is the impact of the possibility of stock-out on search behavior, not the relationship between individual attributes and stock-out, we use the aggregated data to estimate reservation value. Furthermore, given the small sample size of subjects (because of multiple treatments), we need to impose a more stringent assumption to estimate each individual's reservation values. Details are available upon request.

[34] Our main concern is not the difference between the predicted and observed values, but whether the directions of change in the predicted and observed results are the same. Therefore, we used the predicted values based on the discretized distribution used in the experiment, not those based on the continuous distribution.

of match utilities. Estimated $\hat{x}^d(s,q)$ represents subjects' search intensity with given s and q.

Our data are regarded as binary choice process data. We can then express the subjects' choices (whether to continue searching or terminate) by introducing a stochastic error term ϵ that is *iid* in specific treatments and sessions. Let u_t be the highest "number" of the card that has been found in the t-th choice (search). We assume that subjects finish the search at the t-th choice if and only if

$$u_t \geq x^d(s,q) + \epsilon_t \quad \text{where } \epsilon_t \sim N(0, \sigma^d_{s,q}). \tag{13}$$

The probability of a subject terminating the search at the t-th choice is expressed by $\Phi(u_t - x^d(s,q))$. According to this, we estimate $x^d(s,q)$ and $\sigma^d_{s,q}$ using the maximum likelihood estimation method.

4 Empirical results

4.1 Recall rate

First, we ascertained whether each subject followed the optimal search rule. We stipulate that a subject made a "recall" if the number on the last flipped card (including zero) when the subject pressed the "Finish" button was smaller than or equal to the previous highest number. In other words, we regard such behavior as violation of the optimal search rule. We considered that subjects followed the optimal rule only when they observed and terminated the number that was strictly higher than the previous highest number. This is because if subjects are rational and follow the optimal search rule, they should never return to the previous cards. As mentioned above, such an experimental design is quite susceptible to recalls.

We note that in previous studies on laboratory experiments, recalls are often observed. Because the optimal search rule we consider cannot explain such recall behavior, subjects who made recalls may follow other search strategies. For example, Kogut

(1990) and Sonnemans (1998) suggest another hypothesis—that total search cost affects search decisions—to explain such recall behavior. As another example, De los Santos et al. (2012) show that in practice, 38% of people made recalls in the online book market.[35] They suggest a fixed sample size search as an alternative search rule. We discuss the number of searches below.

Table 2: Summary of data

$d = Uni$		(1)	(2)		$d = Exp$		(3)	(4)	
s	$1-q$	Best card	Recall(%)	n	s	$1-q$	Best card	Recall(%)	n
1	0	15.43	0.07	105	1	0	11.03	0.14	100
	0.1	14.21	0.09	105		0.1	11.37	0.10	100
	0.5	12.85	0.08	105		0.5	8.38	0.11	100
2	0	14.64	0.02	105	2	0	9.32	0.12	100
	0.1	13.15	0.06	105		0.1	8.44	0.15	100
	0.5	9.88	0.12	105		0.5	5.20	0.15	100
3	0	13.74	0.03	105	3	0	7.46	0.08	100
	0.1	12.80	0.03	105		0.1	6.04	0.07	100
	0.5	6.37	0.11	105		0.5	2.02	0.19	100

Note: Each subject performed five rounds for each treatment. "Best card" represents the average highest number at which the subjects finished the rounds in each treatment. "Recall" is the dummy variable, which takes a value of 1 when the number of the last flipped card when they finished was less than or equal to the highest number they had ever found in the round. n is the number of rounds we used for the estimation.

Table 2 presents a summary of our data. Before considering recall, we first mention the tendencies in the highest number of cards. From Columns (1) and (3), one can see that most subjects succeeded in finding a better number when the search cost or stock-out probability was low. We note that if a subject follows the optimal search rule, then $x^d(s;q)$ should be lower than those numbers (but should be higher than the second-highest number in the round).

[35] We note that De los Santos et al. (2012) conducted a field study, so various factors (*e.g.*, repeating purchasing behavior) could influence recall behavior. For example, their study found a recall pattern whereby consumers first observed a price at Amazon.com and then eventually returned there. They mention the limitations of their data and difficulty of generalizing their result to other markets.

Let us now consider whether the subjects followed the optimal search rule. Columns (2) and (4) show the population of the whole rounds in which subjects made recall. We can confirm that only a few rounds (9% on average) were inconsistent with the optimal search rule from the data.[36] In that sense, we consider most subject search behavior can be explained by the optimal search rule in our experimental design.

This is a somewhat interesting result because the recall rate seems lower than those reported in previous studies with free-recall designs. For example, Sonnemans (1998) conducted a search experiment with a free-recall design that is most similar to ours; he reports 13.9% of observation recalls in his baseline treatment.[37] One notable difference in our experimental design is that we control for risk-hedging behavior by randomly choosing one round to determine earnings. In the experimental design of Sonnemans (1998), earnings are the sum derived from all rounds, so subjects may have an incentive to hedge their risks. In addition, Sonnemans (1998) set time pressure and high initial fixed costs for subjects, making subjects more timid about searching. In our experimental design, we eliminated the incentive for such risk-hedging behavior and made subjects able to contemplate search decisions, which may have led to a lower recall rate than his study.

In the following analyses, we usually exclude data on recall.

4.2 Reservation value

We now focus on the reservation values of each treatment. The identification strategy is described in Section 3.3.

Table 3 shows a summary of the data. Columns (1) and (3) are the estimated reservation values $\hat{x}^d(s, q)$. From the table, we can confirm the consistency with Proposition

[36] We also calculated the recall rate, excluding rounds that the subject didn't participate was around 11%.

[37] As further examples, Schotter and Braunstein (1981) report a recall rate of around 25%, and Kogut (1990) finds that subjects made recalls in 33.9% of rounds. However, those two studies differ from our experimental design with respect to earnings, like that of Sonnemans (1998).

Table 3: Estimated parameters

$d = Uni$			(1)	(2)		$d = Exp$			(3)	(4)	
s	$1-q$	$\tilde{x}$	$\hat{x}$	$\hat{\sigma}$	n	s	$1-q$	$\tilde{x}$	$\hat{x}$	$\hat{\sigma}$	n
1	0	14.17	12.09	2.39	182	1	0	9.03	8.09	2.76	225
	0.1	13.83	11.32	2.99	227		0.1	8.60	8.58	2.48	292
	0.5	11.56	9.26	2.71	343		0.5	5.91	4.57	1.16	239
2	0	11.56	9.91	1.22	186	2	0	5.91	6.71	2.58	196
	0.1	11.06	8.84	2.72	172		0.1	5.42	5.26	1.98	182
	0.5	7.85	6.96	2.85	181		0.5	2.38	3.16	1.74	129
3	0	9.55	8.05	1.73	147	3	0	3.89	4.14	3.49	123
	0.1	8.94	7.97	2.62	161		0.1	3.35	3.59	2.19	107
	0.5	5.00	5.94	2.80	139		0.5	0.13	3.20	1.72	49

Note: We excluded the rounds where subjects did not flip any cards or made recalls. $\tilde{x}^d(s,q)$ is the theoretical value of $x^d(s,q)$. n indicates the sample size (the number of choices) used for the estimation.

(1), that is, the estimated reservation value $\tilde{x}^d(s,q)$ (i) decreases in search cost s, and (ii) increases in the success probability q (*i.e.*, it decreases the failure (stock-out) probability $1-q$), regardless of the distribution of match utility. Most of the estimated reservation values are lower than the theoretical predicted values,[38] but the difference is not very large (except for $\tilde{x}^{Exp}(3, 0.5)$).[39] The tuple is also consistent.

4.3 Number of searches

In this subsection, we test the theoretical prediction of the number of searches (Proposition 2).[40]

[38] One plausible reason for the lower estimated values is that risk aversion reduces x (the proportion of risk-averse subjects was 59% in our experiment). However, in the exponential session, the estimated x is smaller than the theoretical x only when $s = 1$. Therefore, we must consider factors other than risk aversion (there may be multiple factors, including the special nature of the exponential distribution).

[39] As the theoretical predicted value $\tilde{x}^{Exp}(3, 0.5)$ is almost zero, each subjects participation in the round depends almost entirely on ϵ_0. Thus, we cannot compare the reservation values in this treatment. We will discuss this point in section 4.4.

[40] To confirm the robustness of Proposition 2, we conducted an additional experiment with the distribution which has the log-convex survival function. We used the Pareto distribution with $\beta = 1.2$ (which is discretized and truncated at 30) and confirmed that the number of searches decreases with

Table 4: Number of searches by search costs

$d = Uni$			(1)	(2)					
$1-q$	s	$E(\#\vert\tilde{x})$	$\#$	SD	n				
0	1	3.33	1.86	1.18	98	$N =$	301		
	2	2.22	1.81	1.11	103	$Average =$	1.71	F value=	3.8343
	3	1.82	1.47	0.90	100	$SD =$	1.08	p value=	0.0227
0.1	1	3.17	2.36	1.88	96	$N =$	293		
	2	2.47	1.76	1.08	98	$Average =$	1.91	F value=	7.4941
	3	1.85	1.63	1.17	99	$SD =$	1.45	p value=	0.0007
0.5	1	4.44	3.54	2.27	97	$N =$	231		
	2	3.08	2.35	1.95	77	$Average =$	2.87	F value=	8.1592
	3	2.50	2.44	2.14	57	$SD =$	2.20	p value=	0.0004

$d = Exp$			(3)	(4)					
1-q	s	$E(\#\vert\tilde{x})$	$\#$	SD	n				
0	1	4.54	2.62	1.80	86	$N =$	250		
	2	2.25	2.31	1.99	85	$Average =$	2.18	F value=	8.5368
	3	1.62	1.56	1.08	79	$SD =$	1.73	p value=	0.0003
0.1	1	4.20	3.36	2.79	87	$N =$	237		
	2	2.50	2.27	1.53	80	$Average =$	2.45	F value=	17.0071
	3	1.80	1.53	0.96	70	$SD =$	2.11	p value=	0.0000
0.5	1	4.50	2.95	2.24	81	$N =$	165		
	2	2.75	2.15	1.49	60	$Average =$	2.53	F value=	3.9264
	3	2.00	2.04	1.63	24	$SD =$	1.95	p value=	0.0216

Note: We excluded the rounds in which subjects made recalls or did not flip any cards. $E(\#|\tilde{x})$ is the theoretical expected number of searches calculated using $\tilde{x}^d(s,q)$. $\#$ and SD are the sample mean of the number of searches and sample standard deviation, calculated from the data. n is the number of rounds we used for the estimation, and N is the sum of observations for each $1-q$. *Average* is the mean of the number of searches for each success probability.

Table 4 is a summary of statistics for the number of searches. Note that the theoretically expected number of searches $E(\#|\tilde{x})$ contains the discretization error and error from truncation because we discretized the number of cards in the experiment. Columns (1) and (3) show the average number of searches for each distribution. From Columns (1) and (3), we can confirm Proposition 2-(i): the number of searches de-

the failure probability (stock-out probability), as predicted by theory. See the appendix for more details of the experiment.

creases in the search cost in both sessions. As the search cost increases, subjects tend to finish searches earlier. We statistically tested this hypothesis using an analysis of variance. There is a statistical difference in the number of searches between s at the 1% significance level for both distributions: the number of searches decreases in s when other environments are fixed.[41]

Table 5: Number of searches by success probability

$d = Uni$ s	 $1-q$	 $E(\#\|\tilde{x})$	(1) $\#$	(2) SD	 n				
1	0	3.33	1.86	1.18	98	$N=$	291		
	0.1	3.17	2.36	1.88	96	*Average* =	2.58	F value=	21.4616
	0.5	4.44	3.54	2.27	97	SD=	1.96	p value=	0.0000
2	0	2.22	1.81	1.11	103	$N=$	278		
	0.1	2.47	1.76	1.08	98	*Average* =	1.94	F value=	4.7388
	0.5	3.08	2.35	1.95	77	SD=	1.40	p value=	0.0095
3	0	1.82	1.47	0.90	100	$N=$	256		
	0.1	1.85	1.63	1.17	99	*Average* =	1.75	F value=	9.7890
	0.5	2.50	2.44	2.14	57	SD=	1.41	p value=	0.0001

$d = Exp$ s	 $1-q$	 $E(\#\|\tilde{x})$	(1) $\#$	(2) SD	 n				
1	0	4.54	2.62	1.80	86	$N=$	254		
	0.1	4.20	3.36	2.79	87	*Average* =	2.98	F value=	2.2189
	0.5	4.50	2.95	2.24	81	SD=	2.33	p value=	0.1109
2	0	2.25	2.31	1.99	85	$N=$	225		
	0.1	2.50	2.27	1.53	80	*Average* =	2.25	F value=	0.1560
	0.5	2.75	2.15	1.49	60	SD=	1.70	p value=	0.8557
3	0	1.62	1.56	1.08	79	$N=$	173		
	0.1	1.80	1.53	0.96	70	*Average* =	1.61	F value=	2.0304
	0.5	2.00	2.04	1.63	24	SD=	1.13	p value=	0.1345

Note: We excluded the rounds in which subjects made recalls or did not flip any cards. $E(\#|\tilde{x})$ is the theoretical number of searches expected from $\tilde{x}^d(s, q)$. $\#$ and SD are the sample mean of the number of searches and sample standard deviation, which are calculated from the data. n is the number of rounds used for the estimation, and N is the sum of observations for each cost. *Average* is the mean number of searches for each cost.

[41] The observed number of searches seems to be lower than $E(\#|\tilde{x})$ for some treatments. This appears to be attributable to the underestimation of x, as shown in Table 3.

As Proposition 2-(ii) shows, the effect of a change in the failure probability $1-q$ on the number of searches depends on the distribution of match utilities. Specifically, an increase in $1-q$ increases the expected number of searches with the uniform distribution (because the survival function is log-concave), while it does not affect the number of searches in the exponential distribution (because the survival function is log-linear).

Table 5 shows differences in the effect of the failure probability on the number of searches. The theoretically predicted expected number of searches $E(\#|\tilde{x})$ is decreasing in $1-q$ with a uniform distribution but not in an exponential distribution.[42] From Column (1), as for the theoretical prediction, one can see that there is a statistical difference in the number of searches between different values of q at 1% significance level in the uniform session. On the other hand, the results of the exponential session are insignificant. Because p values are significantly high for all of the treatments in the session, we cannot reject the null hypothesis that the mean of the number of searches differs by the success probability.

These results are consistent with the prediction that the effect of a change in $1-q$ on $E(\#|\tilde{x})$ depends on the distribution of match utilities. With fixed s, an increase in $1-q$ increases the number of searches only with the uniform distribution.

4.4 Participation decision

Finally, we consider the participation decisions presented in Lemma 1 and Proposition 3. Subjects should finish the round without flipping any cards unless the expected benefits of participation are positive. In that case, the result of the round is zero.

Following Caplin et al. (2011), we introduced ϵ_t to estimate $\hat{x}^d(s,q)$. The probability of participation depends on $x^d(s,q)$ and $\sigma^d_{s,q}$, and it decreases in the reservation values $x^d(s,q)$. Whether subjects participate depends on $x^d(s,q)+\epsilon_0$. As we have chosen s

[42] As mentioned above, the variations in $E(\#|\tilde{x})$ in the exponential session of Tables 4 and 5 are caused by the discretization error and error from truncation. From (12), one can see $E(\#|\tilde{x})$ in the exponential distribution converges as the upper-bound of the number increases with given s (but the discretization errors remains).

and $1-q$ so that the theoretical values of $\bar{x}$ are all positive, subjects may choose not to participate when they draw a small (negative) stochastic error term ϵ_0 at the beginning of the rounds.

Table 6: Participation rate

$d = Uni$ s	$1-q$	(1) $\hat{p}$	n	$d = Exp$ s	$1-q$	(2) $\hat{p}$	n
1	0	1.00	105	1	0	1.00	100
	0.1	1.00	105		0.1	0.97	100
	0.5	1.00	105		0.5	0.92	100
2	0	1.00	105	2	0	0.97	100
	0.1	0.99	105		0.1	0.95	100
	0.5	0.86	105		0.5	0.75	100
3	0	0.98	105	3	0	0.87	100
	0.1	0.97	105		0.1	0.77	100
	0.5	0.66	105		0.5	0.43	100

Note: $\hat{p}$ is a sample mean of participation. If a subject had flipped at least one card, we defined this as participation in the round. n is the number of rounds we used for the estimation.

Table 6 shows the proportion of rounds in which the subjects did not search. In some treatments, subjects finished the round immediately and did not click on any cards. In both sessions, subjects tended not to participate as s and $1-q$ increased. This trend is consistent with the theoretical predictions presented in Lemma 1 and Proposition 3.

However, the participation rate $\hat{p}$ for treatments where $1-q = 0.5$ seems to be lower than that for other treatments (*e.g.*, $\hat{p} = 0.43$ in $\tilde{x}^{Exp}(3, 0.5)$ treatment). One possible reason is that the lower the probability of success, the more likely this is to discourage participation. To confirm this, we conducted a statistical test of whether the observed participation rate was different from that expected for the $\tilde{x}^{Exp}(3, 0.5)$ treatment.[43] For the treatment, the theoretical value $\tilde{x}^{Exp}(3, 0.5)$ is positive but almost

[43] Unless parameter σ is known or $x = 0$, we cannot statistically test the sample means of participation $\hat{p}$, because the probability of participation p depends on both x and σ. On the condition that $x = 0$, we can statistically test the hypothesis such that $\hat{p} = p$ because p is independent of σ and $p = 0.5$.

zero. Therefore, the participation decision mostly depends on ϵ_0, and the expected participation rate should be at least 0.5. However, in the experiment, the observed participation rate of $\hat{p} = 0.43$ is statistically lower than 0.5. This implies that the impact of the failure probability on the subject's participation decisions could be much stronger as it increases.

5 Concluding remarks

In this paper, we investigate the effect of stock-out on individual consumers' search behavior through both search-theoretic and experimental approaches.

We show that the stock-out affects consumers' search behavior through three channels. Theoretically, an increase in stock-out probability decreases the expected benefit of further searches, so consumer's search intensity decreases as the stock-out probability increases. On the other hand, the effect of an increase in the stock-out probability on the expected number of searches is ambiguous. An increase in the stock-out probability directly increases the number of expected searches but indirectly decreases it by reducing search intensity. Whether the number of expected searches (and the total search cost) increases or decreases with an increase in the stock-out probability hinges on the shape of the survival function under the given distribution. If the survival function is log-concave (log-convex), the number of searches increases (decreases) in the stock-out probability. Furthermore, an increase in the stock-out probability makes it easier for consumers to decide not to participate in the market.

We confirmed these theoretical predictions by conducting a search-theoretic experiment. Almost all experimental results were consistent with the theoretical predictions. An increase in the stock-out probability induced subjects to decrease their search intensity. We also confirmed that an increase in the stock-out probability increased (decreased) the number of searches with the uniform distribution (the Pareto distribution), which has a log-concave (log-convex) survival function. In addition, it did not affect the expected number of searches with the exponential distribution, which has a log-linear survival function. All of those results are consistent with our predictions. Moreover, an increased stock-out probability tended to reduce the number of participants in the search. However, we found that the reservation values and the number of searches were less than the theoretical predictions, especially in the treatment with low search costs. Risk preference may have led to the result, but further study is required.

We believe that these results, which reveal the impact of stock-out on consumers' search behavior, will be useful to many people, especially to practitioners in retailing. Moreover, as our experiment has shown that our simple consumer search-theoretic framework captured consumers' search behavior well, we believe that the framework will lead to more in-depth discussions about stock-out in the future.

Discussion: risk preference

In this paper, we focus on the impact of the possibility of stock-out on consumers' search behavior as an aggregate behavior rather than individual search behavior. However, each consumer's search behavior (including her response to the stock-out) may be influenced by her risk preference (*cf.* Schunk and Winter (2009)). The relationship between each individual's risk preference and her search behavior with the possibility of stock-out is a fascinating issue (it may also explain our observations of a higher rate of nonparticipation when the failure probability was high). Nevertheless, it is necessary to repeat dozens of experiments with the same treatment for each subject to analyze it. In addition, it may require a structural estimation of search costs. We did not conduct an in-depth analysis of risk preference in this paper because such an analysis could obscure our objective of analyzing the impact of stock-out. Instead, we are planning further analysis of this issue as a new study.

References

Aastrup, J. and H. Kotzab (2010). Forty years of out-of-stock research–and shelves are still empty. *The International Review of Retail, Distribution and Consumer Research 20*(1), 147–164.

Andersen Consulting (1996). *Where to Look for Incremental Sales Gains: The Retail Problem of Out-of-stock Merchandise.* The Coca-Cola Retailing Research Council, Atlanta, GA.

Anderson, E. T., G. J. Fitzsimons, and D. Simester (2006). Measuring and mitigating the costs of stockouts. *Management Science 52*(11), 1751–1763.

Anderson, S. P. and R. Renault (1999). Pricing, product diversity, and search costs: A bertrand-chamberlin-diamond model. *The RAND Journal of Economics 30*(4), 719–735.

Bagnoli, M. and T. Bergstrom (2005). Log-concave probability and its applications. *Economic Theory 26*(2), 445–469.

Balachander, S. and P. H. Farquhar (1994). Gaining more by stocking less: A competitive analysis of product availability. *Marketing Science 13*(1), 3–22.

Brown, M., C. J. Flinn, and A. Schotter (2011). Real-time search in the laboratory and the market. *The American Economic Review 101*(2), 948–974.

Burdett, K. and K. L. Judd (1983). Equilibrium price dispersion. *Econometrica: Journal of the Econometric Society 51*(4), 955–969.

Campo, K., E. Gijsbrechts, and P. Nisol (2000). Towards understanding consumer response to stock-outs. *Journal of Retailing 76*(2), 219–242.

Caplin, A., M. Dean, and D. Martin (2011). Search and satisficing. *The American Economic Review 101*(7), 2899–2922.

Carpio, C. E. and O. Isengildina-Massa (2009). Consumer willingness to pay for locally grown products: The case of south carolina. *Agribusiness: An International Journal 25*(3), 412–426.

Corsten, D. and T. Gruen (2003). Desperately seeking shelf availability: An examination of the extent, the causes, and the efforts to address retail out-of-stocks. *International Journal of Retail & Distribution Management 31*, 605–617.

Corsten, D. and T. Gruen (2004). Stock-outs cause walkouts. *Harvard Business Review 82*(5), 26–28.

De los Santos, B., A. Hortaçsu, and M. R. Wildenbeest (2012). Testing models of consumer search using data on web browsing and purchasing behavior. *The American Economic Review 102*(6), 2955–2980.

Emmelhainz, M. A., J. R. Stock, and L. W. Emmelhainz (1991). Consumer responses to retail stock-outs. *Journal of Retailing 67*(2), 138–148.

Fitzsimons, G. J. (2000). Consumer response to stockouts. *Journal of Consumer Research 27*(2), 249–266.

Heinemann, F., R. Nagel, and P. Ockenfels (2009). Measuring strategic uncertainty in coordination games. *The Review of Economic Studies 76*(1), 181–221.

Hey, J. D. (1987). Still searching. *Journal of Economic Behavior & Organization 8*(1), 137–144.

Holt, C. A. and S. K. Laury (2002). Risk aversion and incentive effects. *The American Economic Review 92*(5), 1644–1655.

Kim, M. and S. J. Lennon (2011). Consumer response to online apparel stockouts. *Psychology & Marketing 28*(2), 115–144.

Kogut, C. A. (1990). Consumer search behavior and sunk costs. *Journal of Economic Behavior & Organization 14*(3), 381–392.

Lippman, S. A. and J. J. McCall (1979). *Studies in the Economics of Search*, Volume 123. North-Holland.

Lusk, J. L. (2003). Effects of cheap talk on consumer willingness-to-pay for golden rice. *American Journal of Agricultural Economics 85*(4), 840–856.

McFadden, D. (1973). Conditional logit analysis of qualitative choice behavior. *Frontiers in Econometrics*, 105–142.

Moraga-González, J. L., Z. Sándor, and M. R. Wildenbeest (2017). Prices and heterogeneous search costs. *The RAND Journal of Economics 48*(1), 125–146.

Peirce, J. W. (2007). Psychopy–psychophysics software in python. *Journal of Neuroscience Methods 162*(1-2), 8–13.

Perloff, J. M. and S. C. Salop (1985). Equilibrium with product differentiation. *The Review of Economic Studies 52*(1), 107–120.

Piketty, T. and E. Saez (2003). Income inequality in the united states, 1913–1998. *The Quarterly Journal of Economics 118*(1), 1–41.

Pizzi, G. and D. Scarpi (2013). When out-of-stock products do backfire: Managing disclosure time and justification wording. *Journal of Retailing 89*(3), 352–359.

Sampaio, M. and C. Machline (2009). Consumer response to stockouts. In *POMS 20th Annual Conference, Orlando, Florida, USA, May.*

Scarpa, R., M. Thiene, and K. Train (2008). Utility in willingness to pay space: A tool to address confounding random scale effects in destination choice to the alps. *American Journal of Agricultural Economics 90*(4), 994–1010.

Schotter, A. and Y. M. Braunstein (1981). Economic search: An experimental study. *Economic Inquiry 19*(1), 1–25.

Schunk, D. and J. Winter (2009). The relationship between risk attitudes and heuristics in search tasks: A laboratory experiment. *Journal of Economic Behavior & Organization 71*(2), 347–360.

Sonnemans, J. (1998). Strategies of search. *Journal of Economic Behavior & Organization 35*(3), 309–332.

Stahl, D. O. (1989). Oligopolistic pricing with sequential consumer search. *The American Economic Review 79*(4), 700–712.

Stigler, G. J. (1961). The economics of information. *Journal of Political Economy 69*(3), 213–225.

Varian, H. R. (1980). A model of sales. *The American Economic Review 70*(4), 651–659.

Weitzman, M. L. (1979). Optimal search for the best alternative. *Econometrica: Journal of the Econometric Society 47*(3), 641–654.

Wolinsky, A. (1986). True monopolistic competition as a result of imperfect information. *The Quarterly Journal of Economics 101*(3), 493–511.

Zhou, C., W. Tang, and R. Zhao (2015). Optimal consumer search with prospect utility in hybrid uncertain environment. *Journal of Uncertainty Analysis and Applications 3*(1), 1–20.

Zinn, W. and P. C. Liu (2008). A comparison of actual and intended consumer behavior in response to retail stockouts. *Journal of Business Logistics 29*(2), 141–159.

Zwick, R., A. Rapoport, A. K. C. Lo, and A. Muthukrishnan (2003). Consumer sequential search: Not enough or too much? *Marketing Science 22*(4), 503–519.

Appendix

Optimal search rule and reservation value

The following procedure follows Weitzman (1979) straightforwardly. Let u^k be the match utility for firm (product) k, which is randomly drawn from the common cumulative distribution function $F(u)$. Let the collection K of N products be partitioned into any set S of sampled products and its complement $\bar{S}$ of unsampled products; *i.e.*, $S \cup \bar{S} = K$ and $S \cap \bar{S} = \varnothing$. Moreover, let x be the maximum match utility for sampled products:

$$x = \max_{k \in S} u^k. \tag{14}$$

Let q be the probability of obtaining new product information for each store. Furthermore, let $V(x, \bar{S}; q)$ be the value function for the consumer who faces the binary options of stopping or continuing the search, given x, $\bar{S}$, and q. Then, the value function for consumer i with search cost s_i must satisfy the following recursive equation for each subset $\bar{S}$ and x:

$$\begin{aligned} &V(x, \bar{S}; q) \\ &= \max \left\{ x, \max_{k \in \bar{S}} \left\{ -s_i + qV(\bar{S} - \{k\}, x) \int_{\underline{u}}^{x} dF(u) + q \int_{x}^{\bar{u}} V(\bar{S} - \{k\}, u^k) dF(u) + (1-q)V(x) \right\} \right\}. \end{aligned} \tag{15}$$

It is known that this sort of dynamic programming format is stationary; at any stage with the state of the system $(x, \bar{S})$, each consumer's optimal policy is to maximize the right-hand side (RHS) of (15) (see Weitzman (1979)).

We now turn to the consumer's search decision in each search period. Let x^t be the "hypothetical" highest match utility at time t. If a consumer stops searching at the time, she obtains

$$x^t. \tag{16}$$

On the other hand, if a consumer visits another store, she can expect a net benefit of

$$-s_i + q\left(x^t \int_{\underline{u}}^{x^t} dF(u) + \int_{x^t}^{\bar{u}} udF(u)\right) + (1-q)x^t. \tag{17}$$

This corresponds to the RHS of (15). The first term in the large parentheses is the expected benefit when the match utility at the new store is less than or equal to the known (and highest) value of x^t. The latter represents the expected incremental benefit when a match utility greater than or equal to x^t is obtained. The third term represents an expected benefit when stock-out occurs.

Now consider a situation where a consumer is indifferent between visiting a new store and purchasing a known option (with the highest match utility). That is, (16) and (17) hold with equality:

$$s_i = q\int_{x^t}^{\bar{u}} (u - x^t)dF(u). \tag{18}$$

Then we obtain the critical value x^t, which satisfies (18) and call it the *reservation match value*, which is mentioned in eq.(1) in the main context. Then, according to the optimal search policy of Weitzman (1979), a continuing search is optimal if $x^{t-1} < x^t$; stopping the search is optimal if $x^{t-1} > x^t$ for any $1 \leq t \leq T$.

Proof of Proposition 2

From (5), the partial derivative of # with respect to s is given by

$$\frac{\partial \#}{\partial s} = \frac{f(x(s;q))}{q[1-F(x(s;q))]^2} \cdot \frac{\partial x}{\partial s}. \tag{19}$$

Because $\partial x/\partial s$ is negative (see the first part of (3)), this expression is nonpositive. Therefore, the expected number of searches is increasing in s.

On the other hand, the partial derivative of # with respect to q is given by

$$\frac{\partial \#}{\partial q} = \frac{f(x(s;q))}{q[1-F(x(s;q))]^2} \cdot \frac{\partial x}{\partial q} - \frac{1}{q^2[1-F(x(s;q))]}. \tag{20}$$

Because the presence of the second term on the RHS, although the signs of $\partial x/\partial s$ and $\partial x/\partial q$ are opposite (recall (3)), the sign of expression (20) is ambiguous. To confirm this, we rearrange this expression as follows:

$$\frac{\partial \#}{\partial q} = \frac{1}{q^2[1-F(x)]^2}\left(\frac{f(x)}{1-F(x)}\int_x^{\bar{u}}[1-F(u)]du - [1-F(x)]\right). \tag{21}$$

Let $\bar{F} = 1-F(x)$ and $H(x)$ be the right-hand integral of $\bar{F}$: $H(x) = \int_x^{\bar{u}} \bar{F} du$. Then, we have $H'(x) = -\bar{F}(x)$, and $H''(x) = f(x)$. Using these notations, (21) can be rewritten as follows:

$$\frac{\partial \#}{\partial q} = \frac{1}{q^2\bar{F}^2}\left(-\frac{H''(x)}{H'(x)}H(x) + H'(x)\right). \tag{22}$$

Therefore, the expected number of searches is increasing in the stock-out probability $1-q$ if the expression in the large parentheses in eq. (22) is *negative*. It would be useful to rewrite this condition as follows:

$$-H'(x) + \frac{H''(x)}{H'(x)}H(x) > 0. \tag{23}$$

(We note again that this inequality means that eq. (22) becomes negative.) Because $H'(x) < 0$, this inequality can be rewritten as

$$\frac{H''(x)}{H(x)} - \left(\frac{H'(x)}{H(x)}\right)^2 < 0. \tag{24}$$

We note that $\bar{F}(x)$ is log-concave if $(ln(\bar{F}(x)))'' < 0$, and if $\bar{F}$ is log-concave, then the right-hand integral $H(x)$ is also log-concave (see Bagnoli and Bergstrom (2005)). Because $\bar{F}(x) = 1-F(x)$, $\bar{F}'(x) = -F'(x)$ and $\bar{F}''(x) = -F''(x)$. Then, we have

$$ln(\bar{F}(x))'' < 0 \Leftrightarrow \frac{-F'(x)^2 - (1-F(x))F''(x)}{(1-F(x))^2} = \frac{\bar{F}''(x)}{\bar{F}(x)} - \left(\frac{\bar{F}'(x)}{\bar{F}(x)}\right)^2 < 0. \tag{25}$$

Hence, if $1-F(x)$ is log-concave, then $H(x)$ is also log-concave, so inequality (24) holds. Therefore, if $1-F(x)$ is log-concave, (22) becomes negative, which means that the expected number of searches is increasing in $1 - q$. The discussion is the same for log-linearity and log-convexity. $\square$

Additional experiment with a log-convex survival function

To confirm the robustness of Proposition 2, we conducted an additional experiment with the distribution of the log-convex survival function.[44]

The experiment was conducted at the Institute of Social Economic Research, Osaka University in September 2020. The subjects were 21 undergraduate students from various departments of Osaka University. Because our focus is on the relationship between stock-out probability and the number of searches, we set the search cost to be constant ($s = 1$) and only varied the success probability, which could be 0, 0.1, or 0.5. The game consisted of 33 rounds (including three practice rounds). A number of each card was chosen based on the Pareto distribution with $\beta = 1.2$, which was discretized and truncated at 30. We gave a graph of the distribution and extra information about probability in the instructions. Unlike the other two experiments, because this type of distribution has a large discretization error and too little expected reward, we set the upper bound of the distribution at 30 and multiplied the result of each round by 200 JPY. We chose a random round from 30 rounds as the earnings. The average earnings were about 1120 JPY (including the participation fee of 500 JPY), and the average time required for the experiment was about 35 minutes. The experiment was computerized, and a recording of instructions was played.

The result is summarized in Table 7. As we have seen in Proposition 2-(ii), the number of searches decreases in the failure probability when the survival function of the match utility is log-convex (*e.g.*, Pareto distribution). Column (1) shows that the observed number of searches is actually decreasing in $1 - q$. There is a statistical difference in the number of searches between different values of q at the 5% significance

[44] Instructions and data are available upon request.

Table 7: Number of searches by success probability

$d = Pareto$			(1)	(2)					
s	$1-q$	$E(\#\vert\tilde{x})$	#	SD	n				
0.5	0	12.21	4.14	4.34	161	$N =$	479		
	0.1	10.90	3.75	3.57	163	$Average =$	3.09	F value=	3.3769
	0.5	7.83	3.09	2.74	155	SD=	2.74	p value=	0.0350

Note: We excluded the rounds in which subjects made recalls or did not flip any cards. $E(\#|\tilde{x})$ is the theoretical number of searches calculated using $\tilde{x}^d(s,q)$. # and SD are the sample mean of the number of searches and sample standard deviation, which are calculated from the data. n is the number of rounds we used for the estimation, and N is the sum of observations. *Average* is the mean of the number of searches.

level. Thus, we can conclude that Proposition 2-(ii) is robust with respect to the shape of the distribution of the survival function.

As in previous studies (*e.g.*, Sonnemans (1998)), we note that the number of searches was significantly lower than the theoretical value, and this tendency is more pronounced with the Pareto distribution than with the uniform or exponential distributions. One possible reason is that the subject's risk preference affects search behavior, and the magnitude of the effect of such risk preferences depends on the convexity of the survival function of the match utility distribution. However, because risk preference is not our main research concern, we do not discuss this further here.

Firm's Problem

Here, we describe the firm's problem and how pricing is affected by a change in q when consumers are heterogeneous in their search costs. Suppose consumer i's search cost s_i is drawn from the common $G(s)$ with the interval $[\underline{s}, \bar{s}]$. In addition, suppose that firms know this.

For the moment, we consider all consumers who decide to participate in the market and suppose that there are N firms. According to the optimal search rule, each consumer stops searching and purchases the product from firm k, which charges price p^k

when the net surplus from purchasing is greater or equal to

$$u^k - p^k \geq x_i(s_i) - p^*.$$

Then the probability of purchase of consumer i is given by

$$q[1 - [F(x_i(s_i) - p^* + p^k)]]. \tag{26}$$

Because each consumer searches in a random order, the probability of each consumer who decides to participate in the market visiting firm k first is $\frac{1}{N}$. Recall that the probability that a consumer will terminate her search in each period is $q(1 - F(x_i(s_i))$. In other words, she will visit the next firm with probability $(1 - q) + qF]x_i(s_i)] = 1 - q[1 - F(x_i(s_i))]$. Likewise, firm k is sampled second with probability $\frac{(1-q)+qF(x_i(s_i))}{N}$, and sampled third with probability $\frac{((1-q)+qF(x_i(s_i)))^2}{N}$. Therefore, the expected demand from such a consumer is given as

$$\frac{q}{N}[1 - [F(x_i(s_i)) - p^* + p^k)]]\frac{1 - [1 - q(1 - F(x_i(s_i)))]^N}{q[1 - F(x_i(s_i))]}. \tag{27}$$

Moreover, a consumer who decides to participate in the market and searches all firms but has not purchased may return and purchase from firm k with probability $(Pr[u^k - p^k > u^l - p^*])^{N-1}$ for $k \neq l$. Hence, the expected (remaining) demand from such a consumer is given by

$$q\int_{\underline{u}}^{x_i(s_i)-p^*+p^k} F(u - p^k + p^*)^{N-1}dF(u).^{45} \tag{28}$$

Now consider the deviant payoff for firm k. The unconditional demand for deviant firm k is given as follows:

$$\begin{aligned} D(s, p^k, p^*, q, N) = {} & \frac{1}{N}[1 - [F(x_i(s_i) - p^* + p^k)]]\frac{1 - [(1 - q) + qF(x_i(s_i))]^N}{[1 - F(x_i(s_i))]} \\ & + q\int_{\underline{u}}^{x_i(s_i)-p^*+p^k} F(u - p^k + p^*)^{N-1}dF(u). \end{aligned} \tag{29}$$

[45] Because the net surplus from purchasing from a firm is zero when the product is unavailable, we can regard the match utility of the firm as zero.

We note that when $D(s,p^k,p^*,q,N)|_{N\to\infty} \to \frac{1-[F(x_i(s_i))-p^*+p^k)]}{1-F(x_i(s_i))}$.[46]

So far, we have considered all consumers to be active in the market. Now we consider the participation decision. From Lemma 1, the conditional demand for firm k with respect to the expected number of consumers is given as follows:

$$D(s,p^k,p^*,q,N) = \int_{\underline{s}}^{s^*} \left\{ \frac{1}{N}[1-[F(x(s)-p^*+p^k)]] \frac{1-[(1-q)+qF(x(s))]^N}{[1-F(x(s))]} + q\int_{\underline{u}}^{x(s)-p^*+p^k} F(u-p^k+p^*)^{N-1} dF(u) \right\} g(s)ds, \quad \text{for} \quad p^k < p^*. \tag{30}$$

Each firm solves the following problem:

$$\max_{p^k} \ \pi = p^k D(s,p^k,p^*,q,N). \tag{31}$$

The FOC, which is evaluated at $p^k = p^*$ is given by

$$0 = D(s,p^k,p^*,q,N) - p^* \int_{\underline{s}}^{s^*} \left\{ \frac{1}{N} \frac{\left(1-[(1-q)+qF(x(s))]^N\right)}{[1-F(x(s))]} f(x(s)) - q\int_{\underline{u}}^{x(s)} F(u)^{N-1} f'(u)du \right\} g(s)ds.$$

Assuming that $D(s,p^*,p^*,q,N) = G\left(q\int_{p^*}^{\bar{u}}(u-p^*)dF(u)\right)$ $(= G(s(p^*)))$, we have

$$p^* = \frac{G\left(q\int_{p^*}^{\bar{u}}(u-p^*)dF(u)\right)}{\int_{\underline{s}}^{s^*} \left\{ \frac{(1-[(1-q)+qF(x(s))]^N)}{[1-F(x(s))]} f(x(s)) - qN\int_{\underline{u}}^{x(s)} F(u)^{N-1} f'(u)du \right\} g(s)ds}. \tag{32}$$

In particular, when there is an infinite number of firms, this expression can be written by

$$p^* = \frac{G\left(q\int_{p^*}^{\bar{u}}(u-p^*)dF(u)\right)}{\int_{\underline{s}}^{s^*} \left\{ \frac{f(x(s))}{[1-F(x(s))]} \right\} g(s)ds} = \frac{G(s(p^*(s,q)))}{\int_{\underline{s}}^{s^*} \left\{ \frac{f(x(s))}{[1-F(x(s))]} \right\} g(s)ds}, \tag{33}$$

[46] Note that when there are infinitely many firms, a consumer who once decided to participate in the market and left firm k will not return to firm k (she finally purchases from another firm). Therefore, once a consumer decides to participate, the demand for each firm is affected by the stock-out probability $1-q$ only through a change in the reservation value.

which is a similar expression to the equilibrium price of Moraga-González et al. (2017) except for the numerator and s^*. Indeed, it is the same as theirs when we set $q = 1$. According to Theorem 1 in their paper, it can be shown that when the function $h(x_i^{-1}(t))$ is log-concave in t and when there are an infinite number of firms, then there exists a unique pure-strategy Nash equilibrium. On the other hand, when there is a finite number of firms, it is difficult to show the uniqueness and existence of the price in (32) in a general form. We note that when the upper bound of s is small, and $q = 1$, (33) will degenerate to the well-known price form $p^* = \frac{1-F(x(s))}{f(x(s))}$.

Again, according to Moraga-González et al. (2017), we emphasize that the equilibrium price p^* can go either way as q changes. This is because a decrease in q makes consumers more reluctant to search, and it induces firms to increase p^*, whereas such a decrease in q drives consumers out of the market, and it induces firms to decrease p^*. The result depends on the density of s. Formally, when the search cost's density function has an ILR property, p^* will decline as q increases. However, as we will see below, the price p^* is determined implicitly, so it is difficult to confirm a comparative static analysis of p^* with respect to q. Instead, we present two examples to illustrate how a change in q affects p^*.

Example 1: uniformly distributed utility and search cost

We first show an example of search cost density, which does *not* have the ILR property. Suppose that u is distributed uniformly such as $F(u) = u$, and s is also distributed uniformly so that $G(s) = u$, both with the interval $[0, 1]$. Suppose also that there is an infinite number of firms. Then, from (8), the critical search cost s^* is given by

$$s^* = q\int_{p^*}^{\bar{u}}(1 - F(u))du = \frac{q}{2}(1 - p^*)^2. \tag{34}$$

Recall that $x(s) = 1 - \sqrt{\frac{2s}{q}}$. Then, (33) can be rewritten as follows:

$$p^* = \frac{G(s^*)}{\int_0^{s^*}[\frac{1}{1-x(s))}]g(s)ds} = \frac{s^*}{\int_0^{s^*}\frac{1}{\sqrt{\frac{2s}{q}}}ds}. \tag{35}$$

Equations (34) and (35) yield the unique equilibrium price p^*:

$$p^* = \frac{1}{3}. \tag{36}$$

It is perhaps surprising that (36) is independent of q. In general, an increase in q has two effects regarding firm's pricing: (i) an increase in the share of each firm through an increase in s^* and (ii) greater search intensity (an increase of $x(s;q)$), which also intensifies price competition. With a uniform distribution, these two effects exactly cancel out, so the price depends only on the shapes of the distribution of search costs and utilities.

On the other hand, the equilibrium profit depends on q because it affects the entire demand. Formally, each firm's profit is given by $\pi = \frac{2q}{27}$, which increases in q. Hence, an increase in the stock-out probability is not desirable for firms.

Example 2: exponentially distributed search cost

Suppose that u is distributed uniformly so that $F(u) = u$ with the interval $[0, 1]$ and s is distributed exponentially so that $G(s) = 1 - e^{-s}$ with the interval $[0, \infty)$. We note that the exponential distribution has the ILR property.[47] The threshold value s^* is the same as (34). Substituting s^*, (33) can be rewritten as follows:

$$p^* = \frac{G(s^*)}{\int_0^{s^*}[\frac{1}{1-x(s)}]g(s)ds} = \frac{1 - e^{-\frac{q}{2}(1-p^*)^2}}{\int_0^{\frac{q}{2}(1-p^*)^2} \frac{1}{\sqrt{\frac{2s}{q}}} e^{-s} ds}. \tag{37}$$

As Figure 6 shows, p^* increases in $1 - q$ with this type of distribution. However, one can see that each firm's profit is slightly decreasing in $1 - q$; that is, stock-out does not benefit the firm in this setting. As the stock-out probability increases, price-sensitive consumers will choose not to participate in the market. However, because it is necessary to reduce the price significantly to induce price-sensitive consumers to enter the market, firms do not lower their prices. Instead, firms set a higher price for the remaining consumers who are insensitive to price changes.

[47] For other examples, normal, binomial, and Poisson distribution also have the ILR property.

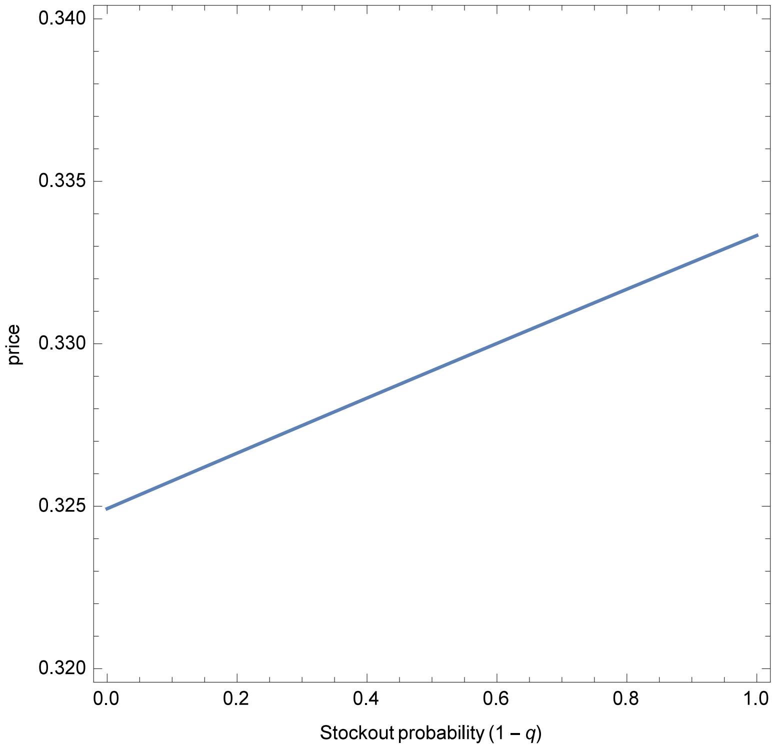

Figure 6: The price increases in the stock-out probability $1 - q$.

As Balachander and Farquhar (1994) have shown, if firms can set the stock-out rate, it may reduce price competition and improve profits. Hence, there remains an open question of whether stock-out can be profitable for firms. To consider the issue of "profitable stock-outs," we may have to extend our model to endogenize stock-out decisions; this issue is beyond the scope of the present study and is not discussed here.