Discussion Paper No. 510

EMERGENCE OF NEW INDUSTRIES AND ENDOGENOUS GROWTH CYCLES

Ryo Horii

June 2000

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

Emergence of New Industries and Endogenous Growth Cycles

Abstract

This paper constructs a growth model in which monopolistically competing firms choose the characteristic of their own product from an unbounded product space. While consumers wish to satisfy various needs by purchasing a diverse range of goods, production costs are lower for those goods that are more similar to existing ones because of spillover effects in the learning-by-doing process. The dynamic interaction of these centrifugal and centripetal forces on the product space leads to sporadic emergence and disappearance of industries, which is a source of cyclical fluctuations in the macroeconomic growth rate.

JEL Classification Numbers: O41, E32, L16.

Keywords: product differentiation, learning-by-doing with spillover, emergence of industries, growth cycles.

Ryo Horii
The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki
Osaka 567-0047 JAPAN
E-mail address: horii@iser.osaka-u.ac.jp

1 Introduction

Beginning with the seminal works of Romer (1987, 1990) and Grossman and Helpman (1989), many models have been presented in which an increase in the number of goods drives economic growth. Most of these models predict that the economy will eventually converge to a balanced growth path, where the economy is always a scaled-up version of what it was years ago. This property, however, clearly depends on various kinds of simplifying assumptions, posed primarily for analytical tractability of the model. Among such simplifications, the most crucial is the fact that many models have ignored relationships between goods, or at least treated them in a separable way.

In a situation where new goods are continually being introduced, such a treatment means we ignore two important aspects of economic growth. First, how consumers value a certain new good depends on the whole bundle of goods they are currently consuming. Thus, the current availability of goods affects the prices of new goods when they are introduced.[1] Second, production of a new good depends to a greater or lesser extent on various kinds of knowledge in the public domain. A large portion of such knowledge was in fact obtained through the past experiences of private firms and has subsequently become publicly available. Therefore, the set of goods that has been produced in the whole economy determines the current distribution of specific knowledge, and thus affects the production cost of the next goods to be introduced.

In these two ways, interactions between new and existing goods affect the kinds of new good introduced by profit maximizing private firms and the time of their introduction. Moreover, there also exist similar interactions among existing goods, which will cause a dynamic evolution of their prices and quantities, and hence affect economic growth rate.

[1]For ease of description, we suppose in this paper that consumers directly purchase differentiated goods, although it is more common to assume there is a final good producer who demands differentiated intermediate goods. Essentially, it is a matter of convenience to take either view, and it is possible to reinterpret our model in terms of intermediate goods.

In this paper, we explicitly introduce these interactions into an endogenous growth model and investigate how the conclusions must be modified. More specifically, in our setting, which will be formally introduced in the next section, there are many firms competing monopolistically among themselves, and each of them is allowed to choose the characteristic of its own differentiated good from an unbounded product space, represented by the real axis. Here, two opposite forces come into play. On the one hand, since each consumer desires to satisfy various needs by purchasing a diverse range of goods, every firm has an incentive to choose characteristics that are matched poorly by other existing goods. This works as a centrifugal force on the product space. On the other hand, we assume that knowledge accumulated by learning-by-doing spills over 'nearby' characteristics, and thus production costs are lower for those goods that have characteristics closer to existing ones. This works as a centripetal force and these two forces together represent the interaction between goods described earlier.

Analyses in the following sections will show that firms tend to agglomerate at several endogenously determined points in the product space.[2] We would like to interpret each mass point as an 'industry', since within each point every firm produces a mutually symmetric, though differentiated, good.[3] Along the path of economic growth, the number of differentiated goods, or firms, in each industry varies continuously. This growth process is simply a multisector extension of the usual endogenous growth models. There is yet another type of dynamic process: the structure of the economy itself changes discretely by the emergence or disappearance of industries. Moreover, the timing of emergence and the characteristics of new industries are endogenously determined, dependent on consumer preferences and production technology.

[2]In this paper, we use the term 'agglomerate' to mean 'gravitate towards a common point in the product space'. Note that it do not mean 'merger' of firms in the takeover sense.

[3]Alternatively, we may well call each point a 'product group'. Indeed, these two ideas are equivalent in our model since each firm produces only one good. However, in the real economy where joint production is possible, such an equivalence does not necessarily hold. We are not concerned here with this point, however.

In this situation, how will such events affect economic growth? We will confirm an intuitive property that recently-emerged new industries explain a disproportionately large portion of economic growth. This property implies that the growth rate will be affected dramatically when industrial structure changes. In fact, we will show that after a new industry emerges, the overall economic growth rate sharply declines, at least in the short term, since a slowdown of knowledge accumulation in currently large industries has a large negative effect on the growth rate. This might be a surprising result since we will also see that in the long run, positive economic growth is not sustainable unless new industries are introduced. As a result, the growth rate persistently fluctuates in parallel with the sporadic emergence of new industries.[4]

There is a large body of literature on product differentiation. As shown in the review by Perloff and Salop (1985), two basic formalizations of product differentiation have been explored in detail: the Hotelling (1929) spatial competition model, and the representative consumer model often associated with Chamberlin (1933). Most current endogenous growth models are based on the representative consumer model, in particular on the work of Dixit and Stiglitz (1977), where a representative consumer (or a final good producer) purchases many goods, varying the proportions of each according to their prices and exogenously given utility weights. Although this formalization has a desirable property of permitting multibrand competition, it does not clearly focus on the characteristic of each good nor the relationship between them. In contrast, Kim (1989), and Kim and Mohtadi (1992) are among the few studies in the endogenous growth literature that are based on the spatial

[4]Hornstein and Krusell (1996) and Greenwood and Yorukoglu (1997) reported the 'productivity slowdown puzzles', which attribute productivity slowdown to the learning cost associated with new technologies. This paper offers an alternative explanation to their hypothesis, and can be viewed as an integration of the puzzles into a theory of cyclical growth. In fact, similar studies already exist. Helpman and Trajtenberg (1998) presented a model in which exogenous sporadic improvements in the GPT (General Purpose Technology) caused recurrent cycles in the growth rate. Matsuyama (1999) is a recent work that focused on the tradeoff between expansion of variety and the growth rate, in the discrete time setting. In our model, the pattern of introduction of new technologies is endogenously determined in the continuous time setting.

competition model. Indeed, they explicitly focus on the good's characteristic, but competition reduces to a localized phenomenon for them, since each consumer purchases only one or two most preferred goods.

We will integrate these two approaches by introducing a two-dimensional product space. Initially, we will concentrate on a monopolistically competitive economy where each firm can ignore its impact on, and hence reaction from, other firms. In such a situation, Hart (1985) shows that there must be at least one dimension of product differentiation where the 'non-neighboring goods property' holds; that is, goods must be sufficiently distinctive that each good is equally far apart from every other good and thus there is no natural ordering over them. In fact, the Dixit-Stiglitz style competition satisfies this desirable property, which can be interpreted in our context as one dimension of competition that represents the differentiation 'within' each industry. We will combine this dimension with another of the Hotelling-style spatial competition that corresponds to the differentiation 'between' industries. This hybrid specification enables us to focus on each good's characteristic while retaining features of multibrand monopolistic competition.[5]

As an engine of economic growth, we will exclusively concentrate on 'learning-by-doing', the incidental process of knowledge accumulation that accompanies the production of goods.[6] Among various aspects of the learning-by-doing process, our model incorporates two important characteristics pointed out by Young (1991, 1993). First, there appear to be substantial spillover

[5]Technically, our formalization of consumer preference is closely related to recent models in urban economics, such as in Fujita, Krugman and Venables (1999), where spatial structures of cities are analyzed in the context of monopolistic competition. However, there still remains a crucial difference in that our model has a representative consumer who actively determines the distribution of her expenditure over the product space, while in urban economics consumers themselves are distributed locationally.

[6]Of course, investment in knowledge often precedes production, for example in the case of R&D, and many workers invest actively in their human capital. Although these processes are also important, there is empirical evidence that supports our specification as a close approximation. Jovanovic (1995) reports that even the most advanced countries spend far more on adoption of existing technologies than on inventing new ones, with his rough estimate in the U.S. that adoption costs outweigh invention costs by 20 or 30 to 1.

effects in the development of knowledge across industries, since many of the technical and managerial advances brought about by experience in the production of certain goods seem to have applications elsewhere. Thus, productivity increases in each industry are not only a consequence of productive activity in that industry, but also the result of spillovers from learning-by-doing in other industries. Our model focuses also on the varying degrees of difficulty of application by formulating the magnitude of the spillover effect as a decreasing function of the characteristic distance between industries.[7]

The second empirical regularity relevant to us is the existence of strong diminishing returns in the learning-by-doing process.[8] In particular, we suppose that learning-by-doing is 'bounded', which means that even if production experience accumulates unboundedly, the production cost of a particular good does not fall below a certain amount. Although the existence of such a bound is an unresolved empirical matter, it has an intuitive implication: positive economic growth will not persist unless new goods are continually introduced. Thus, in our model, the learning-by-doing process within an industry affects the growth rate only transitorily, while its spillover effect contributes to economic growth in the long run by paving the way for the emergence of ever newer industries.[9]

The rest of this paper is organized as follows. After presenting the model in the next section, we go on in Section 3 to investigate the market equilibrium at each instant for fixed knowledge and population. Dynamic evolution of the system of industries is analyzed in Section 4, utilizing results from numerical simulations. Then, the fifth section clarifies why the growth rate

[7]Stoky (1988) presents an alternative formulation of the spillover of knowledge in a more general setting.

[8]For extensive references, see Young (1991, p371-72).

[9]In this respect, our model contrasts clearly with and is complementary to the recent 'hybrid' endogenous growth models presented by Young (1998), Peretto (1998), and Dinopoulos and Thompson (1998), which combine the variety expansion model and the quality ladder model. In their models, the primary determinant of long-term growth is quality improvement within each industry, while the expansion of variety is treated as a transitory adjustment process that does not affect the long-term growth rate.

has cycles. Finally, Section 6 briefly offers an extension that does not require population growth.

2 The Model

In this section, we develop a specific model that explicitly focuses on the relationships between goods. The first subsection formally defines the product space and consumer preference on it, under which firms have an incentive to move apart from each other. The second subsection then specifies production technologies and the processes of learning-by-doing, which induce firms to agglomerate with each other.

2.1 Preference and Product Space

Assume that the economy has many identical consumers, and that the measure of them, N_t, grows at an exogenous positive rate, λ. Here, time is continuous, but in the following, we will omit the time subscript unless it is necessary. Each consumer has potentially unbounded 'needs', represented by the real axis $\mathcal{R} \equiv (-\infty, \infty)$, from which she can attain utility. In particular, let us define a subutility $v(r)$ for each $r \in \mathcal{R}$, which represents the level of satisfaction of the need, r. Then, overall instantaneous utility is

$$U = \left[\int_{-\infty}^{\infty} v(r)^{\frac{\beta-1}{\beta}} dr \right]^{\frac{\beta}{\beta-1}}, \tag{1}$$

where β is the elasticity of substitution between needs. We assume $\beta > 1$, which means that consumers desire to satisfy an increasing range of needs.

There is a continuum of firms and we index each of them by $i \in [0, \bar{n}]$. Here, $\bar{n}$ is the measure of the total number of firms, whose evolution will be determined in equilibrium. Each firm produces one differentiated good, which can be arbitrarily divided to satisfy various needs. We assume that consumers benefit from consuming a large variety of differentiated goods to satisfy each need. Goods are not necessarily symmetric with each other, however; that is, each good has its own 'characteristic' that represents what need it matches best. We formally state these considerations by introducing a two-dimensional product space.

The first dimension, which we call the 'variety dimension', enables goods to be intrinsically differentiated from each other in a way similar to that introduced by Dixit and Stiglitz (1977). In particular, we suppose that, for a given fixed budget, the subutility of a need becomes higher when a greater number of distinct goods are consumed to satisfy it. In this case, a slight rise in the price of a certain good will not eliminate its demand since consumers do not want to give up this good altogether, as long as no pair of firms produce the same good. In fact, we will be able to confirm in the next subsection that every firm produces a distinct good, and therefore the firms compete monopolistically among themselves.

The other dimension has a spatial structure represented by the real axis $\mathcal{R}$. We would like to call the latter the 'need dimension', since there is a one-to-one relationship between this dimension and the space of needs, also represented by the real axis: when a consumer uses a marginal unit of a good that has a characteristic $s \in \mathcal{R}$ in the need dimension to fulfill a need $r \in \mathcal{R}$, the contribution from this marginal consumption to the subutility of the need r is negatively related to the distance between r and s. This specification parallels Hotelling (1929), and combined with the variety dimension, opens up the possibility of an 'industry' where many firms compete among themselves for the same need.

Let $l(i) \in \mathcal{R}$ denote firm i's choice of its characteristic in the need dimension[10] and let $\tilde{c}(r,i)$ denote the density of consumption of firm i's product that is used by the representative consumer to satisfy need r. Then the subutility of need r is

$$v(r) = \left[\int_0^{\bar{n}} \left(\tilde{c}(r,i) e^{-\tau |r - l(i)|} \right)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where σ is the elasticity of substitution between goods, and τ the coefficient that measures how rapidly a good becomes unfit for the need r as its char-

[10] In the following, we will say 'the characteristic of a firm's product' and 'a firm's characteristic' interchangeably, since each firm produces only one kind of good.

acteristic $l(i)$ becomes more distant from r. Substituting (2) for (1), we now have an instantaneous utility function in terms of the list of characteristics of firms' products $l(\cdot)$ and the consumption density $\tilde{c}(\cdot,\cdot)$.

Note that, from the consumer's viewpoint, taking $l(\cdot)$ as given, this is a two-stage CES utility function of $\tilde{c}(\cdot,\cdot)$. Here we assume that the elasticity of substitution between goods within a certain need, σ, is higher than that across needs, β. This is equivalent to supposing that the marginal utility of consuming a good to satisfy a certain need r decreases as the subutility of that need increases. Under this assumption, each firm has an incentive to choose a characteristic that is matched poorly by the other existing goods. It is this centrifugal force that is one of the two major driving forces in this economy.

By making use of the assumption $\sigma > \beta > 1$, we can characterize the utility function in a more convenient manner. Since the instantaneous utility function is strictly quasi-concave,[11] if two firms have chosen the same characteristic, the demands for the products of these two firms would be the same provided that both firms have set the same price. In fact, this will be the case in equilibrium, since both firms confront the same demand curve and the same cost condition. Thus, instead of the index of each firm, i, we can express the consumption density in terms of the characteristic of that firm, s. Formally, let $c(r,s)$ denote the value of $\tilde{c}(r,i)$ if there exists any firm i whose characteristic $l(i)$ coincides with s. If there is no such firm, let $c(r,s) = 0$. Then, (2) can be rewritten as

$$v(r) = \left[\int_{-\infty}^{\infty} \left(c(r,s) e^{-\tau|r-s|} \right)^{\frac{\sigma-1}{\sigma}} dn(s) \right]^{\frac{\sigma}{\sigma-1}}, \tag{2$'$}$$

where $n(\cdot)$ is a distribution function of firms, which is defined as the measure of firms whose characteristics are equal to or to the left of s.

Before closing this subsection, let us consider labor supply and saving behaviors. First, we assume that each consumer inelastically supplies one unit of labor and the wage rate is normalized to unity. We also suppose that

[11]By the assumption $\sigma > \beta > 1$, we have both $(\sigma-1)/\sigma < 1$ and $(\sigma/(\sigma-1))((\beta-1)/\beta) < 1$, which together establish the strict quasi-concavity of U.

every good is perishable and cannot be stored. In addition, as will be seen in the following, there is no opportunity to invest. Thus, in this economy where consumers are homogeneous, the credit market involves no trade, and every consumer spends all of her income at each instant. This implies that she maximizes the instantaneous utility (1) with (2′) under the instantaneous budget constraint

$$\int_{-\infty}^{\infty} q(s)p(s)\, dn(s) = 1, \tag{3}$$

where $p(s)$ and $q(s) \equiv \int_{-\infty}^{\infty} c(r,s)\, dr$ are the price and the amount of purchase, respectively, of each good that has the characteristic s.

2.2 Learning-by-Doing and Production Technology

We now turn to describe the production side. In this economy, there is neither any kind of entry deterrence nor lump sum setup/exit costs, and every firm can choose its product's characteristic freely. At each instant, every firm simultaneously determines whether to enter the market or not, its characteristic, quantity and price to maximize its profit.

There are two factors of production: labor, and knowledge specific to each characteristic. We assume that each specific knowledge accumulates by learning-by-doing processes and has an inappropriable and nonrival nature; therefore, each firm takes it as given.

Since one kind of input factor is publicly available, each firm hires only the remaining factor, namely, labor, at the prevailing wage rate. In particular, when a firm produces Q of its good with a characteristic s, the labor requirement for this firm is

$$L = (MQ + F)a(s). \tag{4}$$

In the above equation, M and F are the marginal and fixed flow cost of production, respectively, in terms of efficiency units of labor. The last term, $a(s)$, represents the number of workers required to generate one efficiency unit. We assume that this requirement depends on the good's characteristic and is negatively related to the knowledge accumulated with respect to it. Specifically, we pose a simple relationship between the unit requirement and the accumulated knowledge,

$$a(s) = 1 + \frac{1}{K(s)}, \tag{5}$$

where $K(s)$ is the accumulated knowledge with respect to a characteristic s. Note that, although efficiency units per person, $1/a(s)$, increase as the knowledge accumulates, there is an upper bound of unity. In other words, as long as the economy operates on the same set of characteristics, the learning-by-doing effect is bounded.

The formula (4) also expresses the total cost of production, since the wage rate is normalized to unity. From the fact that the average cost decreases with the production amount, we can confirm that every firm produces a distinct good, at least in the variety dimension. Suppose that there are two firms producing exactly the same good and each of them runs with zero profit. Then, if one of these firms decides to produce another variant in the variety dimension while keeping its price and characteristic in the need dimension unaffected, this firm will attract twice the demand. In that case, this firm must attain positive profit, since its average cost has fallen because of the increase in demand. Thus, such a situation contradicts the assumption of free entry. Also, in a similar way, we can show that no pair of goods are the same in the variety dimension even though they have distinct characteristics in the need dimension.

Finally, we must specify the process of accumulating knowledge by learning-by-doing. We assume that knowledge of a particular characteristic is proportional to the discounted sum of the past production experience of goods that have characteristics 'near' to the knowledge concerned. Specifically, we formulate the accumulation process as

$$\dot{K}(s) = \gamma \int_{-\infty}^{\infty} e^{-\nu|s-s'|} Q(s')\, dn(s') - \delta K(s), \tag{6}$$

where $Q(s)$ denotes the amount that each firm with characteristic s produces. There are three positive parameters in the above formula: γ captures the overall effectiveness of learning-by-doing; δ represents the depreciation rate of knowledge; and ν is the coefficient of the spillover effect of learning-by-doing across characteristics. Note that ν represents how rapidly the spillover effect diminishes as the characteristic distance increases.[12] We assume in the following that the spillover is not too strong: specifically, $\nu > ((\sigma - 1)/\sigma)\tau$. This assumption is fully satisfied when applying some knowledge to another characteristic is no easier than using some good for another need.

This diminishing spillover effect gives firms an incentive to agglomerate with each other over time. When there is a characteristic chosen historically by many firms, the accumulated knowledge with respect to this characteristic must be greater than neighboring ones. Since the latter implies low production costs for this characteristic, current firms have an incentive to choose it again. Together with the static centrifugal force arising from consumer preference, this temporal centripetal force plays a central role in the economic dynamics.

3 Instantaneous Equilibrium

Before turning to dynamic analyses, this section clarifies static properties of the market equilibrium, taking as given the values of the state variables $K(\cdot)$ and N. The first subsection characterizes the instantaneous equilibrium with a set of conditions, or, alternatively, with two curves representing production costs and demand prices, respectively. After examining global and local properties in the second subsection, the third subsection establishes the optimality, uniqueness and existence of the instantaneous equilibrium, which are needed to justify numerical procedures.

3.1 Equilibrium Conditions

Let us start by deriving the demand function of consumers for each good with various characteristics. To solve a constrained maximization problem of (1),

[12] In particular, $\nu \to \infty$ corresponds to the case where there is no spillover.

$(2')$, with (3), we use a two-step method. In the first step, we maximize the subutility of each need under a given expenditure density and derive the indirect subutility function. Then, in the second step, we maximize the overall instantaneous utility function with respect to the expenditure density under the instantaneous budget constraint.

The first problem is to maximize the subutility of each need $v(r)$, defined by $(2')$, under the constraint

$$\int_{-\infty}^{\infty} p(s)c(r,s)\,dn(s) = y(r), \tag{7}$$

with respect to $c(r,\cdot)$. Here $y(\cdot)$ is the density function of expenditure, which at this point we take as given. For each $r \in \mathcal{R}$, we can calculate the solution to this problem and the maximized value, respectively, as[13]

$$c^*(r,s;y(r)) = y(r)P(r)^{\sigma-1}p(s)^{-\sigma}e^{-(\sigma-1)|r-s|}, \tag{8}$$

$$v^*(r;y(r)) = y(r)/P(r), \tag{9}$$

where $P(r)$ is the 'price index' of the need r:

$$P(r) = \left[\int_{-\infty}^{\infty} \left(p(s)e^{\tau|r-s|}\right)^{-(\sigma-1)} dn(s)\right]^{-\frac{1}{\sigma-1}}. \tag{10}$$

Next, consider the second problem. Substituting the indirect subutility function (9) for (1), we can express the instantaneous utility in terms of the expenditure density, $y(\cdot)$. The problem here is to maximize this utility function subject to the instantaneous budget constraint $\int_{-\infty}^{\infty} y(r)\,dr = 1$. We can calculate the optimal expenditure density and the maximized instantaneous utility, respectively, as

$$y^*(r) = \bar{P}^{\beta-1}P(r)^{-(\beta-1)} \tag{11}$$

$$U^* = 1/\bar{P}, \tag{12}$$

[13]On the subset of the characteristic space where no firm operates, let us define $p(s) = \infty$ so that the optimal value of $c(r,s)$ becomes zero. In fact, this treatment might be redundant, since on such a subset the value of $c(r,s)$ is relevant to neither the subutility $(2')$ nor the budget constraint (3).

where $\bar{P}$ is the 'average price index' over all needs:[14]

$$\bar{P} = \left[\int_{-\infty}^{\infty} P(r)^{-(\beta-1)} dr \right]^{-\frac{1}{\beta-1}}. \tag{13}$$

Collecting these two steps, we can derive each consumer's actual demand. Substituting (11) into (8), the optimal consumption rule turns out to be

$$c(r,s) = \bar{P}^{\beta-1} P(r)^{\sigma-\beta} p(s)^{-\sigma} e^{-(\sigma-1)\tau|r-s|}. \tag{14}$$

The amount of each good purchased can also be calculated by integrating its use over all needs:

$$q(s) = \bar{P}^{\beta-1} p(s)^{-\sigma} \int_{-\infty}^{\infty} P(r)^{\sigma-\beta} e^{-(\sigma-1)\tau|r-s|} dr. \tag{15}$$

With this result in hand, we now turn to investigate firms' profit maximization behavior. Since there is a continuum of firms, any single firm's behavior will not change price indexes $P(r)$ or $\bar{P}$ and thus they take these indexes as given. Here, we again use a two-step method to maximize a firm's profit. Specifically, in the first step, we choose a price-quantity pair subject to the consumer's demand function to maximize the firm's profit, taking its product's characteristic as given. Then, in the second step, comparing the result of the previous step, we find out which characteristic yields the greatest profit.

Consider a firm that has determined its product's characteristic to be s. Even though there already exist some other firms with the same characteristic, this firm has the power to determine its own product's price, since every firm selects a distinct good in the variety dimension of the product space. However, there is a tradeoff between sales quantity, Q, and its price, p. Multiplying the individual demand function (15) by the number of consumers, this tradeoff is expressed as[15]

$$Q = N\bar{P}^{\beta-1} p^{-\sigma} \int_{-\infty}^{\infty} P(r)^{\sigma-\beta} e^{-(\sigma-1)\tau|r-s|} dr. \tag{16}$$

[14] Alternatively, we can interpret $\bar{P}$ as the 'expenditure function' required to attain a unit of instantaneous utility.

[15] Note that the equation (15) has been derived from the reduced form of the subutility function (2′), where every firm with the same characteristic chooses the same price. Here,

Taking its characteristic and the prices of other firms as given, this is a simple function of its own price, with a constant elasticity σ. Thus, under the production technology (4), which exhibits constant marginal cost, it is straightforward to calculate the optimum pricing rule and the maximized profit:

$$\begin{aligned} p^*(s) &= \frac{\sigma}{\sigma-1} Ma(s), \qquad (17) \\ \pi^*(s) &= \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma (Ma(s))^{\sigma-1}} N\bar{P}^{\beta-1} \int_{-\infty}^{\infty} P(r)^{\sigma-\beta} e^{-(\sigma-1)\tau|r-s|} dr - Fa(s). \end{aligned}$$

The next step is to find out which characteristic yields the greatest profit. By the assumption of free entry, however, there must be no opportunity in equilibrium to attain positive profit. This implies that the whole characteristic space, $\mathcal{R}$, can be divided into two subsets: in one subset every operating firm earns zero profit, whereas in the other there is no operating firm since potential profits are negative. Put formally, the free entry equilibrium requires $\pi^*(s) \leq 0$ for all $s \in \mathcal{R}$ with equality on the support of $n(\cdot)$. Using the results of the previous step, the condition $\pi^*(s) \leq 0$ reduces to

$$\left[\frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma M^{\sigma-1} F} N\bar{P}^{\beta-1} \int_{-\infty}^{\infty} P(r)^{\sigma-\beta} e^{-(\sigma-1)\tau|r-s|} dr \right]^{\frac{1}{\sigma}} \leq a(s). \qquad (18)$$

Although the above is a key expression in our model, it may seem somewhat messy. However, if we choose units of measurement appropriately, it simplifies to some extent. First, since our instantaneous utility function exhibits homogeneity of degree in the consumption density, we can freely choose measurement units for output quantity. Let us choose units in such a way that the marginal requirement of efficiency units of labor, M, coincides with $(\sigma-1)/\sigma$. Second, our instantaneous utility is also a homogeneous function in the distribution of firms. Thus, without losing generality, we can freely choose the unit for the measure of firms. In particular, we choose the unit

however, we want to let each firm choose its price independently of the other firms with the same characteristic. Thus, to be precise, the demand function (16) must be derived from the original definition of the subutility function (2) instead of the reduced form (2′).

so that the flow fixed requirement satisfies $F = 1/\sigma$.[16]

These normalizations simplify the optimal pricing rule (17) and the zero profit condition (18) into $p^*(s) = a(s)$ and $\hat{p}(s; n(\cdot), p(\cdot), N) \leq a(s)$, respectively. In the latter expression,

$$\hat{p}(s; n(\cdot), p(\cdot), N) \equiv \left[N\bar{P}^{\beta-1} \int_{-\infty}^{\infty} P(r)^{\sigma-\beta} e^{-(\sigma-1)\tau|r-s|} dr \right]^{\frac{1}{\sigma}} \tag{19}$$

represents the price at which a marginal firm with the characteristic s can sell a unit quantity, when the distribution of the existing firms, their prices and the population are given by $n(\cdot)$, $p(\cdot)$ and N, respectively. In the following, we refer to this value as the 'unit demand price'. Note that the unit demand price is expressed explicitly as a function of $n(\cdot)$ and $p(\cdot)$ in addition to N, because $\bar{P}$ and $P(\cdot)$ in the RHS of (19) depends on $n(\cdot)$ and $p(\cdot)$.

Substituting the optimal pricing rule and the zero profit condition into the demand function (16), we can see that each operating firm's optimal output quantity is always unity. In light of this result, the free entry condition, $\hat{p}(s; \cdot) \leq a(s)$ with equality on the support of $n(\cdot)$, has a natural interpretation. In equilibrium, every firm sells optimally a unit quantity at the price of $a(s)$. This assures zero profit, since $a(s)$ is exactly the cost of producing a unit quantity of a good with the characteristic s.[17] However, if the unit demand price is below the unit cost $a(s)$, it will be impossible to operate without deficits. Thus, there is no operating firm with characteristics such that $\hat{p}(s; \cdot) < a(s)$ holds. Conversely, suppose that there is some characteristic where the unit demand price exceeds the unit cost $a(s)$. Then, there must be an opportunity to acquire positive profit, contradicting the free entry condition. Thus, $\hat{p}(s; \cdot) \leq a(s)$ must hold for all s.

[16] If we double the whole distribution of firms and simultaneously halve the output of each firm and the fixed requirement, the instantaneous utility will be multiplied by $2^{1/(\sigma-1)}$, whereas all the remaining equations hold without any substantial change. For this kind of normalization, see Fujita, Krugman and Venables (1999, Chapter 4).

[17] Remember that the total labor requirement for producing a unit quantity of a good with a characteristic s is $(M + F)\, a(s)$, which is equal to $a(s)$ because of the normalizations of M and F.

Now we are in a position to summarize the instantaneous equilibrium of this economy, in terms of the distribution of firms $n(\cdot)$, their prices $p(\cdot)$ and the consumption density $c(\cdot,\cdot)$. Note that the distribution of knowledge, $K(\cdot)$, has been determined by the past history of the economy, and must be taken as given at each instant. Formally speaking, the instantaneous equilibrium $\{n(\cdot), p(\cdot), c(\cdot,\cdot)\}$ is a function of $K(\cdot)$ and N.[18] The conditions to be satisfied in equilibrium are the following:

(E1) *Free entry:* $\hat{p}(s; n(\cdot), p(\cdot), N) \leq a(s)$ for all $s \in \mathcal{R}$ with equality on the support of $n(\cdot)$.

(E2) *Monopoly pricing:* $p(s) = a(s)$ on the support of $n(\cdot)$, and $p(s) = \infty$ otherwise.

(E3) *Consumption rule:* for all $(r, s) \in \mathcal{R}^2$, $c(r, s)$ is determined by (14).

(E4) *The labor market clears:*

$$\int_{-\infty}^{\infty} \left[M \int_{-\infty}^{\infty} Nc(r, s)\, dr + F \right] a(s)\, dn(s) = N. \tag{20}$$

There are two points to note. First, when the conditions (E1)-(E3) are satisfied, the markets for every differentiated good clear. In this case, the labor market also automatically clears by virtue of Walras' law. Thus, in fact, only the three conditions (E1)-(E3) must be confirmed.

Second, the conditions (E1)-(E2) might at first blush suggest that $n(\cdot)$ and $p(\cdot)$ must be simultaneously determined, since the unit demand price appearing in (E1) depends both on $n(\cdot)$ and $p(\cdot)$. This is, however, not the case. In fact, the unit demand price does not depend on the whole profile of prices $p(\cdot)$, but only on the prices of operating firms, that is, the values of $p(\cdot)$ on the support of $n(\cdot)$, which are equal to the known values of $a(\cdot)$. Thus, we are able to replace (E1) with a simpler version:

(E1′) *Free entry:* $\hat{p}(s; n(\cdot), a(\cdot), N) \leq a(s)$ for all $s \in \mathcal{R}$, with equality on the support of $n(\cdot)$.

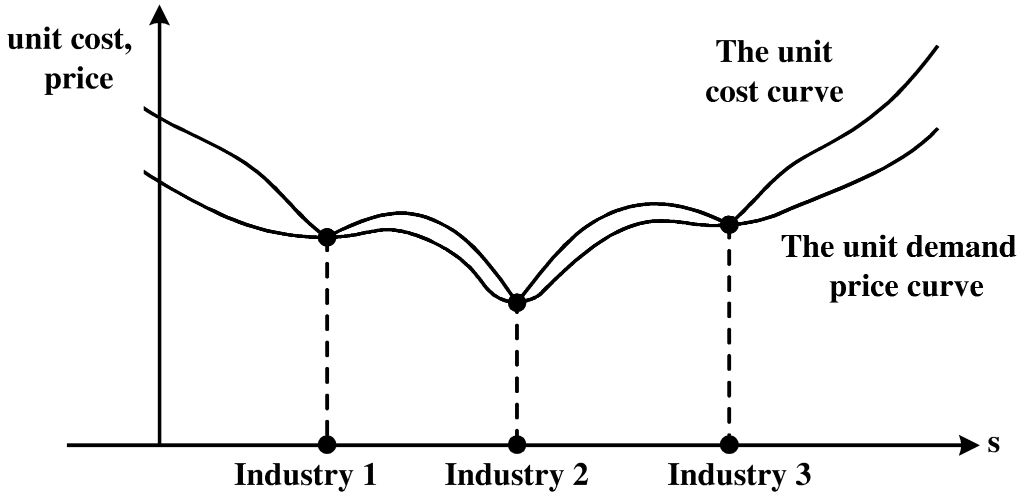

Figure 1: An example of the instantaneous equilibrium

Condition (E1$'$) says that the equilibrium distribution of firms, $n(\cdot)$, is determined in such a way that the resulting curve of $\hat{p}(s; n(\cdot), a(\cdot), N)$ does not go above the curve of $a(s)$ and necessarily touches it on the support of the distribution. Figure 1 gives an example of these two curves that together constitute an instantaneous equilibrium. Once we find the equilibrium distribution of firms, the remaining part of the equilibrium, $p(\cdot)$ and $c(\cdot, \cdot)$, can be calculated straightforwardly, using the conditions (E2)-(E3).

3.2 Global and Local Characterizations

Unfortunately, the key condition (E1$'$) cannot be solved explicitly, except for some special cases. Nonetheless, analytical properties of the two curves, the unit demand price curve and the unit cost curve, enable us to characterize the instantaneous equilibrium both from global and local aspects. Specifically, we investigate in this subsection two important properties of the equilibrium distribution of firms: the boundedness of the distribution's support in a static sense and the possibility of mass points that tend to persist for some time period. More fundamental properties, such as uniqueness and existence, will

[18]Note that $a(\cdot)$ is also exogenous, since it has a one-to-one relationship to $K(\cdot)$.

be treated in the following subsection.

First, let us focus on the global side. We have defined the characteristic space of goods to be an unbounded real axis, and thus, in principle, firms can choose characteristics infinitely distant from others to avoid competition. This consideration suggests that the equilibrium distribution of firms might 'explode' in the characteristic space. In such an event, however, the unit production cost of goods would also rise unboundedly, since the economy has accumulated relatively little knowledge at distant characteristics. In fact, we can show that such an explosion never occurs, as long as the economy has never experienced such a phenomenon before in its whole history. Formally, we will prove the following: if the support of $n_{t'}(\cdot)$ is contained in some finite interval $\mathcal{I} \equiv [\underline{I}, \bar{I}] \subset \mathcal{R}$ for all $t' < t$, then, another finite interval $\mathcal{I}' \equiv [\underline{I}', \bar{I}']$ exists such that the support of $n_t(\cdot)$ is contained in $\mathcal{I}'$.

To prove this, let us examine the slope of the unit demand price function. It turns out that this function is smooth and we can calculate its derivative with respect to s:

$$\frac{\partial \hat{p}(s;\cdot)}{\partial s} = \frac{\sigma-1}{\sigma} \tau \hat{p}(s;\cdot)^{1-\sigma} \int_{-\infty}^{\infty} \operatorname{sgn}(s-r) P(r)^{\sigma-\beta} e^{-(\sigma-1)\tau|r-s|} dr,$$

where we have omitted arguments other than s in $\hat{p}(s; n(\cdot), a(\cdot), N)$. By straightforward comparison of (19) and the above, we can show that there is an upper bound on the slope of the unit demand price function that depends only on the parameters:

$$\left| \frac{1}{\hat{p}(s;\cdot)} \frac{\partial \hat{p}(s;\cdot)}{\partial s} \right| < \frac{\sigma-1}{\sigma} \tau. \tag{21}$$

This result can easily be interpreted: to satisfy a certain need, consumers can substitute for some good another 'poorly matched', but inexpensive one, with an extra 'unfitness' cost that increases exponentially with the distance in the characteristic space; thus, the unit demand price curve cannot have slopes larger than the value that corresponds to this cost.

We utilize this fact to prove the claim. Suppose that the right-hand end of the 'historical frontier' has not gone beyond the characteristic $\bar{I}$ by time t.[19]

[19]That is, there is no support of $n_{t'}(\cdot)$ in $(\bar{I}, \infty)$ for all $t' < t$.

Then, we see from (6) that for characteristics currently 'beyond the frontier', $s \in (\bar{I}, \infty)$, the accumulated knowledge monotonically decreases with s at the rate of ν. This can also be expressed in terms of production cost:

$$a_t(s) = 1 + \big(a_t(\bar{I}) - 1\big)\, e^{\nu(s-\bar{I})} \qquad \text{for all } s \geq \bar{I}. \tag{22}$$

On the other hand, the condition (E1$'$) guarantees that a local inequality $\hat{p}(\bar{I}) \leq a_t(\bar{I})$ always holds in equilibrium. Combining this inequality with the upper bound for the slope (21), we have an upper bound for the unit demand price curve beyond the historical frontier:

$$\hat{p}(s;\cdot) \leq a_t(\bar{I}) e^{\frac{\sigma-1}{\sigma}\tau(s-\bar{I})} \qquad \text{for all } s \geq \bar{I}. \tag{23}$$

Let us compare these two expressions. When the accumulated knowledge at the frontier characteristic is no more than $\nu/(\nu - ((\sigma-1)/\sigma)\tau)$, we can see that the upper bound curve for the unit demand price, given by the RHS of (23), stays below the unit cost curve (22) for all $s \geq \bar{I}$. In this case, the historical frontier does not expand since it cannot be profitable to produce goods with characteristics beyound the current frontier. Even if this is not the case, the unit cost must eventually exceed the unit demand price for a sufficiently large s, from the assumption $\nu > ((\sigma-1)/\sigma)\tau$. In fact, given values for the parameters and $a(\bar{I})$, we can always find a finite value $\bar{I}'$ such that the unit cost curve (22) strictly dominates the RHS of (23) for all $s > \bar{I}'$, which in turn implies that there must be no support of $n_t(\cdot)$ beyond $\bar{I}'$. Obviously, the same argument can be made for $\underline{I}$ and $\underline{I}'$. Thus, we have established that the instantaneous equilibrium distribution is bounded, provided that the whole history has been bounded.[20]

Next, we turn to the local property. Note that our model deals directly with the distribution function of firms $n(\cdot)$, rather than the more analytically convenient density functions. In fact, this somewhat awkward treatment is indispensable, since mass points are likely to emerge in the equilibrium distribution of firms. Moreover, once such points have emerged, they usually persist for some time period.

[20] Note that this is a static property. In a dynamic sense, as we will see in the next section, the historical frontier expands without bounds.

To illustrate these points, it is useful to clarify the curvature of the unit demand price function. For ease of description, let us differentiate $\hat{p}(s;\cdot)^\sigma$ twice, rather than the function itself. This yields

$$\frac{\partial^2 \hat{p}(s;\cdot)^\sigma}{\partial s^2} = -2(\sigma-1)\tau N\bar{P}^{\beta-1}P(s)^{\sigma-\beta} + (\sigma-1)^2\tau^2\hat{p}(s;\cdot)^\sigma. \tag{24}$$

The first term in the RHS shows that the curvature at a certain characteristic is larger when the price index at this characteristic, $P(s)$, is lower. Under the equilibrium condition (E2) and the definition of the price index (10), the term simply states that the curvature becomes larger when there is a greater number of firms operating near this characteristic. Even though a mass of firms may have agglomerated, however, there is also an upper bound on this curvature: because the first term cannot become positive, the curvature cannot exceed the value of the second term in the RHS, which is no greater than $(\sigma-1)^2\tau^2 a(s)^\sigma$ under the equilibrium condition (E1′).

With this fact in hand, we can now explain why a locally isolated mass point may emerge in the equilibrium distribution of firms. Assume that there is an interval $\mathcal{I}$ in the characteristics space, on which firms are distributed continuously. Then, the free entry condition assures that the equality $a(s) = \hat{p}(s;\cdot)$ holds for all $s \in \mathcal{I}$. As knowledge accumulates, however, the shape of the unit cost curve continually changes, and it would not be unusual for the curvature of $a(s)^\sigma$ to eventually exceed $(\sigma-1)^2\tau^2 a(s)^\sigma$ in some subinterval $\mathcal{I}' \subset \mathcal{I}$. Suppose that this occurs and that some firms still remain in this subinterval. Then, the unit demand price curve necessarily touches the unit cost curve at only one point in the interval $\mathcal{I}'$, since the former has a larger curvature than the latter. Moreover, this locally isolated point in the characteristic space generically contains firms with positive measure, since this is the only characteristic in the interval $\mathcal{I}'$ with which firms can operate without deficits.

How, then, can such a mass point persist thereafter? Assume that there already exists a mass point of firms that has persisted for some time period. By the process of knowledge accumulation (6), these firms must have created an upward kink in $K(\cdot)$, which also implies a downward kink in $a(\cdot)$. Since the unit demand curve is everywhere smooth, it tends to touch the curve of $a(s)$ only on the point of kink, at least locally, rather than continuously over an interval that contains the kink. Thus, such a locally isolated mass point tends to persist. Moreover, further accumulation of knowledge by these firms further strengthens the kink in $K(\cdot)$, and, again, makes the mass point sustainable in the future.

In Section 4, we will confirm these global and local properties in a dynamic context by numerical procedures. Before turning to such analyses, however, we must establish some more fundamental properties of the instantaneous equilibrium in the following subsection.

3.3 Optimality, Uniqueness and Existence

Although previous subsections have characterized the instantaneous equilibrium in several ways, it remains to be shown that there actually exists only one equilibrium for each pair of knowledge, $K(\cdot)$, and population, N. This is particularly important in the numerical analyses to follow: even if numerical procedures find a solution, it cannot be assured to occur in reality unless it is the unique solution. We tackle this issue by focusing on another fundamental concern, the welfare properties of the instantaneous equilibrium.

Since the processes of accumulation of knowledge are perfectly external for each firm, it is unlikely that the equilibrium path induces an intertemporally optimal allocation. We can show, however, that the instantaneous equilibrium is 'instantaneously optimal': given the current distribution of knowledge, it attains the maximal instantaneous utility.[21] After proving the exact coincidence of the instantaneous equilibrium with the instantaneously optimal allocation, we will proceed to show that such an allocation uniquely

[21] As is well known, Dixit and Stiglitz (1977) have shown that, in a static model with only one dimension of differentiation, the monopolistically competitive equilibrium is actually the first best solution, provided that the utility function of the representative consumer has a CES form. The instantaneous equilibrium in our model is a generalization of their model, in that if all firms are forced to choose the same characteristic, or differentiation in the need dimension is prohibited, the resulting equilibrium reduces to virtually the same as the Dixit-Stiglitz model. Thus, the first best will result in such a special setting. In the text, we show that this property indeed extends to our generalized economy.

exists. More precisely, this subsection will proceed in the following way. (i) Formulate the problem of maximizing the instantaneous utility subject to the current state of knowledge, $K(\cdot)$, and limited workforce, N. (ii) Prove that the solution to the maximization problem is unique. (iii) Show that any instantaneous equilibrium is a solution to this maximization problem, and vice versa. (iv) Show that such an equilibrium-maximizer always exists. These procedures establish all of the three claims: instantaneous optimality, uniqueness, and existence of the instantaneous equilibrium.

The problem to be formulated first is to maximize the instantaneous utility (1) and (2′) with respect to the distribution of firms $n(\cdot)$ and the consumption density $c(\cdot,\cdot)$, subject to the resource constraint (20). This is a constrained maximization problem, but not a standard one as it includes maximization with respect to a distribution function. Moreover, because of the possibility of mass points in the distribution, we cannot reformulate this problem using a density function of firms. These facts imply that, although we want to show equivalence of the solution to the equilibrium, its direct proof would be exceedingly involved. Rather than attempting this difficult task, let us introduce a supposedly innocuous restriction on firms' behavior. Specifically, we discretize the characteristic space, $\mathcal{R}$, into arbitrarily fine grids and restrict the behavior of firms so that they can select their characteristics only from points on these grids.[22]

Let $\mathcal{S} = \{s_1, s_2, \cdots\}$ be a countable set of distinct points in the real axis, $\mathcal{R}$, from which firms can choose their characteristics. Assume that a small number $\epsilon > 0$ exists such that $|s_i - s_j| \geq \epsilon$ holds for all $i \neq j$, and thus $\mathcal{S}$

[22]Besides the ease of analysis, there are three reason to adopt this method. First, as long as the points in the grids are countable, we can take arbitrarily fine grids. When the grids are made sufficiently fine, we can legitimately expect the model not to exhibit significant differences from the continuum case. Second, because of the temporal centripetal force described earlier, a great majority of firms tend to agglomerate in several points in the characteristic space, rather than to be distributed continuously. Finally, one of the reasons why we want to establish uniqueness is to validate the following numerical analyses, where discretization is inevitable as long as digital computers are used. Alternatively, it is also possible to interpret our original model itself as a limiting case of the discrete setting presented in the text.

has no cluster point. Since firms are distributed exclusively on the points in $\mathcal{S}$, their distribution can be expressed in terms of a sequence of nonnegative numbers $\{n_1, n_2, \cdots\}$, where n_j denotes the measure of firms with characteristic s_j. With these notations, it would be possible to construct a set of standard first order conditions for the instantaneously optimal allocation.

However, there is another subtle issue to be clarified: the first order conditions may not be sufficient, since the objective function (1) with (2′) is not concave, nor is the resource constraint (20) linear. To resolve this issue, let us transform two sets of variables. First, instead of maximizing the instantaneous utility U directly, we maximize its increasing transformation $\tilde{U} \equiv U^{(\beta-1)/\beta}$. Second, in place of $c(\cdot, s_j)$, which indicates the consumption density of each good with characteristic s_j, we introduce $h_j(\cdot) \equiv n_j c(\cdot, s_j)$, which represents the total consumption density of all goods with characteristic s_j.[23] Using these variables, the problem is transformed into

$$\underset{\{n_j, h_j(\cdot)\}_{j=1}^{\infty}}{\text{maximize}} \ \tilde{U} = \int_{-\infty}^{\infty} v(r)^{\frac{\beta-1}{\beta}} dr, \tag{1'}$$

where

$$v(r) = \left[\sum_{j=1}^{\infty} n_j^{\frac{1}{\sigma}} \left(h_j(r) e^{-\tau|r-s_j|} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{2''}$$

subject to the resource constraint

$$\sum_{j=1}^{\infty} \left(MN \int_{-\infty}^{\infty} h_j(r) dr + F n_j \right) a(s_j) = N. \tag{20'}$$

[23]Note that, for any characteristic s outside the grids, trivially $c(\cdot, s) = 0$ holds since we have assumed that there is no firm with such a characteristic. Therefore, only the values of $c(\cdot, s_j)$, $j = 1, 2, \cdots$, need to be specified. Strictly speaking, however, the relationship between $c(\cdot, s_j)$ and $h_j(\cdot)$ is not always one to one, since, when $n_j = 0$ and $h_j(r) > 0$ for some j and r, there is no counterpart of $c(r, s_j)$ to such $h_j(r)$. Thus, we unexpectedly have a broader freedom in this transformed problem. However, this extended freedom will never be exploited, since, when $n_j = 0$, the marginal benefit of increasing $h_j(r)$ is zero, whereas its cost is positive.

Clearly, the constraint (20′) now becomes a linear one, and in Appendix A we prove that $\tilde{U}$ is indeed a concave function with respect to the variables concerned.

To be precise, there are two other sets of inequality constraints; that is, nonnegativity of $\{n_j\}$ and $\{h_j(\cdot)\}$. As will soon become clear, the latter never binds, whereas the former sometimes does. Thus, we must solve an equality and an inequality constrained problem. After considerable calculations, the Kuhn-Tucker condition for the maximizer of this problem turns out to be the existence of some number ξ, the Lagrange multiplier for the resource constraint, such that the following conditions (K1)-(K3) are satisfied.[24]

(K1) *First order condition for the distribution of firms:* for each $j = 1, 2, \cdots$, two inequalities, $n_j \geq 0$ and

$$\frac{\sigma}{\sigma-1}\frac{\beta-1}{\beta}\xi^{-1}\left[N^{1-\sigma}\int_{-\infty}^{\infty} v(r)^{-\frac{\sigma-\beta}{\beta}} e^{-\tau(\sigma-1)|r-s_j|}dr\right]^{\frac{1}{\sigma}} \leq a(s_j),$$

hold. In addition, the latter holds with equality if $n_j > 0$.

(K2) *First order condition for the consumption density:* for each $j = 1, 2, \cdots$ and each $r \in \mathcal{R}$,

$$h_j(r) = \left(\frac{\sigma}{\sigma-1}\frac{\beta-1}{\beta}\frac{1}{\xi N a(s_j)}\right)^{\sigma} n_j v(r)^{-\frac{\sigma-\beta}{\beta}} e^{-\tau(\sigma-1)|r-s_j|}.$$

(K3) *Resource constraint:* equation (20′) holds.

Since all constraints are linear and the objective $\tilde{U}$ is concave, these conditions are both necessary and sufficient, and the set of optima is convex. Moreover, Appendix B shows that the optimum is in fact unique. Combining these facts together, it is assured that the set of variables that satisfies the conditions (K1)-(K3) is unique.

Once the set of conditions for the optimal allocation is found, we are in the position to show its equivalence to the equilibrium conditions (E1)-(E4).

[24] For details about constrained maximization and the Kuhn-Tuchker condition, see, for example, Mas-Colell, Whinston and Green (1995, Section M.K).

Before proceeding, however, remember that we have modified the economy slightly so that firms are allowed to choose their characteristics only from the points in $\mathcal{S}$. Thus, to be consistent, we must also modify the free entry condition (E1) into

(E1$''$) *Free entry:* for each $j = 1, 2, \cdots$, two inequalities, $n_j \geq 0$ and $\hat{p}(s_j; \{n_j\}, p(\cdot), N) \leq a(s_j)$, hold. In addition, the latter holds with equality if $n_j > 0$.

This is simply a weaker version of the original condition, in that, while (E1) demands that the free entry condition holds over the whole characteristic space $\mathcal{R}$, this version requires the condition to hold only over the subset $\mathcal{S} \subset \mathcal{R}$, since in this economy firms cannot choose any point outside $\mathcal{S}$ from the beginning.

Let us start the proof. In one direction, we show that any instantaneous equilibrium is instantaneously optimal. Suppose that we have an instantaneous equilibrium, $\{\{n_j\}, p(\cdot), c(\cdot, \cdot)\}$, which satisfies the conditions (E1$''$) and (E2)-(E4). Then, our task is to show that there exists an appropriate Lagrange multiplier ξ, such that the conditions (K1)-(K3) are satisfied.

First, it is clear that the resource constraint (K3) is equivalent to the labor market clearing condition (E4). Thus, (K3) is trivially satisfied.

Next, using (9) and (12), we can eliminate price indexes from the equilibrium consumption density (E2). Then, by applying the definition of $h_j(r)$, this condition becomes

$$h_j(r) = \left(a(s_j) U^{\frac{\beta-1}{\beta}}\right)^{-\sigma} n_j v(r)^{-\frac{\sigma-\beta}{\beta}} e^{-\tau(\sigma-1)|r-s_j|},$$

which indeed coincides with the condition (K2), provided that we choose the multiplier to be

$$\xi = \frac{\sigma}{\sigma-1} \frac{\beta-1}{\beta} N^{-1} U^{\frac{\beta-1}{\beta}}. \tag{25}$$

This coincidence means that the equilibrium consumer behavior is in fact optimal given the distribution of firms, which is a natural outcome since all consumers have the same preference and there is no contemporaneous externality.

Finally, eliminating price indexes from the free entry condition (E1″), and substituting the multiplier (25) into it shows that its equivalence to (K1) is straightforward. This fact shows that the unit demand price function represents the marginal benefit of increasing the number of firms with each characteristic. Collecting these three results, we have proven that the instantaneous equilibrium is instantaneously optimal.

In the other direction, we show that the instantaneously optimal allocation constitutes an instantaneous equilibrium. Suppose that there is an optimal allocation $\{\{n_j\}, c(\cdot,\cdot)\}$ and a Lagrange multiplier ξ, which together satisfy the Kuhn-Tucker condition (K1)-(K3). We now prove that there exists a set of prices, $p(\cdot)$, such that the equilibrium conditions (E1″) and (E2)-(E4) are satisfied.

Let us choose a set of prices according to the condition (E2). Then, by substituting the condition (K2) for (2″), we can solve for $v(r)$ in terms of ξ, N and $p(\cdot)$. Once again substituting this result into the conditions (K1) and (K2), we have the following two results:

$$\left[\tilde{\xi}^{-\beta} N^{1-\beta} \int_{-\infty}^{\infty} P(r)^{\sigma-\beta} e^{-\tau(\sigma-1)|r-s_j|} dr\right]^{\frac{1}{\sigma}} \leq a(s_j) \tag{26}$$

with equality if $n_j > 0$ and

$$c(r, s_j) = \tilde{\xi}^{-\beta} N^{-\beta} P(r)^{\sigma-\beta} p(s_j)^{-\sigma} e^{-(\sigma-1)\tau|r-s|}. \tag{27}$$

To minimize notation, here we have introduced a transformed Lagrange multiplier $\tilde{\xi} \equiv (\sigma/(\sigma-1))((\beta-1)/\beta)\xi$. Note that the above expressions are equivalent to the equilibrium conditions (E1″) and (E3), respectively, if and only if $\tilde{\xi} = \bar{P}^{-(\beta-1)/\beta}/N$ holds. We next show that this is actually the case under the resource constraint (K3).

The total labor requirement in the economy, represented by the LHS of (20′), consists of two parts, the sum of the marginal requirements and the sum of the fixed ones. Let us calculate the amount of each part, given the (transformed) Lagrange multiplier $\tilde{\xi}$. Utilizing the equation (27), we can calculate the first part as

$$M \sum_{j=1}^{\infty} \left(N \int_{-\infty}^{\infty} n_j c(r, s_j) dr \right) a(s_j) = M \tilde{\xi}^{-\beta} \left(N \bar{P} \right)^{1-\beta}. \tag{28}$$

By integrating the equation (27) with respect to r and applying the condition (26), we have the familiar result that each firm produces a unit quantity. Again, combining this property and (28), the second part becomes

$$F \sum_{j=1}^{\infty} n_j a(s_j) = F \tilde{\xi}^{-\beta} \left(N \bar{P} \right)^{1-\beta}. \tag{29}$$

These two equations show that the total labor requirement is $\xi^{-\beta} \left(N \bar{P} \right)^{1-\beta}$.[25] From this expression, we can conclude that the transformed Lagrange multiplier must have the value $\tilde{\xi} = \bar{P}^{-(\beta-1)/\beta}/N$, at which the labor requirement coincides with the labor supply, N, and thus the resource constraint (K3) is satisfied. Therefore, the conditions (E1$''$) and (E3) hold in the optimal allocation.

Finally, the condition (E4) holds by virtue of (K3). This concludes the proof for this direction. Collecting the results for both directions, we have proven that the instantaneous equilibrium exactly coincides with the optimal allocation. Moreover, this fact implies that the instantaneous equilibrium is unique, since we have already shown that the optimal allocation is unique.

The remaining task is to prove the existence of such an equilibrium allocation. In the previous subsection, we showed that the support of the equilibrium distribution is contained in a certain bounded interval, as long as the whole history has also been bounded. Then, there are only a finite number of points in $\mathcal{S}$ that are also contained in the interval. Without losing generality we can denote these points by $s_1, \cdots, s_J$, where J is a finite number. We assume that this is the case, and refer to the property that $n_j = 0$ holds for all $j > J$ as condition (B).

Note that, since we have established the coincidence of the optimal allocation with the instantaneous equilibrium, the latter also maximizes U subject

[25] Recall that we have normalized M and F into $(\sigma - 1)/\sigma$ and $1/\sigma$, respectively.

to the resource constraint (E4) and, of course, satisfies (E2), (E3) and (B). Keeping this fact in mind, consider a constrained maximization of U with respect to $\{n_j\}$, $c(\cdot)$ and $p(\cdot)$, subject to (E2), (E3), (E4) and (B). Then, if a solution to this problem exists, it actually constitutes the instantaneous equilibrium, since the maximization problem is not altered substantially either by the additional variable $p(\cdot)$ or by the additional constraints (E2), (E3), or (B).

Let us prove the existence of a solution to this transformed problem. From the constraints (E2) and (E3), the instantaneous utility U can be expressed as $1/\bar{P}$. In addition, we can confirm that the output of each firm is unity; thus, the constraint (E4) can be simplified to $\sum_j a(s_j)n_j = N$. These conditions also say that, $p(\cdot)$ and $c(\cdot,\cdot)$ will be passively determined when $\{n_j\}$ is specified. Thus, in effect, this is the problem of maximizing $1/\bar{P}$ with respect to $\{n_j\}$. By virtue of the constraint (B), moreover, we only have to deal with J nonnegative variables. Therefore, the problem simplifies to a constrained minimization of $\bar{P}$ with respect to $n_1, \cdots, n_J$, under the constraints $\sum_{j=1}^{J} a(s_j)n_j = N$ and $n_j \geq 0$ for $j = 1 \cdots J$. This is an intuitive characterization of the instantaneous equilibrium, in which the distribution of firms minimizes the average price index. It is straightforward to confirm that the objective $\bar{P}$ is a continuous function of $n_1, \cdots, n_J$, and that the admissible set for the control variables is compact. Thus, we are assured of a solution. This establishes the existence of the instantaneous equilibrium.

4 Dynamic Evolution

Having finished the necessary arguments, we are now in a position to explore dynamic evolution of the economy. Recall that our model includes two dynamic elements: the process of accumulating knowledge, specified by (6), and exogenous population growth at the rate of λ. As proven in the previous section, given the current set of knowledge $K_t(\cdot)$ and the current population N_t, a unique distribution of firms exists that satisfies the key condition (E1′), which we denote by $\hat{n}(\cdot; K_t(\cdot), N_t)$. Then, utilizing the fact that every firm produces unit quantity in equilibrium, we have an autonomous system with respect to $K(\cdot)$ and N:

$$\dot{K}_t(s) = \gamma \int_{-\infty}^{\infty} e^{-\nu|s-s'|} d\hat{n}(s'; K_t(\cdot), N_t) - \delta K_t(s), \tag{30}$$

$$\dot{N}_t = \lambda N_t. \tag{31}$$

While the above equations fully characterize the evolution of the equilibrium distribution of firms, there is another profound problem: how does the first industry emerge? When there has never been any firm, the accumulation process (6) implies that there is no knowledge at all, and (5) states that any production is prohibited. Thus, it is clear that our model is not at all suited to answer this kind of problem. Although it is not difficult to add some elements to our model to answer this issue, we would like simply to pose a plausible initial condition for the dynamics, since the aim of this paper is to describe how systems of industries interact to exhibit various dynamics, but not to deal with the ultimate origin of them.

As a starting point for the analysis, we adopt an economy with only one small industry, where every firm chooses the same characteristic.[26] Specifically, we assume that the distribution of knowledge at the initial date $t = 0$ is

$$K_0(s) = \bar{K}_0 e^{-\nu|s|}, \tag{32}$$

where $\bar{K}_0$ is a small positive value. Given (32) and an initial population $N_0 > 0$, we now construct the initial instantaneous equilibrium.

Substituting (32) for (5), we can derive the unit cost curve:

$$a_0(s) = 1 + \bar{K}_0^{-1} e^{\nu|s|}. \tag{33}$$

This curve is symmetric with respect to the characteristic 'zero' and has a downward kink there. If every firm chooses this characteristic, the labor

[26]Without losing generality, we can refer to this characteristic as $s = 0$, since the characteristic space, $\mathcal{R}$, is *ex ante* symmetric.

market clearing condition, (E4), requires the number of firms at the characteristic 'zero' to be $N_0(1+\bar{K}_0^{-1})$. Formally, when this is the case, the initial distribution of firms is expressed as

$$n_0(s) = \begin{cases} 0 & \text{for } s < 0, \\ N_0\left(1+\bar{K}_0^{-1}\right) & \text{for } s \geq 0. \end{cases} \tag{34}$$

Then applying (33) and (34) for the equilibrium conditions (E2) and (E3), we can determine the whole set of variables $\{n(\cdot), c(\cdot,\cdot), p(\cdot)\}$, which completely characterizes the instantaneous equilibrium.

However, we must still confirm whether this set of variables truly constitutes an instantaneous equilibrium, by checking the remaining free entry condition (E1′). From (33) and (34), we can calculate the price indexes $P_0(r)$ and $\bar{P}_0$. Substituting these indexes for (19), we have an explicit expression for the unit demand price function,

$$\hat{p}(s; n_0(\cdot), a_0(\cdot), N) = \left(1+\bar{K}_0^{-1}\right)\Psi(s), \tag{35}$$

where $\Psi(s)$ is a symmetric, smooth function that depends only on s and parameters:

$$\Psi(s) = \left[\frac{\int_{-\infty}^{\infty} \exp\left\{(1-\sigma)\tau|r-s| + (\sigma-\beta)\tau|r|\right\} \, dr}{\int_{-\infty}^{\infty} \exp\left\{(1-\beta)\tau|r|\right\} \, dr}\right]^{\frac{1}{\sigma}}. \tag{36}$$

Here, we have to compare the unit demand price curve (33) and the unit cost curve (35). For this purpose, it is useful to define another function that expresses the gap between these two curves at each characteristic, $\Omega(s; n(\cdot), a(\cdot), N) \equiv \hat{p}(s; n(\cdot), a(\cdot), N) - a(s)$. We refer to this function as the 'market potential function', since it represents the profitability of entering the market by producing a good with each characteristic. Using this function, we can restate the free entry condition (E1′) in the following way: the market potential curve should not exceed zero for all $s \in \mathcal{R}$, and must equal zero on the support of $n(\cdot)$.

From (33) and (35), we can derive the market potential curve for the one-industry economy:

$$\Omega(s; n_0(\cdot), a_0(\cdot), N_0) = \Big(\Psi(s) - 1\Big) - \bar{K}_0^{-1}\left(e^{\nu|s|} - \Psi(s)\right). \tag{37}$$

With this expression in hand, we can examine whether this economy satisfies the free entry condition. First, let us examine the profitability of the existing industry, $s = 0$. Since Appendix C shows $\Psi(0) = 1$, both expressions in parentheses in the RHS of (37) become zero, which implies exactly zero market potential for the industry.

On the other hand, what is the profitability of operating outside the existing industry? Appendix C shows $1 < \Psi(s) < e^{\nu|s|}$ for all $s \neq 0$, which means that the two expressions in parentheses in the RHS of (37) are strictly positive and negative, respectively. Thus, for sufficiently small $\bar{K}_0$ (that is, large $\bar{K}_0^{-1}$), we can see intuitively that the second term will dominate for all $s \neq 0$, and thus the condition (E1$'$) will be satisfied. Conversely, if $\bar{K}_0$ is not so small, there may exist some characteristic where the first term dominates, which implies violation of the condition (E1$'$). With such a distribution of the initial knowledge, the true instantaneous equilibrium differs, in fact, from the one-industry economy characterized by (34). In Appendix D, we formally prove that there is a finite positive constant $\bar{K}^*$ such that the condition (E1$'$) is satisfied if and only if $\bar{K}_0 \leq \bar{K}^*$.

In the following, we assume that the initial knowledge is sufficiently small that $0 < \bar{K}_0 < \bar{K}^*$ holds. Then, the initial instantaneous equilibrium involves only one industry and thereafter knowledge accumulates according to the equation (30). In general, this process involves a continuum of variables, the whole set of knowledge for each specific characteristic. As long as the one-industry structure persists, however, we can in fact concentrate on only one of them, $\bar{K}_t \equiv K_t(0)$, since an obvious relationship between them continues to hold:

$$K_t(s) = \bar{K}_t e^{-\nu|s|}. \tag{38}$$

Moreover, it turns out that $\bar{K}_t$ follows rather simple dynamics,

$$\dot{\bar{K}}_t = \frac{\gamma N_t}{1 + \bar{K}_t^{-1}} - \delta \bar{K}. \tag{39}$$

From (39) and (31), it is straightforward to draw a phase diagram in $(N, \bar{K})$ space, given by Figure 2, and together with initial values of $\bar{K}_0$ and

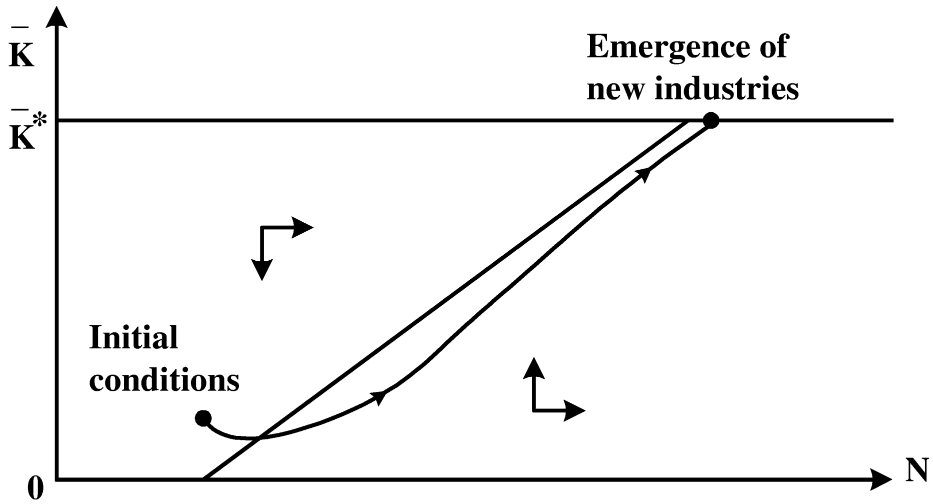

Figure 2: The phase diagram of the one-industry economy

N_0 we can see how this economy evolves. Note that, however, this reduced dynamic system applies only while the economy has the one-industry structure; but for how long does it persist? Since the distribution of knowledge (38) at any time has the same form as the initial distribution (32), we can examine the equilibrium conditions at each instant in exactly the same way as we examined the initial equilibrium; that is, the one-industry structure, similar to (34), constitutes the instantaneous equilibrium if and only if $\bar{K}_t$ does not exceed $\bar{K}^*$. The phase diagram shows that this structure is bound to become unsustainable in finite time, as long as the initial condition satisfies $N_0 > 0$ and $0 < \bar{K}_0 < \bar{K}^*$. We denote by t^* the critical time such that $\bar{K}_{t^*} = \bar{K}^*$ holds.

Before time t^*, the potential curve $\Omega(s; n_t(\cdot), a_t(\cdot), N_t)$ stays below the horizontal axis everywhere except the origin. This means that it is not profitable for any firm to deviate from the characteristic 'zero'. In other words, the centripetal force induced by the learning-by-doing effect globally dominates the centrifugal force induced by the desire of consumers to satisfy a wider range of needs. As knowledge accumulates, however, the centripetal force gradually gets weaker. Because of the spillover effects of learning-by-doing, the unit cost falls not only for the existing industry, but also outside it, and, moreover, the cost falls more rapidly at somewhat distant characteristics, where there still remains a relatively large gap between the current cost and the lower bound. Thus, the potential curve moves upward continually away from the origin and eventually, at time t^*, touches the horizontal axis at some point $s^* \in (0, \infty)$.[27] This means that it would then exactly pay to enter the market by producing a good with characteristic s^*, provided that every existing firm continues to choose the characteristic 'zero'.[28] Then, slightly after the time t^*, if no firm changes its characteristic from zero, the opportunity of operating around s^* provides strictly positive profit, contradicting the assumption of free entry. Thus, some firms must change their characteristic, which leads to the emergence of new industries.

Numerical Simulation

After the critical time t^*, the reduced dynamic system (39) and (31) no longer applies and we must deal directly with the general system (30) and (31). Since the equilibrium distribution of firms at each instant, $\hat{n}(\cdot; K_t(\cdot), N_t)$, could not be derived explicitly, it seems sensible at this point to turn to numerical simulation. To implement such a simulation, two kinds of approximation are necessary. First, since arbitrary distributions of firms on continuous space, $\mathcal{R}$, cannot be handled numerically, we discretize this space with a small fixed interval of 0.01. Second, although the evolution of this economy is characterized by a system of differential equations, its solution requires a finite difference method that in effect replaces a continuous time problem with a discrete time problem. Specifically, we divide a unit time,

[27] See Appendix D for the proof that such $s^* \in (0, \infty)$ exists. Note also that, since the potential curve is symmetric around zero, there is another point where the curve touches the horizontal axis, namely, $-s^*$.

[28] Strictly speaking, at the very critical time t^*, no firm will choose the characteristic s^*. At time t^*, the potential curve, derived from the one-industry distribution, does not exceed zero for all $s \in \mathcal{R}$, and thus the one-industry structure nonetheless remains as an equilibrium. Moreover, there is no other possibility, since the instantaneous equilibrium is unique.

which can be considered as a year, into 20 subunits and solve a system of difference equations.[29]

As a benchmark, we select parameter values for the preference of the representative consumer to be $\beta = 2$, $\sigma = 4$ and $\tau = 1$. For the process of knowledge accumulation, we specify $\gamma = 2.4$, $\delta = 0.24$ and $\nu = 1$. The remaining parameter, the population growth rate, is set to $\lambda = 0.024$. Finally, we must specify initial conditions for this economy, $\bar{K}_0$ and N_0. The initial knowledge for producing goods in the initial industry is set to $\bar{K}_0 = 0.1$ so that the initial price becomes $a_0(0) = 11$. Recall that the lower bound for the unit cost is unity, and thus there remains a considerable margin for cost reductions because of learning-by-doing within the initial industry. We select the initial population to be $N_0 = \delta/\gamma = 0.1$, since, if the population is smaller than δ/γ, the phase diagram, Figure 2, shows that the stock of knowledge is always decreasing and thus there is no possibility of emergence of new industries.

Figure 3 depicts the simulated evolution of the distribution of firms. As the economy always evolves symmetrically around the characteristic 'zero', we show results only for the characteristic $s \geq 0$. In particular, black loci in the figure represent the evolution of the support of $n_t(\cdot)$, from which we can read several dynamic properties of the system of 'industries'.[30] First, at each time after t^*, the economy consists of a finite number of distinguishable industries, rather than one big industry that contains a continuous range of characteristics. This in turn implies that structural changes, namely, emergence or disappearance of industries, occur sporadically at distinct points in

[29]We have confirmed that these kinds of discretization are robust, in that changing the units of discretization does not alter the result substantially. Thus, we claim that none of the substantial results in our simulation depend upon the discrete nature of the procedures.

[30]Here, it is necessary to define formally what we mean by the term 'industry' in our model. At each time t, if the support of the distribution of firms, supp $n_t(\cdot) \subset \mathcal{R}$, is a joint set, we say that all firms belong to one industry. If not, we say that each industry is composed of a set of firms in a joint subset of supp $n_t(\cdot)$. In a dynamic sense, the history of an industry is identified by a joint subset of the graph of supp $n_t(\cdot)$ in (t, s) space.

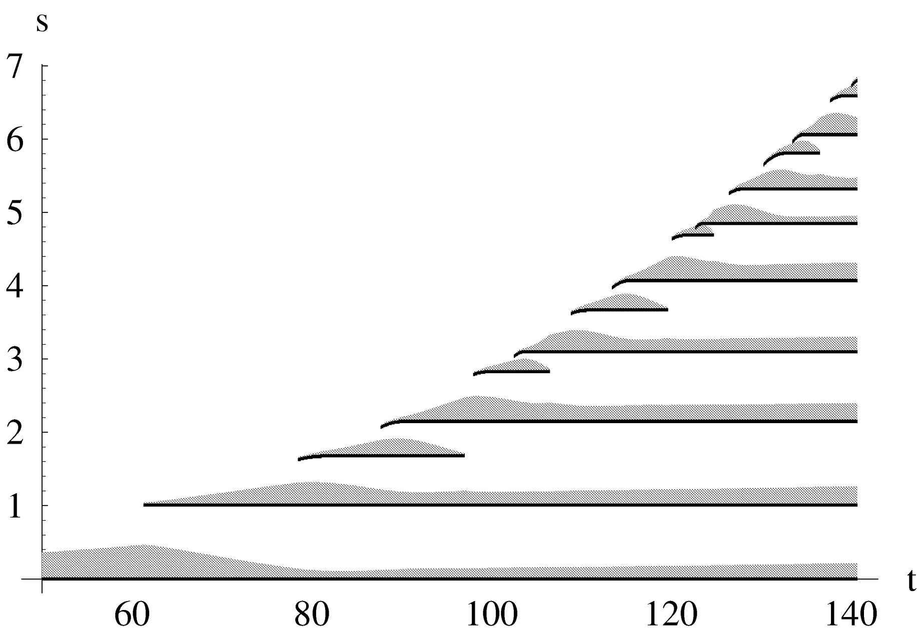

Figure 3: Evolution of industries ($s \geq 0$)

time. Second, except for a short time interval after emergence, all firms in each industry choose the same characteristic, that is, agglomerate in a mass point. Moreover, once they have agglomerated, there seems to be a strong hysteresis in their characteristic.[31] Third, although some industries disappear in finite time, everlasting industries are roughly evenly spaced and their number increases without bound in a dynamic sense.[32] We also anticipate from the figure that there is some cyclicality in the evolution of the industrial structure, although the durations of the cycles seem to become shorter as the economy becomes larger.

[31]During a short time interval after emergence, a newly emerged industry seems to shift its set of characteristics continually in the direction of even newer characteristics. However, as we have seen in Subsection 3.2, once a mass of firms stays in a point on the characteristic space, such a mass point creates a downward kink in the unit cost curve and thereafter tends to persist.

[32]This contrasts to the argument in Subsection 3.2, where we proved that the economy is bounded in a static sense.

The figure also shows the evolution of the size of each industry. Specifically, the height of the gray area above the support of each industry represents the total revenue, or equivalently, the number of workers, in that industry. Until the critical time t^*, the total revenue in the initial industry steadily increases in parallel with the population growth. After the emergence of a couple of new industries, however, the size of the initial industry begins to shrink and eventually the initial industry is dominated by new ones. There are two reasons for this. First, since there remains little margin for cost reduction in the initial industry, the prices of goods in new industries fall more rapidly than those in the initial industry by the learning-by-doing process. Thus, new industries gradually get a larger share of consumer expenditure. Second, since two industries have emerged symmetrically, the initial industry is no longer located on the frontier. While each new frontier industry attracts a large demand from the needs outside the frontier where other industries have small competitive powers, the initial industry must compete with other industries on both sides. Thus, even if all industries post the same prices, the frontier industries attract relatively larger demands. When industries currently on the frontier become sufficiently large, two more new industries emerge and a similar process restarts.

Finally, let us explain why some industries disappear. When a frontier industry emerges, it attracts a large demand from the needs beyond its characteristic. Thus, it may be profitable to produce a new good with a characteristic not so distant from existing industries. When the next frontier industries emerge, however, the demand that the industry previously on the frontier can attract significantly falls since it now has a characteristic too close to other industries on both sides. In some cases, the demand eventually becomes so small that no firm in this industry can reach zero profit. Such a phenomenon occurs partly because no firm is forward-looking, as knowledge is inappropriable. Is, then, such an ephemeral industry completely futile from the viewpoint of economic growth? In fact, it plays an important historical role: the knowledge accumulated by such an industry spills beyond itself and serves as a stepping-stone to new, possibly everlasting, frontier industries.

5 Growth Cycles

In the previous section, the simulation results showed that industries sporadically emerge and disappear as the economy grows. Here, we would like to investigate how such continual changes in industrial structure affect the pattern of the macroeconomic growth rate. Specifically, we proceed in the following way: (i) specify how to measure the macroeconomic growth; (ii) decompose the growth rate into contributions from each industry; and (iii) clarify how structural changes, especially the emergence of new industries, affect the growth rate. From these considerations, it will be clear why the growth rate has cycles.

In our model, we can measure the macroeconomic growth rate in two ways. The first metric is the conventionally measured per capita GDP growth, denoted by g_t. This metric measures how much the total value of output increases for fixed prices. Though it is relatively easy to observe its value in the real economy, there is a shortcoming, as this metric does not account for the benefit resulting from increases in the variety of goods. Alternatively, we can measure growth by the instantaneous utility, which incorporates the benefit from a large variety of goods. However, this alternative also has its intrinsic drawback, as instantaneous utility is not a cardinal metric, and thus is not generally suited to measure the absolute value of welfare improvement. Thus, there seems to be an inevitable tradeoff between these two concepts. As long as growth is confined to the equilibrium path, however, this is not the case; we show in the following that there is a linear relationship between the two metrics, and thus we can in effect use either of them without arbitrariness of the metric and without missing certain benefits.

For this purpose, let us derive a formula for the growth rate of the instantaneous utility, $\dot{U}_t/U_t$. From the equation (12), which states that the instantaneous utility is the reciprocal of the average price index, the growth rate can also be seen as the rate of fall of this index. Since the latter depends on the distribution of firms $n_t(\cdot)$ and the profile of prices $p_t(\cdot)$, we must, in principle, deal with the time variation of these functions. For ease of exposition, however, let us again in this subsection utilize the discrete approximation. That is, we assume that firms are distributed exclusively on grids with countable points, $\{s_1, s_2, \cdots\}$.[33] Then, a fundamental rule of calculus enables us to express the growth rate as[34]

$$\frac{\dot{U}_t}{U_t} = \sum_{j=1}^{\infty} \left(\bar{P}_t^{-1} \frac{\partial \bar{P}_t}{\partial p(s_j)} \right) (-\dot{p}_t(s_j)) + \sum_{j=1}^{\infty} \left(-\bar{P}_t^{-1} \frac{\partial \bar{P}_t}{\partial n_j} \right) \dot{n}_{tj}. \tag{40}$$

The first term in the above expression represents the welfare improvement that comes from the decline in the price of goods, whereas the second term represents the benefit from the increase in the variety of goods.

Applying the equilibrium condition (E1$''$) for (40), the first term in the RHS simply becomes g_t, the per capita GDP growth rate, and the second turns out to be $(\lambda + g_t)/(\sigma - 1)$. Since at each instant the equilibrium allocation maximizes instantaneous utility subject to the resource constraint, these results can be interpreted using the Envelope Theorem.[35] Collecting

[33] See Subsection 3.2.

[34] Strictly speaking, we must show that the time derivatives of $p_t(s_j)$ and n_{tj} actually exist. First, from the equilibrium conditions and (5), we have $\dot{p}_t(s_j) = \dot{a}_t(s_j) = -K_t(s_j)^{-2}\dot{K}_t(s_j)$. Since $\dot{K}_t(s_j)$ is defined by (6), $\dot{p}_t(s_j)$ is also well defined. Next, as shown in Subsection 3.3, the equilibrium firm distribution is the solution to the constrained minimization problem of $\bar{P}_t$. Since the constraint of this problem is compact and changes continuously, we can apply the Theorem of the Maximum to show that the solution $\{n_{t1}, n_{t2}, \dots\}$ changes continuously and thus their time derivatives exist. See Stoky and Lucas (1988, Chapter 3).

[35] Recall that the per capita nominal income is normalized to unity and that the instantaneous utility function is homogeneous of degree one in consumption density. Thus, given the number of firms, a decline in prices induces an increase in the per capita total consumption quantities, which results in a proportional increase in the instantaneous utility. Therefore, the first term, the benefit from falling prices, exactly equals the per capita GDP growth rate. Next, let us focus on the benefit from increasing variety, represented by the second term. It will be shown in Section 6 that population growth affects the instantaneous utility by multiplying the number of firms with every characteristic, and that the elasticity of instantaneous utility to population is $(\sigma - 1)^{-1}$. Thus, the first part of the second term, $\lambda/(\sigma - 1)$, comes from exogenous population growth. The remaining part, $g_t/(\sigma - 1)$, reflects a mixture of the above two effects. That is, an increase in the total quantity because of the decline in prices is absorbed, not by an increase in the output

these effects together, we now have a simple formula:

$$\frac{\dot{U}_t}{U_t} = \frac{\sigma}{\sigma - 1} g_t + \frac{\lambda}{\sigma - 1}. \tag{41}$$

The above states that, as far as the equilibrium path is concerned, there is a linear relationship between the two metrics of the macroeconomic growth rate. Thus, it would be a matter of convenience which of the two concepts is used, and, in the following, we concentrate on the per capita GDP growth rate.[36]

In our model, the per capita GDP growth is driven by the learning-by-doing process in each industry through the reduction of the unit cost of producing a good with each characteristic. Then, how much does each industry contribute to the macroeconomic growth? To clarify the issue, we derive an intuitive decomposition of g_t. Formally, the definition of the per capita GDP growth is

$$g_t \equiv \frac{\sum_{j=1}^{\infty} p_t(s_j) \frac{d}{dt}(Q_t(s_j) n_{tj})}{\sum_{j=1}^{\infty} p_t(s_j) Q_t(s_j) n_{tj}} - \frac{\dot{N}_t}{N_t}, \tag{42}$$

where $Q_t(s_j)n_{tj}$ represents the total amount produced in the industry at characteristic s_j.[37] The above expression simplifies considerably under the equilibrium conditions (E1)-(E4), but we can go one step further. Differentiating both sides of the total expenditure relationship, $\sum_{j=1}^{\infty} a_t(s_j) n_{tj} = N_t$,

quantity of each firm, but by an increase in the number of firms, which induces an additional benefit from the larger variety. For a formal treatment of the Envelope Theorem, see Mas-Colell, Whinston and Green (1995, Section M.L).

[36]Note that, however, the relationship (41) holds only if the equilibrium conditions (E1″)-(E4) are satisfied. For example, industrial policies that accelerate g_t do not necessarily improve the growth rate of the instantaneous utility, since under such policies the equilibrium conditions might not be satisfied.

[37]To simplify the exposition, we proceed in the following text as if the support of every industry contains no more than one point in $\mathcal{S}$. However, more generally, the support of an industry might be a finite number of adjacent points in $\mathcal{S}$. In such a case, we shall interpret this industry's contribution to the growth rate as the sum of the contributions from the firms at all points within its support.

we have an identity,

$$N_t^{-1}\sum_{j=1}^{\infty} a_t(s_j)\dot{n}_{tj} = \lambda - N_t^{-1}\sum_{j=1}^{\infty}\dot{a}_t(s_j)n_{tj}. \tag{43}$$

This formula enables us to express the growth rate in terms of the rate of cost reduction, $-\dot{a}_t(s_j)/a_t(s_j)$, which is directly related to the learning-by-doing process. Let us denote the latter by η_{tj}. Then, the equation (42) reduces to

$$g_t = \sum_{j=1}^{\infty} x_{tj}\eta_{tj}, \tag{44}$$

where x_{tj} is the share of GDP produced by each industry, $a_t(s_j)n_{tj}/N_t$. The above formula gives a clear decomposition of the per capita GDP growth rate into each industry; that is, g_t is the weighted sum of the rates of cost reduction in each industry, where the weights are the shares in the GDP.

While this formula might at first blush suggest that there is a linear relationship between each industry's share and its contribution to the growth rate, this is not the case, since the rate of cost reduction, η_{tj}, also depends on shares through the learning-by-doing process. Specifically, from (5) and (6), we can write down the rate of cost reduction in terms of current shares, unit costs and population:

$$\eta_{tj} = \gamma N_t \frac{(a_t(s_j)-1)^2}{a_t(s_j)}\sum_{m=1}^{\infty}\frac{x_{tm}}{a_t(s_m)}e^{-\nu|s_j-s_m|} - \delta\frac{a_t(s_j)-1}{a_t(s_j)}. \tag{45}$$

Each item within the summation in the first term represents the flow of knowledge that spills from the industry at s_m to the industry at s_j. Note that this item depends negatively on the distance between the two industries and linearly on the industry's share where production experience is gained. Thus, we can see that the rate of cost reduction itself also depends linearly on each industry's share, given $K_t(\cdot)$ and N_t. Combining the two formula (44) and (45), it turns out that, at each instant, the per capita GDP growth depends not linearly but, in fact, quadratically on shares in the GDP.

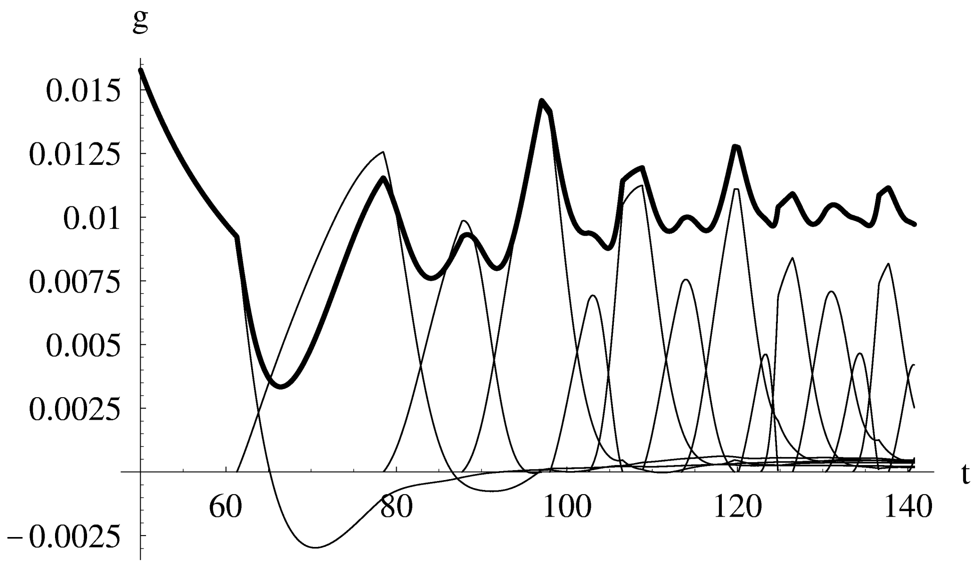

Figure 4: The per capita GDP growth and its decomposition

Numerical Analyses

Now, let us present a simulation result using the same parameter values and initial conditions as in the previous section. Figure 4 depicts the overall per capita GDP growth rate and its decomposition into contributions from each industry, according to formula (44). We can easily see from this figure that, at each point in time, usually only one or two, at most three, young industries represent quite a large portion of the growth rate. This observation is consistent with our formulation of the learning-by-doing process; that is, since there is a lower bound on the unit cost of production, structural changes, especially the emergence of new industries, are necessary to sustain positive economic growth. When we take a closer look at this figure, however, a paradoxical phenomenon can be observed: the overall growth rate falls sharply immediately after the emergence or disappearance of industries, and it is not until newly emerged industries become sufficiently large that the growth rate seems to rebound. In the following, we attempt to explain why structural changes affect the growth rate in such a way, and eventually give rise to cyclical fluctuations in growth.

When a new frontier industry emerges, some portion of consumer expenditure begins to shift from existing industries to the new one. In our model, where knowledge accumulation cannot be separated from production, this event has two distinct effects on the growth rate. To clarify each of them, let us differentiate the formula (44) with respect to each industry's share:[38]

$$\frac{\partial g_t}{\partial x_{tj}} = \eta_{tj} + \sum_{m=1}^{\infty} x_{tm} \frac{\partial \eta_{tm}}{\partial x_{tj}}. \tag{46}$$

The first term, η_{tj}, represents the effect that comes from variation in the composition of GDP: if a larger proportion of GDP is produced by industries in which costs are rapidly falling, the overall GDP growth rate is higher. Figure 5 gives the rates of cost reduction in each industry, and the gap between any two curves indicates the magnitude of the 'GDP composition effect' when a marginal share shifts between these industries. Apparently, the rate of cost reduction is higher in young, recently emerged industries, since their costs are still distant from the lower bound and thus are more effectively reduced by a given amount of production experience. Therefore, when a marginal share shifts to new industries, the resulting variation in the GDP composition has a positive effect on the overall growth.

There is another effect, however, which may overturn the above result. Specifically, the second term in (46) comes from variations in the composition of the learning-by-doing process: variations in each industry's share affect the pattern of knowledge accumulation and thus the pattern of cost reductions, which in turn has an impact on the overall growth rate. Differentiating (45), we can write this 'learning composition effect' as

$$\gamma N_t \sum_{m=1}^{\infty} x_{tm} \frac{(a_t(s_m) - 1)^2}{a_t(s_m)} \frac{1}{a_t(s_j)} e^{-\nu |s_j - s_m|}. \tag{47}$$

[38]The formulas derived earlier, (44) and (45), show that the per capita GDP growth depends on current shares, unit costs and population. Since unit costs are determined by the accumulated knowledge, however, they do not affect the growth rate at each instant in the first order. Also, population is exogenously determined. Thus, each industry's share is the only variable that influences the growth rate in the short term.

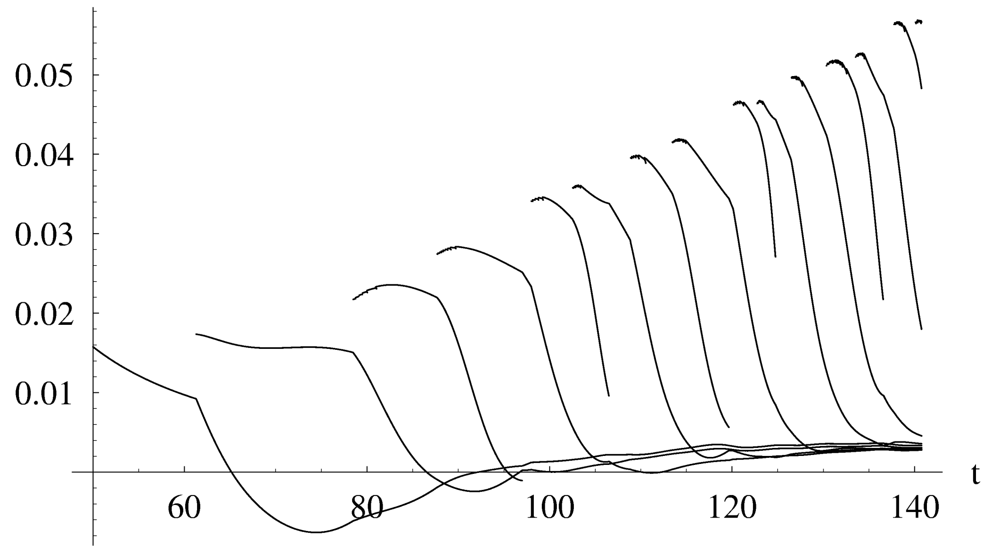

Figure 5: The GDP composition effect

Although a marginal increase in the production experience in an industry s_j leads to cost reductions in every industry through spillovers, the magnitude of the spillover, represented by the term $e^{-\nu|s_j - s_m|}$, diminishes as the characteristic distance gets larger. In fact, the major portion of the contribution comes from the cost reduction in industry s_j itself. Thus, a kind of economy of scale exists: additional experience in a certain industry contributes greatly to the growth rate when its own share is already large, because additional cost reduction has a strong effect on a large portion of the GDP.[39] Figure 6 plots the value of the expression (47) for each industry, and again, the gap between any two curves indicates the magnitude of the learning composition effect. There seems to be a typical pattern: each curve starts from a relatively low value, since a recently emerged industry captures only a small share; then it grows until it attains its peak value around the time another new industry emerges. It then gradually declines as the unit cost approaches

[39]Roughly speaking, when a small industry's share shifts to a large one, the growth rate tends to be enhanced and vice versa.

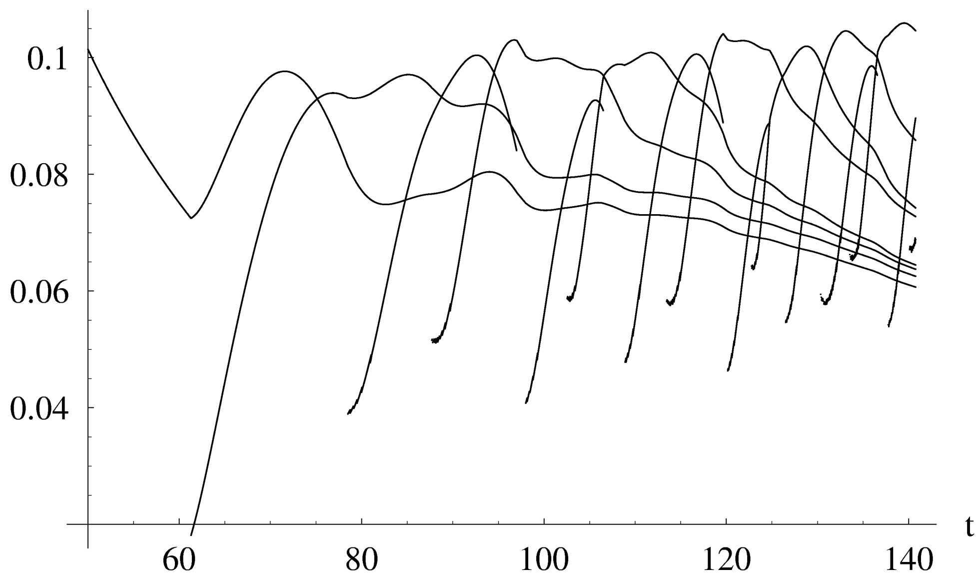

Figure 6: The learning composition effect

the lower bound.

With these two results in hand, we can now explain the cyclical behavior of the growth rate. Immediately after emergence of a new industry, the industry previously on the frontier still gets the largest share.[40] Thus, the overall growth rate is greatly affected by the slowdown in its pace of learning-by-doing when its share shifts to the small new industry. In fact, we can see from the last two figures that the learning composition effect is negative and, in absolute value, dominates the GDP composition effect. This is the reason why the growth rate falls following the emergence of a new industry, at least in the short term.[41] As the knowledge at the new industry accumulates,

[40]See Figure 3 in the previous section.

[41]In a parallel way to the text, we can show why the growth rate tends to fall after the disappearance of existing industries. Let us consider what happens just before an industry disappears. Since its unit cost must have approached the lower bound, the rate of cost reduction in this industry is generally small. Moreover, since this industry is going to disappear in the near future, it must have only a small share. In such a case, a marginal share shift from it to other industries will enhance the growth rate, since both the GDP

however, its share becomes gradually larger, whereas that of the previous one becomes smaller. This implies that the absolute value of the learning composition effect becomes small. Eventually, this negative effect is cancelled by the GDP composition effect, and thereafter the growth rate begins to rebound. Furthermore, when the new industry's share dominates that of the previous one, the learning composition effect itself turns positive, accelerating the growth rate. However, this boom period does not last forever. Another new industry will emerge, which starts another U-shaped pattern in the growth rate.

6 Population Growth and Scale Effect

Prior to this section, we have assumed that the size of the population grows exogenously, which was a necessary condition for positive long-run growth in our model. This assumption is, however, not essential. We show here that this assumption can be dispensed with by reformulating the process of accumulation of knowledge.

Recently, much attention has been paid to the relationship between population growth and long-run growth rate. In his influential paper, Jones (1995) has pointed out that most endogenous growth models have an implausible 'scale effect': in these models the size of population affects the long-run growth rate. This phenomenon comes from a particular specification of the R&D process, in which the current efficiency of R&D depends linearly on the accumulated past experience of R&D. Jones (1995) presented an alternative specification, where efficiency does not depend (or depends less than linearly) on the past experience. However, this involves a tradeoff: the scale effect is eliminated, at the cost of requiring exponential population growth to maintain long-run growth. Although there is a difference between R&D and the learning-by-doing process, our model essentially belongs to this alterna-

composition effect and the learning composition effect are positive. After this industry completely diminishes, however, such enhancing effects also disappear, which tends to lower the growth rate.

tive class of specifications. Is this a flaw? Since it is controversial whether the scale effect is present or not, we do not argue this point in this paper. Rather, we simply illustrate how this model can be extended for the case where population growth is not necessary, but the scale effect is present.[42]

To this end, let us clarify how the population size affects the instantaneous equilibrium in the original model. Observe that the unit demand price function, defined by (19), is homogeneous of degree zero in $\big(n(\cdot), N\big)$. Thus, if we double both the total population and the equilibrium distribution of firms simultaneously, they would continue to constitute an equilibrium, given the distribution of knowledge. While such an event does not alter the price of each good, the consumption density of each good must have been halved, since now there are twice the varieties of goods, whereas the per capita income is unaffected. As a result, the utility of each consumer mildly increases with the returns to scale being $(\sigma - 1)^{-1}$.[43] This kind of scale merit exists in almost all models that incorporate Dixit-Stiglitz style product differentiation. Rather, the major finding here is the fact that, given the distribution of knowledge, the size of the economy does not alter the basic structure of industries, but merely scales it.

From the viewpoint of economic growth, this fact implies that a larger population simply accelerates the learning-by-doing process, and hence the rate of economic growth at each instant.[44] Indeed, the equation (45) shows that the true coefficient that determines the speed of the economy-wide

[42] Some recent 'hybrid' models overcome this tradeoff by combining the quality ladder model with the variety expansion model. These models eliminate the scale effect while, nonetheless, long-run growth is possible with a constant population. In principle, it is possible to achieve the same effect in our model by adding one more dimension of variety expansion to it. See Jones (1999) for a survey.

[43] We see that the utility function, defined by (1) and (2′), is homogeneous of degree $\sigma/(\sigma - 1)$ with respect to the distribution of firms, and of degree one in terms of the consumption density. Therefore, if the population is doubled, the utility will be multiplied by $2^{\sigma/(\sigma-1)}$ because of the doubling of firms, but simultaneously divided by two because of the halving of the consumption density.

[44] Note that this fact does not mean that the size of the population affects the 'long-run' economic growth rate.

learning-by-doing process is not γ itself, but γN_t. If this coefficient were fixed, the long-run growth rate would eventually fall to zero, since the frontier industries' share would get smaller and smaller compared to the growing whole economy. In our original model where N_t grows exogenously, the true coefficient γN_t also grows linearly with the size of the economy, enabling positive long-run growth. Alternatively, suppose that the total population is fixed, but the efficiency of learning-by-doing, γ, grows in accordance with past experience of learning-by-doing, namely, the whole profile of accumulated knowledge. For example, this consideration can be specified as

$$\gamma_t = \int_{-\infty}^{\infty} \Gamma(K_t(s))\, ds,$$

where $\Gamma(\cdot)$ is an increasing function. This specification has virtually the same effect as population growth and resolves the zero growth problem for a fixed population, with the cost that this model now exhibits the scale effect.

Appendix

A Proof of Concavity of $\tilde{U}$

Here we establish concavity of the transformed instantaneous utility function $\tilde{U}$, given by $(1')$ and $(2'')$, in terms of $\{n_j\}$ and $\{h_j(\cdot)\}$.

Consider two sets of variables, $\theta = \{\{n'_j\}, \{h'_j(\cdot)\}\}$ and $\theta' = \{\{n''_j\}, \{h''_j(\cdot)\}\}$. In addition, for arbitrary $\alpha \in (0, 1)$, let us construct a new set of variables $\theta^* = \{\{n^*_j\}, \{h^*_j(\cdot)\}\}$ by setting

$$\left.\begin{aligned} n^*_j &= (1-\alpha)n'_j + \alpha n''_j \\ h^*_j(\cdot) &= (1-\alpha)h'_j(\cdot) + \alpha h''_j(\cdot) \end{aligned}\right\} \quad \text{for } j = 1, 2, \cdots. \tag{A.2}$$

Our task is then to show that $\tilde{U}^* \geq (1-\alpha)\tilde{U}' + \alpha\tilde{U}''$ holds, where $\tilde{U}^*$ is the value of the transformed utility function when we have substituted for θ^*, and so on.

First, from an identity $1/\sigma + (\sigma-1)/\sigma = 1$, we have

$$n^{*\frac{1}{\sigma}}_j h^*_j(r)^{\frac{\sigma-1}{\sigma}} \geq (1-\alpha)n'^{\frac{1}{\sigma}}_j h'_j(r)^{\frac{\sigma-1}{\sigma}} + \alpha n''^{\frac{1}{\sigma}}_j h''_j(r)^{\frac{\sigma-1}{\sigma}}, \tag{A.3}$$

for all j and r. Applying this result for (2″), it is straightforward to show

$$\begin{aligned} v^*(r)^{\frac{\sigma-1}{\sigma}} &= \sum_{j=1}^{\infty} n_j^{*\frac{1}{\sigma}} h_j^*(r)^{\frac{\sigma-1}{\sigma}} e^{-\tau\frac{\sigma-1}{\sigma}|r-s_j|} \\ &\geq (1-\alpha)v'(r)^{\frac{\sigma-1}{\sigma}} + \alpha v''(r)^{\frac{\sigma-1}{\sigma}}, \end{aligned} \tag{A.4}$$

where $v^*(r)$, $v'(r)$ and $v''(r)$ are defined in parallel with $\tilde{U}^*$, and so on. Next, note that from the assumption $\sigma > \beta > 1$, an inequality $\sigma/(1-\sigma) \cdot (\beta-1)/\beta < 1$ holds. Thus, the previous result can be transformed into

$$\begin{aligned} v^*(r)^{\frac{\beta-1}{\beta}} &\geq \left[(1-\alpha)v'(r)^{\frac{\sigma-1}{\sigma}} + \alpha v''(r)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}\frac{\beta-1}{\beta}} \\ &\geq (1-\alpha)v'(r)^{\frac{\beta-1}{\beta}} + \alpha v''(r)^{\frac{\beta-1}{\beta}}. \end{aligned} \tag{A.5}$$

Finally, applying the above inequality for (1′), we have

$$\tilde{U}^* = \int_{-\infty}^{\infty} v^*(r)^{\frac{\beta-1}{\beta}} dr \geq (1-\alpha)\tilde{U}' + \alpha\tilde{U}'', \tag{A.6}$$

which establishes the claim.

B Proof of Uniqueness of Optimum

In the text, we have established that the set of conditions (K1)-(K3) is the necessary and sufficient set of conditions for the instantaneous optimum, and that the set of maximizers is convex. Here we prove that there is no more than one maximizer.

Suppose that there are two sets of variables, θ' and θ'', defined the same way as in Appendix A, and that both of them attain the maximum transformed utility, denoted by $\tilde{U}_M$. When we choose $\alpha \in (0, 1)$ arbitrarily, another set of variables θ^*, defined by (A.2), will also attain $\tilde{U}_M$, since the set of maximizers is convex. This means that the inequality (A.6) holds with equality. By making use of this fact, in the following we show that $\theta' = \theta''$ necessarily holds, which establishes the uniqueness of the maximizer.

For the final inequality (A.6) to hold with equality, the second inequality in (A.5) must hold with equality for all $r \in \mathcal{R}$. This in turn requires $v'(r) =$

$v''(r)$ for all $r \in \mathcal{R}$. On the other hand, since both θ' and θ'' are maximizers, the condition (K2) is satisfied by both of them. Substituting this condition for (2$''$), we have

$$v'(r)^{\frac{\sigma-1}{\beta}} = \left(\frac{\sigma}{\sigma-1}\frac{\beta-1}{\beta}\frac{1}{N\xi'}\right)^{\sigma-1}\sum_{j=0}^{\infty} n'_j a(s_j) e^{-\tau(\sigma-1)|r-s_j|}, \tag{A.7}$$

and a similar relationship for the case of θ''. Note that if we see each term in the summation, $a(s_j)e^{-\tau(\sigma-1)|r-s_j|}$, as a function of r, they are linearly independent. Thus, for the equation $v'(r) = v''(r)$ to hold for all r, all coefficients must be equal to each other: that is, $\xi'^{1-\sigma}n'_j = \xi''^{1-\sigma}n''_j$ holds for all j. This implies that there is a positive constant ζ, such that $n''_j = \zeta n'_j$ holds for all j.

Next, let us turn to the former two inequalities. For the final inequality to hold with equality, the first inequality in (A.5) must also hold with equality for all r. This is equivalent to (A.4) holding with equality, which in turn requires (A.3) to hold with equality for all r and j. Note that from the argument above, if $n'_j = 0$ for some j, then $n''_j = 0$ also holds. In this case, condition (K2) ensures that $h'_j(r) = h''_j(r) = 0$ for all r. If $n'_j > 0$, on the other hand, then $n''_j = \zeta n'_j > 0$ holds and condition (K2) ensures that both $h'_j(r)$ and $h''_j(r)$ are positive for all r. In such a case, for (A.3) to hold with equality, an equality $h'_j(r)/n'_j = h''_j(r)/n''_j$ must hold for all r. After all, $h''_j(r) = \zeta h'_j(r)$ is required for all j and r.

Finally, remember that both θ' and θ'' satisfy the resource constraint (K3). The preceding results, $n''_j = \zeta n'_j$ and $h''_j(r) = \zeta h'_j(r)$ for all j and r, are compatible with this condition if and only if $\zeta = 1$. This establishes the claim.

C Upper and Lower Bounds for $\Psi(s)$

Here we establish upper and lower bounds for the function $\Psi(\cdot)$, defined by (36). Specifically, we prove that an inequality $1 \leq \Psi(s) \leq e^{\nu|s|}$ holds, with equality if and only if $s = 0$.

Let us define a new function $\psi(\cdot, \cdot)$ by

$$\psi(r, s) = (\sigma - \beta)\tau|r| - (\sigma - 1)\tau|r - s|. \tag{A.8}$$

Then, we can express the function concerned, $\Psi(\cdot)$, as

$$\Psi(s) = \left[\frac{\int_{-\infty}^{\infty} \exp \psi(r, s)\, dr}{\int_{-\infty}^{\infty} \exp \psi(r, 0)\, dr}\right]^{\frac{1}{\sigma}}. \tag{A.9}$$

With this expression, it is obvious that $\Psi(0) = 1$ holds, since the numerator and the denominator coincide when $s = 0$.

Next, consider the case of $s > 0$. We can show that

$$\psi(r, s) \leq \psi(r - s, 0) + (\sigma - \beta)\tau s, \tag{A.10}$$

holds with strict inequality when $r < s$. Substituting this result into (A.9), we have

$$\Psi(s) < e^{\frac{\sigma-\beta}{\sigma}\tau s}. \tag{A.11}$$

Recall the assumptions $\sigma > \beta > 1$ and $\nu > ((\sigma - 1)/\sigma)\tau$, which together establish an inequality $\nu > ((\sigma - \beta)/\sigma)\tau$. Applying this for (A.11), the inequality $\Psi(s) < e^{\nu s}$ results. Similarly, we can also show that

$$\psi(r, s) \geq \psi\left(r - \frac{\sigma - 1}{\beta - 1}s, 0\right) \tag{A.12}$$

holds with strict inequality if $0 < r < ((\sigma - 1)/(\beta - 1))s$. Substituting this into (A.9), we have $\Psi(s) > 1$. Collecting both results, the inequality $1 < \Psi(s) < e^{\nu|s|}$ is established for all $s > 0$.

Finally, since $\Psi(s)$ is symmetric around zero, we can show the same result for $s < 0$.

D Existence of the Critical Value $\bar{K}^*$

Here we prove that there is a finite positive constant $\bar{K}^*$ such that the market potential function for the one-industry economy (37) is negative for all $s > 0$ if and only if $\bar{K}_0 \leq \bar{K}^*$.

The negativity of (37) turns out to be equivalent to

$$\bar{K}_0 \leq \frac{e^{\nu s} - \Psi(s)}{\Psi(s) - 1} \equiv \kappa(s) \tag{A.13}$$

for all $s > 0$. Then, if we can show that the minimum of $\kappa(s)$ exists in the interval $(0, \infty)$, the minimized value is $\bar{K}^*$ and we are done.

Note that, since $0 < \Psi(s) < e^{\nu s}$ holds for all $s \in (0, \infty)$, the function $\kappa(s)$ is well defined and strictly positive in this interval. Moreover, the continuity of $\Psi(s)$ ensures the continuity of $\kappa(s)$. The interval $(0, \infty)$ is, however, not compact and thus we must examine how $\kappa(s)$ behaves at both ends of the interval.

First, consider the case when s tends to zero from above. In this case, both the denominator and numerator of $\kappa(s)$ also tend to zero. Then, applying the l'Hôpital's Theorem, we have

$$\lim_{s \to +0} \kappa(s) = \lim_{s \to +0} \frac{\nu - \Psi'(s)}{\Psi'(s)}. \tag{A.14}$$

Note that from (A.8) and (A.9), we can confirm that $\Psi(s)$ is increasing for all $s > 0$. In addition, since $\Psi(s)$ is everywhere differentiable and symmetric around zero, $\Psi'(0) = 0$ apparently holds. Thus, as s tends to zero from above, $\Psi'(s)$ also tends to zero from above. Substituting this result for (A.14), we have $\lim_{s \to +0} \kappa(s) = +\infty$.

Next, let us examine how $\kappa(s)$ behaves as s tends to infinity. From the inequality (A.11) and the assumption that parameter ν is greater than $((\sigma - \beta)/\sigma)\tau$, we can show that $e^{-\nu s}\Psi(s)$ tends to zero from above as s tends to infinity. Similar arguments can be given for $e^{-\nu s}(\Psi(s) - 1)$, since $\Psi(s) > 1$ holds for all $s > 0$. Thus, we have

$$\lim_{s \to \infty} \kappa(s) = \lim_{s \to \infty} \frac{1 - e^{-\nu s}\Psi(s)}{e^{-\nu s}(\Psi(s) - 1)} = +\infty. \tag{A.15}$$

These arguments assure that the minimizer of $\kappa(s)$ exists in the interior of $(0, \infty)$, and its minimized value, $\bar{K}^*$, is finite and strictly positive.

Acknowledgements

We are grateful to Kenn Ariga, Masahisa Fujita, Koichi Futagami, and Akihisa Shibata for their helpful comments and suggestions. The author is responsible for any remaining errors.

References

CHAMBERLIN, E.H. (1993) *The Theory of Monopolistic Competition* (Cambridge, Mass: Harvard University Press).

DINOPOULOS, E. and THOMPSON, P. (1998), "Schumpeterian Growth without Scale Effects", *Journal of Economic Growth*, **3**, 313-35.

DIXIT, A.K. and STIGLITZ, J.E. (1977), "Monopolistic Competition and Optimum Product Diversity", *American Economic Review*, **67**, 297-308.

FUJITA, M., KRUGMAN, P. and VENABLES, A.J. (1999) *The Spatial Economy: Cities, Regions and International Trade* (Cambridge: MIT Press).

GREENWOOD, J. and YORUKOGLU, M. (1997), "1974", *Carnegie-Rochester Conference Series on Public Policy*, **46**, 49-95.

GROSSMAN, G.M. and HELPMAN, E. (1989), "Product Development and International Trade", *Journal of Political Economy*, **97**, 1261-83.

HART, O.D. (1985), "Monopolistic Competition in the Spirit of Chamberlin: a General Model", *Review of Economic Studies*, **52**, 529-46.

HELPMAN, E. and TRAJTENBERG, M. (1998), "A Time to Sow and a Time to Reap: Growth Based on General Purpose Technologies", ch. 3 in E. Helpman (ed.) *General Purpose Technologies and Economic Growth* (Cambridge: MIT Press).

HORNSTEIN, A. and KRUSELL, P. (1996), "Can Technology Improvements Cause Productivity Slowdowns?", in B. S. Bernanke and J. Rotemberg (eds.) *NBER Macroeconomic Annual 1996* (Cambridge: MIT Press).

HOTELLING, H. (1929), "Stability in Competition", *The Economic Journal*, **39**, 41-57.

JONES, C.I. (1995), "R&D-Based Models of Economic Growth", *Journal of Political Economy*, **103**, 759-84.

JONES, C.I. (1999), "Growth: With or Without Scale Effects?", *American Economic Review*, **89**, 139-44.

JOVANOVIC, B. (1995), "Learning and Growth" (NBER Working Paper No. 5383).

KIM, S. (1989), "Labor Specialization and the Extent of the Market", *Journal of Political Economy*, **97**, 692-705.

KIM, S. and MOHTADI, H. (1992), "Labor Specialization and Endogenous Growth", *American Economic Review*, **82**, 404-08.

MAS-COLELL, A., WHINSTON, D. and GREEN, J.R. (1995) *Microeconomic theory* (New York: Oxford University Press).

MATSUYAMA, K. (1999), "Growing Through Cycles", *Econometrica*, **67**, 335-47.

PERETTO, P. (1998), "Technological Change and Population Growth", *Journal of Economic Growth*, **3**, 283-311.

PERLOFF, J.M. and SALOP, S.C. (1985), "Equilibrium with Product Differentiation", *Review of Economic Studies*, **52**, 107-20.

ROMER, P.M. (1987), "Growth Based on Increasing Returns Due to Specialization", *American Economic Review*, **77**, 56-62.

ROMER, P.M. (1990), "Endogenous Technological Change", *Journal of Political Economy*, **98**, S71-102.

STOKY, N.L. (1988), "Learning by Doing and the Introduction of New Goods", *Journal of Political Economy*, **96**, 701-17.

STOKY, N.L. and LUCAS, R.E. Jr., with PRESCOTT, E.C. (1988) *Recursive Methods in Economic Dynamics* (Cambridge: Harvard University Press).

YOUNG, A. (1991), "Learning by Doing and the Dynamic Effects of International Trade", *Quarterly Journal of Economics*, **106**, 369-405.

YOUNG, A. (1993), "Invention and Bounded Learning by Doing", *Journal of Political Economy*, **101**, 443-472.

YOUNG, A. (1998), "Growth without Scale Effects", *Journal of Political Economy*, **106**, 41-63.