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WHEN EDUCATION POLICY AND HOUSING POLICY INTERACT: CAN THEY CORRECT FOR THE EXTERNALITIES?

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We develop a simple spatial equilibrium model with the peer group effect and local public finance to analyze the implications of housing policies such as public housing and housing voucher programs, and education policies such as school finance consolidation. The calibrated model can match several stylized facts of the labor market and the housing market in the United States. Our counterfactual policy analyses suggest that public housing and housing voucher programs have similar welfare implications on the household level. However, within a household, the public housing program tends to benefit the children more than the parents, while the housing voucher program delivers the opposite result. Combining the school finance consolidation policy with the public housing program could improve the well-being of children from poor households without hurting other households' welfare. Some policies' short-run welfare implications can deviate significantly from their long-run counterparts when all choices are optimized.

JEL Classification: H00; I20; R00

Keywords: school finance consolidation, public housing, housing voucher, endogenous sorting mechanism, short-run rigidity versus long-run flexibility.

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1 Introduction

The United States practices "local public finance," where K12 education is substantially financed by local property tax. The public school is then provided to heterogeneous agents within the same community at zero price. Therefore, the housing market is linked to the education of children. The competitions for housing in well-funded school districts, would drive up house prices, and drive out relatively low-income families.[1] The peer group effect literature informs us that quality of education tends to increase with the proportion of students from high-income families (Sacerdote, 2011; Stinebrickner and Stinebrickner, 2006). As a result, offsprings from disadvantaged families could receive lower-quality education and are more likely to remain less privileged as adults.[2]

Can appropriately designed policies mitigate this intergenerational persistence in income phenomenon? To address this question, we develop a simple spatial general equilibrium model with the peer group effect and local public finance in this paper. We show that housing policies such as public housing and housing voucher programs, and education policies such as school finance consolidation interact with one another and sometimes produce unexpected consequences. Thus, we complement the literature on housing policies, which mainly relies on partial equilibrium analysis (Currie and Yelowitz, 2000; Olsen and Ludwig, 2013).[3]

The general equilibrium approach provides an avenue for understanding households' decisions and outcomes in a systematic fashion and allows us to analyze the feasibility of specific policies. For example, housing market policies, such as public housing and housing voucher, often impose additional tax burdens on non-participants, which might bring political resistance. While a partial equilibrium setting may take the tax as given, our model can partially address this issue in two ways. First, the property tax rate is determined through majority voting within the community. Second, we attempt to find a Pareto-improving policy, i.e., to improve some members' welfare and let no other get hurt. Such a policy would be easier to receive support from voters. Notice that a Pareto-improving policy may be feasible in the present context because many forms of market imperfections exist in the housing and education markets.

More specifically, our model is a variant of the Hanushek-Yilmaz (HY, henceforth) framework, which incorporates all the externality and market imperfections discussed above.[4] In an HY economy, households choose among the ex-ante identical communities for residence, given the school

[1] According to Schwartz (2013, p.24~25), "... most children in the U.S., where you live determines where you go to school... As of the 2008-2009 school year, 11 percent of children went to private schools, approximately 3 percent of U.S. public school students attended charter schools, and another 5 percent attended magnet schools. Only 1 percent of public school students enrolled in different school district through interdistrict choice programs, even though 46 percent of school districts reported offering such a program."

The average level of housing costs in household budgets is around 20% in the United States, similar to the average figure (18%) in the OECD countries.

[2] Intergenerational persistence in income is high in the United States. *One* percent increase in parental income is associated with a *0.5* percent increase in offspring's earnings (Solon, 1999; Black and Devereux, 2011). A child from an economically adverse family grows into a luxurious adult is unlikely (Chetty and Hendren, 2018; Currie and Almond, 2011; Hanushek and Yilmaz, 2011).

[3] Using a general equilibrium spatial model, Leung et al. (2012) show that housing vouchers generate higher social welfare under some conditions than public housing. However, their paper does not explicitly model the intra-household allocation of resources. Hence, it is not particularly suitable for analyzing the intergenerational effects of housing policies.

[4] Papers adopting this framework includes Hanushek et al. (2011), HY (2007, 2013, 2015, 2020), and Leung et al. (2012).

quality and property tax rate pair (Tiebout, 1956). Within a community, families also make the tradeoff between commuting costs and spatially-differentiated rents in locational decisions (Alonso, 1964). Thus, the HY framework links the housing market and education through local public finance, embedding the peer group effect and spatial elements in a unifying framework.

Our contribution hinges on making two realistic modifications to the HY framework. Our first modification is to differentiate between the well-being of parents and offspring explicitly. The level of parental investment in offspring well-being becomes a choice variable and can differ across households. Also, parents decide the intra-household allocation of goods and residence space. Hence, different families can respond to policy changes differently, and we can assess both the intra-household and inter-household welfare changes across policy regimes. The second modification is to model the peer group effect as a function of the average quality of the peers (e.g., Blume and Durlauf, 2006), which, in turn, depends on their parents' investment and per capita educational spending of the local government. In contrast, previous Hanushek-Yilmaz papers assume peer group effects only depend on population composition, such as the fraction of educated parents within a community. To highlight the importance of this modification, we repeat our policy analysis under an alternative specification where the peer group effect only depends on the population composition. As we now note in a footnote and show in the appendix, our formulation could lead to quantitatively different policy implications from the case where the peer group effect only depends on population composition.

With both discrete choices (e.g., which community to reside) and externalities (e.g., peer group effect, public education budget) in the model, it may not be feasible to estimate the parameters using a reduced-form approach directly. Instead, we follow the path of existing HY papers. More specifically, we calibrate our model to match several stylized facts of the U.S. economy circa 2010, enabling us to "identify" parameter values that might not be directly observed and ensure that our model is consistent with those stylized facts. We perform counterfactuals based on the calibrated model to analyze the implications of housing and education policies. Concerning the effects of housing policies, we find that building public housing units for the poor can improve the well-being of their offspring and themselves at the cost of hurting the rest of the economy. The average welfare also declines as location choices are restricted, and the incentives to work are distorted. However, we also find that a policy that combines the public housing program with school finance consolidation can be effective (aiding the poor) and efficient (Pareto-improving). Intuitively, this is the case because, in contrast to the results in HY (2007, 2013), we find that school finance consolidation can improve welfare for all households, based on our calibrated model. We provide more discussion and intuition in section 3.

Our policy experiments yield additional results. First, it is better to locate the public housing units on the "edge of the city," i.e., the land that would not be occupied without the public housing program. The intuition is that such an arrangement of public housing units would not reduce the amount of accessible area. Second, on the *household* level, housing voucher provides similar welfare results as public housing policy does. However, their implications on the *individual* level differ. Adequately designed public housing policy can benefit the adult members in low-income families and improve the well-being of their children. In contrast, while we still find an increase in the well-being of the less educated parents under the housing voucher program, their children's well-being is impaired. We also compare the short- and long-run welfare implications of public policies in the context of our model and find that there is a possibility that they would differ. To our knowledge, such comparison is relatively rare in the spatial general equilibrium literature and would hopefully enrich our understanding.

The rest of the paper is organized as follows. We first present a modified version of the HY model and deliver the baseline results. The model-generated rent gradients are compared with the empirical findings as a validity test of the model. We then analyze the welfare consequences of introducing various housing policies and consolidating school finance, both in the short- and long-run. We conclude in the final section and preserve the technical details in the appendix.

2 The Model

We first provide an informal overview of the model, where our subsequent policy analysis would be based. Our model modifies the pioneering work in HY (2007), which combines the insights of Tiebout (1956) and Alonso (1964) in a multi-district spatial model. Our monocentric city is composed of two ex-ante identical jurisdictions, East (E) and West (W), and three types of households (low, medium, and high skill). The two jurisdictions are divided by a line that goes through the Central Business District (CBD), where all households commute to work daily. For simplicity, we assume that a family is formed via assortative matching, and parents make choices to maximize the utility of the whole household. The tradeoff between the quantity and "quality" (or well-being) of children differs across household types. Household members consume housing, a consumption good, and leisure. Parents invest in their children directly (by purchasing a house and consumption goods) and indirectly (through taxes that fund schools). School funding and peer group effects jointly determine school quality, with the school funding itself determined by the property taxes collected by the local government. While both property tax rate and peer group effects are endogenous and jurisdiction-specific, the determination mechanisms are different. The former is determined by majority voting, and the latter is determined by the average educational outcomes within the neighborhood. Figure 1 visualizes all these relationships.

(Figure 1 about here)

2.1 The General Set-up

Our formal model is a variant of the HY framework, with an elaborated form of parental altruism.[5] We use the subscript p for parents and subscript o for offspring. Parents' well-being Ω_p depends on the amount of goods Z_p and lot sizes S_p that they consume and how much leisure time $l \in [0, 24]$ they enjoy. Parents care about both the well-being of each offspring Ω_o^j and the quantity n_o of their offspring, where j is the community the family chooses to live in (Becker, 1991; Hanushek, 1992). The well-being of each offspring Ω_o^j depends on both the *public inputs*, i.e., the education quality in community j, q_j and *private inputs,* i.e., the amount of consumption goods Z_o and the residential space S_o (Goux and Maurin, 2005; Gertler et al., 2004). We also assume that the marginal utility derived from the number of offspring declines as the fertility rate n_o increases. $g(n_o)$ measures the degree of altruism shown toward each child. Following HY (2007, 2013) and others, we assume

[5] See also Becker and Barro (1988). Alternatively, we can assume that parents care about the utility of their children. However, given the non-intergenerational nature of the model, the two formulations are observationally equivalent.

that households' utility function is in the Cobb-Douglas form.[6] Formally, the utility function of a typical household in district $j \in \{W, E\}$ is given by:

$$U(S_p, Z_p, l, S_o, Z_o, n_o) = (\Omega_p)^{k_p}(\Omega_o^j)^{k_o} n_o g(n_o), \tag{1}$$

where $\Omega_p \equiv (S_p^{\alpha_p} Z_p^{\beta_p} l^{\eta})$, $\Omega_o^j \equiv q_j^{\gamma} S_o^{\alpha_o} Z_o^{\beta_o}$, $g(n_o) \equiv g n_o^{-\epsilon}$, with the restrictions on parameters that $\alpha_p + \beta_p + \eta = 1$, $\gamma + \alpha_o + \beta_o = 1$, $k_p + k_o = 1$, $\epsilon < 1$. The parameters k_p and k_o capture the relative importance of the parent part and the offspring part, respectively.

We now describe the budget constraint faced by the household. Consider a household located r miles away from the CBD. The two parents earn all the income for the whole family. w represents the sum of their hourly wages. The parents allocate the hours in their days to work, leisure (l hours), rearing offspring ($C(n_o)$ hours), and commuting (br hours), where b is the time cost per mile of their daily round-trip commute. For simplicity, we assume that $C(n_o) = cn_o$, where c is a constant. Hence, the total income of this household is

$$Income = [24 - l - C(n_o) - br]w.$$

The parents use their income to purchase consumption goods and pay for housing rents. We normalize the price of the composite consumption goods as unity. We assume that the daily round-trip commuting costs a total of ar dollars. Formally, the total expenditure is

$$Expenditure = (S_p(r) + n_o(r)S_o(r))(1 + \tau_j)R_j(r) + Z_p(r) + n_o(r)Z_o(r) + ar,$$

where $S_p(r)$ is the amount of space consumed by the parents, $n_o(r)$ is the number of offspring in the household, $S_o(r)$ is the amount of space by each offspring, τ_j is the property tax rate, $R_j(r)$ is the daily unit rent in the location r. Hence, $(S_p(r) + n_o(r)S_o(r))(1 + \tau_j)R_j(r)$ is the after-tax expenditure on space for the whole household. The total expenditure on consumption goods of this family is $Z_p(r) + n_o(r)Z_o(r)$. We allow all these variables to be functions of r, reflecting that the consumption of space and goods could depend on the location of residence. Our budget constraint then equates the *Income* to the *Expenditure*. We reserve the details in the appendix.

2.1.1 Household Heterogeneity

In practice, households differ in many dimensions. In this paper, we focus on the differences in income and preference on the well-being of offspring. We will show that these differences would generate significant heterogeneity in equilibrium under local public finance practice. Table 1 shows that, in data, more highly educated parents, on average, have higher earnings and lower fertility rates. They spend more resources on fewer children, translating a difference in parents' income into a difference in offspring well-being. Guided by these stylized facts, we classify the households according to their adult members' educational attainment and calibrate our model accordingly. For simplicity, we assume that there are three types of households: "Not a High school graduate" (*N.H.*), "High School to associate degree" (*H.A.*), and "Bachelors degree or above" (*B.A.*). We further assume that a family is formed with assortative matching. Hence, both of the two adults

[6] There are several merits of assuming the Cobb-Douglas form utility function. Previous studies use that as well, and hence we can easily compare our results with them. The Cobb-Douglas form is tractable. The expenditure share of each good is fixed, which seems to be consistent with the evidence (Davis and Ortalo-Magne, 2011).

(wife and husband) have attained the same educational attainment in a given household.[7] Our assumption that $w_{NH} < w_{HA} < w_{BA}$ is consistent with the U.S. data indicating that higher educational attainment leads to higher wage income on average.

The second dimension of heterogeneity among parents is the quality-quantity tradeoff of offsprings (Becker, 1991; Hanushek, 1992). In our model, a higher value of k_o means that the parents care more about the well-being of their offspring than the number of offsprings they have. To capture the fact that adults who attain a higher level of education tend to bear fewer children, we assume that $k_o^{NH} < k_o^{HA} < k_o^{BA}$.[8] With three types of agents and only two communities involved, we have imperfect sorting, consistent with the empirical evidence (Davidoff, 2005; Hardman and Ioannides, 2004).

(Table 1 about here)

2.2 Basic Analysis of the Equilibrium

In this section, we define and characterize the equilibrium of our model. Our analytical characterizations hold for a broad and reasonable set of parameters and are broadly consistent with the empirical evidence. Hence, they provide some validity of our model. These characterizations also assist our calibration in a later section.

2.2.1 Bid-rent Functions and Market Rent Curves

Like many spatial equilibrium models, all households bid for land on a featureless plane. Therefore, we solve for the bid-rent function, which expresses a household's willingness to pay for space with a given utility level $\overline{u}$. For a type $i \in \{NH, HA, BA\}$ household living in district $j \in \{W, E\}$, the maximization problem is as follows:

$$\psi_i(r, \overline{u}_i, q_j, \tau_j) = \max_{S_p, S_o, Z_p, l, Z_o, n_o} \left\{ \frac{Y_i(r) - Z_p - n_o Z_o - w_i l - w_i c n_o}{(1+\tau_j)(S_p + n_o S_o)} | U_i(.) = \overline{u}_i \right\}. \tag{2}$$

Solving this maximization problem, we obtain the following *bid-rent* function:

$$\psi_i(r, \overline{u}_i, q_j, \tau_j) = \frac{1}{1+\tau_j} \left\{ \frac{K_i q_j^{\gamma k_o^i} Y_i(r)^{k_T^i}}{\overline{u}_i} \right\}^{\frac{1}{k_S^i}}, \tag{3}$$

and the following *bid-max lot size* function:

[7] In practice, marital sorting is not as extreme. According to Fernandez et al. (2005), the cross-country average of assortative matching regarding spouse education level is about 0.6. For a review of the literature and new evidence for assortative matching in marriage, see Bruze (2011), among others.

[8] There are at least two ways to interpret the assumption that income is correlated with parents' degree of altruism. According to the warm-glow theory (Andreoni, 1990), a higher income leads to a higher degree of altruism. Chowdhury and Jeon (2014) conduct a field experiment (dictator and recipient) and find support for the warm-glow theory. The second interpretation is that less-educated parents may be less informed on how to "invest" in their offspring.

For a survey of the altruism literature, see Laferrere and Wolff (2006), among others.

$$S_i(r,\overline{u}_i,q_j,\tau_j) \quad = \quad S_p^i(r,\overline{u}_i,q_j,\tau_j)+n_oS_o^i(r,\overline{u}_i,q_j,\tau_j)=\frac{Y_i(r)}{\psi_i(r,\overline{u}_i,q_j,\tau_j)}\frac{k_S^i}{k_T^i}\frac{1}{1+\tau_j}, \tag{4}$$

where K_i, k_T^i, k_S^i, k_n^i are functions of parameters. (Interested readers may refer to the appendix for details).

In the model, all of the lands are rented out via auctions. All three types of households and agricultural workers can bid for any location (r, j).[9] For each location, the right of usage goes to the agent who offers the highest bid. Therefore, the equilibrium rent curve $R_j(r)$ is the upper envelope of the bid rent curves $\psi_i(r,\overline{u}_i,q_j,\tau_j)$ of the three types of households and the agricultural rent R_a. As the household moves away from the CBD, its bid rent declines due to the transportation cost. It means that beyond a certain distance $R_{jf}^\star$, the agricultural rent R_a dominates the bids offered by *all* of the households in the economy. Hence, no one resides there. Within the fringe distance $R_{jf}^\star$, the spatial order of two adjacent types of households is determined by the relative steepness of their bid rent curves at the intersection point. The one with the steeper curve resides closer to the CBD. In other words, the condition for the equilibrium location of Household 1 being further from the CBD than that of Household 2 is $\frac{\partial\psi_1(\cdot)/\partial r}{\partial\psi_2(\cdot)/\partial r}<1$. Furthermore,

$$\frac{\partial\psi_i(\cdot)}{\partial r}=-\psi_i(r,\overline{u}_i,q_j,\tau_j)\frac{k_T^i}{k_S^i}\frac{a+bw_i}{Y_i(r)}.$$

Based on these observations, the following proposition becomes intuitive (all proofs are included in the appendix):

Proposition 1 *If $\alpha_o > \alpha_p$, then in each neighborhood, households with better-educated adults live further from the CBD at the equilibrium.*

There are opposing forces on the households' location choice. As long as $\frac{k_T^i}{k_S^i}>1$ (which is true given $\alpha_o > \alpha_p$), $S_i(r,u,q_j,\tau_j)$ is increasing in the distance to CBD r. Thus, the *income effect* of a higher wage creates more demand for lot size consumption and induces the household to live farther away from the CBD. However, higher hourly wages also increase the opportunity cost of commuting time. This *substitution effect* generates an incentive for the parents to live closer to the CBD and therefore spend less time commuting. In our model, the income effect always dominates. Thus, a higher wage income and a stronger preference for the well-being of their offspring drive better-educated parents to reside farther away from the CBD. This prediction is consistent with a long-lasting stylized fact in the United States that the nation's poor are more likely to reside in central areas of cities. In the year 1990, the majority (59%) of the poor poverty area residents lived in central cities. 28% and 13% of them resided in outer-metropolitan areas and suburban areas, respectively (Bureau of the Census, 1990).

2.2.2 Population and Fertility Decision

The total number of *households* for each type i is *exogenously given* at $\overline{N_i}$ in this model, $i \in \{NH, HA, BA\}$. However, the *total population is endogenous* as the fertility in each family is a

[9] Following the urban economics literature, the agricultural workers are assumed to be self-sustained, except for the participation of the land auction. They would not affect any other aspect of the model economy.

choice variable. Suppose that in equilibrium, the locations r miles away from the CBD in district j are occupied by households of type i, where $j \in \{W, E\}$. Let $L(r)$ represent the amount of land available per unit distance, at distance r. Because the whole land is equally divided into two districts, $L(r) = \frac{1}{2}2\pi r = \pi r$ in each district. The land market is cleared, which means that within the fringe distance $D^{\star}_{jf}$, $L(r) = S_i(r, u^{\star}_i, q_j, \tau_j)m^j_i(r, u^{\star}_i, q_j, \tau_j)$, where $m^j_i(r, u^{\star}_i, q_j, \tau_j)$ is the equilibrium number of households per unit distance in district j assuming that distance r is occupied by type i household and $u^{\star}_i$ is the equilibrium utility of type i household. We introduce the function $t^{\star}_j(r)$ to indicate the type of the residents at distance r of district j. All of the households find locations to reside, implying the following *population constraint:*

$$\int_0^{\infty} m^W_i(r, u^{\star}_i, q_W, \tau_W)I[t^{\star}_W(r) = i]dr + \int_0^{\infty} m^E_i(r, u^{\star}_i, q_E, \tau_E)I[t^{\star}_E(r) = i]dr = \overline{N}_i \ , \tag{5}$$

where $I[.]$ is an indicator function that takes the value 1 when the condition in the bracket is satisfied and 0 otherwise, $i \in \{NH, HA, BA\}$. It is easy to verify that the household number distribution function in district j is

$$m^j(r) = \sum_{i \in \{NH, HA, BA\}} m^j_i(r, u^{\star}_i, q_j, \tau_j)I[t^{\star}_W(r) = i].$$

The total population in this economy consists of adult and child populations, where the latter is endogenous. We denote $n^{ij}_o(r)$ to be the fertility choice of type i parents in district j, located r miles from the CBD. The solution of (2) suggests that

$$n^{ij}_o(r) \ = n^i_o(r) = \frac{k^i_n}{w_i c k^i_T} Y_i(r),$$

which is independent of district j.

Proposition 2 *If $\gamma < \frac{1}{2-\epsilon}$, then parents who care more about their offspring's well-being bear fewer children, other things being equal.*

$$\frac{\partial n^i_o(r)}{\partial k^i_o} < 0.$$

This proposition is rather intuitive. Hence, parents who care more about their offspring lean more heavily towards children's well-being and bear fewer children. Together with our assumption that more altruistic adults have higher wage income, this proposition implies a negative income-fertility relationship, which is in line with the data.

Finally, the offspring population located r miles from the CBD and in district j is

$$m^j_o(r) = \sum_{i \in \{NH, HA, BA\}} n^i_o(r)m^j_i(r, u^{\star}_i, q_j, \tau_j)I[t^{\star}_j(r) = i].$$

2.2.3 Property Taxes, Peer Group Effect and School Quality

In the previous section, households take the property tax and school quality of each district as given in their location choice. This section shows how these subjects are determined. Recall that

each of the two districts finances its school through the property taxes placed on the residential land within that district. Because they do this independently, the education quality and property tax rate packages (q_j, τ_j) may differ between the two districts. As in the U.S., the publicly funded schools in our model are only open to the residents in the same districts and do not charge tuition fees. The local government of district j would have the following budget constraint:

$$X_j N_o^j = \tau_j \int_0^{D_{jf}^\star} R_j(r) L(r) dr, \tag{6}$$

where N_{oi}^j is the population of children from type i household in jurisdiction j, $N_o^j = \sum_{i \in \{NH, HA, BA\}} N_{oi}^j$ is the total population of the children in jurisdiction j, X_j is expenditure per student in district j and τ_j is the property tax rate. Thus, equation (6) states that the total expenditure on students $X_j N_o^j$ needs to be financed by the local property tax collected within the district $\tau_j \int_0^{D_{jf}^\star} R_j(r) L(r) dr$.

In this model, the (local) education quality q_j has two determinants. First, a higher value of X_j means that the local school can afford better instructional facilities and instructors, and hence provide better education quality q_j. Second, a higher value of the peer group effect Π_j, which means having more qualified peers, lifts a student's educational achievement via several channels. For instance, students may learn from their classmates during group works or even casual interactions. Competing with well-educated peers in school may also induce a student's motivation to study. Following the HY (2007, 2013), we assume that the quality is the product of expenditure and peer group effect,[10]

$$q_j = X_j \Pi_j. \tag{7}$$

We now turn to the determination of peer group effect in the community, Π_j, $j \in \{W, E\}$. Some previous studies assume that the peer group effect is a function of the *population composition* (HY, 2007, 2013). A higher proportion of skilled adults in the total population generates a higher positive peer group effect. Such formulation captures the ideas that (1) family has a significant impact on student performance and (2) the abilities of parents and children are positively correlated. However, this formulation implicitly restricts the parental investment on offspring to be identical across households, and hence the population composition would be sufficient to capture the peer group effect. This paper relaxes this assumption and allows parental investment to be an endogenous decision. We also need a formulation that can apply to any finite number of household types. Therefore, we assume that the peer group effect is a function of the *average quality of all students in the community,* as suggested by some previous studies (Blume and Durlauf, 2005; Liu et al., 2014; Sacerdote, 2011).[11] This formulation captures the quality-dependent nature of peer group effects, while it remains tractable in a spatial general equilibrium model with many distortions. Following HY (2013), we assume a similar functional form of $\Pi_j(\overline{\Omega_o^j})$,

$$\Pi_j(\overline{\Omega_o^j}) = c_1 + c_2 \exp(\overline{\Omega_o^j}),\ c_1, c_2 > 0, \tag{8}$$

[10] Notice that there is no fundamental unit for either school quality or peer group quality. Therefore, we can renormalize them so that (7) holds.

[11] For simplicity, we assume that the "quality" of a child as a student is equal to her well-being, $\overline{\Omega_o^j}$, and use the terms "(student) quality" and "offspring well-being" interchangeably.

where $\overline{\Omega_o^j}$ is the average quality of the students in the community, whose detailed specification is provided in the appendix. Given our formulation, it is straightforward to show the following:

Proposition 3 *In each neighborhood, better-educated adults produce higher quality students.*

This proposition is intuitive. Parents with higher education levels have several advantages in producing offsprings who are higher quality students. They earn higher wage income, have fewer children, and spend a substantial proportion of that income on each child. As a result, the expenditure per child increases. Moreover, these households live farther away from the CBD, where land rents are much cheaper. They can afford larger lot sizes with the same budget, which improve the well-being of their offspring. This proposition also agrees with the perceived high intergenerational correlation between income and education in the United States.

To close the model, we now describe how the property tax rate τ_j is determined. All of the *adults* (parents) in district j have the right to vote for their preferred tax rate.[12] Hence, the preferred property tax rate of a particular household i is the tax rate which maximizes the utility, subject to all the constraints. Formally, it is the solution to the following maximization problem,

$$\begin{aligned} \max_{\tau_j^i} V_i(.) &= \frac{K_i q_j^{\gamma k_o^i} Y_i(r)^{k_T^i}}{\left[(1+\tau_j^i) R_j(r)\right]^{k_S^i}}, \\ \text{subject to } q_j &= X_j \Pi_j \text{ and } X_j = \tau_j^i \overline{R_j}, \end{aligned} \tag{9}$$

where $\overline{R_j}$ is the total rent collected in community j. Its detailed expression is presented in the appendix. The solution takes the following simple form,

$$\tau_j^i = \gamma k_o^i / (k_S^i - \gamma k_o^i) = \gamma k_o^i / (\alpha_p k_p^i + \alpha_o k_o^i - \gamma k_o^i). \tag{10}$$

Furthermore, the calibrated set of parameters ensures that τ_j^i is positive for all three types of households.

2.2.4 Stationary Equilibrium

We are ready to define the general equilibrium of this model economy. In the stationary equilibrium, no household has an incentive to relocate after the electoral outcome is realized and observed. It can be formally defined as follows:

Definition 1 *An equilibrium is a set of utility levels* $\{u_{NH}^\star, u_{HA}^\star, u_{BA}^\star\}$*, market rent curves* $\{R_W(r), R_E(r)\}$*, school quality and property tax rate pairs* $\{(q_W, \tau_W), (q_E, \tau_E)\}$*, household number/offspring population distribution functions* $\{(m^W(r), m_o^W(r)), (m^E(r), m_o^E(r))\}$ *and type functions* $\{t_W^\star(r), t_E^\star(r)\}$ *that show the equilibrium occupant of the location at distance* r *in district* j*, producing the following results.*

- *The households offer their bids according to equation (3). The land is rented out through auction. The household that offers the highest bid wins a particular location if it is higher than the agricultural rent. Otherwise, the land is left for agricultural use.*
- *Each household rents a certain amount of land according to equation (4). The land market clears, and the population constraint (5) holds.*

[12] Following Nechyba (1997, 2003), parents are assumed to be "myopic" when voting and do not consider the implications of their votes on the population composition, land prices and the peer group effects in either communities.

• *Households of the same type attain the same utility level.*

• *Each jurisdiction finances its school through property taxes placed on residential land. The property tax rate is determined by majority voting, according to equation (9). The local government budget balances in all districts, as described in equation (6).*

• *School quality depends on both per-student spending and the peer group effect, which is a function of the average quality of the students in the school district, as shown in equation (8).*

• *All adults commute to the CBD for work and earn wage income according to their types. Commuting presents both monetary and time costs.*

2.3 Calibration

2.3.1 Parameter Set

We calibrate the stationary equilibrium of our model to match a large set of stylized facts from the U.S. statistics circa 2010. We divide the parameters of our model into three categories, which are (1) budget constraint parameters w_i, a, b and c, (2) preference parameters α_p, β_p, η, α_o, β_o, γ, k_o^i, k_p^i, and ϵ, and (3) macroeconomic environment parameters R_a, $\overline{N}_i$, c_1 and c_2. Below we describe the calibration of each category of parameters.

We start with the budget constraint parameters. Because we assume that the two parents in one household attain the same level of education, the target wage income of the household type i is the twice the wage income of a type i agent, $i \in \{NH, HA, BA\}$. Hence, the average annual earnings of type *NH*, *HA,* and *BA* households are about $\$51,432$, $\$87,479$, and $\$155,013$, respectively (U.S. Census Bureau, 2009). Based on the U.S. Department of Labor, the daily work time is around 7.64 *hours*, and hence we set the hourly wages to be $w_{NH} = 20$, $w_{HA} = 32$ and $w_{BA} = 55$, accordingly.[13] The monetary cost of commuting per mile in our city is about \$0.55. In a household in which two adults commute to work, the total round-trip pecuniary cost per mile is $a = 2 \times 2 \times \$0.55 = \$2.2$. Assuming the commuting speed in the city is 20 miles per hour, we set $b = 0.1$. Zick and Byrant (1996) estimate that each parent in a wife-husband family with *two* children spends an average of about 1.3607˜1.5110 *hours* on childcare every day. To mimic this fact, we choose $c = \frac{1}{2}\left(\frac{1.3607+1.5110}{2}\right) = 0.7179$.

We then describe how we calibrate the preference parameters. Since we have imposed the restrictions that $\alpha_p + \beta_p + \eta = 1$, $\gamma + \alpha_o + \beta_o = 1$ and $k_p^i + k_o^i = 1$, there are six free parameters to be calibrated. We jointly choose values for these six parameters such that the baseline economy approximates a list of "stylized facts" of the U.S. economy. In particular, we target the following six moments: (1) share of total expenditure on children, (2) share of total expenditure on housing, (3) share of children's expenditure on housing, (4) share of total "budget" on leisure, (5) preferred property tax rate,[14] and (6) fertility rate. Table 2 summarizes these moments and their correspond-

[13] We choose to match the national average hourly wages because, for each of the three types of households, the average hourly wage for urban residents is fairly close to the average hourly wage for the overall population. For *NH* and *HA* households, this is the case because they do not enjoy large urban wage premium. For *BA* household, this is the case because the vast majority (around 90%) of college graduates live in the urban area.

[14] In the model, the property tax rate is the fraction of rents that are collected by the local government. In reality, property taxes are typically based on the value of the house, which can be computed as rents divided by interest rate $inte = 0.025$. Hence, in the numerical section, we report the property tax rate as the fraction of house value that is collected by the local government instead. Mathematically, this property tax rate is equal to $\frac{\gamma k_o^i}{inte(\alpha_p k_p^i + \alpha_o k_o^i - \gamma k_o^i)}$.

ing expressions in our model. Each of these six moments' model counterpart is often a function of a subset of the preference parameters. The only exception is the fertility rate, which depends on budget constraint parameters that are already chosen in previous steps. Therefore, for each of the three types of households, the preference parameters can be determined as the solution to a six-equation, six-unknown equation system. The calibrated preference parameters are reported in Table 3a.

(Table 2, 3a about here)

The last set of parameters to be determined are the macroeconomic environment parameters, i.e., agricultural rent R_a, number of household $\overline{N}_i$ and peer group effect parameters c_1 and c_2. We fix the total number of households at $500,000$. In the data sample that we have access to, about 10%, 55%, 35% of the mothers are of NH, HA, and BA types, respectively. We assume that this ratio also applies to the fathers and expect the proportion of college graduates to be slightly higher in cities than in the national survey. Hence, we set the ratio of NH, HA, and BA type households to be 10%, 50%, and 40%, respectively. Given the total number of households, agricultural rent R_a determines the size of the city. The lower R_a is, the larger the city is. We set agricultural rent $R_a = \$1,237$ per acre per month to match the endogenous calibration targets for the fringe distance, which is around 10˜15 miles. The peer group effect parameters c_1 and c_2 determine the demographic composition of the two communities. We normalize $c_2 = 1$. We show in the appendix that the larger c_1 is, the stronger the sorting pattern is. We set $c_1 = 10$ to match the endogenous target that over 70% of the *BA* type households reside in and constitute the majority of the *West*.[15]

In the "Real data" column of Table 3b, we present some key equilibrium statistics about family, the labor market, and the housing market in the data. Next to it is the "Baseline" column, which reports the counterpart generated by our baseline model. Our model matches the data reasonably well.

(Table 3b about here)

2.3.2 Baseline Equilibrium

This model has multiple equilibria. We focus on the asymmetric equilibrium, which is the stable one.[16] It also permits us to discuss cross-district sorting related to the tradeoff between the well-being of parents and offspring. We summarize the baseline equilibrium outcomes in a series of tables and figures. Figure 2a shows that the market rents decline as the households move away from the CBD. More prosperous families take advantage of the lower housing rents in remote areas to purchase larger lots. Hence, a household's lot size increases along with its distance from the CBD, as shown in Figure 2b. In each district, the NH type agents, who have the lowest wage income, live the closest to the CBD, followed by HA (middle-income) and BA (richest) types. This spatial allocation of the population is a feature of the Alonso-Muth type model (Alonso, 1964;

[15] We choose 70% as the target because it is approximately the lowest fraction to ensure that skilled (BA type) workers can constitute the majority of the West community and determine the property tax rate. In the appendix, we perform robustness checks to examine whether increasing this target will lead to different policy implications. We find that, when we use a higher fraction as the target, SFC policy's positive effect becomes bigger. Consequently, our main finding is that combining public housing and SFC can lead to Pareto Improvement and help the poor become even stronger.

[16] The instability of symmetric equilibrium in spatial equilibrium models has been noticed by the literature (e.g., Fernandez and Rogerson, 1996).

Muth, 1969), which describes a spatial structure similar to many US metropolitan cities. Figure 2c shows that population density decreases as residents move toward suburban areas. Two economic decisions made by households drive this spatial pattern. A household's lot size increases with its distance from the CBD. Moreover, more affluent families who tend to live further away from the city center also tend to have fewer offspring.

(Figure 2 about here)

The three types of households also differ in other ways. Table 4a and the "Baseline" column of Table 3b together indicate a few things. First, parents who attain a higher level of education have higher incomes, achieve a higher level of well-being, and tend to have smaller families. Second, the offspring with better-educated parents tend to be better. Third, the *differences in parents' well-being are much lower than the differences in offspring well-being.* The reason is simple. Better-educated parents earn higher incomes, bear fewer offspring, and spend a substantial proportion of their incomes on their children's consumption, consequently increasing the expenditure per child. They also choose to live further away from the CBD, which allows them to provide more space for each child. All of these effects work together and magnify the difference in offspring well-being Ω_o across households. However, the impact of higher education levels on parents' well-being is ambiguous. Devoting a larger share of expenditure on children leads to a smaller expenditure share for the parents. This intra-household allocation of resources harms parents' well-being. According to our benchmark calibration, the positive effects of a higher total income and lower rents (weakly) outweigh the adverse effects.

(Table 4a about here)

The "Baseline" column of Table 4b indicates (partial) income-sorting or imperfect sorting across districts. Almost half of the households (46.93%) in the West community are BA type, while only 29.40% of families in the East are of type BA. The West has a smaller proportion of NH-type households (7.05%) than the East (14.52%). Consequently, the West's average annual income is higher than that in the East ($\$116,192$ vs. $\$102,106$). Such spatial sorting has several implications. First, the West has more high-quality students and a more substantial peer group effect. Because most of the households in the West comprise BA-type, its property tax rate is 1.4673%, higher than that in the East (1.3992%). A higher property tax rate and better peer group effect make the West a more desirable community that attracts more households than the East (about 60% of the total population).

Given that the two districts are ex-ante identical, it is interesting to note that (1) the population shares of the two communities are so different (40% v.s. 60%) and (2) the equilibrium market rent in the West, the more populated neighborhood, is significantly higher ($\$41,076$ vs. $\$37,703$ in the East). Because the schools are financed through property taxes, parents in the West effectively pay more instructional expenses. As a result, the school quality, which is the product of per-child education expenditure and peer group effects, is much higher in the West than in the East.

(Table 4b about here)

Note that the schools in the West have a better quality that benefits the children in the community. However, such benefit comes at the expense of parents' well-being because they need to pay for higher housing rents. Table 4a also shows how the parents make the tradeoff between the well-being of their offspring and themselves. For each household type, the average well-being of the parents (children) is higher (lower) in the East than in the West.

2.4 Model-implied Rent gradient

We have shown in Table 3b that our model can quantitatively match many targeted moments related to the US economy. To further strengthen the credibility of our model, we check whether it can simultaneously match some moments that are *not* used as calibration targets. Specifically, we ask whether the rent gradient generated by our calibrated model is consistent with what is typically found in the literature. To do so, we use the model to create some "artificial data" of house rent in different locations in the city, and then run a regression that resembles some existing empirical works. We will then compare the model-generated rent gradient with the empirical counterpart. Notice that the rent gradient in the model is *not* targeted in the calibration process.

To calculate the model-implied rent gradient, we draw a random sample that contains $2,800$ observations for both the East and the West from their corresponding population in the model.[17] For simplicity, we adopt a semi-logarithmic regression equation,

$$\log R_i = \beta_0 + \beta_D D_i + \beta_X X_i + u_i, \tag{11}$$

where R_i is the rent, D_i is the distance from CBD, and X_i are other control variables.

Notice that the location choice is endogenous, and hence, in the empirical literature, regression models like (11) often include control variables X_i to mitigate the endogeneity issues. The control variables X_i normally includes (1) variables that reflect the heterogeneity of the landlords/tenants; and (2) variables that represent specific housing unit characteristics. In our model, for simplification, we assume that all housing units are identical except concerning lot-size and location. As the dependent variable is rent rate per square mile, by construction, R_i accounts for the effect of the difference in lot-size. Therefore, we do not need to add other hedonic variables into X_i (Malpezzi, 2003). Also, we assume that adults who purchase/rent the housing units differ only concerning their wage rates w and degree of altruism toward their children k_o. Notice further that k_o is not observable, and it is *perfectly correlated with wage* w in our model. Therefore, it suffices to include wage w in X_i. Thus, (11) can be rewritten as:

$$\log R_i = \beta_0 + \beta_D D_i + \beta_w w_i + u_i. \tag{12}$$

Table 5a summarizes the descriptive statistics of the two samples. The average rent and wage income are higher in the West than in the East because the former attracts more skilled workers than the latter. Other things being equal, the West is a more attractive community because it provides better education to the younger generation. Consequently, more land is occupied in the West. On average, families living in the West reside further away from the CBD than those in the East.

(Table 5a, 5b about here)

We estimate equation (12) for each of the two groups separately. Table 5b shows the regression results. All of the coefficient estimates are highly significant. In the West, an additional dollar in occupants' hourly wage decreases market rents by about 0.11%, and the same change raises housing rents by 0.08% in the East. Our focus is the coefficient of distance from the CBD. The estimate of β_D falls around 0.09, indicating that rental rates are about 9% cheaper at locations 1 *mile* away from the CBD. This estimate is comparable to the empirical finding of Eberts and Gronberg (1982). It suggests that our model, as a first-order approximation of "reality," is reasonably reliable.

[17] We also estimate a large sample (over 100,000 observations) version. The results are almost identical to the small sample version and are therefore omitted here.

3 Policy Analysis

Based on our calibrated model, we conduct a series of counterfactual experiments to analyze the welfare implications of various education and housing policies. In this section, welfare can refer to household utility, the well-being of parents, or offspring, and we will clearly distinguish their differences whenever there is a chance of ambiguity. Our primary goal is to explore the possibility of a Pareto-improving policy package. To facilitate the comparison with the previous literature, the education policy that we study is school finance consolidation. For housing market policies, we consider the provision of public housing units and housing voucher programs. To build our intuitions in this highly complicated environment, we first study each regime separately. Then, we consider some policy packages and their overall effects.

To compare the well-being of parents, offspring, and society's average across different policy regimes, we turn to a widely used consumption-equivalent measure. More specifically, we search for the discount factors/multipliers, χ_t, $t \in \{W, \Omega_o, U\}$, which must be imposed on the *consumption* of parents and children in the new equilibrium to push their well-being back to their levels at the baseline equilibrium. Therefore, $\chi_t > 1$ indicates that the households are worse off in the new equilibrium because we need to multiply the amount of consumption of the household members to bring them back to the baseline utility level. Similarly, $\chi_t < 1$ indicates that the households are better off in the new equilibrium. We report the value of $1 - \chi_t$, $t \in \{W, \Omega_o, U\}$ in the summary table so that the value is positive (negative) when welfare increases (declines). Here is our formal definition.

Definition 2 *For a particular group of households with average utility level, the well-being of parents and offspring equal to U_{base}, W_{base}, and Ω_{obase} in the baseline equilibrium, the welfare measure χ_U, χ_W, and χ_Ω satisfy the following equations:*

$$\begin{aligned} Average(U(\chi_U Z_p^\star, \chi_U Z_o^\star, ...)) &= U_{base}; Average(W(\chi_W Z_p^\star, ...)) = W_{base}; \\ Average(\Omega_o(\chi_\Omega Z_o^\star, ...)) &= \Omega_{obase}, \end{aligned}$$

where $U(Z_p^\star, Z_o^\star, ...)$, $W(Z_p^\star, ...)$ and $\Omega_o(Z_o^\star, ...)$ are the average well-being of the household, parent and offspring in the new equilibrium, respectively.

3.1 School Finance Consolidation (An education policy may impact the housing market)

School Finance Consolidation (SFC), or School District Consolidation, is an apparent post-war trend in the US. The number of school districts that provide elementary and secondary education had dropped from 117,108 in 1939˜1940 to 13,862 in 2006-07 (National Center for Education Statistics). In HY's model setting, the central government moves all students to a single school, which it finances through the property taxes collected from all of the lands in the economy. HY (2007) calibrate their baseline equilibrium to match a representative United States city circa 1997. Based on the parameter set obtained from the calibration, they show that enforcing SFC hurts everyone in the economy. Their finding supports Fischel (2006) arguments, which describes consolidation policy as an external distortion leading to welfare decline. In practice, although the government can *equalize the per-child educational spending*, but may not be able to *equalize the quality of the peer group*, as it depends on the agents' choices made in equilibrium. To complement the literature, we assume

that the two communities have the same per-child educational spending but can differ in school quality when considering SFC.

We summarize the new equilibrium in the "SFC" column of Table 3b and 4b and compare this SFC equilibrium with our baseline equilibrium. After the consolidation of school districts, the property tax rate is voted by all the adult members in the economy. The same tax rate then applies to both the West and the East communities. Because HA-type adults comprise the majority, their preferred property tax rate of 1.3992% will be the equilibrium property tax rate. It is slightly lower than the level preferred by the BA. The drop in property tax rate decreases per-child educational expenditure in the West, which induces BA- and HA- type households, who value their well-being of offspring, to move from the West to the East. Their movement increases (decreases) the average income in the East (West), which boosts (lowers) the rent in the East (West). Because school funding is derived from property taxes, which are proportional to housing rents, the per-child school funding is higher (lower) in the East (West) than before. The movement also tends to decrease the gap in peer-group quality between the two communities.

In summary, the SFC narrows the school quality gap between the West and the East, which substantially reduces cross-districts sorting at the equilibrium. As a result, the population is evenly distributed in the two districts after the SFC policy is imposed. The thick solid lines in Figure 3a shows this pattern explicitly.

(Figure 3a about here)

Our welfare results are summarized in the "SFC" column of Table 6. Unlike in HY (2007), the SFC policy makes all types of households better off. On the one hand, SFC indeed restricts school quality choices to one, potentially resulting in a more substantial average individual deviation from optimal levels of school quality. On the other hand, the SFC policy substantially reduces the gap of school qualities, rents, and population compositions between the two communities. Hence, under the baseline equilibrium, the land in the West is "over-used," while the land in the East is "under-used." In contrast, under the SFC policy, the land in both districts is evenly utilized, which leads to more efficient land use. Indeed, Table 3b shows that the population density is lower under the SFC policy than in the baseline (5.43 v.s. 5.53). Whether the SFC policy is welfare-improving or not depends on which of the two effects dominates the other. It, in turn, depends on how extreme the sorting pattern is in the baseline case since excessive sorting leads to inefficient land use. The *sorting pattern is more extreme in our baseline case* than the counterparts of the previous HY papers.[18] As a result, when we remove sorting, the positive effect from the more efficient use of land dominates the negative impact of restrictions on choices. Our focus here is *not* to overturn the conclusion of HY (2007, 2013) on SFC, but rather to highlight that SFC could potentially lead to Pareto Improvement in a variant of HY.

Furthermore, Table 6 shows that the adults in BA- and HA-type households enjoy better lives while their offspring suffer. In other words, *SFC is Pareto improving at the household level, but not the individual level*. The reasons are simple. First, some of the BA- and HA-type families move to the East where the housing rents are lower than the West, and hence, they can consume larger lot sizes than they would if they stayed in the West. Second, cross-community sorting becomes much weaker. Recall that in our model, parents with a higher level of education tend to produce children who would be better students. Consequently, children from BA- and HA-type households are faced with lower peer group quality on average because they are pooled with children from NH-type families. The converse applies to NH-type families.

[18] For more robustness checks for alternative targets of community compositions, see the appendix.

(Table 6 about here)

3.2 Public Housing and Housing Vouchers (Housing market policies may impact education)

In the previous section, we focus on SFC, an education policy that has implications on the housing market. In this section, we consider two housing aid policies, including government-subsidized public housing and housing vouchers, which would, in turn, affect education quality. These housing aid programs are designed to provide low-income groups with necessary residential spaces. In our model, 10% of the households are of the NH-type. The income in these families is about 60% and 33% of adults' wages in the HA- and BA-type households, respectively. Hence, we assume that public aid programs are only open to NH-type families. We further assume that all NH-type households can receive assistance.[19] A central government finances the programs through the income taxes paid by the adults. We assume that the NH-type adults, who would enjoy the housing policy benefits, do not need to pay the income taxes and that all of the other adults are faced with uniform income tax rate θ. One may interpret this as a form of progressive taxation.[20]

We introduce public housing into the baseline model and study how it affects economic outcomes and social welfare. Under this policy, each participating household enjoys a lot of size S_{PH} and contributes $S_{PH}R_{PH}$ to the program. The government receives contributions from the participants and income taxes from the non-participants. Then, the government purchases land and builds housing units for the program participants. Therefore, a particular program participant's decision problem is reduced to

$$\max_{S_p,S_o,Z_p,l,Z_o,n_o} U_{NH}\left(S_p,Z_p,l,S_o,Z_o,n_o\right)=\left(S_p^{\alpha_p}Z_p^{\beta_p}l^{\eta}\right)^{k_p^{NH}}\left(q^{\gamma}S_o^{\alpha_o}Z_o^{\beta_o}\right)^{k_o^{NH}}gn_o^{1-\epsilon},$$

$$\begin{aligned}\text{subject to } Y_{NH}(r) &= w_{NH}l(r)+w_{NH}cn_o(r)+S_{PH}R_{PH}+Z_p(r)+n_o(r)Z_o(r),\\ S_p+n_o(r)S_o &= S_{PH}.\end{aligned}$$

For the sake of space, we show the exact formula for the new fertility decision, $n_o^{Pub}(r)$, and the latest children's well-being $\Omega^j_{oPub}(r)$ in the appendix.

Recall that one of the main goals of public housing policies is to assist children from economically adverse families and promote intergenerational mobility. Hence, we are interested in the effects of government housing programs on the well-being of children from NH-type families. With the above derivations, it is simple to prove the following propositions.

Lemma 1 *At a given location, NH-type adults under housing program give birth to fewer offspring than they would at the baseline equilibrium if their contribution to the program is more prominent than their expenditures on housing in the baseline situation*

$$\begin{aligned}\textit{If } S_{PH}R_{PH} &> [(1+\tau_j)S_{NH}(r,u^{\star}_{NH},q_j,\tau_j)\psi_{NH}(r,u^{\star}_{NH},q_j,\tau_j)]_{Baseline},\\ \textit{then } n_o^{Pub}(r) &< n_o^{NH}(r)_{Baseline}.\end{aligned}$$

[19] In practice, more families are eligible for public housing programs than receiving assistance in the US. Leung et al. (2012) study the case when the public housing units are "under-supplied," and low-income families can get those units through rationing.

[20] Again, the uniform tax rate across HA- and BA-type households is imposed to simplify the analysis. The crucial point is that NH-type families do not need to pay the income tax while receiving the benefits. The results would carry to the environment when different groups of agents all face different tax rates.

Proposition 4 *Given the same location and school quality, when NH-type households receive a larger lot size and pay more under the public housing program than in the baseline equilibrium, they produce offspring who would become higher-quality students.*

$$\begin{aligned} \textit{If } S_{PH}R_{PH} &> [(1+\tau_j)S_{NH}(r, u^{\star}_{NH}, q_j, \tau_j)\psi_{NH}(r, u^{\star}_{NH}, q_j, \tau_j)]_{Baseline} \\ \textit{and } S_{PH} &> S_{NH}(r, u^{\star}_{NH}, q_j, \tau_j)_{Baseline}, \\ \textit{then } \Omega^j_{oPub}(r) &> \Omega^j_{oNH}(r)_{Baseline}. \end{aligned}$$

The intuitions are straightforward. Facing the government-subsidized rent with public housing units that are larger than they would otherwise rent from the market, the NH-type households choose to have fewer children. Hence, as the number of children decreases, per-children spending could increase, and children from those households could become better students. Moreover, the second inequality guarantees that the public housing policy does not reduce the consumption of lot size. Consequently, the well-being of offspring improves.

3.2.1 Public Housing Policy 1: Units Located at the Middle Ring of the City

In practice, public housing units are *not evenly* distributed within a city. In this paper, we consider the case where public housing units are built in only one district, the East. The central government must decide the locations of those public housing units *within* the neighborhood. We investigate two alternatives. First, we consider a case similar to Leung et al. (2012), where public housing units are located between D_i and D_o miles from the CBD. We assume that the land in this area is rented from the market. Hence, the central government needs to calculate the rent she needs to pay in a competitive rental market. The rental rates are determined at the auction with type i, $i \in \{HA, BA\}$ households, and agricultural workers. Since the analysis is analogous to the baseline case with no public housing, we refer the interested readers to the appendix for details.

In addition to the rental costs, the central government must make a payment known as the Payment in Lieu of Tax (PILOT) to compensate the local government in the East for some of the property tax revenue lost due to the public housing program. Here, we follow Leung et al. (2012) to make the simplifying assumption that the PILOT is equal to the property taxes placed on the public housing recipients' contribution to the program,

$$PILOT = \tau_E \overline{N_{NH}} S_{PH} R_{PH}, \tag{13}$$

where $\overline{N}_{NH}$ is the total number of NH-type households.

This program is financed by the residents' contribution to public housing and the income taxes paid by non-participating households. The former can be calculated by the simple formula $Contribution = \overline{N_{NH}} S_{PH} R_{PH}$. The central government adjust the parameters (θ, S_{PH}, R_{PH}, D_i, and D_o) of the public housing program so that the government budget constraint holds,

$$R_{Pub} + PILOT = Contribution + IT, \tag{14}$$

where R_{Pub} is the sum of rents that the central government needs to pay to the private sector to obtain the land for public housing, IT is the total income tax revenue. We reserve the full expressions of R_{Pub} and IT in the appendix.

To compute the Public Housing Equilibrium, we impose some parameter values. The public housing units are located only in the East and start from 4 *miles* away from the CBD. The family-specific lot size is set to be 0.001 square mile, which is about 25% larger than the average unit

within that band in the baseline equilibrium. We can then determine the outside boundary of the public housing band, which is 6.916 miles. The central government charges an income tax rate $IT = 0.8\%$ to balance its budget, which endogenously matches the calibration target for the participant contribution to the program of \$221.4 per month. The increase in utility that the participants obtain by joining the public housing program is close to the one derived from a 25% *consumption* subsidy in the baseline equilibrium.

The thick dash-dotted lines in Figure 3a and the "PH1" column of Table 4b display powerful sorting in equilibrium.[21] Notice that the NH-type households, who are the public housing recipients, all allocated to the East. They also care the least about the well-being of the offspring. Thus, the public housing program leads to a sharp decline in the peer group effect in the East. In response, all of the BA-type and most of the HA-type households choose to live in the West. This intense sorting makes the two districts significantly distinct. Both the property tax rates and the lot sizes in the two communities are almost identical. However, with the strong sorting effect, the West's market rent exceeds that in the East by a large margin, which leads to much higher expenditure on education in the West. Combined with the peer group effect, school quality in the West is also much higher.

The "PH1" column of Table 6 presents the welfare effects of this policy. At the household level, the public housing residents (NH) are better off, and all of the other types are worse off. This result is intuitive. Under the public housing policy, all of the household location choices deviate from the efficient ones. As a result, the welfare of the whole economy declines. A careful inspection of the results reveals that the improvement of NH-type households' happiness level comes from an increase in parents' well-being. These parents can enjoy large lot sizes without paying more. However, there is no free lunch. The education quality in the East drops dramatically as all NH-type households concentrate in one district. Consequently, public housing policy hurts the well-being of NH-type children.

On the other hand, the proportion of offsprings from BA-type families increase in the West, and through the peer group effect, BA-type children benefit from the outstanding quality of education in the West. However, the public housing policy hurts adults from BA-type families the most for two main reasons. First, the BA-type households all locate in the West and hence drive up the housing rents significantly. Second, because they do not constitute the majority of their community, the electoral outcome of the property tax rate is not their preferred outcome. Both parents and offspring of HA-type households are worse off to a relatively mild extent.

3.2.2 Public Housing Policy 2: Units Located at the Edge of the City

In the previous section, we assumed that public housing units are located in areas that would otherwise be occupied by non-participating households. Consequently, this type of public housing policy decreases the amount of accessible land and probably results in higher market rents and less-efficient land allocation. In this section, we consider an *alternative public housing policy* that would have minimal effect on the already occupied land. More specifically, we assume the public housing units are located at the *edge of the city*. We compare the new equilibrium with the one in the previous section concerning resource allocation and welfare. Under this scenario, the households who are not eligible for public housing would first compete with agricultural use on each piece of land they desire. Then the government builds the public housing unit *outside the fringe of the East*

[21] Although the sorting in our public housing equilibrium is qualitatively more substantial than that shown by Leung et al. (2012), they are mostly similar in spirit.

district. In other words, the public housing units are located in areas that would not be occupied by different households, and thus, the government acquires the land at the agricultural rent R_a. For the sake of comparison, we maintain the assumption that $S_{PH} = 0.001$ square mile and $\theta = 0.8\%$. Each participating family must contribute \$199.4 to the program so that equation (14) holds. The *required contribution is smaller* than that in the previous case, as the market value of the public housing band is less than before. In the equilibrium, this band is located between 6.0871 *miles* and 8.2997 *miles* away from the CBD. The thick dotted lines in Figure 3a show the housing rents and occupants of all locations. For the public housing area, market rent is defined as the participants' contribution (per square mile).

A comparison between the "PH1" column with the "PH2" column of Table 4b highlights the difference between the two public housing policies. Although we still observe no BA-type household in the East, the proportion of HA-type families residing in the East increases from 9.29% to 21.33%. The spatial sorting is weaker. At the same time, the BA-type households outnumber the HA-type households in the West and determine the property tax rate to be 1.4673%. The corresponding "Community Comparison" column of Table 4b summarizes other essential statistics of this equilibrium. As more HA-type households move from the West to the East, the demand for land in the East (West) becomes stronger (weaker), resulting in higher (lower) housing rents. Per-child *education spending increases in both communities*. In the East, this is a consequence of higher rents. The community attracts more HA-type households and has a more substantial peer group effect than before. In the West, although housing rents decrease slightly, the property tax rate and total tax income increase. We observe *better school quality in both communities* in equilibrium. Comparing the "PH2" with "PH1" columns in Table 6, we conclude that all of the family members from all of the household groups are better off after the government puts the public housing units outside the fringe. It confirms the intuition that *decreasing the amount of accessible land has an adverse welfare effect on the whole economy*. In a sense, the government kills two birds with one stone (i.e., implementing the second, at-the-edge type public housing policy, rather than the within-the-city type that is considered in the previous section). First, it can improve the welfare of the most impoverished families (NH) at a lower welfare cost imposed on other households in the economy. Second, it can reallocate resources between adults and children. Relative to the public housing policy where units are located in the middle ring of the city, children are better off, and parents are worse off when the public housing units are located at the edge of the town.

Notice that our results are also broadly consistent with other empirical studies of public housing unit residents. For instance, Olsen and Barton (1983) study the benefits and costs of public housing based on New York City data, and find that *"...the mean benefit of the program to these families is substantial relative to their mean income but small compared with the cost to taxpayers."* Currie and Yelowitz (2000), Jacob (2004), among others, also confirm that public housing per sec can improve the younger generation of public housing residents.[22]

3.2.3 Housing Voucher

Instead of building public housing units and hence directly changing the housing consumption, the government may distribute cash, also known as the housing voucher program (VC). Under this scheme, the government collects income taxes from the skilled workers and redistributes them to the poor in the form of a housing voucher, which can only be used to purchase housing services.

[22] Public housing plays a more critical role in Asia. Among others, see Leung and Tang (2015), and the reference therein.

Hence, program participants can still choose their desired locations, desired lot sizes, etc. We would also examine the welfare implications of the housing voucher policy under the current setup, as we did to other policies.

Here are the details of VC. For non-participating households, the utility maximization problem is the same as that under public housing policy. For participating families, they are exempted from the income tax but instead receive housing vouchers from the government that amounts to v_p, which is for housing consumption only. Hence, the program participant's budget constraint becomes

$$\begin{aligned} Y_{NH}(r) &= w_{NH}l(r) + w_{NH}cn_o(r) + Z_p(r) + n_o(r)Z_o(r) \\ &\quad + \max\{0, (S_p(r) + n_o(r)S_o(r))(1+\tau_j)R_j(r) - v_p\}. \end{aligned} \tag{15}$$

Since the utility function is monotonically increasing in lot size, all participating households spend no less than the amount of their vouchers on housing. Whether they will have a higher expenditure on housing depends on their housing preference and income levels.

The rest of the program is simple. All of the lands are rented out through the same market mechanism, as described in previous sections. The government chooses policy parameters (θ, v_p) correctly so that the income tax revenue IT is equal to the total cost of financing the program, which is $v_p\overline{N}_{NH}$.

The computation of the market equilibrium under the housing voucher program is similar to other cases. To be compatible with previous sections, we keep the income tax rate at 0.8%. In equilibrium, it implies that each participating household receives the housing vouchers that amounts to \$23.12 each day, and all participants spend more than this amount on housing units. As we observe from the thick dashed lines in Figure 3a and the "VC" column of Table 4b, household sorting is more substantial than that at the baseline and weaker than the one under public housing policy (when all public housing units are placed outside of the fringe distance). Although there are no BA-type households in the East under the VC program, we observe NH-type families that amount to 2.69% of the total number of households in the West. Land demand and market rents increase in the West, where all of the affluent families live. They constitute the majority in the West and determine the property tax rate to be 1.4673%, which is the same as the baseline case. Having higher housing rents allows the West's local government to collect more property taxes and provide more funding to the schools.

Conversely, in the East, the average market rent declines from $\$37,703$ to $\$35,172$, indicating that less funding for schools is collected. The change in school funding amplifies the gap in school quality between the two communities. Recall that skilled workers care more about their offspring hence tend to produce children who would become higher-quality students. Under our model setting, schools with higher percentages of students from high-income families provide better peer group quality. Therefore, the gap in school conditions is further enlarged because the West now attracts all BA-type households. As a result, the educational qualities are 741 in the West and 123 in the East. The counterparts in the baseline case are 564 and 280, respectively. More substantial sorting occurs under the VC program, and an amplified gap in school quality is observed. NH-type parents seek a better education for their offspring and are willing to pay higher housing rents.

The "VC" column of Table 6 shows the welfare implications of the housing voucher program. The *overall welfare changes are very similar to those in the public housing case.* The average utility of program participants increases by about 18.5%, while other families' overall welfare declines slightly, as in Leung et al. (2012). The economy-wide average welfare decreases by only 0.19%. Hence, *the housing voucher program incurs less welfare loss than the public housing policy* because the former imposes fewer restrictions than the latter on household choices. Moreover, the increase

in NH-type households' welfare from the housing voucher program is less costly than that from the public housing program, which leads to a smaller welfare loss for HA- and BA-type families.

On the other hand, while the housing voucher program and public housing policy deliver similar aggregate utility, their *intra-household welfare implications* are very different. Recall that the public housing policy improves the well-being of children from low-income families. Hence, the *gaps in children's quality as students between different households are reduced*, potentially increasing intergenerational social mobility. On the other hand, our welfare results show that VC policy *enlarges the gap* between affluent and low-income families. When the school quality of two districts becomes drastically different, skilled parents tend to cluster more heavily in the community with higher rents and better schools. Unskilled parents do the opposite.

3.3 Policy Package: Combination of School Finance Consolidation and Housing Market Policies

The previous section, which studies each of the education and housing policies in isolation, delivers the following lessons. First, when the baseline sorting pattern is extreme, imposing school finance consolidation may increase the aggregate utility. Second, an appropriately designed public housing program can be a handy tool concerning aiding children from low-income families. Third, housing voucher tends to enlarge the quality gap between the children from high income and low-income families. Based on these observations, this section addresses a natural question: is it possible to help children in need while keeping other families at least as well off as before by combining education and housing policies?

(Figure 3b about here)

We consider different policy packages. One possibility involves combining school finance consolidation with a housing voucher program. Based on our model calibration, the housing voucher policy alone does not perform well regarding increasing the living and educational qualities of children in need. However, it is still possible that a housing voucher program with school finance consolidation delivers better outcomes. The thick dashed lines in Figure 3b depicts the rent-distance relationship of this policy package. The "SFC+VC" column of Table 6 summarizes the welfare changes caused by this combination. Comparing this column with the "VC" column, we find that adding a school finance consolidation policy to the housing-voucher-only regime makes all types of households in this economy better off. This finding may not be surprising, given that we observe similar results when adding a school finance consolidation policy to the baseline economy. However, this policy package is *not a Pareto improvement* over the baseline situation at the household level because non-participants are slightly worse off. Moreover, it does not help children from low-income families, as their average well-being slightly decreases (-0.33%) comparing with the baseline case.

Hence, we consider another policy package, which combines the school finance consolidation with public housing. It equalizes the per-child funding in the two districts and puts all public housing units outside the fringe distance in the East. As the thick dotted lines in Figure 3b and the "SFC+PH" column of Table 4b show, the cross-community sorting in this equilibrium is more potent than the baseline case but much milder than the scenario of public housing policy only. This observation confirms our conjecture: SFC makes the two districts less different and weakens the sorting magnitude. The "SFC+PH" column of Table 6 shows the welfare results of this policy package. Comparing with the baseline equilibrium, all three types of households are better off. We also observe a substantial improvement in the well-being of children from NH-type families.

Hence, the government can bring welfare improvement to the economy and increase the well-being of children from low-income families.

3.4 Short- vs. Long-run Analysis

Thus far, our policy analyses have followed the tradition of urban equilibrium models, which assumes that markets are initially in equilibrium, upon which policy is imposed unexpectedly. Agents re-optimize their choices, such as their location and consumption. The markets instantly clear, and we compare the welfare under the new equilibrium with the original one. With our static model, such analysis is interpreted as a long-run assessment of public policies. However, as Quigley and Swoboda (2010) and others recognize, some choices cannot be altered in the short-run.

Consequently, the long-run welfare implications for some policies can be dramatically different from their short-run counterparts. For instance, policy changes typically do not significantly alter the housing supply in the short run, which is often assumed to be fixed. However, the long-run supply of housing should arguably be flexible. Housing decays over time, and economic agents can decide to replace it. As such, it may be vital to re-examine our welfare results while considering the short-run rigidity.[23]

To facilitate the comparison, we make minimal modifications to the current framework.[24] More specifically, we differentiate variables according to their corresponding "flexibilities." In our model, all of the choices are *flexible* in the long-run. In the short-run, some decisions are more flexible than others. Table 7 summarizes the flexibility of different choices. We consider fertility choices to be the most *inflexible*. It is physically impossible to decrease the number of children that parents have already had. It would take at least another year for parents to bear an additional child in any given year.

Furthermore, residential choices cannot be easily adjusted. In many places, the term length of a residential rental lease is typically one year or longer. Depending on the market condition, it may take a similarly lengthy period to sell a house. On the other hand, households are free to decide how many non-durable goods to consume, even in the short-run. We also assume that the amount of leisure can be easily adjusted.

The time allocation in our model is not entirely flexible. For instance, childcare time is a linear function of the fertility rate and is therefore inflexible in the short-run, just like fertility choices. A worker's commute time is also rigid, as it is merely a multiple of the distance from the worker's home to the CBD. However, as some workers might adjust their work hours, the choice of leisure time can be modified to a certain extent. Hence, we treat leisure time as a flexible choice.

(Table 7 about here)

The recognition that households are not allowed to move in the short-run restricts the set of policies for which we can conduct short-run analysis. Since the public housing experiments force program participants to relocate across districts, we perform only a "short- vs. long-run" analysis for the school finance consolidation and housing voucher experiments and the experiment that combines the two.

[23] In this section, the term "goods" is inclusive. All of the parents' choices, including those related to fertility, consumptions, space, and leisure, are considered "goods."

[24] Short- vs. long-run analysis is not a perfect substitute for the transition dynamics analysis, which is feasible only when a fully dynamic general equilibrium (DGE) setting is available. In this paper, we focus on a static setup and leave the DGE for future research. For a review of the related literature, see Leung (2004), Leung and Ng (2019), among others.

Table 8a summarizes some relevant statistics. Household's consumption choices and working hours in the short and long runs are similar. In our model, a household's optimal expenditure on consumption and leisure are constant fractions of their total potential income. Hence they do not depend on their lot size choices. Thus, the fact that a household cannot adjust its residential area in the short run does not affect its consumption and leisure decisions.

Nevertheless, the short- and long-run choices are not identical for at least two reasons. First, a household's location decision affects its potential income and, consequently, its consumption and leisure choices. Therefore, as the economy has significantly different household space distributions in the short- and long-run, the average consumption and leisure time take different values in the two situations. Second, a household's decision on leisure and working hours are subject to time constraints. Households allocate their time endowments to different activities, including childcare, commuting, and leisure and work enjoyment. Both childcare and commute time are fixed in the short-run, as fertility and location choices are set in the short-run. Because agents are allowed to re-optimize these decisions in the long-run, the leisure choice is affected.

(Table 8a about here)

When the school finance consolidation is imposed, school quality almost does not change when the time horizon moves from the short-run to the long-run. Recall that in this model, the school quality of a community is simply the product of average education expenditure and average student quality. Both of these components depend on the population share of highly educated households in the city because their presence tends to drive up the rents, leading to more school funding. Also, higher-quality students tend to be associated with higher-income parents. Under the school finance consolidation regime, there is only one school district whose composition is always the population distribution in both the short and long runs. Consequently, under SFC, school quality does not vary as much as under alternative policies when the economy moves from the short-run to the long-run.

Table 8b reports welfare comparisons. Households in the East are generally better off than those in the West in the short run, especially when school finance consolidation policy is imposed. In the long term, economic agents are mobile. Hence, the welfare of economic agents would depend on their types and not the residential location. Our setup enables us to conveniently decompose the household's total welfare into the well-being of parents and offspring. The extra benefits that residents in the East obtain come mainly in the form of higher well-being of the children. Recall that the East has lower average rental rates and peer group quality. SFC policy enables children from the East to go to better schools in the West, bringing significant welfare improvement to those households. Simultaneously, because the agents are unable to move in the short run, families in the East are effectively given a "free-ride" on the better education provided in the West. In other words, *short-run rigidity prevents the housing market from functioning efficiently, and agents' welfare becomes location-dependent.*

(Table 8b about here)

Another important finding is that long-run equilibria are not always better than short-run equilibria. According to Table 8b, the long-run average welfare of households is higher than the short-run counterpart only when school finance consolidation policy is imposed. When the housing voucher program alone is imposed, the economy's general welfare increases in the short-run (relative to the laissez-faire benchmark) but deteriorates in the long run. In particular, *Group 2 agents (i.e., HA-type) is hurt in the short-run (relative to the laissez-faire benchmark) and hurt even more in*

the long run. This finding is at odds with conventional wisdom. The intuition under the complete market is as follows. Moving from the short-run to the long-run, households have more choices, and economic agents are usually better off. *However, our agents live in a model with many forms of market imperfection.* They include the peer group effect (which is a form of externality) and the non-convexity of consumption caused by the household's location choices. Thus, HA-type agents can be hurt more if the proportion of BA agents in their community drops dramatically in the long run. Our numerical exercises with plausible parameter values suggest that it is indeed the case. It is because when the housing voucher program is imposed, wealthy families cluster more intensively in the West when given such an opportunity, greatly enlarging the difference in community composition.[25]

4 Concluding Remarks

This paper builds a simple spatial general equilibrium model that embeds fertility choice, location choice, and work-leisure choice in a *unifying framework*. Our model distinguishes the welfare of the children from their parents. Our model distinguishes the welfare of the children from their parents. It enables us to differentiate policies that benefit all at the *household level* from those that help all at the *individual level*. Our calibration confirms that the model can simultaneously match specific families, labor markets, and housing market outcomes. While we do not include the rent gradient as a calibration target, our model-implied rent gradient also matches previous empirical estimates. We analyze various educational and housing policies and their combinations. We demonstrate that public housing policy can induce intense sorting among different types of agents. The welfare result depends crucially on whether the public housing units are built on land that would otherwise be used. On the other hand, while the housing voucher program can increase the overall welfare of low-income households, it is the parents who capture the welfare gains, and their offsprings can be hurt.

We also demonstrate that under some parameterization of the model, the *policy package of school finance consolidation with appropriately located public housing units can help children from low-income families without hurting other types' welfare*. We stress that our main intention is to highlight an empirically plausible possibility, rather than producing results that can be directly taken for policy recommendations. In practice, there are many issues to consider. For instance, there is a lag of public housing construction, which means that by the time the public housing units are available for end-users, the supply could be insufficient or in excess. After the public housing units are occupied, the demand conditions continue to vary, and hence both the quantity and the price may fail to adjust to the market. Local governments might also have fiscal commitment issues. As a result, many public housing projects in the U.S. are disappointing (Freeman, 2002; Malpezzi, 2020; Olsen et al., 2005; Wiltz, 2019).

On the other hand, we observe that public housing units are provided in Asia and Europe under different labels, while some form of school finance consolidation is implemented (Silver and Danielowski, 2019). Our analysis justifies this type of policy regime. We leave it to future research to investigate whether the current level of government involvement in those Asian and European countries are indeed at the socially optimal level.

We also compare the short-run (i.e., when some decisions are constrained), versus the long-run

[25] The result here is consistent in spirit with studies of "second-best theory" in which the laissez-faire equilibrium is inefficient.

(i.e., when all arrangements are flexible). To the best of our knowledge, other than the study by Quigley and Swoboda (2010), such short- vs. long-run analysis has been relatively underexplored in the urban economics literature. For instance, we find that the housing voucher program can increase the average welfare of the economy in the short-run but decreases it in the long run with plausible parametrization. This result also points to the possibility that some policies which can bring long-run gains to the economy may not be implemented due to their short-run adverse impact.

Besides, we demonstrate that in some situations, *middle-income agents can lose more in the long run than in the short run after a policy change*. It is because their utility levels depend on the proportion of high-income agents living in the same neighborhood. In the short term, agents stay in the original houses, and hence the welfare loss is simply the direct policy effect. However, in the long run, the high-income agents may move to another community, driving down the original neighborhood's school quality. The middle-income agents either stay and live with the depreciated school quality or migrate with the high-income agents and face possibly higher rents and taxes. Future research should further explore such considerations.

There are critical dimensions to be further explored, as well. For instance, our model has a simple commuting cost structure, while the reality may be more complicated (e.g., Leroy and Sonstelie, 1983, Yilmaz, 2019). Chetty and Hendron (2018b) find that long commute time is associated with low intergenerational mobility. Thus, future research should re-examine the optimal location of public housing units with a more realistic transportation system. Urban policy analysis in those environments remains a challenge to be met.

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Figure 1. Household Decision, Peer Group Effect and Public Expenditure Effect

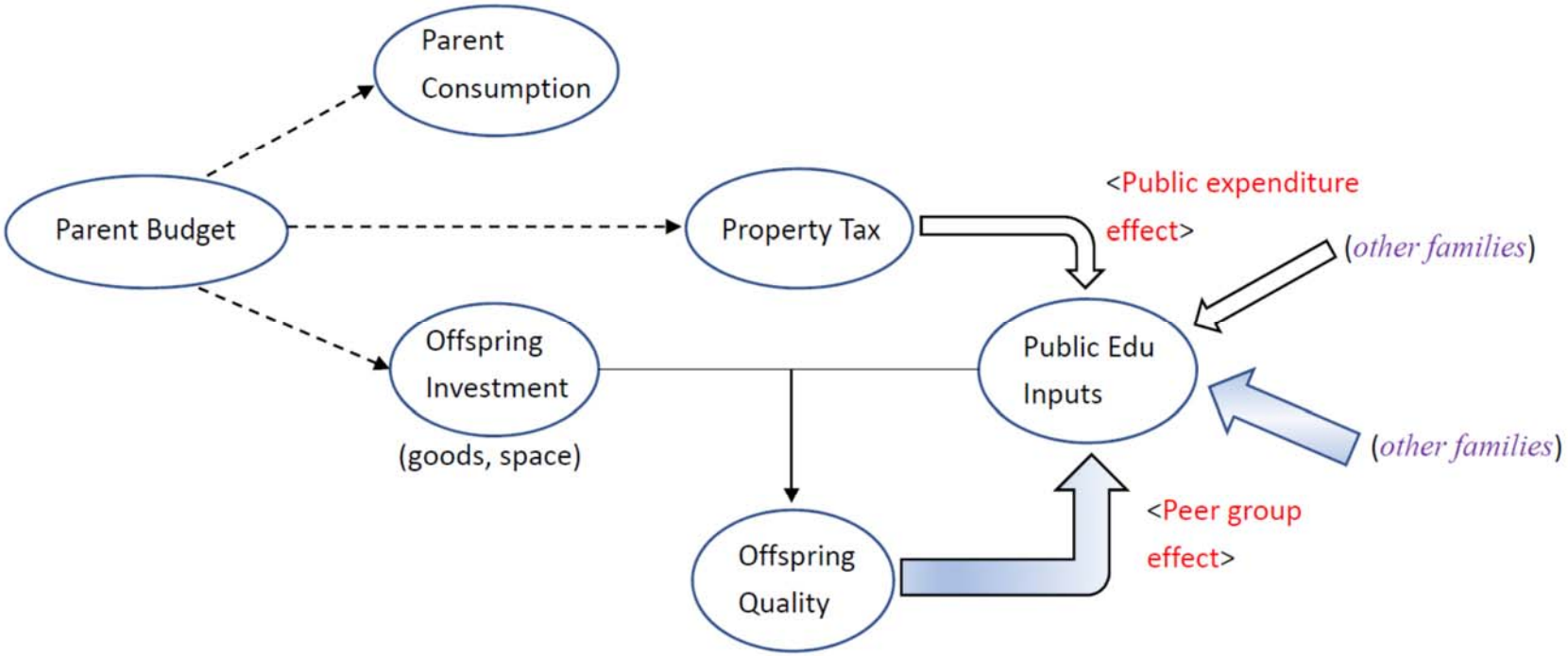
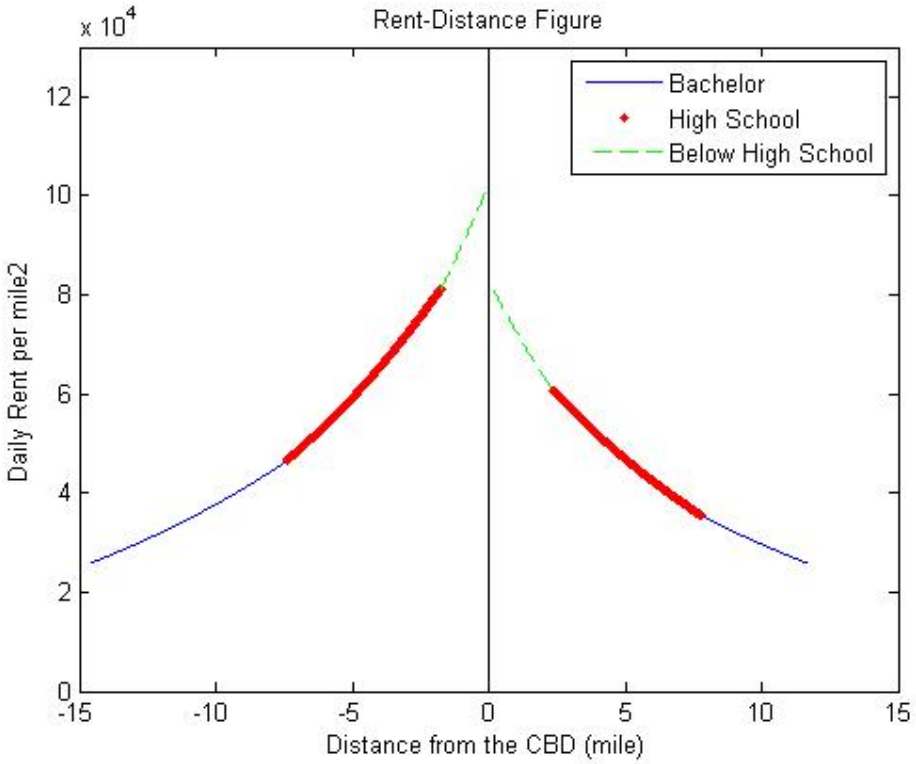
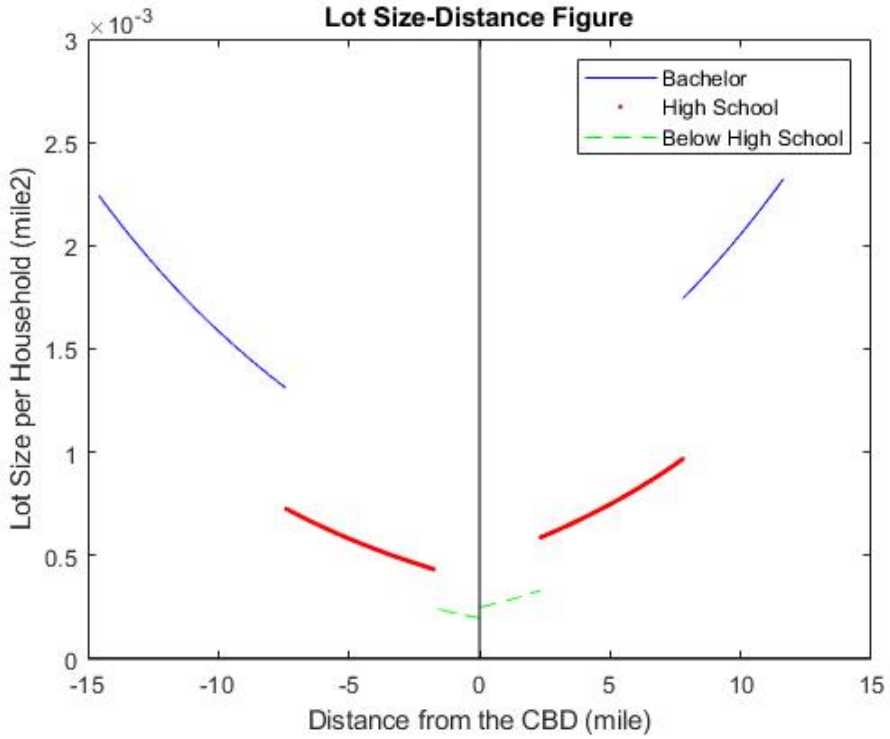


Figure 2. Rent, Lot-size, and Population Density at the Baseline Equilibrium

(a)



(b)


Lot Size-Distance Figure
×10-3
Bachelor
High School
Below High School
Lot Size per Household (mile2)
Distance from the CBD (mile)

(c)

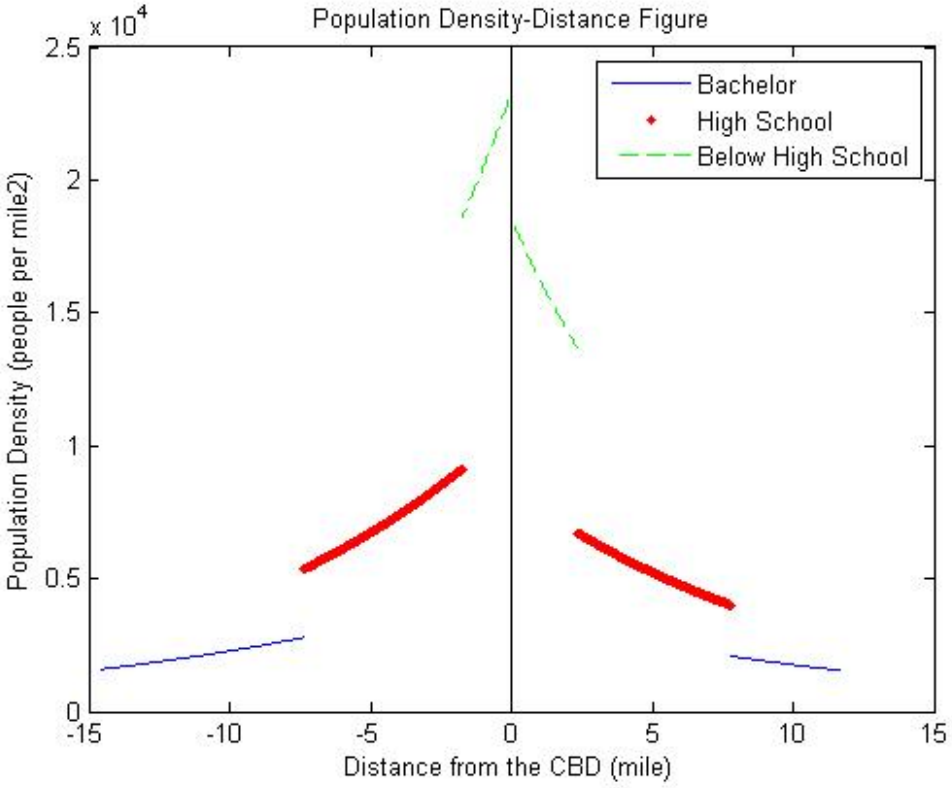

Population Density-Distance Figure
x 10^4
Bachelor
High School
Below High School
Population Density (people per mile2)
Distance from the CBD (mile)

Figure 3a. Rent-Distance Curve under Alternative Policy Regimes

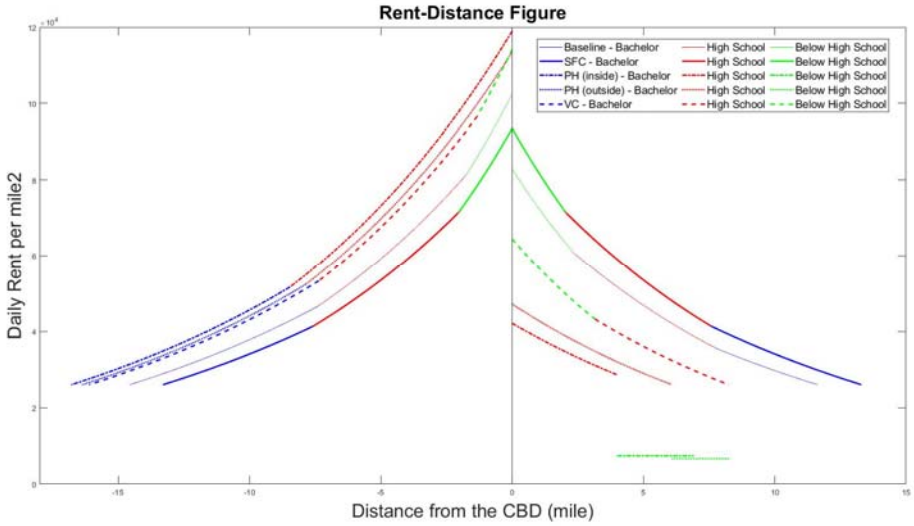


Figure 3b. Rent-Distance Curve under Policy Packages

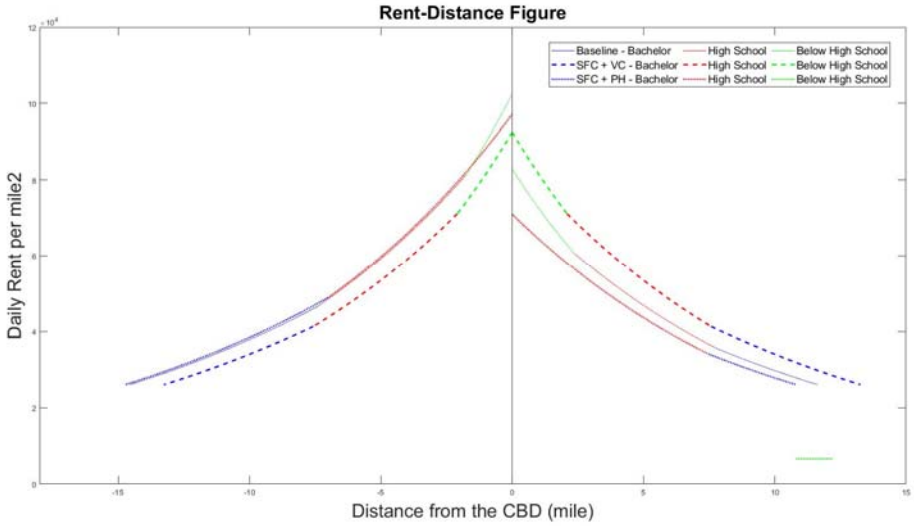


Table 1 Educational Attainment, Annual Income and Fertility Rate

Level of Education	Male Income (in $)	Female income (in $)	Income of Pseudo Household (in $)	Female Fertility Rate	Average Income ($) across Groups	Average Fer. Rate across Groups
Less than 9th grade	26,604	19,588	46,192	2.521	51,432	2.521
9th to 12th grade	33,194	23,478	56,672			
High school graduate	43,140	32,227	75,367	1.954	87,479	1.918
Some college	52,580	36,553	89,133	1.892		
Associate degree	55,631	42,307	97,938	1.869		
Bachelors degree	92,815	62,198	155,013	1.682	155,013	1.652
Graduate or professional degree				1.597		
Total	62,445	44,857	107,302	1.888	N/A	N/A

Note: Income data is in current dollar and is from U.S. Census Bureau (2009). Fertility data is from U.S. Census Bureau (2010). Due to data availability, we calculate the across group average income by taking simple average. The average fertility rate is accurately calculated.

Table 2 Statistics and Expressions

	Statistics	Expression
Share of Total Expenditure on Children	31% - 47%	$\frac{(\alpha_o+\beta_o)k_o^i}{(\alpha_p+\beta_p)k_p^i+(\alpha_o+\beta_o)k_o^i}$
Share of Total Expenditure on Housing	20%	$\frac{\alpha_o k_o^i+\alpha_p k_p^i}{(\alpha_p+\beta_p)k_p^i+(\alpha_o+\beta_o)k_o^i}$
Share of Children's Expenditure on Housing	31%	$\frac{\alpha_o}{\alpha_o+\beta_o}$
Share of Total 'Budget' on Leisure	65% - 70%	$\frac{k_p^i\eta}{1-k_o^i\gamma}$
Property Tax Rate	1.22% - 1.47% (of house value)	$\frac{\gamma k_o^i}{\alpha_p k_p^i+\alpha_o k_o^i-\gamma k_o^i}$
Fertility Rate	2.521 - 1.652	$\frac{k_n^i}{w_i c k_T^i}Y_i(\mathrm{r})$

Table 3a Symbols for Variables and Parameter Values

(Panel 1)

Variable Symbols	Interpretation
S_p (S_o)	lot size for parents (offspring)
Z_p (Z_o)	consumption goods for parents (offspring)
l	Parent leisure time
q	Educational quality of the public school
n_o	number of offspring
$g(n_o) = gn_o^{-\varepsilon}$	degree of altruism toward each child
$C(n_o) = cn_o$	time cost of bearing n_o offspring
r	distance from the CBD

(Panel 2)

Parameter Symbols	Short Description	Parameter Value	Source
a	per mile pecuniary cost	2.2	Similar to Hanushek and Yilmaz (2007)
b	per mile commuting time cost	0.1	Similar to Hanushek and Yilmaz (2007)
c	time cost of bearing each offspring	0.7179	Zick and Byrant (1996)
c_1	(Intercept) parameter in peer group effect function	10	Target: 70% of the population of the West is BA (bachelor degree holders)
c_2	(slope) parameter in peer group effect function	1	Normalization
w_{BA}	Wage rate of the BA (workers with bachelor degrees)	55	Our calculations based on the U.S. Census Bureau and Department of Labor
w_{HA}	Wage rate of the HA (workers who graduated from high school or obtained associate degrees)	32	Our calculations based on the U.S. Census Bureau and Department of Labor
w_{NH}	Wage rate of the NH (workers who did not graduate from high school)	20	Our calculations based on the U.S. Census Bureau and Department of Labor
$\bar{N}_1$	Population of BA	200,000	Our calculations based on the U.S. Census Bureau
$\bar{N}_2$	Population of HA	250,000	Our calculations based on the U.S. Census Bureau
$\bar{N}_3$	Population of NH	50,000	Our calculations based on the U.S. Census Bureau
R_a	Agricultural rent per acre per month	1,237	To match a fringe distance around 10~15 miles, similar to Hanushek and Yilmaz (2007)

(Panel 2, continued)

Parameter Symbols	Short Description	Parameter Value	Target
k_o^{BA}	Utility weight of offspring for BA	0.176	To jointly match six targets, including: (1) share of total expenditure on children, (2) share of total expenditure on housing, (3) share of children's expenditure on housing, (4) share of total "budget" on leisure, (5) preferred property tax rate, (6) fertility rate.
k_o^{HA}	Utility weight of offspring for HA	0.166	
k_o^{NH}	Utility weight of offspring for NH	0.141	
k_p^{BA}	Utility weight of parent for BA, $k_p^{BA} = 1 - k_o^{BA}$	0.824	
k_p^{HA}	Utility weight of parent for HA $k_p^{HA} = 1 - k_o^{HA}$	0.834	
k_p^{NH}	Utility weight of parent for NH $k_p^{NH} = 1 - k_o^{NH}$	0.859	
α_o	Utility weight of log size in offspring direct utility	0.2588	
β_o	Utility weight of consumption goods in offspring direct utility	0.5761	
γ	Utility weight of public school quality in offspring direct utility $\gamma = 1 - \alpha_o - \beta_o$	0.1651	
α_p	Utility weight of log size in parent direct utility	0.04	
β_p	Utility weight of consumption goods in parent direct utility	0.18	
η	Utility weight of leisure in parent direct utility $\eta = 1 - \alpha_p - \beta_p$	0.78	
ϵ	(slope) parameter of the parent altruism towards children	0.8	
g	(scale) parameter of the parent altruism towards children	1	Normalization

Table 3b Statistics and Calibration Results

Target		Real data	Baseline	SFC	PH1	PH2	VC	SFC+PH	SFC+VC
Labor Market-related variables									
Annual Income ($)	Group 1	51,432	51,233	51,219	46,311	46,837	45,366	48,686	45,234
	Group 2	87,479	88,288	88,288	87,583	87,522	87,615	87,492	87,607
	Group 3	155,013	153,394	153,412	152,030	152,075	152,108	152,225	152,232
Time Spent on Working per Day (hour)	Group 1	7.64	7.02	7.02	6.34	6.42	6.21	6.67	6.20
	Group 2		7.56	7.56	7.56	7.55	7.56	7.55	7.56
	Group 3		7.64	7.64	7.63	7.64	7.64	7.64	7.64
Family-related variables									
Fertility Rate	Group 1	2.521	2.566	2.567	2.604	2.568	2.683	2.464	2.691
	Group 2	1.918	1.913	1.913	1.915	1.919	1.912	1.921	1.913
	Group 3	1.652	1.624	1.626	1.604	1.608	1.612	1.625	1.626
Child-care Time Cost per Day (hour)	Group 1	1.3607 ~ 1.5110	1.8421	1.8427	1.8696	1.8437	1.9261	1.7691	1.9320
	Group 2		1.3736	1.3736	1.3745	1.3777	1.3728	1.3793	1.3732
	Group 3		1.1657	1.1670	1.1511	1.1546	1.1573	1.1664	1.1670
Proportion of Expenditure on Children	Group 1	31% ~ 47%	38.39%		35.44%	35.33%	38.39%	35.29%	38.39%
	Group 2		43.03%						
	Group 3		44.77%						
Housing Market-related variables									
Proportion of Total Expenditure on Housing	Group 1	Around 20%	23.10%		5.82%	5.18%	23.10%	4.93%	23.10%
	Group 2		23.70%						
	Group 3		23.92%						

Share of Children's Expenditure on Housing	Group 1	31%	31%	31%	8.45%	7.55%	31%	7.20%	31%
	Group 2			31%					
	Group 3								
Population per Acre		4.63	5.53	5.43	5.82	5.69	5.87	5.24	5.48
Preferred Property Tax Rate	Group 1	About 1.40%	1.22%						
	Group 2		1.40%						
	Group 3		1.47%						

Key 1: Group 1: Not a high school graduate; Group 2: High school graduate to Associate degree; Group 3: Bachelor's degree or above.

Key 2: SFC: School finance consolidation regime; PH1: Public Housing policy regime (Public housing units locate within the fringe distance); PH2: Public Housing policy regime (Public housing units locate outside the fringe distance); VC: Housing Voucher; SFC+PH: School finance consolidation and Public Housing policy (Public housing units locate outside the fringe distance) are imposed simultaneously; SFC + VC: School finance consolidation and Housing Voucher policy are imposed simultaneously

Table 4a Cross-community Welfare Comparison at the Baseline equilibrium

	Total Welfare (Utility level)	Parent Direct Utility		Offspring Quality	
		West	East	West	East
Group 1	10.4342	12.3364	12.4337	0.9857	0.9441
Group 2	11.4934	13.4727	13.5910	2.3647	2.2709
Group 3	13.6434	14.9952	15.2109	5.0705	4.6966
Average	12.2474	14.0249		3.0554	

Key 1: Group 1: Not a high school graduate; Group 2: High school graduate to Associate degree; Group 3: Bachelor’s degree or above.

Table 4b Equilibrium Outcome Summary

Variables		Baseline	SFC	PH1	PH2	VC	SFC+PH	SFC+VC
Household Distribution								
Number of Household in the West	Group 1	4.26%	5%	0%	0%	2.69%	0%	5%
	Group 2	27.83%	25%	45.35%	39.34%	32.72%	27.66%	25%
	Group 3	28.38%	20%	40%	40%	40%	30.62%	20%
Number of Household in the East	Group 1	5.74%	5%	10%	10%	7.31%	10%	5%
	Group 2	22.17%	25%	4.65%	10.66%	17.28%	22.34%	25%
	Group 3	11.62%	20%	0%	0%	0%	9.38%	20%
Community Comparison								
(W) School Quality/ Property Tax Rate		564/ 1.47%	406/ 1.40%	618/ 1.40%	774/ 1.47%	741/ 1.47%	632/ 1.40%	386/ 1.40%
(E) School Quality/ Property Tax Rate		280/ 1.40%	406/ 1.40%	34/ 1.40%	64/ 1.40%	123/ 1.40%	264/ 1.40%	386/ 1.40%
(W) Average Rent ($)		41,076	39,515	44,709	43,861	43,309	41,212	39,513
(E) Average Rent ($)		37,703	39,515	28,488	29,316	35,172	34,405	39,513
(W) Annual Income ($)		116,192	110,631	117,810	120,090	120,269	121,491	109,219
(E) Annual Income ($)		102,106	110,631	59,255	67,752	75,187	92,726	109,219
(W) Annual Edu-Spending ($)		5,321	4,815	5,362	5,668	5,627	4,578	4,757
(E) Annual Edu-Spending ($)		4,308	4,815	957	1,651	2,931	4,578	4,757

Key 1: Group 1: Not a high school graduate; Group 2: High school graduate to Associate degree; Group 3: Bachelor's degree or above.

Key 2: SFC: School finance consolidation regime; PH1: Public Housing policy regime (Public housing units locate within the fringe distance); PH2: Public Housing policy regime (Public housing units locate outside the fringe distance); VC: Housing Voucher; SFC+PH: School finance consolidation and Public Housing policy (Public housing units locate outside the fringe distance) are imposed simultaneously; SFC + VC: School finance consolidation and Housing Voucher policy are imposed simultaneously.

Table 5a Summary Statistics of the Model-Generated Data

Variables	Mean		Minimum		Maximum		Standard Deviation	
	West	East	West	East	West	East	West	East
R_i	52,605	46,692	26,042	26,050	102,408	82,834	20,560	15,812
D_i	7.2857	5.7909	0.0031	0.0015	14.5629	11.6291	4.1937	3.3888
w_i	41.8436	37.1336	20	20	55	55	13.3761	13.3541

Table 5b Regression Results

$$\log R_i = \beta_0 + \beta_D D_i + \beta_w w_i + u_i$$

	Point Estimate (Standard Deviation)	
	West	East
β_0	11.4857 (0.0018)	11.2497 (0.0016)
β_D	-0.0880 (0.0002)	-0.1005 (0.0003)
β_w	-0.0011 (0.0001)	0.0008 (0.0001)
Sample Size	2,800	2,800
R^2	0.9962	0.9951
F-statistics	370,278	286,228

Table 6 Equilibrium Welfare Comparison

Household Type	Baseline	SFC	PH1	PH2	VC	SFC + PH	SFC + VC
Welfare Comparison (Consumption-Equivalent Measure %)							
Panel A: Household Level							
Group 1	Benchmark	+0.02	+19.98	+20.22	+18.59	+17.53	+18.93
Group 2		+0.02	-6.30	-2.69	-2.14	+0.45	-1.62
Group 3		+0.01	-5.55	-2.26	-2.06	+0.28	-1.62
Average		+0.02	-3.46	-0.35	-0.19	+1.92	+0.28
Panel B: Parent Direct Utility (PDU) Only							
Group 1	Benchmark	-0.44	+31.13	+25.81	+25.79	+7.52	+22.89
Group 2		+0.15	-5.59	-2.08	-1.58	+0.87	-0.91
Group 3		+1.33	-10.33	-9.02	-7.83	-0.79	+0.36
Average		+0.61	-3.45	-1.98	-1.27	+0.78	+2.05
Panel C: Offspring Quality (OQ) Only							
Group 1	Benchmark	+0.75	-10.54	+8.12	-8.13	+39.92	-0.33
Group 2		-0.24	-6.96	-3.63	-2.66	-1.02	-2.74
Group 3		-2.28	+3.45	+8.58	+7.14	+1.74	-4.93
Average		-1.30	-1.78	+3.62	+1.85	+3.58	-4.63

Key 1: Group 1: Not a high school graduate; Group 2: High school graduate to Associate degree; Group 3: Bachelor's degree or above.

Key 2: SFC: School finance consolidation regime; PH1: Public Housing policy regime (Public housing units locate within the fringe distance); PH2: Public Housing policy regime (Public housing units locate outside the fringe distance); VC: Housing Voucher; SFC+PH: School finance consolidation and Public Housing policy (Public housing units locate outside the fringe distance) are imposed simultaneously; SFC + VC: School finance consolidation and Housing Voucher policy are imposed simultaneously.

Table 7 Flexibility of Various Choices

Type of Goods	Short-run	Long-run
Fertility Rate	Inflexible	Flexible
Lot Size	Inflexible	Flexible
Rental Rate	Inflexible	Flexible
Residential Location	Inflexible	Flexible
Non-durable Good	Flexible	Flexible
Leisure	Flexible	Flexible
Property Tax Rate	Flexible	Flexible
School Quality	Flexible	Flexible

Table 8a Short-run VS Long-run (Statistics)

Statistics		Baseline	SFC		VC		SFC + VC	
			SR	LR	SR	LR	SR	LR
Annual Income ($)	Group 1	51,233	51,170	51,219	44,990	45,366	44,927	45,234
	Group 2	88,288	88,149	88,288	88,428	87,615	88,289	87,607
	Group 3	153,394	153,090	153,412	153,645	152,108	153,339	152,232
Annual Consumption ($)	Group 1	38,554	38,576	38,567	40,752	40,311	40,773	40,434
	Group 2	64,227	64,279	64,224	63,660	63,678	63,712	63,696
	Group 3	110,175	110,293	110,299	109,197	108,505	109,315	109,415
Hours Worked per Day	Group 1	7.02	7.01	7.02	6.16	6.21	6.15	6.20
	Group 2	7.56	7.55	7.56	7.57	7.56	7.56	7.56
	Group 3	7.64	7.62	7.64	7.65	7.64	7.64	7.64
Voucher Received per Day ($)		N.A.	N.A.	N.A.	23.1249	23.1223	23.1249	23.1326
Property Tax Rate	West	1.47%	1.40%	1.40%	1.47%	1.47%	1.40%	1.40%
	East	1.40%	1.40%	1.40%	1.40%	1.40%	1.40%	1.40%
Annual Edu-Spending ($)	West	5,321	4,761	4,815	5,321	5,627	4,761	4,757
	East	4,308	4,761	4,815	4,308	2,931	4,761	4,757
Peer Quality	West	38.67	30.18	30.75	38.02	48.08	29.87	29.62
	East	23.73	30.18	30.75	23.61	15.11	29.87	29.62
School Quality	West	564	394	406	554	741	390	386
	East	280	394	406	279	123	390	386

Key 1: Group 1: Not a high school graduate; Group 2: High school graduate to Associate degree; Group 3: Bachelor's degree or above.
Key 2: SFC: School finance consolidation regime; VC: Housing Voucher; SFC + VC: School finance consolidation and Housing Voucher policy are imposed simultaneously.
Key 3: SR: Short-run; LR: Long-run.
Key 4: N.A.: Not Applicable.

Table 8b Short-run VS Long-run (Welfare)

Household Type		Baseline	SFC		VC		SFC + VC	
			SR	LR	SR	LR	SR	LR
Welfare Comparison (Consumption-Equivalent Measure %)								
Panel A: Household Level								
Group 1	W	Benchmark	-3.07	+0.02	+19.01	+18.59	+16.55	+18.93
	E		+3.30	+0.02	+19.16	+18.59	+21.79	+18.93
Group 2	W		-3.53	+0.02	-1.30	-2.14	-4.80	-1.62
	E		+3.72	+0.02	-1.17	-2.14	+2.54	-1.62
Group 3	W		-3.70	+0.01	-1.33	-2.06	-5.00	-1.62
	E		+3.88	+0.01	-1.18	N.A.	+2.69	-1.62
Average			-0.70	+0.02	+0.62	-0.19	-0.04	+0.28
Panel B: Parent Direct Utility Only								
Group 1	W	Benchmark	+0.71	+2.05	+25.55	+21.61	+26.06	+24.80
	E		0	-2.32	+25.63	+26.58	+25.63	+21.45
Group 2	W		+0.78	+2.28	-1.25	-3.76	-0.26	+1.25
	E		0	-2.58	-1.26	+3.68	-1.26	-3.66
Group 3	W		+0.80	+3.59	-1.29	-5.26	-0.27	+2.64
	E		0	-4.38	-1.27	N.A.	-1.27	-5.40
Average			+0.48	+0.61	+1.52	-1.27	+2.00	+2.05
Panel C: Offspring Quality Only								
Group 1	W	Benchmark	-10.69	-3.56	+4.92	+1.08	-5.04	-4.68
	E		+9.28	+3.91	+5.26	-10.23	+13.93	+2.87
Group 2	W		-10.68	-3.37	-1.38	+0.16	-11.99	-5.95
	E		+9.28	+3.64	-1.04	-11.02	+8.20	+1.24
Group 3	W		-10.67	-6.22	-1.38	+3.56	-11.99	-8.97
	E		+9.28	+7.00	-1.04	N.A.	+8.20	+4.60
Average			-2.96	-1.30	-0.97	+1.85	-3.88	-4.63

Key 1: Group 1: Not a high school graduate; Group 2: High school graduate to Associate degree; Group 3: Bachelor's degree or above.

Key 2: SFC: School finance consolidation regime; VC: Housing Voucher; SFC + VC: School finance consolidation and Housing Voucher policy are imposed simultaneously.

Key 3: SR: Short-run; LR: Long-run.

Key 4: W: West; E: East.

Key 5: N.A.: No Group 3 household reside in the East under VC policy in the LR.

Appendix

Alternative Calibration Targets

In the paper's main text, we choose parameters in the peer effect function: 70% of BA-type households reside in the West and determine the property tax rate there. To justify this choice, in this appendix, we consider alternative targets (55%, 75%, and 80%) and examine whether our main results are robust to the choice of target. We recalibrate the model for each alternative target and redo the analysis for SFC and SFC + PH. The results are summarized in Table A1 and A2.

We find that 70% is approximately the lowest fraction to ensure that skilled (BA type) workers can constitute the majority of the West community and determine the property tax rate. When we use a higher fraction as the target, the positive effect of SFC policy becomes bigger. Consequently, our main finding is that combining public housing programs and the school finance consolidation policy can lead to Pareto Improvement and help the poor become even more potent. If we further reduce this fraction (from 70% to 55%), the BA-type household no longer constitutes the majority of the West, and the sorting pattern becomes much weaker. In this case, the overall welfare effect of the SFC policy becomes negative, consistent with the findings in previous HY papers. However, *even in this case, we still find that combining SFC and PH can lead to Pareto improvement, suggesting that our main finding is robust to this.*

Table A1 Equilibrium Welfare Comparison (SFC): Different Targets

Household Type	70% (used in the paper)	75%	80%	55%
Welfare Comparison (Consumption-Equivalent Measure %)				
Panel A: Household Level				
Group 1	+0.02	+0.05	+0.08	-0.04
Group 2	+0.02	+0.06	+0.09	-0.03
Group 3	+0.01	+0.05	+0.08	-0.03
Average	+0.02	+0.05	+0.06	-0.06
Panel B: Parent Direct Utility (PDU) Only				
Group 1	-0.44	-0.69	-1.02	-0.04
Group 2	+0.15	+0.18	+0.22	-0.00
Group 3	+1.33	+1.92	+2.67	+0.07
Average	+0.61	+0.86	+1.15	-0.01
Panel C: Offspring Quality (OQ) Only				
Group 1	+0.75	+1.25	+1.86	-0.02
Group 2	-0.24	-0.22	-0.21	-0.08
Group 3	-2.28	-3.29	-4.58	-0.21
Average	-1.30	-1.81	-2.54	-0.23

Key 1: SFC: School finance consolidation regime.
Key 2: Group 1: Not a high school graduate; Group 2: High school graduate to an Associate degree; Group 3: Bachelor's degree or above.
Key 3: The fractions shown in the first row are the fraction of Group 3 households in the West in the baseline case.

Table A2 Equilibrium Welfare Comparison (SFC + PH): Different Targets

Household Type	70% (used in the paper)	75%	80%	55%
Welfare Comparison (Consumption-Equivalent Measure %)				
Panel A: Household Level				
Group 1	+17.53	+17.34	+17.90	+17.42
Group 2	+0.45	+0.52	+0.56	+0.42
Group 3	+0.28	+0.35	+0.38	+0.25
Average	+1.92	+1.99	+2.04	+1.88
Panel B: Parent Direct Utility (PDU) Only				
Group 1	+7.52	+7.04	+7.55	+7.70
Group 2	+0.87	+0.91	+0.94	+0.72
Group 3	-0.79	-0.24	+0.50	-2.01
Average	+0.78	+1.00	+1.35	+0.20
Panel C: Offspring Quality (OQ) Only				
Group 1	+39.92	+40.12	+40.45	+39.63
Group 2	-1.02	-0.92	-0.91	-0.82
Group 3	+1.74	+0.97	-0.22	+3.69
Average	+3.58	+3.26	+2.56	+4.66

Key 1: SFC+PH: School finance consolidation and Public Housing policy (Public housing units located outside the fringe distance) are imposed simultaneously.
Key 2: Group 1: Not a high school graduate; Group 2: High school graduate to an Associate degree; Group 3: Bachelor's degree or above.
Key 3: The fractions shown in the first row are the fraction of Group 3 households in the West in the baseline case.

HY-type Peer Group Effect Function

In the paper's main text, we assume that the peer group effect in a district is determined by the average quality of students residing in that district, which differs from Hanushek and Yilmaz (2007, 2013). They assume that the peer group effect is determined by the fraction of college-educated households in the district. Our formulation implies that the peer group effect depends on parental investment in children and government spending on education, all equilibrium objects. Since the equilibrium cross-district allocation of these objects changes when counterfactual policies are imposed, our formulation can potentially lead to different policy implications from the original HY formulations.

To quantify the importance of our formulation, we redo all of the counterfactual policy analysis considered in the paper, adopting a peer group effect function similar in spirit to the original HY formulations. Expressly, in each of the counterfactual, when computing the peer group effect in a district, we assume that the average quality of students for each household type is the same as its counterpart in the baseline equilibrium. Since the overall average quality of students in a district is just a weighted (by population structure in the neighborhood) average of average qualities of children for each type of household, the peer group effect in a community only depends on the district's population composition under this alternative formulation. We re-solve for the equilibrium using this formulation. The results are summarized in Table A3.

Comparing Table A3 to Table 6 in the paper, *we find both qualitative and quantitative differences.* Perhaps most notably, we find substantially larger positive effects of public housing and housing voucher programs on students' quality from NH (Not a High school graduate) type households under this alternative HY-type formulation of peer group effects. For example, as shown in the VC column of Panel C in Table 6, students' average quality from NH type households drops by 8.13% after the housing voucher program is implemented under our peer group effects. In contrast, the VC column of Panel C in Table A3 shows that the housing voucher program leads to a 0.93% increase in the average quality of students from NH type households under the alternative HY-type formulation. Similar patterns can be found by comparing the PH1 and PH2 columns of the two tables.

The reason behind this difference is as follows. As shown in the last two columns (Annual Edu-Spending) of Table 4b, the three policies (PH1, PH2, and VC) lead to more extreme sorting of households and lower the per capita educational spending the East district, where all or most NH type households reside. As a result, the average quality of students from NH type households decreases in these policy regimes. Under our formulation, this decrease in average student quality has an additional negative effect through the peer group effect. The peer group effect is weaker in the East district, resulting in lower school quality for NH type households who primarily reside in the East. Under the alternative HY-type formulation, this additional negative effect is assumed away.

Table A3 Equilibrium Welfare Comparison (Alternative Peer Group Effects)

Household Type	Baseline	SFC	PH1	PH2	VC	SFC + PH	SFC + VC
Welfare Comparison (Consumption-Equivalent Measure %)							
Panel A: Household Level							
Group 1	Benchmark	+0.17	+20.19	+20.18	+19.28	+20.14	+19.28
Group 2		+0.19	-6.36	-1.78	-1.12	-1.82	-1.12
Group 3		+0.20	-5.64	-1.70	-1.10	-1.74	-1.10
Average		+0.20	-3.51	+0.30	0.77	+0.27	0.77
Panel B: Parent Direct Utility (PDU) Only							
Group 1	Benchmark	-0.44	+29.58	+21.87	+22.89	+21.77	+22.89
Group 2		+0.15	-5.48	-0.10	-0.91	+0.03	-0.91
Group 3		+1.33	-10.24	-7.77	+0.36	-7.65	+0.36
Average		+0.61	-3.58	-1.01	+2.05	-0.92	+2.05
Panel C: Offspring Quality (OQ) Only							
Group 1	Benchmark	+1.20	-4.04	+18.59	+0.93	+18.72	+0.93
Group 2		+0.22	-7.41	-4.79	-1.46	-5.10	-1.46
Group 3		-1.82	+3.10	+7.83	-3.61	+7.68	-3.61
Average		-0.84	-1.82	+3.53	-3.32	+3.33	-3.32

Key 1: Group 1: Not a high school graduate; Group 2: High school graduate to an Associate degree; Group 3: Bachelor's degree or above.

Key 2: SFC: School finance consolidation regime; PH1: Public Housing policy regime (Public housing units located within the fringe distance); PH2: Public Housing policy regime (Public housing units located outside the fringe distance); VC: Housing Voucher; SFC+PH: School finance consolidation and Public Housing policy (Public housing units located outside the fringe distance) are imposed simultaneously; SFC + VC: School finance consolidation and Housing Voucher policy are imposed simultaneously.