Kinked Demand Curves, the Natural Rate Hypothesis, and Macroeconomic Stability*

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Abstract

In the presence of staggered price setting, high trend inflation induces a large deviation of steady-state output from its natural rate and indeterminacy of equilibrium under the Taylor rule. This paper examines the implications of a “smoothed-off” kink in demand curves for the natural rate hypothesis and macroeconomic stability using a canonical model with staggered price setting, and sheds light on the relationship between the hypothesis and the Taylor principle. An empirically plausible calibration of the model shows that the kink in demand curves mitigates the influence of price dispersion on aggregate output, thereby ensuring that the violation of the natural rate hypothesis is minor and preventing fluctuations driven by self-fulfilling expectations under the Taylor rule.

JEL Classification: E31, E52

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1 Introduction

“There is always a temporary trade-off between inflation and unemployment; there is no permanent trade-off.” Thus spoke Milton Friedman (1968, p. 11). Since then the natural rate hypothesis (NRH, henceforth)—in the long run output is at its natural rate regardless of the trend inflation rate—has been widely accepted in macroeconomics. The Calvo (1983) model of staggered price setting, however, fails to satisfy this hypothesis, as McCallum (1998) forcefully criticized. Nevertheless, it has been a leading model of price adjustment for monetary policy analysis in the past decade and a half. One likely reason for this is that the introduction of price indexation makes the Calvo model meet the NRH, as shown in Ascari (2004). In fact, a considerable amount of research incorporates price indexation to trend inflation as in Yun (1996) or to past inflation as in Christiano, Eichenbaum, and Evans (2005). Yet the presence of price indexation raises another issue. The resulting model is not consistent with micro evidence that each period a fraction of prices is kept unchanged under a positive trend inflation rate.\(^1\) Since firms that do not reoptimize prices use price indexation, all prices change in every period.

Another likely reason why the Calvo model has thrived is that its violation of the NRH may be too small to induce grossly misleading implications for monetary policy.\(^2\) However, Ascari (2004), Levin and Yun (2007), and Yun (2005) examine the steady-state relationship between output and inflation in the Calvo model and show that the deviation of steady-state output from its natural rate becomes larger as trend inflation rises. Higher trend inflation widens the dispersion of relative prices of differentiated goods in the presence of unchanged prices, because it causes price-adjusting firms to set a higher price and non-adjusting firms’ relative prices to erode more severely. Therefore, it increases the dispersion of demand for the goods and generates a larger loss in aggregate

\(^1\)Moreover, Cogley and Sbordone (2008) demonstrate that price indexation to past inflation is not empirically important once drift in trend inflation is taken into account.

\(^2\)For tractable models of price adjustment that satisfy the NRH, see, e.g., the sticky information model of Mankiw and Reis (2002) and the P-bar model of McCallum (1994). The leading role of the Calvo model relative to these alternatives thus suggests that its violation of the NRH is generally considered sufficiently small to be of limited consequence for results obtained with it.
The large violation of the NRH in the Calvo model has implications for monetary policy. Higher trend inflation reduces not only steady-state output but also the long-run inflation elasticity of output in the Calvo model. In their analysis of determinacy of equilibrium under the Taylor (1993) rule, Ascari and Ropele (2009), Kurozumi (2011), and Kurozumi and Van Zandweghe (2012) show that this elasticity plays a key role for the determinacy condition called the long-run version of the Taylor principle: in the long run the interest rate should be raised by more than the increase in inflation. Higher trend inflation reduces the elasticity substantially once the trend inflation rate exceeds a certain positive threshold. Then, since the long-run version of the Taylor principle is less likely to be satisfied with a lower value of the elasticity, it imposes a more severe upper bound on the output coefficient of the Taylor rule as trend inflation rises. Moreover, higher trend inflation gives rise to another condition for determinacy that imposes more severe lower bounds on the inflation and output coefficients of the Taylor rule. Therefore, indeterminacy under the Taylor rule is more likely with higher trend inflation. In this context, Coibion and Gorodnichenko (2011) argue that a decline in trend inflation along with an increase in the Fed’s policy response to inflation accounts for much of the U.S. economy’s shift from indeterminacy during the Great Inflation era to determinacy during the Great Moderation era. This argument differs from that of the previous literature, including Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004), who all attribute such a shift solely to the Fed’s change from a passive to an active policy response to inflation.

This paper examines implications of a “smoothed-off” kink in demand curves for the
NRH and macroeconomic stability in the Calvo model. This kink in demand curves has been analyzed by Kimball (1995), Dotsey and King (2005), and Levin, Lopez-Salido, Nelson, and Yun (2008), and generates strategic complementarity in price setting.\footnote{See also Levin, Lopez-Salido, and Yun (2007) and Shirota (2007).} Recent empirical literature emphasizes the importance of such complementarity for reconciling the Calvo model with micro evidence on the frequency of price changes.\footnote{See Bils and Klenow (2004), Klenow and Kryvtsov (2008), and Nakamura and Steinsson (2008) for recent micro evidence on price changes.} The strategic complementarity arising from the smoothed-off kink in demand curves thus gives the New Keynesian Phillips curve (NKPC, henceforth) the flat slope (i.e., the small elasticity of inflation with respect to real marginal cost) reported in the empirical literature, such as Gali and Gertler (1999), Galí, Gertler, and Lopez-Salido (2001), Sbordone (2002), and Eichenbaum and Fisher (2007), keeping the average frequency of price changes consistent with micro evidence.

A calibration of the model that is consistent with both the micro evidence on the frequency of price changes and the empirical literature on the NKPC shows that the presence of the smoothed-off kink in demand curves mitigates the influence of price dispersion on aggregate output, thereby ensuring that the violation of the NRH is minor and preventing fluctuations driven by self-fulfilling expectations under the Taylor rule. As noted above, higher trend inflation widens price dispersion in the presence of unchanged prices, thereby increasing demand dispersion and reducing aggregate output. The kink in demand curves causes demand for a good to become \textit{more} price-elastic for an \textit{increase} in the relative price of the good, thus reducing the desired markup of price-adjusting firms and the output distortion associated with the average markup. Moreover, the kink in demand curves causes demand for a good to become \textit{less} price-elastic for a \textit{decline} in the relative price of the good, which mitigates the increase in demand dispersion due to non-adjusting firms and hence the relative price distortion of output. Because of these two effects the violation of the NRH is minor in the presence of the kink. Moreover, the mitigating effect of the kink in demand curves reverses a decline in the long-run inflation elasticity of output caused by higher trend inflation and thus makes the long-run version
of the Taylor principle much more likely to be met than in the absence of the kink. It also makes irrelevant the other determinacy condition that induces lower bounds on the inflation and output coefficients of the Taylor rule. Consequently, determinacy of equilibrium under the Taylor rule is much more likely in the presence of the kink.\(^6\)

The desirable properties of the smoothed-off kink in demand curves in terms of preventing both large violations of the NRH and indeterminacy of equilibrium under the Taylor rule are not shared by firm-specific labor, which is another source of strategic complementarity. It is shown that in the Calvo model with firm-specific labor the violation of the NRH is much larger and indeterminacy under the Taylor rule is much more likely than in that with the kink in demand curves, using the calibration of each model that is consistent with both the micro evidence on the frequency of price changes and the empirical literature on the NKPC. This is because the influence of price dispersion on aggregate output is mitigated in the latter model as noted above, whereas this mitigating effect is absent in the former model. Coibion and Gorodnichenko (2011) use the Calvo model with firm-specific labor to emphasize the importance of the role of trend inflation for the U.S. economy’s Great Inflation era. However, such a model induces a large violation of the NRH. In particular, it generates a large deviation of steady-state output from its natural rate during the Great Inflation era. The Calvo model with the kink in demand curves, by contrast, brings about a minor violation of the NRH and supports the view of the previous literature that places emphasis only on the role of the Fed’s policy response to inflation during the Great Inflation era.

The remainder of the paper proceeds as follows. Section 2 presents the Calvo model with a smoothed-off kink in demand curves. In this model, Section 3 examines implications of the kink for the NRH, while Section 4 analyzes those for equilibrium determinacy.

\(^6\)The implications of the smoothed-off kink in demand curves for the NRH and equilibrium determinacy would apply qualitatively to the Taylor (1980) model of staggered price setting as well, although they may not be of quantitative importance because Ascari (2004) and Kiley (2002) show that price dispersion is smaller in the Taylor model than in the Calvo model. As for determinacy of equilibrium under the Taylor rule, Hornstein and Wolman (2005) and Kiley (2007) show that higher trend inflation is more likely to induce indeterminacy in the Taylor model.
and shows the relationship between the NRH and the long-run version of the Taylor principle. In Section 5 these implications are compared with those obtained in the model with firm-specific labor. Finally, Section 6 concludes.

2 The Calvo model with a smoothed-off kink in demand curves

A smoothed-off kink in demand curves—which has been studied by Kimball (1995), Dotsey and King (2005), and Levin, Lopez-Salido, Nelson, and Yun (2008)—is introduced in the Calvo model. In the model economy there are a representative household, a representative final-good firm, a continuum of intermediate-good firms, and a monetary authority. Key features of the model are that each period a fraction of intermediate-good firms keeps prices of their differentiated products unchanged, while the remaining fraction reoptimizes its prices in the face of the kinked demand curves of the final-good firm. The behavior of each economic agent is described in turn.

2.1 Household

The representative household consumes $C_t$ final goods, supplies $N_t$ labor, and purchases $B_t$ one-period riskless bonds so as to maximize the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right)$$

subject to the budget constraint

$$P_t C_t + B_t = P_t W_t N_t + i_{t-1} B_{t-1} + T_t,$$

where $E_t$ denotes the expectation operator conditional on information available in period $t$, $\beta \in (0, 1)$ is the subjective discount factor, $\sigma_n \geq 0$ is the inverse of the elasticity of labor supply, $P_t$ is the price of final goods, $W_t$ is the real wage, $i_t$ is the gross interest rate on one-period riskless bonds, and $T_t$ consists of lump-sum public transfers and firm profits.
Combining first-order conditions for utility maximization with respect to consumption, labor supply, and bond holdings yields

\[ W_t = C_t N_t^\sigma, \quad (1) \]

\[ 1 = E_t \left( \frac{\beta C_t}{C_{t+1} \pi_{t+1}} \right), \quad (2) \]

where \( \pi_t = P_t / P_{t-1} \) denotes gross inflation.

### 2.2 Final-good firm

As in Kimball (1995), the representative final-good firm produces \( Y_t \) homogeneous goods under perfect competition by choosing a combination of intermediate inputs \( \{Y_t(f)\} \) so as to maximize profit

\[ P_t Y_t - \int_0^1 P_t(f) Y_t(f) \, df \]

subject to the production technology

\[ \int_0^1 F \left( \frac{Y_t(f)}{Y_t} \right) \, df = 1, \quad (3) \]

where \( P_t(f) \) is the price of intermediate good \( f \in [0, 1] \). Following Dotsey and King (2005) and Levin, Lopez-Salido, Nelson, and Yun (2008), the production technology is assumed to be of the form

\[ F \left( \frac{Y_t(f)}{Y_t} \right) = \frac{\tilde{\theta}}{(1 + \epsilon)(\theta - 1)} \left[ (1 + \epsilon) \frac{Y_t(f)}{Y_t} - \epsilon \right]^{\frac{\theta - 1}{\theta}} + 1 - \frac{\tilde{\theta}}{(1 + \epsilon)(\theta - 1)}, \]

where \( \tilde{\theta} = \theta(1 + \epsilon) \), and \( \theta > 1 \) and \( \epsilon \leq 0 \) are constant parameters. The parameter \( \epsilon \) represents the degree of strategic complementarity, since in the case of \( \epsilon = 0 \) the production technology (3) is reduced to the CES one \( Y_t = [\int_0^1 (Y_t(f))^{(\theta - 1)/\theta} \, df]^{\theta/(\theta - 1)} \), where the parameter \( \theta \) represents the elasticity of demand for each intermediate good with respect to its price.

The first-order conditions for profit maximization yield the final-good firm’s demand for intermediate good \( f \),

\[ Y_t(f) = \frac{1}{1 + \epsilon} Y_t \left[ \left( \frac{P_t(f)}{P_t d_{1t}} \right)^{-\tilde{\theta}} + \epsilon \right], \quad (4) \]
where $d_{1t}$ is the Lagrange multiplier on the production technology (3) in profit maximization, given by

$$d_{1t} = \left[ \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{1-\theta} df \right]^{1/(1-\theta)},$$

and is a measure of price dispersion.

Perfect competition in the final-good market leads to

$$P_t = \frac{1}{1+\epsilon} \left[ \int_0^1 (P_t(f))^{1-\theta} df \right]^{1/(1-\theta)} + \frac{\epsilon}{1+\epsilon} \int_0^1 P_t(f) df \iff 1 = \frac{1}{1+\epsilon} d_{1t} + \frac{\epsilon}{1+\epsilon} d_{2t},$$

where

$$d_{2t} = \int_0^1 \frac{P_t(f)}{P_t} df.$$ (7)

Note that in the case of $\epsilon = 0$, as the production technology (3) becomes the CES one, equations (4), (5), and (6) can be reduced to $Y_t(f) = Y_t(P_t(f)/P_t)^{1-\theta}$, $P_t = \left[ \int_0^1 (P_t(f))^{1-\theta} df \right]^{1/(1-\theta)}$, and $d_{1t} = 1$, respectively.

The final-good market clearing condition is given by

$$Y_t = C_t.$$ (8)

### 2.3 Intermediate-good firms

Each intermediate-good firm $f$ produces one kind of differentiated goods $Y_t(f)$ under monopolistic competition. Firm $f$’s production function is linear in its labor input

$$Y_t(f) = N_t(f).$$ (9)

The labor market clearing condition is given by

$$N_t = \int_0^1 N_t(f) df.$$ (10)

Given the real wage $W_t$, the first-order condition for minimization of production cost shows that real marginal cost is identical among all intermediate-good firms, given by

$$mc_t = W_t.$$
Combining this equation with (1), (4), (8), (9), and (10) yields

\[ mc_t = Y_t^{1+\sigma_n} \left( \frac{s_t + \epsilon}{1 + \epsilon} \right)^{\sigma_n}, \]  

(11)

where \((s_t + \epsilon)/(1 + \epsilon)\) represents the relative price distortion and \(s_t\) is given by

\[ s_t = \int_0^1 \left( \frac{P_t(f)}{P_t d_{1t}} \right)^{-\hat{\theta}} \, df. \]  

(12)

In the face of the final-good firm’s demand (4) and the marginal cost (11), intermediate-good firms set prices of their products on a staggered basis as in Calvo (1983). Each period a fraction \(\alpha\in(0,1)\) of firms keeps previous-period prices unchanged, while the remaining fraction \(1-\alpha\) of firms sets the price \(P_t(f)\) so as to maximize the profit function

\[ E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \frac{1}{1 + \epsilon} Y_{t+j} \left[ \left( \frac{P_t(f)}{P_{t+j} d_{1t+j}} \right)^{-\hat{\theta}} + \epsilon \right] \left( \frac{P_t(f)}{P_{t+j}} - mc_{t+j} \right), \]

where \(q_{t,t+j} = \beta^j C_t/C_{t+j}\) is the stochastic discount factor between period \(t\) and period \(t+j\). In order for this profit function to be well-defined, the following assumption is imposed.

**Assumption 1** The three inequalities \(\alpha\beta\pi^{\hat{\theta} - 1} < 1, \alpha\beta\pi^{\hat{\theta}} < 1, \) and \(\alpha\beta\pi^{-1} < 1\) hold, where \(\pi\) denotes gross trend inflation.

Using (8), the first-order condition for Calvo staggered price setting leads to

\[ E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \prod_{k=1}^j \pi_{t+k}^{\hat{\theta}} \left[ \left( \frac{p_t^*}{\pi_{t+k}} \right)^{\hat{\theta}} \frac{1}{\hat{\theta} - 1} mc_{t+j} \right] d_{1t+j}^{-\hat{\theta}} - \frac{\epsilon}{\hat{\theta} - 1} \left( p_t^* \prod_{k=1}^j \frac{1}{\pi_{t+k}} \right)^{1+\hat{\theta}} = 0, \]  

(13)

where \(p_t^*\) is the real price set by firms that reoptimize prices in period \(t\). Moreover, under the Calvo staggered price setting, the price dispersion equations (5), (7), and (12) can be reduced to, respectively,

\[ (d_{1t})^{1-\hat{\theta}} = (1 - \alpha) \left( p_t^* \right)^{1-\hat{\theta}} + \alpha \left( \frac{d_{1t-1}}{\pi_t} \right)^{1-\hat{\theta}}, \]  

(14)

\[ d_{2t} = (1 - \alpha) p_t^* + \alpha \left( \frac{d_{2t-1}}{\pi_t} \right), \]  

(15)

\[ (d_{1t})^{-\hat{\theta}} s_t = (1 - \alpha) \left( p_t^* \right)^{-\hat{\theta}} + \alpha \left( \frac{d_{1t-1}}{\pi_t} \right)^{-\hat{\theta}} s_{t-1}. \]  

(16)
2.4 Monetary authority

The monetary authority conducts interest rate policy according to a policy rule as in Taylor (1993). This rule adjusts the interest rate \( i_t \) in response to deviations of inflation and output from their steady-state values,

\[
\log i_t = \log i + \phi_\pi (\log \pi_t - \log \pi) + \phi_y (\log Y_t - \log Y),
\]

where \( i \) and \( Y \) are steady-state values of the interest rate and output and \( \phi_\pi, \phi_y \geq 0 \) are the policy responses to inflation and output.

2.5 Log-linearized equilibrium conditions

For the subsequent analysis of equilibrium determinacy, the log-linearized model is presented. Under Assumption 1, log-linearizing equilibrium conditions (2), (6), (8), (11), (13)–(16), and (17) and rearranging the resulting equations leads to

\[
\dot{Y}_t = E_t \dot{Y}_{t+1} - \left( \hat{i}_t - E_t \hat{i}_{t+1} \right),
\]

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \left( \frac{1 - \alpha \pi^{\theta-1}(1 - \alpha \beta \pi^{\hat{\theta}})}{\alpha \pi^{\theta-1}[1 - \hat{\theta}/(\theta - 1 - \epsilon)]} m c_t - \frac{1}{\alpha \pi^{\theta-1}} \left( \hat{d}_t - \alpha \beta \pi^{\hat{\theta}-1} c_t \hat{d}_{t+1} \right) + \hat{d}_{t-1} - \alpha \beta \pi^{\theta-1} \hat{d}_t - \left( [\beta (\pi - 1)(1 - \alpha \beta \pi^{\hat{\theta}}) + \epsilon (1 - \alpha \beta \pi^{\hat{\theta}})] \hat{d}_t + \xi_t + \psi_t, \right.
\]

\[
m c_t = (1 + \sigma_n) \hat{Y}_t + \sigma_n \frac{s}{s + \epsilon} \hat{t}_t,
\]

\[
\hat{t}_t = \frac{\alpha \beta \pi^{\hat{\theta} - 1} (\pi - 1)}{1 - \alpha \pi^{\theta - 1}} \left( \hat{\pi}_t + \hat{d}_t - \hat{d}_{t-1} \right) + \alpha \pi^{\theta} \hat{t}_{t-1},
\]

\[
\hat{d}_t = - \frac{\epsilon \alpha \pi^{-1} (\pi^{\theta-1} - 1 - \alpha \beta \pi^{-1})}{(1 - \alpha \pi^{\theta-1}[1 - \beta \pi^{\hat{\theta}} + \epsilon (1 - \alpha \beta \pi^{\hat{\theta}}])] \hat{\pi}_t + \frac{\alpha \pi^{-1}[1 - \alpha \beta \pi^{\hat{\theta}} + \epsilon (1 - \alpha \beta \pi^{\hat{\theta}})]}{1 - \alpha \beta \pi^{\theta-1} + \epsilon (1 - \alpha \beta \pi^{-1})} \hat{d}_{t-1},
\]

\[
\xi_t = \alpha \beta \pi^{\theta} E_t \xi_{t+1} + \frac{\beta (\pi - 1)(1 - \alpha \pi^{\theta-1})}{1 - \hat{\theta}/(\theta - 1 - \epsilon)} \left[ \hat{\theta} E_t \hat{\pi}_{t+1} + (1 - \alpha \beta \pi^{\hat{\theta}}) \left( E_t m c_{t+1} + \hat{\theta} E_t \hat{d}_{t+1} \right) \right],
\]

\[
\psi_t = \alpha \beta \pi^{-1} E_t \psi_{t+1} + \frac{\epsilon \beta (\pi^{\theta-1} - 1 - \alpha \beta \pi^{\theta-1})}{\pi^{\theta}} \left[ \hat{\theta} - 1 - \epsilon (\theta + 1) \right] E_t \hat{\pi}_{t+1},
\]
\[ \hat{i}_t = \phi_x \hat{x}_t + \phi_y \hat{Y}_t, \]  

(25)

where all hatted variables represent log-deviations from steady-state values, \( \xi_t \) and \( \psi_t \) are auxiliary variables, and

\[ \tilde{\epsilon} = \frac{1 - \alpha \beta \pi^{\theta - 1}}{1 - \alpha \beta \pi^{-1}} \left( 1 - \frac{\alpha \pi^{\theta - 1}}{1 - \alpha} \right)^{-\frac{\theta}{\pi - 1}}, \quad s = \frac{1 - \alpha}{1 - \alpha \pi^\theta} \left( 1 - \frac{\alpha}{1 - \alpha \pi^{\theta - 1}} \right)^{-\frac{\hat{s}}{\pi - 1}}. \]

The strategic complementarity arising from the smoothed-off kink in demand curves reduces the slope of the NKPC (19) by \( 1/[1 - \tilde{\epsilon} \tilde{\theta}/(\tilde{\theta} - 1 - \tilde{\epsilon})] \). Consequently, it allows to reconcile the model with both the micro evidence on the frequency of price changes and the empirical literature on the NKPC.

In the case of no kink in demand curves (i.e., \( \epsilon = \tilde{\epsilon} = 0 \)), (22) and (24) imply that \( \hat{d}_{1t} = 0 \) and \( \psi_t = 0 \), and hence (19), (20), (21), and (23) can be reduced to

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \alpha \pi^{\theta - 1})(1 - \alpha \beta \pi^\theta)}{\alpha \pi^{\theta - 1}} \hat{m}c_t + \xi_t, \]

\[ \hat{mc}_t = (1 + \sigma_n) \hat{Y}_t + \sigma_n \hat{s}_t, \]

\[ \hat{s}_t = \frac{\alpha \theta \pi^{\theta - 1}(\pi - 1)}{1 - \alpha \pi^{\theta - 1}} \hat{s}_t + \alpha \pi^\theta \hat{s}_{t-1}, \]

\[ \xi_t = \alpha \beta \pi^\theta E_t \xi_{t+1} + \beta(\pi - 1)(1 - \alpha \pi^{\theta - 1}) [\theta E_t \hat{\pi}_{t+1} + (1 - \alpha \beta \pi^\theta) E_t \hat{mc}_{t+1}]. \]

Note that these log-linearized equilibrium conditions are the same as those analyzed in Ascari and Ropele (2009) and Kurozumi (2011).

In the case of the zero trend inflation rate (i.e., \( \pi = 1 \)), (21)–(24) imply that \( \hat{s}_t = 0, \hat{d}_{1t} = 0, \xi_t = 0, \) and \( \psi_t = 0 \), and hence (19) and (20) can be reduced to

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha[1 - \epsilon \theta/(\theta - 1)]} \hat{m}c_t, \]  

(26)

\[ \hat{mc}_t = (1 + \sigma_n) \hat{Y}_t. \]

Eq. (26) shows that (19) presents a general formulation of the NKPC.

2.6 Calibration

For the ensuing analysis, an empirically plausible calibration of the model is presented. The benchmark calibration of the quarterly model is summarized in Table 1. The subjective discount factor and the inverse of the elasticity of labor supply are set at the
widely-used values of $\beta = 0.99$ and $\sigma_n = 1$. The probability of no price change is chosen at $\alpha = 0.6$ so that it would be consistent with the micro evidence reported by Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008), who all show that the average frequency of price changes including substitutions (i.e., the median duration until either the regular price changes or the product disappears) is around 7.5 months (i.e., $7.5/3$ quarters $= 1/(1-\alpha)$). The remaining two parameters regarding the elasticity of demand and the strategic complementarity, $\epsilon$, $\theta$, are set in the same way as Levin, Lopez-Salido, Nelson, and Yun (2008). The empirical literature on the NKPC, such as Galí and Gertler (1999), Galí et al. (2001), Sbordone (2002), and Eichenbaum and Fisher (2007), shows that its slope is around $0.025$. The values of $\epsilon$ and $\theta$ are chosen so that these values would give the NKPC (26) (i.e., the one (19) with $\pi = 1$) the slope of $0.025$ (i.e., $(1-\alpha)(1-\alpha\beta)/\{\alpha[1-\epsilon\theta/(\theta-1)]\} = 0.025$) together with the above calibration of $\beta$, $\sigma_n$, and $\alpha$. This paper then considers the calibration of $\theta = 7$, which implies that the price markup under the zero trend inflation rate is 16.7 percent. This calibration of $\theta$ yields $\epsilon = -8.4$. Note that to meet Assumption 1 under the calibration presented above, the annualized trend inflation rate needs to be greater than $-2.1$ percent.

3 Natural rate hypothesis

This section examines implications of a smoothed-off kink in demand curves for the NRH in the Calvo model. Specifically, the (non-linear) steady-state relationship between output and inflation is investigated to analyze how the deviation of steady-state output from its natural rate (i.e., the steady-state output gap) varies with trend inflation in the presence of the kinked demand curves.

Combining (6), (11), (13)–(15), and (16) at a steady state yields the relationship

\[\text{Combining (6), (11), (13)–(15), and (16) at a steady state yields the relationship} \]

\[\text{For a discussion of this empirical literature, see footnote 34 in Woodford (2005).} \]
between steady-state output $Y$ and trend inflation $\pi$

$$Y = \begin{bmatrix} \frac{\hat{\theta} - 1}{\theta} - \frac{1}{\theta} \left( \frac{1 - \alpha}{1 - \alpha \pi^{\theta - 1}} \right) \frac{\hat{\pi}}{\pi - 1} - \frac{1 - \alpha \beta \pi^{\theta}}{1 - \alpha \beta \pi^{\theta - 1}} \right] \right)^{\frac{1}{1+\sigma_n}} \begin{bmatrix} 1 + \epsilon \left( \frac{1 - \alpha}{1 - \alpha \pi^{\theta - 1}} \right) \frac{\hat{\pi}}{\pi - 1} + \epsilon \left( \frac{1 - \alpha}{1 - \alpha \pi^{\theta - 1}} \right) \frac{\hat{\pi}}{\pi - 1} \end{bmatrix}^{\frac{\sigma_n}{1+\sigma_n}}. \end{equation} \tag{27}

In the absence of Calvo staggered price setting, the (steady-state) natural rate of output can be obtained as

$$Y^n = \left( \frac{\theta - 1}{\theta} \right)^{\frac{1}{1+\sigma_n}}. \tag{28}$$

The steady-state output gap is thus given by

$$\log Y - \log Y^n = - \frac{1}{1 + \sigma_n} \left( \frac{\theta - 1}{\theta} \right) \log \left[ \frac{1}{1 + \epsilon} \left( \frac{1 - \alpha}{1 - \alpha \pi^{\theta - 1}} \right) \frac{\hat{\pi}}{\pi - 1} + \epsilon \left( \frac{1 - \alpha}{1 - \alpha \pi^{\theta - 1}} \right) \frac{\hat{\pi}}{\pi - 1} \right]$$

$$- \frac{\sigma_n}{1 + \sigma_n} \log \left. \left( \frac{1 - \alpha}{1 - \alpha \pi^{\theta - 1}} \right) \frac{\hat{\pi}}{\pi - 1} + \epsilon \right). \tag{29}$$

Note that under the zero trend inflation rate (i.e., $\pi = 1$), steady-state output $Y$ is equal to the natural rate of output $Y^n$ and hence the steady-state output gap is zero. Note also that in the absence of the kink in demand curves (i.e., $\epsilon = 0$), eq. (29) can be reduced to eq. (1) of Levin and Yun (2007). As these authors indicate, the steady-state output gap is twofold. The first term in the gap (29) captures the distortion associated with the average markup and the second term represents the relative price distortion.

Fig. 1 displays the effect of the annualized trend inflation rate on the steady-state output gap (29).\(^8\) The thin line in this figure shows that the steady-state output gap is highly sensitive to trend inflation in the absence of the kink in demand curves (i.e., $\epsilon = 0$). This line is obtained by choosing $\alpha = 0.85$ so as to set the slope of the NKPC

\(^8\)Even when analyzing the non-linear relationship between trend inflation and the steady-state output gap, each calibration of model parameters is chosen so that it would give the log-linearized NKPC (26) the slope of 0.025. This allows us to demonstrate in the next section how the deviation of steady-state output from its natural rate in the non-linear relationship is related to the likelihood of satisfying the long-run version of the Taylor principle in the log-linearized model, since the same calibrations are used in the analysis of determinacy.\]
(26) at 0.025 together with the benchmark calibrated values of $\beta$, $\sigma_n$, and $\theta$.\textsuperscript{9} Under this calibration, Fig. 1 illustrates that the steady-state output gap declines exponentially with higher trend inflation, as Ascari (2004), Levin and Yun (2007), and Yun (2005) point out.\textsuperscript{10} As shown in Table 2, a rise in the annualized trend inflation rate from two to four and eight percent reduces the steady-state output gap from $-0.30$ percent to $-2.07$ percent and $-31.25$ percent respectively. Moreover, both components of the steady-state output gap—the distortion associated with the average markup and the relative price distortion—make a substantial contribution to the size of the gap. A rise in trend inflation exponentially enlarges both the distortion associated with the average markup and the relative price distortion.\textsuperscript{11} Therefore, at an empirically plausible value for the slope of the NKPC, the Calvo model without the kink in demand curves is characterized by a large violation of the NRH.

This large violation of the NRH is prevented by the smoothed-off kink in demand curves. In Fig. 1, the solid line represents the steady-state output gap (29) under the benchmark calibration presented in Table 1. It demonstrates only a small steady-state output gap even under high trend inflation. As shown in Table 2, the output gap is 0.24 percent at the annualized trend inflation rate of two percent, 0.62 percent at the trend inflation rate of four percent and 1.50 percent at the trend inflation rate of eight percent. Moreover, the kink in demand curves brings about a substantial reduction in both components of the steady-state output gap, particularly in the relative price

\textsuperscript{9}The calibration of $\alpha = 0.85$ implies that the average duration between price changes is 20 months, which—as the empirical literature on the NKPC stresses in the absence of strategic complementarity—is much longer than micro evidence indicates.

\textsuperscript{10}In the Calvo model Levin and Yun (2007) show that when firms choose the probability of price adjustment, the steady-state output gap remains non-trivial under moderate trend inflation, but it wanes and eventually disappears under much higher trend inflation because the probability approaches the one in the absence of Calvo staggered price setting.

\textsuperscript{11}Higher inflation makes firms choose a higher markup when they adjust their prices, but it also makes the markup of non-adjusting firms erode more severely. Without the kink in demand curves, the effect of adjusting firms dominates for sufficiently high inflation, so that higher inflation is associated with a higher markup (King and Wolman, 1999).
distortion. Consequently, it ensures that the violation of the NRH is minor. An intuition for this is as follows. As noted above, higher trend inflation widens dispersion of prices of differentiated goods in the presence of unchanged prices, thereby increasing dispersion of demand for the goods and inducing a larger loss in aggregate output. The kink in demand curves causes demand for a good to become more price-elastic for an increase in the relative price of the good, thus reducing the desired markup of price-adjusting firms and the distortion associated with the average markup.\footnote{The increase in the steady-state output gap with higher trend inflation implies that with the kinked demand curves the effect of non-adjusting firms’ eroding markups dominates that of adjusting firms’ higher markups.} Moreover, the kink in demand curves causes demand for a good to become less price-elastic for a decline in the relative price of the good, thus mitigating the increase in demand dispersion due to non-adjusting firms and the relative price distortion.

4 Equilibrium determinacy

This section analyzes implications of a smoothed-off kink in demand curves for determinacy of equilibrium in the log-linearized model consisting of (18)–(25). It also sheds light on the veiled relationship between the NRH and the long-run version of the Taylor principle.

4.1 Implications of a smoothed-off kink in demand curves for equilibrium determinacy

For the annualized trend inflation rate of zero, two, four, and eight percent, Fig. 2 displays regions of the Taylor rule’s coefficients \((\phi_\pi, \phi_y)\) that guarantee equilibrium determinacy under the benchmark calibration presented in Table 1. Note that the coefficients estimated by Taylor (1993) are \((\phi_\pi, \phi_y) = (1.5, 0.5/4) = (1.5, 0.125)\)—which is marked by “x” in each panel of the figure—and thus it is reasonable to consider the range of \(0 \leq \phi_\pi \leq 1.5 \times 3 = 4.5\) and \(0 \leq \phi_y \leq 0.125 \times 3 = 0.375\). For each rate of trend inflation,
there is only one region of determinacy within the coefficient range considered. This region is characterized by

$$\phi_\pi + \phi_y \epsilon_y > 1,$$  

(30)

where

$$\epsilon_y = \frac{\alpha \pi^{\theta-1}(1-\theta)(\theta-1-\alpha)}{(1+\sigma)(\theta-1-\alpha)} + \frac{\beta(\pi-1)(1-\alpha \pi^{\theta-1}) + \sigma y(1-\alpha \pi^{\theta-1})}{(1-\alpha \pi^{\theta-1})}$$

This condition can be interpreted as the long-run version of the Taylor principle. From the log-linearized equilibrium conditions (19)–(24), it follows that a one percentage point permanent increase in inflation yields an $\epsilon_y$ percentage points permanent change in output. Thus $\epsilon_y$ represents the long-run inflation elasticity of output. The Taylor rule (25) then implies a $(\phi_\pi + \phi_y \epsilon_y)$ percentage points permanent change in the interest rate in response to a one percentage point permanent increase in inflation. Therefore, the condition (30) suggests that in the long run the interest rate should be raised by more than the increase in inflation. Fig. 2 thus demonstrates that determinacy is likely even under high trend inflation, if the long-run version of the Taylor principle (30) is satisfied, and that this condition is not restrictive because the coefficient estimates by Taylor (1993), i.e., $(\phi_\pi, \phi_y) = (1.5, 0.125)$, ensure determinacy for any trend inflation rate considered.

This result is in stark contrast with that obtained by Ascari and Ropele (2009) and Kurozumi (2011), who show that indeterminacy is more likely with higher trend inflation in the absence of the kink in demand curves. Fig. 3 displays regions of the Taylor rule’s coefficients $(\phi_\pi, \phi_y)$ that guarantee equilibrium determinacy in the absence of the kink (i.e., $\epsilon = 0$, $\alpha = 0.85$). The figure illustrates that higher trend inflation is more likely to induce indeterminacy. In each panel of the figure, there is only one region of determinacy within the coefficient range considered. This region is characterized not only by the Taylor principle (30) but also by another condition.\footnote{For the zero trend inflation rate the region of determinacy is characterized only by the Taylor principle (30).}
generates lower bounds on the inflation and output coefficients $\phi_x, \phi_y$, but it becomes irrelevant in the presence of the kink in demand curves as can be seen in Fig. 2. The Taylor principle (30) yields an upper bound on the output coefficient $\phi_y$, and even in the presence of the kink, it remains a relevant condition for determinacy, although it brings about lower bounds on the inflation and output coefficients as can be seen in Fig. 2.

The Taylor principle (30) is more likely to be satisfied for the Taylor rule’s coefficients $\phi_x, \phi_y \geq 0$ as the long-run inflation elasticity of output $\epsilon_y$ is larger. In the absence of the kink in demand curves (i.e., $\epsilon = 0$), higher trend inflation makes this elasticity decline exponentially, as shown in Table 2. A rise in the annualized trend inflation rate from zero to two, four, and eight percent reduces the elasticity from 0.18 to −1.65, −6.21, and −102.18, respectively. This exponential decline in the elasticity caused by higher trend inflation is reversed by the kink in demand curves. As shown in Table 2, the elasticity increases from 0.20 to 0.67, 0.83, and 0.93 respectively when trend inflation increases from zero to two, four, and eight percent. Therefore, the kink can prevent fluctuations driven by self-fulfilling expectations even under high trend inflation.

4.2 Relationship between the natural rate hypothesis and the long-run version of the Taylor principle

Thus far this paper has shown that the smoothed-off kink in demand curves can prevent both large violations of the NRH and indeterminacy of equilibrium under the Taylor rule. This subsection addresses the question of whether and how the NRH is related to the long-run version of the Taylor principle.

As noted above, the Taylor principle (30) is more likely to be satisfied for the Taylor rule’s coefficients $\phi_x, \phi_y \geq 0$ when the long-run inflation elasticity of output $\epsilon_y$ is larger. By definition, this elasticity—the percentage points permanent change in output in response to a one percentage point permanent increase in inflation—is given by

$$\epsilon_y = \frac{d \log Y}{d \log \pi}.$$  

Since the natural rate $Y^n$ is constant with respect to the trend inflation rate, the derivative of the steady-state output gap with respect to the trend inflation rate equates the
long-run inflation elasticity of output (i.e., \(d (\log Y - \log Y^n)/d \log \pi = d \log Y/d \log \pi = \epsilon_y\)). Thus an increase (a decline) in the derivative is associated with an increase (a decline) in the elasticity. Indeed, this can be seen in Table 2. For instance, in the absence of the kink in demand curves, as the trend inflation rate rises, the steady-state output gap first increases and then declines. This implies that the derivative takes a positive value at the zero trend inflation rate and then decreases with higher trend inflation as the long-run inflation elasticity of output does. Therefore, it follows that a rise in the trend inflation rate is more likely to induce indeterminacy of equilibrium under the Taylor rule, by lowering the upper bound on the rule’s coefficient on output, if and only if such a rise reduces the steady-state output gap at a declining rate. Consequently, by mitigating the influence of price dispersion on aggregate output, the kink in demand curves subdues the size of the derivative of the steady-state output gap with respect to the trend inflation rate and thus ensures that the violation of the NRH is minor. At the same time, the kink subdues the size of the long-run inflation elasticity of output and thus prevents higher trend inflation from inducing indeterminacy of equilibrium under the Taylor rule.\(^{14}\)

5 Comparison with the model with firm-specific labor

The smoothed-off kink in demand curves gives rise to strategic complementarity in price setting. Recent empirical literature on the NKPC emphasizes the role of such complementarity for reconciling the Calvo model with micro evidence on the frequency of price changes. This section thus addresses the question of whether firm-specific labor—which is another source of strategic complementarity—prevents both large violations of the NRH and indeterminacy of equilibrium under the Taylor rule as the kink in demand curves does.

\(^{14}\)The kink in demand curves prevents the derivative of the steady-state output gap with respect to the trend inflation rate and thus the long-run inflation elasticity of output from turning negative. However, the desirable properties of the kink derive from the subdued size rather than the positive signs of the derivative and the elasticity.
5.1 On the natural rate hypothesis

The Calvo model with firm-specific labor imposes the following assumption instead of Assumption 1 in order for intermediate-good firms’ profit functions to be well-defined.\textsuperscript{15}

**Assumption 2** The two inequalities \( \alpha \beta \pi^{\theta-1} < 1 \) and \( \alpha \beta \pi^{\theta(1+\sigma_n)} < 1 \) hold.

Under this assumption, the equilibrium conditions are given by the spending Euler equation (2), the final-good market clearing condition (8), the Taylor rule (17), the first-order condition for Calvo staggered price setting

\[
0 = E_t \sum_{j=0}^{\infty} (\alpha \beta)^j \prod_{k=1}^{j} \frac{\pi_{t+k}^\theta}{p^*_t} \left[ \prod_{k=1}^{j} \frac{1}{\pi_{t+k}^\theta} - \frac{\theta}{\theta - 1} \frac{Y_{t+j}}{Y_{t}} \right]^{\frac{1-\theta}{\theta}},
\]

and the final-good price equation

\[
1 = (1 - \alpha) \left( p^*_t \right)^{1-\theta} + \alpha \left( \frac{1}{\pi_t} \right)^{1-\theta}.
\]

Combining the last two equations at a steady state yields the relationship between steady-state output \( Y \) and trend inflation \( \pi \)

\[
Y = \left[ \frac{\theta-1 - \alpha \beta \pi^{\theta(1+\sigma_n)}}{\theta - 1 - \alpha \beta \pi^{\theta-1}} \right]^{\frac{1}{1+\sigma_n}} \left[ \frac{1-\alpha}{1-\alpha \pi^{\theta-1}} - \frac{1+\sigma_n}{\theta-1} \right].
\]

In the absence of Calvo staggered price setting, the (steady-state) natural rate of output can be obtained as (28).

The steady-state output gap is thus given by

\[
\log Y - \log Y^n = - \frac{1}{1 + \sigma_n} \log \frac{\frac{1-\alpha}{1-\alpha \pi^{\theta-1}} - \frac{1+\sigma_n}{\theta-1}}{\frac{1-\alpha \beta \pi^{\theta(1+\sigma_n)}}{1-\alpha \beta \pi^{\theta-1}}}. \tag{34}
\]

Note that under the zero trend inflation rate, steady-state output \( Y \) is equal to the natural rate of output \( Y^n \) and hence the steady-state output gap is zero. In the model

\textsuperscript{15} For a description of the Calvo model with firm-specific labor, see Kurozumi and Van Zandweghe (2012).
with firm-specific labor the steady-state output gap consists of only one term, which captures distortion associated with the average markup, and there is no relative price distortion term.

Fig. 4 displays the effect of the annualized trend inflation rate on the steady-state output gap. The thick line represents the gap (29) in the model with the kink in demand curves under the benchmark calibration presented in Table 1. The thin line shows the gap (34) in the model with firm-specific labor under the calibration that sets $\theta = 9.8$ by following the same strategy as for the benchmark calibration. The figure illustrates that the violation of the NRH is much larger in the model with firm-specific labor than in the model with the kink in demand curves. Indeed, as shown in Table 2, for the annualized trend inflation rate of two, four, and eight percent, the steady-state output gap is $-0.47$ percent, $-2.67$ percent, and $-22.22$ percent in the former model, whereas the gap is $0.24$ percent, $0.62$ percent, and $1.50$ percent in the latter model. The reason for the much larger violation of the NRH in the model with firm-specific labor is that the influence of price dispersion on aggregate output is mitigated in the model with the kink in demand curves, whereas this mitigating effect is absent in the model with firm-specific labor.\footnote{Specifically, while firm-specific factors dampen the size of firms' price changes, higher trend inflation makes price-adjusting firms choose a higher markup. Consequently, the distortion associated with the average markup increases exponentially.}

This result on the violation of the NRH suggests, from the relationship between the NRH and the long-run version of the Taylor principle, that indeterminacy of equilibrium under the Taylor rule is much more likely in the model with firm-specific labor than in the model with the kink in demand curves. The next subsection compares these two models in terms of equilibrium determinacy.
5.2 On equilibrium determinacy

Under Assumption 2, log-linearizing the equilibrium conditions (2), (8), (17), (31), and (32) and rearranging the resulting equations leads to (18), (25), and

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \alpha \pi^{\theta-1})[1 - \alpha \beta \pi^{\theta(1+\sigma_n)}]}{\alpha \pi^{\theta-1}(1 + \theta \sigma_n)}(1 + \sigma_n)\hat{Y}_t + \xi_t, \tag{35}
\]

\[
\xi_t = \alpha \beta \pi^{\theta(1+\sigma_n)}E_t \xi_{t+1} + \frac{\beta(\pi^{1+\theta\sigma_n} - 1)(1 - \alpha \pi^{\theta-1})(1 + \sigma_n)}{1 + \theta \sigma_n}\left\{\theta E_t \hat{\pi}_{t+1} + [1 - \alpha \beta \pi^{\theta(1+\sigma_n)}]E_t \hat{Y}_{t+1}\right\}. \tag{36}
\]

The strategic complementarity arising from firm-specific labor reduces the slope of the NKPC (35) by \(1/(1 + \theta \sigma_n)\). Thus, it allows to reconcile the Calvo model with both the micro evidence on the frequency of price changes and the empirical literature on the NKPC.

As shown in Proposition 1 of Kurozumi and Van Zandweghe (2012), determinacy of equilibrium in the model with firm-specific labor is obtained under non-negative trend inflation rates if and only if both the long-run version of the Taylor principle (30), where the long-run inflation elasticity of output is now given by

\[
\epsilon_y = \frac{\alpha \pi^{\theta-1}\{1 - \beta\}(1 + \theta \sigma_n)(1 - \alpha \beta \pi^{\theta(1+\sigma_n)})(1 + \theta \sigma_n) - \beta \theta(1 + \sigma_n)(\pi^{1+\theta\sigma_n} - 1)(1 - \alpha \pi^{\theta-1})}{(1 + \sigma_n)(1 - \alpha \pi^{\theta-1})(1 - \alpha \beta \pi^{\theta(1+\sigma_n)}[1 - \alpha \beta \pi^{\theta(1+\sigma_n)}])},
\]

and another condition are satisfied.

For the annualized trend inflation rate of zero, two, four, and eight percent, Fig. 5 displays regions of the Taylor rule’s coefficients \((\phi_\pi, \phi_y)\) that guarantee determinacy of equilibrium in the model with firm-specific labor under the calibration presented in the preceding subsection (i.e., \(\theta = 9.8\)). This figure illustrates that indeterminacy is more likely with higher trend inflation, in line with Coibion and Gorodnichenko (2011) and Kurozumi and Van Zandweghe (2012). For each rate of trend inflation, there is only one region of determinacy within the coefficient range considered. This region is characterized only by the long-run version of the Taylor principle (30) for the trend inflation rate of zero percent, while for the rate of two and four percent it is featured not only by the Taylor principle (30) but also by the other condition. This latter condition induces lower bounds on the inflation and output coefficients \(\phi_\pi, \phi_y\), while the Taylor principle
(30) generates an upper bound on the output coefficient $\phi_y$. The two conditions make determinacy impossible at the trend inflation rate of eight percent.

As noted above, the Taylor principle (30) is more likely to be satisfied for the Taylor rule’s coefficients $\phi_\pi, \phi_y \geq 0$ as the long-run inflation elasticity of output $\epsilon_y$ is larger. Yet higher trend inflation exponentially reduces this elasticity: its value is 0.20, −2.33, −7.08, and −44.92 respectively for the trend inflation rate of zero, two, four, and eight percent, as shown in Table 2. Hence the Taylor principle (30) induces a more severe upper bound on the output coefficient $\phi_y$ as trend inflation rises. The other condition for determinacy generates more severe lower bounds on the inflation and output coefficients $\phi_\pi, \phi_y$ for higher trend inflation. Consequently, under the calibration of the model that is consistent with both the micro evidence on the frequency of price changes and the empirical literature on the NKPC, indeterminacy is much more likely in the model with firm-specific labor than in the model with the kink in demand curves, as the relationship between the NRH and the long-run version of the Taylor principle implies. Thus the reason for the much higher likelihood of indeterminacy in the model with firm-specific labor is, again, that the influence of price dispersion on aggregate output is mitigated in the model with the kink in demand curves, whereas this mitigating effect is absent in the model with firm-specific labor.

6 Concluding remarks

This paper has examined implications of a smoothed-off kink in demand curves for the NRH and macroeconomic stability in the Calvo model, and has shed light on the relationship between the NRH and the long-run version of the Taylor principle. An empirically plausible calibration of the model has shown that the kink in demand curves mitigates the influence of price dispersion on aggregate output, thereby ensuring that the violation of the NRH is minor and preventing indeterminacy of equilibrium under the Taylor rule. Moreover, it has been shown that in terms of preventing both large violations of the NRH and equilibrium indeterminacy, the smoothed-off kink in demand curves possesses much more desirable properties than firm-specific labor, which is another
source of strategic complementarity in price setting.

Coibion and Gorodnichenko (2011) employ the Calvo model with firm-specific labor to emphasize the importance of the role of trend inflation for the U.S. economy’s Great Inflation era, in contrast with the previous literature, such as Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004), which places emphasis on the role of the Fed’s policy response to inflation. Such a model, however, induces a large violation of the NRH. In particular, it generates a large deviation of steady-state output from its natural rate during the Great Inflation era. The Calvo model with the kink in demand curves, in which the violation of the NRH is minor, supports the view of the previous literature.
References


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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<td>$\beta$</td>
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<td>$\sigma_n$</td>
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<td>$\epsilon$</td>
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Table 2: Relationship between steady-state output and trend inflation

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<tr>
<td>A. Steady-state output gap (%)</td>
<td></td>
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<tr>
<td>Kink in demand curves</td>
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<td>B. Distortion associated with average markup (%)</td>
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<td>C. Relative price distortion (%)</td>
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<td>D. Long-run inflation elasticity of output</td>
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Note: To obtain the case of no kink in demand curves the baseline calibration is adjusted by setting $\epsilon = 0$ and $\alpha = 0.85$. The case of firm-specific labor is analyzed by choosing $\theta = 9.8$. 
Figure 1: Effect of trend inflation on steady-state output gap.

Notes: The thick line shows the case of a smoothed-off kink in demand curves, and the thin line shows the case of no kink (i.e., $\epsilon = 0$, $\alpha = 0.85$). Trend inflation is expressed in percent at an annual rate and the steady-state output gap is expressed in percent.
Figure 2: Regions of the Taylor rule’s coefficients ($\phi_\pi, \phi_y$) that guarantee equilibrium determinacy: Benchmark calibration.

Note: In each panel the mark “×” shows Taylor (1993)’s estimates ($\phi_\pi, \phi_y$) = (1.5, 0.5/4).
Annualized trend inflation rate of 0 percent  
Annualized trend inflation rate of 2 percent  
Annualized trend inflation rate of 4 percent  
Annualized trend inflation rate of 8 percent  

Figure 3: Regions of the Taylor rule’s coefficients \((\phi_\pi, \phi_y)\) that guarantee equilibrium determinacy in the case of no smoothed-off kink in demand curves (i.e., \(\epsilon = 0, \alpha = 0.85\)).

Note: In each panel the mark “×” shows Taylor (1993)’s estimates \((\phi_\pi, \phi_y) = (1.5, 0.5/4)\).
Figure 4: Effect of trend inflation on steady-state output gap: smoothed-off kink in demand curves versus firm-specific labor.

Notes: The thick line shows the model with a smoothed-off kink in demand curves and the thin line shows the model with firm-specific labor. Trend inflation is expressed in percent at an annual rate and the steady-state output gap is expressed in percent.
Figure 5: Regions of the Taylor rule’s coefficients (\(\phi_\pi, \phi_y\)) that guarantee equilibrium determinacy in the model with firm-specific labor: \(\theta = 9.8\).

Note: In each panel the mark “\(\times\)” shows Taylor (1993)’s estimates (\(\phi_\pi, \phi_y\)) = (1.5, 0.5/4).