Only the Final Outcome Matters: Persistent Effects of Efforts in Dynamic Moral Hazard

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Abstract

We analyze a dynamic principal–agent problem in which the agent’s effort in each period has strong persistent effects. We show that a simple contract, in the sense that the reward depends only on the final outcome, is explained as the optimal contract derived in the principal’s optimization problem. The paper also discusses that the optimality of such a simple payment scheme crucially depends on the first-order stochastic dominance of the final outcome under various effort sequences.

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Keywords: dynamic moral hazard; history dependence; simple contract; first-order stochastic dominance.

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*This is a substantially revised paper of an earlier version (Ogawa (2003)). Following this earlier version, I became aware of two independent works, Mukoyama and Sahin (2004) and Kwon (2006), the results of which are both incorporated in the paper. The present version is included as Chapter 2 in my Ph. D. thesis (2008), and I would like to thank my advisor, Michihiro Kandori, for his guidance and encouragement. Comments and discussions by Eddie Dekel, Junichiro Ishida, Hideshi Itoh, Minoru Kitahara, Dan Sasaki and Satoru Takahashi much improved the paper. All remaining errors are mine. Financial support from the COE Program CEMANO and the Japan Society for the Promotion of Science is gratefully acknowledged.

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1 Introduction

In long-term principal–agent relationships with complete contracts, the principal writes payment schedules in advance that potentially depend on the agent’s period-by-period performance (that is, performance related to the level of effort), in order to provide proper incentives. In the light of the celebrated Sufficient Statistic Theorem (Holmström [3]), one may expect that using a very detailed history of past performances, each of which are informative of the agent’s efforts, is optimal for the principal in writing payment schedules. In reality, however, we often observe various incentive schemes that are not necessarily dependent on the whole detailed record of performances, but only on a subset of them, where such subsets are sometimes much smaller than the entire set of performances.

From the viewpoint of economic studies, such simple contracts are interpreted in several ways. One argument is that the principal incurs costs in writing complex contracts or in enforcing them and, as a result of taking such costs into consideration, some sort of simple contract is concluded as a suboptimal solution. Another possibility, following the line of Holmstrom and Milgrom [4], is that simple payment schedules are justified by their robustness to the change of model parameters. Understanding that such interpretations provide insights that enable us to grasp important aspects of contracts in reality, the present paper is devoted to studying a third way of explaining simple contracts.

Consider an environment where the agent’s present efforts have persistent effects over the future performances. In such environments, a contract that provides strong incentives in the future induces the agent to work hard in the present, and the role of such future incentives appears to be more important than in situations where there are no persistent effects from the agent’s efforts. However, this is not to say that providing incentives only in the future is sufficient: the agent’s present performance is still informative about present efforts,\(^1\) and we may expect, from

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\(^1\)In the paper, we assume that performances are statistically independent between one period and the next. See Section 2 for the formal model.
arguments such as the Sufficient Statistic Theorem, that every informative performance should be included in the optimal contract, no matter how low their informativeness may be. The present paper shows that this is not always the case, i.e., we show that an incentive scheme that depends only on the final performance is optimal if the agent’s effort in each period has strong persistent effects.

Theorems 1–3 of the paper provide sufficient conditions for such simple contracts to be optimal in various models of dynamic moral hazard circumstances, in which the cost of efforts is the same in all periods. The common feature of our sufficient conditions is simply summarized as follows: the probability distribution of the final outcome when the agent shirks only in the final period first-order stochastically dominates (FOS-dominates hereafter) the distribution when the agent shirks in any other period in such a way that the expected number of shirking events is one. To grasp the idea behind this condition intuitively, consider the two-period model in which the agent’s first-period action also affects the probability distribution of the second-period outcome. Let \((a,a')\) denote the action profile in which the first element (second element) indicates the agent’s first-period action (second-period action), and let \(\bar{a}\) (\(a\)) denote a strong effort (a shirk). Then, the sufficient condition has the following two requirements (Theorem 1).

(i) The probability distribution of the second-period outcome when the agent shirks only in the second period \((\bar{a},a)\) FOS-dominates the distribution when the agent shirks only in the first period \((a,\bar{a})\).

(ii) The probability distribution of the second-period outcome when the agent shirks only in the second period \((\bar{a},a)\) FOS-dominates the half-and-half randomization of \((a,a)\) the distribution when the agent shirks in both periods \((a,a)\) and (b) the distribution when the agent never shirks in any period \((\bar{a},\bar{a})\).

Requirement (i) ensures that shirking in the first period \((a,\bar{a})\) always makes the agent worse off than would shirking in the second period \((\bar{a},a)\). As a result of
the first-order statistical dominance (FOSD hereafter), the agent obtains a larger expected payoff from wages in $(\bar{a}, \bar{a})$ than in $(\bar{a}, \bar{a})^2$, and as the number of efforts is the same in both action profiles, the agent obtains a larger overall expected payoff if the agent undertakes $(\bar{a}, \bar{a})$ rather than $(\bar{a}, \bar{a})$. Thus, in designing the optimal contract, the principal need not take into account the possibility that the agent may shirk in the first period $(a, \bar{a})$. Requirement (ii) ensures that shirking in both periods $(a, a)$ makes the agent worse off compared with shirking in the second period $(\bar{a}, \bar{a})$. As the amount of effort differs between the two alternatives, the FOSD condition should be arranged in such a way that the expected number of times that the agent makes an effort is set to be the same. In requirement (ii), this is achieved by setting the expected number of efforts to be one on both sides $(1 = 0.5 \times 2 + 0.5 \times 0)^3$. Thus, in designing the optimal contract, the principal need not take into account the possibility that the agent may shirk in both periods $(\bar{a}, \bar{a})$.

Under (i) and (ii), the principal need not take into consideration any possibilities that the agent shirks in the first period whatsoever $(a, \cdot)$. Therefore, the principal’s interest is concentrated on incentivizing the agent’s second-period effort only, which induces the simple optimal contract that depends only on the final outcome. It is noteworthy that requirement (ii) together with (i) is summarized as follows: the probability distribution of the final outcome when the agent shirks only in the final period FOS-dominates the distribution when the agent shirks in any other period in such a way that the expected number of shirking events is one. Such arguments in relation to the role of FOSD and the expected number of efforts also apply to $T$-period models, and sufficient conditions are provided in a similar manner (Theorems 2–3).

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2As is presented formally in Section 3, we assume that the distribution of outcomes has the monotone likelihood ratio property. Therefore, in the optimal contract, the wage scheme is an increasing function of outcomes, which enables us to make a comparison between expected payoffs from wages by means of FOSD.

3The reader may wonder why $(\bar{a}, \bar{a})$, which is irrelevant in the comparison between $(\bar{a}, \bar{a})$ and $(a, a)$, appears in requirement (ii). This is because the incentive compatibility constraint between $(\bar{a}, \bar{a})$ and $(\bar{a}, \bar{a})$ is binding (indifferent to the agent) in the optimum. See Section 3 for the detail.
Simple models corresponding to Theorems 1 and 3 in the present paper have been studied in independent works by Kwon (2006), Mukoyama and Sahin (2004) and Ogawa (2003). In their studies, performances take only two values, “success” and “failure”, and simplified versions of our requirements (i) and (ii) are presented as the sufficient condition for the simple contract. However, as in other moral hazard studies with risk-averse agents, investigating a model with only two levels of performances often provides limited implications and interpretations about the mechanics of incentives in the equilibrium, as well as the function of the simple contracts.\textsuperscript{4} In the present paper, we study a model with \( N \) possible performances and the proof is given in an organized manner, which provides clear and rigorous interpretations of the earlier works.\textsuperscript{5}

Strong persistent effects of efforts as characterized by the FOSD are the main sources of our result. Historical dependence of this sort is often seen in real economic environments. For example, if an effort has a time-lag effect into the next period, as well as the direct effect in the current period, then the probability of success in period 2 is influenced by the effort level in period 1. If the production technology involves irreversibility, then the model becomes history dependent in a similar manner.\textsuperscript{6}

A brief review of the related literature is as follows. The results of the paper (Theorems 1–3) contrast with those of the repeated moral hazard literature, which state that payments in an optimal long-term contract should be dependent on the whole history of past performances (Lambert [6], Rogerson [13], Malcomson and

\textsuperscript{4}Although the distinction between the FOSD and the MLRC plays an important role in the implications and interpretations of moral hazard models, in the model with only two levels of performances, the two conditions are reduced to the same inequality:

\[
\Pr[\text{“success”} \mid \bar{a}] > \Pr[\text{“failure”} \mid \bar{a}].
\]

Such excessively simple inequalities often provide misleading interpretations of the equilibrium, and make the result seem artificial.

\textsuperscript{5}The aforementioned works have individual advantages. Mukoyama and Sahin (2004) provided some numerical analyses for the case in which the theoretical approach is difficult. The highlight of Kwon (2006) is the empirical analysis using health insurance data.

\textsuperscript{6}These examples are examined in detail in Section 4.
Spinnewyn [9] and Chiappori et al. [1]). In that literature, it is assumed that there are no exogenous links between one period and the next, and the complementarity between incentives discussed in the preceding paragraphs does not emerge.

The relationship between Holmström [3]’s *Sufficient Statistic Theorem* and our result casts an interesting light on the interpretation of “informativeness” in economic studies. In the view of the Sufficient Statistic Theorem, every statistically informative signal is useful (should be used) in the optimal contract, whereas in our view, statistically informative signals are useless sometimes in providing incentives. Our result explains that there is a difference between “statistical informativeness” and “economic usefulness”, and states that the former does not always bring about the latter property in the theory of incentives.

The remainder of the paper is organized as follows. Section 2 describes the basic model of two-period dynamic moral hazard. Section 3 provides the main result of the paper. It is shown that the optimal long-term contract is dependent only on the final outcome and a sufficient condition for the result is presented (Theorem 1). Section 4 provides some examples of environments in which the sufficient condition is satisfied. In Section 5, we extend the basic model to $T$-periods and present sufficient conditions for the optimal contract to be simple, as in Theorem 1. Section 6 contains some concluding remarks.

## 2 The Basic Model

We study a simple dynamic moral hazard model with “history dependence”. The relationship between a principal (she) and an agent (he) lasts for two periods ($t = 1, 2$).

In each period, the agent chooses his action $a^t$ from the action space $A = \{a, \bar{a}\}$. These actions are unobservable to the principal. We may find it convenient to interpret those actions as effort levels, and say that the agent works hard (shirks) when he chooses $\bar{a}$ ($a$).

In period $t$, after the agent has chosen his action $a^t$, the outcome $x^t \in \{x_1, \cdots, x_N\}$
is realized according to probabilities that depend on the history of the agent’s actions; that is, the distribution of $x^1$ depends on $a^1$, whereas that of $x^2$ depends on the pair $(a^1, a^2)$. These outcomes are immediately observed by both parties (and assumed to be verifiable to third parties, such as a court). We may regard these outcomes as performances, and identify each of them with a corresponding revenue received by the principal.

We assume that $x^1$ and $x^2$ are independently distributed\(^7\); hereafter, we write the distributions as follows:

\[
\begin{align*}
p^1_i(a^1) &= \Pr[x^1 = x_i \mid a^1] & (i = 1, \ldots, N), \\
p^2_i(a^1, a^2) &= \Pr[x^2 = x_i \mid (a^1, a^2)] & (i = 1, \ldots, N).
\end{align*}
\]

Throughout the paper, we assume that the distributions are of full supports:

\[
\begin{align*}
p^1_i(a^1) > 0 & \quad \text{for all } (i, a^1) \in \{1, \ldots, N\} \times A, \\
p^2_i(a^1, a^2) > 0 & \quad \text{for all } (i, a^1, a^2) \in \{1, \ldots, N\} \times A^2.
\end{align*}
\]

At the beginning of the game (i.e., before $t = 1$), the principal and the agent sign a contract in the manner described in detail below.

First, the principal offers a long-term contract $w = (w^1, w^2)$, where $w^1 = (w^1(x^1))_{x^1 \in X}$ and $w^2 = (w^2(x^1, x^2))_{(x^1, x^2) \in X^2}$ are payment schedules for periods 1 and 2, respectively, under outcome realizations $(x^1, x^2)$. Such a contract stipulates $N + N^2$ possible payments, depending on the realizations of outcomes. Next, the agent decides whether to accept or refuse the contract offered by the principal. If the agent refuses the offered contract, both parties receive their reservation utilities, and the game comes to an end. If the agent accepts the contract, the game enters into the two-times moral hazard repetition discussed above.

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\(^7\)This assumption says that the realized value of $x^1$ does not influence the distribution of $x^2$, so that the former yields no information on the current likelihood of any particular production levels in period 2. The “history dependence” discussed in this paper deals with the case where $x^2$ is affected by $a^1$, but not by the realization of $x^1$. 

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We assume that the principal can commit to the long-term contract that she has offered before \( t = 1 \) and so, once the contract is accepted by the agent, the principal cannot change the payment schedule \( w \) and must make the payment each period according to the history of outcome realizations up to the date of payment. In addition, we assume that the agent must commit to his participation in the game and so, once he accepts the contract, he cannot exit in the midst of the game and must participate in it until the end of period 2.

In each period, the agent attains a payoff of \( u(w) - c(a) \), where \( u \) is strictly increasing and strictly concave (the agent is risk averse) and \( c(a) < c(\bar{a}) \) (harder work involves a greater cost). We normalize this as \( c(a) = 0 \) and \( c(\bar{a}) = C \).

Given a long-term contract \( w \), the agent’s strategy consists of two parts: one is the action he takes in the first period, \( a^1 \), and the other is the action schedule for the second period \( a^2 = (a^2_i)_{i=1}^N \), each of which specifies the action he takes in period 2 under the outcome realization of \( x^1 \) in period 1.\(^8\) Let \( U_i(a^1, a^2_i, w^2) \) denote the expected utility in period 2 for the agent when he chose \( a^1 \) and the outcome was \( x_i \) in the first period:

\[
U_i(a^1, a^2_i, w^2) = \sum_{j=1}^N p_i^2(a^1, a^2_i)u(w^2(x_i, x_j)) - c(a^2_i).
\]

Using this notation, the intertemporal expected utility for the agent \( U(a^1; a^2; w) \) under the agent’s strategy \( (a^1; a^2) \) is written as

\[
U(a^1; a^1, \ldots, a^2_N; w) = \sum_{i=1}^N p_i^1(a^1) \left[ u(w^1(x_i)) + U_i(a^1, a^2_i, w^2) \right] - c(a^1). \tag{9}\]

\(^8\)Accordingly, we allow the agent to change his action in period 2 after he observes the outcome realization in period 1, which is one of the standard assumptions in the literature. Once we cease this assumption and assume that the agent has to commit to a pair of actions \( (a^1, a^2) \) \textit{ex ante}, then the model reduces to a one-shot multitask incentive problem. We shall make the sequentiality assumption to focus on the dynamics of the model, but note that the main result of the paper (Theorems 1–3) also apply to the one-shot multitask model.

\(^9\)We assume that both the principal and the agent have a common discount factor of one. If the common discount factor was less than one (but positive) and the outcome space consisted of three elements or more, we could not attain the plausible sufficient conditions as in Assumption 1,
The optimization problem for the principal when she wishes to implement an action profile \((a^1, a^2)\) is written as:

\[
\min_w \sum_{i=1}^N p^1_i(a^1) \left[ w^1(x_i) + \sum_{j=1}^N p^2_j(a^1, a^2_i)w^2(x_i, x_j) \right],
\]

subject to

\[
U(a^1, a^2, w) \geq U(a', a'', w), \quad a' \neq a^1, \quad \forall a'' \in A^N, \quad (IC1)
\]

\[
U_i(a^1, a^2_i, w^2) \geq U_i(a', a^2_i, w^2), \quad a' \neq a^2_i, \quad i = 1, \ldots, N, \quad (IC2)
\]

\[
U(a^1, a^2, w) \geq 2\bar{u}, \quad (PC)
\]

where \(\bar{u}\) denotes the reservation utility for the agent.

At this point, we should emphasize how the optimization problem (P) differs from the one for repeated moral hazard models. When the model is just a repetition of two moral hazard stages, the action taken in period 1, \(a^1\), does not affect the probability distribution of outcomes in period 2, so that \(U_i(a', a^2_i, w^2) = U_i(a'', a^2_i, w^2)\) for any \(a' \neq a''\). This reduces the incentive constraints for the first period (IC1) to:

\[
U(a^1, a^2, w) \geq U(a', a^2, w), \quad (a' \neq a^1), \quad (IC1^{ind})
\]

under which we must only take into account the deviation strategies from \(a^1\) to the other \(a'\), with \(a^2\) fixed. For the dynamic model that we investigate in this paper, this is not sufficient: we must take into account all possibilities of deviation that the agent might make during the two periods, as it is no longer assured that he always takes \(a^2\) regardless of the action he takes in period 1, even if (IC2) is satisfied for the \(a^1\).

which relies on the nature of \((p^1_i(\cdot))\) and \((p^2_i(\cdot, \cdot))\). Mukoyama and Sahin [9] showed that in the case of \(N = 2\), an extension of Assumption 1 is a sufficient condition for \(w^1(x^1)\) to be constant, in a similar model in which both players have a common discount factor of less than one.
3 Simple Contract

In this section, we show that the optimal long-term contract is dependent only on the second-period outcome if the probability distribution of the second-period outcome satisfies certain conditions, as briefly discussed in the Introduction. The result (Theorem 1) contrasts with that of the repeated moral hazard literature, where the optimal long-term contract is always dependent on the whole history of past outcomes.

The following assumption gives the sufficient condition for such simple contracts. We may regard this assumption as relating to “strong persistent effects” in the sense that the action chosen in period 1 has a stronger influence on the outcome in period 2 than does the action chosen in period 2.

**Assumption 1.** $p^2_i(a^1, a^2)$ satisfies the following three conditions:

(i) $p^2_i(a^1, \bar{a})/p^2_i(a^1, a)$ is increasing in $i$ for all $a^1$. (MLRC)

(ii) $\sum_{i=1}^I p^2_i(a, \bar{a}) \geq \sum_{i=1}^I p^2_i(\bar{a}, a)$ for all $I \in \{1, \ldots, N\}$.

(iii) $\frac{1}{2} \sum_{i=1}^I (p^2_i(a, a) + p^2_i(\bar{a}, \bar{a})) \geq \sum_{i=1}^I p^2_i(\bar{a}, a)$ for all $I \in \{1, \ldots, N\}$.

In Assumption 1, (ii) says that the action profile $(\bar{a}, a)$ stochastically dominates the action profile $(a, \bar{a})$ in the distribution of $x^2$, whereas (iii) says that $(\bar{a}, \bar{a})$ stochastically dominates the half-and-half randomization between $(a, a)$ and $(\bar{a}, \bar{a})$. We should note that neither (ii) nor (iii) in Assumption 1 is satisfied in repeated moral hazard models.

**Theorem 1.** Suppose that the probability distribution of the second-period outcome satisfies Assumption 1. Then, the optimal long-term contract $w$ that implements $a^1 = \bar{a}$ and $a^2 = (\bar{a}, \ldots, \bar{a})$ is such that:

(a) $w^1(x^1)$ is a constant for all $x^1$,

(b) $w^2(x^1, x^2)$ is independent of $x^1$, and is increasing in $x^2$. 

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Proof. The proof proceeds in two steps. In the first step, we solve a “relaxed” optimization problem as follows:

\[
\min_w \sum_{i=1}^N p_i^1(a^1) \left[ w^1(x_i) + \sum_{j=1}^N p_j^2(a^1, a_i^2)w^2(x_i, x_j) \right], \quad (P')
\]

subject to

\[
U_i(a^1, a_i^2, w^2) \geq U_i(a^1, a_i', w^2), \quad a_i' \neq a_i^2, \quad i = 1, \ldots, N, \quad (IC2)
\]

\[
U(a^1, a^2, w) \geq 2\bar{a}, \quad (PC)
\]

and show that the solution satisfies the properties (a) and (b). In the second step, we verify that (any) contract satisfying properties (a) and (b) is always compatible with the constraint (IC1). By these two steps, we conclude that the solution to the “original” optimization problem (P) satisfies properties (a) and (b).

1. The first-order condition for \( w^1(x_i) \) in the “relaxed” problem (P') is:

\[
\frac{1}{u'(w^1(x_i))} = \nu \quad \text{for all } x_i,
\]

where \( \nu \) is the Lagrange multiplier with respect to (PC). Thus, \( w^1(x_i) \) is a constant for all \( x_i \).

The first-order condition for \( w^2(x_i, x_j) \) is:

\[
\frac{1}{u'(w^2(x_i, x_j))} = \frac{\mu_i}{p_i(\bar{a})} \left[ 1 - \frac{p_j^2(\bar{a}, a_i)}{p_j^2(\bar{a}, \bar{a})} \right] + \nu,
\]

where \( \mu_i \) is the Lagrange multiplier with respect to (IC2) for the corresponding \( i \). Here, \( w^2(x_i, x_j) \) is independent of \( i \) (otherwise the principal could be strictly better off by offering the certainty equivalence \( \tilde{w}_j \), such that \( u(\tilde{w}_j) = \sum_i p_i^1(\bar{a})u(w^2(x_i, x_j)) \), without affecting the remaining constraints (IC2) and (PC)). Hence, the ratio \( \mu_i/p_i(\bar{a}) \) is a constant for all \( i \).

If \( \mu_i = 0 \), then \( w^2(x_i, x_j) \) is a constant for all \( j \), which violates (IC2) for \( i \).
Hence, $\mu_i > 0$ should be satisfied for all $i$, which means that (IC2) is binding in the optimum. Therefore, from Assumption 1 (i) and the concavity of $u(\cdot)$, $w^2(x_i,x_j)$ must be increasing in $j$.

2. First, we check that (IC1) is satisfied for two deviation strategies, $(a^1; a^2) = (a; \bar{a}, \cdots, \bar{a})$ and $(a^1; a^2) = (a; a, \cdots, a)$, under the optimal contract derived in step 1. Here, we write $w^1(x_i) = w^1$ and $w^2(x_i, x_j) = w^2_j$, as the contract is not dependent on $x_i$.

As shown in step 1, (IC2) is binding at the optimum. Therefore:

$$C = N \sum_{j=1}^{N} p^2_j(\bar{a}, a)u(w^2_j) - \sum_{j=1}^{N} p^2_j(\bar{a}, a)u(w^2_j) \quad (1)$$

Substituting (1) into (IC1) yields:

$$\sum_{j=1}^{N} p^2_j(\bar{a}, a)u(w^2_j) \geq \sum_{j=1}^{N} p^2_j(a, \bar{a})u(w^2_j)$$

$$2 \sum_{j=1}^{N} p^2_j(\bar{a}, a)u(w^2_j) \geq \sum_{j=1}^{N} p^2_j(\bar{a}, \bar{a})u(w^2_j) + \sum_{j=1}^{N} p^2_j(a, a)u(w^2_j).$$

The former inequality is ensured by the Assumption 1 (ii), whereas the latter is ensured by the Assumption 1 (iii).

Finally, we check that (IC1) is satisfied for any deviation strategies, $(a^1; a^2) = (a; a_1^2, \cdots, a_N^2)$. Suppose the agent undertakes $a_i^2 = \bar{a}$ if $i \in I \subset \{1, \cdots, N\}$ and $a_i^2 = a$ if $i \in I = \{1, \cdots, N\} \setminus \bar{I}$. The intertemporal payoff to the agent following this deviation strategy satisfies:

$$u(w^1) + \sum_{i \in I} p^1_i(a) \left[ \sum_{j=1}^{N} p^2_j(a, \bar{a})u(w^2_j) - C \right] + \sum_{i \in \bar{I}} p^1_i(a) \left[ \sum_{j=1}^{N} p^2_j(a, a)u(w^2_j) \right]$$

$$\leq u(w^1) + \max \left\{ \sum_{j=1}^{N} p^2_j(a, \bar{a})u(w^2_j) - C, \sum_{j=1}^{N} p^2_j(a, a)u(w^2_j) \right\}$$

$$= \max \{ U(a; \bar{a}, \cdots, \bar{a}; w), U(a; a, \cdots, a; w) \}$$
\[ \leq U(\bar{a}; \bar{a}, \cdots, \bar{a}; w), \]

where the last inequality comes from the previous result that (IC1) is satisfied both for \((a^1; a^2) = (\bar{a}; \bar{a}, \cdots, \bar{a})\) and for \((a^1; a^2) = (a; a, \cdots, a)\). Hence, (IC1) is satisfied for any deviation strategy \((a^1; a^2) = (a_1^2; \cdots, a_N^2)\).

The intuition behind the proof is as follows. For the principal who intends to induce the agent to exert the positive effort \(\bar{a}\) in period 2, it is necessary to make the second-period payment \(w^2(x_i, x_j)\) dependent on the second-period outcome \(x_j\), as this is the only source of incentive power available. However, such a payment schedule induces the agent to work hard in period 1 because the distribution of second-period outcomes is affected not only by \(a^2\) but also by \(a^1\). Moreover, this gives the agent a sufficient incentive to work hard in period 1 under Assumption 1. Assumption 1 (ii) ensures that the agent always obtains a larger gross expected payoff in terms of wages by undertaking action profile \((\bar{a}, a)\) than by undertaking \((a, \bar{a})\) as a result of the FOSD. In addition, as the cost of effort, \(C\), is the same in both periods, the agent obtains a larger net expected payoff as well. Thus, if the contract is to induce hard work by the agent in the second period, it automatically provides the agent with the incentive to work hard in the first period. Assumption 1 (iii), on the other hand, ensures that the agent does not deviate to a strategy of shirking in both periods (i.e., to \((a, a)\)). half-and-half randomization of the two probability distributions, \((\bar{a}, \bar{a})\) and \((a, a)\), gives the agent’s gross expected payoff from taking \((a, a)\) in accordance with the benefit of effort cost reduction normalized to \(C\) (a one-time shirk). Thus, if the contract is to induce hard work in the second period, it automatically makes the agent worse off if he shirks in both periods, \((a, a)\).

To summarize, if the probability distribution of the second-period outcome when the agent shirks only in the second period \((\bar{a}, a)\) FOS-dominates the distribution when the agent shirks in any other periods in such a way that the expected number of shirkings is one, providing incentives to work hard in the second period becomes sufficient to induce the agent to make strong efforts in both periods.
As we will see in Section 5, such arguments regarding FOSD and one-time shirking play central roles in T-period models as well. We also discuss the sufficient conditions for simple contracts for T-period models in a similar manner.

4 Examples

In this section, we provide a few examples in which $p^2_T(\cdot, \cdot)$ satisfies Assumption 1. These examples incorporate “strong persistent effects”, in the sense that the action chosen in period 1 has a stronger influence on the outcome of period 2 than does the action chosen in period 2. Under such circumstances, the optimal long-term contract is simple, by which we mean that the payment schedule is dependent only upon the second-period outcome.

In the following examples, we suppose that $N = 2$ (“success” and “failure”) and let $\pi_t(\cdot)$ denote the probability of “success” in period $t$; that is, $\pi_t(\cdot) = p^t_2(\cdot)$ and $1 - \pi_t(\cdot) = p^t_1(\cdot)$.

Example 1 (Time lag). There is a time lag between the effort and its effect.

If the agent works hard in period $t$, this not only increases the probability of success in the same period by $\alpha$ but also increases the probability of success in the following period by $\beta$. We assume that $0 < \alpha < \beta$, and regard $\beta$ as a “full effect” of the effort and $\alpha$ as a “partial effect” of the effort. Let $\pi$ denote the probability of success when the agent has never taken any positive efforts. Then, we write $\pi_t(\cdot)$ as follows:

\[
\begin{align*}
\pi_1(a) &= \pi, & \pi_1(\bar{a}) &= \pi + \alpha, \\
\pi_2(a, a) &= \pi, & \pi_2(a, \bar{a}) &= \pi + \alpha, \\
\pi_2(\bar{a}, a) &= \pi + \beta, & \pi_2(\bar{a}, \bar{a}) &= \pi + \alpha + \beta.
\end{align*}
\]

Assumption 1 (ii) is satisfied since $\pi_2(\bar{a}, a) > \pi_2(a, \bar{a})$; this is the “time-lag effect” because the positive effort $\bar{a}$ taken in period 1 has a greater influence ($\beta$) than it has if taken in period 2 ($\alpha$). Assumption 1 (iii) is also satisfied since
\( \pi_2(\bar{a}, a) > \frac{1}{2} [\pi_2(\bar{a}, \bar{a}) + \pi_2(a, a)] \).

**Example 2** (Irreversibility). The agent has to make a positive effort in every period to maintain the highest probability of success \( \bar{\pi} \). If he shirks, the probability of success declines by \( \gamma \) and this degree of success is be recovered, even if the agent makes a positive effort in the following period:

\[
\begin{align*}
\pi_1(a) &= \bar{\pi} - \gamma, & \pi_1(\bar{a}) &= \bar{\pi}, \\
\pi_2(a, a) &= \bar{\pi} - 2\gamma, & \pi_2(a, \bar{a}) &= \bar{\pi} - \gamma \\
\pi_2(\bar{a}, a) &= \bar{\pi} - \gamma, & \pi_2(\bar{a}, \bar{a}) &= \bar{\pi}.
\end{align*}
\]

It is straightforward that the distribution satisfies Assumptions 1 (ii) and (iii) with equalities.

## 5 Extensions

In this section, we extend the basic model to a \( T \)-period setup, and show that similar results as in Theorem 1 are obtained. As in Section 2, we let \( a^t \) and \( x^t \) denote the agent’s actions and outcomes in each period \( t = 1, \cdots, T \), respectively. The distribution of each outcome \( x^t \) is dependent on the whole past history of actions \( a^t = (a_1; \cdots; a_t) \). We write the distributions as follows:

\[
p_{it}(a^t) = \Pr [x_t = x_i | a^t], \quad i = 1, \cdots, N.
\]

For simplicity, throughout this section, we assume that the agent decides his whole action profile \( a^T = (a^1, \cdots, a^T) \) at the beginning of period 1 and that he never changes this profile after observing the outcomes in each period.\(^{10}\) We split

\(^{10}\)In the basic model of Section 2, it is assumed that the agent makes his second period action \( a^2 \) after observing the first period outcome \( x^1 \); therefore, the action profile consists of \( N + 1 \) components \((a^1, a^2_1, \cdots, a^2_N)\), where \( a^2_i \) denotes the second-period action when the first-period outcome is \( x_i \). For the \( T \)-period model in this section, we may also consider the possibility that the agent’s actions depend on past outcomes (in which case, the action profile consists of \((N^T - 1)/(N - 1)\))
the agent’s action space $A^T = A \times \cdots \times A$ into partitions $(\mathcal{A}_0, \ldots, \mathcal{A}_T)$, where:

$$\mathcal{A}_k = \{(a^1, \ldots, a^T) \mid \# \{t \mid a^t = \bar{a}\} = k\}, \quad k = 0, \ldots, T.$$  

That is, $\mathcal{A}_k$ is the set of the action profile $a^T$ in which there are $k$ weak efforts $\bar{a}$ (and hence, $T - k$ strong efforts $\bar{a}$). For instance, if $T = 3$, then the above notation gives us $\mathcal{A}_0 = \{(\bar{a}, \bar{a}, \bar{a})\}$, $\mathcal{A}_1 = \{(\bar{a}, \bar{a}, a), (\bar{a}, a, \bar{a}), (a, \bar{a}, \bar{a})\}$, etc.

Let $V(a, w)$ denote the agent’s gross expected payoff from payment schedule $w$ when he takes action profile $a$. Then, the agent’s net expected payoff is written as:

$$V(a, w) - C \cdot m(a),$$

where $C$ is the cost of a strong effort $\bar{a}$ (as in Section 2) and $m(a)$ is the number of strong efforts in action profile $a$. Then, the incentive compatibility constraint in the $N$-period model is simplified as follows: for $k = 1, \ldots, T$:

$$V(\bar{a}, \ldots, \bar{a}, w) - C \cdot k \geq V(a', w), \quad \text{for all } a' \in \mathcal{A}_k. \quad \text{(IC}_k\text{)}$$

Here, we offer two models of $N$-period dynamic moral hazard.

**Extension 1 (Summary outcome in the final period)**

Suppose that the agent’s actions result in mutually independent outcomes in each period but that, in the final period, $t = T$, there is a “summary” outcome that is dependent on all of the past actions $a^T = (a_1, \cdots, a_T)$. To be specific:

$$p^T_{it}(a^t) = \Pr[x^t = x_i \mid a^t], \quad t = 1, \cdots, T - 1,$$

$$p^T_{jT}(a^T) = \Pr[x^T = x_j \mid a^T].$$

components), but such a consideration does not change the result in Theorems 2–3. See the Appendix for more on this point.

11It is obvious that $(\mathcal{A}_0, \ldots, \mathcal{A}_T)$ satisfies $\mathcal{A}_k \cap \mathcal{A}_l = \emptyset$ for any $k, l$, and $\bigcup_k \mathcal{A}_k = A$. Hence, $(\mathcal{A}_0, \cdots, \mathcal{A}_T)$ is a partition of $A$. 

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In this setup, \( t \)-period action \( a' \) is assumed to affect the current outcome \( x' \) as well as the final outcome \( x_T \). Examples of this model include the relationship between week-by-week homework and a final exam. Each week, students are assigned homework concerning the topic they have just studied but, in the end, the students must face the challenge of a final exam concerning the entire topic that they were taught during the semester.

**Theorem 2.** Suppose that \( p_{jt}(a^T) \) satisfies the following two conditions:

1. \( p_{jt}(a^{T-1}, \bar{a}) / p_{jt}(a^{T-1}, a) \) is increasing in \( j \) for all \( a^{T-1} \) (MLRC), and

2. For all \( k = 1, \ldots, T \) and all \( a' \in \mathcal{A}_k \), the inequality

\[
\sum_{j=1}^{J} \left[ (k-1) p_{jt}(\bar{a}, \ldots, \bar{a}, \bar{a}) + p_{jt}(a') \right] \geq \sum_{j=1}^{J} p_{jt}(\bar{a}, \ldots, \bar{a}, a)
\]

holds for all \( J \in \{1, \ldots, N\} \).

Then, payments in the optimal long-term contract are dependent only on the final outcome \( x_T \).

First, note that condition (2) is a generalization of conditions (ii) and (iii) in Assumption 1. Substituting \( k = 1 \) into equation (2) gives:

\[
\sum_{j=1}^{J} p_{jt}(a') \geq \sum_{j=1}^{J} p_{jt}(\bar{a}, \ldots, \bar{a}, a)
\]

for all \( a' \in \mathcal{A}_1 \) and \( J \),

which states that the distribution of the final outcome, \( x_T \), when the agent takes action profile \( (\bar{a}, \ldots, \bar{a}, a) \) \( \text{FOS-dominates} \) that of any action profile \( a' \) in which the agent makes a weak effort only in one period; this is an exact extension of condition (ii) in Assumption 1. Similarly, substituting \( k = 2 \) into inequality (2) provides an extension of condition (iii) in Assumption 1. In general, inequality (2) is seen as the following condition: shirking in the final period (right-hand side) FOS-dominates any shirking, the expected number of which is exactly one (left-hand side).
Proof. As in the proof of Theorem 1, we show that the optimal long-term contract is independent of outcome history $x_{T-1}$ up to the period $T-1$, if all incentive constraints are not binding with the exception of:

$$V(\bar{a}, \ldots, \bar{a}, \bar{a}, w) - C \geq V(\bar{a}, \ldots, \bar{a}, a, w)$$  \hspace{1cm} (3)

In the following, we show that the derived contract, which is dependent only on the final outcome $x_T$, automatically satisfies all the incentive compatibility constraints.

As the derived contract $w^*$ satisfies (3) with equality, we have:

$$C = V(\bar{a}, \ldots, \bar{a}, \bar{a}, w^*) - V(\bar{a}, \ldots, \bar{a}, a, w^*).$$  \hspace{1cm} (4)

For each $k = 1, \ldots, T$, substituting (4) into (IC$_k$) yields:

$$k \cdot V(\bar{a}, \ldots, \bar{a}, a, w^*) \geq (k-1) \cdot V(\bar{a}, \ldots, \bar{a}, a, w^*) + V(a', w^*).$$

Condition (2) ensures this inequality.

Extension 2 (Human capital investment)

Suppose that the distributions of outcomes in each period are dependent not on the detail of past actions, but on the number of strong efforts that the agent has taken to date. To be specific, we let $p_i(k)$ denote the probability distribution when the agent has undertaken $k$ strong efforts:

$$p_i(k) = \Pr[x_t = x_i | \# \{t \mid a_t = \bar{a} \} = k], \quad k = 0, \ldots, T.$$  

Examples of this model includes the agent’s human capital investment or the learning-by-doing effect of the agent’s effort.

Theorem 3. Suppose that distributions $p_i(k)$, $k = 0, \ldots, T$, satisfy the following two conditions:


1. $p_i(T)/p_i(T-1)$ is increasing in $i$, and

2. For all $k = 2, \cdots, T$, the inequality

$$\sum_{j=1}^{I} \left[ \frac{(k-1)p_j(T) + p_j(T-k)}{k} \right] \geq \sum_{j=1}^{I} p_j(T-1)$$

holds for all $I \in \{1, \ldots, N\}$.

Then, the optimal long-term contract is dependent only on the final outcome $x^T$.

We should note that substituting $k = 2$ into condition 2 yields the counterpart of condition (iii) in Assumption 1.\(^{12}\) As in Extension 1, condition 2 is seen as the following condition: shirking in the final period (right-hand side) FOS-dominates any shirkings, the expected number of which is one (left-hand side).

It is also important to note that condition 2 is a reasonable assumption because it is a weaker condition of nonincreasing marginal returns to investment:

$$\sum_{j=1}^{I} [p_j(k+1) - p_j(k)] \geq \sum_{j=1}^{I} [p_j(k) - p_j(k-1)].$$

This inequality states that the marginal “benefit” in the probability distribution of one additional effort is decreasing in $k$. To see that the “nonincreasing marginal returns” implies condition 2, replace $k$ with $T - k + l$ and multiply the inequality (5) by $l$ as follows:

$$l \sum_{j=1}^{I} [p_j(T-k+l+1) - p_j(T-k+l)]$$

$$\geq l \sum_{j=1}^{I} [p_j(T-k+l) - p_j(T-k+l-1)],$$

\(^{12}\)Note that the counterpart of condition (ii) in Assumption 1 is an identity in this extension since two action profiles $(\bar{a}, \ldots, \bar{a}, a)$ and $(\bar{a}, \ldots, a, \bar{a})$ yield the same distribution $p_i(T-1)$ in the final period $T$.  

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Then, by summing up both sides for \( l = 1, \ldots, k - 1 \), we have:

\[
(k - 1) \sum_{j=1}^{J} p_j(T) + \sum_{j=1}^{J} p_j(T - k) \geq k \sum_{j=1}^{J} p_j(T - 1),
\]

which is equivalent to condition 2.\(^{13}\)

**Proof.** As in the proof of Theorem 1, we show that the optimal long-term contract is independent of outcome history \( x^{T-1} \) up to the period \( T - 1 \), if all incentive constraints are not binding, with the exception of:

\[
V(\bar{a}, \ldots, \bar{a}, \bar{a}, w) - C \geq V(\bar{a}, \ldots, \bar{a}, a, w)
\]

(6)

In the following, we show that the derived contract, which is dependent only on the final outcome \( x^T \), automatically satisfies all the incentive compatibility constraints. As the derived contract \( w^* \) satisfies (6) with equality, we have:

\[
C = V(\bar{a}, \ldots, \bar{a}, \bar{w}^*) - V(\bar{a}, \ldots, \bar{a}, a^*, w^*).
\]

(7)

For each \( k = 1, \ldots, T \), substituting (7) into \((IC_k)\) yields:

\[
k \cdot V(\bar{a}, \ldots, \bar{a}, a^*, w^*) \geq (k - 1) \cdot V(\bar{a}, \ldots, \bar{a}, a^*, w^*) + V(a', w^*).
\]

\(^{13}\)Kwon [5] investigated a similar model with binary outcomes \((N = 2)\) and showed that the optimal long-term contract is dependent only on the final outcome under the “nonincreasing marginal returns” assumption. Although the nonincreasing marginal returns is an easy assumption to interpret economically, it is rather a strong assumption if \( T \) is a substantially large number, as it requires that the inequality (5) is satisfied for all pairs of adjacent periods, \( k = 1, \ldots, T - 1 \). On the other hand, consider the following distributions when \( T = 3 \) and \( N = 2 \):

\[
p_1(0) = 0.8, \quad p_1(1) = 0.7, \quad p_1(2) = 0.4, \quad p_1(3) = 0.3,
\]

\((p_1(\cdot) \) represents the probability of “failure” when \( N = 2 \). This is not “nonincreasing returns”, but it is compatible with our FOSD condition. Our result (Theorem 3) suggests that what is central to the incentives in simple contracts is the FOSD relationship, and “nonincreasing marginal returns” is one example of the condition. In addition, note that the two conditions coincide if (and only if) \( T = 2 \).
Condition 2. of the Theorem ensures this inequality.

6 Concluding Remarks

This paper has examined the role of history dependence in a dynamic moral hazard model. It is shown that, under certain conditions on the probability distributions of outcomes, the optimal long-term contract is such that the payment schedules are not contingent upon the realization of past outcomes. This finding contrasts strikingly with the results in repeated moral hazard models, where the optimal long-term contracts are generally dependent on the whole history of past outcomes.

In reality, there are a variety of circumstances where effort has persistent effects, and the result of the paper that payments do not fully reflect the realization of past outcomes under such circumstances is persuasive. However, the assumption of full commitment may be too strong in some of these economic contexts. In the study of moral hazard problems, renegotiation-proof contracts have been investigated by Fudenberg and Tirole [2], Ma [7, 8] and Park [12]. In this respect, the study of dynamic moral hazard requires further research on renegotiation.

As we have discussed in the Introduction, the difference between the Sufficient Statistics Theorem and our result is regarded as the gap between the “statistical informativeness” and the “economic usefulness” of the signal. Our result that the two properties do not always coincide may not be surprising in general, but the relationship between each is not straightforward.

This paper has favored the simplest models to focus upon the role of history dependence. In particular, we have assumed two actions and independent distributions over periods in the paper. Generalizations of this model also deserve further investigation.
Appendix

In this Appendix, we provide some mathematical arguments for footnote 8 in Section 5.

Suppose that the agent is to undertake actions dependent on past outcomes. We think of such an agent’s strategy as a sequence of “behavior strategy”; for example:

\[ \alpha = (\alpha^1, \alpha^2(x^1), \alpha^3(x^1, x^2), \ldots, \alpha^T(x^1, \ldots, x^{T-1})) \]

where each \( \alpha^t : \{1, \ldots, N\}^{t-1} \to A \) is a mapping from the history of past outcomes (up to period \( t-1 \)) to the action in period \( t \).

The problem in footnote 8 is whether the agent improves his payoff by selecting such a history-dependent strategy \( \alpha \) (rather than a history-independent strategy \( a = (a^1, a^2, \ldots, a^T) \)).

**Theorem 4.** If the contract is simple, then the agent cannot improve his payoff by selecting a history-dependent strategy \( \alpha \).

**Proof.** The expected payoff to the agent undertaking such a strategy \( \alpha \) is written as:

\[
\sum_{a \in A^T} \left\{ \left( \sum_{(x^1, \ldots, x^{t-1}) \in I(a, \alpha)} \prod_{t=1}^{T-1} \Pr[x^t | \alpha^t(x^1, \ldots, x^{t-1})] \right) \times \left( \sum_{i=1}^{T-1} u(w^i) + \sum_{j=1}^{N} p_j^T(a)u(w_j^T) - C \cdot m(a) \right) \right\} 
\]

(8)

where:

\[ I(a, \alpha) = \{ (x^1, \ldots, x^{T-1}) | \alpha^t(x^1, \ldots, x^{t-1}) = a^t \text{ for } t = 1, \ldots, T \} \]

That is, \( I(a, \alpha) \) is the set of historical outcomes with a positive probability that action profile \( a \) is played under strategy \( \alpha \).

Because every history \( (x^1, \ldots, x^{T-1}) \) generates exactly one action profile, \( a \) given \( \alpha \), the first parentheses in (8) is seen as a probability distribution of \( a \) over
$A^T$; that is,
\[
\sum_{a \in A^T} \left( \sum_{(x^1, \ldots, x^{t-1}) \in I(a,a)} \prod_{t=1}^{T-1} \Pr[x^t | \alpha_t(x^1, \ldots, x^{t-1})] \right) = 1 \quad \text{for any } \alpha.
\]

As the expected value of random variables does not exceed the maximum of the variables, we establish that:
\[
(8) \leq \max_{a \in A^T} \left( \sum_{t=1}^{T-1} u(w_t) + \sum_{j=1}^{N} p_j^T(a)u(w_j^T) - C \cdot m(a) \right)
\]
\[
= \max_{a \in A^T} (V(a, w) - C \cdot m(a)).
\]

\[\square\]

References


