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ON DECREASING IMPATIENCE

Ken-ichi Hirose
and
Shinsuke Ikeda

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The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

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Hirose, Ken-Ichi* **and Shinsuke Ikeda†**

Ritsumeikan University and Osaka University

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Abstract

In the theory of endogenous time preference, one of the most common and most controversial assumptions is that the degree of impatience, measured by the rate of time preference, is increasing in wealth. Although this empirically-unjustified assumption often helps ease dynamic analyses by ensuring stability, it has never been discussed why decreasing impatience is theoretically problematic. We first show that under certain conditions there exists no optimal solution when impatience is decreasing. By considering problem-proof, well-behaved models, we examine implications of decreasing impatience for three issues that have been discussed in the literature: (i) long-run tax incidence of capital taxation; (ii) the effect of inflation on growth; and (iii) wealth distribution dynamics in the two-country world.

Keywords: Decreasing impatience, time preference, capital taxation, the Tobin effect, wealth distribution.

JEL classification: D90, E00, F41.

*The Department of Economics, Ritsumeikan University, 1-1-1, Noji-Higashi, Kusatsu, Shiga 525-8577, Japan. Telephone: +81(797)84-9633. E-mail: k1hirose@hotmail.com

†Corresponding author. The Institute of Social and Economic Research, Osaka University, 6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan. Telephone: +81(6)6879-8568. Fax: +81(6)6878-2766. E-mail: ikeda@iser.osaka-u.ac.jp

1 Introduction

In the theory of endogenous time preference, one of the most common and, at the same time, the most controversial assumptions is that the degree of impatience, measured by the rate of time preference, is increasing in wealth. Although there is no empirical evidence to the assumption of increasing impatience,[1] no research has so far been conducted on theoretical or economic implications of decreasing impatience, notwithstanding the huge accumulation of research on time preference and/or recursive preferences. Why is the assumption of decreasing impatience theoretically problematic? Whether or not can we consider well-behaved models of decreasing impatience that are immune to the "theoretical problem"? And if such models are available, what would be economic implications of decreasing impatience? All of these problems are left unsolved. For example, it has often been stressed that the assumption of increasing impatience is required to ease dynamic analyses by ensuring stability (see, e.g., Epstein (1987a,b), Lucas and Stokey (1984), and Obstfeld (1990)). However, Svensson and Razin (1983) criticized this reasoning since unstable equilibrium time paths of the economy cannot be ruled out a priori. We should consider what would be problematic when deceasing impatience destabilizes dynamics.

This paper tackles these problems regarding decreasing impatience. The main aims are: (i) to clarify its theoretical difficulty; and (ii) to examine economic implications of decreasing impatience by considering well-behaved models. We first show that instability resulting from decreasing impatience is inconsistent with optimality in a certain situation. In particular, with appropriate regularity conditions such as concavity, divergent consumption paths under decreasing impatience are shown to make, sooner or later, the discount rate negative, thereby violating the transversality condition for the discount factor when the rate of interest is constant.

This difficulty does not occur in two cases: (a) when there exists some decreasing-return properties that stabilize economic dynamics; and/or (b) when there are upper bounds for consumption resources. As a typical example of case (a), we incorporate capital accumulation with the neoclassical production function. The resultant well-behaved decreasing-impatience mod-

[1]See Lawrance (1991), Becker and Mulligan (1997), and Samwick (1997). They commonly reported that the degree of impatience is decreasing in wealth or income. See also Ogawa (1993), which reported that the shapes of the time preference schedule are internationally different.

els are used to elucidate implications of decreasing impatience for two policy issues which has been discussed in the literature of increasing impatience (e.g., Epstein and Hynes (1983) and Obstfeld (1990)): (i) the effects of capital taxation; and (ii) the effects of inflation on economic growth. As for (b), we consider a two-country model, in which consumption resources are bounded by a constant world output level. By using the model, we examine (iii) wealth distribution dynamics in a two-country world economy.

Main implications we show on issues (i) though (iii) are as follows. In the neoclassical model with decreasing impatience, an increase in capital taxes raises the long-run interest rate, so that the long-run tax incidence on capital are larger than in the cases of constant time preference and of increasing impatience. When impatience is decreasing, a decrease in the capital stock caused by capital taxation makes consumers less patient, which raises the long-run interest rate irrespective of the capital tax increase, and thereby causes a further reduction in the capital stock. This property is contrasted sharply to what is obtained in the literature by assuming increasing impatience (e.g., Epstein and Hynes (1983)).

When money is introduced into the neoclassical model with decreasing impatience, an increase in the rate of nominal money growth reduces the steady state capital stock, that is the Tobin effect does not hold. This implication, which again contrasts to the case of increasing impatience (e.g., Epstein and Hynes (1983)), is consistent with many empirical studies reporting of growth-harming inflation (e.g., Barro (1995) and Bruno and Easterly (1998)).

In the two-country context, decreasing impatience leads to various wealth-distribution dynamics. If impatience is decreasing in both of the two countries, the equilibrium time path of the world economy is unstable. When one country displays decreasing impatience while the other displays increasing impatience, the equilibrium time path can be either stable or unstable, depending on the relative degrees of the increasingness and decreasingness of impatience. In the stable case, an upward shift of the subjective discount rate schedule in the country with increasing impatience lowers the steady-state interest rate. With decreasing impatience, there can be multiple steady-state equilibria.

The rest of the paper is organized as follows. Section 2 presents an intertemporal utility maximization problem with decreasing impatience and points out the difficulty of decreasing impatience. In Section 3, the effects of capital taxation are examined. In Section 4, the effects on capital accumulation of monetary growth are examined. In Section 5, wealth distribution in a two-country economy is investigated. Section 6 concludes the paper.

2 Difficulty under decreasing impatience

2.1 Regular utility maximization problems

To clarify theoretical difficulties of decreasing impatience, begin with checking a necessary and sufficient condition for optimal consumption in the standard model of endogenous time preference. Consider an infinitely-lived consumer who maximizes lifetime utility by choosing the time profile of n-commodity consumptions $\{c(t)\}_{t=0}^{\infty} = \{(c_1(t), \cdots, c_n(t))\}_{t=0}^{\infty}$ and total asset holding $\{a(t)\}_{t=0}^{\infty}$. His or her problem is specified as problem (P) in the following:

$$\text{(P)} \qquad \max \int_0^{\infty} u(c(t)) \exp(-\Delta(t)) \mathrm{d}t, \tag{1}$$

subject to:

$$\dot{a}(t) = h(a(t), c(t)), a(0) = a_0 \text{ (constant)}, a(t) \geq 0,$$

$$\dot{\Delta}(t) = \delta(c(t)), \Delta(0) = 0,$$

where a dot represents the time derivative; $u(c)$ represents the felicity function; Δ denotes a cumulative discount rate with the instantaneous discount rate being given by $\delta(c)$: $\Delta(t) = \int_0^t \delta(c(\tau)) \mathrm{d}\tau$; and $h(a, c)$ is the law of motion for a, which is assumed to be concave in (a, c).[2]

The regularity conditions to make problem (P) well-defined should be treated carefully since decreasing impatience may be inconsistent with those conditions. With the present-value Hamiltonian function defined by

$$H = u(c) \exp(-\Delta) + \lambda^p h(a, c) + \eta^p \delta(c),$$

where λ^p and η^p represent the present-value shadow prices of $\dot{a}$ and $\dot{\Delta}$, respectively,[3] we can prove the following proposition:

Proposition 1: *Suppose that intertemporal utility maximization problem (P) satisfies:*

[2] Asset holding a includes both financial wealth and human capital. We can rewrite the positivity condition for a as the no-Ponzi game condition to bond holdings.

[3] We abbreviate the notation of time when there is no risk of confusion.

1. *$u < 0$;*
2. *u is concave;*
3. *$-u$ is* log-*convex; and*
4. *δ is concave,*

then, the first-order condition:

$$\frac{\partial H}{\partial c} = 0; \frac{\partial H}{\partial a} = -\dot{\lambda}^{p}; \frac{\partial H}{\partial \Delta} = -\dot{\eta}^{p};$$

and the transversality conditions:

$$\lim_{t \to \infty} a\left(t\right) \lambda^{p}\left(t\right) = 0; \lim_{t \to \infty} \Delta\left(t\right) \eta^{p}\left(t\right) = 0,$$

are also sufficient for the optimal consumption plan.

Proof. See Appendix A. ■

Remark 1: Under the conditions given in Proposition 1, the solution to problem (P) may not be unique. The uniqueness of the solution is ensured if conditions 2 through 4 are rewritten by using the "strict" versions of concavity and convexity.

The degree of impatience is increasing or decreasing in wealth as δ is increasing or decreasing in consumption. In what follows, impatience is said to marginally increasing (decreasing), or simply increasing (decreasing), when δ is increasing (decreasing) in c.

Throughout the paper, we assume that regularity conditions 1 through 4 in Proposition 1 are satisfied. Note, however, that these conditions are not related to whether the subjective discount rate δ is increasing or decreasing. What is required to the discount function δ is just concavity: it can be (i) monotonically increasing; (ii) monotonically decreasing; or (iii) hump-shaped as illustrated in Figure 1, where the one-commodity case is depicted. Several researchers (e.g., Fukao and Hamada (1991) and Jafarey and Park (1998)) consider U-shaped nonmonotonic discount-rate functions. Proposition 1 implies that with their discount-rate functions the usual first-order conditions do not ensure optima.

2.2 Difficulties of decreasing impatience

With only few exceptions, it has been assumed that impatience is increasing in the literature. One main reason is that decreasing impatience is likely to destabilize dynamics. Svensson and Razin (1983) criticize this treatment, pointing out that stability cannot be regarded a priori as more reasonable than instability. As a result of the instability, however, there may be no optimal solution when decreasing impatience, as we shall show now.

Under the regularity conditions given in Proposition 1, consider consumer problem (P) with one-commodity by assuming:

- (A1) impatience is decreasing: $\delta_c(c) < 0$; and
- (A2) the rate of interest r is constant,

where $\delta_c(c)$ represents $\mathrm{d}\delta(c)/\mathrm{d}c$.

Let his or her flow budget constraint be given by

$$\dot{a} = ra - c. \tag{2}$$

When we define the generating function g as

$$g(c, \phi) = u(c) - \phi\delta(c),$$

where $\phi(t)$ represents the lifetime utility $U(t)$ from the consumption stream after time t, g generates ϕ by the low of motion,

$$\dot{\phi} = -g(c, \phi) \ \text{ s.t. } \lim_{t\to\infty} \phi(t)\exp(-\Delta(t)) = 0.$$

We assume that $g_c > 0$ and $g_{cc} < 0$.

Let λ represent the current-value shadow price for savings. With the lifetime utility function (1), the optimal conditions are given by[4]

$$g_c(c, \phi) = \lambda, \tag{3}$$

$$\dot{\lambda} = (\delta(c) - r)\lambda, \tag{4}$$

[4] Letting Ω denote the discount facor $\Omega \equiv \exp(-\Delta)$, the current-value shadow price λ can be related to λ^p in Proposition 1 as $\lambda = \lambda^p/\Omega$. The ϕ is the shadow price or the costate variable for the evolution of the discount facor $\Omega : \dot{\Omega} = -\delta\Omega$. It can be related to the present-value costate variable η^p in Proposition 1 as $\eta^p = -\phi\Omega$.

$$\dot{\phi} = -g(c,\phi), \tag{5}$$

$$\lim_{t\to\infty} \exp(-\Delta(t))\,\lambda(t)\,a(t) = 0, \tag{6}$$

$$\lim_{t\to\infty} \exp(-\Delta(t))\,\phi(t) = 0. \tag{7}$$

Define the rate of time preference ρ as $\rho = -d\ln\Omega(t)/dt|_{\dot{c}=0}$, where $\Omega \equiv g_c(c,\phi)\exp(-\Delta)$ represents the present-value marginal utility of c. Then, from equation (3), equation (4) can be rewritten as

$$\dot{c} = -\frac{g_c(c,\phi)}{g_{cc}(c,\phi)}\left(r - \rho(c,\phi)\right), \tag{8}$$

where

$$\rho(c,\phi) = \delta(c) - \frac{g(c,\phi)}{g_c(c,\phi)}\delta_c(c). \tag{9}$$

The optimal path for (c,ϕ,a) must be jointly generated by equations (2), (5), and (8). In particular, local dynamics around steady state should be determined from:

$$\begin{pmatrix} \dot{c} \\ \dot{\phi} \\ \dot{a} \end{pmatrix} = \begin{pmatrix} 0 & \frac{r\delta_c}{g_{cc}} & 0 \\ -g_c & r & 0 \\ -1 & 0 & r \end{pmatrix} \begin{pmatrix} \hat{c} \\ \hat{\phi} \\ \hat{a} \end{pmatrix},$$

where the coefficient matrix is evaluated at the steady state; and a hat $\hat{x}$ denotes a deviation from the steady state value: $\hat{x} = x - x^*$. The characteristic roots for this system are:

$$\begin{aligned} r &> 0,\ \omega' \equiv (1/2)\left\{r + \left(r^2 - \frac{4r\delta_c}{g_{cc}g_c}\right)^{1/2}\right\} > 0,\ \text{and} \\ \omega &\equiv (1/2)\left\{r - \left(r^2 - \frac{4r\delta_c}{g_{cc}g_c}\right)^{1/2}\right\} > 0. \end{aligned} \tag{10}$$

Note that any time paths governed by the roots other than ω cannot satisfy the transversality condition (6). The optimal solution, *if it exists,* is thus given by the eigenvector associated with smallest root ω. As seen from

(10), decreasing impatience implies a positive ω:[5] the resultant saddle path governed by ω is unstable, which is consistent with what has been pointed out in the literature. Given the concavity of δ, this unstable saddle path cannot satisfy the TVC (7) if the initial wealth holding a_0 is larger than the steady state stock $\bar{a}$, as we shall demonstrate just below.

Let c' be any consumption level such that $\delta(c') > 0$. Due to concavity, δ satisfies that for $c > c' - \frac{\delta(c')}{\delta_c(c')}$,

$$\begin{aligned} \delta(c) &< \delta(c') + \delta_c(c')(c - c') \\ &< 0, \end{aligned}$$

implying that δ is negative for the consumption levels which are higher than a finite critical value (see Figure 1). Suppose that $a(0)$ is larger than a^*. Then, if consumption evolves along the saddle trajectory governed by ω, c grows at that rate for ever. After some point in time, δ continues to take negative values. This consumption time-profile definitely violates the transversality condition (7):

Proposition 2 (*difficulty of decreasing impatience*)**:** *Under (A1) and (A2), problem (P) has no optimal solution if* $a(0) > a^*$.

Remark 2: As an important corollary, decreasing impatience is not suitable for analyzing small country models since, in the constant interest-rate economy, decreasing impatience may prevent us from having optimal solution. Intuitively, when consumption divergently grows, the subjective discount rate falls into the negative region, so that the transversality condition (7) is violated.

Remark 2 has an important implication for what Svensson and Razin (1983) discussed. They criticize Obstfeld's result that the Harberger-Laursen-Metzler (HLM) effect cannot occur by claiming that it depends crucially on the assumption of increasing impatience and showing that with decreasing impatience the HLM effect definitely takes place. However, their analysis is limited to a small country model, in which the assumption of decreasing impatience is inconsistent with the existence of the optimal solution, as shown by Remark 2.

Proposition 2 does not imply that we cannot analyze decreasing impatience models in any situations. There are two cases in which we can treat

[5] Root ω can be imaginary. In that case, the real part of ω is positive.

well-behaved models with decreasing impatience: that is, (i) when the models are stabilized by some other decreasing properties in production technology and/or preferences, for which case unstable time paths are excluded; and (ii) when there is some upper bound for consumption resources, which enables even divergent time paths to satisfy the transversality conditions. As a typical example of (i), we incorporate capital accumulation with a usual decreasing-return technology in Sections 3 and 4. The resultant well-behaved models are used to analyze the implications of decreasing impatience for the effects of capital taxation and inflation. As for (ii), Section 5 considers a two-country model, in which consumption resources are bounded by a constant world output level.

3 The effects of capital taxation

Let us examine the implication of decreasing impatience for the effect of capital taxation. Epstein and Hynes (1983) showed that with endogenous time preference capital taxes reduce the steady state capital stock less than under constant time preference. However this result depends crucially on the assumption of increasing impatience. When impatience is decreasing, the result is drastically changed, as we shall show below.

Consider the usual neoclassical model: there are two production factors, labor and capital; there is a single multi-purpose commodity; it is produced by using a constant-to-scale technology F; and firms are competitive. Consumers supply one unit of labor at each instant inelastically. Their preferences are specified just as in the previous section. In particular, we assume decreasing impatience, $\delta_c(c) < 0$. The government levies tax τ on capital income and pays back the revenue to consumers in the lump-sum manner. Letting k represent the capital labor ratio and f a percapita production function satisfying $f_k > 0$ and $f_{kk} < 0$, we can easily obtain a reduced dynamic system as follows:

$$\begin{aligned}
\dot{c} &= -\frac{g_c(c,\phi)}{g_{cc}(c,\phi)}\left((1-\tau)f_k(k) - \rho(c,\phi)\right), \\
\dot{\phi} &= -g(c,\phi), \qquad (11) \\
\dot{k} &= f(k) - c,
\end{aligned}$$

where the steady state equilibrium (c^*, k^*) is jointly determined by:

$$\delta(c^*) = (1-\tau) f_k(k^*), \tag{12}$$

$$f(k^*) = c^*. \tag{13}$$

Assuming the existence of a steady state equilibrium, consider the local dynamics around the steady state. Then, we can prove the following saddle-point stability condition:

Property 1: *The equilibrium time path is uniquely given by the stable arm if and only if:*

$$\delta_c(c^*) f_k(k^*) > (1-\tau) f_{kk}(k^*).$$

Proof. See Appendix B. ■

Assumption 1: $\delta_c(c) f_k(k) > (1-\tau) f_{kk}(k)$.

Remark 3: As shown in Appendix B, if Assumption 1 were not met, the three characteristic roots would be all positive, so that the difficulty shown by Proposition 2 would occur, insofar as the distortion τ is sufficiently small (close to zero) or sufficiently large (close to one).[6]

With Assumption 1, we can illustrate the determination of the steady state capital stock k^* and the effect of capital taxation on it by using Figure 2. From (12) and (13), k^* is determined by

$$\delta(f(k^*)) = (1-\tau) f_k(k^*).$$

As shown in Figure 2, $(1-\tau) f_k(k^*)$ can be depicted as a downward-sloping schedule in the (r, k) plane. With decreasing impatience, $\delta(f(k^*))$ can also be expressed by a downward-sloping schedule, where, from Assumption 1, the $(1-\tau) f_k(k^*)$ schedule is steeper than the $\delta(f(k^*))$ schedule. The steady-state capital stock k^* is given at the intersection, say point E_0, of the two schedules.

[6]When the distortion is neither sufficiently small nor sufficiently large, however, we cannot rule out a priori the (very little) possibility that without Assumption 1 two of the three characteristic roots are negative. In that case, there exist multiple equilibria: we have to be confronted with the indeterminacy problem, instead of the no-existence problem.

Now suppose that capital tax τ is raised from τ_0 to τ_1. It shifts the $(1-\tau) f_k(k^*)$ schedule downward, thereby bringing the steady state point from point E_0 to E_1. Consequently, k^* decreases in response to the tax increase. Note that this reduction in k^* and hence the long-run tax incidence on capital are larger than in the case of constant time preference: if δ were constant, the reduction would stop at k'. This property is contrasted sharply to what Epstein and Hynes (1983) stressed by assuming increasing impatience. When impatience is decreasing, a decrease in k^* makes consumers impatience, which raises the long-run interest rate irrespective of the capital tax increase, and thereby causes a further reduction in k^*.

Implication 1: *When impatience is decreasing, an increase in capital taxes raises the long-run interest rate, so that the long-run tax incidence on capital is larger than in the cases of constant time preference and of increasing impatience.*

4 Inflation and growth with decreasing impatience

Many empirical studies have reported that inflation harms capital accumulation (e.g., Fischer (1993), De Gregorio (1992, 1993), Barro (1995), and Bruno and Easterly (1998)), whereas theoretical models including Epstein and Hynes (1983) often predict the Tobin effect that inflation promotes growth by shifting real money balances away in favor of consumption. As the second implication of decreasing impatience, we shall show that an increase in the rate of nominal money growth reduces the steady state capital stock.

Let us extend the model in section 3 by introducing money in the "money-in-the-utility-function" framework. For brevity we specify the subjective discount rate as a function of the felicity level, $\delta = \delta(u)$ as in Uzawa (1968), where decreasing impatience is assumed by setting $\delta'(u) < 0$. The consumer's problem is now given by

$$\max \int_0^\infty u(c, m) \exp(-\Delta) \mathrm{d}t, \tag{14}$$

subject to:

$$\dot{\Delta} = \delta(u(c, m)), \tag{15}$$

$$\dot{a} = ra + w - c - (r + \pi) m + x, \tag{16}$$

$$a = k + m, \tag{17}$$

where a represents total financial wealth; m real money holdings; w the wage rate; π the rate of inflation; and x the lump-sum transfer payments from the government.

Assume that the regularity condition given in Proposition 1 is satisfied. Letting $g(c, m, \phi)$ be $u(c, m) - \phi\delta(u(c, m))$, the optimal consumption plan should satisfy:

$$g_c(c, m, \phi) = \lambda,$$

$$\chi(c, m) \equiv \frac{u_m(c, m)}{u_c(c, m)} = r + \pi, \tag{18}$$

$$\dot{\lambda} = (\delta(u(c, m)) - r)\lambda,$$

$$\dot{\phi} = -g(c, m, \phi),$$

and the transversality conditions. In the same way to obtain (8), the first-order conditions can be reduced to:

$$\frac{\dot{c}}{c} = -\frac{g_c(c, m, \phi)}{c g_{cc}(c, m, \phi)}(r - \rho(c, m, \phi)),$$

$$\frac{\dot{m}}{m} = -\frac{g_m(c, m, \phi)}{m g_{mm}(c, m, \phi)}(r - \rho(c, m, \phi)), \tag{19}$$

where $\rho(c, m, \phi)$ is defined using $g(c, m, \phi)$ as in (9). We assume that $g_c > 0$, $g_m > 0$, $g_{cc} < 0$, $g_{mm} < 0$, and $g_{cm} = 0$.

Money is supplied by the government in the form of "helicopter money:"

$$\mu m = x, \tag{20}$$

where μ denotes the growth rate of nominal money supply.

From the profit-maximizing behavior of firms, r and w are given as

$$w = f(k) - k f_k(k); r = f_k(k). \tag{21}$$

Substituting (17), (20), and (21) into (16) yields

$$\dot{k} = f(k) - c.$$

By combining $\frac{\dot{m}}{m} = \mu - \pi$ with (18) and (19), we obtain

$$\frac{g_m}{mg_{mm}}(\rho(c, m, \phi) - f_k(k)) = \mu + f_k(k) - \chi(c, m),$$

which can be solved for m as

$$m = \upsilon(c, \phi, k; \mu).$$

By using this, the dynamic system is finally reduced to:

$$\dot{c} = -\frac{g_c\left(c, \upsilon(c, \phi, k; \mu), \phi\right)}{g_{cc}\left(c, \upsilon(c, \phi, k; \mu), \phi\right)}\left(f_k(k) - \rho\left(c, \upsilon(c, \phi, k, \mu), \phi\right)\right),$$

$$\dot{\phi} = -g\left(c, \upsilon(c, \phi, k; \mu), \phi\right),$$

$$\dot{k} = f\left(k\right) - c.$$

The steady state equilibrium (c^*, m^*, k^*) is determined by

$$\delta\left(u\left(c^*, m^*\right)\right) = f_k(k^*),$$

$$\chi(c^*, m^*) = f_k(k^*) + \mu, \tag{22}$$

$$f\left(k^*\right) = c^*.$$

By analyzing the local dynamics around the steady state, we can show the following:

Property 2: *The equilibrium time path is saddle-point stable only if*

$$\Lambda \equiv (1 - \frac{\chi\chi_c}{\chi_m})\delta_u u_c f_k - f_{kk} + \frac{\chi}{\chi_m}\delta_u u_c f_{kk} > 0.$$

Proof. See Appendix C. ■

Assumption 2: $\Lambda > 0$.

Remark 4: Assumption 2 is a necessary condition for the system to be saddle-point stable, under which the determinant of the coefficient matrix is negative. As shown in Appendix C, the trace of the matrix, $2r + \frac{g_m r \delta_u u_m}{m g_{mm} \chi_m}$, should be positive for the saddle-point stability. Otherwise all of the three roots might be negative, implying that there might be multiple equilibria.

Let us now examine the effect of a permanent increase in the core rate of inflation μ. From (22), the steady state equilibrium is determined by

$$\delta\left(u\left(f\left(k^*\right), m^*\right)\right) = f_k(k^*), \tag{23}$$

$$\chi(f\left(k^*\right), m^*) = f_k(k^*) + \mu. \tag{24}$$

Figure 3 depicts the determination, where schedule KK' represents (23) and MM' (24). The MM' schedule is positively sloping and, under Assumption 2, the slope of the KK' schedule is also positive and gentler than that of MM'. The steady state is determined at the intersection of the two schedules. An increase in μ shifts the MM' schedule upward, bringing the steady state point from E_0 to E_1. As illustrated, the steady state capital stock decreases in response to the rise in the inflation rate. In fact, from (23) and (24), we can derive

$$\frac{\mathrm{d}k^*}{\mathrm{d}\mu} = -\frac{\delta_u u_m}{\chi_m \Lambda} < 0. \tag{25}$$

This result can be summarized as follows:

Implication 2: *When impatience is decreasing, an increase in the core rate of inflation harms capital accumulation, that is the Tobin effect does not take place.*

Remark 5: As seen from (25), whether an increase in μ increases or decreases k^* depends crucially on the sign of $\partial\delta/\partial m$, but not on $\partial\delta/\partial c$, that is on whether impatience with respect to m is increasing or decreasing, but not on whether impatience with respect to c is increasing or decreasing. When δ depends on (c, m) independently of u, and hence when preferences are weakly non-separable in the sense of Shi (1994), the same result as in implication 2 is valid even if impatience is increasing in c.

5 Wealth distribution dynamics in a two-country world economy

Let us finally consider the implication of decreasing impatience in a two-country model. Suppose that the world economy is composed of countries

1 and 2, each of which is populated with infinitely-lived identical agents. The representative agent in each country consumes a single consumption good and holds wealth in the form of bonds. Both the goods and bonds are traded freely in international markets. For brevity, the representative agents in country i $(i = 1, 2)$ are assumed to be endowed with constant amounts y^i. The budget constraint for the representative agent is given by

$$\dot{b}^i = rb^i + y^i - c^i, \tag{26}$$

where b^i represents net foreign assets held by country i.

By assuming the same preference structure as specified in section 2.2, the optimal conditions can be obtained in exactly the same way as in (3) through (7). They reduce to, for $i = 1, 2$,

$$\dot{c}^i = \sigma^i \left\{ r - \rho^i(c^i, \phi^i) \right\}, \tag{27}$$

$$\dot{\phi}^i = \delta^i \left(c^i \right) \phi^i - u^i \left(c^i \right),$$

and the transversality conditions, where $\sigma^i = -g^i_c / g^i_{cc}$.

The market-clearing conditions are given by

$$c^1(t) + c^2(t) = Y, \tag{28}$$

$$b^1(t) + b^2(t) = 0,$$

where Y represents the aggregate output, $y^1 + y^2$. The interest rate is endogenously determined by the market clearing condition: by differentiating (28) and substituting (27) into the result, we can obtain

$$r = \frac{\sigma^1}{\sigma^1 + \sigma^2} \rho^1(c^1, \phi^1) + \frac{\sigma^2}{\sigma^1 + \sigma^2} \rho^2(c^2, \phi^2), \tag{29}$$

implying that the equilibrium interest rate is determined as a weighted sum of the rates of time preference in the two countries.

From (28) and (29), the dynamic system (26) through (27) can be reduced to:

$$\dot{c}^1 = \frac{\sigma^1 \sigma^2}{\sigma^1 + \sigma^2} \left\{ \rho^2(Y - c^1, \phi^2) - \rho^1(c^1, \phi^1) \right\},$$

$$\dot{\phi}^1 = \delta^1 \left(c^1 \right) \phi^1 - u^1 \left(c^1 \right),$$

$$\dot{\phi}^2 = \delta^2\left(Y - c^1\right)\phi^2 - u^2\left(Y - c^1\right),$$

$$\dot{b}^1 = \left\{\frac{\sigma^1}{\sigma^1+\sigma^2}\rho^1(c^1,\phi^1) + \frac{\sigma^2}{\sigma^1+\sigma^2}\rho^2(Y - c^1,\phi^2)\right\} b^1 + y^1 - c^1,$$

the steady state of which is determined by

$$\delta^1\left(c^{1*}\right) = \delta^2\left(Y - c^{1*}\right) = r^*,$$

$$\phi^{1*} = u^1\left(c^{1*}\right)/r^*, \phi^{2*} = u^2\left(Y - c^{1*}\right)/r^*,$$

$$c^{1*} = r^* b^{1*} + y^1, c^{2*} = r^* b^{2*} + y^2.$$

By analyzing the local dynamics around the steady state, we can obtain the following property:

Property 3: *The steady-state point is locally saddle-point stable if and only if*

$$\delta_c^1\left(c^{1*}\right) + \delta_c^2\left(Y - c^{1*}\right) > 0. \tag{30}$$

Proof. See Appendix D. ■

Remark 6: Unlike in the previous sections, stability (in the saddle-point sense) is not necessary for an equilibrium path to exist. Even if (30) is not satisfied and hence even if the steady state is unstable, the time-path that governed by the smallest root can be an equilibrium. In the previous sections, consumption is not bounded from the above, so that any divergent consumption paths, sooner or later, make the discount rate negative and thereby violate the transversality condition. In the present two country setting, in contrast, consumption is bounded by the total quantity Y of world output, which allows a divergent equilibrium consumption time-path. In particular, this is the case when $\min\left(\delta^1(Y), \delta^2(Y)\right) > 0$, for which case the discount rates remain positive for any divergent time-paths of the economy.

As implied by Property 3, if impatience is decreasing in both of the two countries, the equilibrium time path is unstable. For a steady-state equilibrium to be saddle-point stable, at least one of the two countries should display increasing impatience.[7] Suppose that one of the two, say country 2, exhibits increasing impatience, whereas the other's, i.e., country 1's impatience is decreasing. For a steady-state equilibrium allocation to be saddle-point stable, a marginal transfer from country 1 to country 2 should make the recipient country 2 less patient.[8] Figures 4(a) and 4(b) illustrate the determination of the steady-state equilibrium and equilibrium dynamics in the (c^1, r)-plane in the stable and unstable cases, respectively. The δ^1 and δ^2 are both depicted as decreasing functions in c^1. The steady-state equilibrium is determined at the intersection E of the two schedules. As is shown, steady state is stable if and only if the δ^1-schedule cuts the δ^2-schedule from the below.[9]

Using Figure 4(a), consider the effect of an upward shift of country 2's subjective discount rate schedule δ^2. This brings the steady state point from E to E', lowering the steady state interest rate. The upward shift in δ^2, *ceteris paribus,* makes country 2 less patience than country 1, which induces wealth movements from country 2 to 1 and thereby reduces the interest rate along the δ^1-schedule.

Implication 3: *Suppose that a country exhibits decreasing impatience whereas the other displays increasing impatience, and that saddle-point stability condition (30) is met. Then, an upward shift of the subjective discount rate schedule in the country with increasing impatience lowers the steady-state interest rate.*

With decreasing impatience, there can be multiple steady-state equilibria. Figures 5(a) and 5(b) illustrate typical possibilities. First, consider the case as illustrated in Figure 5(a). Only when the initial wealth distribution is sufficiently equal, the economy will converge to the steady state point

[7] For the analysis of the two-country world economy with increasing impatience, see Devereux and Shi (1991).

[8] The same condition is derived by Jafarey and Park (1998).

[9] In Figure 4 (b), the divergent time paths towards $(c_1, c_2) = (Y, 0)$ and $(0, Y)$ are equilibria, if

$$\lim_{c^i \to 0} \frac{g_c^i\left(c^i, u^i\left(c^i\right)/\delta^i\left(c^i\right)\right)}{g_{cc}^i\left(c^i, u^i\left(c^i\right)/\delta^i\left(c^i\right)\right)} = 0, i = 1, 2,$$

for which case points $(Y, 0)$ and $(0, Y)$ are steady state equibria that are saddle-point stable. It can be shown that a sufficient condition for the above equation to be vaild is: (i) δ^i is a function of u^i; (ii) u^i displays constant relative risk aversion; and (iii) $\phi\delta_{uu}/\left(1 - \phi\delta_u\right)$ is bounded.

S with both countries consuming positive quantities of goods. If the initial wealth is distributed quite unequally, the economy will asymptotically converge to the steady state point $(c_1, c_2) = (Y, 0)$ or $(0, Y)$, at which only initially wealthy country has positive consumption. In this sense, co-prosperity needs sufficiently equal initial wealth distribution. Figure 5(b) illustrates another multiple-equilibrium case, where there is unstable steady state point U with fairly equal wealth distribution, placed between two equilibria S_1 and S_2 with uneven distributions. In this case, unequal wealth distributions are both stable and long-run wealth distribution is likely to be uneven. There is a threshold level for initial wealth holding that determines which unequal distribution of S_1 and S_2 takes place.

6 Conclusions

This paper has examined theoretical as well as policy implications of decreasing impatience. We have first shown that in constant interest-rate economies divergent consumption time-paths resulting from decreasing impatience are often inconsistent with optimality. This difficulty disappears (i) when there are some deceasing-return properties built in the model; and/or (ii) when there is some upper bound for consumption resources. Based on this observation, we have considered well-behaved decreasing-impatience models and investigated their policy implications.

In the neoclassical model with decreasing impatience, an increase in capital taxes raises the long-run interest rate and the long-run tax incidence on capital are larger than in the case of constant time preference and of increasing impatience. When money is introduced into this model, the Tobin effect does not hold and inflation harms capital accumulation, as many empirical studies have reported. In the two-country context, decreasing impatience leads to various interesting multiple steady-state equilibria. We have shown the existence of an equilibrium in which an upward shift of a country's discount rate function lowers the world interest rate.

A Proof of Proposition 1:

Let us define $q(c,\Delta)$ as

$$q(c,\Delta) = u(c)\exp(-\Delta),$$

and the Hamiltonian as

$$H = q(c,\Delta) + \lambda^p h(a,c) + \eta^p \delta\left(c\right).$$

Suppose that (c^*, a^*, Δ^*) satisfies the first-order condition and the transversality condition of problem (P):

$$\left(\frac{\partial H}{\partial c_i} =\right) q_{c_i}\left(c^*,\Delta^*\right) + \lambda^p h_{c_i}\left(a^*,c^*\right) + \eta^p \delta_{c_i}\left(c^*\right) = 0,\ i = 1,\cdots,n, \tag{A.1}$$

$$\left(\frac{\partial H}{\partial a} =\right) \lambda^p h_a\left(a^*,c^*\right) = -\dot{\lambda}^p, \tag{A.2}$$

$$\left(\frac{\partial H}{\partial \Delta} =\right) q_\Delta\left(c^*,\Delta^*\right) = -\dot{\eta}^p, \tag{A.3}$$

$$\lim_{t\to\infty} a^*\left(t\right)\lambda^p\left(t\right) = 0, \tag{A.4}$$

$$\lim_{t\to\infty} \Delta^*\left(t\right)\eta^p\left(t\right) = 0. \tag{A.5}$$

Then, the following lemmas hold.

Lemma A.1: q is concave in (c,Δ) if and only if (i) $u < 0$; (ii) u is concave, and (iii) $-u$ is log-convex.

Proof. Because $q_{c_ic_j} = u_{c_ic_j}\exp(-\Delta)$, $q_{c_i\Delta} = -u_{c_i}\exp(-\Delta)$, $q_{\Delta\Delta} = u\exp(-\Delta)$, the necessary and sufficient condition for the concavity of q (or the convexity of $-q$) is that

$$X \equiv \begin{pmatrix} -u_{c_1c_1} & \cdots & -u_{c_nc_1} & u_{c_1} \\ \vdots & \ddots & \vdots & \vdots \\ -u_{c_1c_n} & \cdots & -u_{c_nc_n} & u_{c_n} \\ u_{c_1} & \cdots & u_{c_n} & -u \end{pmatrix}$$

is a positive semi-definite matrix, i.e., any determinant of the principal submatrix of X is non-negative:

$$\left|X_{\{i_1,\cdots,i_m\}}\right| \geq 0, \text{ for any } \{i_1,\cdots,i_m\} \text{ satisfying } 1 \leq i_1 < \cdots < i_m \leq n+1,$$

where $X_{\{i_1,\cdots,i_m\}}$ denotes the principal submatrix of X which lies in rows $i_1,\cdots,i_m$ and columns $i_1,\cdots,i_m$ $\left(\text{e.g., } X_{\{1,n+1\}} = \begin{pmatrix} -u_{c_1c_1} & u_{c_1} \\ u_{c_1} & -u \end{pmatrix}\right)$. Then, we can prove the following properties:

(a) $\left|X_{\{n+1\}}\right| \geq 0$ iff $u < 0$;

(b) $\left|X_{\{i_1,\cdots,i_m\}}\right| \geq 0$ for any $\{i_1,\cdots,i_m\}$ satisfying $1 \leq i_1 < \cdots < i_m \leq n$ iff u is concave; and

(c) $\left|X_{\{i_1,\cdots,i_m,n+1\}}\right| \geq 0$ for any $\{i_1,\cdots,i_m\}$ satisfying $1 \leq i_1 < \cdots < i_m \leq n$ iff $-u$ is log-convex.

Since (a) and (b) are obvious, let us sketch the proof of (c). The necessary and sufficient condition for the log-convexity of $-u$ is that

$$Z \equiv \begin{pmatrix} uu_{c_1c_1} - u_{c_1}u_{c_1} & \cdots & uu_{c_nc_1} - u_{c_n}u_{c_1} \\ \vdots & \ddots & \vdots \\ uu_{c_1c_n} - u_{c_1}u_{c_n} & \cdots & uu_{c_nc_n} - u_{c_n}u_{c_n} \end{pmatrix}$$

is a positive semi-definite matrix, i.e., any determinant of the principal submatrix of Z is non-negative:

$$\left|Z_{\{i_1,\cdots,i_m\}}\right| \geq 0, \text{ for any } \{i_1,\cdots,i_m\} \text{ satisfying } 1 \leq i_1 < \cdots < i_m \leq n.$$

Now, we can show that

$$\left|X_{\{i_1,\cdots,i_m,n+1\}}\right| = (-u)^{-m+1}\left|Z_{\{i_1,\cdots,i_m\}}\right|.$$

For example,

$$\begin{aligned} \left|X_{\{1,\cdots,n,n+1\}}\right| &= \begin{vmatrix} -u_{c_1c_1} & \cdots & -u_{c_nc_1} & u_{c_1} \\ \vdots & \ddots & \vdots & \vdots \\ -u_{c_1c_n} & \cdots & -u_{c_nc_n} & u_{c_n} \\ u_{c_1} & \cdots & u_{c_n} & -u \end{vmatrix} \\ &= (-u)^{-n}\begin{vmatrix} uu_{c_1c_1} & \cdots & uu_{c_nc_1} & u_{c_1} \\ \vdots & \ddots & \vdots & \vdots \\ uu_{c_1c_n} & \cdots & uu_{c_nc_n} & u_{c_n} \\ -uu_{c_1} & \cdots & -uu_{c_n} & -u \end{vmatrix} \end{aligned}$$

$$\begin{aligned}
&= (-u)^{-n} \begin{vmatrix} uu_{c_1c_1} - u_{c_1}u_{c_1} & \cdots & uu_{c_nc_1} - u_{c_n}u_{c_1} & u_{c_1} \\ \vdots & \ddots & \vdots & \vdots \\ uu_{c_1c_n} - u_{c_1}u_{c_n} & \cdots & uu_{c_nc_n} - u_{c_nc_n} & u_{c_n} \\ 0 & \cdots & 0 & -u \end{vmatrix} \\
&= (-u)^{-n+1} \begin{vmatrix} uu_{c_1c_1} - u_{c_1}u_{c_1} & \cdots & uu_{c_nc_1} - u_{c_n}u_{c_1} \\ \vdots & \ddots & \vdots \\ uu_{c_1c_n} - u_{c_1}u_{c_n} & \cdots & uu_{c_nc_n} - u_{c_n}u_{c_n} \end{vmatrix} \\
&= (-u)^{-n+1} \left| Z_{\{1,\cdots,n\}} \right|.
\end{aligned}$$

Properties (a), (b), and (c) imply lemma A.1. ∎

Therefore, Proposition 1 follows from Lemma A.2 below:

Lemma A.2: Suppose that the following conditions are met:

1. q is concave in (c, Δ);
2. δ is concave in c; and
3. h is concave in (a, c).

Then, we have

$$D \equiv \int_0^\infty \{q(c^*, \Delta^*) - q(c, \Delta)\} \, \mathrm{d}t \geq 0$$

for any feasible paths (c, a, Δ).

Proof. D satisfies

$$\begin{aligned}
D &= \int_0^\infty \{q(c^*, \Delta^*) - q(c, \Delta)\} \, \mathrm{d}t \\
&\geq \int_0^\infty \left\{ \sum_{i=1}^n (c_i^* - c_i) q_{c_i}(c^*, \Delta^*) + (\Delta^* - \Delta) q_\Delta(c^*, \Delta^*) \right\} \mathrm{d}t \text{ (the concavity of } q) \\
&= \int_0^\infty \left\{ \sum_{i=1}^n (c_i^* - c_i) \left(-\lambda^p h_{c_i}(a^*, c^*) - \eta^p \delta_{c_i}(c^*)\right) + (\Delta^* - \Delta)(-\dot{\eta}^p) \right\} \mathrm{d}t \\
&= I_1 + I_2,
\end{aligned}$$

where I_1 and I_2 are:

$$\begin{aligned}
I_1 &= \int_0^\infty \left\{ \sum_{i=1}^n (c_i^* - c_i) \left(-\lambda^p h_{c_i}(a^*, c^*)\right) \right\} \mathrm{d}t, \\
I_2 &= \int_0^\infty \left\{ \sum_{i=1}^n (c_i^* - c_i) \left(-\eta^p \delta_{c_i}(c^*)\right) + (\Delta^* - \Delta)(-\dot{\eta}^p) \right\} \mathrm{d}t.
\end{aligned}$$

I_1 and I_2 are both positive as I shall show now. I_1 satisfies:

$$\begin{aligned}
I_1 &= \int_0^\infty \left\{ \sum_{i=1}^n (c_i^* - c_i) \left(-\lambda^p h_{c_i}(a^*, c^*)\right) + (a^* - a)\left(-\lambda^p h_a(a^*, c^*) - \dot{\lambda}^p\right) \right\} \mathrm{d}t \\
&\geq \int_0^\infty \left\{ -\lambda^p \left(h(a^*, c^*) - h(a, c)\right) - (a^* - a)\dot{\lambda}^p \right\} \mathrm{d}t \text{ (the concavity of } h) \\
&= \int_0^\infty \left\{ -(\dot{a}^* - \dot{a})\lambda^p - (a^* - a)\dot{\lambda}^p \right\} \mathrm{d}t \\
&= -\left[(a^* - a)\lambda^p\right]_0^\infty \\
&= \lim_{t\to\infty} a(t)\lambda^p(t) \;\; (a^*(0) = a(0) = a_0) \\
&\geq 0 \; (\lambda^p(t) \geq 0).
\end{aligned}$$

As for I_2, we have:

$$\begin{aligned}
I_2 &= \int_0^\infty \left\{ \sum_{i=1}^n (c_i^* - c_i)\left(-\eta^p \delta_{c_i}(c^*)\right) + \left(\dot{\Delta}^* - \dot{\Delta}\right)\eta^p \right\} \mathrm{d}t - \left[(\Delta^* - \Delta)\eta^p\right]_0^\infty \\
&= \int_0^\infty \eta^p \left\{ \delta(c^*) - \delta(c) - \sum_{i=1}^n (c_i^* - c_i)\left(\delta_{c_i}(c^*)\right) \right\} \mathrm{d}t - \left[(\Delta^* - \Delta)\eta^p\right]_0^\infty \\
&\geq -\left[(\Delta^* - \Delta)\eta^p\right]_0^\infty \;\; \text{(the concavity of } \delta) \\
&= \lim_{t\to\infty} \Delta(t)\eta^p(t) \\
&\geq 0 \; (\eta^p(t) \geq 0 \text{ (See footnote 3 in the text.)}).
\end{aligned}$$

This implies $D \geq 0$. ■

B Proof of Property 1

By linearizing system (11) around the steady state, the local dynamic system can be obtained as

$$\begin{pmatrix} \dot{c} \\ \dot{\phi} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} 0 & \frac{\delta\delta_c}{g_{cc}} & -\frac{g_c(1-\tau)f_{kk}}{g_{cc}} \\ -g_c & \delta & 0 \\ -1 & 0 & f_k \end{pmatrix} \begin{pmatrix} \hat{c} \\ \hat{\phi} \\ \hat{k} \end{pmatrix},$$

where the coefficient matrix is evaluated at the steady state. For this coefficient matrix,

$$\text{trace} = \delta + f_k > 0, \det = \frac{\delta g_c}{g_{cc}} \left(\delta_c f_k - (1-\tau) f_{kk} \right).$$

The linear system thus has two positive and one negative roots if and only $\delta_c f_k - (1-\tau) f_{kk} > 0$, as stated in Property 1.

When $\delta_c f_k - (1-\tau) f_{kk} < 0$, the determinant of the coefficient matrix is positive, implying that (i) the three roots are all positive; or that (ii) two roots are negative and the other one is positive.. It is easy to show that case (ii) cannot occur insofar as the capital tax rate τ is sufficiently small or sufficiently large, as in Remark 3. For example, when $\delta_c f_k - (1-\tau) f_{kk} < 0$, the three roots are all positive if

$$\frac{g_c f_{kk}}{g_{cc}} \left(\delta_c f_k - (1-\tau) f_{kk} \right) + \delta f_k - \tau \frac{g_c}{g_{cc}} \delta_c f_k > 0.$$

This inequality indeed holds valid when $\tau = 0$ or 1.

C Proof of Property 2

Noting that

$$\begin{aligned} \upsilon_c &= -\frac{\chi_m}{\chi_c}, \\ \upsilon_\phi &= -\frac{r\delta_u u_m}{m g_{mm} \chi_m}, \\ \upsilon_k &= -\frac{f_{kk}}{\chi_m} - \frac{g_m}{m g_{mm} \chi_m} f_{kk}, \end{aligned}$$

the local dynamic system can be obtained as

$$\begin{pmatrix} \dot{c} \\ \dot{\phi} \\ \dot{k} \end{pmatrix} = A \begin{pmatrix} \hat{c} \\ \hat{\phi} \\ \hat{k} \end{pmatrix};$$

$$A = \begin{pmatrix} 0 & \frac{r\delta_u u_c}{g_{cc}} & -\frac{g_c}{g_{cc}} f_{kk} \\ -g_c - g_m(-\frac{\chi_m}{\chi_c}) & r - g_m(-\frac{r\delta_u u_m}{m g_{mm} \chi_m}) & -g_m(\frac{f_{kk}}{\chi_m} + \frac{g_m}{m g_{mm} \chi_m} f_{kk}) \\ -1 & 0 & r \end{pmatrix},$$

where coefficient matrix A is evaluated at the steady state. Then, the linear system has two positive and one negative roots only if

$$\det(A) = \frac{g_m}{g_{cc}\chi_m} r\delta_u u_c f_{kk} + \frac{g_c r}{g_{cc}}\left[(1 - \frac{g_m}{g_c}\frac{\chi_c}{\chi_m}) r\delta_u u_c - f_{kk}\right] < 0,$$

or alternatively stated, only if

$$\Lambda = (1 - \chi\frac{\chi_c}{\chi_m}) r\delta_u u_c - f_{kk} + \frac{\chi}{\chi_m}\delta_u u_c f_{kk} > 0,$$

as in Property 2. Note also that if $\Lambda > 0$ and if

$$(\text{trace}\,(A) =)\; 2r + \frac{g_m r\delta_u u_m}{m g_{mm}\chi_m} > 0,$$

the system indeed has two positive and one negative roots, and is saddle-point stable, as demonstrated in Remark 4.

D Proof of Property 3

By linearizing the dynamic system around the steady state, we obtain

$$\begin{pmatrix} \dot{c}^1 \\ \dot{\phi}^1 \\ \dot{\phi}^2 \\ \dot{b} \end{pmatrix} = \begin{pmatrix} B & O_{3\times1} \\ C & r \end{pmatrix} \begin{pmatrix} \hat{c}^1 \\ \hat{\phi}^1 \\ \hat{\phi}^2 \\ \hat{b} \end{pmatrix};$$

$$B = \begin{pmatrix} 0 & -\left(\frac{\sigma^1\sigma^2}{\sigma^1+\sigma^2}\right)\frac{r\delta_c^1}{g_c^1} & \left(\frac{\sigma^1\sigma^2}{\sigma^1+\sigma^2}\right)\frac{r\delta_c^2}{g_c^2} \\ -g_c^1 & r & 0 \\ g_c^2 & 0 & r \end{pmatrix},$$

where the coefficient matrix is evaluated at the steady state. The roots of this linear system are thus r and three eigenvalues of B. Because trace$(B) = 2r > 0$, B has two positive and one negative eigenvalues when

$$\det(B) = -r^2\frac{\sigma^1\sigma^2}{\sigma^1+\sigma^2}\left(\delta_c^1 + \delta_c^2\right) < 0.$$

Therefore, if $\delta_c^1 + \delta_c^2 > 0$, the linear system has three positive and one negative roots.

References

Barro, R.J., 1995, Inflation and economic growth, Bank of England Quarterly Bulletin, May, 166-176.

Becker, G.S. and C.B. Mulligan, 1997, The endogenous determination of time preference, Quarterly Journal of Economics 112 (3), 729-758.

Bruno, M. and W. Easterly, 1998, Inflation crises and long-run growth, Journal of Monetary Economics 41 (1), 3-26.

De Gregorio, J., 1992, The effects of inflation on economic growth, European Economic Review 36 (2-3), 417-424.

De Gregorio, J., 1993, Inflation, taxation and long-run growth, Journal of Monetary Economics 31, 271-298.

Devereux, M. and S. Shi, 1991, Capital accumulation and the current account in a two-country model, Journal of International Economics 30, 1-25.

Epstein, L.G., 1987a, A simple dynamic general equilibrium model, Journal of Economic Theory 41, 68-95.

Epstein, L.G., 1987b, The global stability of efficient intertemporal allocations, Econometrica 55, 329-355.

Epstein, L.G. and J.A. Hynes, 1983, The rate of time preference and dynamic economic analysis, Journal of Political Economy 91, 611-625.

Fischer, S., 1993, The role of macroeconomic factors in growth, Journal of Monetary Economics 32 (3), 485-511.

Fukao, K. and K. Hamada, 1991, The Fisherian time preference and the long-run capital ownership pattern in a global economy, NIRA Research Output 4, no.1, 86-97.

Jafarey, S. and H. Park, 1998, The dynamics of optimal wealth distributions with recursive utility, Economic Letters 61, 149-158.

Lawrance, E.C., 1991, Poverty and the rate of time preference, Evidence from panel data, Journal of Political Economy 99, 54-77.

Lucas, R. and N. Stokey, 1984, Optimal growth with many consumers, Journal of Economic Theory 32, 139-171.

Obstfeld, M., 1982, Aggregate spending and the terms of trade: Is there a Laursen-Metzler effect? Quarterly Journal of Economics 97, 251-270.

Obstfeld, M., 1990, Intertemporal dependence, impatience, and dynamics, Journal of Monetary Economics 26, 45-76.

Ogawa, K., 1993, Economic development and time preference schedule: The case of Japan and East Asian NICs, Journal of Development Economics 42, 175-195.

Samwick, A.A., 1997, Discount rate heterogeneity and social security reform, NBER Working Paper 6219.

Shi, S., 1994, Weakly nonseparable preference and distortionary taxes in a small open economy, International Economic Review 35, 411-428.

Svensson, L.E.O. and A. Razin, 1983, The terms of trade and the current account: The Harberger-Laursen-Metzler effect, Journal of Political Economy 91, 97-125.

Uzawa, H., 1968, Time Preference, the consumption function and optimum asset holdings, in: J.N. Wolfe, ed., Value Capital and Growth: Papers in Honour of Sir John Hicks (Chicago: Aldine).

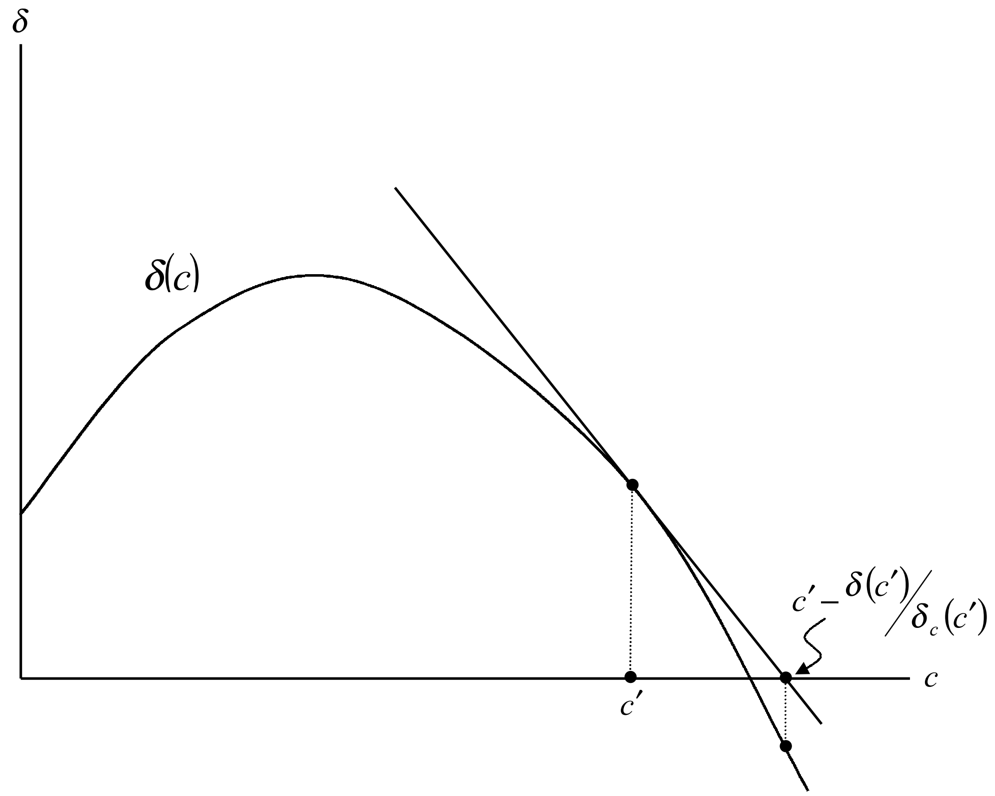

Figure 1. Non-monotonic impatience

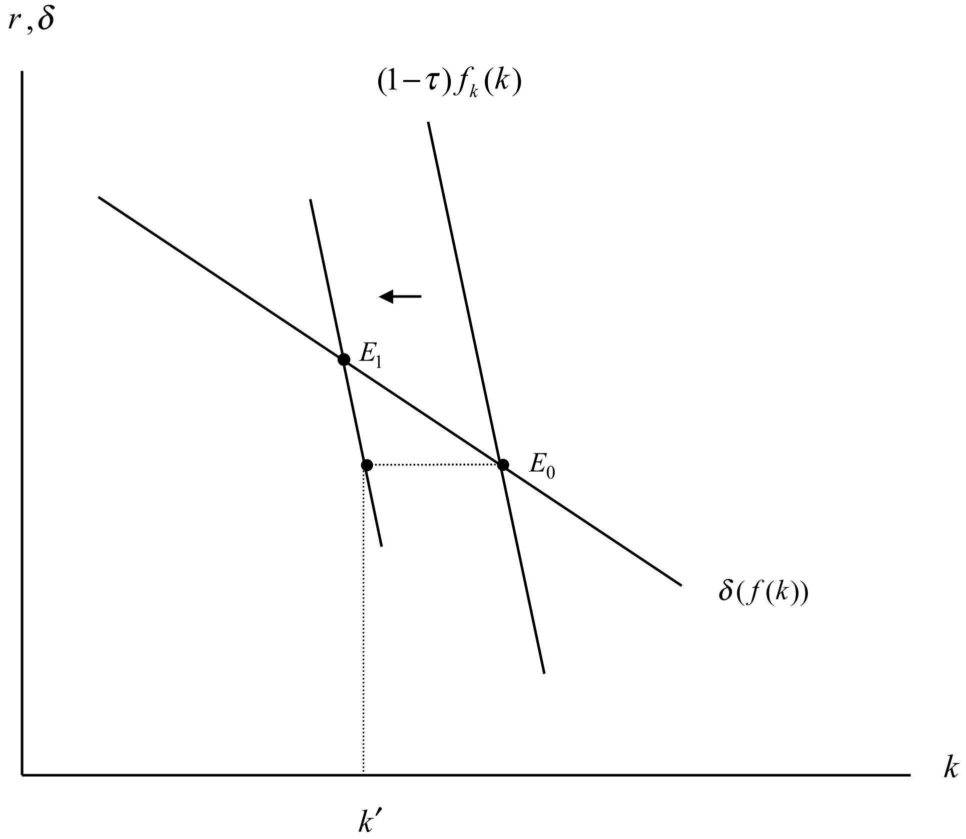

Figure 2. Capital taxation under decreasing impatience

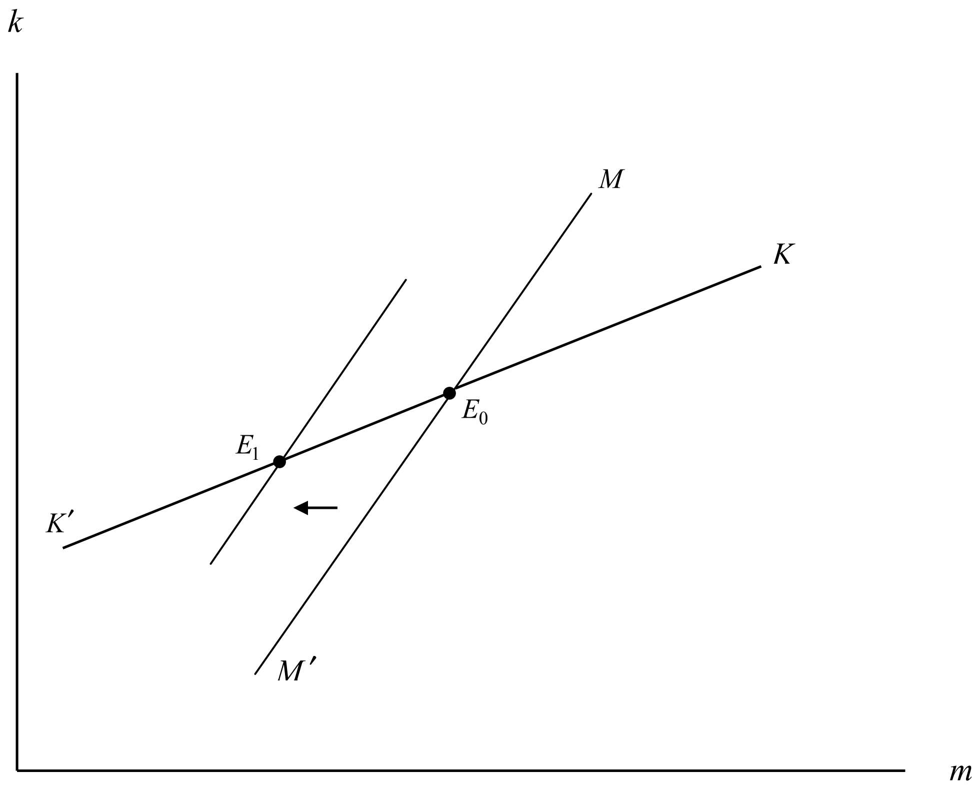

Figure 3. Inflation and capital stock under decreasing impatience

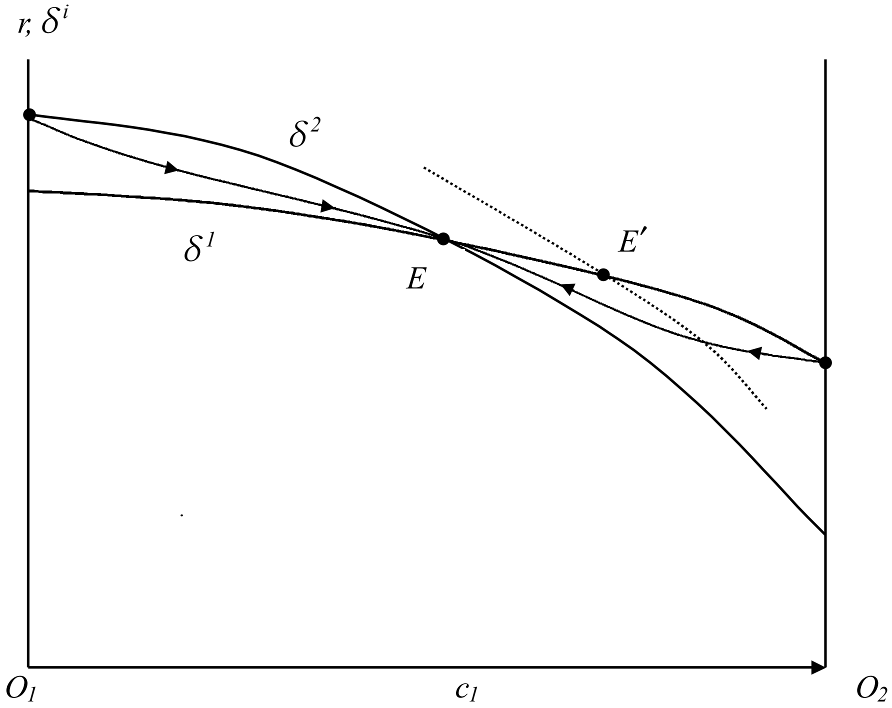

Figure 4 (a). Two country equilibrium:
The case of saddle-point stability

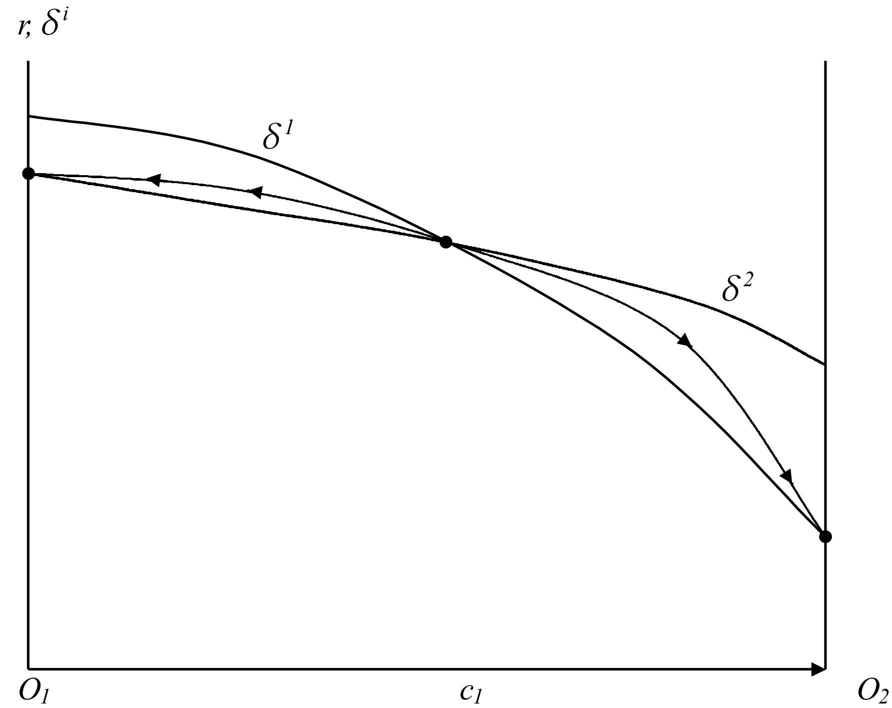

Figure 4 (b). Two country equilibrium:
The case of unstable steady state

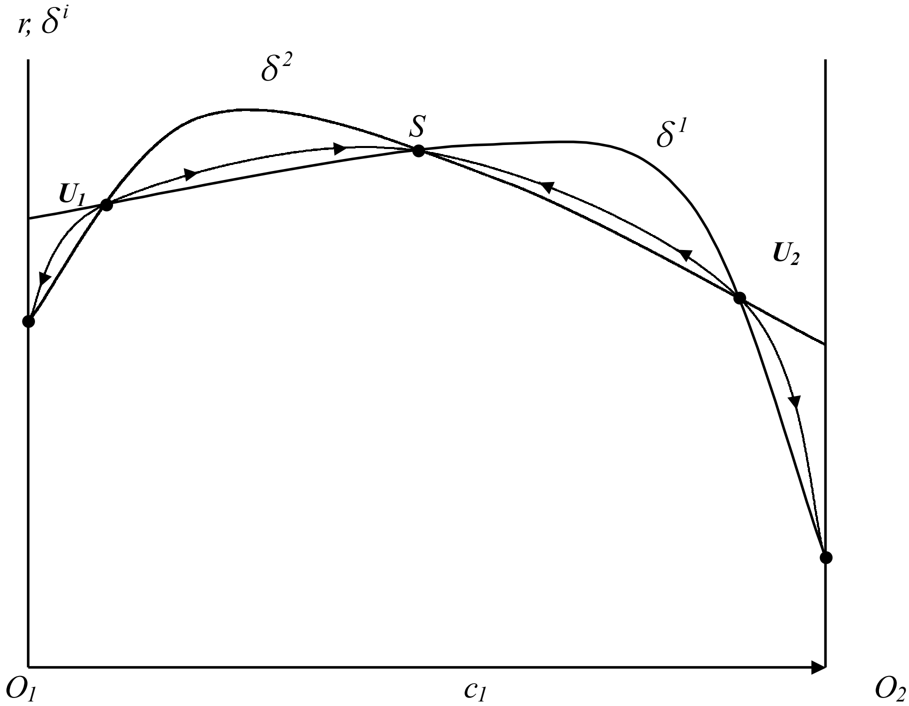

Figure 5 (a). Multiple steady-state equilibria

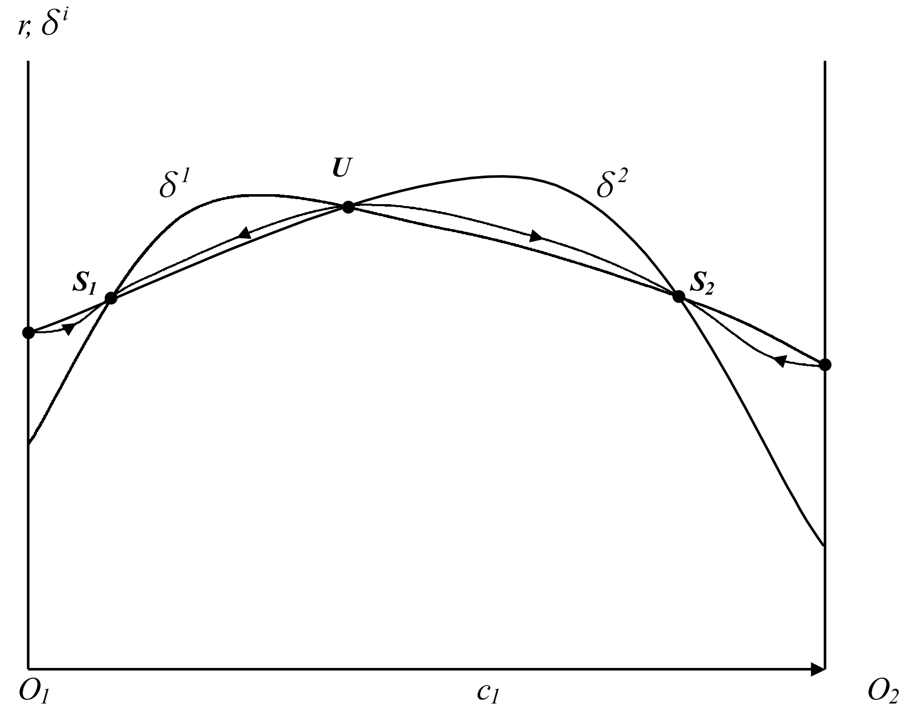

Figure 5 (b). Multiple steady-state equilibria