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POWER OF VOTERS AND DOMAIN OF PREFERENCES WHERE VOTING BY COMMITTEES IS STRATEGY–PROOF

by

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Section 1 Introduction. When a society consisting of several agents has to select from a set of alternatives, in order to choose a desirable alternative in any reasonable sense, the procedure must take into accounts agents' preferences on alternatives. So procedures are formally represented as functions from the set of possible preference profiles into the set of alternatives, and they are called social choice functions or schemes. However, preferences are, usually, privately known. Therefore possibly, selfish agents try to strategically misrepresent their preferences to manipulate the final outcome. As a result of such strategic behaviors, actual outcome may be far from satisfactory from the social point of view. Thus it is important to know whether a social choice function is immune to strategic behaviors, and if it is immune, then it is said strategy–proof. Gibbard (1973) and Satterthwaite (1975) showed a negative result : when there are more than three alternatives and the domain of preferences is large enough, no nondictatorial scheme is strategy–proof.

However there are many important cases in which the restriction on the preference domain is economically plausible, and so, which fall outside the purview of the Gibbard–Satterthwaite theorem. One recent such example is Barbera, Sonnenschein, and Zhou (1991, hereafter B, S & Z). In B, S & Z, there are n voters and k issues. These issues are interpreted as bills considered for adoption, public goods etc. The feasible set is the set of any combinations of issues : some issues are adopted and the others are not. Each voter has preferences on the feasible set. B, S & Z established the following. (i) A class of voting schemes, which they call voting by committees, is strategy–proof on an important domain of preferences, which they call separable preferences. (ii) These schemes are the only strategy–proof schemes on this domain that satisfy voter sovereignty[1]. (iii) This domain is the maximal domain in which these schemes are strategy–proof except for what they call "extreme" cases. In B, S & Z, if an issue is interpreted as a public good, its available levels

[1] A voting scheme satisfies voter sovereignty if any combination of issues is assigned to some preference profile.

are zero or one. Barbera, Gul and Stacchetti (1991, hereafter B, G & S) extended this model so as to permit multiple levels of each public good. And B, G & S established results parallel to (i) and (ii) of B, S & Z.

In this paper, studying how the condition of strategy–proofness restricts the domain of preferences, we generalize (iii) of B, S & Z in the extended model of B, G & S. However, if the power of a voter is extremely strong or weak, the condition of strategy–proofness does not bite. For example, dictatorship is strategy–proof on the universal domain. Or consider the case when some voter can not influence the outcome at all. Then his preference is not restricted by strategy–proofness. Actually the restriction on the preference domain of a voter by strategy–proofness depends on his power. Next we specify the preferences to be used. In general, the most preferred level of any good depends on the levels of the other goods. A preference relation is peak–separable if the most preferred level of any good is independent of the consumption levels of the other goods. A preference relation is goodwise single–peaked if for each good, the induced preference relation on the available levels of that good obtained by fixing the levels of the other goods can be represented by a single–peaked function. A preference relation is cross–shaped if it is peak–separable and goodwise single–peaked. The main result of this paper is as follows : *If a scheme of voting by committees without a dummy voter*[2] *is strategy–proof on some rich*[3] *domain, then any preference in the domain must be cross–shaped on some subset of the feasible set. Furthermore the width of such subset depends on the power of voters.* As a corollary, we have that the domain of cross–shaped preferences is the maximal one in which the schemes of voting by committees are strategy–proof except for "extreme" cases. This corollary is the result parallel to (iii) of B, S & Z.

In Section 2, we set up the model and state the characterization of voting by

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[2] A voter is a dummy voter if he cannot influence on the decision being considered.

[3] A domain is rich if for any alternative, each voter has a preference such that he prefers it most.

committees by B, G & S. In Section 3, we establish our main result.

Section 2 Model and Characterization of Voting by Committees. The set of voters is $N = \{1,2,\dots,n\}$, $n \geqq 2$. The set of issues is $X = \{1,2,\dots,k\}$. The set of available levels of $x \in X$ is $L_x = \{0,1,\dots,m_x\}$, $m_x \geqq 1$ and the feasible set is $L = \prod_{x \in X} L_x$. The set of strict[4] preference relations on L is $\mathscr{P}$. Given the preference relation $P_i \in \mathscr{P}$, let R_i be the weak preference relation associated with P_i (for any ℓ and $\ell' \in L$, $\ell \; R_i \; \ell'$ if and only if $\ell' = \ell$ or $\ell P_i \ell'$). Given a preference relation $P_i \in \mathscr{P}$ and a set $L' \subsetneq L$, $B(P_i,L')$ is the best element of P_i on L'. Let $B(P_i) = B(P_i,L)$, and $B_x(P_i,L')$ be the x–th coordinate of $B(P_i,L')$.

Definition : A social choice function or scheme is a function from $\mathscr{P}^n$ to L.

Definition : A scheme f satisfies voter sovereignty on $\mathscr{P}' \subset \mathscr{P}^n$ if

$$\forall \ell \in L, \exists P \in \mathscr{P}' \text{ such that } f(P) = \ell$$

Definition : A set of winning coalitions for $\ell_x \in L_x \backslash \{0\}$ is a subset $\mathscr{W}_x(\ell_x)$ of 2^N such that

$$\text{i) } \mathscr{W}_x(\ell_x) \neq \emptyset \quad \text{ii) } \emptyset \notin \mathscr{W}_x(\ell_x) \quad \text{iii) } W \in \mathscr{W}_x(\ell_x) \ \& \ W' \supsetneq W \Rightarrow W' \in \mathscr{W}_x(\ell_x)$$

Definition : A list $\left(\mathscr{W}_x(\ell_x)\right)_{x \in X, \ell_x \in L_x \backslash \{0\}}$ of sets of winning coalitions is monotonic if

$$\forall x \in X, \forall \ell_x, \ell_x' \in L_x \backslash \{0\}, \quad [\ell_x \leqq \ell_x' \Rightarrow \mathscr{W}_x(\ell_x') \subsetneq \mathscr{W}_x(\ell_x)]$$

Definition : A scheme $f: \mathscr{P}^n \longrightarrow L$ is voting by committees[5] if there is a monotonic list

[4] In this paper, the strictness of preference relation means completeness, asymmetricity, irreflexivity and transivity.

[5] The schemes of voting by committees are called generalized median voter schemes in

$\left(\mathscr{W}_x(\ell_x) \right)_{x \in X, \ell_x \in L_x \setminus \{0\}}$ of sets of winning coalitions such that

$$\forall x \in X, \forall \ell_x \in L_x \setminus \{0\}, \forall P \in \mathscr{P}^n,$$
$$f_x(P) \geqq \ell_x \Longleftrightarrow \{i \in N \mid B_x(P_i) \geqq \ell_x\} \in \mathscr{W}_x(\ell_x)$$

Definition : Let $\mathscr{P}' = \mathscr{P}_1' \times \ldots \times \mathscr{P}_n' \subseteq \mathscr{P}^n$. A voting scheme $f: \mathscr{P}^n \to L$ is strategy–proof on $\mathscr{P}'$ if

$$\forall P \in \mathscr{P}', \forall i \in N, \; \forall P_i' \in \mathscr{P}_i', \; f(P) \; R_i \; f(P_i', P_{-i}).$$

We now define the domains of preferences that we will use in the characterization of voting by committees. In general, a voter's preferences on the available levels of a particular good depend on the levels of the other goods. "Separability" defined below says that preferences on the available levels of each good are independent of the levels of the other goods. A weaker form of separability is "peak–separability", which says that the most preferred level of each good is independent of the levels of the other goods. Given a preference relation $P_i \in \mathscr{P}$ and $\ell_{-x} \in L_{-x}$, let $\bar{\ell}_x(\ell_{-x}) = B_x(P_i, \{(\ell_x, \ell_{-x}) \mid \ell_x \in L_x\})$.

Definition : A preference relation $P_i \in \mathscr{P}$ is separable on $L' \subset L$ if

$$\forall x \in X, \forall \ell_{-x}, \ell_{-x}' \in L_{-x}, \forall \ell_x, \ell_x' \in L_x \text{ s.t. } (\ell_x, \ell_{-x}), (\ell_x, \ell_{-x}'), (\ell_x', \ell_{-x}), (\ell_x', \ell_{-x}') \in L',$$
$$(\ell_x, \ell_{-x}) \; P_i \; (\ell_x', \ell_{-x}) \implies (\ell_x, \ell_{-x}') \; P_i \; (\ell_x', \ell_{-x}') .$$

A preference relation $P_i \in \mathscr{P}$ is separable if it is separable on L.

Denote by $\mathscr{P}^S \subseteq \mathscr{P}$ the class of separable preferences.

Definition : A preference relation $P_i \in \mathscr{P}$ is peak–separable on L' if

$$\forall x \in X, \forall \ell_{-x} \in L_{-x}, [(\bar{\ell}_x(\ell_{-x}), \ell_{-x}) \in L' \implies \bar{\ell}_x(\ell_{-x}) = B_x(P_i)].$$

A preference relation $P_i \in \mathscr{P}$ is peak–separable if it is peak–separable on L.

B, G & S. And a list of sets of winning coalitions in this definition corresponds to a right–hand coalition system in B, G & S.

Denote by $\mathscr{P}^{PS} \subseteq \mathscr{P}$ the class of peak–separable preferences.

Note that $\mathscr{P}^{S} \subseteq \mathscr{P}^{PS}$. But if for each $x \in X$, $L_x = \{0,1\}$, then $\mathscr{P}^{S} = \mathscr{P}^{PS}$, and both notions coincide with the notion of separable preferences of B, S & Z.

Definition : A preference relation $P_i \in \mathscr{P}$ is goodwise single–peaked on L′ if

$$\forall x \in X, \forall \ell_{-x} \in L_{-x}, \forall \ell_x, \ell'_x \in L_x \text{ s.t. } (\ell_x, \ell_{-x}) \in L', (\ell_x', \ell_{-x}) \in L'$$

$$[\ell_x < \ell'_x \leqq \bar{\ell}_x(\ell_{-x}) \text{ or } \ell_x > \ell'_x \geqq \bar{\ell}_x(\ell_{-x})] \Rightarrow (\ell'_x, \ell_{-x})\, P_i\, (\ell_x, \ell_{-x}).$$

A preference relation is goodwise single–peaked if it goodwise single–peaked on L.
Denote by $\mathscr{P}^{GS} \subseteq \mathscr{P}$ the class of goodwise single–peaked preferences.

Definition : A preference relation $P_i \in \mathscr{P}$ is cross–shaped on L′ if P_i is peak–separable and goodwise single–peaked on L′. A preference is cross–shaped[6] if it is cross–shaped on L.
Denote $\mathscr{P}^{C} = \mathscr{P}^{PS} \cap \mathscr{P}^{GS}$.

Note that if for each $x \in X$, $L_x = \{0,1\}$, then goodwise single–peakedness is automatically satisfied. Thus if for each $x \in X$, $L_x = \{0,1\}$, then since $\mathscr{P}^{PS}$ coincides with the notion of separable preference of B, S & Z, so does $\mathscr{P}^{C}$. B, G & S characterized voting by committees on the domain $\mathscr{P}^{C}$ of the cross–shaped preferences.

Theorem 1 (Barbera, Gul, and Stacchetti, 1991) : A social choice function is a scheme of voting by committees if and only if it satisfies voter sovereignty and strategy–proofness on the domain $(\mathscr{P}^{C})^n$ of cross–shaped preferences.

[6] Cross–shaped preferences are called multidimensional single–peaked preferences in B, G & S.

Section 3 Strategy–Proofness of Voting by Committees, Power of Voters, and the Cross–Shaped Preferences. In this section, we study how the strategy–proofness of voting by committees restricts the domain of preferences. However, if the powers of some voter is "extreme," then the condition of strategy–proofness does not bite. For example, dictatorship is strategy–proofness on the universal domain. Or consider the case when some voter can not influence the outcomes at all. Then his preference domain is not restricted by strategy–proofness of voting by committees. Thus the strength of this restriction depends on voters' power.

Definition : Given a set $\mathscr{W}_x(\ell_x)$ of winning coalitions, denote the minimal winning coalitions by $\{N_\alpha \subseteq N\}$. A voter $i \in N$ is a veto voter in $\mathscr{W}_x(\ell_x)$ if $i \in \bigcap_{W \in \mathscr{W}_x(\ell_x)} W$. A voter $i \in N$ is a decisive voter in $\mathscr{W}_x(\ell_x)$ if $\{i\} \in \mathscr{W}_x(\ell_x)$. A voter $i \in N$ is a dummy voter in $\mathscr{W}_x(\ell_x)$ if $i \notin \cup N_\alpha$. And $\mathscr{W}_x(\ell_x)$ is extreme if $\mathscr{W}_x(\ell_x)$ contains a veto voter or a decisive voter or a dummy voter.

Remark : (i) A voter is a dictator if and only if for any $x \in X$ and for any $\ell_x \in L_x \backslash \{0\}$, he is a veto and decisive voter in $\mathscr{W}_x(\ell_x)$ at the same time. (ii) If a voter is a veto voter in $\mathscr{W}_x(\ell_x)$, then he is also a veto voter in $\mathscr{W}_x(\ell_x')$ for any $\ell_x' \geqq \ell_x$. Conversely (iii) if a voter is a decisive voter in $\mathscr{W}_x(\ell_x)$, then he is also a decisive voter in $\mathscr{W}_x(\ell_x')$ for any $\ell_x' \leqq \ell_x$. (iv) There is no such logical relation for dummy voter (see Example 1 below). However (v) if a voter is a veto or decisive voter in $\mathscr{W}_x(\ell_x)$, then he is not a dummy voter.

Example 1 : Let $N = \{1, 2, 3, 4\}$, $X = \{x\}$, $L_x = \{0, 1, 2, 3\}$, $\mathscr{W}_x(1) = \{W \subseteq N \mid \{1, 2\} \subseteq W \text{ or } \{3, 4\} \subseteq W\}$, $\mathscr{W}_x(2) = \{W \subseteq N \mid \{2, 3, 4\} \subseteq W\}$, and $\mathscr{W}_x(3) = \{N\}$. Then voter 1 is a dummy in $\mathscr{W}_x(2)$, but not in $\mathscr{W}_x(1)$ or in $\mathscr{W}_x(3)$.

Definition : A social choice function is a scheme of voting by committees without a dummy voter (a scheme of voting by nonextreme committees) if it is a scheme of voting by committees and no set of winning coalitions has a dummy voter (no set of winning coalitions is extreme).

Since if the domain is too small, strategy–proofness is meaningless, we consider the admissible domains that admit a sufficient variety of preferences.

Definition : A domain $\mathscr{P}' = \mathscr{P}_1' \times \cdots \times \mathscr{P}_n' \subsetneq (\mathscr{P}^C)^n$ is rich if

$\forall i \in N, \forall \ell \in L, \exists P_i \in P_i$ such that $\ell = B(P_i)$

Theorem 2 : Let f be a scheme voting by committees without a dummy voter, and let $\mathscr{P}' = \mathscr{P}_1' \times \ldots \times \mathscr{P}_n' \subsetneq \mathscr{P}^n$ be a rich domain on which f is strategy–proof. Let $i \in N$. For any $x \in X$, let $\hat{\ell}_x$ is the maximal level such that voter i is a decisive voter in $\mathscr{W}_x(\hat{\ell}_x)$, and let $\hat{\ell}_x'$ be the minimal level such that he is a veto voter in $\mathscr{W}_x(\hat{\ell}_x')$. Then any $P_i \in \mathscr{P}_i'$ is cross–shaped on $L' = \{\ell \in L \mid \forall x \in X, \min\{\hat{\ell}_x, B_x(P_i)\} \leqq \ell_x \leqq \max\{\hat{\ell}_x', B_x(P_i)\}\}$.

Remark : For each $x \in X$, let $\bar{L}_x \subsetneq L_x$ be the set of levels such that voter i is a dummy voter in $\mathscr{W}_x(\ell_x)$, $D_x = \{\ell \in L \mid \ell_x \in \bar{L}_x\}$ and $D = \bigcup_{x \in X} D_x$. If f is a scheme of voting by committees with dummy voter i, then some $P_i \in \mathscr{P}_i'$ may not cross–shaped even on $L'' = \{\ell_x \in L \setminus D \mid \forall x \in X, \min\{\hat{\ell}_x, B_x(P_i)\} \leqq \ell_x \leqq \max\{\hat{\ell}_x', B_x(P_i)\}\}$, which is shown in Example 2 below.

Example 2 : Let $N = \{1, 2, 3, 4\}$, $X = \{x, y\}$, $L_x = L_y = \{0, 1, 2, 3\}$, $\mathscr{W}_x(1) = \mathscr{W}_y(1) = \{W \subsetneq N \mid \#W \geqq 2\}$, $\mathscr{W}_x(2) = \mathscr{W}_y(2) = \{W \subsetneq N \mid \#(W \setminus \{1\}) \geqq 2\}$, $\mathscr{W}_x(3) = \mathscr{W}_x(3) = \{W \subsetneq N \mid \#W \geqq 3\}$. Let $\hat{P}_1$ be such that $(3, 3)\ \hat{P}_1\ (1, 1)\ \hat{P}_1\ (1, 0)\ \hat{P}_1\ (0, 1)\ \hat{P}_1\ (0, 0)\ \hat{P}_1$

(1, 3) $\hat{P}_1$ (0, 3) $\hat{P}_1$ (3, 1) $\hat{P}_1$ (3, 0) $\hat{P}_1$ (2, 3) $\hat{P}_1$ (3, 2) $\hat{P}_1$ (2, 2) $\hat{P}_1$ (2, 1) $\hat{P}_1$ (2, 0) $\hat{P}_1$ (1, 2) $\hat{P}_1$ (0, 2). Let $\mathscr{P}'_1 = \mathscr{P}^C \cup \{\hat{P}_1\}$, $\mathscr{P}'_2 = \mathscr{P}'_3 = \mathscr{P}'_4 = \mathscr{P}^C$. Then D = {(2, 0), (2, 1), (2, 2), (2, 3), (2, 0), (2, 1), (2, 3)} and although $\hat{P}_1$ is not cross–shaped on L\D, the scheme of voting by committees defined above is strategy–proof on $\mathscr{P}_1' \times \mathscr{P}_2' \times \mathscr{P}_3' \times \mathscr{P}_4'$.

<u>Corollary</u>[7] : Let f be a scheme of voting by nonextreme committees, and let $\mathscr{P}' = \mathscr{P}'_1 \times \dots \times \mathscr{P}'_n \subseteq \mathscr{P}^n$ be a rich domain on which f is strategy–proof. Then for all $i \in N$, $\mathscr{P}_i' \subseteq \mathscr{P}^C$.

The proof of Theorem 2 consists of Lemma 1 and Lemma 2.

<u>Lemma 1</u> : Let f be a scheme of voting by committees without a dummy voter. If $\mathscr{P}' = \mathscr{P}'_1 \times \cdots \times \mathscr{P}_n' \subseteq \mathscr{P}^n$ is a rich domain on which f is strategy–proof, then for all $i \in N$, any $P_i \in \mathscr{P}_i'$ is peak–separable on $L' = \{\ell \in L \mid \forall x \in X, \min\{\hat{\ell}_x, B_x(P_i)\} \leqq \ell_x \leqq \max\{\hat{\ell}'_x, B_x(P_i)\}\}$.

<u>Lemma 2</u> : Let f be a scheme of voting by committees without a dummy voter. If $\mathscr{P}' = \mathscr{P}'_n \times \dots \times \mathscr{P}'_n \subseteq \mathscr{P}^n$ is a rich domain on which f is strategy–proof, then for all $i \in N$, any $P_i \in \mathscr{P}_i'$ is goodwise single–peaked on $L' = \{\ell \in L \mid \forall x \in X, \min\{\hat{\ell}_x, B_x(P_i)\} \leqq \ell_x \leqq \max\{\hat{\ell}'_x, B_x(P_i)\}\}$.

<u>Proof of Lemma 1</u> Let $P_i \in \mathscr{P}'$, $x \in X$, $\ell_{-x} \in L_{-x}$ $\bar{\ell}_x = B_x(P_i, \{(\ell_x, \ell_{-x}) \mid \ell_x \in L_x\})$ and $(\bar{\ell}_x, \ell_{-x}) \in L'$. We will show that $\bar{\ell}_x = B_x(P_i)$.

Suppose, by contradiction, that $\bar{\ell}_x \neq B_x(P_i)$. Without loss of generality, we may assume that $\bar{\ell}_x < B_x(P_i)$, since we can derive a contradiction in the opposite case by the same argument. Let $\ell'_x = \bar{\ell}_x + 1$. Then $\bar{\ell}_x < \ell'_x \leqq B_x(P_i)$. Since voter i is not a dummy in

[7] This is the result in our model parallel to (iii) of B, S & Z.

$\mathscr{W}_x(\ell'_x)$, there is $W \in \mathscr{W}_x(\ell'_x)$ such that $i \in W$ & $W \setminus\{i\} \notin \mathscr{W}_x(\ell'_x)$. Because $\mathscr{P}'$ is rich, we can choose P_j for $j \in N\setminus\{i\}$ such that

$$B(P_j) = (\ell'_x, \ell_{-x}) \quad \text{if } j \in W\setminus\{i\}$$

$$B(P_j) = (\tilde{\ell}_x, \ell_{-x}) \quad \text{otherwise.}$$

Let $y \neq x$. Note that $\{j \in N \mid B_y(P_j) = \ell_y\} \supseteq N\setminus\{i\}$. Since $(\tilde{\ell}_x, \ell_{-x}) \in L'$, if $B_y(P_i) > \ell_y$, then voter i is not a decisive voter in $\mathscr{W}_y(\ell'_y)$ for any $\ell'_y > \ell_y$, and if $B_y(P_i) < \ell_y$, then he is not a veto voter in $\mathscr{W}_y(\ell'_y)$ for any $\ell'_y < \ell_y$. Thus $f_y(P_i, P_{-i}) = \ell_y$, and so we have : (1) $f_{-x}(P_i, P_{-i}) = \ell_{-x}$. By $W \in \mathscr{W}_x(\ell'_x)$, $f_x(P_i, P_{-i}) \geqq \ell'_x$. Furthermore since $(\tilde{\ell}'_x, \ell_{-x}) \in L'$ and $\tilde{\ell}_x < \ell'_x \leqq B_x(P_i)$ imply that voter i is not a decisive voter in $\mathscr{W}_x(\ell'_x)$, it follows from $\{j \in N \mid B_x(P_j) \geqq \ell'_x + 1\} \subsetneq \{i\}$ that $f_x(P_i, P_{-i}) < \ell'_x + 1$. Thus we have : (2) $f_x(P_i, P_{-i}) = \ell'_x$.

Let P_i' be such that $B_x(P_i') = \tilde{\ell}_x$. Similarly to (1), we have: (3) $f_{-x}(P_i', P_{-i}) = \ell_{-x}$. On the other hand, $W\setminus\{i\} \notin \mathscr{W}_x(\ell'_x)$ implies that $f_x(P_i', P_{-i}) < \ell'_x$. Note that when voter i announces P_i', it follows from $\{j \in N \mid B_x(P_j) \geqq \tilde{\ell}_x\} = N$ that $f_x(P_i', P_{-i}) \geqq \tilde{\ell}_x$. Thus we have : (4) $\tilde{\ell}_x = f_x(P_i', P_{-i}) < \ell_x'$.

Remember that by the definition of $\tilde{\ell}_x$, $(\tilde{\ell}_x, \ell_{-x})\ P_i\ (\ell'_x, \ell_{-x})$. Therefore by (1), (2), (3) and (4), $f(P_i', P_{-i})\ P_i\ f(P)$. This contradicts strategy–proofness. Hence we conclude that $\tilde{\ell}_x = B_x(P_i)$. Q.E.D.

<u>Proof of Lemma 2</u> Given $P_i \in \mathscr{P}^{PS}$, $x \in X$, and $\ell_{-x} \in L_{-x}$, let $\ell_x, \ell'_x \in L_x$ and $\tilde{\ell}_x = B_x(P_i)$, $(\ell_x, \ell_{-x}) \in L'$ and $(\ell'_x, \ell_{-x}) \in L'$. By Lemma 1, we need to prove :

$$\ell_x < \ell'_x \leqq \tilde{\ell}_x \quad \Rightarrow (\ell'_x, \ell_{-x})\ P_i\ (\ell_x, \ell_{-x})$$

$$\text{and} \quad \ell_x > \ell'_x \geqq \tilde{\ell}_x \quad \Rightarrow (\ell'_x, \ell_{-x})\ P_i\ (\ell_x, \ell_{-x}).$$

However we prove the first part only , since the proof of the second part is similar.

Note that by the transitivity of preference we have only to show that if for some $t = 1, 2, \ldots, \tilde{\ell}_x$, $\ell_x = \tilde{\ell}_x - t$ and $\ell'_x = \tilde{\ell}_x - (t-1)$, then $(\ell'_x, \ell_{-x})\ P_i\ (\ell_x, \ell_{-x})$.

Assume that $t \in \{1, \ldots, \tilde{\ell}_x\}$, $\ell_x = \tilde{\ell}_x - t$, $\ell'_x = \tilde{\ell}_x - (t-1)$. Since voter i is not a dummy voter in $\mathscr{W}_x(\ell'_x)$, there is $W \in \mathscr{W}_x(\ell'_x)$ such that $i \in W$ and $W\setminus\{i\} \notin \mathscr{W}_x(\ell'_x)$. Because $\mathscr{P}'$ is rich, we choose P_j for $j \in N\setminus\{i\}$ such that

$B(P_j) = (\ell'_x, \ell_{-x})$ if $j \in W\setminus\{i\}$

$B(P_j) = (\ell_x, \ell_{-x})$ otherwise.

Then note that similarly to (1) and (2) in the proof of Lemma 1 we have :

(1) $f(P) = (\ell'_x, \ell_{-x})$.

Let P'_i be such that $B_x(P'_i) = \ell_x$. Then similarly to (3) and (4) in the proof of Lemma 1 we have :

(2) $f_x(P'_i, P_{-i}) = (\ell_x, \ell_{-x})$

Since the strategy–proofness of f implies $(P'_i, P_{-i})\ P_i\ (P)$, it follows from (1) and (2) that $(\ell'_x, \ell_{-x})\ P_i\ (\ell_x, \ell_{-x})$ Q.E.D.

We have completed the proof of Theorem 2.

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