ISER Seminar Series
1/19/12 (Thu) 4:00-6 PM

Expectation formation functions and price dynamics: an application to the U.S. hog market

SaangJoon Baak*

Professor
School of International Liberal Studies
Waseda University
1-6-1 Nishi-Waseda, Shinjuku-ku
Tokyo 169-8050, Japan

email: baak@waseda.jp
Fax: 81-3-3208-8401
Phone: 81-3-5286-9866

Sep. 1, 2011

ABSTRACT

This paper explores the dynamics of the U.S. hog market with three different dynamic models that are distinguished only by their assumptions with regard to the market participants' expectations of future prices. The first model assumes that all the producers in the market have rational expectations. The second model assumes that a constant fraction of the producers have static expectations. The third model, the main focus of the paper, assumes producers choose either rational expectations or static expectations every year based on the past performances of the two different types of expectations. Empirical tests such as log-likelihood ratio type specification tests of GMM estimations and one-step ahead forecasts with the annual data covering 1945-2007 indicate that the third model best captures the movements of the price and quantity data in the hog market, even though the empirical value added by the third model over the second is not great. Simulation experiments using estimated parameter values illustrate that the market will reach a steady state in the framework of the first and the second model if external shocks are zero. In contrast, in the framework of the third model, the market does not reache a steady state but generates cycling movements of the price. If the intensity of choice is increased, the price dynamics can be even chaotic.

JEL Classification: C61, C62, E32, E37

Keywords: Heterogeneous expectations, Bounded rationality, Economic cycles, Non-linear dynamics, Chaos

1. Introduction

This paper investigates the price dynamics of the U.S. hog market with three different dynamic models that are distinguished only by their assumptions with regard to market participants' expectations of future prices. Specifically, the paper aims to determine which model best captures the movements of the actual data of the market. Subsequently, the price dynamics of the market are explored using the three models, with particular attention given to the best model.

Model 1 has the scheme of a typical linear quadratic investment (or, inventory) decision model such as in Sargent (1987, Ch. 14) and in West (1993). Producers intend to maximize their present and future stream of profits by optimal investment. They are fully rational in the sense that their expectations are equivalent to mathematical expectations. Jarvis (1974) and Rosen et al. (1994) employ these kind of models to explain the dynamics of cattle markets.

The other two models (Model 2 and Model 3) presented in this paper are identical to Model 1 with the exception of the assumption regarding the expectations of future prices. Model 2 assumes that a constant fraction of the producers have static expectations. Therefore, there exist two different types of economic agents, rational and boundedly rational, in the market. Using this type of model, Baak (1999, 2000) and Chavas (1999, 2000) estimated the fractions of boundedly rational economic agents in agricultural markets and reported that the fractions are significantly different from zero. Aadland (2002) shows that the data of the U.S. beef cattle market can be best explained by his dynamic model when the fraction of boundedly rational ranchers is assumed to be around 0.3 as is the estimated value in Baak (1999).

In the meantime, Model 3 assumes, following Brock and Hommes (1997), a producer chooses either a rational or a boundedly rational expectation formation function every year based on the observed relative performances of the two different expectation formation functions. Since the producers can freely switch their expectation formation function every year and since their choices are not homogeneous, the fraction of boundedly rational economic agents can fluctuate between zero and one.

DeGrauwe and Grimaldi (2006) and Jong, Verschoor and Zwinkels (2010) applied the idea of Brock and Hommes (1997) to foreign currency markets and showed that the model allowing market participants to switch their expectation formation functions better explained the foreign exchange rate data. The present paper applies the same idea to the U.S. hog market in Model 3.[1]

The parameter values of the three models, estimated by the GMM, are all reasonable and statistically significant. The fraction of boundedly rational agents is estimated to be 38.4 percent in Model 2. That fraction varies every year in Model 3, depending on the relative performance of the boundedly rational expectations and the intensity of choice.[2] The estimation results of Model 3 indicate that the fraction moves between 13.7 percent and 49.5 percent for the period from 1948 to 2007. However, the average of the fraction for the period is 36.8 percent, close to the value estimated by Model 2.

Empirical tests such as the log-likelihood ratio type specification tests of GMM estimations and one-step ahead forecasts indicate Model 3 best captures the movements of the price and quantity data in the U.S. hog market. Therefore, the main focus of this paper is given to this model when price dynamics are examined by simulations. It should be noted, however, that the empirical value added of Model 3 over Model 2 is not great, while Model 2 performs significantly better than Model 1.

Simulation experiments using estimated parameter values illustrate that the market will reach a steady state in the framework of Model 1 and Model 2 if external shocks are zero. The price settles at the steady state value more slowly in Model 2. In contrast, when the artificial data are generated by Model 3, the market does not reach a steady state but generates cyclical movements of the hog price, indicating that the market is inherently volatile. The dynamics of the market described by Model 3 become even chaotic if the value of the intensity of choice is increased higher than a certain value. On the contrary, if

[1] Baak (1999, 2000), Chavas (1999, 2000) and Aadland (2002) adopted a heterogeneous expectations model inspired by Brock and Hommes (1997). However, their models assumed that the fraction of boundedly rational agents is fixed. As far as I know, no paper exploring the agricultural market or employing an investment decision model has allowed market participant to switch their expectation formation functions as the present paper tries in Model 3. .

[2] The value of the intensity of choice is not estimated but assumed in the way to minimize the GMM function. More detailed explanation is presented in section 3.

the value of the intensity of choice is decreased lower than a certain value, the market reaches a steady state just as in the other two models.

2. Models

The three models presented in the paper adopt the typical framework of a linear quadratic investment decision model applied to the agricultural sector. Similar models can be found in Jarvis (1974), Chavas and Klemme (1986), Rosen et al. (1994), Chavas (1999, 2000), and Baak (1999, 2000). The basic scheme of the models in the present paper is similar to those found in Chavas (1999) and Baak (2000) which also analyzed the U.S. hog market.

Model 1 in the present paper assumes that all the producers in the market are fully rational. Model 2 assumes, following Chavas (1999, 2000) and Baak (1999, 2000), that a constant fraction of the producers have static expectations. It implies that there are two different types of producers in the market with regard to expectations of future prices. In addition, neither rational nor boundedly rational producers change their expectation formation functions over time.

Following Brock and Hommes (1997), Model 3 assumes that producers choose either rational expectations or boundedly rational expectations every year based on the past relative performances of the two different types of expectations. Since the relative performance of the two expectations change over time, the fraction of boundedly rational economic agents fluctuates between zero and one in the model. The form of the boundedly rational expectation will be specified later in this section.

With the exception of the assumption regarding the expectations of future prices, the three models are identical.

Production technology

The final output of the hog industry at time t is the sum of adult hogs that have survived the previous year and one-year old animals which have just joined the adult stock.

$$y_t = (1-\delta)k_{t-1} + (1-\delta)x_{t-1} \tag{1}$$

$$x_{t-1} = gk_{t-1} \tag{2}$$

where y_t is the final output, k_{t-1} is the adult stock at time $t-1$, x_{t-1} is the pig crop at time $t-1$, δ is the natural death rate, and g is the birth (fertility) rate per sow. Following Chavas (1999), piglets are assumed to become adults after one period because they are able to join the breeding stock at 8 months.[3]

In this industry, the amount of capital (breeding) stock at time t equals the amount of investment at time t due to the fact that the final output is either sold in the market or held as an investment for future production (i.e. breeding). Therefore,

$$y_t = i_t + c_t \tag{3}$$

$$k_t = i_t \tag{4}$$

where i_t is investment, and c_t consumption (or sales) at time t, The final output held for future production is all female since all young males are sold (or slaughtered) at maturity. Therefore, although c_t contains both males and females, $i_t (= k_t)$ contains just female stock.

Two types of producers

[3] Refer to Chavas (1999) for a more detailed explanation on the production technology of hog markets.

The first model of this paper assumes homogeneous producers. In contrast, the second and the third model assume heterogeneous producers. Specifically, there are two types of producers and they have different expectation formation functions of the future hog price. One type (a rational producer) is assumed to have a rational expectation. He fully utilizes all available information including the existence and the fraction of the other type of producers to predict future hog prices. Therefore, his prediction will be the same as that of the model.

The other type (a boundedly rational producer) is assumed, as in Grandmont (1994), Nerlove and Fornari (1998), Hommes and Sorger (1998) and Chavas (1999), to formulate his hog price expectations based on time series observations. For example, Chavas (1999) assumed that their expectation formation function is an AR(3) equation, since the hog price data is well fitted to the AR(3) equation. However, in the present paper the price and quantity data are detrended by the Hodrick-Prescott filter, and the detrended hog price data do not fit to any AR process. Instead, they look like random walks to the naked eye. Therefore, a boundedly rational producer is assumed to have a static expectation in the present paper. That is, he holds Ezekiel (1938)'s cobweb expectation. Figures 1-1 and 1-2 illustrate the nominal and real hog price data from 1945 to 2007, respectively. The CPI data whose base year is 1967 was used to obtain the real hog price data. In the meantime, Figure 1-3 illustrates the detrended real hog price data, which is used in the estimations in section 3.

Objective function

The integral sum of all producers in the present model is assumed to be unity without loss of generality. The fraction of boundedly rational producers at time t is denoted by n_t. By definition, n_t is between zero and one. The goal of a producer, whether rational or boundedly rational, is assumed to be to maximize the expected present discounted value of current and future profits over an infinite horizon of time. Specifically, the objective function that a producer intends to maximize is the following:

$$\max_{i_{j,t}} E_{j,0}\left\{\sum_{t=1}^{\infty}\beta^t\left[p_t c_{j,t} - \rho_h h_t\left(k_{j,t} + x_{j,t}\right) - \frac{\psi_c}{2}c_{j,t}^2 - \frac{\psi_0}{2}k_{j,t}^2\right]\right\} \tag{5}$$

$$\text{s.t.} \quad c_{j,t} = (1-\delta)k_{j,t-1} + (1-\delta)gk_{j,t-1} - i_{j,t}$$

$$k_{j,t} = i_{j,t}$$

$$x_{j,t} = gk_{j,t}$$

where p_t is the market price of an adult animal and $\rho_h h_t$ is the one-period holding costs for a breeding animal and a piglet. The quadratic term $\frac{\psi_0}{2}k_t^2$ captures the increasing costs of holding adult animals and $\frac{\psi_c}{2}c_t^2$ captures the increasing costs of preparing them for slaughter. The discount factor β is assumed to be positive yet less than one. The parameters ψ_c and ψ_0 are assumed to be positive.

The subscript *j* in the endogenous state variables c, k, and the control variable i denotes that the variables are associated with a type *j* (*j*=1 or 2) producer. Hereafter, subscripts 1 and 2 represent rational and boundedly rational economic agents, respectively. For example, $c_{1,t}$ and $c_{2,t}$ are the quantities supplied by a rational and boundedly rational producer, respectively.

The holding costs $\{h_t\}_{t=0}^{\infty}$ are exogenous state variables, while the price stream $\{p_t\}_{t=0}^{\infty}$ is determined by the competitive market equilibrium.

Demand

The demand for hogs is assumed to be a linear function of the market price as in other agricultural articles such as Rosen et al. (1994) and Chavas (1999).

$$c_t = \alpha_0 - \alpha_1 p_t + \rho_d d_t + \varepsilon_t^d, \text{ where } \alpha_0 > 0, \alpha_1 > 0 \tag{6}$$

where d_t is a demand shifter and ε_t^d is white noise. It is typical to assume a linear market demand function in investment (or, inventory) decision models.[4]

Euler equation

The Euler equation is obtained by differentiating the objective function (5) by the control variable, i_t. Since the two types of producers have different price expectations, they have different Euler equations. The Euler equation of a rational producer is

$$E_t \begin{bmatrix} -p_t + \psi_c c_{1,t} - \rho_h h_t (1+g) \\ + \beta p_{t+1}(1-\delta+g) - \beta \psi_c c_{1,t+1}(1-\delta+g) \end{bmatrix} = 0$$

(7)

On the other hand, the Euler equation of a boundedly rational producer is

$$E_t \begin{bmatrix} -E_{2,t}(p_t) + \psi_c c_{2,t} - \rho_h h_t (1+g) \\ + E_{2,t}\left(\beta p_{t+1}(1-\delta+g)\right) - \beta \psi_c c_{2,t+1}(1-\delta+g) \end{bmatrix} = 0 \qquad (8)$$

where $E_{2,t}$ denotes the expectations operator of a boundedly rational producer. As mentioned earlier, he has static expectations, implying $E_{2,t}(p_{t+i}) = p_{t-1}$ where $i = 0, 1, 2$ …

Accordingly, the Euler equation of a boundedly rational rancher can be re-written in the following way.

$$E_t \begin{bmatrix} -p_{t-1} + \psi_c c_{2,t} - \rho_h h_t (1+g) \\ + \beta p_{t-1}(1-\delta+g) - \beta \psi_c c_{2,t+1}(1-\delta+g) \end{bmatrix} = 0 \qquad (8)`$$

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[4] See West (1993)

The aggregate (market) Euler equation is obtained by adding the two Euler equations, (7) and (8)`, after multiplying each equation by the fraction of each type of producer. Specifically, $(1-n_t)$ and $n_t$ are multiplied to equation (7) and equation (8)` respectively. Then, the aggregate Euler equation is the following.

$$E_t\begin{bmatrix} -(1-n_t)p_t + \psi_c c_t - \rho_h h_t(1+\gamma_0 g) \\ +(1-n_t)\beta p_{t+1}(1-\delta+g) - \beta\psi_c c_{t+1}(1-\delta+g) \\ -n_t p_{t-1} + n_t \beta p_{t-1}(1-\delta+g) \end{bmatrix} = 0 \qquad (9)^5$$

From the constraints in the objective function (5), the aggregate (market) constraints can be obtained in the same way.

$$c_t = (1-\delta)k_{t-1} + (1-\delta)gk_{t-1} - k_t \qquad (10)$$

$$x_t = gk_t \qquad (11)$$

Then, the market as a whole is described by the aggregate Euler equation (9), the aggregate constraints (10) and (11), the demand function (6), and the way that the fraction of boundedly rational agents, n_t, is determined. The three models presented in this paper are different only with regard to the assumption about the determination of n_t.

Model 1

Model 1 assumes that producers are homogeneous and fully rational. Therefore, the fraction of boundedly rational agents, n_t, is zero.

Model 2

[5] Note that $c_t = n_t c_{1,t} + (1-n_t)c_{2,t}$.

Model 2 assumes that a fraction of producers possess static expectations and that the fraction is constant. Therefore, the fraction of boundedly rational agents, n_t, is a constant number between zero and one. That is, $n_t = \bar{n}$. If $\bar{n}$ happens to be zero, Model 2 is exactly the same as Model 1. Therefore, Model 2 can be regarded as a generalized version of Model 1.

<u>Model 3</u>

Following Brock and Hommes (1997), Model 3 assumes that producers choose either rational expectations or static expectations every year based on the past performances of their expectations.[6] Accordingly, the fraction of boundedly rational economic agents (n_t) can be any number between zero and one in the model. For simplicity and empirical tractability, it is assumed that the functional form of n_t is not known to the economic agents in the market but that its value is observed by rational producers at the beginning of time *t*.

3. Estimations and Forecasts

The three models of this paper are fully described by equations 6, 9, 10 and 11 with different assumptions regarding n_t. In this section, those equations in each model are estimated using GMM,[7] and the three models are compared on the basis of the estimation results. In addition, the forecast performance of each of the three models are also presented and compared.

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[6] For econometric tractability, the functional form of n_t of Brock and Hommes (1997) is simplified in this paper. The functional form of n_t employed in Model 3 in the present paper is presented in the following section.

[7] The present paper uses GMM following Chavas (1999) who estimated heterogeneous expectations models of the US hog market.

The estimations use the data set containing annual observations during the period 1945-2007 for the pig crop (x_t), the number consumed (c_t), the breeding stock (k_t), the price of an adult animal (p_t), the corn price, and the beef cattle price in the U.S. market. The corn price and the beef cattle price are used as proxies for the holding cost (h_t) and the demand shifter (d_t), respectively. All the data are detrended by the Hodrick-Prescott filter. In order to make the detrended quantity and price data take on positive values, the difference between the first observations in the original and in the detrended time series were added to the detrended time series in each of the six time series. Table 1 presents some information on the data.

As previously stated, the hog market model described by equations 6, 9, 10, and 11 in section 2 takes on three forms depending on the assumption about the fraction of boundedly rational economic agents, n_t. In Model 1, n_t is assumed to be zero. Therefore, n_t disappears from the equations listed above, and is not estimated in Model 1. In Model 2, n_t is assumed to be a fixed real number between zero and one, and its value is estimated along with the other deep parameters of the equations. Accordingly, Model 2 has one more estimate than Model 1.

In Model 3, n_t is assumed to be determined by the past relative performance of the rational and boundedly rational expectations, following the idea of Brock and Hommes (1997). In the paper of Brock and Hommes (1997),

$$n_t = \frac{e(\lambda \pi_{2,t-1})}{e(\lambda \pi_{1,t-1}) + e(\lambda \pi_{2,t-1})}$$

where λ = intensity of choice, $\pi_{1,t-1}$ = performance measure of rational expectations at time *t-1*, $\pi_{2,t-1}$ = performance measure of boundedly rational expectations at time *t-1*, and *e* is the exponential function.

In the present paper, a simplified version of the above function is used to generate the data for n_t for empirical tractability.

$$n_t = \frac{e(\pi_{2,t-1})}{1+e(\pi_{2,t-1})} \qquad (12)$$

where $\pi_{2,t-1} = aF_{t-1} + bF_{t-2} + c$, $F_{t-1} = -|p_{t-1} - p_{t-2}|$, $F_{t-2} = -|p_{t-2} - p_{t-3}|$. The parameters a, b and c are positive values, and they play the role of the intensity of choice. This is why λ is not in equation 12. If the value of *a* or *b* increases, n_t fluctuates more. If the value of *c* increases, the average value of n_t also increases.

The data for F_{t-1} and F_{t-2} are computed from the actual price data. A number of sets of values are assigned to the parameters (a, b, and c) to generate the data for n_t. Then, those series of n_t are used one by one in the GMM estimation of Model 3 to examine which series of n_t minimizes the objective function of the GMM estimation. More specifically, in empirical experiments, 0 was assigned to the three parameters at first, then their values were increased by 0.1 up to 1.0, generating 1331 (=11x11x11) sets. Accordingly, 1331 different series of n_t were generated. Then, each of the n_t series was used in the GMM estimation of Model 3. When *a*=0.2, *b*=0.0, and *c*=0.0, the objective function of GMM was minimized. As the next step, the values of the three parameters were increased by 0.05 from 0 to 0.5, generating another 1331 different series of n_t. In this second experiment, the objective function of GMM was minimized still when *a*=0.2, *b*=0.0, and *c*=0.0. Finally, the parameter values were increased by 0.02 from zero to 0.3. Then, the n_t series generated from the parameter set, *a*=0.22, *b*=0.0 and *c*=0.0, produced the smallest GMM objective function value, 17.066. Therefore, that specific intensity of choice values and the series of n_t generated by them was finally chosen to estimate Model 3. More discussions on the values of the intensity of choice parameters are provided in the following section when the estimation and simulation results of Model 3 are presented.

Since the calculation of n_t according to equation 12 involves lagged values of the hog price up to p_{t-3} and since the hog price data starts from 1945, n_t can be computed from

1948 to 2007. Given the computed data for n_t, along with the data for other variables, the deep parameters of equations 6, 9, 10 and 11 are estimated by the GMM just like Models 1 and 2. Therefore, Model 3 has the same number of estimates as Model 1, but it has one more time series, n_t, than the other two models.

Some parameter values in the estimated equations are set a priori to reduce the number of parameters to be estimated in the GMM estimation. The discount factor (β) is assumed to be 0.96 following AHMS and Baak (1999). Agricultural literature usually does not contain quadratic cost terms as in Rosen et al (1994) and Chavas (1999). These terms are included in the present paper to make the objective function linear quadratic. Therefore, the coefficients of the quadratic cost terms are assumed to be close to zero: $\psi_0 = 0.0001$, $\psi_c = 0.0001$.[8] The remaining parameters are estimated by the GMM.[9]

All the parameter values are significantly estimated by the three models at the 5% significance level, and turn out to be within reasonable boundaries. In Table 2, the estimates and the standard errors of the parameters are reported.

The fraction of the boundedly rational agents, n_t, in Model 2 is estimated to be 0.384, indicating that around 40 percent of the producers are boundedly rational. This fraction is not fixed but changeable in Model 3, and it is assumed to be determined by the intensity of choice parameters (*a*, *b,* and *c*) and the performances of static expectations as shown in equation 12. As explained in the previous section and shown in Table 2, the intensity of choice parameters are assumed to have the following values: a=0.22, b=0.0, c=0.0, because Model 3 showed the best performance when the intensity of choice parameters have those values in preliminary empirical experiments. The fraction of boundedly rational agents, n_t, computed by equation 12 using the assumed intensity of choice values, is illustrated in Figure 2. The average of n_t is 0.368, and it is different only by a very small margin from 0.384 which is the fixed fraction of boundedly rational agents estimated by Model 2.

[8] Anderson et al. (1996) also makes the same assumption when they estimate the model of Rosen et al (1994).
[9] The Hansen-Heaton-Ogaki GMM codes explained in Ogaki (1993) were used for the estimation. The method of Andrews (1991) was adopted to compute the covariance matrix.

Table 3 shows that the mean and the standard deviation of n_t is affected by the value of *a*. As the value of *a* decreases, economic agents change their expectations formation functions less vigorously. This explains why the standard deviation of n_t decreases as the value of *a* decreases.

The minimized value of the GMM function, J_T, is the highest in Model 1 and the lowest in Model 3 implying Model 3, may perform better than Model 2, and Model 2 better than Model 1. However, the difference between Model 2 and Model 3 is much smaller than the difference between Model 1 and Model 2, implying that the empirical value added by Model 3 over Model 2 may not be great compared to the empirical value added by Model 2 over Model 1. More scientific comparisons of the relative performance of the three models are introduced in the following section.

4. Comparisons of the Models, Forecasts and Simulations

Since Model 2 has one more parameter, *n*, than Model 1, and Model 1 is a special case of Model 2 when *n*=0, a specification test can be done to statistically examine the relative performance between the two models.

The specification test of the two models using a log-likelihood ratio type statistic, suggested in Ogaki (1993), supports Model 2 (a heterogeneous expectations model) over Model 1 (a rational expectations model). The null and the alternative hypothesis of this test are:

$$H_0 : n = 0$$
$$H_A : n > 0$$

The statistic is $T(J_T(\hat{\theta}) - J_t(\widetilde{\theta}))$, where $J_T(\hat{\theta})$ is the minimized value of the GMM function of the restricted model (Model 1) and $J_T(\widetilde{\theta})$ is the minimized value of the GMM function of the unrestricted model (Model 2). The degree of freedom, r, is the number of restrictions. In Model 1, the restriction is $n = 0$ and thus the number of restrictions is 1. Since the parameter space is restricted under the alternative hypothesis, the test statistic is distributed as a mixture of Chi-squared distributions.[10] Specifically, under the null hypothesis, the distribution of the statistic is $\frac{1}{2}\chi^2(0)+\frac{1}{2}\chi^2(1)$. The critical value at the 5% significance level is 2.71. The test statistic exceeds the critical value, rejecting the null hypothesis of homogeneous full rationality.

In the meantime, since a constant parameter, $\overline{n}$, in Model 2 is replaced by a time series, n_t, in Model 3,the difference between the two models does not lie with parameter value restrictions. Therefore, the specification test used above cannot be applied to this case. Instead, the forecast performances of the two models are compared using the methodology of Diebold and Marino (1995).[11] Specifically, one-step ahead forecasts of the hog price and quantity data are done using Model 2 and Model 3 respectively, and the squared-errors of their forecasts are compared to statistically determine which performs better.

To forecast future hog prices, the solution of the hog market model presented in section 2 should be obtained. The solution of the model is the dynamics of the control variable, i_t, which is a function of the endogenous (k_t) and exogenous (d_t, h_t) state variables. This solution is obtained from equations 6, 9, 10, and 11 using lag operator manipulations.

$$i_t = C + (\eta_1 + \eta_2)k_{t-1} - \eta_1\eta_2 k_{t-2} + Hh_t + Dd_t + D'd_{t-1} \qquad (13)$$

where $C = \left[\dfrac{\eta_1\eta_2}{(1-\delta)(1+g)-1}\right]\left[\dfrac{(1-n)-\beta(1-n)(1-\delta)(1+g)}{n[1-\beta(1-\delta)(1+g)]}+1\right]\alpha_0$,

$$D = \frac{\eta_1\eta_2}{(1-\delta)(1+g)-\rho_d}\left(\frac{(1-n)[1-\beta(1-\delta)(1+g)\rho_d]}{n[1-\beta(1-\delta)(1+g)]}+\frac{1}{(1-\delta)(1+g)}\right),$$

[10] See Gourieroux, Holly and Monfort (1982) and Andrews (1996).

[11] The test statistic of Diebold and Mariano (1995) cannot be used to compare two models one of which is a Hansen-Heaton-Ogaki GMM codes explained in Ogaki (1993) were used for the estimation. The method of Andrews (1991) was adopted to compute the covariance matrix.

$D' = \frac{\eta_1\eta_2}{(1-\delta)(1+g)}$, $H = \left[\frac{\eta_1\eta_2}{(1-\delta)(1+g)-\rho_h}\right]\frac{(1+g)\alpha_1}{n[1-\beta(1-\delta)(1+g)]}$, and η_i's are the roots of the following polynomial equation,

$$\left(\eta^2 - \frac{1}{\beta(1-\delta)(1+g)}\eta - \frac{n[1-\beta(1-\delta)(1+g)]}{\beta(1-\delta)(1+g)(1-n)}\right) = 0.$$

Using $i_t = k_t$ and equations 6 and 10 along with the solution for i_t, the hog price, p_t, can be also described as a function of lagged endogenous state variables (k_{t-1} and k_{t-2}) and current/lagged exogenous state variables (h_t, d_t and d_{t-1}).

$$p_t = \frac{1}{\alpha_1}\left[\alpha_0 + C + ((\eta_1+\eta_2) - (1-\delta)(1+g))k_{t-1} - \eta_1\eta_2 k_{t-2} + Hh_t + (1+D)d_t + D'd_{t-1}\right] \quad (14)$$

The mean-squared-errors (MSEs) of the one-step-ahead price and quantity forecasts of Model 2 and Model 3 are reported in Table 4. As seen in the table, Model 3 generates lower mean-squared errors than Model 2 in all four variables.

The test statistic of Diebold and Marino (1995) is obtained by regressing the difference between the squared errors of the forecasts of the two models on the constant. When the null hypothesis is that Model 2 and Model 3 show the same forecast performance and the alternative is that Model 3 is better than Model 2, the p-value of the test statistic of the four variables are all higher than 0.1. Therefore, the null is accepted even at the 10 percent significance level indicating the difference between the forecast performances of the two models is not statistically evident, even though Model 3 shows lower forecast errors in all four variables.[12]

As the last task in this paper, the artificial data of the hog price (p_t) are generated using the three models and their estimated parameter values. In these simulation experiments, exogenous variables are fixed to be constant to see the dynamics of the hog price when external shocks are zero.

[12] The test of Diebold and Mariano (1995) cannot be used to compare Model 1 and Model2, because Model 1 is nested by Model 2. See Clark and McCracken (2010).

As shown in Figures 3 and 4, the market reaches a steady state in the framework of Models 1 and 2, though Model 2 requires a much longer period to arrive at a steady state than Model 1. It implies that these two models attribute the volatility of the market price mainly to external shocks. In contrast, the market does not reach a steady state in the framework of Model 3. Instead it generates cyclical dynamics when the intensity of choice parameters are assumed to be a=0.22, b=0.0, and c=0.0 as they are in the estimation (Figure 5-1). Therefore, the hog price is inherently unstable in Model 3.

Until the value of a is increased to 0.24, the price dynamics is still cyclical (Figure 5-2). However, when the value of a becomes 0.25, the hog price illustrates chaotic dynamics (Figure 5-3). On the contrary, when the value of a is decreased to 0.17, the market reaches a steady state in the long-run just as in the other two models (Figure 5-5). These simulation results illustrate that the dynamics of the hog price highly depends on the intensity of choice.

5. Conclusion

This paper presented three investment decision models of the U.S. hog market that are distinguished only by their assumptions with regard to market participants' expectations of future prices. Then it investigated which model best captures the movements of the actual data of the market. Model 3, in which market participants switch their expectation formation functions, better explained the dynamics of the U.S. hog data than the models in which the fraction of boundedly rational economic agents is zero (Model 1) or a positive constant (Model 2).

Simulation experiments performed by Models 1 and 2 illustrated that the economy would reach the steady state, as long as external shocks were zero. In contrast, Model 3 generated chaotic or cyclical movements depending on the value of the intensity of choice even without any external shocks, implying price volatility is an inherent phenomenon in the hog market.

The findings of the paper shows the possibility that economic models incorporating heterogeneous expectations may explain our economy better than conventional rational expectations models. In addition, they imply that our economy may be inherently unstable.

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[Table 1] Sample Means and Standard Deviations

Variable	Mean	Std. Dev.
x_t	85.337	5.415
k_t	12.903	0.611
c_t	72.994	4.653
p_t	30.899	2.621
h_t	2.438	0.185
d_t	26.962	2.894

[Table 2] GMM Estimation Results

Parameters	Model 1 ($n_t = 0$)		Model 2 ($n_t = \bar{n}$)		Model 3 (a=0.2, b=0.0, c=0.14)	
	Estimates	Std. error	Estimates	Std. error	Estimates	Std. error
α_0	117.045	3.625	115.102	3.209	114.000	2.946
α_1	1.960	0.083	1.901	0.080	1.865	0.062
g	6.603	0.016	6.596	0.016	6.595	0.015
δ	0.117	0.002	0.118	0.002	0.120	0.002
ρ_h	0.896	0.006	0.904	0.004	0.901	0.005
ρ_d	0.610	0.083	0.613	0.061	0.614	0.060
$\bar{n}$	n.a.	n.a.	0.384	0.026	n.a.	n.a.
J_T [1)]	20.048		17.581		17.066	

1) The minimized value of the GMM function

[Table 3] Intensity of Choice and the Fraction of Boundedly Rational Agents

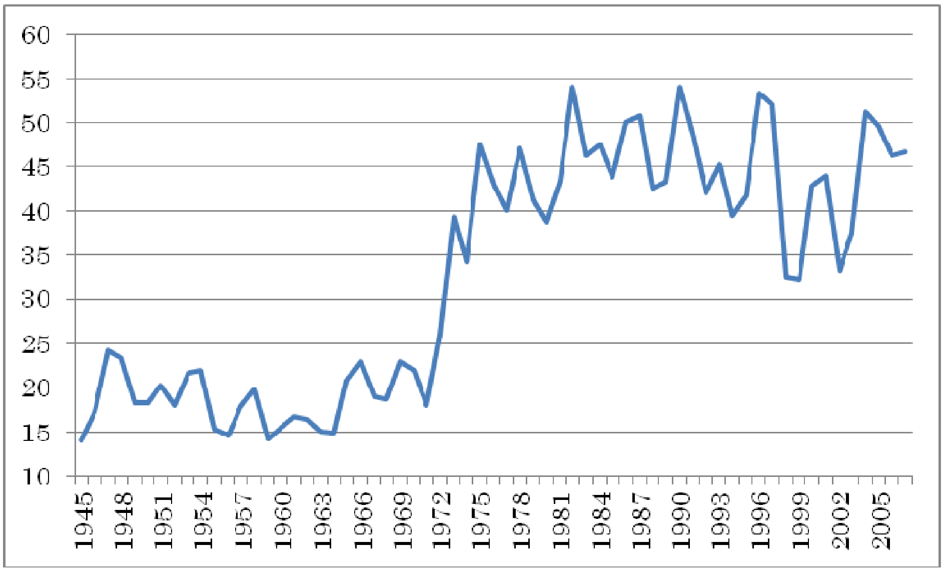
a	0.16	0.24	0.28
Mean	0.401	0.368	0.339
Stdev	0.077	0.098	0.114

1) a=c = 0.0.

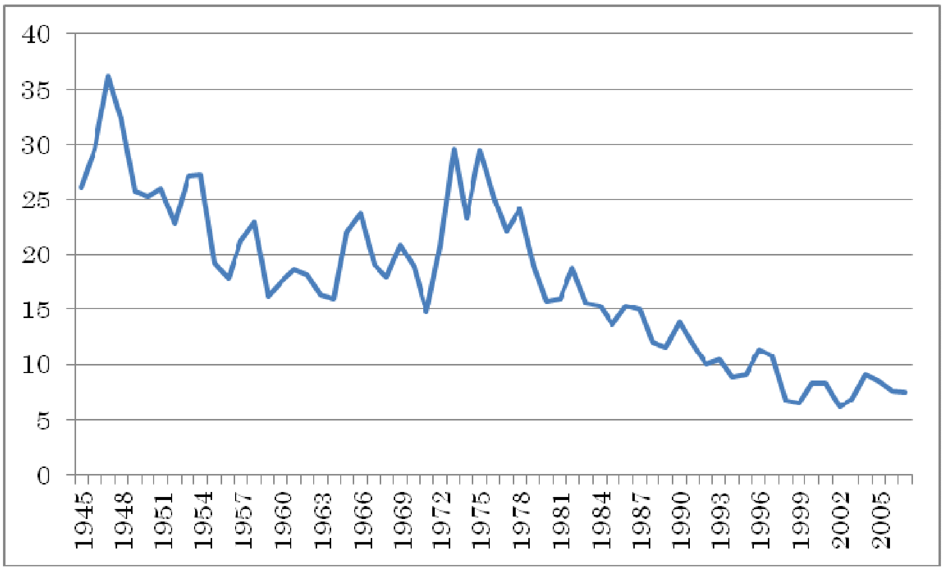
[Table 4] MSE of one-step ahead forecasts

	Model 2	Model 3
x_t	71.237	70.497
k_t	1.510	1.441
c_t	19.613	19.125
p_t	7.900	7.832

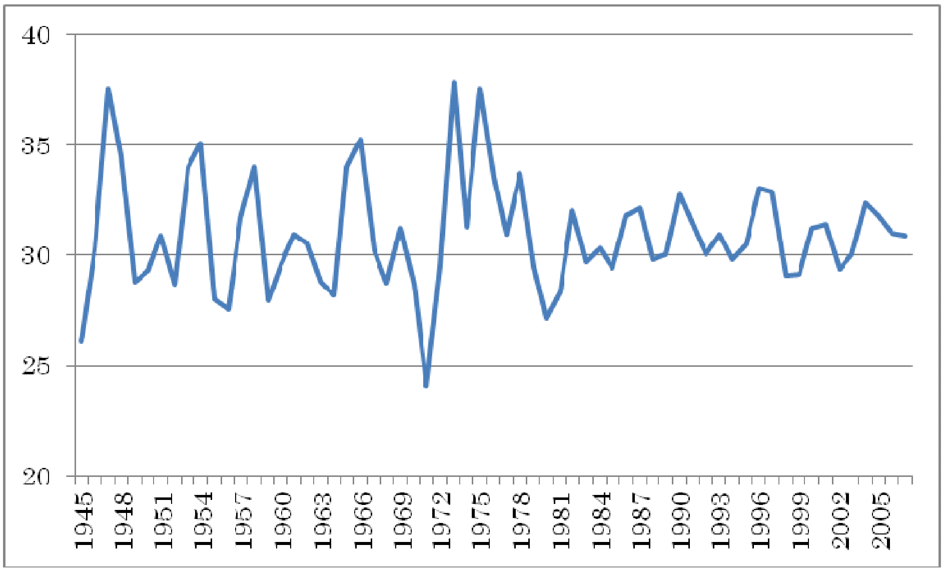
[Figure 1-1] Nominal Hog Prices (US $ per 100 pounds)



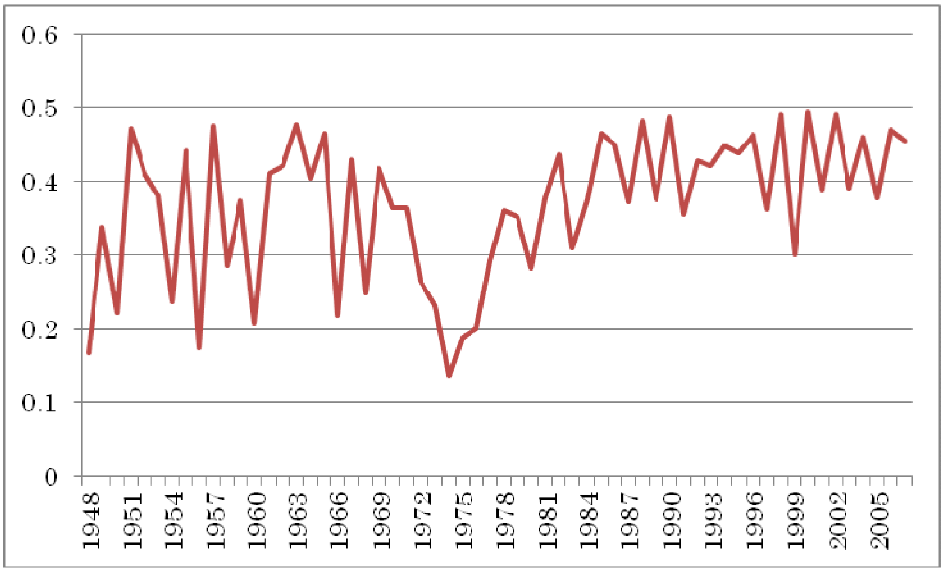
[Figure 1-2] Real Hog Prices (US $ per 100 pounds, Base Year = 1967)



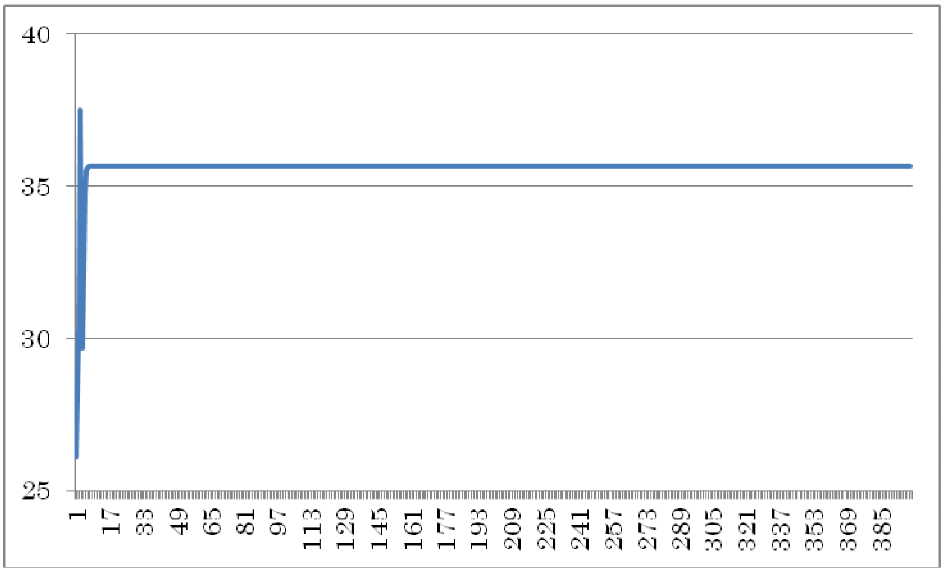
[Figure 1-3] Detrended Real Hog Prices (US $ per 100 pounds)



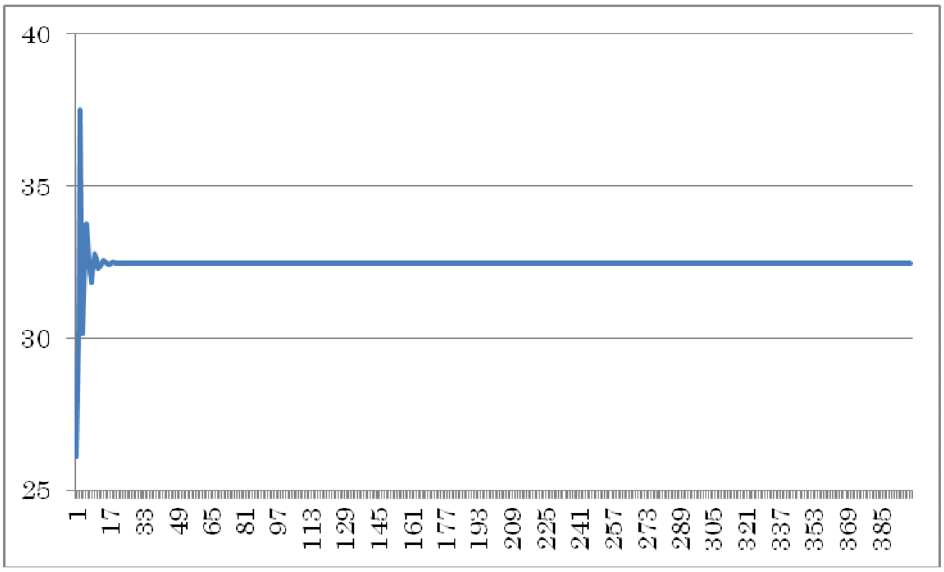
[Figure 2] Fraction of Boundedly Rational Producers (a=0.22, b=0.0, c=0.0)



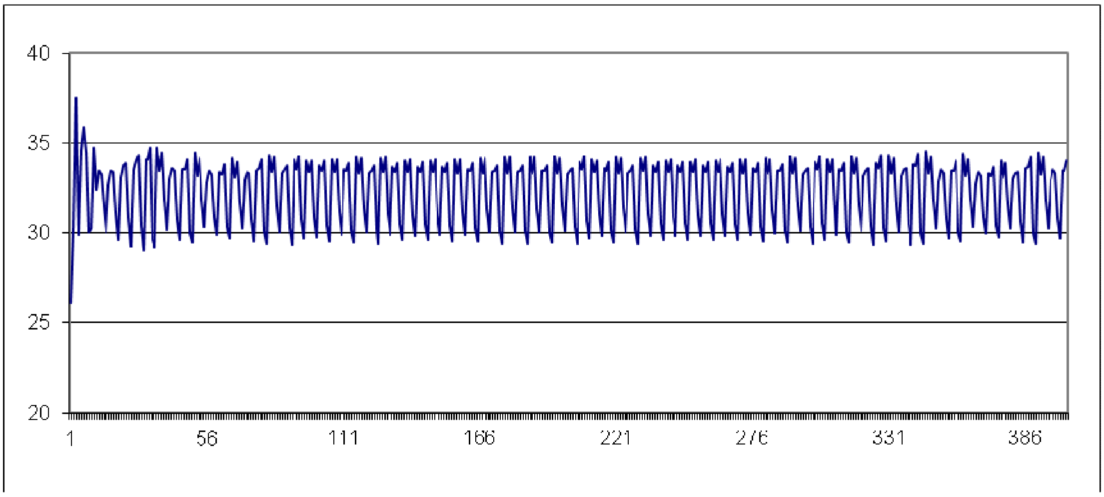
[Figure 3] Simulated data by Model 1



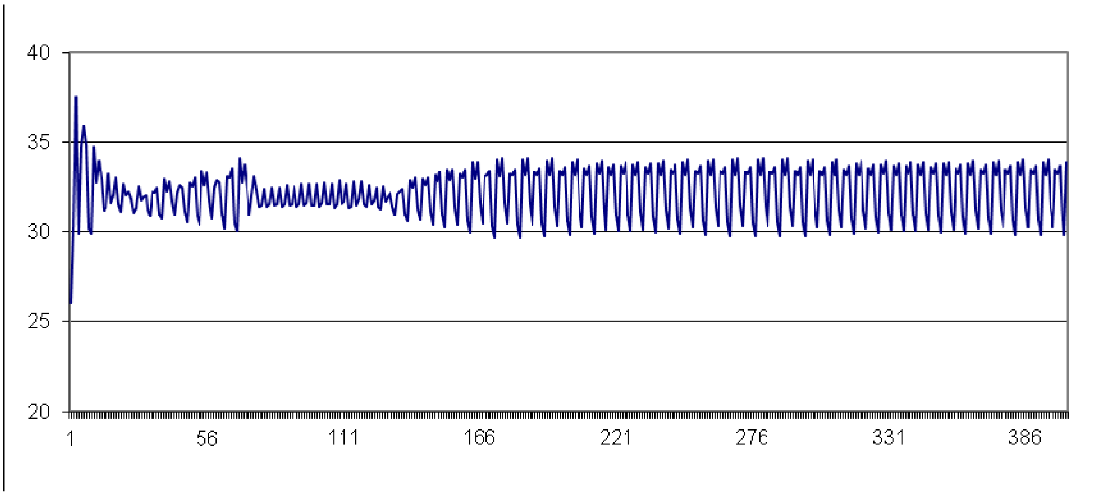
[Figure 4] Simulated data by Model 2



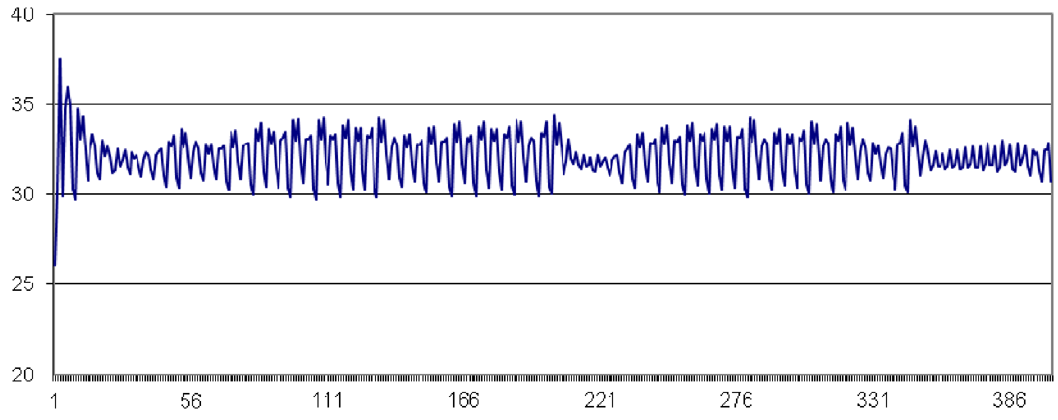
[Figure 5-1] Simulated data by Model 3 (a=0.22; b=0.0; c=0.0)



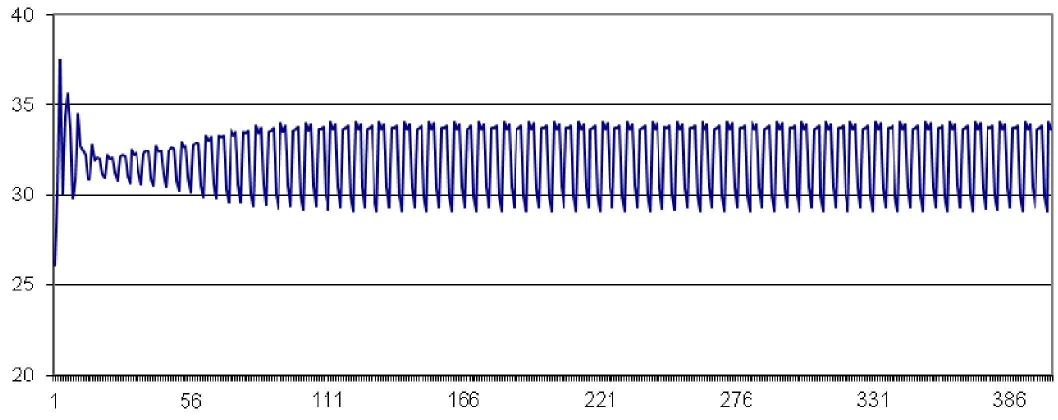
[Figure 5-2] Simulated data by Model 3 (a=0.24; b=0.0; c=0.0)



[Figure 5-3] Simulated data by Model 3 (a=0.25; b=0.0; c=0.0)



[Figure 5-4] Simulated data by Model 3 (a=0.18; b=0.0; c=0.0)



[Figure 5-5] Simulated data by Model 3 (a=0.17; b=0.0; c=0.0)