

# Pecuniary Externalities, Bank Overleverage, and Macroeconomic Fragility\*

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## Abstract

Pecuniary externalities in models with financial friction justify macroprudential policies for preventing economic agents' excessive risk taking. We extend the Diamond and Rajan (2012) model of banks with production factors and explore how a pecuniary externality affects a bank's leverage. We show that the laissez-faire banks in our model take on excessive risks compared with the constrained social optimum. Our numerical simulations suggest that the crisis probability is 2–3 percentage points higher in the laissez-faire economy than in the constrained social optimum.

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# 1 Introduction

Since the global financial crisis of 2007–08, one of the challenges for policymakers has been how to avoid systemic financial crises. Policymakers and researchers have discussed a pecuniary externality as a rationale for preventing economic agents’ excessive risk taking. For example, Mendoza (2010), Korinek (2018), and Jeanne and Korinek (2019) use the small open economy model with the collateral constraint. They show that households’ overborrowing takes place because households fail to internalize the general equilibrium effect of the price of the collateral on the collateral constraint.<sup>1</sup> However, as argued in Dávilla and Korinek (2017), it is “remarkably difficult to provide general results on the direction of inefficiency.” In other words, even if a model of financial crises has a pecuniary externality, it does not guarantee that economic agents will engage in excessive risk taking. An example is the small open economy model with tradables and nontradables by Benigno, Chen, Otrok, Rebucci, and Young (BCORY, 2013). They show that, once they change the endowment economy model into the production economy model, the model of overborrowing predicts underborrowing in normal times. This example suggests that it is important to check the direction of inefficiency, namely, whether a pecuniary externality creates excessive risk taking in a model of a financial crisis.

In this paper, we take a model of bank runs to investigate the direction of inefficiency. We develop a production economy model with maturity-mismatching banks. Here, the model with banks is a variant of Diamond and Rajan (2012, DR) and is discussed in Kato and Tsuruga (2016), in which a liquidity preference shock precipitates a banking crisis. We ask how a pecuniary externality affects allocations in a competitive equilibrium with *laissez-faire* (LF) banks. In particular, we introduce the prices of production factors (e.g., the capital price and wages) to ask how pecuniary externalities generate inefficiency. This extension is similar to that in BCORY because we also change the endowment economy model into the production economy model. In our model, banks fail to internalize the effect of their risk taking on factor prices. Our question is whether the pecuniary externality leads to “underleverage” or “overleverage” in a model of banks.

The pecuniary externality in our model leads to inefficient allocations compared with the constrained social optimum that a social planning (SP) agent can achieve. However, our model differs from the abovementioned small open economy model with the collateral

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<sup>1</sup>See also other related studies such as Lorenzoni (2008), Bianchi (2010, 2011), Stein (2012), Benigno, Chen, Otrok, Rebucci, and Young (2016), and Bianchi and Mendoza (2018).

constraint in a few aspects. First, the economic agent that engages in inefficient borrowing is a bank rather than a household. Second, the pecuniary externality in our model operates through the constraint on the bank's solvency rather than the collateral. The solvency constraint arises as a natural consequence of the maturity-mismatching banks that issue non-state-contingent short-term debt and invest them in illiquid assets.

Key elements of the model are (i) the bank's issuance of non-state-contingent debt and (ii) the dependence of the banks' asset value on factor prices. The first element enables banks to raise funds and to promote liquidity creation in the absence of complete markets, as discussed in Diamond and Rajan (2001a, b). However, this benefit of non-state-contingent debt comes at the cost of potential insolvency of banks. The second element is incorporated with our overlapping generations (OLG) framework, which explicitly includes factor markets. In our model, banks' balance sheets directly depend on the price of capital goods. Under this dependence of banks' balance sheets on capital price, the banks fail to internalize the effect of capital prices on their solvency.

Our main finding is that banks leverage to an excessive level because of a pecuniary externality. Using numerical analysis, we compute the crisis probabilities in the competitive equilibrium. The crisis probability is inefficiently high and the magnitude of the inefficiency is not negligible. In our benchmark simulations, the crisis probability is 6.6 percent under the competitive equilibrium, compared with 4.5 percent under the constrained social optimum. Our OLG framework also generates sharp declines in economic activity in the aftermath of a crisis. When a liquidity shock precipitates a crisis, banks are required to liquidate all illiquid loans to entrepreneurs to repay depositors. The liquidation stops capital goods production and results in a significant contraction in consumption and output.

Our numerical analysis suggests that the primary source of overleverage is the price-taking behavior by banks. Atomistically small banks take factor prices as given because none of them can affect prices individually. However, in equilibrium, all banks make the same decisions, and factor prices are in fact affected by the banks' synchronized actions. Specifically, when the LF banks raise their leverage, they do not internalize the extra marginal increase in the crisis probability via the synchronized decisions. As a result, banks' leverage and the crisis probability are inefficiently elevated compared with the socially optimal levels.

We quantify the importance of the pecuniary externality stemming from the banks' solvency because it directly influences the crisis probability. The banks' solvency depends on the capital price, which the LF banks take as given. We perform a counterfactual experiment in which the LF banks internalize changes in the capital price that appear in the solvency

constraint while ignoring all other general equilibrium effects stemming from factor prices. The result suggests that the LF banks would keep their leverage close to the constrained social optimum if they could internalize the pecuniary externality stemming from the solvency constraint.

Our paper makes a contribution to the literature by incorporating the pecuniary externality into the theory of banking. Our model extends Allen and Gale (1998) and DR in modeling banks.<sup>2</sup> At the same time, our model borrows the idea of the pecuniary externality from models with the collateral constraint, such as Lorenzoni (2008), Bianchi (2011), and Jeanne and Korinek (2019), among others. In our model, the banks' solvency plays a crucial role in generating bank runs or financial crises. Our model further explores the possibility of inefficient financial crises via the dependence of the banks' solvency on factor prices.<sup>3</sup>

Our paper is also related to the macroeconomic literature on models with financial constraints. Macroeconomic models with banks have focused on how financial frictions amplify business cycles (e.g., Gertler and Karadi 2011, Gertler and Kiyotaki 2011, Meh and Moran 2010). Our focus contrasts with these studies because we examine the efficiency of the competitive equilibrium. Angeloni and Faia (2013) incorporate banks à la Diamond and Rajan (2000, 2001a) into the dynamic stochastic general equilibrium model. While their model has the endogenous probability of bank insolvency, the probability of bank insolvency can be interpreted as a measure of individual bank fragility rather than the probability of systemic financial crises. Our model is also close to Gertler and Kiyotaki (2015) and Boissay, Collard, and Smets (2016). In Gertler and Kiyotaki (2015), whether an equilibrium with bank runs exists depends on macroeconomic fundamentals. However, financial crises per se are precipitated by self-fulfilling expectations, as in Diamond and Dybvig (1983). In our model, fundamental shocks (liquidity preferences) precipitate financial crises. In this regard, our model is broadly in line with empirical findings of the business cycle view of Gorton (1988) and Allen and Gale (1998). Boissay, Collard, and Smets (2016) focus on the moral hazard in the interbank markets and successfully replicate systemic financial crises. On the other hand, we stress the fragility of the maturity mismatch of banks, as highlighted by the Gorton and Metrick (2012) "run-on-repo" view.

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<sup>2</sup>Our model assumes that banks issue demand deposits following DR. In our model, however, we broadly interpret banks as financial intermediaries that raise funds via short-term debt such as repo and commercial paper and transform maturities on their balance sheet. Demand deposits are an extreme case of short-term debt.

<sup>3</sup>Stein (2012) introduces pecuniary externalities into the macroeconomic model with banks to focus on the risk of fire sales. Ikeda (2018) and Ikeda and Matsumoto (2021) study banks in a global game context and explore banks' excessive risk taking in the general equilibrium model.

The remainder of the paper proceeds as follows. Section 2 illustrates the model. In Section 3, we compare the competitive equilibrium in Section 2 with the constrained social optimum achieved by the SP banks. Section 4 discusses the numerical results and explains why the banking sector tends to be overleveraged. Section 5 concludes.

## 2 A Macroeconomy with Banks

### 2.1 Agents, Endowments, Preferences, and Technology

We consider an infinite-horizon OLG model with banks. Each generation of agents consists of households, entrepreneurs, and bankers. Each period, generation  $t$  is born at the beginning of period  $t$  and lives for two periods,  $t$  and  $t + 1$ . Each agent is identical and constant in the population.<sup>4</sup>

Households are risk averse and subject to a liquidity shock that affects their preference for consumption over the two periods. The liquidity shock is an aggregate shock and the only source of uncertainty in the model. Following DR, households are endowed with a unit of consumption goods at birth and do not consume the initially endowed consumption goods at the beginning of period  $t$ . The households deposit all initial endowments at banks operating in the same generation.<sup>5</sup> They receive wages  $w_t$  in the competitive labor market by supplying one unit of labor in both periods,  $t$  and  $t + 1$ .

Entrepreneurs are risk neutral and have access to capital-producing technology. They launch long-term investment projects at the beginning of period  $t$  by borrowing household endowments via the banks in the same generation. The investment project requires one period for gestation, and capital goods are produced in period  $t + 1$ . We call this capital-producing technology a “project.” Entrepreneurs sell the capital goods in the competitive market for the capital price  $q_{t+1}$ .

Banks raise funds from households and lend them to entrepreneurs at the beginning of period  $t$ .<sup>6</sup> In principle, we follow Diamond and Rajan (2001a) to model banks. Banks are risk neutral and competitive at raising and lending funds in the markets. They issue demand deposits (short-term debt) and commit to repaying the households. As the nature of

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<sup>4</sup>We assume that an initial old generation lives for one period.

<sup>5</sup>We implicitly assume the intraperiod perishability of endowments. More precisely, all endowments perish before the realization of the liquidity shock.

<sup>6</sup>We assume intragenerational banking, which effectively means that all bankers of generation  $t$  die out at the end of period  $t + 1$ .

demand deposits, banks can provide insurance against depositors' liquidity shocks. However, when households demand repayment before the completion of the entrepreneurs' projects, banks must liquidate premature projects to meet the demand for repayment. This maturity mismatch, represented by the combination of long-term assets and short-term liabilities, leaves banks exposed to risks of a default because, depending on the amount of withdrawals in the interim period, the banks' solvency is endangered.

The technology to produce consumption goods  $Y_t$  is represented by a standard constant-returns-to-scale Cobb–Douglas production function:

$$Y_t = F(K_t, ZH_t) = K_t^\alpha (ZH_t)^{1-\alpha},$$

where  $K_t$ ,  $H_t$ , and  $Z$  denote the capital stock, hours worked, and labor-augmenting technological progress, respectively. Demand for labor and capital satisfies

$$w_t = F_{H,t} = (1 - \alpha) \left( \frac{K_t}{ZH_t} \right)^\alpha Z \tag{1}$$

$$q_t = F_{K,t} = \alpha \left( \frac{K_t}{ZH_t} \right)^{\alpha-1}. \tag{2}$$

In what follows, we describe each agent's decisions (consumption, withdrawal, and liquidation of the entrepreneurs' projects) *after* the liquidity shock is realized. Then, we move on to the bank's decision on its leverage *before* the realization of the liquidity shock. Table 1 summarizes the sequence of events in each generation.

## 2.2 Households

Under the competitive banking sector, each household accepts the banks' offer on deposit face value  $D_t$  at the beginning of period  $t$ , and observes the liquidity shock  $\theta_t$  in the middle of period  $t$ . The liquidity shock is common across all households in the same generation and has the probability density function  $f(\theta_t)$  with a support of  $[0, 1]$ . This shock represents households' preference for consumption when young and signals the need for liquidity in period  $t$ .

After the realization of  $\theta_t$ , households make their decisions for consumption smoothing

without uncertainty. In period  $t$ , they try to withdraw  $g_t$  to maximize

$$\begin{aligned} U(C_{1,t}, C_{2,t+1}) &= \theta_t \log C_{1,t} + (1 - \theta_t) \log C_{2,t+1} \\ \text{s.t. } C_{1,t} &= w_t + g_t \\ C_{2,t+1} &= w_{t+1} + R_t (D_t - g_t), \end{aligned} \quad (3)$$

where  $C_{1,t}$  and  $C_{2,t+1}$  denote the consumption of households born in period  $t$  when young and old, respectively. Each household supplies a unit of labor in each period and receives wage income  $w_t$  in period  $t$  and  $w_{t+1}$  in period  $t + 1$ . Here,  $R_t$  denotes the one-period gross interest rate from period  $t$  to  $t + 1$ .

The intertemporal first-order condition for consumption is

$$\frac{\theta_t}{1 - \theta_t} \left( \frac{C_{1,t}}{C_{2,t+1}} \right)^{-1} = R_t. \quad (4)$$

The withdrawals can be written as

$$g_t = \theta_t \left( \frac{w_{t+1}}{R_t} + D_t \right) - (1 - \theta_t) w_t, \quad (5)$$

which implies that large  $\theta_t$  and  $D_t$  increase their withdrawals. To make the model nontrivial,  $g_t$  needs to be strictly positive. To ensure this condition, we assume that wage income in period  $t$  is sufficiently low relative to the initial endowment.

In our model, a financial crisis takes place endogenously, depending on  $\theta_t$  and  $D_t$ . When the banks cannot honor the debt, households run to the banks and their withdrawals equal the liquidation value of premature projects,  $X$  ( $< 1$ ), in period  $t$ , and nothing is left in period  $t + 1$ . In the case of a crisis, their consumption ends up with  $C_{1,t} = w_t + X$  and  $C_{2,t+1} = w_{t+1}$ .

### 2.3 Entrepreneurs

Entrepreneurs maximize their expected lifetime utility represented by  $E(C_{1,t}^e + C_{2,t+1}^e)$ , where  $C_{1,t}^e$  and  $C_{2,t+1}^e$  denote entrepreneurs' consumption when young and old, respectively. They use a unit of consumption goods financed by banks for their capital goods production, and this production technology takes one period for gestation before its completion. In period  $t + 1$ , the project yields a random capital goods output  $\tilde{\omega}$ , which is distributed

over  $[\omega_L, \omega_H]$  with the probability density function  $h(\tilde{\omega})$ .<sup>7</sup> If this project is prematurely liquidated in period  $t$ , the transformation from consumption goods into capital is incomplete. As a result, output is reduced to  $X$  units of consumption goods and is repaid fully to banks in period  $t$ . When the project is completed in period  $t + 1$ , however, entrepreneurs can sell their output in the capital goods market for the capital price  $q_{t+1}$ .

Each entrepreneur can borrow from a bank that has, or until the project matures, can learn knowledge about an alternative yet less profitable method to operate the project. The bank's specific knowledge allows it to generate  $\gamma q_{t+1} \tilde{\omega}$  from a project outcome with  $\gamma < 1$ . Once a bank has lent, no one else (including other banks) can learn this alternative way to operate the project. As a result, entrepreneurs accept the financing contract with each bank and repay  $\gamma q_{t+1} \tilde{\omega}$ . They are left with  $1 - \gamma$  of the share of their profit and enjoy their own consumption based on their linear utility. We assume that entrepreneurs are endowed with  $\underline{I}$  units of capital goods at the beginning of period  $t + 1$ .<sup>8</sup> They sell this endowment capital together with the newly created capital made from the consumption goods.

## 2.4 Banks

Banks maximize their expected lifetime utility  $E(C_{1,t}^b + C_{2,t+1}^b)$ , where  $C_{1,t}^b$  and  $C_{2,t+1}^b$  denote the consumption of banks when young and old, respectively. We borrow the microfoundation of modeling banks from Diamond and Rajan (2001a, b, 2012). Banks have no initial endowment at birth, and thus need to raise funds from households. As the relationship lender, a bank knows how to operate the entrepreneurs' project, which is knowledge that cannot be transferred to households. Banks issue demand deposits (short-term debt) as a commitment device to compensate for their lack of knowledge transferability (i.e., collection skills).<sup>9</sup> The demandable nature of deposit contracts creates a collective action problem for depositors: depositors run to the banks whenever depositors anticipate that the banks cannot honor the debt. The deposit contract is predetermined before observing the liquidity shock. In particular,  $D_t$  is predetermined at the beginning of period  $t$ , and a liquidity shock is realized in the middle of period  $t$ .

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<sup>7</sup>Following DR, we assume that there is no aggregate uncertainty in the project outcome.

<sup>8</sup>For simplicity, we assume a 100 percent depreciation rate in the law of motion for capital. The introduction of the endowment of capital goods here guarantees a finite capital price in the aftermath of a financial crisis in which all projects are scrapped because of full liquidation.

<sup>9</sup>Diamond and Rajan (2001b) discuss the microfoundation of maturity-mismatching banks and explain why these demand deposits can promote liquidity creation under the lack of transferability of their collection skill.



Each bank attracts many entrepreneurs through a competitive loan offer, resulting in an identical portfolio shared by all the symmetric banks. This assumption implies that each bank and the aggregate economy face an identical distribution of entrepreneurs. In period  $t$ , the banks receive signals  $\omega$  that perfectly predict the realized value of  $\tilde{\omega}$  in period  $t + 1$ . With this information  $\omega$  and the households' liquidity demand observed in period  $t$ , each bank chooses one of the following options: (i) to liquidate projects in period  $t$ , obtaining  $X$  of consumption goods per project; or (ii) to collect a fraction  $\gamma q_{t+1}\omega$  from a completed project in period  $t + 1$ . The bank liquidates the project if the outcome falls short of  $\tilde{\omega}_{t+1}$ , defined as a function of  $R_t/q_{t+1}$ :<sup>10</sup>

$$\tilde{\omega}_{t+1} = \frac{X}{\gamma} \frac{R_t}{q_{t+1}}. \quad (6)$$

Otherwise, the bank continues the project and receives repayment of  $\gamma q_{t+1}\omega$ , and then entrepreneurs consume the remaining fraction of outcome,  $(1 - \gamma) q_{t+1}\omega$ , per project. After repaying the full amount of the households' withdrawals, the banks consume their capital.

Let the banks' assets be  $A(R_t/q_{t+1})$ . The banks' assets at the beginning of period  $t$  (i.e., prior to the withdrawals) can be expressed as

$$\begin{aligned} A\left(\frac{R_t}{q_{t+1}}\right) &= \int_{\omega_L}^{\tilde{\omega}_{t+1}} X h(\omega) d\omega + \frac{\gamma q_{t+1}}{R_t} \int_{\tilde{\omega}_{t+1}}^{\omega_H} \omega h(\omega) d\omega \\ &= L\left(\frac{R_t}{q_{t+1}}\right) + \frac{\gamma q_{t+1}}{R_t} I\left(\frac{R_t}{q_{t+1}}\right). \end{aligned} \quad (7)$$

Note that  $h(\omega)$  is interchangeable with  $h(\tilde{\omega})$  owing to perfect signaling. The banks' assets denoted in (7) can be decomposed into two components: the values of the prematurely liquidated projects  $L_t = L(R_t/q_{t+1}) \equiv \int_{\omega_L}^{\tilde{\omega}_{t+1}} X h(\omega) d\omega$ , which is used to meet the liquidity demand (i.e., withdrawals) from the households, and the banks' share of the investment output (measured by the present value of consumption goods), denoted as  $\gamma q_{t+1} I_t / R_t$ , where  $I_t = I(R_t/q_{t+1}) = \int_{\tilde{\omega}_{t+1}}^{\omega_H} \omega h(\omega) d\omega$ .

The banks are subject to the *solvency constraint*:

$$D_t \leq A(R_t/q_{t+1}). \quad (8)$$

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<sup>10</sup>Equation (6) can be reinterpreted as follows:  $\gamma q_{t+1}\omega/X$  corresponds to the marginal rate of transformation (MRT) between the period- $t$  consumption goods (i.e., liquidation) and the period- $t + 1$  consumption goods (i.e., continuation of projects). Here, the MRT is compared with the marginal rate of substitution of the households observed as the interest rate,  $R_t$ .

If  $\omega$  follows a uniform distribution,  $A(\cdot)$  monotonically decreases with  $R_t/q_{t+1}$ . We then define the relative price  $R_t^*/q_{t+1}^*$  that satisfies the solvency constraint with equality:

$$D_t = A\left(\frac{R_t^*}{q_{t+1}^*}\right). \quad (9)$$

We refer to  $R_t^*$  and  $q_{t+1}^*$  as the *threshold* interest rate and capital price, respectively. Hereafter, we denote a variable with an asterisk as the variable on the threshold. For the purpose of subsequent discussion, we note that given  $A(R_t/q_{t+1})$ , the bank leverage  $D_t/(A_t - D_t)$  is uniquely determined once  $D_t$  is chosen, and hence, we refer to  $D_t$  as leverage hereafter.

## 2.5 Market Clearing Conditions

Four markets need to clear in the competitive equilibrium: (i) liquidity; (ii) consumption goods; (iii) capital goods; and (iv) labor. The liquidity market clearing condition is given by

$$L\left(\frac{R_t}{q_{t+1}}\right) = \theta_t \left(\frac{w_{t+1}}{R_t} + D_t\right) - (1 - \theta_t) w_t. \quad (10)$$

Next, the market clearing condition for consumption goods is

$$Y_t + L\left(\frac{R_t}{q_{t+1}}\right) = C_{1,t} + C_{2,t} + C_{1,t}^e + C_{2,t}^e + C_{1,t}^b + C_{2,t}^b. \quad (11)$$

The left-hand side of (11) includes the supply of goods from the liquidated projects. On the right-hand side of (11),  $C_{2,t}$ ,  $C_{2,t}^e$ , and  $C_{2,t}^b$  are consumption when generation  $t - 1$  is old.

The capital goods market clearing condition is

$$K_{t+1} = \begin{cases} \underline{I} + I(R_t/q_{t+1}) & \text{at normal times} \\ \underline{I} & \text{at crises,} \end{cases} \quad (12)$$

where  $\underline{I}$  denotes the capital good endowment. Here, the equation suggests that the capital goods supply sharply declines in the aftermath of a crisis. Throughout the paper, we use  $\underline{w}$  and  $\underline{F}_H$  to denote the wage rate and marginal product of labor evaluated at  $K_{t+1} = \underline{I}$ , respectively.

Finally, both young and old generations supply a unit of labor in each period. Therefore,  $H_t$  equals two for all  $t$ .

## 2.6 Optimal Bank Leverage

We now consider how the banks choose their optimal leverage individually. In the next section, we compare the optimal leverage chosen individually to the constrained social optimum. The banks are competitive at issuing demand deposits, and household endowments are scarce compared with entrepreneurs' projects. As a result of competition, the banks make a competitive offer of deposits for households, aiming to maximize household welfare (Allen and Gale 1998, 2007), while, in fact, they are maximizing their profits. Maximizing household utility via the deposit offers means that banks determine the offer taking the optimal behavior of the price-taking households into account.

In making the competitive offer to households, the banks take possible changes in the crisis probability  $\pi_t$  into account. To understand how the banks' choice of  $D_t$  affects  $\pi_t$ , we take three steps. First, we define a function  $R_{LF}^*$  as

$$R_{LF}^*(D_t) = q_{t+1}^* A^{-1}(D_t), \quad (13)$$

from (9). Second, using (10), we define a function  $\theta_{LF}^*$  as

$$\theta_{LF}^*(D_t) = \frac{L(R_t^*/q_{t+1}^*) + w_t}{w_t + D_t + w_{t+1}^*/R_t^*}, \quad (14)$$

where  $R_t^* = R_{LF}^*(D_t)$  and  $w_t$ ,  $w_{t+1}^*$ , and  $q_{t+1}^*$  are taken as given for the LF banks. Third, we connect  $\theta_t^*$  to the crisis probability  $\pi_t$ . Equation (13) means that any changes in  $D_t$  give rise to changes in  $R_t^*/q_{t+1}^*$ . Once  $R_t^*/q_{t+1}^*$  is determined, we can calculate the liquidity demand  $L(R_t^*/q_{t+1}^*)$  in which the solvency constraint is satisfied with equality. For the liquidity market to clear, we obtain the threshold level of the liquidity shock on the brink of a financial crisis. Namely, when  $\theta_t$  is strictly greater than  $\theta_t^*$ , the banks turn out to be insolvent and a crisis is precipitated. Thus, the crisis probability  $\pi_t$  has a one-to-one relationship with  $\theta_t^*$  via the probability density function  $f(\theta_t)$ :

$$\pi_t = \int_{\theta_t^*}^1 f(\theta_t) d\theta_t. \quad (15)$$

We are now ready to set up the optimization problem for the banks to determine the level of their leverage.

**Problem LF** *The laissez-faire banks maximize the household expected utility*

$$\begin{aligned} & \max_{D_t} \int_0^{\theta_t^*} \{\theta_t \ln(w_t + L_t) + (1 - \theta_t) \ln[w_{t+1} + R_t(D_t - L_t)]\} f(\theta_t) d\theta_t \\ & + \int_{\theta_t^*}^1 [\theta_t \ln(w_t + X) + (1 - \theta_t) \ln(\underline{w})] f(\theta_t) d\theta_t, \end{aligned} \quad (16)$$

subject to

$$\begin{aligned} L\left(\frac{R_t}{q_{t+1}}\right) &= \theta_t \left(\frac{w_{t+1}}{R_t} + D_t\right) - (1 - \theta_t) w_t, \\ \theta_{LF}^*(D_t) &= \frac{L(R_t^*/q_{t+1}^*) + w_t}{w_t + D_t + w_{t+1}^*/R_t^*}, \end{aligned}$$

where  $R_t^* = R_{LF}^*(D_t)$  from (13) and  $\theta_t^* = \theta_{LF}^*(D_t)$  from (14).

The first-order condition for their leverage is

$$\begin{aligned} & \{[\theta_t^* \ln(\theta_t^* m_t^*) + (1 - \theta_t^*) \ln(R_t^* (1 - \theta_t^*) m_t^*)] \\ & - [\theta_t^* \ln(w_t + X) + (1 - \theta_t^*) \ln(\underline{w})]\} \frac{d\pi_t}{d\theta_t^*} \theta_{LF}'(D_t) \\ & = \int_0^{\theta_t^*} \left[ \frac{1}{m_t} \left(1 - \frac{w_{t+1}}{R_t^2} R_{LF}'(D_t, \theta_t)\right) + (1 - \theta_t) \frac{R_{LF}'(D_t, \theta_t)}{R_t} \right] f(\theta_t) d\theta_t, \end{aligned} \quad (17)$$

where  $m_t \equiv w_t + D_t + w_{t+1}/R_t$  is the lifetime income of households and  $m_t^* \equiv w_t + D_t + w_{t+1}^*/R_t^*$ , accordingly. More importantly,  $\theta_{LF}'(D_t)$  is calculated from (14), taking capital prices and wages as given. Likewise,  $R_{LF}'(D_t, \theta_t)$  is calculated from (10), taking capital prices, wages, and  $\theta_t$  as given. Here, we abuse notations slightly. While we use the prime for  $\theta_{LF}'(D_t)$  and  $R_{LF}'(D_t, \theta_t)$  to denote marginal changes in  $\theta_t^*$  and  $R_t$  with respect to  $D_t$ , they are *not* the total derivative that internalizes the general equilibrium effect of  $D_t$  on factor prices. We compare these notations in the next section.

Equation (17) provides an economic intuition. The terms in curly brackets on the left-hand side of (17) represent the loss of utility in a crisis compared with the threshold. From (15), the term outside the curly brackets indicates the marginal changes in a crisis probability with respect to bank leverage. The left-hand side of the equation is the expected loss of utility multiplied by the marginal change in the crisis probability. Simply put, the left-hand side of (17) is the marginal cost of increasing  $D_t$ .

The right-hand side of (17) consists of the effects of increasing leverage on the expected

households' utility through their lifetime income. On the one hand, the increase in  $D_t$  has a positive effect on households' income: the higher the leverage, the larger the withdrawal, allowing households to enjoy more consumption. On the other hand, the increase in  $D_t$  leads to a higher interest rate via liquidity shortage, discounting the households' labor income in period  $t+1$  and reducing returns on forgoing withdrawal until period  $t+1$ . Therefore, as long as the positive effect on lifetime income exceeds the effect on the interest rate, the higher leverage is beneficial to households. Simply put, the right-hand side of (17) is the marginal benefit of increasing  $D_t$ .

Before defining the equilibrium, some remarks are in order. In characterizing the equilibrium, bank leverage  $D_t$  plays a key role, as discussed in Sections 2.4 and 2.6. This is in the context of the liquidity market, where demand and supply are given by  $g_t$  and  $L(R_t/q_{t+1})$  in (10), respectively, which is at the center of our model. In the liquidity market, only households demand liquidity because both banks and entrepreneurs choose not to consume in the first period because of the linear utility. In equilibrium, all markets, that is, markets for liquidity, consumption goods, capital goods, and labor, must clear. We are now ready to define the competitive equilibrium as follows.

**Definition (A competitive equilibrium)** *A competitive equilibrium consists of allocations and prices  $\{g_t, D_t, L_t, K_t, I_t, H_t, R_t, q_t, w_t\}_{t=0}^{\infty}$  such that (i) withdrawal decisions are given by (5) for  $\theta_t \leq \theta_t^*$ ; (ii) banks' leverage satisfies (17); (iii) banks' liquidity supply is determined by (6); and (iv) all markets clear.*

### 3 Social Planning Banks

Next, we consider the SP banks that choose their leverage as the constrained social planner. We call the allocations for the SP banks the constrained social optimum and compare the allocations with those for the LF banks. To lead off the analysis, we clarify the constraint to which the SP banks are subject. The SP banks must make all their decisions before observing  $\theta_t$ . After realizing  $\theta_t$ , they are left with no options. In other words, the SP banks are subject to the constraint that they cannot control household consumption in response to a realized value of  $\theta_t$ . The allocations chosen by the SP banks must be distinguished from the unconstrained, first-best optimum. The SP banks precommit to payment on their debt regardless of the states realized following their commitment. The extra ability given to the SP banks compared with the LF banks is that the former can internalize all price effects in

all markets when they make decisions regarding their leverage.

Formally, Problem SP replaces  $q_t$  and  $w_t$  in Problem LF with  $F_{K,t}$  and  $F_{H,t}$ , respectively. SP banks are faced with the solvency constraint  $D_t \leq A(R_t/F_{K,t+1})$ . The newly introduced solvency constraint for the SP banks has different effects on the threshold because (9) and (13) are now replaced with

$$D_t = A\left(\frac{R_t^*}{F_{K,t+1}^*}\right), \quad (18)$$

$$R_{SP}^*(D_t) = F_{K,t+1}^* A^{-1}(D_t). \quad (19)$$

We summarize the SP banks' problem as follows.

**Problem SP** *The social planning banks maximize household expected utility,*

$$\begin{aligned} & \max_{D_t} \int_0^{\theta_t^*} \{\theta_t \ln(F_{H,t} + L_t) + (1 - \theta_t) \ln[F_{H,t+1} + R_t(D_t - L_t)]\} f(\theta_t) d\theta_t \\ & + \int_{\theta_t^*}^1 [\theta_t \ln(F_{H,t} + X) + (1 - \theta_t) \ln \underline{F}_H] f(\theta_t) d\theta_t, \end{aligned}$$

subject to

$$L\left(\frac{R_t}{F_{K,t+1}}\right) = \theta_t \left(\frac{F_{H,t+1}}{R_t} + D_t\right) - (1 - \theta_t) F_{H,t} \quad (20)$$

$$\theta_{SP}^*(D_t) \equiv \frac{L(R_t^*/F_{K,t+1}^*) + F_{H,t}}{F_{H,t} + D_t + F_{H,t+1}^*/R_t^*}, \quad (21)$$

where  $R_t^* = R_{SP}^*(D_t)$  from (19) and  $\theta_t^* = \theta_{SP}^*(D_t)$  from (21).

The first-order condition for the SP banks' leverage is

$$\begin{aligned} & \{[\theta_t^* \ln(\theta_t^* m_t^*) + (1 - \theta_t^*) \ln(R_t^* (1 - \theta_t^*) m_t^*)] \\ & - [\theta_t^* \ln(F_{H,t} + X) + (1 - \theta_t^*) \ln \underline{F}_H]\} \frac{d\pi_t}{d\theta_t^*} \theta_{SP}'^*(D_t) \\ & = \int_0^{\theta_t^*} \left[ \frac{1}{m_t} \left(1 - \frac{w_{t+1}}{R_t^2} R_{SP}'(D_t, \theta_t)\right) + (1 - \theta_t) \frac{R_{SP}'(D_t, \theta_t)}{R_t} \right] f(\theta_t) d\theta_t, \end{aligned} \quad (22)$$

where  $m_t \equiv F_{H,t} + D_t + F_{H,t+1}/R_t$  and  $m_t^* \equiv F_{H,t} + D_t + F_{H,t+1}^*/R_t^*$ . Importantly, the SP banks factor in all general equilibrium effects when they calculate  $\theta_{SP}'^*(D_t)$  and  $R_{SP}'(D_t, \theta_t)$ .

We define the allocations chosen by the SP banks as follows.

**Definition (Constrained social optimum)** *Allocations and prices under the constrained social optimum are  $\{g_t, D_t, L_t, K_t, I_t, H_t, R_t, q_t, w_t\}_{t=0}^{\infty}$  such that (i) withdrawal decisions are given by (5) for  $\theta_t \leq \theta_t^*$ ; (ii) banks' leverage satisfies (22); (iii) banks' liquidity supply is determined by (6); and (iv) all markets clear.*

In general, the allocations achieved by the SP banks differ from those achieved by the LF banks because  $\theta_{SP}^*(D_t)$  and  $R'_{SP}(D_t, \theta_t)$  are not necessarily the same as  $\theta_{LF}^*(D_t)$  and  $R'_{LF}(D_t, \theta_t)$ , respectively. In fact, because the two problems are subject to the same constraints, any discrepancy in the first-order conditions results in different allocations between the two problems. Moreover, as long as the LF banks calculate  $\theta_{LF}^*(D_t)$ , taking capital prices and wages as given, the LF banks cannot achieve *the constrained social optimum* that the SP banks can achieve. We summarize the result in the following proposition.

**Proposition 1** *The allocations under the competitive equilibrium are less efficient than those under the constrained social optimum.*

While the presence of the inefficiency in the competitive equilibrium is clear, the above result does not necessarily imply overleverage by the LF banks. The next subsection focuses on the direction of the inefficiency, relying on numerical simulations.

## 4 Numerical Results

### 4.1 Calibration

This section provides numerical simulations of the model to explore the inefficiency in the competitive equilibrium. We numerically consider the following questions: (i) Do the LF banks take on excessive risks?; (ii) If yes, what is the primary source of overleverage?; and (iii) How quantitatively important is the solvency constraint for overleverage?

Our calibration mostly follows DR. We set the value of prematurely liquidated project  $X$  at 0.95. We assume that  $\omega$  follows the uniform distribution over a range of  $[\omega_L, \omega_H] = [0.5, 3.5]$ , similar to the original parameter values in DR. The degree of banks' special collection skills  $\gamma$  is set at 0.9. In addition to parameterization of DR, we need to set several other parameters. We calibrate the capital share in the production function,  $\alpha$ , to 1/3. The capital goods endowment received by entrepreneurs,  $\underline{I}$ , and technological progress,  $Z$ ,

determine the size of the scarring effect of a financial crisis. Here, we set  $\underline{I} = 1$  and  $Z = 4$ . More importantly, we assume that the liquidity shock  $\theta_t$  follows the beta distribution with a mean of 0.50 and a standard deviation of 0.07. This parameterization implies a symmetric bell-shaped distribution.

To simulate the model, we numerically solve the nonlinear system of the equations consisting of the first-order conditions and resource constraints.<sup>11</sup> The system of equations includes equations that define the threshold variables, as well as those describing normal times. For example, the system includes the solvency constraint (9), which defines the relative price  $R_t^*/q_{t+1}^*$  on the brink of a crisis.

Before interpreting the numerical results, we reconfirm the economic interpretations of  $D_t$ . In the context of our model,  $D_t$  represents the gross return to bank deposits. On the other hand,  $D_t$  cannot be translated into an annual percentage rate or an interest rate *per annum*, because the model does not specify the length of each period of time (e.g., 1 year, 10 years). To focus more clearly on the economic interpretations,  $D_t$  needs to be translated into a timeless measure such as bank leverage.

## 4.2 Excessive Risk Taking by LF Banks

We now summarize the benchmark results in Table 2. The results suggest that the LF banks take on excessive risks in determining their leverage. Thus, the direction of inefficiency can support macroprudential policy. More specifically, the upper panel of the table reports that the LF banks choose a higher level of leverage  $D_t$  than do the SP banks. As the first row shows, the LF banks set the level of  $D_t$  at 1.061, which is 1.2 percentage points higher than the constrained social optimum of 1.049. This overleverage of the LF banks leads to a higher crisis probability. Our calibration points to a 6.59 percent crisis probability under the competitive equilibrium compared with 4.50 percent under the constrained social optimum. These numbers may be comparable with some recent empirical studies on crises.<sup>12</sup> The lower panel of Table 2 compares the bank capital ratio, defined as  $(A_t - D_t)/A_t$ , and the output of consumption goods  $Y_{t+1}$  under the competitive equilibrium and the constrained social optimum when the realized value of  $\theta_t$  takes a mean of 0.5. The LF banks are undercapitalized by 1.1 percentage points compared with the constrained social optimum. Nevertheless,

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<sup>11</sup>For details, see the Appendix.

<sup>12</sup>Based on data going back to the 1800s, Reinhart and Rogoff (2013) report that the frequencies are 7.2 percent for the advanced economies. The Basel Committee on Banking Supervision (2010, BCBS) uses multiple datasets on banking crises. According to the BCBS, the frequency ranges from 3.6 to 6.8 percent.



production does not substantially differ between the two allocations, provided that a financial crisis does not take place. The above exercise indicates that the welfare loss primarily arises from inefficiently elevated crisis probability.

It would be helpful to see our results graphically. The higher  $\pi_t$  in the competitive equilibrium means that the economy undergoes too frequent crises created by pecuniary externalities. To illustrate this, we run the model over 100 periods by randomly generating liquidity shocks. The upper panel of Figure 1 plots the dynamic paths of output  $Y_t$  generated by the same random shock sequences across two economies. The red lines correspond to the competitive equilibrium, while the blue lines point to the constrained social optimum.

Comparisons between the two economies reveal that the dynamic paths of the production of consumption goods are almost identical except for more frequent crises under the competitive equilibrium. Under the competitive equilibrium, crises take place in periods 5, 16, and 94, and  $Y_t$  falls sharply in each subsequent period. This simulation result indicates that the last two crises should have been prevented if the banks had taken risks at the optimal level. However, the first crisis takes place even in the constrained social optimum with the SP banks. The crisis in this allocation should not be avoided in the context of the optimal financial crises (see Allen and Gale 1998).<sup>13</sup>

In the model, the liquidity shock  $\theta_t$  precipitates financial crises. As shown in the lower panel of Figure 1, financial crises in our model take place only when  $\theta_t$  exceeds the threshold  $\theta_t^*$ . For example, the liquidity shock  $\theta_t$  in period 5 exceeds the curves of  $\theta_t^*$  for both the LF and SP banks, and the output of consumption goods falls in period 6. However,  $\theta_t^*$  for the LF banks is always lower than  $\theta_t^*$  for the SP banks, which implies that the financial system is more vulnerable to liquidity shocks under the competitive equilibrium than under the constrained social optimum.

The overleveraged LF banking system that results in inefficient financial crises remains a robust outcome across a range of calibrated parameters (e.g.,  $X$  and distributions of  $\omega$  and  $\theta_t$ ).<sup>14</sup> In addition to robustness, sensitivity analysis regarding the volatility of  $\theta_t$  has noteworthy implications for the inherent fragility of the banking system. Table 3 examines how bank leverage and crisis probabilities are affected by the volatility of  $\theta_t$ . In each simulation, the standard deviation of  $\theta_t$  changes from the benchmark value of 0.07 to either 0.02 or 0.10, with other parameters held unchanged at the benchmark calibration. The table, together

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<sup>13</sup>Their model includes a single liquidity market where the banks internalize all the effects through the single price changes. By contrast, our model includes multiple markets (e.g., the capital market), in which banks fail to internalize their general equilibrium effect.

<sup>14</sup>The tables of the sensitivity analysis are available upon request.

with the benchmark results, confirms that the crisis probability rises monotonically along with the increase in the standard deviation of  $\theta_t$ . Note that smaller volatility of the liquidity shock does not fully wipe out the financial crisis because the banks raise their leverage under the less volatile liquidity demand. As a result, a financial crisis always remains a nonzero probability event.

### 4.3 The Primary Source of Overleverage

To understand the LF banks' overleverage in more detail, we numerically compare the optimality condition for  $D_t$  under the two allocations. Figure 2 plots the marginal cost and benefit of increasing the leverage under the two allocations. In the figure, we compute the curves based on (17) and (22).<sup>15</sup> The solid blue line represents the marginal cost for the SP banks (the right-hand side of (22)), while the blue dashed line represents the marginal benefit for the SP banks (the left-hand side of (22)). The red solid and dashed lines are the marginal cost and benefit for the LF banks, respectively, calculated from (17). Under the competitive equilibrium,  $D_t$  is 1.061 at the intersection of the two red lines. In Figure 2, however, the marginal benefit for the SP banks is not as high as that for the LF banks. Numerically, if we evaluate the SP banks' marginal benefits at  $D_t$  in the competitive equilibrium, the marginal benefit for the SP banks is 4.5 percent lower than that for the LF banks. By contrast, the distortion in the marginal cost is much larger than that in the marginal benefit. The marginal cost for the SP banks is not as low as that for the LF banks. Numerically, if we evaluate the SP banks' marginal cost at  $D_t$  in the competitive equilibrium, the marginal cost for the SP banks is 29.2 percent higher than that for the LF banks.

Given that the distortion is substantially large in the marginal cost, it would be useful to focus on the marginal cost as the primary source of overleverage. We are interested in the difference between the left-hand side of the equations (17) and (22). At a given level of  $D_t$  in the figure, our observation is as follows.

$$\begin{aligned}
& \{[\theta_t^* \ln(\theta_t^* m_t^*) + (1 - \theta_t^*) \ln(R_t^* (1 - \theta_t^*) m_t^*)] \\
& - [\theta_t^* \ln(w_t + X) + (1 - \theta_t^*) \ln(\underline{w})]\} \frac{d\pi_t}{d\theta_t^*} \theta_{SP}'(D_t) \\
> & \{[\theta_t^* \ln(\theta_t^* m_t^*) + (1 - \theta_t^*) \ln(R_t^* (1 - \theta_t^*) m_t^*)] \\
& - [\theta_t^* \ln(F_{H,t} + X) + (1 - \theta_t^*) \ln(\underline{F}_H)]\} \frac{d\pi_t}{d\theta_t^*} \theta_{LF}'(D_t)
\end{aligned}$$

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<sup>15</sup>For details of the computations, see the Appendix.

All the difference between the marginal cost for the SP and LF banks arises from the terms outside of the curly brackets because the marginal products replace all factor prices. Because the expression inside the curly brackets is naturally positive,<sup>16</sup> the above expression can further be simplified to

$$\frac{d\pi_t}{d\theta_t^*}\theta_{SP}^{*'}(D_t) > \frac{d\pi_t}{d\theta_t^*}\theta_{LF}^{*'}(D_t). \quad (23)$$

The above equation indicates that the primary source of the overleverage in our model is the banks' evaluation of the marginal changes in the crisis probability with respect to  $D_t$ . The LF banks take other banks' decisions as given. However, the crisis probability is affected by the synchronized decisions by the banking sector as a whole. Equation (23) means that the LF banks do not take the extra marginal increase in the crisis probability into account when they increase their leverage individually. However, because all banks take the same actions in equilibrium, the LF banks are overleveraged compared with the socially optimal level. In our numerical simulations,  $(d\pi_t/d\theta_t^*)\theta_{SP}^{*'}(D_t) = 1.993$  and  $(d\pi_t/d\theta_t^*)\theta_{LF}^{*'}(D_t) = 1.544$ . If leverage is increased by 1 percent from the allocation in the competitive equilibrium, each LF bank expects that the crisis probability increases up to 8.22 percent by its individual decision, but the banking sector as a whole is, in fact, exposed to a higher crisis probability of 8.69 percent.<sup>17</sup>

#### 4.4 The Importance of the Pecuniary Externality Stemming from the Solvency Constraint

The tightness of the solvency constraint is closely related to the crisis probability. Recall that the solvency constraint of the LF banks is  $D_t = A(R_t^*/q_{t+1}^*)$ , or equivalently  $R_{LF}^*(D_t) = q_{t+1}^*A^{-1}(D_t)$ . When the LF banks determine the leverage, they internalize  $R_t^*$ . However, they take  $q_{t+1}^*$  as given because  $q_{t+1}^*$  is the price outside the liquidity market. The overestimation of the banks' solvency leads to their overleverage and the inefficiently high crisis probability.

To assess the importance of the pecuniary externality stemming from the solvency con-

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<sup>16</sup>For the expression inside the curly brackets, note that  $\theta_t^* \ln(\theta_t^* m_t^*) + (1 - \theta_t^*) \ln(R_t^* (1 - \theta_t^*) m_t^*)$  is the lifetime utility on the brink of a financial crisis, while  $[\theta_t^* \ln(w_t + X) + (1 - \theta_t^*) \ln(\underline{w})]$  is the lifetime utility at crisis evaluated at  $\theta_t = \theta_t^*$ . The first-order conditions are satisfied at  $\theta_t = \theta_t^*$  in the former, but not in the latter. As long as  $X < 1$ , the expression inside the curly brackets is strictly positive.

<sup>17</sup>These increases in the probability are obtained by transforming the marginal changes in the probability into the semielasticity of the probability:  $(d\pi_t/d\theta_t^*)\theta_{LF}^{*'}(D_t) \times D_{LF,t} = d\pi_t/(dD_t/D_{LF,t}) = 1.64$  and  $(d\pi_t/d\theta_t^*)\theta_{SP}^{*'}(D_t) \times D_{LF,t} = d\pi_t/(dD_t/D_{LF,t}) = 2.11$ . Together with 6.58 percent of the crisis probability, the crisis probability perceived by the LF banks is 8.22 ( $= 6.58 + 1.64$ ), but the true probability is 8.69 ( $= 6.58 + 2.11$ ).

straint, we perform a counterfactual experiment. Suppose that hypothetical banks internalize changes in  $q_{t+1}^*$  in the solvency constraint  $D_t = A(R_t^*/q_{t+1}^*)$ , while they do not internalize other wedges caused by the pecuniary externalities (e.g., wages). In particular, the hypothetical banks calculate the direct effects of changes in  $q_{t+1}^*$  on their solvency in the same way the SP banks do.<sup>18</sup> Apart from the effects on their solvency, they take other prices as given, as in the LF banks' decision-making. While the allocations implied by the hypothetical banks are not sustainable as an equilibrium, this counterfactual experiment helps assess the importance of the pecuniary externality stemming from the solvency constraint.

The pecuniary externality stemming from the solvency has a substantial impact on allocations. In the simulations, the banks choose their leverage at  $D_t = 1.047$ . The leverage in the counterfactual allocation is lower than the leverage in the competitive equilibrium (1.061). As a result, the crisis probability is also low (4.18 percent) compared with 6.58 percent in the competitive equilibrium. This level of the leverage ( $D_t = 1.047$ ) is close to that in the constrained social optimum (1.049). Consequently, the crisis probability of 4.18 percent is close to the probability under the constrained social optimum (4.50 percent). The proximity to the constrained social optimum implies that the welfare level is also close to the constrained social optimum.

Figure 3 additionally plots the marginal cost against  $D_t$  for the hypothetical banks. For comparison, we replicate the curves for the marginal cost and the marginal benefit shown in Figure 2. The marginal benefit for hypothetical banks is not shown in the figure because it remains the same as the marginal benefit for the LF banks. All the difference from the competitive equilibrium is the internalization of the pecuniary externality stemming from the solvency constraint. In the figure, the internalization leads to the upward shift of the curve for the marginal cost. The marginal cost for the hypothetical banks is 41.6 percent higher than that for the LF banks if evaluated at  $D_t$  in the competitive equilibrium.

We emphasize that our experiment is counterfactual. The LF banks have no incentive to internalize this pecuniary externality in the solvency constraint. From the households' viewpoint,  $D_t = 1.047$  is less attractive than  $D_t = 1.061$  in the competitive equilibrium. Each LF bank is selfish and lacks incentives to coordinate with other banks. The competition across the LF banks ill-incentivizes banks to take on excessive risks systemically.

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<sup>18</sup>While the straightforward empirical counterpart of  $q_t$  is stock prices, any market price of financial assets held by price-taking banks could have similar effects via their solvency constraints.

## 5 Conclusions

We extended the DR model of banks with production factors and explore how a pecuniary externality affects a bank's leverage. This extension generates a pecuniary externality because the banks do not internalize their general equilibrium effect of the factor prices on the economy. As argued in the existing literature on the pecuniary externality, however, the direction of inefficiency is model-dependent. In other words, the pecuniary externality itself does not justify the macroprudential policies.

Using numerical simulations, we showed that, compared with the constrained social optimum, banks in the competitive equilibrium could take on excessive risks. In the benchmark calibration, the crisis probability is 6.58 percent in the competitive equilibrium, higher than the 4.50 percent under the constrained social optimum. Our findings include the importance of the pecuniary externality stemming from the bank's solvency for generating their overleverage. We also performed counterfactual analysis in which banks internalize only the pecuniary externality stemming from the solvency constraint. In this case, the chosen leverage is very close to the constrained social optimum.

The analysis in this paper can be extended in a number of directions. First, it would be useful to examine how newly introduced aggregate shocks (e.g., shocks to the asset side of banks' balance sheets) affect the economy's exposure to crisis risks. Second, the optimal macroprudential policy could be analyzed. Third, our framework may be translated into an infinitely-lived agent model for integration with quantitative business cycle studies. All of these directions would provide important avenues for future research.

## References

- [1] Allen, F. and D. Gale (1998). “Optimal Financial Crises,” *Journal of Finance*, 53(4), pp. 1245-1284.
- [2] Allen, F. and D. Gale (2007). *Understanding Financial Crises*, Oxford University Press.
- [3] Angeloni, I. and E. Faia (2013). “Capital Regulation and Monetary Policy with Fragile Banks,” *Journal of Monetary Economics*, 60(3), pp. 311-324.
- [4] Basel Committee on Banking Supervision (2010). “An Assessment of the Long-Term Economic Impact of Stronger Capital and Liquidity Requirements,” <http://www.bis.org/publ/bcbs173.htm>
- [5] Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. R. Young (2013). “Financial Crises and Macro-prudential Policies,” *Journal of International Economics*, 89(2), pp. 453-470.
- [6] Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. R. Young (2016). “Optimal Capital Controls and Real Exchange Rate Policies: A Pecuniary Externality Perspective,” *Journal of Monetary Economics*, 84, pp. 147-165.
- [7] Bianchi, J. (2010). “Credit Externalities: Macroeconomic Effects and Policy Implications,” *American Economic Review*, 100(2), pp. 398-402.
- [8] Bianchi, J. (2011). “Overborrowing and Systemic Externalities in the Business Cycle,” *American Economic Review*, 101(7), pp. 3400-3426.
- [9] Bianchi, J. and E. G. Mendoza (2018). “Optimal Time-Consistent Macroprudential Policy,” *Journal of Political Economy*, 126(2), pp. 588-634.
- [10] Boissay, F., F. Collard, and F. Smets (2016). “Booms and Banking Crises,” *Journal of Political Economy*, 124(2), pp. 489-538.
- [11] Dávila, E. and A. Korinek (2017). “Pecuniary Externalities in Economies with Financial Frictions,” *Review of Economic Studies*, 85(1), pp. 352-395.
- [12] Diamond, D. W. and P. Dybvig (1983). “Bank Runs, Deposit Insurance, and Liquidity,” *Journal of Political Economy*, 91(3), pp. 401-419.

- [13] Diamond, D. W. and R. G. Rajan (2000). “A Theory of Bank Capital,” *Journal of Finance*, 55(6), pp. 2431-2465.
- [14] Diamond, D. W. and R. G. Rajan (2001a). “Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking,” *Journal of Political Economy*, 109(2), pp. 287-327.
- [15] Diamond, D. W. and R. G. Rajan (2001b). “Banks and Liquidity,” *American Economic Review*, 91(2), pp. 422-425.
- [16] Diamond, D. W. and R. G. Rajan (2012). “Illiquid Banks, Financial Stability, and Interest Rate Policy,” *Journal of Political Economy*, 120(3), pp. 552-591.
- [17] Gertler, M. and P. Karadi (2011). “A Model of Unconventional Monetary Policy,” *Journal of Monetary Economics*, 58(1), pp. 17-34.
- [18] Gertler, M. and N. Kiyotaki (2011). “Financial Intermediation and Credit Policy in Business Cycle Analysis,” *Handbook of Monetary Economics*, B. Friedman and M. Woodford (eds.), 3A, pp. 547-599.
- [19] Gertler, M. and N. Kiyotaki (2015). “Banking, Liquidity and Bank Runs in an Infinite Horizon Economy,” *American Economic Review*, 105(7), pp. 2011-2043.
- [20] Gorton, G. (1988). “Banking Panics and Business Cycles,” *Oxford Economic Papers*, 40(4), pp. 751-781.
- [21] Gorton, G. and A. Metrick (2012). “Securitized Banking and the Run on Repo,” *Journal of Financial Economics*, 104(3), pp. 425-451.
- [22] Ikeda, D. (2018). “Bank Runs, Prudential Tools and Social Welfare in a Global Game General Equilibrium,” Staff Working Paper No. 732, Bank of England.
- [23] Ikeda, D. and H. Matsumoto (2021). “Procyclical Leverage and Crisis Probability in a Macroeconomic Model of Bank Runs,” IMES Discussion Paper No. 2021-E-1, Bank of Japan.
- [24] Jeanne, O. and A. Korinek (2019). “Managing Credit Booms and Busts: A Pigouvian Taxation Approach,” *Journal of Monetary Economics*, 107, pp. 2-17.

- [25] Kato, R. and T. Tsuruga (2016). “The Safer, the Riskier: A Model of Financial Instability and Bank Leverage,” *Economic Modelling*, 52, pp. 71-77.
- [26] Korinek, A. (2018). “Regulating Capital Flows to Emerging Markets: An Externality View,” *Journal of International Economics*, 111, pp. 61-80.
- [27] Lorenzoni, G. (2008). “Inefficient Credit Booms,” *Review of Economic Studies*, 75(3), pp. 809-833.
- [28] Meh, C. and K. Moran (2010). “The Role of Bank Capital in the Propagation of Shocks,” *Journal of Economic Dynamics and Control*, 34(3), pp. 555-576.
- [29] Mendoza, E. (2010). “Sudden Stops, Financial Crises and Leverage,” *American Economic Review*, 100(5), pp. 1941-1966.
- [30] Reinhart, C. M. and K. S. Rogoff (2013). “Banking Crises: An Equal Opportunity Menace,” *Journal of Banking & Finance*, 37(11), pp. 4557-4573.
- [31] Stein, J. C. (2012). “Monetary Policy as Financial-Stability Regulation,” *Quarterly Journal of Economics*, 127(1), pp. 57-95.



## A System of Equations

Given  $K_t$ , the system of equations consists of 15 equations and 15 unknown variables  $D_t$ ,  $\pi_t$ ,  $R_t$ ,  $K_{t+1}$ ,  $q_{t+1}$ ,  $w_t$ ,  $w_{t+1}$ ,  $m_t$ ,  $H$ ,  $R_t^*$ ,  $\theta_t^*$ ,  $K_{t+1}^*$ ,  $q_{t+1}^*$ ,  $w_{t+1}^*$ , and  $m_t^*$ . Of the 15 equations, nine specify the economy in normal times, and six define the threshold variables. Below, we solve the system of equations given  $K_t$ . After solving the system of equations, we can compute other endogenous variables such as output  $Y_t = F(K_t, ZH_t)$  and the bank capital ratio  $(A_t - D_t)/A_t$ .

- Equations at normal times

- The first-order condition for the leverage:<sup>19</sup>

$$\begin{aligned} & \{[\theta_t^* \ln(\theta_t^* m_t^*) + (1 - \theta_t^*) \ln(R_t^* (1 - \theta_t^*) m_t^*)] \\ & - [\theta_t^* \ln(w_t + X) + (1 - \theta_t^*) \ln(\underline{w})]\} \frac{d\pi_t}{d\theta_t^*} \theta_{LF}'(D_t) \\ & = \int_0^{\theta_t^*} \left[ \frac{1}{m_t} \left( 1 - \frac{w_{t+1}}{R_t^2} R_{LF}'(D_t, \theta_t) \right) + (1 - \theta_t) \frac{R_{LF}'(D_t, \theta_t)}{R_t} \right] f(\theta_t) d\theta_t, \end{aligned} \quad (24)$$

where  $\underline{w}$  is the wage in the crisis:  $\underline{w} = (1 - \alpha) [I/(ZH)]^\alpha Z$ . The probability density function  $f(\theta_t)$  is computed from the beta distribution. We use numerical integration for the right-hand side of (24).

- The crisis probability:

$$\pi_t = \int_{\theta_t^*}^1 f(\theta_t) d\theta_t, \quad (25)$$

which implies that the marginal change in the crisis probability with respect to  $\theta_t^*$  is  $d\pi_t/d\theta_t^* = -f(\theta_t^*)$ .

- Liquidity market clearing condition:

$$L\left(\frac{R_t}{q_{t+1}}\right) = \theta_t \left( \frac{w_{t+1}}{R_t} + D_t \right) - (1 - \theta_t) w_t. \quad (26)$$

- Capital market clearing condition for  $\theta_t \leq \theta_t^*$ :

$$K_{t+1} = \underline{I} + I \left( \frac{R_t}{q_{t+1}} \right). \quad (27)$$

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<sup>19</sup>We use the numerical integration for the right-hand side of (24).

If  $\theta_t > \theta_t^*$ ,  $K_{t+1} = \underline{I}$ .

– Capital demand from firms:

$$q_{t+1} = \alpha \left( \frac{K_{t+1}}{ZH} \right)^{\alpha-1}. \quad (28)$$

– Labor demand from firms in period  $t$  and  $t + 1$ :

$$w_t = (1 - \alpha) \left( \frac{K_t}{ZH} \right)^\alpha Z, \quad (29)$$

$$w_{t+1} = (1 - \alpha) \left( \frac{K_{t+1}}{ZH} \right)^\alpha Z. \quad (30)$$

– Lifetime income:

$$m_t = w_t + D_t + \frac{w_{t+1}}{R_t}. \quad (31)$$

– Labor supply:

$$H = 2. \quad (32)$$

• Equations for the threshold variables

– The solvency constraint with equality:

$$D_t = L \left( \frac{R_t^*}{q_{t+1}^*} \right) + \frac{\gamma q_{t+1}^*}{R_t^*} I \left( \frac{R_t^*}{q_{t+1}^*} \right). \quad (33)$$

– Liquidity market clearing condition:

$$L \left( \frac{R_t^*}{q_{t+1}^*} \right) = \theta_t^* \left( \frac{w_{t+1}^*}{R_t^*} + D_t \right) - (1 - \theta_t^*) w_t. \quad (34)$$

– Capital market clearing condition:

$$K_{t+1}^* = \underline{I} + I \left( \frac{R_t^*}{q_{t+1}^*} \right). \quad (35)$$

– Capital demand from firms:

$$q_{t+1}^* = \alpha \left( \frac{K_{t+1}^*}{ZH} \right)^{\alpha-1}. \quad (36)$$

– Labor demand from firms:

$$w_{t+1}^* = (1 - \alpha) \left( \frac{K_{t+1}^*}{ZH} \right)^\alpha Z. \quad (37)$$

– Lifetime income:

$$m_t^* = w_t + D_t + \frac{w_{t+1}^*}{R_t^*}. \quad (38)$$

Leverage and the crisis probability in Tables 2 and 3 are calculated under the steady-state value of  $K_t = K_{t+1}$  for any  $t$ . To generate the dynamic paths of  $Y_t$  in Figure 1, we use the steady-state value of capital under the competitive equilibrium as the initial value and simulate  $\theta_t$ . Once  $\theta_t$  is realized, we can update the capital stock in the next period and solve the system of the equations consisting of (24)–(38). If  $\theta_t \leq \theta_t^*$ ,  $Y_{t+1} = K_{t+1}^\alpha (ZH)^{1-\alpha}$  where  $K_{t+1}$  is given by (27). Otherwise,  $Y_{t+1} = \underline{I}^\alpha (ZH)^{1-\alpha}$ . To plot the marginal cost and the marginal benefit in Figures 2 and 3, we compute the threshold values of  $K_{t+1}^*$  over different values of  $D_t$ . The initial value of capital is the steady-state value of  $K_t$  under the competitive equilibrium. After computing the sequence of  $K_{t+1}^*$  that corresponds to the value of  $D_t$ , we calculate  $q_{t+1}^*$ ,  $w_{t+1}^*$ , and  $\theta_t^*$  from (36), (37), and (34), respectively. These threshold variables allow us to compute the marginal cost given by the left-hand side of the equations (17) or (22). Regarding the marginal benefit, we log-linearize the equation for  $K_{t+1}$  around the steady state and compute  $R_t$  and  $w_{t+1}$  along with the partial derivative of  $R_t$  with respect to  $D_t$ . Given  $\theta_t^*$ , we take the numerical integration over  $\theta_t$ .

Table 1: Sequence of events for generation  $t$

Period $t$	
1.	Households receive endowments.
2.	Banks offer deposits to households and loans to entrepreneurs.
3.	Entrepreneurs launch their projects.
4.	Households supply labor and receive wages $w_t$ determined by that labor market conditions along with the old generation's labor supply.
5.	Liquidity shock $\theta_t$ is realized, and banks receive signals of project outcomes.
6.	Households decide the withdrawal amount $g_t$ .
7.	Banks decide which projects to discontinue and supply liquidity $L_t$ .
(i)	If $g_t > L_t$ , a financial crisis is precipitated and households receive repayment of $X$ .
(ii)	Otherwise, households can transfer their wealth into the period $t + 1$ .
8.	All agents consume.
Period $t + 1$	
1.	Entrepreneurs receive endowments.
2.	Entrepreneurs' projects are completed, and they sell their capital goods for $q_{t+1}$ and make repayment to banks.
3.	Households supply labor and receive wages $w_{t+1}$ determined by that labor market conditions along with the young generation's labor supply.
4.	Households fully withdraw deposits, if any.
5.	All agents consume.

Table 2: Crisis probabilities and allocations under LF and SP banks

	SP banks	LF banks
Leverage and crisis probabilities		
$D_t$	1.049	1.061
$\pi_t$ (%)	4.499	6.585
Bank capital and output		
Bank capital ratio (%)	15.097	13.952
$Y_{t+1}$	5.459	5.457

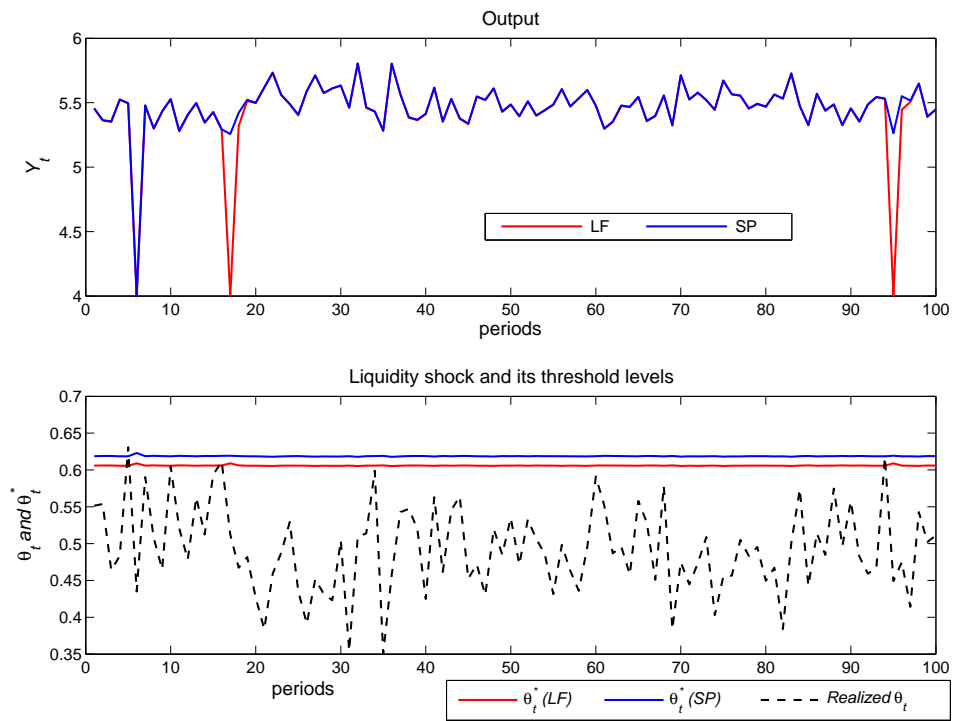
Note: Simulation results are based on the assumption that the liquidity shock  $\theta_t$  follows the beta distribution. The level of bank leverage  $D_t$  and the crisis probability  $\pi_t$  are obtained from Problems LF and SP, respectively. The bank capital ratio is  $(A_t - D_t)/A_t$ .

Table 3: Financial crisis probabilities and bank leverage

	SP banks	LF banks	SP banks	LF banks
Leverage and probabilities	std( $\theta_t$ ) = 0.02		std( $\theta_t$ ) = 0.10	
$D_t$	1.129	1.132	1.014	1.030
$\pi_t$ (%)	1.275	1.703	5.997	9.433

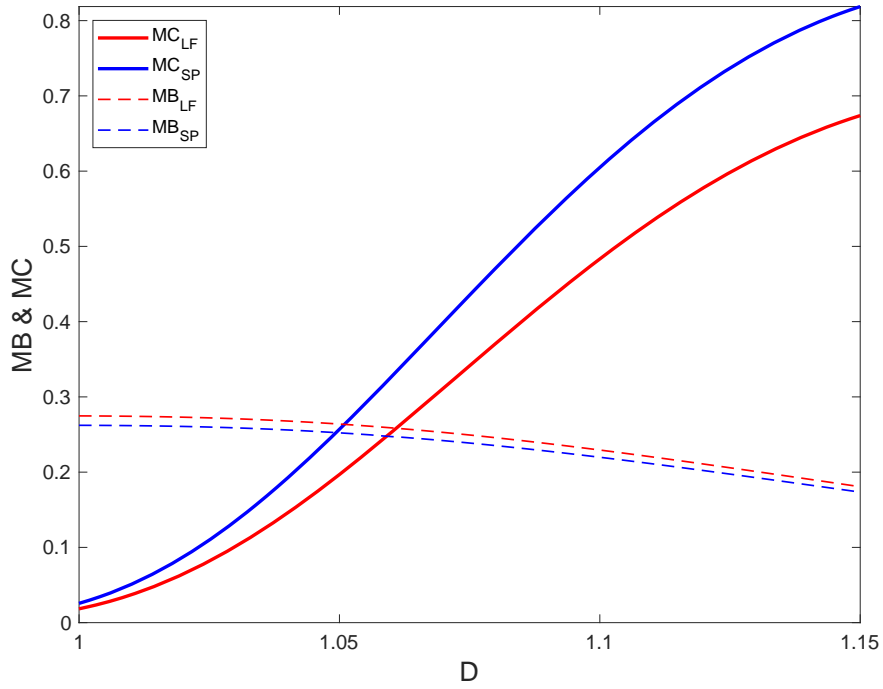
Note: Simulation results are based on various values of the standard deviation of the liquidity shock  $\theta_t$ . The other details can be seen in the note for Table 2.

Figure 1: Simulated paths of output and the liquidity shock



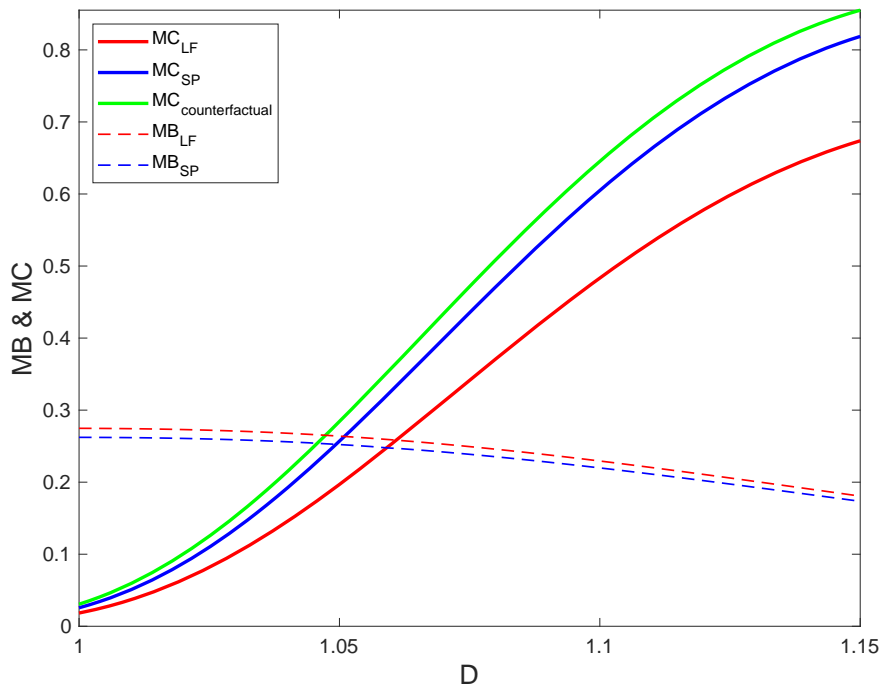
Note: The blue lines (denoted by SP) are the constrained social optimum with the SP banks. The red lines (denoted by LF) are the competitive equilibrium with the LF banks. The upper panel shows the simulated dynamic paths of output  $Y_t$ . The liquidity shock plotted as the dashed black line in the lower panel is generated from the beta distribution with a mean of 0.50 and a standard deviation of 0.07. The solid blue and red lines in the lower panel are the threshold level of the liquidity shock that satisfies the solvency constraint with equality.

Figure 2: Marginal cost and benefit against  $D_t$



Note: The blue and red solid lines represent the marginal cost under the SP banks calculated from the left-hand side of (22) and the marginal cost under the LF banks calculated from the left-hand side of (17), respectively. The blue and red dashed lines refer to the marginal benefit under the SP banks, calculated from the right-hand side of (22), and the marginal benefit under the LF banks, calculated from the right-hand side of (17), respectively.

Figure 3: Marginal cost for the hypothetical banks



Note: The blue and red lines replicate the curves for marginal cost and marginal benefit in Figure 2. The newly added curve represents the marginal cost for the banks in the counterfactual experiment. The marginal benefit in the counterfactual experiment coincides with the marginal benefit for the LF banks.