Detecting Large-Scale Collusion in Procurement Auctions

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Abstract

This paper documents evidence of widespread collusion among construction firms using a novel dataset covering most of the construction projects procured by the Japanese national government from 2003 to 2006. By examining rebids that occur for auctions when all (initial) bids fail to meet the reserve price, we identify collusion using ideas similar to regression discontinuity. We identify about 1,000 firms whose conduct is inconsistent with competitive behavior. These bidders were awarded about 7,600 projects, or close to one fifth of the total number of construction projects in our sample. The value of these projects totals about $8.6 billion.

Key words: Collusion, Procurement Auctions, Antitrust
JEL classification: D44, H57, K21, L12

1 Introduction

One of the central themes of competition policy is to deter, detect, and punish collusion. While there is almost universal agreement among economists that collusion among firms

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is socially undesirable, firms often have private incentives to engage in collusive behavior absent regulatory sanctions. Therefore, it is crucial to ensure that the antitrust agencies have the authority and the resources to detect and punish collusion in order to promote competition among firms. To the extent that collusive activities remain undetected or unpunished, collusion may become the norm rather than the exception, with potentially large detrimental effects on the economy.

In this paper, we propose a way of detecting bidding rings in procurement auctions using bidding pattern in reauctions. In procurement settings, it is quite common for government agencies to reauction projects after an unsuccessful auction in which no bid meets the secret reserve price. Our method of detecting collusion focuses on projects that are put out for a bid multiple times. We then apply our method to a large dataset of procurement auctions in Japan. Our novel dataset, which covers April 2003 through December 2006, accounts for most of the construction projects procured by Japan’s national government during this period. About 20% of the projects in our data are put out for a bid multiple times. We document patterns of collusion that persist across regions, across types of construction projects and across time.

In order to illustrate our approach for detecting collusion, we begin by describing a road resurfacing auction in New York discussed at some length in Porter (1994). In February 1983, the New York Department of Transportation (DoT) held a procurement auction for resurfacing 0.8 miles of road. The lowest bid in the auction was $4 million and the DoT decided not to award the contract because the bid was deemed too high relative to its own cost estimates. The same project was put up for a reauction in May 1983 in which all bidders from the initial auction participated. The lowest bid in the reauction was 20% higher than in the initial auction, submitted by the previous low bidder. Again, the contract was not awarded. The DoT held an auction for the third time in February 1984, and the set of bidders was the same as the initial auction. The lowest bid in the third auction was 10% higher than the second time, again, submitted by the same bidder. The DoT apparently thought this was suspicious: “It is notable that the same firm submitted the low bid in each of the auctions. Because of the unusual bidding patterns, the contract was not awarded through 1987.” (Porter 1994, p523)

As this example illustrates, it is quite intuitive that a pattern of bidding in which the same bidder submits the lowest bid multiple times is suggestive of collusion. If there is a bidding ring and the project has been preallocated to one of its members, the designated winner can be expected to submit the lowest bid in the initial auction as well as in the
subsequent reauctions. The detection method we propose in this paper is based on this intuition.

While the preceding argument has intuitive appeal, the key challenge in putting the previous argument into practice is to control for inherent cost differences across bidders. If one bidder has sufficiently low costs relative to the rest of the bidders, the lowest bidder in the initial auction may bid the lowest in subsequent reauctions even under competition. The contribution of our paper is to operationalize the preceding argument in a way that allows us to differentiate between persistence in the identity of the lowest bidder generated by inherent cost differences and persistence generated by collusive agreements. The way in which we do so is based on ideas similar to regression discontinuity: We focus on auctions in which the second-lowest bidder loses to the lowest bidder by a very small amount in the initial auction. If the two lowest bids are sufficiently close, which bidder turns out to be the lowest/second-lowest bidder in the initial auction is as good as random. Then, the two bidders can be thought of as symmetric with regard to their costs. For these set of auctions, the second lowest bidder is as likely to be the lowest bidder in subsequent reauctions as the lowest bidder under competition, while collusion would still predict persistence. By focusing on auctions where the second-lowest bidder narrowly loses to the lowest bidder, we can separate persistence due to competition and persistence due to collusion.

The approach we propose in this paper may be useful to law enforcement agencies in a variety of settings. First, having a predetermined winner is quite a common feature of bidding rings. While bidding rings can be organized in a variety of ways depending on whether collusion is tacit or explicit, whether or not there are side-payments, etc., it is common for bidding rings to pick a predetermined winner among its members beforehand and reduce competitive pressure in the actual auction. Preallocating the project to a designated bidder can also have significant cost advantages from the bidding ring’s perspective, especially in procurement auctions. Procurement auctions typically involve substantial bidding costs due to the need for estimating the cost of the project beforehand (e.g., estimating the cost of material, labor, machinery, etc.), which are often borne only by the designated winner in a bidding ring. That is, the non-designated bidders can avoid having to pay the project

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1 Examples include the “great electrical conspiracy” (Smith 1961, McAfee and McMillan 1992), bidding ring of stamp dealers (Asker 2010), bidding ring of construction firms in Nassau and Suffolk counties (Porter 1992), etc.

2 For example, a study by the Japan Federation of Bar Associations in 2001 reports that the project estimation costs are borne only by the designated winner in many bidding rings based on the facts that became clear during criminal bid-rigging cases (JFBA 2001).
estimation costs. One important implication of this is that changing the designated winner between the initial auction and the reauction may be quite difficult for the bidding ring.

Second, reauctions are quite commonly observed in procurement settings. This is because the auctioneer often lacks commitment power; the auctioneer cannot commit never to try to reauction the same object when bids fail to meet the reserve price. The lack of commitment power is especially salient in procurement auctions. When the auction is for the procurement of key infrastructure, for instance, it would seem impossible for the procuring agencies to commit never to have a reauction after an unsuccessful auction. Examples in which the auctioneer routinely holds multiple auctions for the same object include timber auctions by the U.S. Forest Services, procurement auctions by the U.S. State Department of Transportation and U.S. offshore gas and oil lease auctions.  

The detection method that we propose, then, is potentially useful for a wide range of settings. Our approach is also quite simple, requiring only bidding data. Moreover, the tests we propose do not rely on parametric assumptions on the primitives of the model.

We apply our idea to a large dataset of Japanese construction projects procured by the Ministry of Infrastructure Land and Transportation. Our data contain more than 40,000 auctions worth more than $42 billion in total. On an annual basis, the total size of the award amount is close to $14 billion, or about 3% of Japan’s national tax revenue.

The auction format used in our data is a first-price sealed bid (FPSB) auction, with the additional possibility of rebidding when all the bids fail to meet the secret reserve price. As long as the lowest bid is below the secret reserve price, the project is awarded to the lowest bidder. If none of the bids is below the reserve price, however, the auctioneer reveals the lowest bid to all the bidders and solicits a second round of bids. The auctioneer reveals only the lowest bid and none of the other bids (the identity of the lowest bidder and the secret reserve price are not revealed). The second round bidding takes place typically 30 minutes after the initial round with the same set of bidders and the same (secret) reserve price. If the lowest bid in the second round is still higher than the secret reserve price, there is a third round of bidding.  

Approximately 20% of all auctions advance to the second round

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3 See McAfee and Vincent (1997) for a description of resales of timber auctions, Li and Ji (2008) for DoT auctions in Indiana and Porter (1995) for offshore gas and lease auctions. Ji and Li (2008) examine DoT auctions in which the auctioneer sets a secret reserve price and there are multiple auctions for the same project when none of the bids meets the secret reserve price. In their sample, they find that about 12.5% of auctions have two rounds. In his analysis of wildcat tracts, Porter (1995) reports “A total of 233 high bids, or 10 percent were rejected on these tracts. On the tracts with rejected bids, 47 percent were subsequently reoffered.”

4 After the third round, a bilateral negotiation takes place between the buyer and the lowest third-round
and about 3% advance to the third round in our data.

In order to identify collusion, we look for patterns in the data where the same bidder submits the lowest bid across multiple rounds of a given auction beyond what competitive behavior can explain. To be concrete, let \( i(1) \) and \( i(2) \) be the lowest and the second-lowest bidders, respectively, in the first round. We examine the second-round bids of \( i(1) \) and \( i(2) \) for the set of auctions where the first-round bids of \( i(1) \) and \( i(2) \) are only \( \varepsilon \) apart. Conditional on the first-round bids being very close to each other, \( i(1) \) and \( i(2) \) can be thought of as symmetric, in terms of costs, risk aversion, beliefs over the distribution of the reserve price, etc. Thus, absent information asymmetry that exists between \( i(1) \) and \( i(2) \) given that only the first-round lowest bid is revealed to the participants, the likelihood that \( i(2) \) outbids \( i(1) \) in the second round should be close to 50% as long as \( \varepsilon \) is small enough. It turns out that when we factor in the information asymmetry, it makes it even more likely that \( i(2) \) outbids \( i(1) \) in the second round under competitive behavior.\(^5\)

To summarize, as long as the first-round bids of \( i(1) \) and \( i(2) \) are sufficiently close, we would expect \( i(2) \) to win at least as often as \( i(1) \) in the second round under competitive bidding.\(^6\) However, we find that \( i(2) \) rarely outbids \( i(1) \) in the second round in the actual data. For example, when we set \( \varepsilon \) to be 1% of the reserve price, \( i(2) \) outbids \( i(1) \) only about 2.6% of the time (56 out of 2,160 auctions).\(^7\) The probability that \( i(1) \) remains the lowest bidder in the second round is around 96.4%.\(^8\)

Of course, it is possible that our findings are still driven by inherent cost differences among the firms; i.e., the bandwidth we use (e.g., \( \varepsilon = 1\% \) of the reserve price) is not small enough to adequately control for differences in costs, etc., among the bidders. In order to rule out this possibility, we compare the second-round bids of \( i(2) \) and \( i(3) \) (the second-bidder. The same secret reserve price is used in all three rounds.

\(^5\)The buyer reveals the lowest bid, but none of the other bids. Hence, \( i(1) \) knows only that its bid was the lowest among the first-round bids, while \( i(2) \) gains knowledge of two bids – its own bid and the lowest bid. This means that conditional on the two lowest bids being very close to each other, \( i(2) \) has an information advantage in the second round: While \( i(1) \) does not know that there is another bidder who bid just above its bid, \( i(2) \) knows that it was outbid by a small margin.

\(^6\)More precisely, we expect \( i(2) \) to do no worse than \( i(1) \). This does not logically imply that \( i(2) \) should win more often if profit margins are very thin. We address this concern by examining auctions that go to the third round and in which \( i(2) \) submits substantially lower bids in the third round. For this set of auctions, we have an upper bound on \( i(2) \)'s costs or, equivalently, a lower bound on \( i(2) \)'s profit margin. Even for these auctions, we find that \( i(2) \) almost never outbids \( i(1) \) in the second round.

\(^7\)This includes 7 ties in which \( i(1) \) and \( i(2) \) bid exactly the same amount in the second round.

\(^8\)The reason why this probability is lower than \( 97.4\% \) (\( = 100\% - 2.6\% \)) is because, occasionally, the third-lowest bidder in the first round (or the fourth-lowest, etc.) becomes the lowest bidder in the second round.
and the third-lowest bidders in the first round). In contrast to the case of \( i(1) \) and \( i(2) \), we find that \( i(3) \) outbids \( i(2) \) in the second round close to 50% of the time. For example, when we examine the second-round bids of \( i(2) \) and \( i(3) \) for the set of auctions where the bid difference between \( i(2) \) and \( i(3) \) in the first round is less than 1% of the reserve price, we find that \( i(3) \) outbids \( i(2) \) in about 49.0% of the cases (2,555 out of 5,218 auctions).\(^9\) This gives assurance that the bandwidth we choose for \( \varepsilon \) is sufficiently small for purging much of the inherent differences among the bidders. Our results, thus, suggest that there is much more persistence – across multiple rounds within the same auction – in the identity of the lowest bidder than competition can explain.

In order to provide further evidence that this persistence is unlikely to occur under competition, we next examine the shape of the distribution of \( \Delta_{12} \) (i.e., the difference between the second-round bids of \( i(1) \) and \( i(2) \), normalized by the reserve price).\(^10\) In the left panel of Figure 1, we plot the histogram of \( \Delta_{12} \) for the set of auctions in which the first-round bids of \( i(1) \) and \( i(2) \) are within 1% of the reserve price. The right panel of Figure 1 plots the histogram of \( \Delta_{23} \) (i.e., the difference between the second-round bids of \( i(2) \) and \( i(3) \), normalized by the reserve price) for the set of auctions in which the first-round bids of \( i(2) \) and \( i(3) \) are within 1% of the reserve price.\(^11\) A noticeable feature of Figure 1 is that there appears to be a discontinuous jump in the distribution of \( \Delta_{12} \) at exactly zero. That is, when we focus on a small band around zero, we find hundreds of auctions in which \( \Delta_{12} \) falls just to the right of zero (i.e., \( \Delta_{12} \in (0, t) \) for some small positive \( t \)), whereas we find very few auctions in which \( \Delta_{12} \) falls just to the left of zero (i.e., \( \Delta_{12} \in (-t, 0) \)). This implies that there are many auctions in which \( i(2) \) loses to \( i(1) \) in the second round by a tiny margin, but almost no auctions in which \( i(2) \) wins by a tiny margin. The distribution for \( \Delta_{23} \), on the other hand, is continuous and symmetric around zero with a fair amount of variance.

The discontinuity exhibited in the distribution of \( \Delta_{12} \) at zero strongly suggests that the persistence in the identity of the lowest bidder is generated by collusive agreements. In particular, we take the discontinuity as evidence that the bidders know how each other will bid in the second round and, moreover, that auction participants designate a predetermined winner in advance. To see this, suppose the contrary: If \( i(1) \) and \( i(2) \) were uncertain as to...
how each other will bid in the second round, then one should observe a similar number of auctions in which \( i(2) \) outbids \( i(1) \) by a tiny margin as auctions in which \( i(1) \) outbids \( i(2) \) by a tiny margin. Hence, the fact that the distribution of \( \Delta_{12} \) seems discontinuous at zero suggests that the bidders are aware of how each other will bid. But if this is the case, why else would \( i(2) \) lose by a small margin (rather than win by a small margin) other than to yield to the predetermined winner?

In order to formally reject the null of competitive behavior, we next study the persistence in the identity of the lowest bidder vis-à-vis the optimality of \( i(2) \)’s bid in the second round. In particular, we test if \( i(2) \) can substantially increase its profits by decreasing its second-round bids by a small margin. To the extent that this is possible, it implies that \( i(2) \) is not best responding to the strategies of the other bidders, violating the necessary condition of Nash Equilibrium.\(^{12}\)

To test the optimality of \( i(2) \)’s second-round bid, we need to compare \( i(2) \)’s current profits with its counterfactual profits from employing an alternative strategy in the second round. While we cannot directly compute the counterfactual profits as bidder costs are not observable, we can still obtain a lower bound on the counterfactual profits using the idea

\(^{12}\)More precisely, we test the optimality of the second-round bidding strategy employed by bidders who narrowly lose in the first round. Note that \( i(2) \) does not necessarily know that it came in second at the beginning of the second round.
that $i(2)$’s third-round bid puts an upper bound on its costs under certain conditions. Our approach here is based on ideas similar to Haile and Tamer (2003) in which the authors obtain an upper bound on the value of bidders in an incomplete model of English auctions using the assumption that bidders do not bid above their value.

When we apply our test to our data, we find that bidders who narrowly lose in the first round can increase its profits by up to 2 million yen on average by simply decreasing its bid in the second round uniformly by 1%. The results formally reject the null that bidders are playing a Nash Equilibrium of a competitive auction game under the assumption that a bidder’s third-round bid puts a bound on its costs.

Lastly, we develop a test statistic of collusive behavior that we can apply to each firm. The test formalizes the idea that $\Delta_{12}$ should not be discontinuous at zero under competitive behavior. Our test statistic is composed of two parts: a measure of how sharply the distribution of $\Delta_{12}$ changes around zero, and the variance of $\Delta_{23}$. Under the null of competitive bidding, the variance of $\Delta_{23}$ puts a bound on how sharply the distribution of $\Delta_{12}$ can change around zero. Thus, our test statistic compares the change in the distribution of $\Delta_{12}$ around zero with the variance of $\Delta_{23}$. We then apply this test to each firm in our dataset by computing the test statistic using just the sample of auctions in which the firm participated.

In our baseline result, we find about 1,000 construction firms for whom we reject the null hypothesis of competitive behavior at the 95% confidence level. This is in contrast to the fact that the antitrust authorities (JFTC) brought only four collusion cases against a total of 92 construction firms in connection with the procurement projects in our sample. The number of auctions won by the firms that we identify as uncompetitive totals 7,600, or close to one fifth of the total number of auctions in our sample. These auctions range from small scale projects such as painting and paving to large and complex projects such as building tunnels and bridges. The total award amount of these auctions is about $8.6 billion. We estimate that, absent collusion by these firms, taxpayers could have saved about $721 million. While this is already a large number, it is worth mentioning that a large fraction of firms that we identify as uncompetitive are also active in other public procurement projects, such as municipal and prefectural projects. Given that the total value of these public projects is close to ten times the size of our dataset, the overall impact of collusion on taxpayers can be staggering.
1.1 Related Literature

This paper is most closely related to the empirical literature on the detection of collusion.\textsuperscript{13} Existing empirical studies of collusion tend to take advantage of known episodes of cartel activity, e.g., paving in highway construction in Nassau and Suffolk counties (Porter and Zona 1993); school milk in Ohio (Porter and Zona 1999); school milk in Florida and Texas (Pesendorfer 2000); and collectible stamps in North America (Asker 2010). While none of our analysis requires information on known bidding rings, it is still useful to study the bidding behavior of known cartels for validation purposes. We do this in Section 5 for the four known bidding cartels that were prosecuted by the JFTC in connection to our sample.

There is another strand of literature that tests for collusion in the absence of any prior knowledge of bidder conduct. Examples include bidding in seal coat contracts in three states in the U.S. Midwest (Bajari and Ye 1999); U.S. Forest Service timber sales (Baldwin, Marshall and Richard 1997; Athey, Levin and Seira 2011); Offshore gas and oil lease (Hendricks and Porter 1988; Haile, Hendricks, Porter and Onuma 2013); roadwork contracts in Italy (Conley and Decaloris, 2013); and public-works consulting in Japan (Ishii, 2009). Ishii (2009) studies 175 auctions for design consultant contracts in Naha, Okinawa and analyzes how the winner of the auctions can be explained by exchange of favors. While her identification is based on bid patterns across auctions, our identification strategy focuses on how bidders bid within a given auction. Our study also looks at most of the construction projects procured by the national government, whereas she studies a specific local market.\textsuperscript{14}

Lastly, this paper is related to the literature on sequential auctions. McAfee and Vincent (1997) studies the problem of a seller who can post a reserve price, but cannot commit never to attempt to resell an object if it fails to sell. In a recent paper, Skreta (2013) solves for the seller’s optimal mechanism and shows that both first and second price auctions with optimally chosen reserve prices maximize the seller’s revenue when the bidders are ex-ante identical. The auctions in our dataset have the feature that the seller cannot commit never to resell, but can commit to the same secret reserve price. Ji and Li (2008) is a paper that structurally estimates a model of multi-round procurement auctions with a secret reserve price using data on procurement auctions organized by the Indiana DoT. The Indiana DoT

\textsuperscript{13}For a brief survey, see Asker (2010a). For a more comprehensive study, see, e.g., Marshall and Marx (2012).

\textsuperscript{14}For a more general overview of bidding rings among procurement firms in Japan, see McMillan (1991). See, also, Ohashi (2009), who discusses how the change in auction design in Mie Prefecture affected collusion.
also maintains the same secret reserve price throughout the multiple rounds as in our setting. Ji and Li (2008) recover the private cost distributions of the bidders assuming that the observed bids are competitive.

2 Institutional Background

Auction Mechanism  The auction mechanism used in our sample is a variant of the first-price sealed bid (FPSB) auction with a secret reserve price. In fact, the auction mechanism is exactly the same as the FPSB auction as long as the lowest bid is below the secret reserve price, in which case, the lowest bidder becomes the winner with price equal to the lowest bid, and the auction ends. If none of the bids is below the reserve price, however, the buyer reveals the lowest bid to all the bidders and solicits a second round of bids. The buyer reveals only the lowest bid and none of the other bids (the identity of the lowest bidder and the secret reserve price are not revealed). The second round bidding takes place typically 30 minutes after the initial round, with the same set of bidders and the same (secret) reserve price.\footnote{See, e.g., Bidding Guidelines of Ministry of Land, Infrastructure, and Transportation, Chugoku Regional Developing Bureau.} This means that when bidding in the second round, the bidders know that the secret reserve price is lower than the lowest first-round bid.

The second round proceeds in the same manner as the initial round; if the lowest bid is below the reserve price, the auction ends, and the lowest bidder wins. Otherwise, the buyer reveals the lowest second-round bid to the bidders, and the auction goes to the third round. The third round is the final round. If no bid meets the reserve price in the third round, bilateral negotiation takes place between the buyer and lowest third-round bidder. The same secret reserve price is used in all three rounds.\footnote{The reserve price, the identity of the bidders, and all the bids in each round are made public after the auction ends.}

Bidder Participation  As is the case in many countries, participation in procurement auctions in Japan is not fully open. A contractor that wishes to participate must first go through screening to be pre-qualified. Because pre-qualification occurs at the regional level, a contractor needs to be pre-qualified for each region in which it wishes to bid on projects.\footnote{Our data set is divided into nine regions.}

In addition to pre-qualification, there may be additional restrictions on participation:
Depending on how restrictive they are, the auctions can be divided into four categories: The first and the second categories are the most restrictive, with participation by government invitation. In these two categories, the government typically invites ten bidders from the pool of pre-qualified contractors. The difference between the two categories is that in the first category, the invited bidders are chosen randomly from the pool, while in the second category, the government chooses bidders based on contractors’ preferences over project type, project location, etc., submitted by the contractors in advance.\footnote{Each pre-qualified contractor submits a form to the government to express its preferences over the type and location of projects it wishes to bid on.}

The third and fourth categories are less restrictive. The set of potential bidders is still restricted to the pool of pre-qualified contractors, but any pre-qualified contractor can participate in the bidding. The difference between the third and fourth categories is that in the third category, the government reserves the right to exclude potential bidders from participating in the auction under certain conditions.

Collusive Behavior In principle, bidding rings can be organized in a variety of ways, depending on whether or not members engage in side-payments, whether explicit communication between the members is feasible, etc. Whatever the exact arrangement, however, a very common feature of bidding rings is that the ring picks a predetermined winner in advance and that the rest of the ring members help the predetermined winner win. Almost all of the existing evidence indicates that bidding rings in the construction industry in Japan are organized in this manner.

There are also two other documented features of prosecuted bidding rings in the construction industry that are worth mentioning: The first feature is that, typically, the designated winner alone incurs the cost of estimating the project cost.\footnote{See, e.g., the criminal bid-rigging case regarding the construction of a sewage system in Hisai city (Tsu District Court, No. 165 (Wa), 1997), the bid-rigging case regarding the construction of a waste incineration plant in Nagoya city (Nagoya District Court, No. 1903 (Wa), 1995), etc. Based on the facts that became clear in these cases, a study by the Japan Federation of Bar Associations in 2001 (JFBA 2001) concludes that the project estimation costs are borne only by the designated winner in many bidding rings.} Estimating the project cost can be quite expensive, and the non-designated bidders typically avoid incurring this cost.\footnote{Estimating the project cost involves understanding the specifications of the project, assessing the quantity and quality of materials required, negotiating prices for construction material and arranging for available subcontractors. These costs are often quite substantial.} Note that this makes it risky for a non-designated bidder to accidentally win the auction. The second feature is that the designated winner of a bidding ring would often communicate to other members how it would bid in each of the three rounds (as opposed...}
to communicating how it would bid just in the first round).\textsuperscript{21}

3 Data

We use a novel dataset of auctions for public construction projects obtained from the Ministry of Land, Infrastructure and Transportation, the largest single procurement buyer in Japan. The dataset spans April 2003 through December 2006 and covers most of the construction works auctioned by the Japanese national government during this period. After dropping scoring auctions, unit-price auctions, and those with missing or mistakenly recorded entries, we are left with 42,561 auctions with a total award amount of more than $42 billion.\textsuperscript{22} The award amount is close to $14 billion annually, accounting for about 3\% of the national government tax revenue.

The data include information on all bids, bidder identity, the (secret) reserve price, auction date, auction category (which corresponds to how restrictive bidder participation is), location of the construction site, and the type of project.\textsuperscript{23} The data also contain information on whether the auction proceeded to the second round or the third round, as well as all the bids in each round. Table 1 provides summary statistics of the data. In the table, we report the reserve price of the auction (Column (1)), the winning bid (Column (2)), the ratio of the winning bid to the reserve price (Column (3)), the lowest bid in each round as a percentage of the reserve price (Columns (4)-(6)), and the number of bidders (Column (7)). The sample statistics are reported separately by whether the auction concluded in Round 1, Round 2, or Round 3.

In the first and second columns of the table, we find that the average reserve price of the auctions is about 98 million yen and the average winning bid is about 93 million yen. In the third column, we find that the winning bid ranges between 92\% and 97\% of the reserve price. In the next three columns, we report the lowest bid in each round as a fraction of the reserve price. Note that for auctions that conclude in the first round, Column (4) is equal to Column (3). For auctions that conclude in the second or third round, the numbers reported in Column (4) are higher than unity by construction. Column (7) reports the average number of bidders, and Column (8) reports the sample size. We find that 16.9\% are...
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<th>Reserve (W)</th>
<th>Reserve (W)/(R)</th>
<th>Lowest Bid/Reserve (R)</th>
<th># Bidders (7)</th>
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<td>98.455</td>
<td>92.795</td>
<td>0.934</td>
<td>0.955</td>
<td>0.962</td>
</tr>
<tr>
<td></td>
<td>(234.49)</td>
<td>(223.11)</td>
<td>(0.079)</td>
<td>(0.102)</td>
<td>(0.035)</td>
</tr>
</tbody>
</table>

Note: The first row corresponds to the summary statistics of auctions that ended in the first round; the second row corresponds to auctions that ended in the second round; and the third row corresponds to auctions that went to the third round. The last row reports the summary statistics of all auctions. The numbers in parentheses are the standard deviations except for the last column, where we report the fraction of auctions that ended in the first, second, and third rounds. First and second columns are in millions of yen.

Table 1: Sample Statistics.

of the auctions go to the second round, and 2.9% advance to the third round.

4 Analysis

4.1 Persistence of the Identity of the Lowest Bidder

Persistence in the Second Round We begin our analysis by studying the extent to which the lowest bidder in the first round is also the lowest bidder in later rounds for a given auction. Recall that a typical feature of bidding rings is that there is a designated winner and that ring members other than the designated winner submit bids in such a way as to ensure that the designated bidder is the lowest bidder. Because, in the setting we study, the reserve price is unknown from the perspective of the bidding ring, the ring members must make sure that the designated bidder is the lowest bidder in each successive round if the auction takes multiple rounds. This is especially important if the designated bidder is the only one that has estimated the project cost. This implies that we should observe persistence in the identity of the lowest bidder across rounds under bidder collusion.

In Table 2, we report how the rank of the bidders changes from the first round to the
second round for all auctions that proceed to the second round with five or more participants \((N = 8,089)\). The \((i,j)\) element of the matrix corresponds to the probability that a bidder submits the \(j\)-th lowest bid in the second round conditional on submitting the \(i\)-th lowest bid in the first round; i.e., \(\text{Pr}(j\text{-th lowest} | i\text{-th lowest})\). Thus, the diagonal elements correspond to the probability that a given bidder remains in the same rank in both rounds. Note that the horizontal sum of the probabilities is one.

What is striking about this table is the probability in the \((1,1)\) cell. We find that in 96.70\% of cases, the lowest bidder in the first round is still the lowest bidder in the second round. The flip side of this is that if a bidder is not the lowest bidder in the first round, the bidder is almost never the lowest bidder in the second round. For example, the conditional probability that a second-lowest bidder in Round 1 becomes the lowest bidder in Round 2 is only 1.59\%. Note, also, that the diagonal elements other than the \((1,1)\) element are much smaller: the probability that the second-lowest bidder in the first round remains the second-lowest bidder is just 26.62\%. There is very strong persistence in the identity of the lowest bidder, but not necessarily for other positions.

In order to illustrate this point further, we examine more closely how the three lowest bidders in the first round behave in the second round. In what follows, we let \(i(k)\) denote the identity of the bidder who submits the \(k\)-th lowest bid in Round 1. We also denote the (normalized) bid of bidder \(i(k)\) in round \(t\) by \(b^t_{i(k)}\). Because there is considerable variation in project size, we work with the normalized bids by dividing the actual bids by the reserve price of the auction. Hence, \(b^2_{i(1)}\), for example, denotes the second-round bid of the first-
round lowest bidder as a percentage of the reserve price.

In the top left panel of Figure 2, we plot the histogram of $\Delta_{12}^2 \equiv b_{i(2)}^2 - b_{i(1)}^2$ for the set of auctions that go to the second round. That is, we plot the difference in the (normalized) second-round bids of $i(1)$ and $i(2)$.$^{24}$ Note that almost all of the mass lies to the right of zero, which confirms what we report in Table 2: A flip in the ordering between the lowest and the second-lowest bidders almost never happens across rounds. In the top right panel of Figure 2, we plot the histogram of $\Delta_{23}^2 \equiv b_{i(3)}^2 - b_{i(2)}^2$, i.e., the difference in the normalized rebids of $i(2)$ and $i(3)$, for the set of auctions that go to the second round. In stark contrast to the left panel, the shape of the histogram for $\Delta_{23}^2$ is quite symmetric around zero. This implies that the ranking between $i(2)$ and $i(3)$ flips in the second round with almost 50% probability. This also seems consistent with our previous finding that there is much less persistence in the ranking for the second and third places.

So far, the results that we have presented correspond to all of the auctions that proceeded to the second round. However, it is possible that our results are driven by inherent differences among firms such as costs, risk attitude, beliefs over the reserve price, etc. For instance, if there are significant cost differences between the lowest bidder and all of the other bidders, our results may be generated by competitive bidding. In order to rule out this possibility, we perform the same analysis by conditioning on the set of auctions in which the first-round bids are close to each other. The idea is that if, for example, the first-round bids of $i(1)$ and $i(2)$ are sufficiently close (i.e., $b_{i(2)}^1 - b_{i(1)}^1 < \varepsilon$ for some small $\varepsilon$), there should be little inherent differences among them, on average. In fact, if $\varepsilon$ is small enough, which bidder turns out to be the lowest/second-lowest bidder in the first round is as good as random. Hence, $i(1)$ and $i(2)$ should be interchangeable, in terms of costs, risk attitude, beliefs over the reserve price, etc.

In the second row of Figure 2, we plot $\Delta_{12}^2$ and $\Delta_{23}^2$ for the subset of auctions for which the bids in the first round are within 5% of each other.$^{25}$ In particular, we plot the histogram of $\Delta_{12}^2$ for the set of auctions in which $b_{i(2)}^1 - b_{i(1)}^1 < 0.05$ in the left panel and the histogram of $\Delta_{23}^2$ for the set of auctions with $b_{i(3)}^1 - b_{i(2)}^1 < 0.05$ in the right panel. Note that the shape of the distribution of $\Delta_{12}^2$ in the left panel is still very skewed and asymmetric around zero,

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$^{24}$The sample sizes are different between the top left and the top right panels because in some auctions, $i(1)$ or $i(3)$ does not bid in the second round.

$^{25}$The sample sizes are different between the two panels because there are more auctions in which $b_{i(3)}^1 - b_{i(2)}^1 < 0.05$ than auctions in which $b_{i(2)}^1 - b_{i(1)}^1 < 0.05$. Similarly for the two panels in the third row. The difference in the sample sizes in the fourth row is due to the fact that in some auctions, $i(1)$ or $i(3)$ does not bid in the second round.
Figure 2: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels). The first row is the histogram for the set of auctions that reach the second stage; and $i(1)$ and $i(2)$ (or $i(2)$ and $i(3)$) submit valid bids in the second round. The second to fourth rows plot the same histogram, but only for auctions in which the differences in the first-round bids are relatively small.

while the distribution of $\Delta_{23}^2$ in the right panel remains symmetric around zero. The fact that the distribution of $\Delta_{23}^2$ is symmetric around zero and very similar to the top panel suggests that cost differences between bidders do not seem to play a large role: If cost differences were driving the skewed bid pattern for $\Delta_{12}^2$ in the left panel, we should also expect to see a distribution of $\Delta_{23}^2$ that is skewed to the right of zero. The third row plots the distribution of $\Delta_{12}^2$ and $\Delta_{23}^2$, but now conditioning on auctions with $b_{i(2)}^1 - b_{i(1)}^1 < 0.01$ and $b_{i(3)}^1 - b_{i(2)}^1 < 0.01$, respectively. Lastly, the bottom row shows the distribution of $\Delta_{12}^2$.
and $\Delta_{23}^2$ conditional on the event that the three lowest bids in the first round are all within 1% of each other, $b_{i(3)}^1 - b_{i(1)}^1 < 0.01$.26 Taken together, Figure 2 suggests that it is not differences in costs, etc. that are driving the persistence in the identity of the lowest bidder.

In the Online Appendix, we explore whether the distributions of $\Delta_{12}^2$ and $\Delta_{23}^2$ exhibit similar patterns when we condition the sample by various auction characteristics, such as region, auction category, project type, and year. We find that the distributions of $\Delta_{12}^2$ and $\Delta_{23}^2$ often look very similar to those shown in Figure 2: The distribution of $\Delta_{12}^2$ is skewed to the right and displays what appears to be a discontinuity at zero, while the distribution of $\Delta_{23}^2$ is symmetric around zero. In the Online Appendix, we also plot the second-round bid differences of $i(1)$ and $i(2)$ and $i(2)$ and $i(3)$ without normalizing the bids by the reserve price. The graphs also appear similar to Figure 2.

**Information Advantage of $i(2)$** Recall from Section 2 that the lowest bid is announced in each round, but none of the other bids are. This means that while $i(1)$ only gains knowledge that it was the lowest bidder in the first round, $i(2)$ learns exactly what the lowest bidder bid in the first round in addition to what it bid itself. This implies that conditional on the two lowest bids being very close to each other, $i(2)$ has an information advantage over $i(1)$ in the second round. To see this, consider the case in which $i(1)$ and $i(2)$ bid almost exactly the same amount, say $\$Z$. The information revealed to $i(1)$ at the end of the first round is that $\$Z$ is the lowest bid and that it bid the lowest. The information revealed to $i(2)$, on the other hand, is that $\$Z$ is the lowest bid and that (at least) one other firm beside itself bid $\$Z$. Clearly, $i(2)$ has a bigger information set at the end of the first round.

So far, we have documented that the ordering between $i(1)$ and $i(2)$ is very persistent across rounds, while the ordering between $i(2)$ and $i(3)$ is not. Given $i(2)$’s information advantage, however, the fact that the ordering between $i(1)$ and $i(2)$ does not change is even more surprising. Once we condition on auctions in which $i(1)$ and $i(2)$ bid close to each other in Round 1, $i(2)$ should be aware that by bidding a little more aggressively, it can beat $i(1)$ in the next round with high probability. Hence, given the informational advantage of $i(2)$, we would normally expect the order of $i(1)$ and $i(2)$ to flip more, and not less, frequently than 50% under competitive behavior. Hence, the persistence in the identity of the lowest bidder seems at odds with competitive behavior.

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26 Note that $b_{i(3)}^1 \geq b_{i(2)}^1 \geq b_{i(1)}^1$, by construction. Hence, $b_{i(3)}^1 - b_{i(1)}^1 < 0.01$ implies $b_{i(2)}^1 - b_{i(1)}^1 < 0.01$ and $b_{i(3)}^1 - b_{i(2)}^1 < 0.01$.  

17
Persistence in the Third Round  For the subset of auctions that go to the third round, we can further examine whether a similar pattern continues to hold in the third round. In the top two panels of Figure 3, we plot the difference in the third-round bids of $i(1)$ and $i(2)$, i.e., $\Delta^3_{12} \equiv b^3_{i(2)} - b^3_{i(1)}$ (left panel), and the difference in the third-round bids of $i(2)$ and $i(3)$, i.e., $\Delta^3_{23} \equiv b^3_{i(3)} - b^3_{i(2)}$ (right panel) for all auctions that advance to the third round. In rows two to four of Figure 3, we plot the histogram conditioning on the set of auctions in which the first-round bids were sufficiently close. Focusing on the left panels, the second row plots $\Delta^3_{12}$ for the set of auctions in which $b^1_{i(2)} - b^1_{i(1)} < 0.05$; the third row plots $\Delta^3_{12}$ for which $b^1_{i(2)} - b^1_{i(1)} < 0.01$; and the last row plots $\Delta^3_{12}$ for which $b^1_{i(3)} - b^1_{i(1)} < 0.03$. Similarly, the second through the fourth panels in the right column plot $\Delta^3_{23}$ for the set of auctions in which $b^1_{i(3)} - b^1_{i(2)} < 0.05$, $b^1_{i(3)} - b^1_{i(2)} < 0.01$ and $b^1_{i(3)} - b^1_{i(1)} < 0.03$, respectively.

4.2 Discontinuity of the distribution of $\Delta^2_{12}$ at Zero

One striking feature of the distribution of $\Delta^2_{12}$ (and $\Delta^3_{12}$) is that there is what appears to be a discontinuous jump at exactly zero. This is in stark contrast to the distribution of $\Delta^2_{23}$ (and $\Delta^3_{23}$), which is symmetric and continuous around zero. We argue that this discontinuous jump in the distribution of $\Delta^2_{12}$ is further evidence that the persistence cannot have been generated by competitive behavior.

Consider, first, the distribution of $\Delta^2_{23}$ in the right panels of Figure 2. Note that, even among bidders that submit almost identical first-round bids, there is a certain amount of variance in $\Delta^2_{23}$. To the extent that these bids are generated under competitive behavior, this seems to indicate that for many auctions, there is a reasonable amount of idiosyncrasy among the bidders with regard to the beliefs over the distribution of the reserve price, risk preference, etc., inducing variance in the second-round bids. In other words, idiosyncratic reasons seem to induce at least a certain amount of uncertainty in the second-round bidding for many auctions even among bidders that submit almost identical first-round bids.

Now consider the distribution of $\Delta^2_{12}$ in the left panels of Figure 2. As long as there exists a reasonable amount of idiosyncrasy among the bidders, $i(2)$ should outbid $i(1)$ in the second round by a narrow margin just as often as $i(1)$ outbids $i(2)$ by a narrow margin. That is, there should be a similar number of observations in which $\Delta^2_{12} \in [-t, 0]$ and $\Delta^2_{12} \in [0, t]$ for small values of $t$ – a feature which we clearly do not see in any of the histograms of the left panels of Figure 2. This is inconsistent with competitive behavior.
Figure 3: Difference in the Third-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Third-Round Bids of $i(2)$ and $i(3)$ (Right Panels). The first row corresponds to all auctions that reached the third round and $i(1)$ and $i(2)$ (in the case of the left panel) or $i(2)$ and $i(3)$ (in the case of the right panel) submitted valid bids in the third round. The second to fourth rows plot the same histogram, but only for auctions in which the differences in the first-round bids are relatively small.

In fact, the discreteness exhibited in the histogram of $\Delta^2_{12}$ at zero suggests that the bidders know exactly how the other bidders will bid in the second round. If, on the contrary, $i(1)$ and $i(2)$ were both uncertain about each other’s bid, there should be just as many cases where $i(2)$ won by a tiny margin as cases where $i(2)$ lost by a tiny margin. Hence, the discontinuity of $\Delta^2_{12}$ suggests that the bidders have prior knowledge about how each other will bid and that $i(2)$ is deliberately losing by submitting a slightly higher bid than $i(1)$ (rather than winning by slightly underbidding $i(1)$).
Regarding whether ring members can achieve such coordination without communication, it seems unlikely. There is large heterogeneity in project size, specification, etc., between auctions. This makes it hard for bidders to predict a particular price that could serve as an obvious anchor of tacit (i.e., no communication) collusion, in general. Therefore, the observed bid pattern seems to indicate communication.\textsuperscript{27}

Note that our findings also suggest that bidding rings communicate beforehand how each ring member should bid in the second round – not just how to bid in the first round. This is natural given that a substantial fraction of auctions go to the second round and that there are only 30 minutes between rounds. In fact, this is consistent with the feature of bidding rings documented in court rulings (See, e.g., Nagoya District Court, No. 1903 (Wa), 1995).

4.3 Optimality of Second-Round Bidding Strategy

Recall that there are many cases in which \( i(2) \) could have outbid \( i(1) \) in the second round by lowering its second-round bid by a tiny margin. For example, focusing on the left panel of the second row in Figure 2, we find that about 15.75\% and 38.67\% of the distribution lies within \([0, 0.01]\) and \([0, 0.02]\), respectively. On the other hand, the probability that the distribution lies to the left of zero is only 1.73\%. This suggests that \( i(2) \) can increase the probability of outbidding \( i(1) \) substantially by decreasing its bid only slightly, raising the question of whether \( i(2) \)'s second-round bid is optimal.\textsuperscript{28}

Based on this observation, we formally test whether the observed bidding pattern is consistent with a competitive equilibrium in this section. While we have not characterized the equilibrium under competitive bidding, it is still possible to test whether the observed bids are consistent with a competitive equilibrium. A necessary condition for a competitive equilibrium is that each firm bids optimally given the strategies played by everybody else. Thus, if we can find an alternative bidding strategy for one of the bidders which yields higher expected profits for that bidder, it implies that the observed bidding patterns are inconsistent with competitive behavior. We apply this argument to the second-round bidding strategy employed by bidders who barely lose in the first round. In particular, we show that bidders who barely lose in the first round can substantially increase their expected profits by decreasing their second-round bid by a small margin.

\textsuperscript{27}But see Section 5 for an example of a bidding ring which used the first-round lowest bid as an anchor.

\textsuperscript{28}Strictly speaking, \( i(2) \) does not know that it came in second at the time of rebidding (it only learns that it came close to being first). The analysis below takes this into consideration.
The key idea behind our analysis in this section is that the firm’s third-round bid can provide an upper bound on its costs under private values. Using this idea, we can compute a lower bound on the bidder’s profits from playing an alternative bidding strategy in the second round without fully characterizing the equilibrium. Our approach is similar in spirit to Haile and Tamer (2003) in which the authors obtain an upper bound on the value of bidders in an incomplete model of English auctions using the fact that bidders do not bid above their value.

In what follows, we compare the expected profits from using the current second-round strategy and the expected profits from using an alternative second-round strategy for bidders that barely lose in the first round. The alternative strategies that we consider are of the form, \( x b^2_i \), where \( x \) is some number less than 1 (e.g., 0.99) and \( b^2_i \) is the bidder’s current (unnormalized) second-round strategy. Just for this section, we work with the raw bids without normalizing by the reserve price. We show below that, for a range of values of \( x \), the expected profits actually increase.

First, consider bidder \( i \)’s expected profits from using the current strategy, \( b^2_i \), conditional on advancing to the second round. The expected profits consist of two components, the expected profits from winning in the second round and the expected profits from being the lowest bidder in the third round if the auction advances to the third round. We denote by \( W^2 \) the event that bidder \( i \) wins in the second round and \( W^3 \) the event that bidder \( i \) is the lowest third round bidder,

\[
W^2 = \{ b^2_i < \min\{ r, \min_{j \neq i} b^2_j \} \}
\]

\[
W^3 = \{ b^3_i < \min_{j \neq i} b^3_j \land \min_{j \neq i} b^2_j > r \},
\]

where \( r \) is the secret reserve price and \( b^3_i \) is the current third-round bidding strategy. Note that \( W^3 \) includes both the event that bidder \( i \)’s third round bid, \( b^3_i \), is below \( r \) as well as the event that it is above \( r \). We now express bidder \( i \)’s expected profits under \( b^2_i \):

\[
\pi_i(c_i, b^1_i, \min_j b^1_j) = \Pr(W^2)(E[b^2_i - c_i|W^2]) + \Pr(W^3) E[\text{profits}|W^3],
\]

where \( c_i \) is bidder \( i \)’s costs. The expected profits in event \( W^3 \) is either \( b^3_i - c_i \), if \( b^3_i \) is lower than \( r \), or some number less than \( r - c_i \) (which depends on how the negotiation plays out), if \( b^3_i \) is higher than \( r \).\(^{29}\) In either case, the expected profits in event \( W^3 \) is less than

\(^{29}\)The negotiated price after the bargaining stage is always less than \( R \) (and, hence, less than \( b^3_i \)) because
\( \mathbb{E}[b_i^3|W^3] \). Thus, we can bound the above expression from above as follows,

\[
\pi_i(c_i, b_i^1, \min_j b_j^1) \leq \Pr(W_2)\mathbb{E}[b_i^2 - c_i|W_2] + \Pr(W^3) \mathbb{E}[b_i^3|W^3].
\]

This bound is not very tight, because we are setting bidder \( i \)'s costs equal to zero for event \( W^3 \).

Now consider the expected profits, \( \bar{\pi}_i \), from an alternative second-round bidding strategy which discounts current second-round bids by some factor \( x \in (0, 1) \). As before, \( \bar{\pi}_i \) consist of two components, the expected profit from the second round as well as the expected profit from the third round,

\[
\bar{\pi}_i(c_i, b_i^1, \min_j b_j^1) = \Pr(\tilde{W}^2)\mathbb{E}[b_i^2 - c_i|\tilde{W}^2] + \mathbb{E}[\text{third round profits}],
\]

where \( \tilde{W}^2 \) is the event in which bidder \( i \) wins in the second round using strategy \( xb_i^2 \), i.e., \( \{xb_i^2 < \min\{r, \min_{j\neq i} b_j^2\}\} \). Because we are only interested in obtaining a lower bound for \( \pi'_i \), it is not necessary to specify \( \mathbb{E}[\text{third round profits}] \) other than to say that it is nonnegative. We obtain the lower bound on \( \pi'_i \) as follows:

\[
\bar{\pi}_i(c_i, b_i^1, \min_j b_j^1) \geq \Pr(\tilde{W}^2)\mathbb{E}[xb_i^2 - c_i|\tilde{W}^2].
\] (1)

Expression (1) is derived by setting \( i \)'s expected profits equal to zero if the auction goes to the third round.

We now compare the change in expected profits, \( \Delta \pi \), from bidding \( xb_i^2 \) instead of \( b_i^2 \). Using the bounds obtained above, \( \Delta \pi \) can be bounded below as follows:

\[
\Delta \pi = \bar{\pi}_i - \pi_i \geq \Pr(\tilde{W}^2 - W^2)\mathbb{E}[xb_i^2|\tilde{W}^2 - W^2] - \Pr(\tilde{W}^2 - W^2)\mathbb{E}[c_i|\tilde{W}^2 - W^2]
- \Pr(W^2)\mathbb{E}[(1 - x)b_i^2|W^2] - \Pr(W^3)\mathbb{E}[b_i^3|W^3],
\] (2)

where \( \tilde{W}^2 - W^2 = \tilde{W}^2 \cap (W^2)^C \).\(^{30}\) \( \tilde{W}^2 - W^2 \) is the event in which bidder \( i \) wins in the second round with \( xb_i^2 \) but not with \( b_i^2 \). Because we consider \( x \in (0, 1) \), we have \( \tilde{W}^2 \supseteq W^2 \). Note that the potential gain from using strategy \( xb_i^2 \) instead of \( b_i^2 \) occurs only in event \( \tilde{W}^2 - W^2 \), and the amount of gain is \( (xb_i^2 - c_i) \).\(^{31}\) The first two terms of expression

\(^{30}\)\( R \) is also the threat point for the auctioneer in the negotiation.

\(^{31}\)To derive expression (2), note that \( \Pr(\tilde{W}^2)\mathbb{E}[xb_i^2 - c_i|\tilde{W}^2] = \Pr(\tilde{W}^2 - W^2)\mathbb{E}[xb_i^2 - c_i|\tilde{W}^2 - W^2] + \Pr(W^2)\mathbb{E}[xb_i^2 - c_i|W^2] \).

\(^{31}\)If \( c_i \) is higher than \( xb_i^2 \), this will not be a gain, but a loss.
(2) correspond to the gain. The third term of (2) corresponds to the loss in bidder $i$’s profits in event $W^2$. In event $W^2$, using $x b_i^2$ is less profitable to $b_i^2$ because bidder $i$ is already winning with a bid of $b_i^2$.

All of the terms in expression (2) except for $E[c_i|\bar{W}^2 - W^2]$ can be evaluated directly from the data in the sense that sample analogues can be constructed. For example, for any given value $x$, $E[x b_i^2|\bar{W}^2 - W^2]$ can be evaluated by taking the sample average of $x b_i^2$ for auctions in which bidder $i$ did not win in the second round, but would have won if it bid $x$ (e.g., 0.99) of the original bid. The only term that we cannot evaluate directly is $E[c_i|\bar{W}^2 - W^2]$ because we do not know $c_i$. However, under the private values assumption, it turns out that we can bound this term using bidder $i$’s third round bids. We discuss this issue next.

Recall that $\bar{W}^2 - W^2$ corresponds to the event in which bidder $i$ wins in the second round with $x b_i^2$ but not with $b_i^2$. The event $\bar{W}^2 - W^2$ includes two possibilities, one in which $r$ happens to be below the lowest second round bid ($\{r < \min_j b_j^2\}$), and the other in which $r$ happens to be above the lowest second-round bid ($\{r \geq \min_j b_j^2\}$). Figure 4 depicts the two situations. Note that for Case 1, the auction proceeds to the third round, and we observe $b_i^3$. Hence, we can bound $c_i$ from above by the observed third round bid, $b_i^3$ in this case. For Case 2, however, the auction ends in the second round, and we do not observe third round bids.

In order to bound costs for the second case, note that whether the auction proceeds to the third round depends, in part, on the random (from the bidders’ perspective) realization

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32For example, $\Pr(W^2|E_x)$ can be evaluated by computing the proportion of auctions that $i$ wins in the second round conditional on being outbid by less than $\varepsilon$ by the lowest bidder in the first round. Similarly, for any given value $x$, $E[(1-x)b_i^2|W^2 \cap E_x]$ can be evaluated by taking the sample average of $(1-x)b_i^2$ for auctions in which (1) $i$ was outbid by less than $\varepsilon$ by the lowest bidder in the first round and (2) $i$ won in the second round.
of \( r \). That is, whether or not the auction ends in the second round is, to some extent, independent of the bidders’ costs. Lemma 1 makes this statement precise; it establishes that conditional on \( b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j \), whether or not the auction goes to the third round is independent of bidder \( i \)'s costs. In other words, if we have two auctions with the same realizations of \( b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j \), but one ending in the second round and the other proceeding to the third round, bidder \( i \)'s costs must be the same in the two auctions.\(^{33}\) This means that we can bound bidder \( i \)'s costs for auctions that end in the second round (i.e., Case 2 in Figure 4) by using the third round bid of auctions that go to the third round (conditional on \( b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j \)).

**Lemma 1** Assume that bidders have private costs \( c = (c_1, \cdots, c_N) \) with density \( g(c) \); that the secret reserve price, \( r \), is distributed \( F_r(\cdot) \), and that \( c \perp r \). Then,

\[
\mathbb{E}[c_i|\tilde{W}^2 - W^2 \cap \{r \geq \min_j b^2_j\}, b^1_i, b^2_i, \min_j b^2_j, \min_j b^1_j]
\]

\[
= \mathbb{E}[c_i|b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j]
\]

\[
= \mathbb{E}[c_i|\tilde{W}^2 - W^2 \cap \{r < \min_j b^2_j\}, b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j].
\]

(3)

Moreover,

\[
\mathbb{E}[c_i|\tilde{W}^2 - W^2 \cap \{r \geq \min_j b^2_j\}]
\]

\[
\leq \mathbb{E}[\mathbb{E}[b^1_i|\{r < \min_j b^2_j\}, b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j]|\tilde{W}^2 - W^2 \cap \{r \geq \min_j b^2_j\}].
\]

(4)

**Proof.** See Appendix. \( \blacksquare \)

The first part of the Lemma states that the expected cost of bidder \( i \) in Case 1 is the same as the expected cost of bidder \( i \) in Case 2, conditional on \( b^1_i, \min_j b^1_j, b^2_i, \min_j b^2_j \). It turns out that it is necessary to condition the expectation on \( b^1_i \) and \( \min_j b^1_j \) in addition to \( b^2_i \) and \( \min_j b^2_j \) because \( b^1_i \) and \( \min_j b^1_j \) are in the information set of bidder \( i \) in the second round. The second part of the Lemma states that we can bound bidder \( i \)'s costs, \( c_i \), in Case 2 by the observed third round bid in Case 1: Note that the inner expectation on the right hand side of (1) is conditional on the event \( \{r < \min_j b^2_j\} \), whereas the outer expectation

\(^{33}\)More precisely, the distribution of \( c_i \) must be the same in two auctions. If bidder \( i \) plays mixed strategies, the set of values \( c_i \) for which bidder \( i \) bids a particular amount \( b^1_i, b^2_i \) may not be a singleton.
is conditional on the event \( \{ r \geq \min_j b_j^2 \} \). Note that the second part of Lemma 1 is a direct consequence of the first part: We can obtain expression (1) by considering the inequality obtained by replacing \( c_i \) in the second line of expression (1) with \( b_3^i \) and integrating both sides with respect to \( b_1^i, \min_j b_j^1, b_2^i, \min_j b_j^2 \) conditional on \((\tilde{W}^2 - W^2) \cap \{ r \geq \min_j b_j^2 \}\).

The reason why the Lemma is useful is because the right hand side of expression (1) can be computed using observed data. The inner expectation can be obtained by regressing \( b_3^i \) on \( b_1^i, \min_j b_j^1, b_2^i, \min_j b_j^2 \) for the subset of auctions such that \( \{ r < \min_j b_j^2 \}\). Note that \( b_3^i \) is observed for this set of auctions. We then take the regression coefficients on \( b_1^i, \min_j b_j^1, b_2^i, \min_j b_j^2 \) and use them to predict the value of \( b_3^i \) for auctions such that \((\tilde{W}^2 - W^2) \cap \{ r \geq \min_j b_j^2 \}\). The average of the predicted value of \( b_3^i \) for this sample corresponds to the outer expectation on the right hand side of expression (1).

Before we present our results, we briefly discuss the two assumption of Lemma 1, namely, that bidders have private values and that costs and the reserve price are independent. We start with the private values assumption. By assuming that bidders have private values, \( c_i \) becomes constant throughout the three rounds ensuring that \( b_3^i \) is a valid upper bound for \( c_i \) at round two. If, instead, bidders have common values, bidders may update their costs in the third round based on the observed lowest second-round bid. If the bidders revise their cost estimates downward in the third round, \( b_3^i \) may no longer be an upper bound on the costs of bidder \( i \) perceived at the time of the second round. For our purposes, whatever assumption – private values or otherwise – that guarantees that \( b_3^i \) is a valid upper bound for \( c_i \) at the time of the second round suffices. That is, even if bidders have common values, we would expect \( b_3^i \) to be a valid upper bound on \( c_i \) at round two for mild forms of interdependence. In this case, our argument would go through without any problems.

Next we discuss the independence of \( c \) and \( r \). One might be inclined to argue that the independence assumption is violated based on, for example, the observation that \( c \) and \( r \) are both low for simple jobs (e.g., road paving) and that they are both high for complicated jobs (e.g., bridges). This is not a valid argument. In this example, the costs and the reserve price are being drawn from two separate distributions, one for simple jobs and the other for complicated jobs. Our assumption requires that \( c \) and \( r \) be independent only among auctions for which costs and the reserve price are being drawn from some common distribution.

We are now ready to evaluate \( \Delta \pi \), the difference in expected profits from using \( xb_i^2 \).
instead of $b_i^2$ in the second stage. Using expressions (2) and (1), we obtain the following bound

\[
\Delta \pi \geq \Pr(\tilde{W}^2 - W^2)E[xb_i^2|\tilde{W}^2 - W^2] - \Pr(W^2)E[(1-x)b_i^2|W^2] - \Pr(W^3)E[b_i^3|W^2] - E[E[b_i^3]\{r < \min_j b_j^2\}, b_i^1, \min_j b_j^1, b_i^2, \min_j b_j^2]|(\tilde{W}^2 - W^2) \cap \{r \geq \min_j b_j^2\}].
\]

As we explained earlier, we can construct sample analogues of all terms on the right hand side of (4.3) and evaluate them using data.\(^{35}\) If the right hand side is strictly positive, (4.3) implies that $\Delta \pi > 0$, which contradicts that bidders are playing according to a competitive equilibrium.

We present our results in Table 3. The table presents the estimated values of $\Delta \pi$ for different values of $x$ ($x \in \{99\%, 98.5\%, 98\%, 97.5\%\}$) and for different values of $\delta$ ($\delta \in \{1\%, 3\%, 5\%\}$), where $\delta$ corresponds to how close bidder $i$’s first-round bid is to the lowest first-round bid. Note that inequality (4.3) still holds even when we condition on the event that bidder $i$ loses by less than some margin $\delta$ in the first round ($b_i^1 - \min_j b_j^1 < \delta \min_j b_j^1$). This allows us to focus on just the set of bidders who narrowly lost in the first round. Hence, each cell in Table 3 represents the expected gain in profits from using $xb_i^2$ instead of $b_i^2$ for a bidder who loses the first round by less than $\delta$. Note that all of the cells in Table 3 are positive (and statistically significant at 95% except for $(x, \delta) = (98.5\%, 1\%)$), implying that firms would gain by decreasing their bid by a small margin. In terms of magnitude, the numbers seem quite large, considering how loose our inequality is. For example, looking at $(x, \delta) = (97.5\%, 1\%)$, we see that the bidder can increase its expected profit by more than 2 million yen by decreasing its second-round bid by 1%. Relative to the mean reserve price of 81 million yen for auctions that proceed to the second round (See Table 1), this seems substantial. Our results suggest that bidders are not bidding competitively.

Before we conclude this section, we make one final remark. It is possible to conduct the same exercise conditional on various observables such as year, location, project type, auction category, etc., because inequality (4.3) holds even when we condition on these observables. Note, however, that the conditional test is weaker than the unconditional test. If inequality (4.3) is violated unconditionally, then it must be violated conditionally for

\(^{35}\)We compute the inner conditional expectation of $b_i^3$ (second line of (4.3)) by a linear regression of $b_i^3$ on $b_i^1, \min_j b_j^1, b_i^2, \min_j b_j^2$, for auctions such that $\{r < \min_j b_j^2\}$. We then predict the value of $b_i^3$ for auctions such that $(\tilde{W}^2 - W^2) \cap \{r \geq \min_j b_j^2\}$ using the estimated regression coefficients. The average of the predicted values corresponds to the second line of (4.3). We compute all the other terms of (4.3) using the sample frequencies or the sample means.
Mean $\Delta \pi$ (Yen)

<table>
<thead>
<tr>
<th>$x$</th>
<th>99.0%</th>
<th>98.5%</th>
<th>98.0%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = 1%$</td>
<td>1,150,355</td>
<td>1,251,340</td>
<td>1,860,170</td>
<td>2,060,583</td>
</tr>
<tr>
<td></td>
<td>(217,830)</td>
<td>(1,038,032)</td>
<td>(630,095)</td>
<td>(500,323)</td>
</tr>
<tr>
<td>$3%$</td>
<td>455,847</td>
<td>711,004</td>
<td>1,106,434</td>
<td>1,334,383</td>
</tr>
<tr>
<td></td>
<td>(153,421)</td>
<td>(298,644)</td>
<td>(257,065)</td>
<td>(237,663)</td>
</tr>
<tr>
<td>$5%$</td>
<td>331,728</td>
<td>589,454</td>
<td>921,495</td>
<td>1,089,442</td>
</tr>
<tr>
<td></td>
<td>(109,105)</td>
<td>(191,652)</td>
<td>(178,324)</td>
<td>(201,520)</td>
</tr>
</tbody>
</table>

Note: Samples are conditioned in the way that the percentage difference between $i$’s first-round bid and the lowest first-round bid is less than $\delta$, i.e., $(b_i - \min_j b_j) / \min_j b_j < \delta$. The numbers in parentheses are the standard deviations.

Table 3: Expected gain in profits from using $xb_i^2$ for bidders that lose by less than $\delta$ in the first round.

some of the conditioning variables.

5 Case Study

In this section, we analyze four collusion cases that were implicated by the JFTC during our sample period. The four cases that we examine are the bidding ring of (A) prestressed concrete providers; (B) firms installing traffic signs; (C) builders of bridge upper structure; and (D) floodgate builders.\(^{36}\) In all of these cases, firms were found to have engaged in activities such as deciding on a predetermined winner for each project and communicating among the members how each bidder will bid.\(^{37}\) All of the implicated firms in cases (B), (C) and (D) admitted wrongdoing soon after the start of the investigation, but none of the firms implicated in case (A) admitted any wrongdoing initially, and the case went to trial.\(^{38}\)

Before we analyze these four cases, we point out one interesting feature of the bidding ring in case (A): According to the ruling in case (A), an internal rule existed among the

\(^{36}\)See JFTC Recommendation #27-28 (2004) and Ruling #26-27 (2010) for case (A); JFTC Recommendation and Ruling #5-8 (2005) for case (B); JFTC Recommendation and Ruling #12 (2005) for case (C); and JFTC Cease and Desist Order #2-5 (2007) for case (D).

\(^{37}\)The ring members took turns being the predetermined winner. The determination of who would be the predetermined winner depended on factors such as whether a given firm has an existing project that is closely related to the auction in question and the number of auctions a given firm has won in the past.

\(^{38}\)Out of 20 firms that were initially implicated in Case (A), one firm was acquired by another firm, one was acquitted, and the rest of the firms eventually settled with the JFTC after going to trial.
subset of the ring members operating in the Kansai region, which prescribed that 1) the predetermined winner should aim to bid below the reserve price in the first round; 2) if the predetermined winner did not bid below the reserve price in the first round, the predetermined winner should submit a second-round bid that is less than some prespecified fraction (e.g., 97%) of its first-round bid (e.g., \( b_{i(1)}^2 < 0.97 \times b_{i(1)}^1 \)); and 3) the rest of the ring members should submit second-round bids that are higher than the prespecified fraction of the predetermined winner’s first-round bid (e.g., \( b_{i(k)}^2 > 0.97 \times b_{i(1)}^1 \) for \( k \geq 2 \)). The prespecified fraction used in the ring was 96% for auctions with an expected value less than 100 million yen, 97% for auctions with an expected value between 100 million yen and 500 million yen, and 97.5% for auctions expected to worth more than 500 million yen.\(^{39}\) One consequence of this internal rule is that we would observe the same lowest bidder in Round 1 and Round 2.

In Figure 5, we plot the winning bid (lowest bid of the concluding round as a percentage of the reserve price) against the calender date for all auctions in which the winner is a member of one of the implicated bidding rings. We have also drawn a vertical line that corresponds to the “end date” of collusion. The “end date” is the date in the JFTC’s ruling after which the ring members were deemed to have stopped colluding. Note that in panels (B) and (C) of Figure 5, there exist periods after the collusion end date during which no ring member wins an auction. This reflects the fact that implicated ring members in cases (B) and (C) were banned from participating in public procurement projects for a period of up to 18 months.\(^{40}\)

We see that for cases (B), (C), and (D), there is a general drop in the winning bid of about 8.3%, 19.5%, and 5.3%, respectively, after the collusion end date. However, there is almost no change in the winning bid for case (A) before and after the end date. Also, it is worth mentioning that, even for cases (B), (C), and (D), there are some auctions in which the winning bid is extremely high after the end date. In fact, about 24.4% of auctions after the end date have a winning bid higher than 95% for cases (B), (C) and (D). While the investigation and the ruling of the JFTC seemed to have made collusion harder, it is far from clear whether the prices after the end date are truly at competitive levels. Hence, the

\(^{39}\)There is evidence that ring members actively communicated with each other on what the prespecified fraction should be. For example, a memo which was obtained by the JFTC from one of the ring members records a discussion among the members over the prespecified fraction. According to the memo, one of the members wanted to use different fractions for auctions worth, for example, 10 million yen and 99 million yen (JFTC Ruling #27 (2010)).

\(^{40}\)The ring members involved in cases (A) and (D) were banned from bidding in procurement auctions for certain periods in 2010 and 2007, respectively.
price drops that we see in Figure 5 may be a conservative estimate of the effect of collusion. We discuss this point more below.

We now examine the second-round bids of \( i(1), i(2), \) and \( i(3) \) during the period in which the firms were colluding. If the distinctive shapes of the distribution of \( \Delta_{12}^2 \) and \( \Delta_{23}^2 \) that we found in Section 4 are indeed evidence of collusion, we should expect to see the same pattern among the second-round bids of these colluding firms. Figure 6 plots the histogram of \( \Delta_{12}^2 \) and \( \Delta_{23}^2 \) before the collusion end date for each of the four bidding rings. The samples used for the figure correspond to the set of auctions in which \( b_{1i(2)}^1 - b_{1i(1)}^1 < 5\% \) for the left column and \( b_{1i(3)}^1 - b_{1i(2)}^1 < 5\% \) for the right column, i.e., \( \epsilon = 0.05 \). We see that
Figure 6: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels) Before the Collusion End Date. We use $\varepsilon = 0.05$; hence, the differences in the first-round bids are relatively small.

for all four bidding rings, $\Delta_{12}^2$ is asymmetric around zero, while $\Delta_{23}^2$ is symmetric around zero, as before. Thus, Figure 6 suggests that the distinctive shapes of the distributions of $\Delta_{12}^2$ and $\Delta_{23}^2$ are a hallmark of collusive bidding.

We next examine the second-round bids of the ring members, but for auctions occurring after the collusion “end date.” To the extent that ring members stopped colluding after the “end date,” we should expect to see the distribution of $\Delta_{12}^2$ distributed to the left of zero. Figure 7 plots the histogram of $\Delta_{12}^2$ and $\Delta_{23}^2$ for each of the four bidding rings with $\varepsilon = 5\%$. Although the sample size is very small, the distribution of $\Delta_{12}^2$ and $\Delta_{23}^2$ in Figure 7 are similar to those in Figure 6. That is, $\Delta_{12}^2$ is distributed to the right of zero while $\Delta_{23}^2$ is
Figure 7: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels) After the Collusion End Date. We use $\varepsilon = 0.05$; hence, the differences in the first-round bids are relatively small.

distributed symmetrically around zero. This may seem to cast doubt on our analysis – why do the distinctive patterns in the distribution of $\Delta^2_{12}$ and $\Delta^2_{23}$ persist even after the collusion end date, when firms presumably started behaving competitively?

Our view is that asymmetry in the distribution of $\Delta^2_{12}$ should be taken as evidence that firms may have been able to continue colluding at least on some auctions even after the “end date.” While the bidding rings seem to have changed their behavior around the time of the “end date,” – as the drop in the winning bid suggests in Figure 5 – this does not necessarily mean that the firms completely ceased to collude. For example, in the ruling on case (A) issued in 2010, more than five years after the start of the investigation,
the judges ordered the ring members, among other things, to take various measures to prevent collusion from recurring. This is because the judges determined that there were still circumstances conducive to collusion even after the “end date” and that ring members needed to take steps to ensure that they do not collude. Moreover, many firms that were implicated in these cases are repeat offenders. For example, one firm involved in case (A) had been found guilty in four previous collusion cases. A number of firms implicated in case (C) were also subsequently charged and found guilty of collusion in a separate case by the JFTC. It seems that being implicated by the JFTC is no guarantee that a firm will behave competitively thereafter; firms may have been able to continue colluding well beyond the “end date,” at least for some auctions.

With respect to case (A), there is additional evidence that the ring members continued to collude beyond the end date, by following the formula for rebids that we described earlier. Recall that a subset of the prestressed concrete ring members in the Kansai region had a prespecified discount (96% for auctions valued at less than 100 million yen, 97% for auctions valued between 100 million yen and 500 million yen, and 97.5% for auctions valued at more than 500 million yen) that they used when rebidding in the second round. Figure 8 plots the second-round bids of the ring members in the Kansai region as a fraction of the lowest first-round bid. The top panel corresponds to auctions with a reserve price below 100 million yen; the middle corresponds to those with a reserve price between 100 and 500 million yen; and the last panel corresponds to those with a reserve price of more than 500 million yen. The horizontal axis in the figure corresponds to the calendar date. The vertical line in each panel corresponds to the collusion end date. Thus, auctions that took place before the end date appear to the left of this line. The circles represent $b_{i(1)}^2/b_{i(1)}^1$, and the Xs represent $b_{i(k)}^2/b_{i(1)}^1$ for $k \geq 2$. We have drawn a horizontal line at 96% (top panel), 97% (middle panel), and 97.5% (bottom panel).

While the top and the bottom panels are not very informative, note that all of $i(1)$’s second-round bids in the middle panel of Figure 8 are below 97% of $i(1)$’s first-round bid. Moreover, the bids of all of the others are above 97% of $i(1)$’s first-round bid, except for one auction. If we focus on auctions after the collusion end date, the second-round bids of $i(k)$ ($k \geq 2$) are all above 97%. The bidding pattern in Figure 8 suggests that bidders continued to use the prespecified discount as the threshold value for submitting second-

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41JFTC Ruling #26-27 (2010). In the ruling, the firms were ordered to take preventative measures such as periodic auditing by a legal officer, etc.
round bids. It seems quite likely that the ring members were able to maintain collusion even after the “end date.”

6 Detection of Collusive Bidders

In this section, we develop a formal statistical test of collusive behavior based on the idea we discussed in Section 4.2, namely, the distribution of $\Delta_{12}^2$ should not be discontinuous at zero under competitive bidding. We then apply our test to each firm in order to examine...
whether or not its bidding behavior is consistent with competitive bidding.

**Test Statistic** Recall from Section 4.2 that there is a reasonable amount of variance in $\Delta^2_{23}$ even among bidders that submit almost identical first-round bids. To the extent that bids are generated by competitive behavior, this means that there is a reasonable amount of bidder-specific idiosyncrasy with regard to the beliefs over the distribution of the reserve price, risk preference, etc., that induce variance in the second-round bids. This, in turn, implies that $i(1)$ cannot be outbidding $i(2)$ in the second round by a small margin all the time under competitive bidding. If $i(1)$ wins some, it has to lose some. Thus, the amount of idiosyncrasy measured by the variance of $\Delta^2_{23}$ puts a bound on how sharply the distribution of $\Delta^2_{12}$ can change around zero. The test statistic that we propose below formalizes this idea by looking for violations of this bound.

We begin by specifying the second-round bids of $i(2)$ and $i(3)$ as follows:

\[
\begin{align*}
    b^2_{i(2)} &= X + u_2 \\
    b^2_{i(3)} &= X + u_3,
\end{align*}
\]

where $X$ is a common component, and $u_2, u_3$ are bidder-specific idiosyncratic shocks distributed independently and identically according to $F_u$. As long as we condition on auctions in which the first-round bids of $i(2)$ and $i(3)$ are close enough, this specification seems natural: Both $i(2)$ and $i(3)$ should have similar cost structures and similar information, which is captured in the common component, $X$. Note that $X$ is a random variable whose distribution can arbitrarily depend on the object being auctioned, information revealed in the first round, etc. Basically, $X$ captures all observed and unobserved common factors between $i(2)$ and $i(3)$. The error terms, $u_2$ and $u_3$, are independent bidder-specific idiosyncrasies that result from differences in the bidders’ beliefs over the secret reserve price, heterogeneity in the bidders’ risk preferences, etc. We assume that $u_2$ and $u_3$ are independent of $X$. Now, given that $\Delta^2_{23}$ is just the difference between $b^2_{i(3)}$ and $b^2_{i(2)}$, we have

\[
\Delta^2_{23} \equiv b^2_{i(3)} - b^2_{i(2)} = u_3 - u_2.
\]

Given our i.i.d. assumptions on $(u_2, u_3)$, we can recover $F_u$ from realizations of $\Delta^2_{23}$.

We now consider putting bounds on the distribution of $\Delta^2_{12}$ using $F_u$. Let us denote by
Y the second-round bid of \(i(1)\):
\[ b_{i(1)}^2 = Y. \]

Given that \(i(1)\) has a different information set than all of the other bidders (as well as, perhaps, having different costs), we do not impose any restrictions on the distribution of \(Y\) other than independence with respect to \((u_2, u_3)\); i.e., \(Y \perp (u_2, u_3)\). In particular, \(Y\) can have arbitrary correlation with respect to \(X\).

Note that \(\Delta_{12}^2 = X + u_2 - Y\), given that \(\Delta_{12}^2 = b_{i(2)}^2 - b_{i(1)}^2\). Now, we define \(d(t)\) \((t \in \mathbb{R}^{++})\), a measure of how discontinuous the distribution of \(\Delta_{12}^2\) is around zero:
\[
d(t) = \Pr(\Delta_{12}^2 \in [0, t]) - \Pr(\Delta_{12}^2 \in [-t, 0]).
\]

\(\Pr(\Delta_{12}^2 \in [-t, 0])\) is just the probability that \(\Delta_{12}^2\) falls within \([-t, 0]\), and \(\Pr(\Delta_{12}^2 \in [0, t])\) is the probability that \(\Delta_{12}^2\) falls within \([0, t]\). Hence, \(d(t)\) is the difference between the probability that \(\Delta_{12}^2\) falls just to the right of zero and the probability that \(\Delta_{12}^2\) falls just to the left of zero.

We can derive a simple bound on \(d(t)\) using \(F_u\) after some algebra,
\[
d(t) = \Pr(\Delta_{12}^2 \in [0, t]) - \Pr(\Delta_{12}^2 \in [-t, 0])
\]
\[
= \int 1_{\{X + u_2 - Y \in [0, t]\}} dF_{X,Y}(X,Y) dF_u(u_2)
\]
\[
- \int 1_{\{X + u_2 - Y \in [-t, 0]\}} dF_{X,Y}(X,Y) dF_u(u_2)
\]
\[
= \int F_u(Y - X + t) - F_u(Y - X) dF_{X,Y}(X,Y)
\]
\[
- \int F_u(Y - X) - F_u(Y - X - t) dF_{X,Y}(X,Y)
\]
\[
= \int F_u(Y - X + t) + F_u(Y - X - t) - 2F_u(Y - X) dF_{X,Y}(X,Y)
\]
\[
\leq \sup_x \| F_u(x + t) + F_u(x - t) - 2F_u(x) \|,
\]

where the second line uses independence of \(u_2\) with respect to \(X\) and \(Y\) and \(F_{X,Y}\) is the joint cumulative distribution function of \(X\) and \(Y\).

\(^{44}\)Note that our formulation incorporates specifications such as \(b_{i(1)}^2 = Y + u_1\).
Our test statistic simply compares \(d(t)\) with the bound derived from \(F_u\). Define \(\tau(t)\) as

\[
\tau(t) \equiv \sup_x \| F_u(x + t) + F_u(x - t) - 2F_u(x) \| - d(t).
\]

Given that we can estimate \(F_u\) and \(d(t)\), we can estimate \(\tau(t)\). Under the null hypothesis of competitive behavior, \(\tau(t)\) should be nonnegative.

**Detecting Collusive Bidders** We now apply this test to each firm that we observe in the data. In particular, for a given firm, we collect all auctions in which the firm participated. We then estimate \(d(t)\) and \(F_u\) parametrically, for each firm, using realizations of \(\Delta^2_{2}\) and \(\Delta^2_{23}\) from a subset of these auctions where 1) the auction proceeded to the second round; and 2) the first-round bids of \(i(2)\) and \(i(3)\) were sufficiently close to each other, i.e., \(b^1_{i(3)} - b^1_{i(2)} < \varepsilon\).\(^{45}\) We use a frequency estimator for \(d(t)\) and a maximum likelihood estimator for \(F_u\) by specifying \(F_u\) to be a mean-zero Normal distribution with parameter \(\sigma_u (u \sim N(0, \sigma_u^2))\). While our test statistic can easily accommodate a nonparametric estimate of \(F_u\), we impose functional form assumptions on \(F_u\) because the number of auctions per firm is not very large. In practice, we estimate \(\tau(t)\) for every firm that participated in at least five auctions that meet the two criteria mentioned above.\(^{46}\) Given our parametric assumption on \(F_u\), \(\tau(t)\) has an asymptotically Normal distribution.

In the top left panel of Figure 9, we plot the estimates of \(\tau(t)\) for each firm for \(t = 1\%\) and \(\varepsilon = 5\%.\) As shown in the panel, the estimated distribution of \(\tau(t)\) lies somewhat to the right of zero, but there is also a substantial mass below zero. Under the null hypothesis of competitive bidding, the value of \(\tau(t)\) should be positive; thus, a negative estimate of \(\tau(t)\) raises concerns about possible collusive behavior. In the top right panel, we plot the \(t\)-statistic for each firm. Again, we find that the estimated \(t\)-statistic is negative for a substantial fraction of firms. In particular, there are 674 firms (out of 3,998 firms) whose \(t\)-statistic is less than \(-1.65\), which is the one-sided critical value for rejecting the null hypothesis of competitive behavior at the 95\% confidence level. The set of 674 firms includes 21 firms (out of a total of 92 firms) that were implicated in one of the four bid-rigging cases.

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\(^{45}\)Note that we condition on the set of auctions where the second- and third-lowest bids in the first round are within \(\varepsilon\), given the assumptions on \(u_2\) and \(u_3\). Note, also, that we drop auctions if \(\Delta^2_{23}\) is bigger than 30\% to make sure that we exclude misrecordings, etc. This biases against finding collusion.

\(^{46}\)A total of 21,622 construction firms are observed in our analysis, among which 3,998 (3,073) firms participated in at least five auctions that proceeded to the second round with \(b^1_{i(3)} - b^1_{i(2)} < 5\%\) (\(b^1_{i(3)} - b^1_{i(2)} < 1\%\)).
and \( \varepsilon = 5\% \). The results are qualitatively similar. For this case, we find that 578 firms have a \( t \)-statistic less than \(-1.65\).

In the bottom two panels of Figure 9, we repeat the same exercise with \( \varepsilon = 1\% \). The panels in the third row correspond to \( t = 1\% \), \( \varepsilon = 1\% \), and the bottom panels correspond to \( t = 2\% \), \( \varepsilon = 1\% \). In the third row, there are 403 firms (out of 3,073 firms) whose estimated \( t \)-statistic is less than \(-1.65\), and in the fourth row, we find that 314 firms have an estimated \( t \)-statistic less than \(-1.65\).

It should be clear from the construction of the test statistic that the value of \( \tau(t) \) should be nonnegative for all values of \( t \) under competitive bidding. Hence, we next conduct a joint hypothesis test. In particular, we pick \( t = 1\% \) and \( t = 2\% \) and test whether \((\tau(1\%), \tau(2\%))\) is jointly nonnegative. Under the joint hypothesis test, we find that we can reject the null for 1,008 firms for \( \varepsilon = 5\% \) (586 firms for \( \varepsilon = 1\% \)). The joint hypothesis test for \( \varepsilon = 5\% \) picks out 25 firms out of 92 firms (27 firms for \( \varepsilon = 1\% \)) that were implicated in one of the four bid-rigging cases.

To get a sense of the magnitude of our findings, note that the total number of auctions awarded to the 1,008 “suspicious” firms that we identify (in the joint hypothesis test for \( \varepsilon = 5\% \)) is about 7,600, or close to one fifth of the total number of auctions in our sample. The total award amount of these auctions equals about $8.6 billion. Given that the four case studies show about a 8.4% average drop in the winning bid after the bidding rings were implicated, our results suggest that taxpayers could have saved about $721 million in the absence of collusion. Moreover, if we consider the fact that the total award amount of municipal and prefectural construction projects in Japan is close to ten times the total value of the auctions in our dataset, the impact of collusion can even be bigger as a whole. There is also ample reason to believe that collusion is just as rampant among municipal and prefectural construction projects, given that some of the same construction firms in our dataset participate in these auctions, as well.

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47 In practice, we estimate the joint (2-dimensional) distribution of \((\tau(1\%), \tau(2\%))\). We then simulate 500 draws of \((2 \times 1)\) random vectors according to the estimated joint distribution. We test whether there are more than 25 (= 5% of 500) draws whose elements are both positive.

48 The drop in the winning bid after the “collusion end date” was 0.6% for (A) prestressed concrete; 8.3% for (B) traffic signs; 19.5% for (C) bridge upper structure; and 5.3% for (D) floodgates. A simple average of the four numbers yields 8.4%.
Figure 9: Estimate of $\tau(t)$ (Left Panel) and $t$-Statistic (Right Panel). We estimated $\tau(t)$ for each firm using only the subset of auctions in which it participated. Top two panels plot the histogram for $t = 1\%$ and $t = 2\%$ with $\varepsilon = 5\%$. Bottom two panels plot the histogram for $t = 1\%$ and $t = 2\%$ with $\varepsilon = 1\%$.

7 Conclusion

In this paper, we document large-scale collusion among construction firms in Japan using bidding data from government procurement projects. We find evidence of collusion across regions, types of construction projects and time. We then test, for each firm, whether its bidding behavior is consistent with competitive behavior. Our test identifies about 1,000 “suspicious” firms that won a total of about 7,600 auctions, or about one fifth of the total number of auctions during our sample period.

The detection method we propose in this paper is very simple and requires only bid data. While our test is not a definitive proof of collusion, we believe that our method can
be useful for law enforcement agencies in identifying possible cases of bid rigging.

References


[16] Judicial District Court Records: Tsu District Court, No. 165 (Wa), 1997, Nagoya District Court, No. 1903 (Wa), 1995


Appendix

Proof of Proposition 1

We first prove that the distribution of bidder $i$'s cost, $c_i$, conditional on $\{b_1^i, \min_j b_1^j, b_2^i, \min_j b_2^j\}$ is independent of whether or not the auction ends in the second round. Let $g(\cdot)$ denote the density of $c_i$ and $F_r(\cdot)$ denote the distribution function of the reserve price. Note that the probability that the auction proceeds to the second round conditional on $\min_j b_1^j$ is $F_r(\min_j b_1^j)$ and the probability that the auction proceeds to the third round conditional on $\{\min_j b_1^j, \min_j b_2^j\}$ is $F_r(\min_j b_2^j)/F_r(\min_j b_1^j)$. Then,

$$g(c_i | b_1^i, \min_j b_1^j, b_2^i, \min_j b_2^j, \text{Third Round})$$

$$= \frac{\Pr(c_i, b_1^i, \min_j b_1^j, b_2^i, \min_j b_2^j, \{\min_j b_2^j > r\})}{\Pr(b_1^i, \min_j b_1^j, b_2^i, \min_j b_2^j, \{\min_j b_2^j > r\})}$$

$$= \frac{\Pr(c_i, b_1^i, \min_j b_1^j, b_2^i, \min_j b_2^j) \times F_r(\min_j b_2^j)/F_r(\min_j b_1^j)}{\Pr(b_1^i, \min_j b_1^j, b_2^i, \min_j b_2^j) \times F_r(\min_j b_2^j)/F_r(\min_j b_1^j)}$$

$$= \frac{\Pr(c_i, b_1^i, \min_j b_1^j, b_2^i, \min_j b_2^j) \times (1 - F_r(\min_j b_2^j))/F_r(\min_j b_1^j)}{\Pr(b_1^i, \min_j b_1^j, b_2^i, \min_j b_2^j) \times (1 - F_r(\min_j b_2^j))/F_r(\min_j b_1^j)}$$

$$= g(c_i | b_1^i, \min_j b_1^j, b_2^i, \min_j b_2^j, \text{Second Round}).$$

Using the fact that the event $(\tilde{W}^2 - W^2)$ is a coarser partitioning than the partition generated by $\{b_1^i, \min_j b_1^j, b_2^i, \min_j b_2^j\}$, it is easy to see that the first part of Lemma 1 is true. The
second part of Lemma 1 is a direct consequence of the first part.
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Analysis of Collusive Behavior by Region, Auction Category, Project Type, and Time

In this Appendix, we show that the shape of the distributions of $\Delta^2_{12}$ and $\Delta^2_{23}$ in Figure 1 is robust to conditioning on region, auction category, project type, and year. We also show that the shape of the distribution is robust to whether or not we normalize the bids by the reserve price. Note that for all of the figures in this section (Figures A.1 - A.5), we set $\varepsilon$ equal to 5%, i.e., the figures plot auctions in which $b_{i(2)}^1 - b_{i(1)}^1 < 5\%$ (left panels) or $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$ (right panels).

By Region

Figure A.1 plots the histogram of $\Delta^2_{12}$ and $\Delta^2_{23}$ for four of the nine regions of Japan with the largest number of auctions. The regions that we show are Hokkaido, Kanto, Kansai and Chubu, in decreasing order of number of total auctions.

By Auction Category

Figure A.2 plots the histogram of $\Delta^2_{12}$ and $\Delta^2_{23}$ for each of the four auction categories that we discussed in Section 2. Category 1 corresponds to auctions with the most restrictions on participation, and category 4 corresponds to auctions with the least restrictions.

By Project Type

In Figure A.3, we plot the histogram of $\Delta^2_{12}$ and $\Delta^2_{23}$ for the four types of projects with the largest number of auctions. The four types of projects are civil engineering, repair and maintenance, paving, and communication equipment, in decreasing order of number of total auctions.

By Year

In Figure A.4, we plot the histogram of $\Delta^2_{12}$ and $\Delta^2_{23}$, by year.
Figure A.1: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Region. The left panels plot $\Delta_{12}^2$ for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot $\Delta_{23}^2$ for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

**Raw Bids**

Finally, in Figure A.5, we plot the raw difference in the second-round bids without normalizing by the reserve price. The left panels plot the second-round bid differences of $i(1)$ and $i(2)$. The right panels plot the second-round bid differences of $i(2)$ and $i(3)$. The top panels correspond to auctions whose reserve price is between 20-22 million yen. The middle and bottom panels correspond to auctions with a reserve price between 60-66 million yen.
Figure A.2: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Auction Category. The left panels plot $\Delta_{12}$ for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot $\Delta_{23}$ for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

and 90-99 million yen, respectively.\footnote{The length of the bandwidth we use (i.e, 2 million, 6 million, and 9 million yen, respectively) is roughly 10% of the reserve price in each row.} The auctions in each row roughly correspond to the 25%, 50% and 75% quantiles in terms of project size.
Figure A.3: Difference in the Second-Round Bids of \(i(1)\) and \(i(2)\) (Left Panel) and the Difference in the Second-Round Bids of \(i(2)\) and \(i(3)\) (Right Panel), by Project Type. The left panels plot \(\Delta_{12}\) for the set of auctions in which the first-round bids of \(i(1)\) and \(i(2)\) are within 5%. The right panels plot \(\Delta_{23}\) for the set of auctions in which the first-round bids of \(i(2)\) and \(i(3)\) are within 5%.
Figure A.4: Difference in the Second-Round Bids of \(i(1)\) and \(i(2)\) (Left Panel) and the Difference in the Second-Round Bids of \(i(2)\) and \(i(3)\) (Right Panel), by Year. The left panels plot \(\Delta_{12}\) for the set of auctions in which the first-round bids of \(i(1)\) and \(i(2)\) are within 5%. The right panels plot \(\Delta_{23}\) for the set of auctions in which the first-round bids of \(i(2)\) and \(i(3)\) are within 5%.
Figure A.5: Raw Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Raw Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels). The left panels plot the raw difference in bids for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5% of the reserve price. The right panels plot the raw difference in bids for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5% of the reserve price.