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Six Approval Rules Whose Outcomes are Exactly the Same as the Mate Choice Rule under BEWDS

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1. Introduction

This note provides six approval rules whose outcomes are exactly the same as the mate choice rule introduced by Saijo, Okano and Yamakawa (2011). Definitions that are not in this note can be found in this article.

2. Unanimous and Generous Rules

Let (A,B,v,w) = (subject 1's choice between C and D, subject 2's choice between C and D, subject 1's approval decision between y and n, subject 2's approval decision between y and n), and Let $p_i(A,B,v,w)$ be subject i's payoff under (A,B,v,w).

Unanimous Rule (U): If both approve the strategy choices in the first stage, then the outcome is what they choose in the first stage. Otherwise, the outcome is (10,10).

Generous Rule (G): If both approve the strategy choices in the first stage, then the outcome is what they choose in the first stage. If a subject approves the other choice and the other subject does not under (C,C), the former's payoff is 17 and the latter's payoff is 7. Otherwise, the outcome is (10,10).

Consider (C,C,n,y). Both subjects choose cooperation in the first stage, and subject 1 does not approve subject 2's decision. In other words, subject 1 says, "thank you very much for your cooperation, but I would like to contribute 10 and please do not do that." In this sense, subject 1 is generous.

Natural Rules (N):

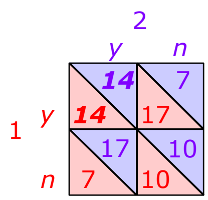
(1) If a subject approves the strategy choice of the other, the other's final contribution is either 0 or 10 under C and 0 under D.

(2) If a subject does not approve the strategy choice of the other, the other's final contribution is 0.

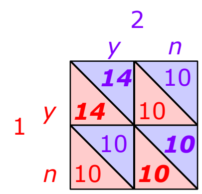
(3) $p_i(A,B,v,w) = p_j(B,A,w,v)$ for all i, j, A, B, w and v.

Property 1: *Suppose that the approval stage rules of the prisoner's dilemma game are natural. Then there are exactly six approval rules including unanimous and generous rules where one of the subgame perfect equilibrium outcomes is* (*14,14*).

Rough proof: (*a*) Consider subgame *a*. Since both choose *C* in the first stage and we focus upon (14,14), it is sufficient to consider the case with (14,14) under (*y*,*y*) due to (*N*1). In order for (14,14) to be a Nash equilibrium in subgame *a*, the outcome (*k*,*l*) under (*n*,*y*) must have a property $k \leq 14$. Since the total number of contribution is either 0 or 10 under (*n*,*y*) with (*N*1), the outcome (*k*,*l*) must be either (7,17) or (10,10) as shown in the following.

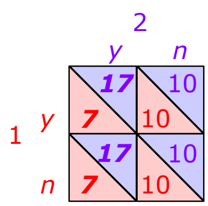


(*a*1)

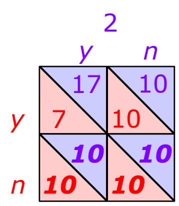


(*a*2)

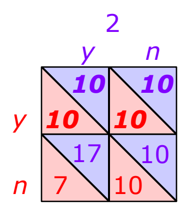
(b) Consider subgame *b*. Applying (*N*1), we have two outcomes (7,17) and (10,10) under (*C*,*D*,*y*,*y*) and (*C*,*D*,*n*,*y*) as shown in the following.



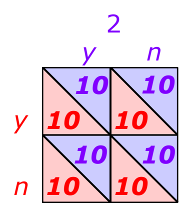
(*b*1)



(*b*2)

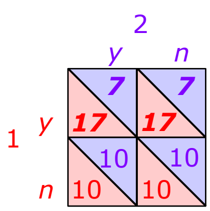


(*b*3)

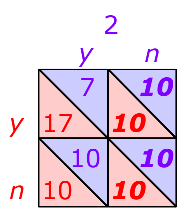


(*b*4)

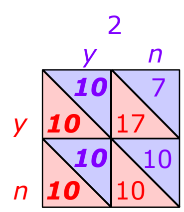
(c) Consider subgame *c*. This case is a symmetric counterpart of subgame *b*.



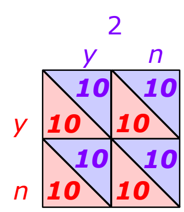
(*c*1)



(*c*2)

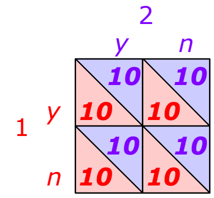


(c3)



(*c*4)

(d) Due to (*N*-*b*), we have the following payoff table.



(*d*)

Due to (*N*3), (*bi*) must be paired with (*ci*) for all *i*. Since (*ai*), (*b*1), (*c*1) and (*d*) with (14,14) form a prisoner's dilemma game for *i* = 1 and 2, the outcome is not (14,14). Therefore, we have six rules: {(*a*1), (*b*2), (*c*2), (*d*)}; {(*a*1), (*b*3), (*c*3), (*d*)}; {(*a*1), (*b*4), (*c*4), (*d*)}; {(*a*2), (*b*2), (*c*2), (*d*)}; {(*a*2), (*b*3), (*c*3), (*d*)}; and {(*a*2), (*b*4), (*c*4), (*d*)}. The three games containing (*a*1) satisfy the generous rule condition, {(*a*2), (*b*2), (*c*2),(*d*)} is the unanimous rule. See Saijo, Okano and Yamakawa (2011) for the definitions of mate choice flat, forthrightness, and axioms 1, 2 and 3 in the following property.

Property 2:

(i) *In each of six rule of Property 1,* (*C*,*C*,*y*,*y*) *is the equilibrium path under BEWDS*.

(ii) *Each rule has the following properties:*

1) {(*a*1), (*b*2), (*c*2), (*d*)}: *Subgame* (*C*,*C*) *does not have the mate choice flat.*

2) {(*a*1), (*b*3), (*c*3), (*d*)}: *Subgames* (*C*,*C*), (*C*,*D*) *and* (*D*,*C*) *do not have the mate choice flat, and forthrightness is violated at* (*C*,*D*) *and* (*D*,*C*).

3) {(*a*1), (*b*4), (*c*4), (*d*)}: *Subgame* (*C*,*C*) *does not have the mate choice flat, and forthrightness is violated at* (*C*,*D*) *and* (*D*,*C*).

4) {(*a*2), (*b*2), (*c*2), (*d*)}: *Forthrightness is satisfied and axioms 1, 2 and 3 are satisfied.*

5) {(*a*2), (*b*3), (*c*3), (*d*)}: *Subgames* (*C*,*D*) *and* (*D*,*C*) *do not have the mate choice flat, and forthrightness is violated at* (*C*,*D*) *and* (*D*,*C*).

6) {(*a*2), (*b*4), (*c*4), (*d*)}: *Forthrightness is violated at* (*C*,*D*) *and* (*D*,*C*).

Rule {(*a*2), (*b*2), (*c*2), (*d*)} satisfying axioms 1, 2 and 3 is the mate choice mechanism in Saijo, Okano and Yamakawa (2011).

Reference

Tatsuyoshi Saijo, Yoshitaka Okano and Takafumi Yamakawa, "The Approval Mechanism Experiment: A Solution to Prisoner's Dilemma," mimeo., 2011.