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A THEORY OF OPTIMUM TARIFF UNDER REVENUE CONSTRAINT

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A theory of optimum tariff under revenue constraint

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Abstract

This paper analyzes the revenue-constrained optimum tariff problem. When a fixed level of tax revenue has to be collected only from tariffs, an efficient resource allocation can not be achieved by any tariff structure. Thus we need to find the optimum tariff structure as the second best resource allocation. We will consider a small open economy with a non-tradable good, and with full technological substitutability between each good. Then, the optimal tariff structure is characterized by following: (i) the optimum tariff rate is lower for the importable that is the closer substitute of the untariffed goods, and (ii) the stronger is the cross-substitutability between importables, the closer is the optimal tariff to uniformity. We also show that the Inverse Elasticity Rule no longer holds in this model.

Key words: Revenue-constrained optimum tariff, Optimal tax rules, Non-tradable, Corlett and Hague Rule, Cross Substitutability Rule, Inverse Elasticity Rule

JEL classification: F1, H21

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1. Introduction

The World Bank has often recommended reduction of highest tariff rates to developing countries.[1] However, tariff is the main revenue source of many developing countries, and hence reduction of highest tariffs forces these countries to increase lower tariffs. The World Bank has in effect recommended these countries to make the tariff structure closer to uniformity.

On the other hand, the literature of revenue-constrained optimum tariff problem, such as Dahl, Devarajan, and van Wijnbergen (1994), Panagariya (1994) and Mitra (1994), has pointed out that the optimum tax rules must be applied to tariff rates in developing countries where tariff is the main source of the government revenue. In particular, this literature has emphasized the importance of the Inverse Elasticity Rule as a conceptual guidance for tariff reform.

In the models employed in this literature, which have only tradable goods, a change in a tariff rate does not affect the prices of the untariffed goods.[2] This is because the only uatariffed goods in these models are exportables, whose prices are exogenously given from the assumption of a small country. In an economy with non-tradable goods, however, a change in a tariff rate affects the prices of the untariffed goods, i.e., the non-tradables. The present paper shows which rules are robust and which rules are not in such an economy.

The purpose of the present paper is four fold. The first is to derive optimal tariff rules in a model with full technological substitutability. In order to

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[1] See Rajaram (1994) for a review of the World Bank's tariff recommendation.

[2] The non-tradable goods here include (i) intrinsically non-tradable goods due to the preference of each country, (ii) goods with prohibitive transportation cost, and (iii) goods with

analyze the economy where prices of the non-tradables are flexible, we need to consider a model with full technological substitutability. The second is to show that the Inverse Elasticity Rule does not hold in an economy with a non-tradable good. The third is to establish that the Corlett and Hague Rule continues to hold in an economy with a non-tradable good. Thus the tariff rate has to be higher for the imports that is more complementary with the compound of the non-tariffed goods. The fourth is to demonstrate that the so called Cross Substitutability Rule also continues to hold even when a non-tradable good is introduced. Thus substitutability between the imports tends to make the tariffs more uniform in the non-tradable good economy as well as in the economy with a non-traded good.

When a fixed level of tax revenue has to be collected only from tariffs, i.e., when a lump-sum tax is not available, an efficient resource allocation can not be achieved by any tariff structure.[3] Thus we need to find the tariff structure that attains the second best resource allocation. We will call this *the revenue-constrained optimal tariff problem in a small country.*

This problem arises under the institutional framework that a fixed level of revenue has to be collected only by tariffs. This problem is entirely different

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prohibitive tariff rates or restrictions.

[3] The reason is as follow: An efficient resource allocation would require that the domestic prices of all the tradables be proportional to their world prices. Given import tariffs, the domestic prices of importables are higher than the world prices. Thus the first best policy would require that the domestic prices of exportables be higher than the world prices, implying that subsidies must be given to all the exportables. Besides, the ad volorem rates of subsidies on exportables and tariffs on importables must be exactly equal. But then the tariff revenue would be zero. See Appendix 1 for this.

This would conflict with the original constraint of raising fixed revenue. Hence, to raise a given tariff revenue, price distortions are inevitable. Problem is what is the optimal distorted structure of tariff is.

from the more familiar optimal tariff problem in a large country, studied by Graaff (1949-50) and Johnson (1953-54) among others, where a lump-sum tax is available. In a large country, the optimal tariffs have to be non-uniform even if a lump-sum tax is available.

The revenue-constrained optimal tariff problem in a small country is an extension of the optimal taxation problem pioneered by Ramsey (1927) and Daimond and Mirrlees (1971).[4] The exportable in the optimal tariff model plays the role of the leisure in the optimal commodity tax model as the untaxed good.[5] Dasgupta and Stiglitz (1974) started the field of optimal tariff in a small economy. Devarajan et al(1986), Heady and Mitra (1987) numerically analyzed optimal tariff rates for a few developing countries. More recently, this problem has been studied by Devarajan et al (1994), Panagariya (1994), Chambers(1994), Mitra (1994) and Hatta (1994).

The most prominent among the optimal tariff rule is the Inverse Elasticity Rule. For example, Dusgupta and Stiglitz (1974), Devarajan et al (1994), and Panagariya (1994) discussed this rule. The Inverse Elasticity holds, however, only under the stringent condition that the imports (i.e., tariffed goods) are independent of each other in consumption and production, and its practical value is limited. The present paper shows that in an economy where a non-tradable good exists, the Inverse Elasticity Rule no longer holds even under the condition

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[4] In R-D-M model labor supply is endogenous, and the distortion is generated in between goods and leisure. Commodity taxes and wage subsidy at a uniform rate get rid of this distortion, and tax revenue, however, is zero. At this point, the optimal commodity tax problem is generated. In the optimal tariff model, we will make labor supply constant, and hence can disregard this distortion.

[5] See Hatta (1994) for a simple comparison of the two theories.

that import goods are independent of each other.[6]

The main message of the present paper is that the Corlett and Hague Rule and the Cross-Substitutability Rule are more relevant than the Inverse Elasticity Rule as the conceptual guidance for the revenue-constrained optimal tariff design in the realistic situation where non-tradables are abundant.

Section 2 presents the model that has only tradable goods. Section 3 will analyzes the optimal tariff problem with a non-tradable. Section 4 proves the main theorem.

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[6] Chambers (1994) showed the sufficient condition of uniform tariff structure. Its implication is similar to that of Sadka (1977) in the optimal commodity tax problem. Mitra (1994) derived the Samuelson rule in the optimal tariff problem.

2. The T-Economy

2.1. The model

We consider an economy that satisfies the following assumptions.

Assumption 1. The economy is small and open. It has perfectly competitive markets for goods and factors.

Assumption 2. The economy produces three goods, one exportable good and two importable goods. The only inputs of the economy are endowed factors. We will denote the exportable good by 0 and the importable goods by 1 and 2.

Assumption 3. There is only one consumer. Initially he possesses all of the factors, whose endowments are fixed. All of his income is obtained from factor markets.

Assumption 4. The consumer consumes all of the three final goods. He has a well-behaved utility function $u(\mathbf{x})$, where $\mathbf{x}' = (x_0,\ x_1,\ x_2)$ is the demand vector of the final goods, and chooses the commodity bundle that maximizes his utility level under given prices and income.[7]

The budget constraint of the consumer is given by

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[7] Since the level of the public good provision is fixed throughout the analysis, it does not enter the utility function as an explicit argument. A function is well behaved if it is (i) increasing

$$\mathbf{q}'\mathbf{x} = m \tag{1}$$

where $\mathbf{q}' = (q_0,\ q_1,\ q_2)$ is the domestic-price vector and m is the consumer's income.

His compensated demand function for the i-th good is given by

$$x_i = x_i(\mathbf{q},\ u), \qquad (i = 0, 1, 2) \tag{2}$$

where u is utility level.

Assumption 5. A producer maximizes his profit, taking prices as given.

The aggregate of the net revenue of the all firms, and hence of all industries is equal to the income of the consumer. Thus the aggregate budget equation of the producers is given by

$$\mathbf{q}'\mathbf{y} = m, \tag{3}$$

where $\mathbf{y}' = (y_0,\ y_1,\ y_2)$ is the output vector.

The supply function of the economy gives the commodity bundle that maximizes the total revenue $\mathbf{q}'\mathbf{y}$, of the production sector, under the given

in each argument, (ii) strictly quasi-concave, and (iii) twice continuously differentiable.

technology and prices. Its i-th element is given by[8]

$$y_i = y_i(\mathbf{q}) \qquad (i = 0, 1, 2). \tag{4}$$

Assumption 6. Tariffs are imposed on the two importables.

The relationship among the world prices, the domestic prices, and import tariffs is given by

$$\mathbf{q} = \mathbf{p} + \mathbf{t}, \tag{5}$$

where $\mathbf{p}' = (p_0,\ p_1,\ p_2)$ is the world price vector, and $\mathbf{t}' = (0,\ t_1,\ t_2)$ is the specific tariff vector.

Assumption 7. The only revenue source of the government is import tariffs. In particular, the government can not levy commodity taxes and income taxes. The government spends all of the tariff revenue on the purchase of the public good which is imported from a foreign country.[9]

Thus, the budget equation of the government can be written as

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[8] Since the domestic factors are fully employed by production sector and its supply level is fixed, it does not enter the supply function as an explicit argument.

[9] Note that the world price of the public good that the government imports is fixed, though we

$$\mathbf{t}'(\mathbf{x}-\mathbf{y}) = r \tag{6}$$

where r is the government spending in the public good and $\mathbf{x}-\mathbf{y}$ represents the net import vector of the private goods.

Assumption 8. The international balance of payment is in equilibrium, i.e.,

$$\mathbf{p}'(\mathbf{x}-\mathbf{y}) + r = 0\,. \tag{7}$$

The left hand side represents the sum of the international value of the net import of private goods and that of the public good.

Assumption 9. The exportable good is chosen as the numerair good: $q_0 = 1$.

Equation (7) is the market equilibrium condition. Equations (1), (3) and (6) are the budget equations of the economic agents. However, equations (1) and (3) can be combined into

$$\mathbf{q}'\mathbf{x} = \mathbf{q}'\mathbf{y}\,. \tag{8}$$

This equation is the budget constraint of the private sector, and implies that the consumers' expenditure equals the producers' revenue. Since equations (7) and (8) immediately yield (6), we will represent this economy by equations (2), (4), (5),

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do not denote it explicitly.

(7) and (8) in the following.

Definition 1. The economy satisfying Assumptions 1 through 9 is called the *T-Economy*. When equation (2), (4), (5), (7) and (8) are all satisfied, we say that *the T-Economy is in the full equilibrium.*

Define the excess demand functions

$$z_i(\mathbf{q},\ u) \equiv x_i(\mathbf{q},\ u) - y_i(\mathbf{q}) \qquad (i = 0, 1, 2) \tag{9}$$

and

$$\mathbf{z}(\mathbf{q},\ u) \equiv \mathbf{x}(\mathbf{q},\ u) - \mathbf{y}(\mathbf{q}), \tag{10}$$

where $\mathbf{z}'(\mathbf{q},\ u) = (z_0(\mathbf{q},\ u),\ z_1(\mathbf{q},\ u),\ z_2(\mathbf{q},\ u))$. By substituting these functions for and $\mathbf{x} - \mathbf{y}$ in (7) and (8), we have

$$\mathbf{q}'\mathbf{z}(\mathbf{q},\ u) = 0, \tag{11}$$

$$\mathbf{p}'\mathbf{z}(\mathbf{q},\ u) + r = 0. \tag{12}$$

In term of this notation, the T-Economy is in full equilibrium if and only if it satisfies equations (5), (11) and (12). Two equations (11) and (12) contain three variables, q_1, q_2, u, since r is fixed by assumption. When one of the three variables are exogenously given, the two equations determine the remaining variables. For example, if u is given, the model determines the remaining variables $(q_1,\ q_2)$. Therefore, from equation (5) we can find the tariff vector $\mathbf{t}$ that maximizes the utility level u. We now define the following.

Definition 2. The tariff combination $(t_1,\ t_2)$ that maximizes the utility level u in the model of (5), (11) and (12) for a fixed level of r is called *the optimum tariff* of the T-Economy.

2.2. The optimal tariff in the T-Economy

We will derive the optimal tariff in the T-Economy. Let us make a few definitions. Let $z_{ij} = \partial z_i / \partial q_j$. Define the import elasticity of the i-th good with respect to the price q_j by $\eta_{ij} = q_j z_{ij} / z_i$ and ad valorem tariff rate of the i-th good by $\tau_i = t_i / q_i$. Then we have following:

Theorem 1. In the T-Economy, the optimal tariff rate is expressed as

$$\begin{aligned} \tau_1 &= (\eta_{20} + \eta_{12} + \eta_{21})\theta, \\ \tau_2 &= (\eta_{10} + \eta_{12} + \eta_{21})\theta, \end{aligned} \tag{13}$$

for some scalar $\theta > 0$.

Proof. We must first choose $\mathbf{q}$ that maximizes the utility level in the model of

(11) and (12) for the fixed level of r [10]:

$$\max_{\mathbf{q},\ u} \quad u$$
$$\text{s.t} \quad \mathbf{q}'\mathbf{z}(\mathbf{q},\ u) = 0, \tag{14}$$
$$\mathbf{p}'\mathbf{z}(\mathbf{q},\ u) + r = 0.$$

The Lagrangian of this maximization problem is

$$L = u - \lambda(\mathbf{q}'\mathbf{z}(\mathbf{q},\ u)) - \delta(\mathbf{p}'\mathbf{z}(\mathbf{q},\ u) + r)$$

where λ and δ are Lagrangian multipliers. Its first-order conditions with respect to q_i are

$$-\lambda z_i - \delta\mathbf{p}'\mathbf{z}_i = 0, \qquad (i = 1, 2),$$

where $\mathbf{z}_i = (\partial\mathbf{z}/\partial q_i) = (z_{0i},\ z_{1i},\ z_{2i})$. By using the Homogeneity condition: $\mathbf{q}'\mathbf{z}_i = 0$ of the compensated demand function, this equation can be rewritten as

$$-\lambda z_i + \delta\mathbf{t}'\mathbf{z}_i = 0 \qquad (i = 1, 2). \tag{15}$$

To derive equation (13) from equation (15), see Auerbach (1985, p.92).[11] As to

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10 See Hatta (1993) for this formation of maximization problem.

11 Alternatively (13) is derived as a special case of (25), for which a full proof is given in

$\theta > 0$, see Daimond-Mirrlees (1971, p.262). *Q.E.D.*

The term $\eta_{12} + \eta_{21}$ is called *the cross-elasticity between the importables.* We will say that the i-th good is *more substitutable* for the k-th good than the j-th good is, if $\eta_{ik} > \eta_{jk}$, and importable goods are *independent* of each other, if $(\partial z_1 / \partial q_2) = (\partial z_2 / \partial q_1) = 0$, i.e., $\eta_{12} = \eta_{21} = 0$. We are in a position to state and prove the following Proposition.

Proposition 1. The following holds in the T-Economy:

(a) *The optimal tariff rate is lower for the importable good that is the closer substitute of the exportable good.*

(b) *The stronger is the cross substitutability between importables, the closer is the optimal tariff to uniformity when all of the cross elasticities involving the expert good are kept constant.*

(c) *The optimal tariff rate is inversely proportional to the own elasticity of excess demand if the importables are independent of each other.*

Proof. From equation (13), we immediately obtain

$$\tau_1 - \tau_2 = (\eta_{20} - \eta_{10})\theta \, , \tag{16}$$

Appendix 2 of the present paper.

$$\frac{\tau_1}{\tau_2} = \frac{\eta_{20} + \eta_{12} + \eta_{21}}{\eta_{10} + \eta_{12} + \eta_{21}}.^{12} \tag{17}$$

Proposition 1-(a) is obvious from equation (16). Proposition 1-(b) is derived from equation (17). The cross elasticity $\eta_{12} + \eta_{21}$ is common in both numerator and dominator of equation (17). When η_{10} and η_{20} are constant, the lager is the cross elasticity, the closer is the value of numerator and dominator, that is, the ratio of tariff to uniformity.[13]

Finally, consider the special case where importable goods are independent of each other. Then, since $\eta_{12} = \eta_{21} = 0$, $\eta_{i0} = -\eta_{ii}$ holds for $i = 1, 2$.[14] Therefore, (17) reduces to

$$\frac{\tau_1}{\tau_2} = \frac{\eta_{22}}{\eta_{11}}. \tag{18}$$

From (18), we have Proposition 1-(c). Q.E.D.

Proposition 1-(a) implies that the optimal tariff rate is higher for the good that is more complementary with the exportable. The exportable is the untaxed

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[12] Harberger (1964) first indicates this formation.

[13] This equation also yields Proposition 1-(a). If $\eta_{10} = \eta_{20}$, $\tau_1/\tau_2 = 1$, that is, the optimal tariff rate is uniformity. This corresponds to Sadka (1977) in the optimal commodity tax problem.

[14] Homogeneity condition yields $\eta_{i0} + \eta_{i1} + \eta_{i2} = 0$. If $\eta_{ij} = 0$ for $i \neq j$ and $i, j \neq 0$, we have $\eta_{i0} = -\eta_{ii}$.

good, and hence it is over-consumed. Taxation on the good that is more complementary with the exportable partially offsets the over-consumption of the exportable. Namely, Proposition 1-(a) shows that the ranking of tariff rates depends upon the relative degree of complementarity between the taxed goods (importable goods) and the untaxed good (exportable good). Since this was first shown by Corllet and Hague (1953) in the context of commodity taxation, we will call this the *Corllet and Hague rule*.

Proposition 1-(b) is called the *Cross Substitutability Rule*. The stronger is cross substitutability between the taxed goods (i.e. the importable goods) creates the stronger is the distortion.

Proposition 1-(c) is called the *Inverse Elasticity Rule*. This rule has been them widely used in empirical estimates in the literature of optimal tariffs under revenue constraints.

3. The N-Economy

3.1. The model

The T-Economy consumes and produces only-tradables. We now incorporate a non-tradable good in the T-Economy. To this end, we substitute Assumptions 2’ and 4’ listed below for Assumptions 2 and 4, respectively. We also add Assumption 10 also listed below.

Assumption 2’. The economy produces four goods: one exportable good, two importable goods and one non-tradable good, which is not traded with foreign country. The only inputs of the economy are endowed factors. We will denote the non-tradable by n, while other goods have same indexes as T-Economy.

Assumption 4’. The consumer consumes all of the four final goods. He has a well-behaved utility function $u(\mathbf{x},\ x_n)$, where x_n is the demand of non-tradable and chooses the commodity bundle that maximizes his utility level under given prices and income.

Assumption 10. The market for the non-tradable good is in equilibrium:

$$x_n = y_n. \tag{19}$$

From Assumptions 3, 4’ and 5, the compensated demand and supply functions for the i-th good are given by[15]:

$$x_i = x_i(\mathbf{q},\ q_n,\ u) \qquad (i = 0, 1, 2, n), \tag{2'}$$

$$y_i = y_i(\mathbf{q},\ q_n) \qquad (i = 0, 1, 2, n), \tag{4'}$$

where q_n is the price of the non-tradable. Since Assumptions 6 and 8 are satisfied,

$$\mathbf{q} = \mathbf{p} + \mathbf{t}\text{, and} \tag{5}$$

$$\mathbf{p}'(\mathbf{x} - \mathbf{y}) + r = 0 \tag{7}$$

continues to hold. Assumptions 2', 3, 4' and 5 yield the budget constraint of the private sector:

$$\mathbf{q}'\mathbf{x} + q_n x_n = \mathbf{q}'\mathbf{y} + q_n y_n\ . \tag{8'}$$

The N-Economy consists of (2'), (4'), (5), (7), (8') and (19).

Equations (8') and (19) yield the equation (8). Equation (8) in this economy implies that the revenue from the sale of the tradable goods equal the consumer's spending on tradable goods. Thus the set of (8') and (19) is equivalent to the set

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[15] Note that the utility function $u(\mathbf{x}, x_n)$ is different from $u(\mathbf{x})$ in T-Economy. However, we will use the same notation to simplify the analysis. This is also applied to the compensated demand, supply and excess demand function defined below. The vector of each variable in this economy expresses the tradable good.

of (8) and (19). We will therefore represent this economy by equations (2'), (4'), (5), (7) (8) and (19).

Definition 3. The economy satisfying Assumptions 1, 2', 3, 4', 5-9 and 10 is called the *N-Economy*. When equation (2'), (4'), (5), (7), (8) and (19) are all satisfied, we say that the *N-Economy is in the full equilibrium*.

The excess demand function in this economy is rewritten as

$$z_i(\mathbf{q},\ q_n,\ u) \equiv x_i(\mathbf{q},\ q_n,\ u) - y_i(\mathbf{q},\ q_n), \qquad (i = 0, 1, 2, n). \tag{9'}$$

By substituting these functions for $x_i - y_i$ and $\mathbf{x} - \mathbf{y}$ in (7), (8) and (19), we have

$$z_n(\mathbf{q},\ q_n,\ u) = 0, \tag{20}$$

$$\mathbf{q}'\mathbf{z}(\mathbf{q},\ q_n,\ u) = 0, \tag{21}$$

$$\mathbf{p}'\mathbf{z}(\mathbf{q},\ q_n,\ u) + r = 0. \tag{22}$$

Note that $\mathbf{z}' = (z_0,\ z_1,\ z_2)$ is a vector of tradables, and do not contain z_n. In term of this notation, the N-Economy is in full equilibrium if and only if it satisfies equations (5), (20), (21) and (22).

Equations (20), (21) and (22) contain four variables q_1, q_2, q_n and u since r is fixed by assumption. When one of the four variables are exogenously given, the three equations determine the remaining variables. For example, if u is given, the model determines the remaining variables $(q_1,\ q_2,\ q_n)$. Therefore, from equation (5) we can find the tariff vector $\mathbf{t}$ that maximizes the utility level u.

3.2. Optimal tariff in the N-Economy(I) : the general formula

We now define the following.

Definition 4. The tariff combination $(t_1,\ t_2)$ that maximizes the utility level u in the model of (5), (20), (21) and (22) for a fixed level of r is called *the optimum tariff* of the N-Economy.

The optimal tariffs in the N-Economy will be different from those in the T-Economy because of the following differences between the models.

First, the N-Economy has two untaxed goods, i.e., the exportable and the non-tradable, while the T-Economy has only one, i.e., the exportable good.

Second, a change in a tariff rate does not affect the price of the untaxed good in the T-Economy, while it affects the price of an untaxed good, i.e., the non-tradable, in tariffs in the N-Economy.

In view of the second difference, the optimal tax formula has to contain the term representing the impact of a tariff change upon the price of the non-tradable. To this end, we have to express q_n as a function of q_1 and q_2. Equation (20) determines the price q_n of the non-tradable good when $\mathbf{q}$ and u are exogenously given. If equation (20) satisfies the conditions of the Implicit Function Theorem, it can be solved for q_n. The resulting function may be written as

$$q_n = q_n(\mathbf{q},\ u). \tag{23}$$

Define the elasticity of the price of non-tradable with respect to the price of i -the importable by

$$\hat{q}_{ni} = \frac{q_i}{q_n}\frac{\partial q_n}{\partial q_i}, \tag{24}$$

and, the elasticity of the importable good i with respect to the price of non-tradable by

$$\eta_{in} = \frac{q_n z_{in}}{z_i}.$$

Then the optimal tariff in the N-Economy can be stated in terms of this notation.

Theorem 2. In the N-Economy, the optimal tariff rate is expressed as

$$\begin{aligned} \tau_1 &= (\eta_{20} + \eta_{21} + \eta_{12} + \eta_{2n} + (\eta_{1n} - \eta_{2n})\hat{q}_{n2})\theta^*, \\ \tau_2 &= (\eta_{10} + \eta_{21} + \eta_{12} + \eta_{1n} + (\eta_{2n} - \eta_{1n})\hat{q}_{n1})\theta^*, \end{aligned} \tag{25}$$

for some scalar $\theta^* > 0$.

Proof. See Appendix 2 *Q.E.D.*

One difference between (13) and (25) is that (25) contains the terms $(\eta_{jn} - \eta_{in})\hat{q}_{ni}$ representing the effect a change in a tariff rate upon the price of the untaxed good through the change in the price of the non-tradable.

Taking the difference between the optimal tariff rates of the two goods in (25), we have

$$\tau_1 - \tau_2 = \left[\{(\eta_{20} + \eta_{2n}) - (\eta_{10} + \eta_{1n})\} + (\eta_{1n} - \eta_{2n})(\hat{q}_{n1} + \hat{q}_{n2})\right]\theta^* . \tag{26}$$

3.3. Optimal tariff in the N-Economy (II) : independence between the importables and the non-tradable

Let us first consider the special case where price changes of the importables do not affect the prices of the non-tradables. Then

$$\hat{q}_{n1} = \hat{q}_{n2} = 0 , \tag{27}$$

and hence

$$\tau_1 - \tau_2 = \left((\eta_{20} + \eta_{2n}) - (\eta_{10} + \eta_{1n})\right)\theta \tag{28}$$

holds.

The expression $\eta_{20} + \eta_{2n}$ in the RHS of (28) represents the percentage increase in the excess demand for the second importable when the prices of both of the untariffed goods (i.e., 0 and n) is simultaneous increased by one percent. Thus it indicates the closeness of substitutability between the second importable and the compound good consisting of the two untariffed goods. We can give a similar interpretation to $\eta_{10} + \eta_{1n}$. Equation (28) implies, therefore, that when (27) holds in the N-Economy, the ranking of tariffs for the importables depends upon their relative closeness of substitutability with the *compound of the two untariffed goods* in this economy.[16] This is a natural extension of Proposition 1-(a), which shows that in the T-Economy, the ranking of tariffs for the importables depends upon their relative closeness of substitutability with the untaxed good.

Equation (28) motivates the following definition.

Definition 5. The first importable good is the *closer direct-substitute of the untaxed goods* than the second importable is, if

$$\eta_{10} + \eta_{1n} > \eta_{20} + \eta_{2n},$$

For equation (28) to hold, however, (27) is unnecessarily strong. It holds if

$$\hat{q}_{n1} + \hat{q}_{n2} = 0 \tag{29}$$

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[16] Consider an economy where an additional exportable good is added to the T-Economy without adding the non-tradable. If we denote the second exportable in this economy by n, then it can be readily shown that the counterpart of the optimal tariff formula of (13) becomes

$$\tau_1 = (\eta_{20} + \eta_{21} + \eta_{12} + \eta_{2n})\theta,$$
$$\tau_2 = (\eta_{10} + \eta_{21} + \eta_{12} + \eta_{1n})\theta,$$

for some scalar $\theta > 0$. The only change from (13) is that the terms η_{1n} and η_{2n} are added.

is satisfied. Totally differentiating (20) in q_n and q_i, while keeping other variables constant, we obtain

$$\frac{\partial q_n}{\partial q_i} = -\frac{z_{ni}}{z_{nn}}. \tag{30}$$

Thus we have

$$q_1 z_{n1} + q_2 z_{n2} = 0 \tag{31}$$

if and only if (29) holds.

Definition 6. It is said that the *composite of the importable goods is independent of the non-tradable good* if (31) holds.

It is needless to say that if (31) holds, a proportional increase in the prices of the importable goods does not affect the net demand for the non-tradable good.

In terms of this definition, the following proposition holds:

Proposition 2. If the composite of the importable goods is independent of the non-tradable, the following holds in the N-Economy:

(a) *The optimal tariff rate is lower for the importable good that is the closer*

From these, (28) holds also in this economy, as the counterpart of (15).

direct-substitute of the untaxed goods.

(b) *The stronger is the cross-substitutability between importables, the closer is the optimal tariff to uniformity when other elasticities are kept constant.*

(c) *The optimal tariff rate is inversely proportional to the own elasticity of excess demand if the importables are independent of each other.*

Proof. Proposition 2-(a) follows directly from (26) and (29). From (25) we have

$$\frac{\tau_1}{\tau_2}=\frac{\eta_{20}+\eta_{2n}+\eta_{12}+\eta_{21}+(\eta_{1n}-\eta_{2n})\hat{q}_{n2}}{\eta_{10}+\eta_{1n}+\eta_{12}+\eta_{21}+(\eta_{2n}-\eta_{1n})\hat{q}_{n1}}, \tag{32}$$

which proves (b). Next, consider the special case where importable goods are independent of each other. Then, since $\eta_{12}=\eta_{21}=0$, $\eta_{i0}+\eta_{in}+\eta_{ii}+0$ holds for $i=1,2$. Substituting this and (29) into (32), we have

$$\frac{\tau_1}{\tau_2}=\frac{\eta_{22}+(\eta_{1n}-\eta_{2n})\hat{q}_{n2}}{\eta_{11}+(\eta_{1n}-\eta_{2n})\hat{q}_{n2}},$$

which proves (c). *Q.E.D.*

It is readily seen that if the independence between the composite of the importable and the non-tradable is not assumed, Proposition 2-(a) and (c) no longer hold.

3.4. Optimal tariff in the N-Economy (III) : the main proposition

Equation (26) may be rewritten as

$$\tau_1 - \tau_2 = [(\eta_{20} - \eta_{10}) + (\eta_{2n} - \eta_{1n}) + (\eta_{1n} - \eta_{2n})(\hat{q}_{n1} + \hat{q}_{n2})]\theta^* \tag{33}$$

〔the original effect〕〔direct effect〕 〔indirect effect〕

Comparing (16) and (33), we find that the introduction of non-tradable brings about two new terms: the *direct effect*, $(\eta_{2n} - \eta_{1n})$ and *indirect effect*, $(\eta_{1n} - \eta_{2n})(\hat{q}_{n1} + \hat{q}_{n2})$. These terms represent the difference in substitutability between importable goods and non-tradable goods. The problem is that the sign of the sum of the two new terms is not apparently clear, since the two terms can have opposite signs. Indeed, we have

the direct effect $0 < (\eta_{2n} - \eta_{1n})$ and

the indirect effect $0 > (\hat{q}_{n1} + \hat{q}_{n2})(\eta_{1n} - \eta_{2n})$,

if $\hat{q}_{ni}$ are positive and $\eta_{2n} > \eta_{1n}$. Does the indirect effect upset the direct effect?

The answer can be derived from the following.

Theorem 3. In the N-Economy, the optimal tariff rate is expressed as

$$\begin{aligned} \tau_1 &= (\eta_{20} + \eta_{12} + \eta_{21} + \eta_{2n}\hat{q}_{n0} + \eta_{1n}\hat{q}_{n2} + \eta_{2n}\hat{q}_{n1})\theta^*, \\ \tau_2 &= (\eta_{10} + \eta_{12} + \eta_{21} + \eta_{1n}\hat{q}_{n0} + \eta_{1n}\hat{q}_{n2} + \eta_{2n}\hat{q}_{n1})\theta^*, \end{aligned} \tag{34}$$

for some scalar $\theta^* > 0$.

Proof. First apply Euler's Theorem to (20) to find

$$1 = \hat{q}_{n0} + \hat{q}_{n1} + \hat{q}_{n2}.$$

From (this) we have

$$\begin{aligned} &\eta_{2n}\hat{q}_{n0} + \eta_{1n}\hat{q}_{n2} + \eta_{2n}\hat{q}_{n1} \\ &= \eta_{1n}\hat{q}_{n2} + \eta_{2n}(1 - \hat{q}_{n2}) \\ &= \eta_{2n} + (\eta_{1n} - \eta_{2n})\hat{q}_{n2}. \end{aligned}$$

Applying this to (25) yields the theorem. *Q.E.D.*

From the proof (34) it is clear that (25) and (34) are equivalent.

By taking the difference between τ_1 and τ_2 in (34), we have

$$\tau_1 - \tau_2 = [(\eta_{20} + \eta_{2n}\hat{q}_{n0}) - (\eta_{10} + \eta_{1n}\hat{q}_{n0})]\theta^* \tag{35}$$

A comparison between (35) and (33) shows that

$$(\eta_{2n} - \eta_{1n}) + (\eta_{1n} - \eta_{2n})(\hat{q}_{n1} + \hat{q}_{n2}) = q_{n0}(\eta_{2n} - \eta_{1n}).$$

From this, it follows that if $\hat{q}_{n0} > 0$, the sum of the direct and indirect effect has the same as the direct effect, and hence the indirect effect does not quite upset the sign of the direct effect.

Note that $\hat{q}_{n0} > 0$ holds if the non-tradable is substitutable for the exportable good, as is clear from (24) and (30).

Equation (35) motivates the following definition

Definition 7. The first importable good is the *closer substitute of the untaxed goods* than the second importable is if

$$\eta_{10} + \eta_{1n}\hat{q}_{n0} > \eta_{20} + \eta_{2n}\hat{q}_{n0}. \tag{36}$$

Hence optimal tariff rates can be ranked in terms of Definition 7.

We are in a position to state and prove the following Proposition.

Proposition 3. The following holds in the N-Economy:

(a) *The optimal tariff rate is lower for the importable that is the closer substitute of the untaxed goods.*

(b) *The stronger is the cross-substitutability between importables, the closer is the optimal tariff to uniformity when other cross elasticities are kept constant.*

(b') *The stronger is the cross-substitutability between the importable and the non-tradable, the closer is the optimal tariff to uniformity, when other cross elasticities are kept constant, provided that untaxed goods are independent of each other*

(c) *The optimal tariff rate is inversely proportional to the own elasticity of excess demand if and only if the importables are independent of each other, i.e.,* $\eta_{12} = \eta_{21} = 0$*, and the importables are equally substitutable for the non-tradable, i.e.,* $\eta_{1n} = \eta_{2n}$.

Proof. Equation (35) and Definition 7 immediately yield (a). From equation (34), we also obtain

$$\frac{\tau_1}{\tau_2} = \frac{\eta_{20} + \eta_{12} + \eta_{21} + \eta_{2n}\hat{q}_{n0} + \eta_{2n}\hat{q}_{n1} + \eta_{n1}q_{n2}}{\eta_{10} + \eta_{12} + \eta_{21} + \eta_{1n}\hat{q}_{n0} + \eta_{2n}\hat{q}_{n1} + \eta_{n1}q_{n2}}.$$

Since the expressions in parentheses are common is both the numerator and the denominator, the larger this term, the closer is the optimal tariff rates to uniformity. This proves (b) and (b').

If we assume that the importables are independent of each other, then $\eta_{12} = \eta_{21} = 0$ hold, and hence we have $\eta_{ii} + \eta_{i0} + \eta_{in} = 0$. By substituting these equations into (25), we have

$$\frac{\tau_1}{\tau_2} = \frac{\eta_{22} + (\eta_{1n} - \eta_{2n})\hat{q}_{n2}}{\eta_{11} + (\eta_{2n} - \eta_{1n})\hat{q}_{n1}}. \tag{37}$$

Equation (37) proves (c). *Q.E.D.*

Proposition 3-(a) is in line with the spirit of the Corlett and Hague Theorem. The ranking of the optimal tariff rates of the two importable is again determined by the ranking of substitutability between the two importables and the untaxed goods(i.e., the exportable and the non-tradable). Proposition 3-(b) and (b') show that strong cross substitutability between the importables and between theuntariffed goods tends to make the import tariffs to become closer to uniformity. Proposition 3-(c) shows that if $\eta_{2n} \neq \eta_{1n}$, the Inverse Elasticity Rule does not hold.[17]

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17 See Hatta (1986) and (1994).

5. Concluding remark

The optimal tariff problem in small open economy has been studied for models with one untaxed good. The present paper introduced the second untaxed good whose domestic price changes endogenously. It was observed that the celebrated Inverse Elasticity Rule is no longer valid in this situation. Often this rule is used as practical guide to obtain a rough estimate of the optimal tariff rates. In the economy with endogeneous price change, however, the rule fails to give such a guide.

On the other hand, it was observed that the Cross Substitutability Rule and the Corlett and Hague Rule are quite robust in the new situation. These rules seem to give qualitative and quantitative insights into the optimal tariff rates in practical situation.

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Appendix 1: uniform and proportional tariffs

In this appendix, we will prove that no tariff structure for a given government revenue can achieve an efficient resource allocation.

A tariff stricture is called *uniform*, if all importable goods share an identical ad valorem tariff rate, i.e., if $t_0 = 0$, and if

$$\frac{t_i}{q_i} = \beta, \qquad (i = 1, 2), \tag{A1-1}$$

holds for some scalar β.

Under uniform tariff structure, domestic prices of the exportable goods are equal to world prices, while those of the importable goods are proportionally higher than the world prices. Thus a uniform tariff structure generates distortions, and the resource allocation is not efficient. However, a uniform tariff can raise a given tariff revenue. Its resource allocation is, however, not efficient, because it is not proportional.

On the other hand, a tariff structure is called *proportional* if all tradable goods i.e., including exportables share an identical ad valorem tariff rate, i.e., if

$$\frac{t_i}{q_i} = \beta, \qquad (i = 0, 1, 2), \tag{A1-2}$$

holds for some scalar β. Under a proportional tariff structure, domestic prices of both exportables and importables are proportional to their world prices.

Subsidies are given to the export of the exportable goods at the same rate as import tariffs. Thus the domestic prices of the exportables are higher than the world prices. The proportional tariff attains an efficient resource allocation.

A proportional tariff structure, however, yields zero revenue. Under the proportional tariff, from (A1-2) and (5), we have

$$\mathbf{q} = \psi \mathbf{p} \qquad \text{(A1-3)}$$

where $\psi = 1/1+\beta$. Substituting (A1-3) into (7) yields

$$\mathbf{p}'\mathbf{z} = 0. \qquad \text{(A1-4)}$$

From (A1-4), we obtain $r = 0$.

This implies that the revenue collected by import tariffs is all spent on export subsidies. Namely, collecting a given tariff revenue necessarily generates a distortion. It is the revenue-constrained optimal tariff structure that minimizes this distortion.

Appendix 2: the proof of Theorem 2

In this Appendix 2, we prove Theorem 2. To this end, we must find the solution of $(\mathbf{q},\ q_n)$ in the following maximization problem:

$$\begin{aligned} \max_{\mathbf{q},\ q_n,\ u} \quad & u \\ \text{s.t} \quad & z_n(\mathbf{q},\ q_n,\ u) = 0, \\ & \mathbf{q}'\mathbf{z}(\mathbf{q},\ q_n,\ u) = 0, \\ & \mathbf{p}'\mathbf{z}(\mathbf{q},\ q_n,\ u) + r = 0. \end{aligned} \tag{A2-1}$$

We can directly obtain Theorem 2 by solving this problem. But we will take a short cut by proving Theorem 3, the equivalent of Theorem 2, after transforming the problem (A2-1) into a manageable form.

Substituting (23) for q_n in the compensated excess demand function yields the excess demand function which does not depend on the price of non-tradable good:

$$\mathbf{z}^*(\mathbf{q},\ u) = \mathbf{z}(\mathbf{q},\ q_n(\mathbf{q},\ u),\ u), \tag{A2-2}$$

where $\mathbf{z}^{*\prime} = (z_0^*(\mathbf{q},\ u),\ z_1^*(\mathbf{q},\ u),\ z_2^*(\mathbf{q},\ u))$. We will call $z_i^*(\mathbf{q},\ u)$ *the reduced-form of excess demand function,* or more simply, *the reduced form.*

In terms of the reduced form, the market equilibrium condition (20)-(22) can be rewritten as

$$\mathbf{q}'\mathbf{z}^*(\mathbf{q},\ u) = 0\,, \tag{A2-3}$$

$$\mathbf{p}'\mathbf{z}^*(\mathbf{q},\ u) + r = 0\,. \tag{A2-4}$$

These two equations contain three variables, q_1, q_2 and u. The market equilibrium conditions (A2-3) and (A2-4) in terms of the reduced form are equivalent to those in terms of (20), (21) and (22).

Thus (A2-1) is transformed into

$$\begin{array}{lll} \max\limits_{\mathbf{q},\,u} & u & \\ \text{s.t} & \mathbf{q}'\mathbf{z}^*(\mathbf{q},\ \mathrm{u}) = 0, & \\ & \mathbf{p}'\mathbf{z}^*(\mathbf{q},\ u) + r = 0. & \end{array} \tag{A2-5}$$

This is formally identical to (14), and hence we immediately obtain the following optimal tariff rates exactly in the same manner as in Theorem 1.

Lemma 1. In the N-Economy, the optimal tariff rate is expressed as

$$\begin{aligned} \tau_1 &= (\eta_{20}^* + \eta_{12}^* + \eta_{21}^*)\theta^*, \\ \tau_2 &= (\eta_{10}^* + \eta_{12}^* + \eta_{21}^*)\theta^*, \end{aligned} \tag{A2-6}$$

where $\theta^* = -\alpha^* z_1^* z_2^* \big/ q_1 q_2 (z_{11}^* z_{22}^* - (z_{12}^*)^2)$ and η_{ij}^* is the elasticity of the i-th reduced with respect to the price q_j.

Let us now decompose η_{ij}^* into terms involving η_{ij} and η_{in}. By partially differentiating (A2-2) with respect to q_j, we have

$$\frac{\partial z_i^*}{\partial q_j} = \frac{\partial z_i}{\partial q_j} + \frac{\partial z_i}{\partial q_n} \cdot \frac{\partial q_n}{\partial q_j}. \tag{A2-7}$$

The first term in the RHS represents the effect of the price change of j-th good upon the excess demand of i-th good. We will call this the *direct effect*. The second term represents the effect of the price change of j-th good upon i-th good through the price change of the non-tradable. We will call this the *indirect effect*. Thus, the LHS of (A2-7) represents total effect including direct and indirect effect.

By rewriting (A2-7), we obtain

$$\eta_{ij}^* = \eta_{ij} + \eta_{in}\hat{q}_{nj}. \tag{A2-8}$$

The term η_{ij} and $\eta_{in}\hat{q}_{nj}$ correspond to the direct and indirect effect, respectively.

Substituting equation (A2-8) into (A2-6), we have equation (34), and hence (25). This result and the following Lemma prove Theorem 2.

Lemma 2. $\theta^* > 0$. [18]

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[18] We are grateful to Professor Suzuki of Kwansei Gakuin University for suggesting this proof.

Proof. The term θ^* has the same sign as government revenue r, if $Z^* = \begin{bmatrix} z_{11}^* & z_{12}^* \\ z_{21}^* & z_{22}^* \end{bmatrix}$ is negative semi-definite. See Daimond and Mirrlees (1971, p.262). From equation (A2-7),

$$Z^* = \begin{bmatrix} z_{11} - \dfrac{z_{1n}z_{n1}}{z_{nn}} & z_{12} - \dfrac{z_{1n}z_{n2}}{z_{nn}} \\ z_{21} - \dfrac{z_{2n}z_{n1}}{z_{nn}} & z_{22} - \dfrac{z_{2n}z_{n2}}{z_{nn}} \end{bmatrix}.$$

From the property of matrix, we find

$$\left|Z_1^*\right| = z_{11} - \frac{z_{1n}z_{n1}}{z_{nn}} = \frac{1}{z_{nn}}\begin{vmatrix} z_{11} & z_{1n} \\ z_{n1} & z_{nn} \end{vmatrix}, \text{ and}$$

$$\left|Z_2^*\right| = \begin{vmatrix} z_{11} - \dfrac{z_{1n}z_{n1}}{z_{nn}} & z_{12} - \dfrac{z_{1n}z_{n2}}{z_{nn}} \\ z_{21} - \dfrac{z_{2n}z_{n1}}{z_{nn}} & z_{22} - \dfrac{z_{2n}z_{n2}}{z_{nn}} \end{vmatrix} = \frac{1}{z_{nn}}\begin{vmatrix} z_{11} & z_{12} & z_{1n} \\ z_{21} & z_{22} & z_{2n} \\ z_{n1} & z_{n2} & z_{nn} \end{vmatrix}.$$

If $z_i(\mathbf{q},\ q_n,\ u)$ is concave with respect to $(\mathbf{q},\ q_n)$, $\left|Z_1^*\right| < 0$ and $\left|Z_2^*\right| > 0$. Thus, since Z^* is the negative semi-definite, θ^* is positive. *Q.E.D.*

Incidentally, inequality (36) can be equivalently written as

$$\eta_{10}^* > \eta_{20}^*.$$

From (A2-8), In Definition 7, therefore, (36) can be replaced by the above inequality.

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