Strategic Ignorance as a Self-Disciplining Device

Juan D. Carrillo and Thomas Mariotti

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Hyperbolic discounting

- The traditional branch of economics assumes *exponential discounting* to evaluate the stream of payoffs realized over time $(u_1, u_2, ...)$: $\sum_{t=1}^{\infty} \delta^{t-1} u_t$.
- Preferences with exponential discounting are *time-consistent*: an optimal choice in period s remains to be optimal at subsequent periods $t > s$.
- Consider the following: (a) the choice between receiving \$100 now ($t = 1$) and \$110 tomorrow ($t = 2$); (b) between receiving \$100 one year from now ($t = 366$) and \$110 one year and one day from now ($t = 367$).

Hyperbolic discounting

- These two questions are equivalent under exponential discounting.
- Why? In (a), you choose to receive \$100 today if $100 \geq 110\delta \Leftrightarrow \frac{10}{11} \geq \delta$.
- In (b), you choose to receive \$100 one year from now if $100\delta^{365} \geq 110\delta^{366} \Leftrightarrow \frac{10}{11} \geq \delta$.
- Tractable but ... *a priori* no reason for why we would or should evaluate future events in that manner.

Hyperbolic discounting

- In hyperbolic discounting, valuations fall very rapidly for initial periods, but then fall slowly later.
- One tractable and elegant way to capture this is (β, δ) preferences: $u_1 + \beta \sum_{t=2}^{\infty} \delta^{t-1} u_t$, where $\beta < 1$ and $\delta \leq 1$.
- Under hyperbolic discounting, the preference exhibits *time-inconsistency*.

Hyperbolic discounting

- In (a), you choose to receive \$100 today if $100 \geq 110\beta\delta$.
- In (b), you choose to receive \$100 one year from now if $100\beta\delta^{365} \geq 110\beta\delta^{366} \Leftrightarrow \frac{10}{11} \geq \delta$.
- This means that your answer depends on when you evaluate the alternatives – time-inconsistency for some range of β.
- If $\delta > \frac{10}{11} \geq \beta\delta$, you prefer to wait one more day in period $t = 366$, evaluating today, but cannot when you face the same choice in period $t = 366$.

Introduction

- This paper analyzes the decision of an agent with time-inconsistent preferences to consume a good that exerts an externality on future welfare.
- The extent of the externality is initially unknown but may be learned via a costless sampling procedure.
- Would it always be optimal to obtain this additional, more precise, information?
- If not, then why?

Introduction

- An examples: assessing the risk of smoking.
- It is shown that people overestimate the risk of smoking.
- Why don't we get ourselves updated with the most accurate information available?
- The cost of information acquisition? Studies on the effect of tobacco are widely publicized and freely available.

Model

- **Actors:** Time is discrete and indexed by $t = 0, 1, 2, ...$
  - The consumer is a countable collection of risk-neutral incarnations, with one incarnation per period.
  - The consumer's incarnation at date t is called self-t.
- **Actions:** In every period, one unit of a free indivisible good is available for consumption. Let $x_t \in \{0, 1\}$ denote the amount consumed in period t.

Model

- **Externalities:** Consumption increases the instantaneous utility but decreases the future payoffs (externalities).
  - A positive consumption level at any t lowers the per-period payoffs of all subsequent selves $t+\tau$, $\tau \geq 1$, by $\lambda^{\tau-1}C > 0$ with probability θ.
  - λ is a depreciation factor.
  - On the whole, the expected negative externality I_t imposed on self-t is $I_t = \sum_{\tau=0}^{t-1} \lambda^{t-\tau-1} x_\tau \theta C$.

Model

- **Information:** The probability of exerting the externality θ is unknown to the players.
  - It is distributed according to some distribution π_0 with continuous density f_0.
  - However, each self can costlessly acquire information about θ and update his beliefs accordingly.
  - I_t is not observable at any t.

Model

- **Instantaneous payoffs:** $u_t = x_t - l_t$ (instantaneous gains, delayed losses).
- **Intertemporal payoffs:** $U_t = E_t(u_t + \beta \sum_{\tau=1}^{\infty} \delta^{\tau} u_{t+\tau})$.
  - β represents the salience of current payoffs (present-biased).
  - δ is the discount factor that applies for all dates.
  - An important assumption: the consumer perfectly anticipates his dynamically inconsistent behavior (sophisticated vs naive).

The main result

- The main result of the model can be illustrated with a three-period example with limited learning opportunities.
- Suppose that there are three periods $t \in \{0, 1, 2\}$.
- The individual may either consume or abstain in periods 0 and 1, and learn the true value of θ before his consumption decision.
- For simplicity, (i) $\delta = 1$; (ii) the externality is exerted only in the period after consumption; and (iii) $1/\beta C < 1$.

The main result

- The intertemporal utility from the perspective of each self is
  - $U_0(x_0, x_1) = x_0(1 - \beta\theta C) + x_1\beta(1 - \theta C)$,
  - $U_1(x_0, x_1) = -x_0\theta C + x_1(1 - \beta\theta C)$.
  - $U_2(x_0, x_1) = -x_1\theta C$.

The main result

- Self-0 would like to:
  - consume in both periods if $\theta \in [0, 1/C]$,
  - consume only in period 0 if $\theta \in (1/C, 1/\beta C)$,
  - abstain in both periods if $\theta \in [1/\beta C, 1]$.
- However, he cannot commit to future decisions: to discipline the future selves, self-0 may need to manipulate information.

The main result

- If self-0 learns the true value of θ, the individual will end up:
  - consuming in both periods if $\theta < 1/\beta C$;
  - abstaining in both periods if $\theta \geq 1/\beta C$.
- If self-0 does not, the individual will end up:
  - consuming in both periods if $E_{\pi_0}(\theta) < 1/\beta C$;
  - abstaining in both periods if $E_{\pi_o}(\theta) \geq 1/\beta C$.

The main result

- The expected payoff if self-0 learns θ is
  - $V_L = \pi_0(\theta < 1/\beta C)[1 + \beta - 2\beta E_{\pi_0}(\theta \mid \theta < 1/\beta C)C]$.
- The expected payoff if self-0 does not learn is
  - $V_{NL} = 1 + \beta - 2\beta E_{\pi_0}(\theta)C$ if $E_{\pi_0}(\theta) < 1/\beta C$;
  - $V_{NL} = 0$ if $E_{\pi_0}(\theta) \geq 1/\beta C$.
- It is then immediate from these that
  - If $E_{\pi_0}(\theta) < 1/\beta C$, then $V_L > V_{NL}$.
  - If $E_{\pi_0}(\theta) \geq 1/\beta C$, then $V_{NL} > V_L$ if and only if $E_{\pi_0}(\theta \mid \theta < 1/\beta C) > (1 + \beta)/2\beta C$.

The intuition

- The source of the problem lies in the range $\theta \in (1/C, 1/\beta C)$ where self-0 would like to consume only in period 0 but ends up consuming in both periods.
- A necessary condition for ignorance is that it induces abstention in period 1, which is the case when $E_{\pi_0}(\theta) \geq 1/\beta C$.
- This is not enough for ignorance being valuable because it also entails several costs.
  - Ignorance and abstention is not optimal for self-0 in period 0 if $\theta \in [0, 1/\beta C)$ and in period 1 if $\theta \in [0, 1/C]$.
  - When $\theta \in [1/\beta C, 1]$, ignorance has neither costs or benefits: the individual abstains in both periods.
- The benefits outweigh the costs if, conditional on $\theta < 1/\beta C$, θ is more likely to be close to $1/\beta C$ than to 0.

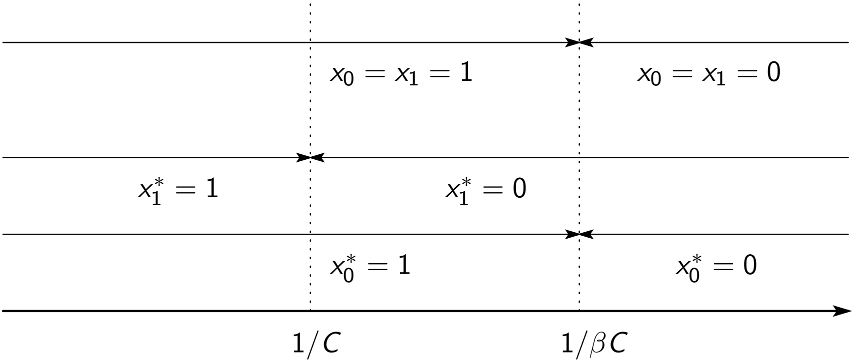
More intuition

- In equilibrium, whether self-0 learns or not, either $(x_0 = 0, x_1 = 0)$ or $(x_0 = 1, x_1 = 1)$. The critical threshold is always $1/\beta C$
- If $E_{\pi_0}(\theta) < 1/\beta C$, the expected cost is too small and $(x_0 = 1, x_1 = 1)$ without leaning.
- Ignorance cannot help in this case because:
  - If $\theta < 1/\beta C$, the individual consumes in both periods anyway (no change);
  - If $\theta \geq 1/\beta C$, the individual changes the choice and abstains in both periods, but this is optimal for self-0.

More intuition

- If $E_{\pi_0}(\theta) \geq 1/\beta C$, the expected cost is too large and $(x_0 = 0, x_1 = 0)$ without leaning.
- Staying ignorant about θ could help here:
  - If $\theta < 1/\beta C$, the individual changes the choice and consumes in both periods, whereas he would like self-1 to abstain when $\theta \in (1/C, 1/\beta C)$.
  - If $\theta \geq 1/\beta C$, the individual abstains in both periods anyway (no change).
- Ignorance has some value when the true value of θ lies in $(1/C, 1/\beta C)$.

More intuition



Conclusion

- The time-inconsistent nature of the preferences amount to a conflict within a self – an intrapersonal game.
- The structure of the game is thus analogous to a multi-person game as we are normally accustomed to.
- This setup is analogous to a situation where the information obtained by one player becomes automatically public.
- The assumption is hard to motivate in general, but is very natural in this intrapersonal setup – intrapersonal games could yield new perspectives.