The Benefit of Mixing Private Noise into Public Information in Beauty Contest Games

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May 2010

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The Benefit of Mixing Private Noise into Public Information in Beauty Contest Games*

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May 27, 2010

Abstract

The authorities often disclose their economic forecasts to the public ambiguously. The more ambiguous the way in which they present the information, the more variously market participants interpret such announcements. Hence, an ambiguous public announcement adds some private noise to the authorities’ economic forecasts. Using Keynesian beauty contest games, as in Morris and Shin (2002), we find that the addition of private noise to public information is often socially beneficial. We also derive the optimal information dissemination policy and find that the authorities should acquire information that is as precise as possible, but they should make ambiguous announcements to a degree that reflects the uncertainty in their forecast.

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*We would like to thank Hideshi Itoh, Noritaka Kudoh, Makoto Saito, Tadashi Sekiguchi, and Akihisa Shibata; the participants of Winter Institute 2010 sponsored by the global COE program of Hitotsubashi University, Young Economist Conference by the global COE program of Osaka University, and the Japanese Economic Association Autumn Meeting 2009 at Senshu University; and the seminar participants at Hitotsubashi University, Senshu University, Tokyo Institute of Technology, Tokyo Metropolitan University, and Bank of Japan for their helpful comments. Any remaining errors are the sole responsibility of the authors. The authors are also grateful for research grants provided by the research project entitled: Understanding Inflation Dynamics of the Japanese Economy, funded by JSPS Grant-in-Aid for Creative Scientific Research (18GS0101), and the global COE programs of Hitotsubashi University and Osaka University.

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1 Introduction

Central bankers as well as governments are often required to disclose transparent information about the underlying economic fundamentals. It is widely believed that if market participants have more accurate information, they can behave more accurately. In some situations, this is true. The provision of such additional information by the authorities allows the participants to predict the fundamentals with greater accuracy. Hence, if the participants are concerned only about the fundamentals, their behavior tends to reflect the fundamentals more accurately, and thus, in this situation, it is socially beneficial for them to possess more information.

However, in many situations, it is observed that market participants behave strategically, and for this reason, the participants are concerned about not only the fundamentals but also other participants’ behavior. Morris and Shin (2002) consider the problem of public information dissemination in such a situation using “beauty contest games.” In their model, each agent has a coordination motive arising from strategic complementarity, but the coordination motive becomes socially inefficient through the zero-sum structure. Each agent receives two signals about the underlying fundamentals. One is a public signal from the authorities, which is shared by all agents. In this model, a public signal is a signal that is perfectly correlated among agents. Another is private information, which is perfectly uncorrelated among agents and cannot be observed by other agents. In this situation, Morris and Shin (2002) show that the authorities’ public information dissemination could lower social welfare. The public information serves as a focal point for the belief of the other agents. Equivalently, for each agent, the public information is more useful than private information for the formation of “higher-order expectations,” that is, an agent’s expectation about others’ expectations of the others’ expectations of · · · of fundamentals. Hence, when there is strategic complementarity, the agents place more weight on public information than does the social planner when choosing his/her action. In other words, the agents overreact to public information. Because of this, when the public information is highly noisy, an agent’s action deviates from the fundamentals on average as a result of the public information dissemination. In consideration of this behavior, the authorities should not disclose information if its precision is sufficiently low relative to the agents’ private information and should disclose the information as precisely as possible otherwise. Therefore, in their model, the optimal announcement policy has a “bang-bang” solution feature.

We may regard the stock market to be a case described by the model studied by Morris and Shin (2002). Certainly, the dividends of stocks reflect the fundamentals (the productivity of firms) to some extent. Therefore, market participants are concerned about the fundamentals. In addition, there exists strategic complementarity
in the market, because when many agents buy a stock, its price rises, and hence, an agent who has already bought the stock obtains capital gain. Hence, agents are also concerned about others’ actions. However, such a coordination motive arising from strategic complementarity is not always socially beneficial, because such behavior may be the source of bubbles and financial crises. Thus, if this information significantly affects the agents’ behavior, the authorities should handle the information dissemination carefully.

As an argument against Morris and Shin (2002), it is rare that the authorities do not disclose any routine information in a given case. Instead, the authorities often announce their information ambiguously. Alan Greenspan, the former Chairman of the Federal Reserve, made the following statement in the Wall Street Journal:

> Since I have become a central banker I’ve learned to mumble with great incoherence. If I seem unduly clear to you, you must have misunderstood what I said. (Alan Greenspan, Wall Street Journal, 1987)

How should we understand this quotation? It is theoretically true that agents overreact to perfectly correlated information in the economy with strategic complementarities. If a coordination motive arising from strategic complementarity is socially undesirable, a clear announcement by the authorities, which provides purely public information to the agents, could result in a worse outcome because agents overreact to public information. For this reason, we conjecture that Greenspan’s statement is correct. Ambiguous announcements by the authorities could have varied interpretations among agents. Thus, an ambiguous announcement by the authorities corresponds to mixing private noise into public information. Hence, an ambiguous announcement by the authorities could prevent an overreaction by the agents. However, preventing an overreaction by an ambiguous announcement of the authorities would not always improve social welfare, because an ambiguous announcement implies a degradation of information about the fundamentals. Hence, the welfare effects of ambiguous announcements should be investigated formally. Moreover, when the authorities are allowed to disclose their information vaguely, it is not clear how they should acquire information; that is, the welfare effects of common noise among agents should be reconsidered. In this paper, we address these two questions.

We use the same payoff structure as Morris and Shin (2002) but extend the information structure so that both signals are allowed to be imperfectly correlated. To do this, we assume that both the signals received by agents contain two kinds of noise, common noise and private noise. This assumption of the information structure can be interpreted as follows: all agents receive information about the fundamentals from two information sources. For example, One is a private think tank and another is the authorities (the government or the central bank).

\footnote{Myatt and Wallace (2008) use a similar information structure, but study a Lucas-Phelps island economy. Hence, their results are partially different from ours.}
information source researches the fundamentals. If a source has a sufficiently good ability to conduct such research, it can predict the fundamentals with small error. Then, the source releases its prediction to market participants. If the source states the prediction very clearly, market participants would receive common information from the source. Hence, the common noise of the signal results from the imperfect nature of the source’s research. In other words, (the reciprocal of) the variance of the common noise reflects the source’s research ability. Alternatively, if the source states its prediction ambiguously, the market participants would interpret the message in various ways. In other words, the ambiguity of the announcement by the source is equivalent to an additional private noise. Hence, (the reciprocal of) the variance of the private noise reflects the degree of clarity of the source’s announcement.

Using our model, we consider the welfare effect of research ability and clarity of public predictions. We find that the welfare effect of the ability can be U-shaped, as found in Morris and Shin (2002). Therefore, if authorities can only choose research ability, the bang-bang solution still exists. However, we also find that if the authorities can choose clarity as well as ability, then the bang-bang solution disappears. Our model indicates that it is optimal that the authorities always acquire information with as high a precision as possible but disclose it with the ambiguity appropriate to its research ability. This means that there is a benefit of mixing private noise with public information. In our model, the control of clarity is given to the authorities as a free option. According to our results, even in the situation where Morris and Shin (2002) conclude that the authorities should hide their information, the authorities should use the option to disclose information ambiguously in order to improve welfare.

Related Literature

In this literature, most studies assume a restrictive information structure, which consists of purely private and purely public information. They investigate the welfare effect under various payoff structures and find implications against Morris and Shin (2002). Angeletos and Pavan (2004) assume payoffs that have investment externalities, and Hellwig (2005) assumes a monopolistic competition payoff structure. They conclude that purely public information always improves social welfare. Angeletos and Pavan (2007) consider a more general market environment. They parameterize the cases when purely public information improves the welfare.

The exceptions are Cornand and Heinemann (2008) and Myatt and Wallace (2008). They focus on the welfare effects under a different information structure from Morris and Shin (2002).

Cornand and Heinemann (2008) study the optimal dissemination range of public announcement. They conclude that the authorities should always choose as high a precision of public information as possible and public information should be disseminated to only some of the agents. Hence, from Cornand and Heinemann (2008)
and our study, we find that disseminating public information to some of the agents and mixing private noise into public information have a similar welfare implication. Myatt and Wallace (2008) extend a standard information structure. Their extension is similar to ours. However, their study mainly focuses on a Lucas-Phelps island economy. They emphasize that in the Lucas island economy, the output gap is an appropriate measure of macroeconomic performance. On the other hand, as in Morris and Shin (2002), we define the welfare measure as the heterogeneity of equilibrium action. We agree that the output gap is a more correct welfare measure of the macroeconomy. However, we focus on the stock market, and believe that the welfare measure of heterogeneity is appropriate for three reasons. First, in stock markets, the authorities would care about the realized price as well as the stability of the price. We can divide the welfare measure, as in Morris and Shin (2002), into two parts, the loss from volatility (the average deviation of average action from the fundamentals) and dispersion (the average deviation of each agents’ action from average action). The volatility part corresponds to the gap between the realized price and fundamentals, and the dispersion of the actions represents the dispersion of the bid prices. It can be interpreted as the market stability measure. Hence, we think that heterogeneity of actions is appropriate for the efficiency measure of stock markets. Second, in reality, the central bank communicates to stock market participants rather than the production sectors. Hence, when central bank communication is considered, the stock market efficiency measure of heterogeneity is more appropriate for the objective function. Third, stock market conditions strongly affect macroeconomic performance through the firms’ and households’ financial positions and transmission mechanism of the monetary policy. Hence, even if we think about macroeconomic performance, stock market conditions are still important for the central bank and the government.

2 The Model

2.1 Payoff Structure

In the economy, there is one measure of agents indexed by \(i \in [0, 1]\). Each agent chooses an action \(a_i \in \mathbb{R}\). We write \(a\) for the action profile over all agents. As in Morris and Shin (2002), we assume the following payoff structure that describes Keynesian beauty contest games:

\[
u_i(a, \theta) = -(1 - r)(a_i - \theta)^2 - r(L_i - L),
\]

where \(\theta\) is the state of the economy (fundamentals), \(r \in [0, 1]\) is a constant, and

\[
L_i \equiv \int_0^1 (a_j - a_i)^2 \, dj, \quad L \equiv \int_0^1 L_j \, dj.
\]

The first term of \(i\)’s payoff represents that agent \(i\) suffers a loss from the distance between his action \(a_i\) and economic fundamentals \(\theta\). \(L_i\) is referred to as beauty
The term implies that agent \( i \) incurs a loss from the distance between his action and the others’ action. Hence, agents have a coordination motive. Then, we have agent \( i \)’s best response function:

\[
a_i = (1 - r)E_i(\theta) + rE_i(\bar{a}),
\]

where \( E_i \) represents the agent \( i \)’s expectation operator conditional on his available information and \( \bar{a} \equiv \int_0^1 a_j dj \) represents the average action over all agents. From (2), parameter \( r \) can be understood as the degree of agent \( i \)’s coordination motive in this economy.

We assume that social welfare is defined as the normalized aggregation of payoff over the whole population:

\[
W(a|\theta) \equiv \frac{1}{1-r} \int_0^1 u_i di = - \int_0^1 (a_i - \theta) di.
\]

Note that the utility loss generated by the beauty contest term disappears in this social welfare function; hence, the socially optimal action is

\[
a_{i,\text{opt}} = E_i(\theta).
\]

This implies that, in this model, the action that reflects only the fundamentals is socially optimal.

If the information is complete, \( a_i = a_{i,\text{opt}} = \theta \). Hence, under complete information, the equilibrium allocation in this economy is efficient and can achieve the first-best allocation. We consider the case of incomplete information below.

### 2.2 Information Structure

For simplicity, we assume that the agents have an improper prior distribution of the fundamentals \( \theta \). In Morris and Shin (2002), each agent potentially receives two kinds of signals regarding \( \theta \). One is a purely private signal that is independent among agents, \( x_i = \theta + \epsilon^x_i \) with \( \epsilon^x_i \sim N(0, \sigma_{x,an}^2) \). The other is a purely public signal that is perfectly correlated among agents, \( y = \theta + \eta^y \) with \( \eta^y \sim N(0, \sigma^2_{y,ac}) \).

We extend this information structure. First, consider public signal \( y \). Morris and Shin (2002) interpret this as the information released by the authorities. This assumption implies that the authorities acquire information and disclose it as it is. However, in reality, information acquisition and disclosure are different policy tools. Statistical staffs or researchers obtain the information and report to top officials. The top officials decide not only whether to release it but also with how much clarity they should disclose it to the public. To separate these two policies, we assume the signal received by agent \( i \) from the authorities is such that

\[
y_i \equiv y + \epsilon^y_i = \theta + \eta^y + \epsilon^y_i, \quad \text{with} \quad \epsilon^y_i \sim N(0, \sigma^2_{y,an}).
\]
As explained in Section 1, we can interpret $\eta_y$ as an information acquisition error, and $\sigma_{y;ac}^2$ as the authorities’ research ability.\footnote{Note that high $\sigma_{y;ac}^2$ means low ability.} We assume the authorities can control $\sigma_{y;ac}^2 \in [\bar{\sigma}_{y;ac}^2, \infty)$ and $\bar{\sigma}_{y;ac}^2 > 0$. The second error term $\epsilon_{yi}^2$ can be regarded as announcement error and the authorities can choose the value of $\sigma_{y;an}^2 \in [0, \infty)$. This means that the authorities can freely choose the expression of disseminating information to the public and control the variance of the interpretations of her message. We refer to $\sigma_{y;an}^2$ as announcement clarity.\footnote{Note that high $\sigma_{y;an}^2$ means low clarity.}

To make our analysis general, we assume that the signal from the other information source (for example, a private think tank), $x_i$, is such that $x_i \equiv \theta + \eta_x + \epsilon_x^i$, with $\eta_x \sim N(0, \sigma_{x;ac}^2)$, and assume that $\sigma_{x;ac}^2$ and $\sigma_{x;an}^2$ are given for the authorities.

### 3 Efficient and Equilibrium Information Allocation

In this section, we derive an equilibrium strategy and socially optimal information allocation under the payoff and information structure given in the previous section.

Define $\psi_k \equiv (\sigma_{k;ac}^2 + \sigma_{k;an}^2)^{-1}$, $k \in \{x, y\}$, as signal $k$’s precision. By using $\psi_k$, agent $i$’s expectation of the value of $\theta$ can be derived as:

$$
E_i(\theta) = E(\theta|x_i, y_i) = \frac{\psi_x x_i + \psi_y y_i}{\psi_x + \psi_y}.
$$

Hence, from (4), agent $i$’s socially optimal information allocation is

$$
a_{i,opt}(x_i, y_i) = \frac{\psi_x x_i + \psi_y y_i}{\psi_x + \psi_y}.
$$

In equilibrium, however, agents do not behave according to (6) because they are concerned about the average action as well as the fundamentals. Then, the signals’ correlations among agents play an important role to estimate average action and hence, an equilibrium strategy for the following reason. If the signal is uncorrelated among agents, agent $i$ cannot know the realized value of the signal that other agents receive. On the other hand, if the signals are correlated among agents, then the signal that agent $i$ receives has some information about the realized values of the signal that other agents receive. Because all agents know that the others move by using such correlated signals, the correlated signal can use to predict the average action. Define $\rho_k \equiv \sigma_{k;ac}^2/(\sigma_{k;ac}^2 + \sigma_{k;an}^2)$ as signal $k$’s correlation among agents and we then have the following proposition about an equilibrium strategy.
**Proposition 1.** The equilibrium exists, is unique, and is linear in \( x_i \) and \( y_i \). Such an equilibrium strategy can be written as

\[
a_i(x_i, y_i) \equiv \frac{\beta_x \psi_i x_i + \beta_y \psi_i y_i}{\beta_x \psi_x + \beta_y \psi_y},
\]

where \( \beta_k \equiv (1 - r \rho_k)^{-1} \).

**Proof.** See Appendix.

Since \( r > 0 \), \( \beta_k \) is increasing with \( \rho_k \). Hence, we find that the agents react to the relatively correlated signal more strongly than the Bayesian (socially optimal) weight.

From (6) and (7), we find that if \( \beta_x \neq \beta_y \), agents allocate the two signals inefficiently. Recall that \((\sigma_{y,ac}^2, \sigma_{y,an}^2)\) is the authorities’ information policy. Hence, the authorities can control \( \beta_y \) and set the equilibrium weight near the socially optimal weight. However, changing \( \beta_y \) involves a change of \( \psi_y \). If the authorities have to lower \( \psi_y \) in order to set \( \beta_y \) near \( \beta_x \), the welfare effect is ambiguous because such a policy makes it difficult for agents to predict the fundamentals. Hence, what we have to do next is to check the marginal welfare effects of ability and clarity and find their optimal pairs.

## 4 Welfare and the Optimal Policy

Substituting (7) into (3), the expected welfare is given as

\[
E[W(a|\theta)] = -E \left[ \int_0^1 (a_i - \theta)di|\theta \right] = -\frac{\psi_x \beta_x^2 \psi_y \beta_y^2}{(\psi_x \beta_x + \psi_y \beta_y)^2} \equiv -\Omega(\sigma_{y,ac}^2, \sigma_{y,an}^2).
\]

Examining (8), we can know the welfare effect of authorities’ ability and announcement clarity. First, we consider the effect of ability given clarity:

\[
\frac{\partial \Omega}{\partial \sigma_{y,ac}^2} < 0
\]

\[
\Leftrightarrow \sigma_{y,an}^2 < \frac{-(1 - r)\sigma_{x,ac}^2 + (2r - 1)\sigma_{x,an}^2}{1 + 2r} \sigma_{y,ac}^2 - \frac{[(1 - r)\sigma_{x,ac}^2 + \sigma_{x,an}^2]^2}{(1 + r)[(2r + 1)\sigma_{x,ac}^2 + (1 - r)^{-1}\sigma_{x,an}^2]}.
\]

We can verify that when \((2r - 1)/(1 - r) > \sigma_{x,ac}^2/\sigma_{x,an}^2\), the welfare effect of ability is U-shape. Figure 1 shows the welfare effect of ability graphically.
Note that we can translate the case of $\sigma_{y,ac}^2 = \infty$ as no announcement. Because the signal that has zero precision means that such an information is junk, no agent uses that signal to determine his action. Hence, the authorities determine her announcement policy as to whether the welfare surpasses $EW(\sigma_{y,ac}^2 = \infty)$. We can easily obtain the critical value $\tilde{\sigma}_{y,ac}^2$ defined as $EW(\sigma_{y,ac}^2 = \tilde{\sigma}_{y,ac}^2) = EW(\sigma_{y,ac}^2 = \infty)$:

$$\tilde{\sigma}_{y,ac}^2 \equiv \sigma_{x,ac}^2 (1 - r + \frac{\sigma_{x,an}^2}{\sigma_{x,ac}^2})^2 - (1 - r)^2 + (3r - 1)\sigma_{x,an}^2 \frac{(1 + r)\sigma_{x,ac}^2 + \sigma_{x,an}^2}{\sigma_{x,ac}^2}. \tag{10}$$

The next proposition sums up the comparative statics about the ability.

**Proposition 2.** Assume that $(2r - 1)/(1 - r) > \sigma_{x,ac}^2/\sigma_{x,an}^2$ and fix the clarity. If $\tilde{\sigma}_{y,ac}^2 \geq \tilde{\sigma}_{y,ac}^2$, the welfare improves by her announcement. On the other hand, if $\tilde{\sigma}_{y,ac}^2 < \tilde{\sigma}_{y,ac}^2$, the welfare is degraded relative to no announcement.

Note that Morris and Shin (2002) is the case of perfect clarity, $\sigma_{y,ac}^2 = 0$. Then the “bang-bang solution” is optimal: disclose their information if the ability is sufficiently high, or hold it back otherwise.

Next, we consider the welfare effect of clarity:

$$\frac{\partial \Omega}{\partial \sigma_{y,an}^2} < 0 \iff \sigma_{y,an}^2 < \frac{-(1 - r)^2\sigma_{x,ac}^2 + (3r - 1)\sigma_{x,an}^2}{(1 + r)\sigma_{x,ac}^2 + \sigma_{x,an}^2} \tilde{\sigma}_{y,ac}^2 - \frac{[(1 - r)\sigma_{x,ac}^2 + \sigma_{x,an}^2]^2}{(1 + r)\sigma_{x,ac}^2 + \sigma_{x,an}^2}. \tag{10}$$

When $(3r - 1)/(1 - r)^2 > \sigma_{x,ac}^2/\sigma_{x,an}^2$ and

$$\sigma_{y,ac}^2 > \tilde{\sigma}_{y,ac}^2 = \frac{[(1 - r)\sigma_{x,ac}^2 + \sigma_{x,an}^2]^2}{-(1 - r)^2\sigma_{x,ac}^2 + (3r - 1)\sigma_{x,an}^2},$$

the welfare effect of clarity is an inverted U-shape with respect to $\sigma_{y,an}^2$. We depict the fact in Figure 2.

The intuition is as follows. A more ambiguous announcement by the authorities lowers its correlation and precision. When the clarity of the authorities is high, $y_i$ is more correlated than $x_i$ (so that $\beta_y > \beta_x$). Then, a more ambiguous announcement
moves its correlation toward another signal’s correlation. Hence, the inefficiency of information use is reduced. This positive welfare effect exceeds the negative one by the information degradation when \( y_i \) is much more correlated than \( x_i \). However, as the clarity decreases, the positive welfare effect by the lower correlation becomes weaker, and becomes even more negative when \( y_i \) is less correlated than \( x_i \). Hence, the total marginal welfare effect by the more ambiguous announcement is positive when the clarity is high but negative when the clarity is low.

**Proposition 3.** Assume that \( (3r-1)/(1-r)^2 > \sigma^2_{z,ac}/\sigma^2_{x,an} \) and \( \sigma^2_{y,ac} > \tilde{\sigma}^2_{y,ac} \), and fix the ability of the authorities. Then, the welfare effect of disseminating ambiguous information to the public is an inverted U-shape. Hence, the optimal clarity \( \tilde{\sigma}^2_{y,an} \) is strictly positive.

Note that the clarity in our model is regarded as the option that is granted to the authorities in the model of Morris and Shin (2002) at no cost. Therefore, if welfare is decreased by using this option, there is no need to exercise it. However, Proposition 3 states that the optimal clarities exist in the interval \([0, \infty)\). This means that this option has the ability to improve social welfare.

[Insert Figure 3 here.]

Here, we bundle the two effects of the two policies in one figure. Figure 3 illustrates the social welfare contours in \((\sigma^2_{y,ac}, \sigma^2_{y,an})\)-space. Note that from (9) and (10), there is the line that represents \( \partial EW/\partial \sigma^2_{y,an} = 0 \) upper than the line represents \( \partial EW/\partial \sigma^2_{y,ac} = 0 \). The curves represent iso-welfare sets. The arrows show the direction of welfare improvement. Recall that this beauty contest game is efficient and achieve the first best in the case of complete information. Therefore, the origin is the point of first best. Recall also that the authorities can freely choose clarity \( \sigma^2_{y,an} \) in \([0, \infty)\), but the choice of ability \( \sigma^2_{y,ac} \) is restricted in \([\bar{\sigma}^2_{y,ac}, \infty)\). Finally, we have our main proposition.

**Proposition 4.** Assume \( \sigma^2_{y,ac} \in [\bar{\sigma}^2_{y,ac}, \infty) \) and \( \sigma^2_{y,an} \in [0, \infty) \). Then, (i) the optimal ability is equal to \( \bar{\sigma}^2_{y,ac} \); (ii) if \( (3r-1)/(1-r)^2 > \sigma^2_{z,ac}/\sigma^2_{x,an} \) and \( \sigma^2_{y,ac} > \tilde{\sigma}^2_{y,ac} \), then the optimal clarity \( \tilde{\sigma}^2_{y,an} \) is strictly positive; and (iii) \( \tilde{\sigma}^2_{y,an} \) is (weakly) increasing in \( \bar{\sigma}^2_{y,ac} \).

(i) says that the authorities should acquire as a precise information as possible. This means that the “bang-bang solution” of Morris and Shin (2002) disappears. (ii) implies that it is desirable that the authorities announce her information ambiguously to some extent when the ability is low. (iii) means that the lower the authorities’ ability, the more ambiguous she should be in her announcement. We think that proposition 4 supports Greenspan’s comments.
5 Conclusion

We extend the information structure as in Morris and Shin (2002) and show that the authorities’ ambiguous announcement could be beneficial to social welfare. Our main conclusions are presented in proposition 4. An ambiguous announcement means adding private noise into the purely public information. If the agents overreact to the public information because of strategic complementarities, then adding private noise to public information prevents the agents from overreacting. Such an announcement policy could be desirable when the authorities have an imperfect information acquisition ability. Moreover, our result suggests that the lower the ability, the greater the benefit from the more ambiguous announcement.

There are some open questions in this paper. First, we assume a full-commitment announcement policy. Allowing that the authorities act strategically, she may tell a lie to improve ex post social welfare after the value of signal is realized. Second, the complete information and the common knowledge about ability and clarity are strong assumptions. In reality, it is natural that market participants cannot know these precise values. Finally, our result may depend on the specification of the payoff structure, as in Morris and Shin (2002). In this literature, it is known that the welfare implication of information is highly dependent on the payoff structure.\footnote{See Angeletos and Pavan (2007).}

However, we think that this research offers some kind of contribution. First, our model has simple and clear implications and some potential for extension. Second, our conclusion that an ambiguous announcement can be beneficial suggests that we need to consider in more detail not only whether the government should disclose but also how the government should disclose.

References


Appendix: Proof of Proposition 1

The existence, uniqueness, and linearity are the same reasons as in this standard literature. Here, we show the derivation of (7).

Define $x \equiv \int_0^1 x_i di = \theta + \eta^x$. We can translate $y_i$ into

$$y_i = \theta + \eta^y - \eta^x + \eta^y + \epsilon^x_i = x - \eta^x + \eta^y + \epsilon^x_i.$$

This means that $y_i$ has information about $x$. Similarly,

$$x_i = \theta + \eta^y - \eta^x + \eta^y + \epsilon^y_i = y - \eta^y + \eta^x + \epsilon^y_i.$$

For each $k \in \{x, y\}$, define $\psi_k \equiv (\sigma_{k,ac}^2 + \sigma_{k,an}^2)^{-1}$, and $\rho_k \equiv \sigma_{k,ac}^2 / (\sigma_{k,ac}^2 + \sigma_{k,an}^2)$.

We have

$$E_i(\theta) = \frac{\psi_x x_i + \psi_y y_i}{\psi_x + \psi_y},$$

$$E_i(x) = \frac{(\psi_x + \psi_y \rho_x) x_i + \psi_y (1 - \rho_x) y_i}{\psi_x + \psi_y}, \text{ and}$$

$$E_i(y) = \frac{\psi_x (1 - \rho_y) x_i + (\psi_y + \psi_x \rho_y) y_i}{\psi_x + \psi_y}.$$

Because of the assumptions of the normal distribution for all error terms and the quadratic payoff function, we can use the method of undetermined coefficients, as in Morris and Shin (2002). We assume

$$a_i = \lambda x_i + (1 - \lambda) y_i. \quad (11)$$

Then, the average action is

$$\bar{a} = \lambda x + (1 - \lambda) y.$$

Hence, from (2),

$$a_i = (1 - r) E_i[\theta] + r E_i[\bar{a}]$$

$$= (1 - r) E_i(\theta) + r \lambda E_i(x) + r \lambda E_i(y). \quad (12)$$

Substituting each expected value into (12) and comparing (12) with (11), we have

$$\lambda = \frac{\beta_x \psi_x}{\beta_x \psi_x + \beta_y \psi_y}, \text{ and} \quad 1 - \lambda = \frac{\beta_y \psi_y}{\beta_x \psi_x + \beta_y \psi_y},$$

where $\beta_k \equiv (1 - r \rho_k)^{-1}$.

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Figure 1: welfare effect of information acquisition

Figure 2: welfare effect of ambiguous announcement
Figure 3: optimal announcement policy