Discussion Paper No. 489

TECHNOLOGICAL COMPARATIVE ADVANTAGE AND BEHAVIOR OF FACTOR PRICES WITH TRADE

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December 1999

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Technological Comparative Advantage and Behavior of Factor Prices With Trade

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JEL classification: F11

keywords: factor-price convergence, comparative advantage, technological comparative advantage, skill premium

We introduce technological differences in a Heckscher-Ohlin model and study how the technology and endowment differences interact to determine the effects of trade on factor prices. When the endowment effect is dominant in determining the autarky relative factor prices, the relative factor prices of trading countries adjust in converging directions with trade if and only if the capital-rich country has a comparative advantage in the capital-intensive sector. Adjustments in converging directions could be excessive. Relative factor prices tend to converge if the technological comparative advantage is small for given relative endowments or if the relative endowment difference is large for a given technological comparative advantage.

*This paper was mostly written while the first author was at the University of Tokyo and ISER, Osaka University, 1998-1999. He thanks the institutions for warm hospitality. We would like to thank the participants of the Far Eastern Meeting of the Econometric Society, Japanese Economic Association Meetings, the Midwest International Economics Meetings and workshops at Kobe University, the University of Tokyo and Osaka University for many valuable comments. Special thanks are due to Tatsuo Hatta, Koji Shimomura and particularly to Masahiro Okuno-Fujiwara whose suggestions greatly benefited the paper.

1 Introduction

Do factor prices of trading countries get closer as a result of international trade? This question has taken on new policy significance in recent years as a result of the simultaneous increase of the skill premium in the United States and its trade volume with NIEs.[1] If the ratios of rent (including the skill premium) to wage of the trading countries converge, the lower rent to wage ratio – typically in the capital-rich country – will increase after trade. Motivated by the increasing USA–NIE trade, we study the behavior of factor prices of trading countries in a simple framework that nevertheless explicitly features technological differences among countries. We introduce Hicks neutral technological differences in a 2–country, 2–good, 2–factor Heckscher-Ohlin trade model and study how the technology and endowment differences interact to determine the circumstances in which the relative factor prices converge or diverge.[2]

[1] There is a closely related but distinct question of the *extent* to which trade with NIEs is increasing the skill premium in the United States. See Burtless[1995], S.J. Davis[1992], Freeman[1995], Krugman[1995][1997]. In Korea, trade volume grew from 21% of GDP in 1963 to 61% of GDP in 1994 while the skill premium increased briefly in the first half of the 1970s and then gradually decreased. The downward trend seems to have bottomed out.

[2] Previous papers that combine endowment and technological differences include Davis[1995] and Xu[1993]. Davis shows how technological differences in intra-industry goods among countries can generate intra-industry trade even under constant returns to scale. Xu, combining two papers by Dornbusch-Fischer-Samuelson[1977][1980], studies complete specialization equilibria in Cobb-Douglas economies with 2 countries and many goods. Neither of these studies focuses on the effect of trade on relative factor prices, however.

Ohlin[1933] proposed a thesis that international trade in goods tends to move factor prices toward equalization.[3] Samuelson[1948], in trying to prove Ohlin's thesis, found that under his conditions, the movement was complete rather than partial. Later, Samuelson[1971] claimed that Ohlin's thesis could be vindicated in a specific factors model. Uzawa[1959] attempted to prove Ohlin's thesis in a model with many goods and factors but he needed a very specialized model to do that. Deardorff[1986] showed that relative factor prices always converged with trade in goods in a Cobb-Douglas international economy when there were no technological differences among countries. Dixit and Norman[1980] suggested a convergence criterion in terms of correlation but then showed that such a result could be hoped for only under very special conditions. Land[1959] and Stewart[1976], on the other hand, gave a graphical example where the relative factor prices of countries diverged after the international trade in goods opened. Their 'counter-example' came in a two-country model with three goods and two factors and no technological differences between the countries. However, perhaps the most natural instances of such examples come from a model that allows technological differences among nations as we show in this paper.

The trade equilibrium relative output price forms between autarky relative output prices. With non-reversal of factor intensities, the rent/wage ratio

[3] Ohlin writes(page 66, [1933]), "The tendency toward equalization of factor prices is explained as follows: goods containing a large proportion of relatively abundant and cheap factors are exported and these factors become more scarce, whereas goods containing a large proportion of scantily supplied and expensive factors are imported and the latter becomes less scarce. Trade consequently acts as a substitute for the movement of productive factors and reduces the disadvantages arising from their immobility."

in a country is an increasing function of the relative price of the capital-intensive good. Thus, rent/wage of a country increases (resp. falls) after trade opens if the country has a comparative advantage in the capital-intensive good (resp. the labor-intensive good). A capital-rich country has a comparative advantage in the capital-intensive good unless it has a strong technological comparative advantage in producing the labor-intensive good.

The relative factor prices of trading countries adjust in converging directions with trade if the country with a lower autarky rent/wage has a comparative advantage in the capital-intensive good and in diverging directions if the comparative advantage is in the labor-intensive sector. When the converging movements of the relative factor prices are small, the relative factor prices converge in a clear-cut way. When the movements are larger but not excessive, the relative factor prices converge as a result of trade but the rank order of relative factor prices among countries may reverse. When the adjustments in converging directions are large, the relative factor price can diverge with trade. If the country with lower autarky rent/wage has a comparative advantage in the labor-intensive sector, the relative factor prices move away from each other, thus diverging in a clear-cut way.

Relative factor prices tend to converge if the technological comparative advantage is not strong for given relative endowments or if the relative endowment difference is large for a given technological comparative advantage. Excessive differences in relative technology or endowments cause one or both countries to specialize in one good. With specialization, however, further expansion of technological differences may not worsen relative factor price differentials.

Section 1.1 describes the concept of relative factor price convergence. Section 2 describes the model and assumptions. Section 3 studies the determination of autarky relative factor prices and the relative factor prices at a diversification trade equilibrium. Section 4 studies a trade equilibrium where a country's production is specialized in one good. Section 5 introduces the relative factor price function and describes its properties. Section 6 gives the main results on the convergence and divergence of relative factor prices with trade. Section 7 studies the welfare effects of trade on factors. The Stolper-Samuelson theorem covers such effects in the diversification range. When we expand the discussion to include specialization, we get a variety of possible trade effects on wages.

1.1 Relative Factor Price Convergence

Let (q_i^a, s_i^a) and (q_i, s_i) be the positive autarky and after-trade prices of goods and factors in country $i = A, B$, in terms of a numeraire good. After trade, q_A is equal to q_B. If factor prices equalize after trade ($s_A = s_B$), they equalize independent of the choice of the numeraire good; the factor prices have converged without any ambiguity.

If they do not, however, we need to compare the difference of s_A^a and s_B^a versus that of s_A and s_B. But such comparisons are sensitive to the choice of the numeraire good. Suppose that, in Japan (country A), the autarky relative price of oranges (good 2) in terms of apples (good 1) is higher than in Taiwan (country B). Using oranges as numeraire, suppose $(q_A^a, s_A^a) = (\frac{1}{2}, 1, 1, 1)$, $(q_B^a, s_B^a) = (2, 1, 1, 1)$. Expressed in orange units, there is no room for further factor price convergence since $s_A^a = s_B^a$. However, in apple units, the prices become $(1, 2, 2, 2)$ and $(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ respectively. In contrast, if we isolate the factor prices, we can compare normalized versions of these and say that the relative autarky factor prices are the same.

For this reason, we use *relative factor prices,* factor prices expressed in terms of a factor and consider (s_A^a, s_B^a) versus (s_A, s_B) independent of the goods' prices.[4] Expressed in the units of the second factor, s_i becomes $(\rho_i, 1)$. When $\rho_i^a < \rho_{i'}^a$, the relative factor prices move *in converging directions* with trade if $\rho_i^a < \rho_i$ and $\rho_{i'} < \rho_{i'}^a$; *in diverging directions* with trade if $\rho_i < \rho_i^a < \rho_{i'}^a < \rho_{i'}$. Relative factor prices *converge* with trade if $\frac{\rho_i^a}{\rho_{i'}^a} < \frac{\rho_{i'}}{\rho_i} < \frac{\rho_{i'}^a}{\rho_i^a}$ and *diverge* if $\frac{\rho_{i'}}{\rho_i} < \frac{\rho_i^a}{\rho_{i'}^a}$ or $\frac{\rho_{i'}^a}{\rho_i^a} < \frac{\rho_{i'}}{\rho_i}$.[5] It is clear that relative factor prices diverge with trade if they move in diverging directions with trade.

2 Economy

There are two countries A, B indexed by i and two goods $1, 2$ indexed by j and two factors, labor L and capital K, whose rental rates are w and r respectively. We may interpret L as unskilled labor and K as a composite representing human and physical capital. Country i has the production function $y_{ij} = t_{ij} f_j(K_{ij}, L_{ij})$ of good j, where $t_{ij} > 0$ and f_j satisfies constant returns to scale and the marginal products are positive. Production functions are different across countries by efficiency factors t_{ij} only. Unit

[4] Uzawa[1959] uses the Euclidean distance of the factor prices of the two countries expressed in terms of a good. This may explain why he needs such stringent assumptions as the linearity of excess demand functions.

[5] In particular, if both ρ_A and ρ_B are between ρ_A^a and ρ_B^a, relative factor prices converge with trade according to the definition. For example, $\rho_A^a < \rho_A \leq \rho_B < \rho_B^a$ implies $\frac{\rho_A^a}{\rho_B^a} < 1 < \frac{\rho_B}{\rho_A} < \frac{\rho_B^a}{\rho_A^a}$.

cost function of y_{ij} is denoted by $c_{ij}(r,w)$. It will be convenient to work with a unit cost function $\phi_j(r,w)$ corresponding to f_j since $\phi_j(r,w)$ is common across countries. Since production of $\frac{1}{t_{ij}}$ units of f_j produces one unit of y_{ij}, $c_{ij}(r,w) = \frac{1}{t_{ij}}\phi_j(r,w)$. Let $\left(\widetilde{K}_j, \widetilde{L}_j\right)$ be the factor requirements of producing one unit of f_j, $j = 1,2$. Then, $\phi_j(r,w) = r\widetilde{K}_j + w\widetilde{L}_j$, $j = 1,2$. For good j, $\widetilde{k}_j \equiv \frac{\widetilde{K}_j}{\widetilde{L}_j}$; for country i, K_i, L_i are endowments and $k_i \equiv \frac{K_i}{L_i}$. Sector 1 employs more capital-intensive method of production than sector 2 for each set of factor prices (no factor intensity reversals). We denote by p_i the price of good 1 in country i in the units of good 2. Country i has a *comparative advantage* in good 1 (resp. in good 2) if her autarky relative price p_i^a is less than (resp. greater than) the autarky relative price $p_{i'}^a$ of country i'.

Let $t_A \equiv \frac{t_{A2}}{t_{A1}}$, $t_B \equiv \frac{t_{B2}}{t_{B1}}$ and $t_1 \equiv \frac{t_{A1}}{t_{B1}}$, $t_2 \equiv \frac{t_{A2}}{t_{B2}}$. It follows that $\frac{t_A}{t_B} = \frac{t_2}{t_1}$. Country i has a *technological comparative advantage*(TCA) in good 1 (resp. in good 2) if her relative technological efficiency t_i is less than (resp. greater than) the relative technological efficiency $t_{i'}$ of country i'. Both countries share a homothetic welfare function that is strictly quasi-concave and has positive partial derivatives. The homotheticity assumption implies that the ratio of the demands for goods at country i is independent of her income level and thus is a function of relative output price p_i. When international trade opens, only goods may be traded. Transportation cost is zero. The markets are competitive. We shall develop a method that allows us to analyze the diversification equilibrium (where both countries produce both goods) and the specialization equilibrium (where some country produces only one good) in a single framework. We assume that countries diversify at autarky.

3 Diversification

When country i $(= A, B)$ produces both goods, her factor prices satisfy: $p_i t_{i1} = \phi_1(w_i, r_i)$, $t_{i2} = \phi_2(w_i, r_i)$. We denote the relative factor price of country i by $\rho_i \equiv \frac{r_i}{w_i}$ and the autarky relative output and factor prices of country i by p_i^a and ρ_i^a respectively. When country i diversifies, $p_i \frac{t_{i1}}{t_{i2}} = \frac{\phi_1(w_i,r_i)}{\phi_2(w_i,r_i)} = \frac{\phi_1(1,\rho)}{\phi_2(1,\rho)}$ since $\left\{\phi_j\right\}$ are homogeneous of degree 1. It will be convenient to work with the *relative cost function* $\varphi(\rho) \equiv \frac{\phi_1(1,\rho)}{\phi_2(1,\rho)}$. When country i diversifies,

$$p_i = t_i \varphi(\rho_i)$$

We use the following properties of φ. The proof is given in the appendix.

Lemma 1 *(1)* $\varphi'(\rho) > 0$, $\varphi''(\rho) < 0$ *and* $\frac{d}{d\rho}\frac{\varphi(\rho)}{\rho} < 0$, *(2)* $\varphi(\lambda\rho) < \lambda\varphi(\rho)$ *and* $\lambda\varphi^{-1}(p) < \varphi^{-1}(\lambda p)$ *for* $\lambda > 1$.

3.1 Autarky relative prices

We simplify autarky equilibrium conditions by stating them in terms of the relative factor prices. In the following, homotheticity of the welfare function gives δ, an increasing function of p_i^a. We derive the conditions in Autarky Equilibrium Conditions in Appendix. For $i = A, B$ separately,

$$\begin{aligned}
\delta(p_i^a) &= \frac{y_{i2}}{y_{i1}} \\
p_i^a &= t_i \varphi(\rho_i^a) \\
K_i &= y_{i1}\frac{1}{t_{i1}}\widetilde{K}_1(\rho_i^a) + y_{i2}\frac{1}{t_{i2}}\widetilde{K}_2(\rho_i^a) \\
L_i &= y_{i1}\frac{1}{t_{i1}}\widetilde{L}_1(\rho_i^a) + y_{i2}\frac{1}{t_{i2}}\widetilde{L}_2(\rho_i^a)
\end{aligned} \tag{1}$$

Solving the last two equations and substituting, $\frac{y_{i2}}{y_{i1}} = -t_i \frac{\widetilde{K}_1 - k_i \widetilde{L}_1}{\widetilde{K}_2 - k_i \widetilde{L}_2}$, while the first two equations yield $\frac{y_{i2}}{y_{i1}} = \delta(t_i \varphi(\rho_i^a))$. Thus, autarky relative factor price ρ_i^a is determined by:

$$-t_i \frac{\widetilde{K}_1 - k_i \widetilde{L}_1}{\widetilde{K}_2 - k_i \widetilde{L}_2} = \delta(t_i \varphi(\rho_i^a)) \tag{2}$$

When the countries share a Cobb-Douglas welfare function, $w_i = x_i^b y_i^{1-b}$ where x_i and y_i are consumptions of good 1 and 2, respectively, for country i, the first equation of (1) becomes $p_i^a \frac{y_{i1}}{y_{i2}} = \frac{b}{1-b}$. In this case, (2) becomes $-\varphi(\rho_i^a) \frac{\widetilde{K}_2 - k_i \widetilde{L}_2}{\widetilde{K}_1 - k_i \widetilde{L}_1} = \frac{b}{1-b}$. Since the latter equation does not involve technology efficiency factors, ρ_i^a is independent of them. Lemma 2 shows that when welfare function is Cobb-Douglas, the capital-rich country has a lower autarky rent/wage ratio. This fact makes the relationship between technological comparative advantage and the factor price convergence particularly simple to describe in the Cobb-Douglas welfare function case.

Lemma 2 *The autarky equilibrium relative factor price ρ_i^a (as well as p_i^a) for a given* (t$_i$,k$_i$) *is unique. If the shared welfare function is Cobb-Douglas, the autarky relative factor prices are independent of relative efficiency factors t_A, t_B and the capital-rich country has a lower autarky rent/wage ratio ($k_A > k_B$ implies $\rho_A^a < \rho_B^a$).*

Proof. See Appendix. ■

3.2 Trade equilibrium with diversification

We first study the case where both countries produce both goods. When the countries share a homothetic welfare function, the equilibrium conditions at a diversification equilibrium are:[6]

$$\begin{aligned}
\delta(p) &= \frac{y_{A2}+y_{B2}}{y_{A1}+y_{B1}} \\
p &= t_i\varphi(\rho_i),\ i=A,B. \\
K_i &= y_{i1}\frac{1}{t_{i1}}\widetilde{K}_1(\rho_i)+y_{i2}\frac{1}{t_{i2}}\widetilde{K}_2(\rho_i),\ i=A,B. \\
L_i &= y_{i1}\frac{1}{t_{i1}}\widetilde{L}_1(\rho_i)+y_{i2}\frac{1}{t_{i2}}\widetilde{L}_2(\rho_i),\ i=A,B.
\end{aligned} \tag{3}$$

Homotheticity of the welfare function gives the first equation where δ is an increasing function. From the last two equations for $i = A, B$, output levels $\{y_{ij}\}$ are determined as functions of endowments and relative factor prices. Then, from the first and second equations, $i = A, B$, the values of ρ_A, ρ_B, p are determined. At a diversification equilibrium, the second equations, $i = A, B$, give a direct relationship of $\frac{t_B}{t_A} = \frac{\varphi(\rho_A)}{\varphi(\rho_B)}$. If production functions are Cobb-Douglas: $y_{ij} = t_{ij}K_{ij}^{\alpha_j}L_{ij}^{1-\alpha_j}$, $i = A, B$ and $j = 1, 2$ and $\alpha_1 > \alpha_2$, it can be shown that $\frac{\varphi(\rho_A)}{\varphi(\rho_B)} = \left(\frac{\rho_A}{\rho_B}\right)^{\alpha_1-\alpha_2}$. Thus, in this case, $\frac{\rho_A}{\rho_B} = \left(\frac{t_B}{t_A}\right)^{\frac{1}{\alpha_1-\alpha_2}}$.

Lemma 3 *Suppose $p_i^a < p_{i'}^a$ and let p be the trade equilibrium price (of good 1). Then, $p_i^a < p < p_{i'}^a$.*

Proof. It is enough to consider the case of $p_A^a < p_B^a$. The other cases are handled in the same way. Suppose that $p_A^a < p_B^a \leq p$. If the countries diversify at the trade equilibrium, then, from the equations on the second line in 3, $\rho_A^a < \rho_A$ and $\rho_B^a \leq \rho_B$. Country A employs less capital-intensive techniques in the production of both goods and, to satisfy resource requirements, must

[6] These conditions may be derived from the equilibrium conditions involving individual factor prices along the same line as in the Appendix A.2.

produce more of good 1, whose sector is capital-intensive, and less of good 2. Similarly, country B produces more of good 1 and less of good 2. Thus, the supply of good 1 increases and the supply of good 2 decreases. However, since the price of good 1 increases, this contradicts the first equilibrium equation in 3. If a country specializes, she specializes in good 1 (see Lemma 4 below). So, the same argument applies. ■

4 Specialization

An equilibrium factor price (w_i, r_i) corresponding to output price p satisfies $p \leq \frac{1}{t_{i1}}\phi_1(w_i, r_i)$, $1 \leq \frac{1}{t_{i2}}\phi_2(w_i, r_i)$, where a strict ineqality for good $j = 1, 2$ means that good j is not produced by country i. If country i specializes in the production of good j, the endowment vector of country i is orthogonal to the unit cost curve of j at the equilibrium factor prices by the Shepard lemma. If country i diversifies, the endowment vector lies in the diversification cone generated by the gradients of ϕ_1 and ϕ_2 at the equilibrium factor prices.

Figure 1 describes how equilibrium factor prices and production pattern change as the (relative) price of good 1 increases from p to p' in country i. The two radially parallel curves are the level curves of $p = \frac{1}{t_{i1}}\phi_1(w_i, r_i)$ and $p' = \frac{1}{t_{i1}}\phi_1(w_i, r_i)$ while the steeper curve is that of $1 = \frac{1}{t_{i2}}\phi_2(w_i, r_i)$. Initially, country i diversifies at C and the slope of OC is the equilibrium relative factor price. The endowment vector of country i is in the diversification cone at C. At p_i', the new factor prices must lie somewhere on ABD'. Since the endowment vector was in the diversification cone at C initially, it cannot be orthogonal to a unit cost curve at any point on the curve segment AB (not including B) nor at any point on the curve segment $C'D'$ (not including C').

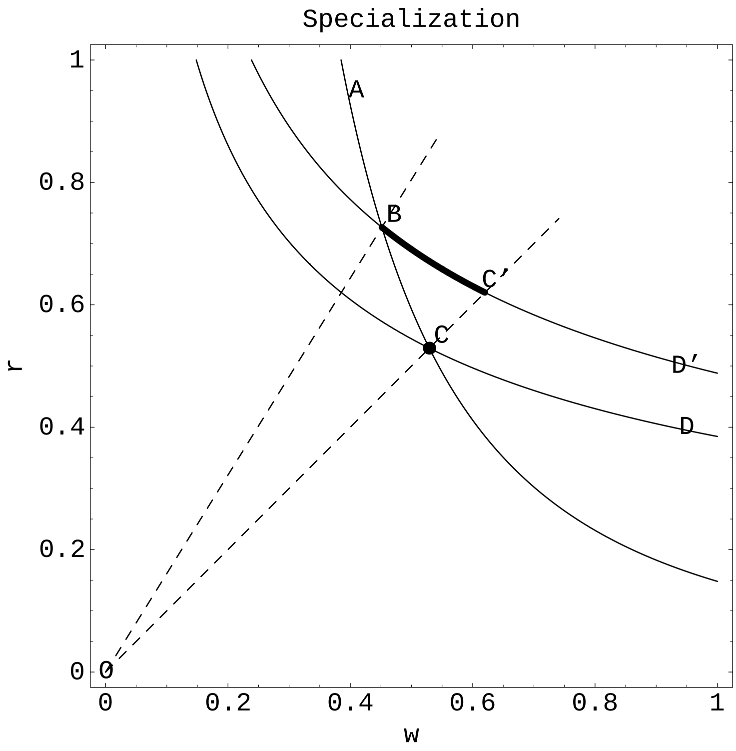

Figure 1:

Thus, we can narrow down the possible factor prices to the arc BC. If country i's endowment vector is in the diversification cone at B, she diversifies her production at p_i'. Otherwise, country i specializes in good 1 at p_i'. The new relative factor prices is give by the slope of a ray between OB and OC.

If country i specializes in good 1 at price p, then her endowment vector is orthogonal to the unit cost curve of good 1 at a point on CD. When the price increases to p', the same condition will be met at a radial extension of the original point on $C'D'$. Thus, the relative factor price does not change. Symmetric statements hold when the price of good 1 goes down.

Lemma 4 *Suppose that the price of good 1 in country i increases from p to p'. If country i diversified at p, the relative factor price increases with the new equilibrium. If country i specializes in production at p', she specializes in good* 1. *If country i specialized in good* 1 *at p, she continues to specialize in good* 1 *at p' and the relative factor price does not change.*

When country i specializes in say, good 1, the factor intensity in sector 1 equals the relative factor endowment of country i ($\widetilde{k}_1(\rho_i) = k_i$). Since the equation does not involve technology, the relative factor price of a country specializing in a good does not change as technologies change so long as the country continues to specializes in the good. In particular, when both countries specialize in production, a change in technology does not affect ρ_A, ρ_B so long as they remain in the same specialization pattern. This is a nice contrast to the factor price equalization theorem where factor prices stay constant under certain changes in endowments that maintain a diversification equilibrium. For a given endowment, those technologies that generate a type of complete specialization generate the same relative factor prices.

5 Relative Factor Price Function

The relative factor price function, $\rho(p;t_i,k_i)$, gives the equilibrium relative factor prices corresponding to relative output prices. From Lemma 4, there are prices $\underline{p}_i$, $\overline{p}_i$, $\underline{p}_i < \overline{p}_i$ for country $i(=A,B)$ such that she specializes in the production of good 1 if $p \geq \overline{p}_i$, in good 2 if $p \leq \underline{p}_i$ and diversifies if $\underline{p}_i < p < \overline{p}_i$. The relative factor price of country i is determined once p, t_i, k_i are given and is constant over a specialization zone.[7] For country $i(=A,B)$, let $\underline{\rho}_i$ and $\overline{\rho}_i$ be defined by $\widetilde{k}_1(\overline{\rho}_i) = k_i$, $\widetilde{k}_2(\underline{\rho}_i) = k_i$. Then,

$$\rho(p;t_i,k_i) \equiv \begin{cases} \overline{\rho}_i & \text{if } p \geq \overline{p}_i \\ \varphi^{-1}(\frac{p}{t_i}) & \text{if } \underline{p}_i < p < \overline{p}_i \\ \underline{\rho}_i & \text{if } p \leq \underline{p}_i \end{cases} \tag{4}$$

When the values of t_i, k_i are fixed, we write $\rho_i(p) \equiv \rho(p;t_i,k_i)$, $i = A, B$. Since φ is strictly increasing due to the factor intensity condition, ρ_i is an increasing function of p and is strictly increasing in the diversification range.

We illustrate by an example how the function $\rho_i(p)$ may be used to determine the behavior of relative factor prices with trade. Consider the case of no TCA ($t_A = t_B$). If country A is capital-rich ($k_A > k_B$), then, $\rho_A(\cdot) \leq \rho_B(\cdot)$ (see Fig. 2 below). Since we show in Lemma 6 that $\frac{d\rho^a}{dk} < 0$, $p^a_A = t_A\varphi(\rho^a_A) < t_B\varphi(\rho^a_B) = p^a_B$. Since the trade equilibrium price p is between p^a_A and p^a_B (Lemma 3), $\rho_A(p^a_A) < \rho_A(p) \leq \rho_B(p) < \rho_B(p^a_B)$. The strict inequalities hold since the countries A, B diversify at autarky where $\{\rho_i\}$ are

[7] Written explicitly,

$$\rho(p;t_i,k_i) \equiv \begin{cases} \widetilde{k}_1^{-1}(k_i) & \text{if } p \geq t_i\varphi(\widetilde{k}_1^{-1}(k_i)) \\ \varphi^{-1}(\frac{p}{t_i}) & \text{if } t_i\varphi(\widetilde{k}_2^{-1}(k_i)) < p < t_i\varphi(\widetilde{k}_1^{-1}(k_i)) \\ \widetilde{k}_2^{-1}(k_i) & \text{if } p \leq t_i\varphi(\widetilde{k}_2^{-1}(k_i)) \end{cases}$$

strictly increasing. Thus, relative factor prices always converge after trade when the countries have the same relative technological efficiencies regardless of whether one or both countries specialize at the trade equilibrium. Deardorff[1986] shows that the relative factor prices converge when there are no welfare and technology differences among countries and all welfare and production functions are Cobb-Douglas. In the $2 \times 2 \times 2$ model, our result generalizes that of Deardorff.[8]

We show how the graph of $\rho(p; t_i, k_i)$ shifts as (t_i, k_i) changes. As k_i increases, the graph of $\rho(p; t_i, k_i)$ 'slides down' that of $\varphi^{-1}\left(\frac{\cdot}{t_i}\right)$. As t_i decreases, the graph of $\rho(p; t_i, k_i)$ 'shifts' to the left. Proposition 5 makes this precise. The situation is illustrated in Figures 2 and 3. In the figures, CC is a benchmark. In Figure 2, CC shifts to BB as the capital/labor endowment ratio decreases. In Figure 3, C shifts to A as the capital-intensive sector 1 becomes more efficient relatively to sector 2 (a decrease in t).

Lemma 5 *As country i $(= A, B)$ becomes more capital-rich, $\overline{\rho}_i$, $\underline{\rho}_i$ and $\overline{p}_i$, $\underline{p}_i$ decrease and $\rho(\cdot; t_i, k_i)$ decreases. As country i becomes more efficient in producing good 1 relative to good 2 (a decrease in t_i), $\overline{\rho}_i$, $\underline{\rho}_i$ do not change, $\overline{p}_i$, $\underline{p}_i$ decrease and $\rho(\cdot; t_i, k_i)$ increases. If $k_A > k_B$ and $t_A > t_B$, $\rho(\cdot; t_A, k_A) < \rho(\cdot; t_B, k_B)$.*

Proof. Since φ is strictly increasing and $\widetilde{k}_1$ is strictly decreasing, $\widetilde{k}_1^{-1}$, $\varphi \circ \widetilde{k}_1^{-1}$ are strictly decreasing functions. Then, the first two statements are immediate from definition. Consequently, when $k_A > k_B$ and $t_A > t_B$,

[8] Deardorff allows any finite number of goods. However, the result does not generalize to this case as the Land[1959] example shows.

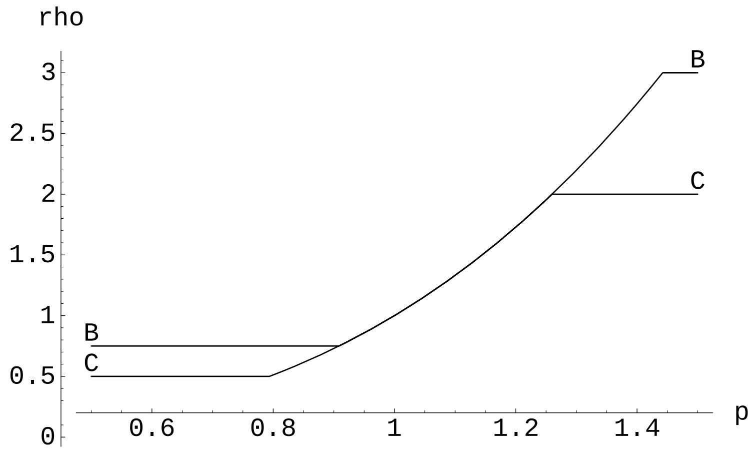

Figure 2:

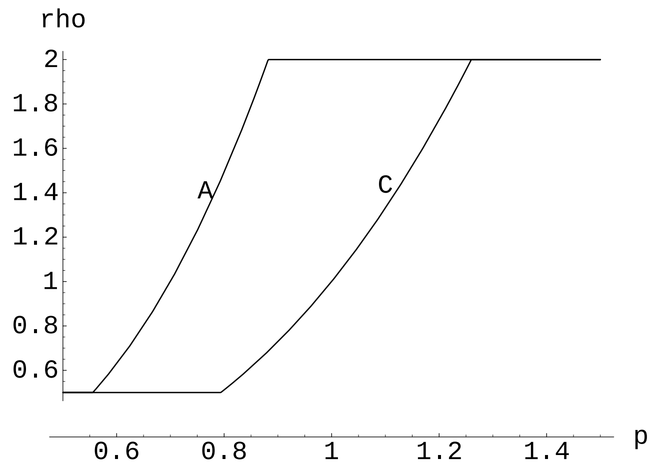

Figure 3:

$\rho(\cdot; t_A, k_A) \leq \rho(\cdot; t_B, k_A) \leq \rho(\cdot; t_B, k_B)$. The second inequality is strict except for those p at which the economies represented by (t_B, k_A) and (t_B, k_B) diversify in production. However, for these values of p, the first inequality is strict. Thus, $\rho(\cdot; t_A, k_A) < \rho(\cdot; t_B, k_B)$. Figure 7 illustrates the situation. ■

6 Main Results

We first determine how autarky relative factor and output prices react as relative factor endowment or relative technological efficiency changes. It turns out that we can sign $\frac{\partial \rho^a}{\partial k}$, $\frac{\partial p^a}{\partial t}$, $\frac{\partial p^a}{\partial k}$, but not $\frac{\partial \rho^a}{\partial t}$ without further assumptions. Lemma 2 shows that $\frac{\partial \rho^a}{\partial t} = 0$ when the welfare function is Cobb-Douglas.

Lemma 6 *Under the factor intensity assumption* $\widetilde{k}_1 > \widetilde{k}_2$, *we have* $\frac{\partial \rho^a}{\partial k} < 0$, $\frac{\partial p^a}{\partial t} > 0$, $\frac{\partial p^a}{\partial k} < 0$.

Proof. See Appendix. ■

Lemma 7 *At autarky,* $\rho_i^a < \rho_{i'}^a$ *if and only if* $\frac{t_i}{t_{i'}} > \frac{p_i^a}{p_{i'}^a}$.

Proof. The result follows from $\varphi(\rho_i^a) = \frac{p_i^a}{t_i}$ and $\varphi(\rho_{i'}^a) = \frac{p_{i'}^a}{t_{i'}}$ since φ is strictly increasing. ■

When a capital-rich country A has just enough TCA in the labor-intensive sector ($t_A = \bar{t}_A > t_B$), the countries end up with no comparative advantage ($p_A^a = p_B^a$). The following proposition compares, wherever possible, (a) autarky relative output prices, (b) autarky relative factor prices and (c) after-trade relative factor prices of countries depending on the location of t_A relative to t_B and $\bar{t}_A$.

Proposition 8 *Assume that country A is capital-rich* ($k_A > k_B$)*, and fix the value of* t_B. (a) *There is* $\bar{t}_A > t_B$ *such that* $p_A^a = p_B^a$ *at* $t_A = \bar{t}_A$, $p_A^a < p_B^a$ *for* $t_A < \bar{t}_A$ *and* $p_A^a > p_B^a$ *for* $t_A > \bar{t}_A$. (b) *There is an open interval containing* $[t_B, \bar{t}_A]$ *on which* $\rho_A^a < \rho_B^a$. (c) *If* $t_A = t_B$*, then* $\rho_A = \rho_B$ *at a diversification trade equilibrium and* $\rho_A < \rho_B$ *when a country specializes. If* $t_B < t_A$*,* $\rho_A < \rho_B$ *at any trade equilibrium. If* $t_A < t_B$*,* $\rho_A > \rho_B$ *at a diversification trade equilibrium.*[9]

Proof. (a) Starting from (k_B, t_B), increase k_B to k_A. Since $\frac{\partial p^a}{\partial k} < 0$, $p_A^a < p_B^a$ when $k_A > k_B$ and $t_A = t_B$. On the other hand, for t_A large enough, $p_B^a < t_B\varphi(\widetilde{k}_1^{-1}(k_B)) < t_A\varphi(\widetilde{k}_2^{-1}(k_A)) < p_A^a$. Thus, there is a level $\bar{t}_A$ of t_A at which $p_A^a = p_B^a$. Since p_A^a strictly increases as t_A increases, $\bar{t}_A > t_B$ and $p_A^a < p_B^a$ for $t_A < \bar{t}_A$ and $p_A^a > p_B^a$ for $t_A > \bar{t}_A$. (b) From Lemma 7, $\rho_A^a < \rho_B^a$ if and only if $\frac{t_A}{t_B} > \frac{p_A^a}{p_B^a}$. Since $\frac{p_A^a}{p_B^a} < 1 \leq \frac{t_A}{t_B}$ for $t_B \leq t_A < \bar{t}_A$ and $\frac{p_A^a}{p_B^a} = 1 < \frac{t_A}{t_B}$ at $t_A = \bar{t}_A$, $\rho_A^a < \rho_B^a$ for t_A on $[t_B, \bar{t}_A]$. Since p_A^a is continuous in t_A, the inequality $\frac{p_A^a}{p_B^a} < \frac{t_A}{t_B}$ continues to be satisfied in an open interval containing $[t_B, \bar{t}_A]$. (c) If $k_A > k_B$ and $t_A = t_B$, $\rho(\cdot; t_A, k_A) \leq \rho(\cdot; t_B, k_B)$ from Lemma 5 and thus $\rho_A \leq \rho_B$. Since the curves of $\rho(\cdot; t_A, k_A)$ and $\rho(\cdot; t_B, k_B)$ overlap in this case precisely in the diversification range, $\rho_A = \rho_B$ if and only if at a diversification trade equilibrium. Lemma 5 shows that if $k_A > k_B$ and $t_B < t_A$, $\rho(\cdot; t_A, k_A) < \rho(\cdot; t_B, k_B)$. Thus, $\rho_A < \rho_B$ in this range. Finally, since $t_A\varphi(\rho_A) = p = t_B\varphi(\rho_B)$ at a diversification trade equilibrium, $t_A < t_B$ implies $\varphi(\rho_A) > \varphi(\rho_B)$. Since φ is increasing, $\rho_A > \rho_B$. ■

[9] It is also possible here that country A specializes in good 1 or country B specializes in good 2 and ρ_A is greater than ρ_B.

Proposition 9 determines exactly when the rent/wage of a country increases or decreases.

Proposition 9 *Suppose that country A is capital-rich ($k_A > k_B$). Then, the rent/wage of country A increases and that of country B decreases as a result of trade ($\rho_A^a < \rho_A$ and $\rho_B < \rho_B^a$) if and only if $t_A < \bar{t}_A$. Similarly, $\rho_A^a > \rho_A$ and $\rho_B > \rho_B^a$ if and only if $t_A > \bar{t}_A$. In particular, relative factor prices of countries never increase or decrease together when trade opens.*

Proof. From Proposition 8, $p_A^a < p_B^a$ if and only if $t_A < \bar{t}_A$. Since the trade equilibrium p forms between autarky prices ($p_A^a < p < p_B^a$) and the relative factor price function is increasing (strictly increasing in the diversification range), $\rho_A^a < \rho_A$ and $\rho_B < \rho_B^a$. ■

Suppose that country A has a lower autarky rent/wage than country B. Then, lemma 10 shows that the relative factor prices of countries adjust in converging directions with trade if country A has a comparative advantage in the capital-intensive sector and in diverging directions if the comparative advantage is in the labor-intensive sector.

Lemma 10 *Suppose (without loss of generality) that $\rho_i^a \le \rho_{i'}^a$, then*
(1) $p_{i'}^a < p_i^a \iff \frac{t_i}{t_{i'}} > \frac{\varphi(\rho_{i'}^a)}{\varphi(\rho_i^a)} \iff \rho_i < \rho_i^a \le \rho_{i'}^a < \rho_{i'}$,
(2) $p_i^a < p_{i'}^a \iff \frac{t_i}{t_{i'}} < \frac{\varphi(\rho_{i'}^a)}{\varphi(\rho_i^a)} \iff \rho_i^a < \rho_i$ and $\rho_{i'} < \rho_{i'}^a$.

Proof. (1) Since countries diversify at autarky, $p_i^a = t_i\varphi(\rho_i^a)$ and $p_{i'}^a = t_{i'}\varphi(\rho_{i'}^a)$. Thus, $p_{i'}^a < p_i^a \iff t_{i'}\varphi(\rho_{i'}^a) < t_i\varphi(\rho_i^a) \iff \frac{\varphi(\rho_{i'}^a)}{\varphi(\rho_i^a)} < \frac{t_i}{t_{i'}}$. Since $\rho_i^a \le \rho_{i'}^a$ and $p_{i'}^a < p < p_i^a$, $\rho_i(p) < \rho_i(p_i^a) = \rho_i^a \le \rho_{i'}^a = \rho_{i'}(p_{i'}^a) < \rho_{i'}(p)$. Conversely, $\rho_i(p) = \rho_i < \rho_i^a = \rho_i(p_i^a)$ implies that $p < p_i^a$. Similarly, $p_{i'}^a < p$, so $p^a_{i'} < p^a_i$. (2) As in (1), $p^a_i < p^a_{i'} \iff \frac{t_i}{t_{i'}} < \frac{\varphi(\rho^a_{i'})}{\varphi(\rho^a_i)}$. It is immediate that $p^a_i < p < p^a_{i'}$ if and only if $\rho_i(p^a_i) < \rho_i(p)$ and $\rho_{i'}(p) < \rho_{i'}(p^a_{i'})$. ■

We shall show that when t_A is close to t_B, the relative factor prices converge with trade. Lemma 11 proves a part of it.

Lemma 11 *Suppose country A is capital-rich ($k_A > k_B$) and fix t_B, the relative technological efficiency of country B. If the TCA of country A in the capital-intensive sector is sufficiently weak (i.e., $t_A < t_B$ and t_A is sufficiently close to t_B), then $\rho^a_A < \rho^a_B$ and the relative factor prices converge with trade.*

Proof. See Appendix. ■

Lemma 12 *If $t_A \leq t_B$, then at a diversification trade equilibrium, $\frac{\rho_B}{\rho_A} \leq \frac{t_A}{t_B}$.*

Proof. From Proposition 8 and Lemma 3, $p^a_A < p < p^a_B$. From Lemma 1, $\frac{t_B}{t_A}\varphi^{-1}\left(\frac{p}{t_B}\right) \leq \varphi^{-1}\left(\frac{t_B}{t_A}\frac{p}{t_B}\right)$ for $\frac{t_B}{t_A} \geq 1$. Thus, $\frac{\rho_B}{\rho_A} = \frac{\varphi^{-1}\left(\frac{p}{t_B}\right)}{\varphi^{-1}\left(\frac{p}{t_A}\right)} = \frac{\varphi^{-1}\left(\frac{p}{t_B}\right)}{\varphi^{-1}\left(\frac{t_B}{t_A}\frac{p}{t_B}\right)} \leq \frac{t_A}{t_B}$. ■

Theorems 13, 14 describe, in terms of technology and autarky prices – information available before trade starts – how relative factor prices behave with trade. Figure 4–Figure 8 illustrate Theorems 13, 14.[10] Country A is capital-rich in the figures. In Figure 4, country A has a strong TCA in the capital-intensive sector (sector 1). As we move from Figure 4 to Figure 8,

[10]The graphs are drawn for the production functions of $y_{ij} = t_{ij}K_{ij}^{\alpha_j}L_{ij}^{1-\alpha_j}$ and welfare function $w_i = x_i^b y_i^{1-b}$, $i = A, B$; $j = 1, 2$. Here, x_i and y_i are consumptions of country i of good 1 and 2 respectively, The values of $\{K_A = 1, L_A = 1, K_B = \frac{1}{1.5}, L_B = 1, t_B = 1, \alpha_1 = \frac{2}{3}, \alpha_2 = \frac{1}{3}, b = \frac{1}{2}\}$ are constant and t_A is changed for different graphs. Equilibrium autarky and trade prices are computed by a program.

country A loses this advantage continuously until she has a strong TCA in the labor-intensive sector (sector 2). In the figures, short dotted lines map autarky relative output prices to autarky relative factor prices. The long dotted lines map trade equilibrium relative output prices to relative factor prices of countries A and B.

Figure 4, drawn for the values of $\{t_A = 0.7, t_B = 1\}$, shows that the relative factor price for country A rises sharply after trade and that of country B falls sharply. The movements are in converging directions. However, the movements are large and the relative factor prices diverge with trade. Also, the rank order of relative factor prices change after trade. Note that country B is specializing in good 2 at the trade equilibrium. In Figure 5, drawn for the values of $\{t_A = 0.8, t_B = 1\}$, the situation is similar to that of Figure 4 except that both countries diversify at the trade equilibrium. In Figure 6, drawn for the values of $\{t_A = 0.9, t_B = 1\}$, the relative factor prices converge somewhat after trade. Nevertheless, for individual countries, there are large movements of relative factor prices and the rank order of relative factor prices change after trade. These first three graphs, where $t_A < t_B$, may represent plausible values for the USA trade with the newly industrializing economies. In Figure 7, drawn for the values of $\{t_A = 1.1,\ t_B = 1\}$, the relative factor prices move a bit in converging directions and the relative factor prices converge clearly. In Figure 8, drawn for the values of $\{t_A = 1.2, t_B = 1\}$, relative prices move in diverging directions.

The capital-rich country has a lower autarky relative factor prices ρ^a if the TCA is not strong (Proposition 8b). The proof of Proposition 8 also makes clear that the range of t_A on which the capital-rich country A has a lower

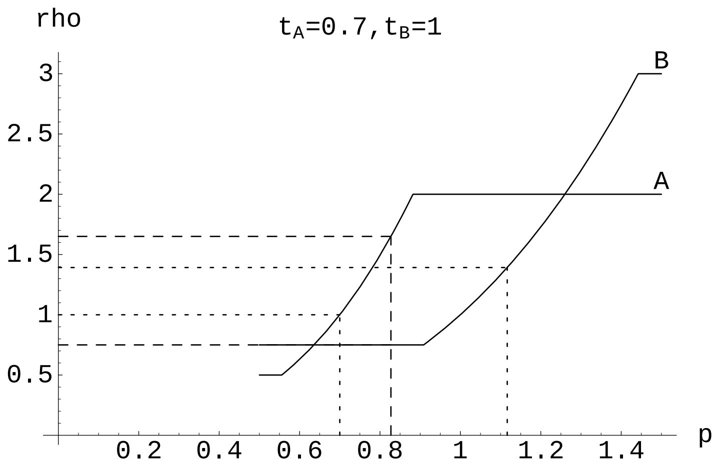

Figure 4:

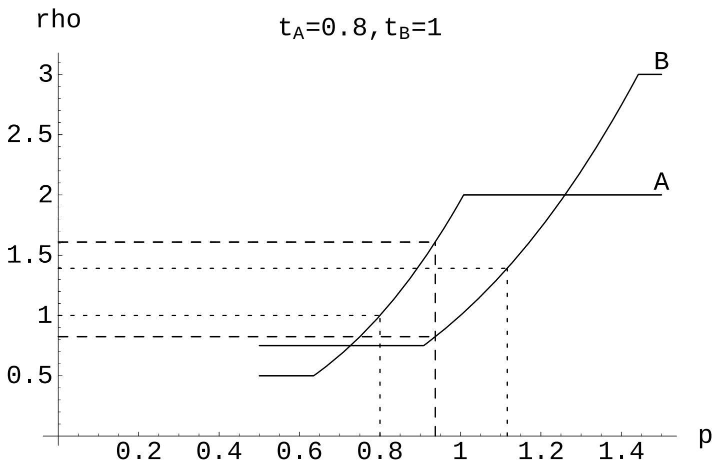

Figure 5:

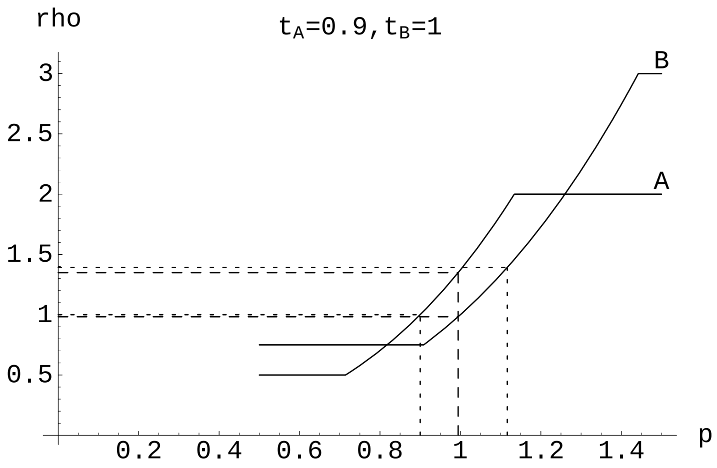

Figure 6:

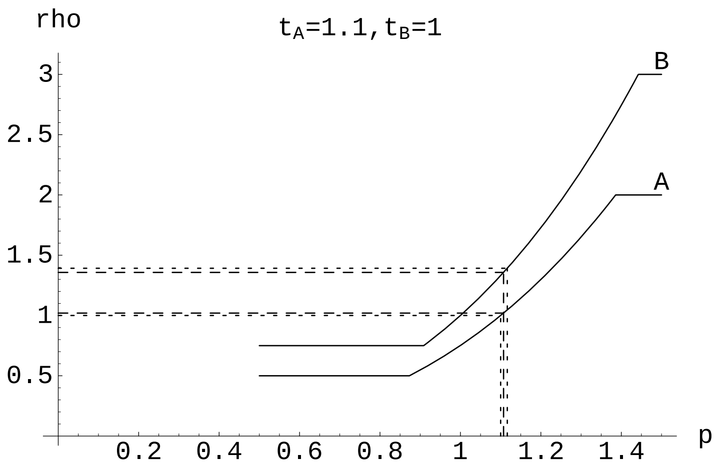

Figure 7:

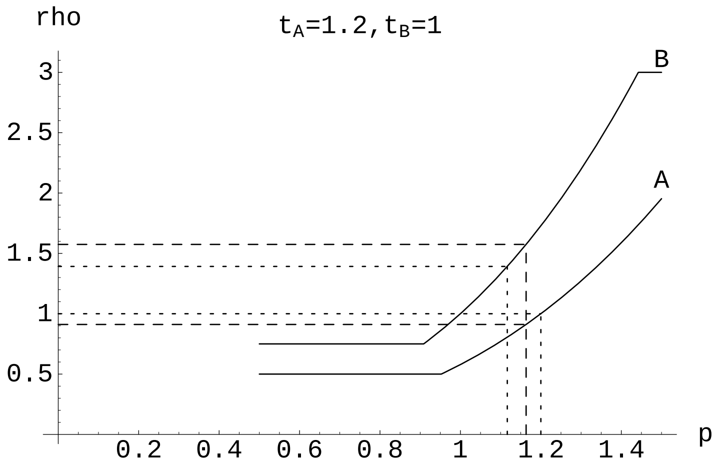

Figure 8:

autarky relative factor prices ρ^a extends to a considerably larger area than $[t_B, \bar{t}_A]$. When t_A is very different from t_B, however, we cannot make a definite statement since the influence of technology on autarky relative factor prices is ambiguous in general. For such values of t_A, we take the neutral technology effect of a Cobb-Douglas welfare function case as a base and assume that the capital-rich country has a lower autarky rent/wage. In Theorems 13 and 14, relative factor prices move in converging directions with trade if the capital-rich country A has a comparative advantage in the capital-intensive sector while they move in diverging directions if she has a comparative advantage in the labor-intensive sector. Even in the former case, however, the relative factor prices could diverge; the movements can be excessive if the TCA of country A in the capital-intensive sector is strong unless specialization mitigates the movements. Theorem 13 shows that relative factor prices converge with trade if TCA is not too strong. Suppose that country A is capital abundant ($k_A > k_B$) and fix the relative efficiency factor t_B for country B. If t_A is between t_B and $\bar{t}_A$, we get a nice convergence $\rho_A^a < \rho_A \leq \rho_B < \rho_B^a$ (Theorem 13 a and b). If t_A is smaller than t_B but is not much smaller, the relative factor prices also converge with trade (Theorem 13c).

Theorem 13 (factor price convergence) *Suppose country A is capital-rich* ($k_A > k_B$). (a) *If there is* no TCA ($t_A = t_B$), *then,* $\rho_A^a < \rho_A \leq \rho_B < \rho_B^a$. (b) *If country A has a weak TCA in the labor-intensive sector that country A has a comparative advantage in the capital-intensive sector* ($t_B < t_A < \bar{t}_A$), *then* $\rho_A^a < \rho_A < \rho_B < \rho_B^a$. (c) *If country A has a weak TCA in the capital-intensive sector* ($t_A < t_B$ but t_A is sufficiently close to t_B), *then* $\rho_A^a < \rho_B^a$ *and the relative factor prices converge with trade.*

Proof. (a) Proposition 8 gives $\rho_A^a < \rho_B^a$, $\rho_A \leq \rho_B$ and $p_A^a < p_B^a$. From $p_A^a < p_B^a$, $\rho_A^a < \rho_A$ and $\rho_B < \rho_B^a$. (b) From Lemma 10, $\rho_A^a < \rho_A$ and $\rho_B < \rho_B^a$. From Proposition 8, $\rho_A < \rho_B$. (c) This is shown in Lemma 11. ■

Theorem 14 shows that relative factor prices diverge with trade if the TCA is strong. As t_A increases, p_A^a increases and if t_A is greater than $\bar{t}_A$, p_A^a becomes greater than p_B^a. If ρ_A^a continues to be less or equal to ρ_B^a here, relative factor prices diverge (Theorem 14a). In the other direction, if t_A is small compared to t_B such that $\frac{t_A}{t_B} < \frac{\rho_A^a}{\rho_B^a} \leq 1$ obtains, the relative factor prices diverge at a diversification equilibrium (Theorem 14b).[11]

Theorem 14 (factor price divergence) *Suppose country A is capital-rich* ($k_A > k_B$) *and assume* $\rho_A^a \leq \rho_B^a$ *in the ranges of* t_A *considered below.* (a)

[11]In Theorem 13, the relationship $\rho_A^a < \rho_B^a$ is derived. In Theorem 14a, there is an open interval greater than $\bar{t}_A$ on which $\rho_A^a < \rho_B^a$ holds; otherwise, the relationship is assumed.in Theorem 14.

If country A has a strong TCA in the labor-intensive sector that she has a comparative advantage in the sector ($\bar{t}_A < t_A$), *then* $\rho_A < \rho_A^a \le \rho_B^a < \rho_B$.
(b) *If country A has a strong TCA in the capital-intensive sector such that* $\frac{t_A}{t_B} < \frac{\rho_A^a}{\rho_B^a}$, *then the relative factor prices diverge with trade at a diversification equilibrium.*

Proof. (a) This is shown in Lemma 10. (b) From Lemma 12, $\frac{\rho_B}{\rho_A} \le \frac{t_A}{t_B} < \frac{\rho_A^a}{\rho_B^a} \le 1$. ■

If a capital-rich country enjoys TCA in the capital-intensive sector or if there is no TCA ($t_A \le t_B$), then the strong divergence case in Theorem 14a cannot occur. In Theorem 14b, it is possible that factor prices converge if a country specializes. There is an example where the relative factor prices even equalize ($\rho_A = \rho_B$) with very small $\frac{t_A}{t_B}$ when one country specializes. Also in Theorem 14b, the order of relative factor prices change from $\rho_A^a < \rho_B^a$ at autarky to $\rho_A > \rho_B$ after trade.[12]

For Theorems 15 and 16, we suppose that country A has a TCA in the capital-intensive sector. In contrast to Theorems 13 and 14, Theorems 15 and 16 show that for a given TCA, the farther apart the relative endowment of country A is from that of country B, the more likely the relative factor prices converge with trade in goods. We start with given values of t_A, t_B, $t_A < t_B$ and k_B. Since $\frac{\partial p^a}{\partial t} > 0$ from Lemma 6, $k_A = k_B$ and $t_A < t_B$ imply $p_A^a < p_B^a$. Since $\frac{\partial p^a}{\partial k} < 0$, p_A^a increases as k_A decreases. Suppose that $\overline{k}_A$

[12]When ρ_A^a, ρ_B^a are independent of t_A, t_B as in the case of Cobb-Douglas welfare functions, Theorems 13 and 14 can be interpreted in terms of the ratio $\frac{t_A}{t_B}$. Since $k_A > k_B$, we have $\rho_A^a < \rho_B^a$. Let $c \equiv \frac{\overline{k}_A}{k_B} = \frac{\varphi(\rho_B^a)}{\varphi(\rho_A^a)} > 1$. Then, Theorem 13 (a), (b) and (c) correspond respectively to the cases of $\frac{t_A}{t_B} = 1, 1 < \frac{t_A}{t_B} < c, \frac{t_A}{t_B} < 1$ but close to 1 while Theorem 14 (a) and (b) correspond to $\frac{t_A}{t_B} > c$ and $\frac{t_A}{t_B} < \frac{\rho_A^a}{\rho_B^a} < 1$.

($< k_B$) is the level of k_A at which p_A^a equals p_B^a. Then, $p_A^a > p_B^a$ if and only if $k_A < \overline{k}_A$. Theorem 15 shows that the relative factor prices converge with trade if $k_A < \overline{k}_A$ or k_A is large relative to k_B.

Theorem 15 *Suppose that country A has a TCA in the capital-intensive sector* ($t_A < t_B$). (a) *If k_A is sufficiently smaller than k_B that country A has a comparative advantage in the labor-intensive sector* ($k_A < \overline{k}_A$), *then* $\rho_B^a < \rho_B < \rho_A < \rho_A^a$. (b) *If k_A is sufficiently larger than k_B, then $\rho_A^a < \rho_B^a$ and the relative factor prices converge with trade.*

Proof. (a) Since $\frac{t_B}{t_A} > 1$, $p_B^a < p_A^a$ implies $\frac{t_B}{t_A} > \frac{p_B^a}{p_A^a}$. From Lemma 7, $\rho_B^a < \rho_A^a$. We have the same situation as Theorem 13b with country indices A and B exchanged. The result follows in the same way. (b) If $k_A > \widetilde{k}_1(\underline{\rho}_B) = \widetilde{k}_1(\widetilde{k}_2^{-1}(k_B))$, $\overline{\rho}_A = \widetilde{k}_1^{-1}(k_A) < \underline{\rho}_B$, since $\widetilde{k}_1$ is a strictly decreasing function. Then, $\rho_A \leq \overline{\rho}_A < \underline{\rho}_B \leq \rho_B$. On the other hand, from Proposition 8, $k_A > k_B$ and $t_A < t_B$ imply $p_A^a < p_B^a$. Thus, $\rho_A^a = \rho_A(p_A^a) < \rho_A < \rho_B < \rho_B(p_B^a) = \rho_B^a$. ■

Theorem 16 shows that if the relative endowments are similar for a given TCA, the relative factor prices tend to diverge with trade. Theorem 16a covers the case of $\overline{k}_A < k_A \leq k_B$. In Theorem 16$b$, k_A increases from the value of k_B. If the increase is small, endowment difference plays a minor role in a possible departure of ρ_A^a from ρ_B^a. If the impact of technological efficiencies on autarky relative factor prices is small, we will have $\frac{t_A}{t_B} < \frac{\rho_A^a}{\rho_B^a} < \frac{t_B}{t_A}$. In this case, the relative factor prices diverge at a diversification equilibrium.

Theorem 16 *Suppose that country A has a TCA in the capital-intensive sector* ($t_A < t_B$). (a) *If country A is labor-abundant but k_A is sufficiently close* *to* k_B *that country* A *has a comparative advantage in the capital-intensive sector* ($\overline{k}_A < k_A \leq k_B$) *and if the endowment effect is dominant in determining the autarky relative factor prices* ($\rho_B^a \leq \rho_A^a$), *then* $\rho_B < \rho_B^a \leq \rho_A^a < \rho_A$. (b) *If* k_A *is close to* k_B *and* $\frac{t_A}{t_B} < \frac{\rho_A^a}{\rho_B^a} < \frac{t_B}{t_A}$, *then at a diversification trade equilibrium,* $\frac{\rho_B}{\rho_A} < \frac{\rho_A^a}{\rho_B^a} < \frac{\rho_A}{\rho_B}$; *the relative factor prices diverge.*

Proof. (a) For the fixed TCA, $t_A < t_B$, $p_A^a < p_B^a$ for $\overline{k}_A < k_A \leq k_B$. Thus, $\rho_A^a < \rho_A$ and $\rho_B < \rho_B^a$. If $\rho_B^a \leq \rho_A^a$, $\rho_B < \rho_B^a \leq \rho_A^a < \rho_A$. (b) If $\frac{t_A}{t_B} < \frac{\rho_A^a}{\rho_B^a} < \frac{t_B}{t_A}$ at a diversification equilibrium, Lemma 12 gives $\frac{\rho_B}{\rho_A} \leq \frac{t_A}{t_B} < \frac{\rho_A^a}{\rho_B^a} < \frac{t_B}{t_A} \leq \frac{\rho_A}{\rho_B}$. Thus, $\frac{\rho_B}{\rho_A} < \frac{\rho_A^a}{\rho_B^a} < \frac{\rho_A}{\rho_B}$. ■

7 Welfare

Previously, we plotted relative factor prices of a country as a function of relative prices. We can plot the individual factor prices in the same way. In Figure 9, w, the wage (of labor) in terms of good 2, is plotted as a function of p (the price of good 1 in units of good 2). Similarly, $\frac{w}{p}$ is the wage in terms of good 1. The economy diversifies in the middle region. In the diversification region, both w and $\frac{w}{p}$ are decreasing in p as the Stolper-Samuelson theorem states.[13] Given different values of t_A and t_B, we can invoke the Stolper-Samuelson theorem to assess welfare consequences of trade on a particular factor in a country if a diversification equilibrium obtains. If country A is capital-rich, country A is more likely to have a comparative advantage in the capital-intensive sector ($p_A^a < p_B^a$). Suppose, for example, that the United States(country A) and China(country B) are equally efficient in the labor-

[13] Recall that sector 1 is capital-intensive.

intensive sector (sector 2) but that the United States is more efficient in the capital-intensive sector (sector 1). Then, the United States has the TCA in the capital-intensive sector since $\frac{t_A}{t_B} = \frac{t_{B1}}{t_{A1}} < 1$. From Proposition 8, $p_A^a < p_B^a$. In this case, the relative price of good 1 increases with trade in the United States. At a diversification trade equilibrium, the labor in the United States unequivocally loses[14] as a result of trade.[15]

When an economy specializes in good 1, a further increase in p leaves the relative factor price ρ unchanged. As p increases in this specialization region, w and r increase in the same proportion. Thus, the wage in terms of good 1 is constant whereas the wage in terms of good 2 increases. If the United States is already specializing in good 1 or is sufficiently close to its specialization (at least as far as the trade sector is concerned, as some argue), a further increase in p would increase rather than decrease the welfare of labor. In Figure 9, the labor loses unequivocally as the relative price of good 1 increases from p_a to p_b. If the price increases from p_b to p_c, however, the wage in terms of good 1 decreases by a little whereas the wage in terms of good 2 increases sharply, possibly leading to an increase of welfare for labor.

A Appendix

A.1 Proof of Lemma 1

(1) Denote the factor requirements of producing one unit of f_j as $\widetilde{K}_j$ and $\widetilde{L}_j$ and let $\widetilde{k}_j = \frac{\widetilde{K}_j}{\widetilde{L}_j}$, $j = 1, 2$. Using the Shepard's lemma ($\phi_i' = \widetilde{K}_i$) and the assumption that sector 1 employs more capital-intensive production method

[14]In the sense that both w and $\frac{w}{p}$ decline.

[15]If $p_A^a > p_B^a$ (this can happen only if $t_A > t_B$), however, p_A^a decreases to p with trade and thus the labor of country A unequivocally gains at a diversification equilibrium.

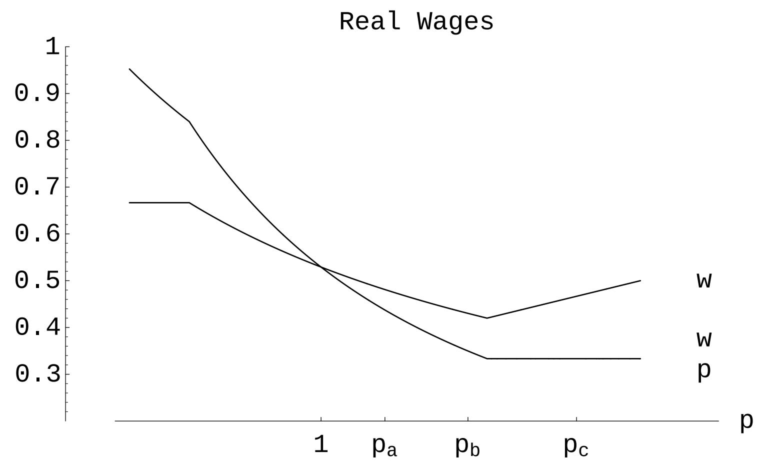

Figure 9:

than sector 2, $\varphi'(\rho) = (\frac{1}{\phi_2})^2\,(\phi_1'\phi_2 - \phi_2'\phi_1) = (\frac{1}{\phi_2})^2\left(\widetilde{K_1}(\widetilde{L}_2 + \rho\widetilde{K_2}) - \widetilde{K_2}(\widetilde{L}_1 + \rho\widetilde{K_1})\right)$
$= (\frac{1}{\phi_2})^2\left(\widetilde{K_1}\widetilde{L}_2 - \widetilde{K_2}\widetilde{L}_1\right) = (\frac{1}{\phi_2})^2\widetilde{L}_1\widetilde{L}_2\left(\widetilde{k}_1 - \widetilde{k}_2\right) > 0$. Also,
$\varphi''(\rho) = -2\phi_2\widetilde{K_2}\widetilde{L}_1\widetilde{L}_2\,(k_1 - k_2) < 0$. Next, $\frac{\varphi(\rho)}{\rho} - \varphi'(\rho) = \frac{\phi_1}{\phi_2}\frac{1}{\rho} - (\frac{1}{\phi_2})^2\widetilde{L}_1\widetilde{L}_2\,(k_1 - k_2)$
$= (\frac{1}{\phi_2})^2\left(\left(\widetilde{L}_1 + \rho\widetilde{K_1}\right)\left(\widetilde{L}_2 + \rho\widetilde{K_2}\right)\frac{1}{\rho} - \widetilde{K_1}\widetilde{L}_2 + \widetilde{K_2}\widetilde{L}_1\right)$
$= (\frac{1}{\phi_2})^2\left(\frac{1}{\rho}\widetilde{L}_1\widetilde{L}_2 + 2\widetilde{K_2}\widetilde{L}_1 + \rho\widetilde{K_1}\widetilde{K_2}\right) > 0$. Thus,
$\frac{d}{d\rho}\frac{\varphi(\rho)}{\rho} = \frac{1}{\rho}\left(\varphi'(\rho) - \frac{\varphi(\rho)}{\rho}\right) < 0$.

(2) From $\frac{\varphi(\rho)}{\rho} > \varphi'(\rho)$, $\varphi(\lambda\rho) < \lambda\varphi(\rho)$ when $\lambda > 1$. Writing $p = \varphi(\rho)$ and applying φ^{-1} to both sides of the inequalities, $\lambda\varphi^{-1}(p) < \varphi^{-1}(\lambda p)$, for $\lambda > 1$ since φ^{-1} is an increasing function. ∎

A.2 Equilibrium Conditions

Let $\overline{y}_{i1}$ and $\overline{y}_{i2}$ stand for the demands for good 1 and 2 respectively. Since the welfare function is homothetic and strictly quasi-concave, $\dfrac{\overline{y}_{i2}}{\overline{y}_{i1}} = \delta(p_i^a)$ for some function δ of the relative price p_i^a. This together with the budget condition $p_i^a\overline{y}_{i1} + \overline{y}_{i2} = r_i^a K_i + w_i^a L_i$, give the demand functions $\overline{y}_{i1} = \dfrac{r_i^a K_i + w_i^a L_i}{p_i^a + \delta(p_i^a)}$,

$\overline{y}_{i2} = \dfrac{\delta(p_i^a)\,(r_i^a K_i + w_i^a L_i)}{p_i^a + \delta(p_i^a)}$. Then, the equilibrium conditions are:

$$\begin{aligned}
p_i^a &= \frac{1}{t_{i1}}\phi_1(r_i^a, w_i^a),\ 1 = \frac{1}{t_{i2}}\phi_2(r_i^a, w_i^a) \\
y_{i1} &= \frac{r_i^a K_i + w_i^a L_i}{p_i^a + \delta(p_i^a)},\ y_{i2} = \frac{\delta(p_i^a)\,(r_i^a K_i + w_i^a L_i)}{p_i^a + \delta(p_i^a)} \\
K_i &= y_{i1}\frac{1}{t_{i1}}\widetilde{K}_1(\rho_i^a) + y_{i2}\frac{1}{t_{i2}}\widetilde{K}_2(\rho_i^a) \\
L_i &= y_{i1}\frac{1}{t_{i1}}\widetilde{L}_1(\rho_i^a) + y_{i2}\frac{1}{t_{i2}}\widetilde{L}_2(\rho_i^a)
\end{aligned} \tag{5}$$

The first two equations are profit maximization conditions. The second two are product market equilibrium conditions. The last two equations are factor market equilibrium conditions. There are five independent equations (by the Walras' law, we can eliminate one product market equilibrium condition) and five variables. The first two equations yield the second equation in 1. And the second two equations yield the first equation in 1. Conversely, given a solution to the equations in 1, we can construct a solution to 5. The value of $\rho_i^a \left(= \frac{r_i^a}{w_i^a}\right)$, together with $p_i^a = \frac{1}{t_{i1}}\phi_1(r_i^a, w_i^a)$, gives the values of (r_i^a, w_i^a). Then, $p_i^a = t_i\varphi(\rho_i^a)$ implies that $1 = \frac{1}{t_{i2}}\phi_2(r_i^a, w_i^a)$. The first two and the last two equilibrium conditions in 5 together imply $p_i^a y_{i1} + y_{i2} = r_i^a K_i + w_i^a L_i$. Together with $\dfrac{y_{i2}}{y_{i1}} = \delta(p_i^a)$, we obtain the second two equations in 5.

A.3 Proof of Lemma 2

Since we assume that a country diversifies at autarky and since $p_i^a = t_i\varphi(\rho_i^a)$, it is enough to show that ρ_i^a is unique. Dropping the country index, the autarky equilibrium conditions yield $-t\frac{\widetilde{K}_1 - k\widetilde{L}_1}{\widetilde{K}_2 - k\widetilde{L}_2} = \delta(t\varphi(\rho))$. Define:$F \equiv t\left(\widetilde{K}_1 - k\widetilde{L}_1\right) + \delta(t\varphi)\left(\widetilde{K}_2 - k\widetilde{L}_2\right)$. Then, using $\delta' > 0, \varphi'(\rho) > 0$, $\widetilde{K}_2 - k\widetilde{L}_2 = \widetilde{L}_2\left(\widetilde{k}_2 - k\right) < 0$, $\widetilde{K}_2' < 0, \widetilde{K}_1' < 0, \widetilde{L}_1' > 0, \widetilde{L}_2' > 0$, we have: $\frac{\partial F}{\partial \rho} = \delta' t\varphi'\left(\widetilde{K}_2 - k\widetilde{L}_2\right) + \delta(t\varphi)\left(\widetilde{K}_2' - k\widetilde{L}_2'\right) + t\left(\widetilde{K}_1' - k\widetilde{L}_1'\right) < 0$. Since $F = 0$ at an equilibrium ρ, the equilibrium ρ is unique. Next, observe $\frac{\partial F}{\partial k} = -\delta(t\varphi(\rho))\widetilde{L}_2 - t\widetilde{L}_1 < 0$. Thus, $\frac{\partial \rho}{\partial k} < 0$. When shared welfare functions are Cobb-Douglas, the equilibrium condition becomes $-\varphi(\rho)\frac{\widetilde{K}_2 - k\widetilde{L}_2}{\widetilde{K}_1 - k\widetilde{L}_1} = \frac{b}{1-b}$. Thus, the equilibrium ρ is independent of t. Since $\frac{\partial \rho}{\partial k} < 0$, $k_A > k_B$ implies $\rho_A^a < \rho_B^a$. ■

A.4 Proof of Lemma 6

Dropping the country index, we start with an autarky equilibrium condition 2:$-t\frac{\widetilde{K}_1 - k\widetilde{L}_1}{\widetilde{K}_2 - k\widetilde{L}_2} = \delta(t\varphi(\rho^a))$. Let $F \equiv t\left(\widetilde{K}_1 - k\widetilde{L}_1\right) + \delta(t\varphi(\rho^a))\left(\widetilde{K}_2 - k\widetilde{L}_2\right)$. We can compute: $\frac{\partial F}{\partial k} = -t\widetilde{L}_1 - \delta(t\varphi(\rho^a))\widetilde{L}_2 < 0$. Note $k = \frac{K_1}{L_1}\frac{L_1}{L} + \frac{K_2}{L_2}\frac{L_2}{L} = \widetilde{k}_1\frac{L_1}{L} + \widetilde{k}_2\frac{L_2}{L}$, where $\widetilde{k}_j = \frac{\widetilde{K}_j}{\widetilde{L}_j}$ and K_j, L_j are factors employed in j_{th} industry. Since a country diversifies at autarky and $\widetilde{k}_1 > \widetilde{k}_2$, $\widetilde{K}_1 - k\widetilde{L}_1 > 0$ and $\widetilde{K}_2 - k\widetilde{L}_2 < 0$. Since $\widetilde{K}_1'$, $\widetilde{K}_2'$ are negative and $\widetilde{L}_1'$, $\widetilde{L}_2'$, δ', $\varphi'(\rho^a)$ are positive, $\frac{\partial F}{\partial \rho^a} = t\left(\widetilde{K}_1' - k\widetilde{L}_1'\right) + \delta'(t\varphi(\rho^a))t\varphi'(\rho^a)\left(\widetilde{K}_2 - k\widetilde{L}_2\right) + \delta(t\varphi(\rho^a))\left(\widetilde{K}_2' - k\widetilde{L}_2'\right) < 0$. Thus, $\frac{\partial \rho^a}{\partial k} = -\frac{\partial F}{\partial k}/\frac{\partial F}{\partial \rho^a} < 0$. From $p^a = t\varphi(\rho^a)$, $\frac{\partial p^a}{\partial k} = \frac{\partial}{\partial \rho^a}t\varphi(\rho^a)\frac{\partial \rho^a}{\partial k} < 0$. Now, express F in terms of t, p^a : $G(t, p^a) \equiv t\left(\widetilde{K}_1(\rho^a) - k\widetilde{L}_1(\rho^a)\right) + \delta(p^a)\left(\widetilde{K}_2(\rho^a) - k\widetilde{L}_2(\rho^a)\right)$, where $\rho^a = \varphi^{-1}(\frac{p^a}{t})$. Then, $\frac{\partial G}{\partial p^a} = \frac{\partial F}{\partial \rho^a}\frac{\partial}{\partial p^a}\varphi^{-1}(\frac{p^a}{t}) < 0$ and $\frac{\partial G}{\partial t} = \widetilde{K}_1 - k\widetilde{L}_1 > 0$. Thus, $\frac{\partial p^a}{\partial t} > 0$. ■

A.5 Proof of Lemma 11

From Proposition 8, $k_A > k_B$ and $t_A < t_B$ imply $p_A^a < p_B^a$. If at the trade equilibrium price p, $\rho_A(p) \leq \rho_B(p)$ (Proposition 8 shows that this can happen only if a country specializes), $\rho_A^a = \rho_A(p_A^a) < \rho_A(p) \leq \rho_B(p) < \rho_B(p_B^a) = \rho_B^a$. Thus, the relative factor prices converge. Now, consider the case of $\rho_B(p) < \rho_A(p)$. From the autarky equilibrium conditions $-t_A\frac{\widetilde{K}_1 - k_A\widetilde{L}_1}{\widetilde{K}_2 - k_A\widetilde{L}_2} = \delta(t_A\varphi(\rho_A^a))$, let $F(\rho_A^a, t_A) \equiv t_A\left(\widetilde{K}_1 - k_A\widetilde{L}_1\right) + \delta(t_A\varphi(\rho_A^a))\left(\widetilde{K}_2 - k_A\widetilde{L}_2\right)$. From the proof of Lemma 2, $\frac{\partial}{\partial \rho_A^a}F(\rho_A^a, t_A) \neq 0$. By the implicit function theorem, ρ_A^a is locally a continuous function of t_A, denoted as $\rho_A^a(t_A)$, near t_B. Since $k_A > k_B$ implies $\rho_A^a(t_B) < \rho_B^a(t_B)$ by Lemma 6, $\rho_A^a(t_A) < \rho_B^a(t_B)$ for t_A close to t_B. Recall the definitions of $\overline{\rho}_A$, $\underline{\rho}_B$ in 4. One can check that $\rho_A(p) > \rho_B(p)$ can happen only if p is in $I \equiv [t_A\varphi(\underline{\rho}_B), t_B\varphi(\overline{\rho}_A)]$. As t_A increases to t_B, $\overline{\rho}_A, \underline{\rho}_B$ do not change while the interval I monotonically decreases if it is not empty. For any $\varepsilon > 0$, $\rho_A - \rho_B \leq \max_{p\in I}[\varphi^{-1}(\frac{p}{t_A}) - \varphi^{-1}(\frac{p}{t_B})] < \varepsilon$, as t_A increases sufficiently to t_B. Since $p_A^a < p_B^a$, $\rho_A^a(t_A) < \rho_A$, $\rho_B < \rho_B^a(t_B)$. In the case $\rho_B < \rho_A$, $\rho_A^a(t_A) - \varepsilon < \rho_B < \rho_B^a(t_B)$ and thus $1 < \frac{\rho_A}{\rho_B} < 1 + \frac{\varepsilon}{\rho_B} < 1 + \frac{\varepsilon}{\rho_A^a(t_A) - \varepsilon} \to 1$ as t_A increases to t_B. At the same time, $\frac{\rho_A^a(t_A)}{\rho_B^a(t_B)} \to \frac{\rho_A^a(t_B)}{\rho_B^a(t_B)} < 1$. Thus, $\frac{\rho_A^a}{\rho_B^a} < \frac{\rho_A}{\rho_B} < \frac{\rho_B^a}{\rho_A^a}$

for all t_A ($< t_B$) sufficiently close to t_B. ■

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