Discussion Paper No. 559

# VOLUNTARY PARTICIPATION IN THE DESIGN OF NON-EXCLUDABLE PUBLIC GOODS PROVISION MECHANISMS

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October 2001

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# Research was partially supported by the Grant in Aid for Scientific Research 1143002 of the Ministry of Education, Science and Culture in Japan, the Nissan Foundation and the Sumitomo Foundation.

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Abstract

Groves-Ledyard (1977) constructed a mechanism attaining Pareto efficient allocations in the presence of public goods. After this path-breaking paper, many mechanisms have been proposed to attain desirable allocations with public goods. Thus, economists have thought that the free-rider problem is solved, in theory. Our view to this problem is not so optimistic. Rather, we propose fundamental impossibility theorems with public goods. In the previous mechanism design, it was implicitly assumed that every agent must participate in the mechanism that the designer provides. This approach neglects one of the basic features of public goods: non-excludability. We explicitly incorporate non-excludability and then show that it is impossible to construct a mechanism in which every agent has an incentive to participate.

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## 1. Introduction

The provision of public goods has an incentive problem called the free-rider problem. As Samuelson (1964) pointed out, it is impossible to attain a Pareto efficient allocation through a decentralized fashion, in particular, a decentralized pricing system. On the contrary, Groves and Ledyard (1977) proposed an explicit procedure, called a mechanism, in which the Nash equilibrium allocation is Pareto efficient. Participants can pursue their own self-interest being free riders if they choose in the mechanism, but the mechanism is free from these incentives. In this sense, as the subtitle of their paper shows, they found a solution to the free-rider problem.

Although the allocation of the Groves-Ledyard mechanism is Pareto efficient, the mechanism is not individual rational. That is, the allocation does not satisfy the condition where it is at least as good as each participant's initial endowment. Following the path-breaking paper by Groves and Ledyard (1977), Hurwicz (1979) and Walker (1981) fixed this problem and succeeded in implementing the Lindahl correspondence in Nash equilibria, which satisfies both Pareto efficiency and individual rationality. Subsequently, numerous mechanisms have been proposed that satisfy additional desirable properties such as individual feasibility and balancedness[1].

Most mechanisms developed thus far share one undesirable property, however: participants in the mechanisms do not have freedom not to participate. As Olson (1965) noticed, any non-participant can obtain benefit of a public good that is provided by

[1] See Groves and Ledyard (1987) and Hurwicz (1994). For the dominant strategy equilibrium concept, we have impossibility results: Pareto efficiency, individual rationality, and incentive compatibility (i.e., truth-telling is a dominant strategy) are inconsistent. See Hurwicz (1972) without public goods and Ledyard and Roberts (1974) with public goods. Saijo (1991) showed an impossibility result without requiring Pareto efficiency: a slightly stronger individually rational condition called an autarkically individual rationality and

others. This is due to the nature of a public good called *non-excludability*. In other words, Groves and Ledyard and their followers found solutions to the free-rider problem once every participant decided to participate in the mechanisms, but not solutions to the problem when agents have the ability not to participate.

This participation problem is important in many practical situations, such as for international treaties. The Kyoto Protocol to cope with global warming and climate change is a specific, recent example. It took years to agree on the basic framework, the United Nations Framework Convention on Climate Change (UNFCCC), to reduce the green house gases. UNFCCC was adopted in 1992 and entered into force in 1994. The parties of UNFCCC adopted the Kyoto Protocol in 1997. The Protocol is a mechanism in our terminology to attain the goal of UNFCCC. The number of signatories including the U.S.A. exceeded 186 in 2000, but in March 2001, President Bush announced not to ratify the Protocol since it is harmful to the U.S. economy. Therefore, the effectiveness of the Protocol remains in doubt. In our framework, ratification is equivalent to participation. Another example is the League of Nations. Following World War I President Woodrow Wilson strongly supported the League, but the U.S. Congress never ratified the Treaty of Versailles[2].

Thus, our fundamental question is: is there any mechanism satisfying the condition that every agent always chooses participation strategically, called the *voluntary participation condition*? In order to answer this question, we first restrict our attention to the Lindahl allocations as a goal of our society. That is, our first question is whether or

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incentive compatibility are inconsistent. However, our impossibility results described below do not depend on the choice of equilibrium concepts.

not any mechanism attaining Lindahl allocations, called a Lindahl mechanism, can survive if we allow agents to choose participation in the mechanism voluntarily. What we found is striking. Each agent has an incentive not to participate in the mechanism in a wide class of environments.

Based upon the preliminary result, we return to the fundamental question with the two-agent economy. We find that no voluntary participation mechanism exits under mild regularity conditions. Furthermore, this result is independent of the choice of equilibrium concepts.

The picture is still bleak even if the number of participants is at least three. Imposing Pareto efficiency on a mechanism, we again find a negative result. The reason why we obtain the negative result might come from Pareto efficiency on which we impose. We have a partial answer to this question. The voluntary contribution mechanism, which cannot attain Pareto efficiency, does not satisfy the voluntary participation condition, either.

Moulin (1986), Palfrey and Rosenthal (1984), and Saijo and Yamato (1999) also analyzed the issue of an incentive to participate in a mechanism for the provision of a public good.[3] Moulin and Palfrey-Rosenthal focused on specific mechanisms: Moulin studied the pivotal mechanism in discrete public goods economies with quasi-linear preferences, whereas Palfrey and Rosenthal considered a simple mechanism for the provision of a binary public good with binary contributions. Saijo and Yamato examined

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[2] Voluntary public goods provision -- such as for public broadcasting -- also faces the participation problem. For example, part of public broadcasting in Japan is supported by the public broadcasting fee. Every family must pay the fee by law, but many choose not to since punishment is practically non-existent.

[3] The participation problem in an institution has been examined mainly in the context of voting and cartel formation (e.g., see Brams and Fishburn (1983), Dixit and Olson (2000), Ledyard (1984), Okada (1996), Palfrey and Rosenthal (1983,1985), and Selten (1973)).

a specific two sage game: the first stage is a decision stage of participation in a mechanism, and only the participants in the first stage play the second stage. On the other hand, we investigate participation incentive properties of a large class of mechanisms in economic environments with a continuous public good.

The paper is organized as follows. In Section 2, we explain an example illustrating our basic idea. In Section 3, we introduce notation and definitions. We establish an impossibility result on participation incentives for the case of two agents in Section 4 and that for the case of at least three agents in Section 5. In Section 6, we investigate the voluntary contribution mechanism. In the final section, we make concluding remarks.

**2. An Example: Lindahl Mechanisms**

We analyze the following symmetric two-agent economies with one private good $x$ and one pure public good $y$ that is non-excludable and non-rival. The public good can be produced from the private good by means of a constant return to scale technology, and let $y = x$ be the production function of the public good. A consumption bundle for agent $i$ is denoted by $(x_i, y) \in \Re_+^2$ where $x_i \in \Re_+$ is the level of private good she consumes on her own, and $y \in \Re_+$ is the level of public good. Two agents have the same preferences that can be represented by a Cobb-Douglas utility function $u_i^{\alpha}(x_i, y)$ = $x_i^{\alpha} y^{1-\alpha}$, where $0 < \alpha < 1$ and $i$ = 1,2. Each agent's initial endowment is also the same and given by $(\omega_i, 0)$ = (10, 0) for $i$ = 1,2. We investigate situations in which the true value of the preference parameter α is unknown to the mechanism designer, but the initial endowment and the production technology are known.

Consider any mechanism implementing the Lindahl correspondence in Nash equilibria (see Hurwicz (1979), Walker (1981), Hurwicz, Maskin, and Postlewaite (1984), Tian (1990), and so on).[4] Suppose that each agent is able to choose whether she participates in the mechanism. Then in order to achieve the desired Lindahl equilibrium allocation by using the mechanism, every agent must choose participation. Therefore, we ask a crucial question of whether each agent always have an incentive to participate in the mechanism. Unfortunately, our answer to this question is negative.

To see why, let $T \subseteq \{1,2\}$ be the set of agents who participate in the mechanism.[5] An equilibrium allocation of the mechanism when the agents in $T$ participate in it is denoted by $((x_i^T)_{i\in T}, y^T)$.[6] If two agents decide to participate in the mechanism, then $(x_1^{\{1,2\}}, x_2^{\{1,2\}}, y^{\{1,2\}})$ is a Lindahl allocation of the economy with two agents, since the mechanism implements the Lindahl correspondence.[7] It is straightforward to check that there exists a unique Lindahl allocation given by $(x_1^{\{1,2\}}, x_2^{\{1,2\}}, y^{\{1,2\}})$ = (10$\alpha$, 10$\alpha$, 20(1-$\alpha$)).

Now suppose that some agent $i$ does not participate in the mechanism, while the other agent $j \neq i$ does, i.e., $T = \{j\}$. Then $(x_j^{\{j\}}, y^{\{j\}})$ is a unique Lindahl allocation of the economy consisting of only one agent $j$. It is easy to see that $(x_j^{\{j\}}, y^{\{j\}})$ = (10$\alpha$, 10(1-$\alpha$)). Notice that non-participant $i$ can enjoy her initial endowment, $\omega_i$, as well as the

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[4] The Lindahl correspondence is the same as the *constrained* Lindahl correspondence (Hurwicz, Maskin, and Postlewaite (1984)) under the present assumptions.

[5] A "participant" stands for an agent who chooses participation in a mechanism, while an "agent" represents any member who belongs to an economy.

[6] A mechanism specifies a strategy set of each participant in $T$ and an outcome function for each $T \subseteq \{1,2\}$. This definition of a mechanism is more general than the usual one.

non-excludable public good produced by agent $j \neq i$, $y^{\{j\}}$. On the other hand, she is no longer able to affect the decision on the provision of the public good. The following condition should be satisfied if each agent has an incentive to participate in the mechanism:

(1) $u_i^{\alpha}(x_i^{\{1,2\}}, y^{\{1,2\}}) \geq u_i^{\alpha}(\omega_i, y^{\{j\}})$ for $i, j = 1,2$, $j \neq i$,

where $u_i^{\alpha}$ is any Cobb-Douglas utility function. We call condition (1) the *voluntary participation condition*.[8] We show that no mechanism implementing the Lindahl correspondence satisfies this condition. This fact can be illustrated for the case of α = 0.6 by using Kolm's triangle. See Figure 1. Point *A* denotes the Lindahl equilibrium allocation when both agents participate in the mechanism: $A = (x_1^{\{1,2\}}, x_2^{\{1,2\}}, y^{\{1,2\}})$ = (6, 6, 8). Point *B* represents the allocation when agent 1 does not participate in the mechanism, but agent 2 does: $B = (\omega_1, x_2^{\{2\}}, y^{\{2\}})$ = (10, 6, 4). Since $u_1^{0.6}(x_1^{\{1,2\}}, y^{\{1,2\}}) \approx$ 6.73 < $u_1^{0.6}(\omega_1, y^{\{2\}}) \approx$ 6.93, agent 1 would not participate in the mechanism when agent 2 does. The same thing holds for agent 2.

Figure 1 is around here.

## 3. Notation and Definitions

[7] A mechanism implements the Lindahl correspondence if for each set of participants $T \subseteq \{1,2\}$ and each economy consisting of the participants in $T$, every equilibrium allocation is a Lindahl allocation and every Lindahl allocation is an equilibrium allocation.

[8] The voluntary participation condition is different from the individually rational condition which requires that $u_i^{\alpha}(x_i^{\{1,2\}}, y^{\{1,2\}}) \geq u_i^{\alpha}(\omega_i, 0)$ for $i$ = 1,2. Since $u_i^{\alpha}(\omega_i, y^{\{j\}}) \geq u_i^{\alpha}(\omega_i, 0)$, the voluntary participation condition is stronger than the individually rational condition.

In the previous section, we saw that any Lindahl mechanism fail to satisfy the voluntary participation condition in symmetric economies with two agents when Nash equilibrium is an equilibrium concept. We will show below that a similar negative result holds for any equilibrium concept and any mechanism meeting mild conditions.

First of all, we introduce notation and definitions. In Section 2, we investigated two-agent economies with one private good, one pure public good, and a constant return to scale technology. We study the same situations with many agents. Let $N = \{1,2,...,n\}$ be the set of agents, with generic element *i*. We assume that each agent $i's$ preference relation admits a numerical representation $u_i : \Re^2_+ \to \Re$ which is continuous, concave, and monotonic. Let $U_i$ be the class of utility functions admissible for agent *i*. Let $P(N)$ be the collection of all no-empty subsets of *N*. For $T \in P(N)$, let $u_T \equiv (u_i)_{i\in T} \in U_T \equiv \times_{i\in T} U_i$ be a preference profile for the agents in *T*.

Agent *i*'s initial endowment is denoted by $(\omega_i, 0)$. That is, there is no public good initially. Let a distribution of initial endowments of the private good $(\omega_i)_{i\in N}$ be given. Given $T \in P(N)$, a *feasible allocation for T* is a list $(x_T, y) \equiv ((x_i)_{i\in T}, y) \in \Re^{\#T+1}_+$ such that $\sum_{i\in T}(\omega_i - x_i) = y$. The set of feasible allocations for *T* is denoted by $A^T$.

A *mechanism* is a function $\Gamma$ that associates with each $T \in P(N)$ a pair $\Gamma(T)$ = $(S^T, g^T)$, where $S^T = \times_{i\in T} S_i^T$ and $g^T : S^T \to \Re^{\#T+1}$. Here $S_i^T$ is the *strategy space of agent i* $\in T$ and $g^T$ is the *outcome function when the agents in T play the mechanism*. Given $g^T(s)$ = $(x_T, y)$, let $g_i^T(s) \equiv (x_i, y)$ for $i \in T$ and $g_y^T(s) = y$.

An *equilibrium correspondence* is a correspondence $\mu$ which associates with each mechanism $\Gamma$, each set of agents $T \in P(N)$, and each preference profile $u_T \in U_T$, a set of strategy profiles $\mu(\Gamma, T, u_T) \subseteq S^T$, where $(S^T, g^T) = \Gamma(T)$. We simply write $\mu(\Gamma, T, u_T)$ as $\mu_\Gamma(u_T)$. Examples of equilibrium correspondences include dominant strategy equilibrium correspondence, Nash equilibrium correspondence, and strong Nash equilibrium correspondence. The set of *$\mu$-equilibrium allocations of* $\Gamma$ *for* $T$ *at* $u_T$ is denoted by $g^T \circ \mu_\Gamma(u_T) \equiv \{(x_T, y) \in \Re^{\#T+1} \mid$ there exists $s \in S^T$ such that $s \in \mu_\Gamma(u_T)$ and $g^T(s) = (x_T, y)\}$, where $(S^T, g^T) = \Gamma(T)$.

## 4. The Case of Two Agents

Let an equilibrium correspondence $\mu$ be given. We introduce several conditions on a mechanism.

*Definition 1.* The mechanism $\Gamma$ satisfies *non-emptiness under* $\mu$ if for all $T \in P(N)$ and all $u_T \in U_T$, $g^T \circ \mu_\Gamma(u_T) \neq \emptyset$.

*Definition 2.* The mechanism $\Gamma$ satisfies *feasibility under* $\mu$ if for all $T \in P(N)$ and all $u_T \in U_T$, $g^T \circ \mu_\Gamma(u_T) \subseteq A^T$.

Non-emptiness says that there always exists an equilibrium. Feasibility demands that an equilibrium allocation of the mechanism should always be feasible. Note that we require feasibility only at equilibrium, but not out of equilibrium. Moreover, a feasible mechanism does not necessarily satisfy individual feasibility (i.e., for all $T \in P(N)$ and all $s \in S^T$, $g^T(s) \in \Re_+^{\#T+1}$) nor balancedness (i.e., for all $T \in P(N)$ and all $s \in S^T$, $g^T(s) \in A^T$).

*Definition 3.* The mechanism $\Gamma$ satisfies the *voluntary participation condition under* $\mu$ if for all $u_N \in U_N$, all $(x^N, y^N) \in g^N \circ \mu_\Gamma(u_N)$, and all $i \in N$,

$$u_i(x_i^N, y^N) \geq u_i(\omega_i, y_{\min}^{N-\{i\}}),$$

where $y_{\min}^{N-\{i\}} \in \underset{y^{N-\{i\}} \in g_y^{N-\{i\}} \circ \mu_\Gamma(u_{N-\{i\}})}{\operatorname{Arg\,min}} u_i(\omega_i, y^{N-\{i\}})$ .

Since there is one public good and preferences satisfy monotonicity, $y_{\min}^{N-\{i\}}$ is the minimum equilibrium level of public good when all agents except $i$ participate in the mechanism. Consider an agent who decides not to participate in the mechanism. Then she can enjoy the non-excludable public good produced by the other agents without providing any private good, while she cannot affect the decision on the provision of the public good. The voluntary participation condition requires that no agent can benefit from such a free-riding action. Note that when an agent chooses non-participation, she has a pessimistic view on the outcome of her action: an equilibrium outcome that is most unfavorable for her will occur.[9] Moulin (1986) proposed a similar condition, called the *No Free Ride* axiom, to characterize the pivotal mechanism when public goods are discrete and costless, and preferences are quasi-linear.

[9] A stronger condition on voluntary participation is conceivable for the case in which the non-participant has a more optimistic view that a better equilibrium outcome will happen. However, we will derive impossibility results regarding this weak condition on voluntary participation, and hence our results hold for other stronger versions.

*Definition 4.* The mechanism $\Gamma$ satisfies the *Robinson Crusoe condition under* $\mu$ if for all $i$ $\in N$ and all $u_i \in U_i$, if $(x_i^{\{i\}}, y^{\{i\}}) \in g^{\{i\}} \circ \mu_\Gamma(u_i)$, then $(x_i^{\{i\}}, y^{\{i\}}) \in \underset{(x_i, y) \in A^{\{i\}}}{\text{Arg max}} \; u_i(x_i, y)$.

The Robinson Crusoe condition means that if only one agent participates in the mechanism, then she chooses an outcome that is best for her.

We establish an impossibility result that three conditions mentioned above are incompatible in the case of two agents. Let $U^{SCD} = \{(u_i)_{i \in N} \mid \forall i \in N,\ u_i(x_i, y) = u_i^{\alpha}(x_i, y)$ $= \alpha \ln x_i + (1-\alpha)\ln y,\ \alpha \in (0,1)\}$ be the class of symmetric Cobb-Douglas utility profiles[10].

*Theorem 1. Let n = 2 and* $\mu$ *be an arbitrary equilibrium correspondence. Suppose that* $U \supseteq U^{SCD}$ *and for all i ∈ N,* $\omega_i = \omega > 0$. *If a mechanism satisfies non-emptiness, feasibility, and the Robinson Crusoe condition under* $\mu$, *then it fails to satisfy the voluntary participation condition under* $\mu$.

The proof of Theorem 1 is illustrated in Figure 2. Consider the case in which both agents have the same Cobb-Douglas utility function with $\alpha = 0.6$. By the Robinson Crusoe condition, a unique equilibrium allocation of the mechanism when only agent 2 (resp. agent 1) participates in it is given by Point C (resp. Point D) in Figure 2. Moreover, if the mechanism satisfies the voluntary participation condition, then at equilibrium, agent 1 (resp. agent 2) should receive a consumption bundle in her weak upper contour

[10] Here a Cobb-Douglas utility function is denoted by a natural logarithmic function, while it is an exponential function in the example described in Section 2. The results in this paper hold independent of which function is used.

set at C (resp. D) when both agents choose participation. These upper contour sets are denoted by the shaded areas in Figure 2. However, since they are disjoint, the feasibility condition is violated. A formal proof of Theorem 1 is given as follows:

Figure 2 is around here.

*Proof of Theorem 1.* Suppose by way of contradiction that the mechanism satisfies the voluntary participation condition. Consider $(u_1^{\alpha}, u_2^{\alpha}) \in U^{SCD}$ with $\alpha = 0.6$.[11] It is easy to check that by the Robinson Crusoe condition, a unique equilibrium allocation of the mechanism for one agent economy is given by $(x_i^{\{i\}}, y^{\{i\}})$ = $(0.6\omega, 0.4\omega)$, $i = 1,2$. Let $V((\omega_i, y^{\{j\}}), u_i^{0.6}) \equiv \{(x_i, y) \in \Re_+^2 \mid u_i^{0.6}(x_i, y) \geq u_i^{0.6}(\omega_i, y^{\{j\}})\}$ be agent *i*'s weak upper contour set at $(\omega_i, y^{\{j\}})$ for $u_i^{0.6}$, where $(\omega_i, y^{\{j\}})$ = $(\omega, 0.4\omega)$ and $j \neq i$. Pick any $(x_1^{\{1,2\}}, x_2^{\{1,2\}}, y^{\{1,2\}}) \in g^{\{1,2\}} \circ \mu_\Gamma(u_1^{0.6}, u_2^{0.6})$. By the voluntary participation condition,

(2) $$(x_i^{\{1,2\}}, y^{\{1,2\}}) \in V((\omega_i, y^{\{j\}}), u_i^{0.6}) \qquad (i, j = 1,2\,; j \neq i).$$

We claim that

(3) $$\forall (x_i, y) \in V((\omega_i, y^{\{j\}}), u_i^{0.6}), \quad 2x_i + y \neq 2\omega \qquad (i, j = 1,2\,; j \neq i).$$

Suppose that (3) does not hold. Then for some *i* and some $(\bar{x}_i, \bar{y}) \in \Re_+^2$, $u_i^{0.6}(\bar{x}_i, \bar{y}) \geq u_i^{0.6}(\omega_i, y^{\{j\}})$ and $2\bar{x}_i + \bar{y} = 2\omega$. Let $(x_i^*, y^*)$ be a maximizer of the utility function $u_i^{0.6}(x_i, y)$ = $0.6 \ln x_i + 0.4 \ln y$ subject to the constraint $2x_i + y = 2\omega$. It is easy to see that $(x_i^*, y^*)$ = $(0.6\omega, 0.8\omega)$ and $u_i^{0.6}(x_i^*, y^*) - u_i^{0.6}(\omega_i, y^{\{j\}})$ = $0.6 \ln 0.6 + 0.4 \ln 2$ <

- 0.30 + 0.28 < 0.  Thus, $u_i^{0.6}(\bar{x}_i, \bar{y}) \geq u_i^{0.6}(\omega_i, y^{\{j\}}) > u_i^{0.6}(x_i^*, y^*)$, which contradicts the fact that $(x_i^*, y^*)$ is the maximizer of $u_i^{0.6}(x_i, y)$ subject to $2x_i + y = 2\omega$.

However, by (2) and (3), $x_1^{\{1,2\}} + x_2^{\{1,2\}} + y^{\{1,2\}} \neq 2\omega$.  This contradicts the feasibility condition on the mechanism.  Q.E.D.

**5.  Pareto Efficient Mechanisms**

In this subsection, we show an impossibility result on the voluntary participation condition in the case of at least three agents.  We propose the following two conditions on a mechanism.  Let an equilibrium correspondence $\mu$ be given.

*Definition 5.*  The mechanism $\Gamma$ satisfies *symmetry under* $\mu$ if for all $T \in P(N)$ and all $u_T \in U_T$, if $u_i = u_j$ and $\omega_i = \omega_j$ for all $i, j \in T$ and $(x_T, y) \in g^T \circ \mu_\Gamma(u_T)$, then $x_i = x_j$ for all $i, j \in T$.

*Definition 6.*  The mechanism $\Gamma$ satisfies *Pareto efficiency only for participants under* $\mu$ if for all $T \in P(N)$ and all $u_T \in U_T$, $g^T \circ \mu_\Gamma(u_T) \subseteq \mathcal{P}(u_T)$, where $\mathcal{P}(u_T) \equiv \{(x_T, y) \in A^T \mid$ there does not exist $(x'_T, y') \in A^T$ such that $u_i(x'_T, y') \geq u_i(x_T, y)$ for all $i \in T$ and $u_i(x'_T, y') > u_i(x_T, y)$ for some $i \in T\}$.

[11] By using an argument similar to the below, it is not hard to check that the four conditions mentioned in Theorem 1 are incompatible for any $\alpha \in (0.5,1)$.

Symmetry requires that if all participants have the same preferences and endowments, then they receive the same consumption bundle at equilibrium. Therefore, every participant pays the same amount of the private good for the provision of the public good. Pareto efficiency only for participants means that every equilibrium allocation of the mechanism should be Pareto efficient for participants, but not necessarily efficient with respect to all agents.

*Theorem 2. Let $n \geq 3$ and $\mu$ be an arbitrary equilibrium correspondence. Suppose that $U \supseteq U^{SCD}$ and for all $i \in N$, $\omega_i = \omega > 0$. If a mechanism satisfies non-emptiness, feasibility, symmetry, and Pareto efficiency only for participants under $\mu$, then it fails to satisfy the voluntary participation condition under $\mu$.*

*Proof.* Take $(u_i^{\alpha})_{i \in N} \in U^{SCD}$. Take any equilibrium allocation $((x_i^N)_{i \in N}, y^N) \in g^N \circ \mu_{\Gamma}(u_N)$. By symmetry, $x_i^N = x_j^N$ for all $i, j \in N$. By feasibility and Pareto efficiency only for participants, $(x_i^N, y^N)$ is a maximizer of the utility function $\alpha \ln x + (1-\alpha)\ln y$, subject to $nx + y = n\omega$. It is easy to check that $(x_i^N, y^N) = (\omega\alpha, n\omega(1-\alpha))$. In a similar way, we can show that $(x_i^{N-\{i\}}, y^{N-\{i\}}) = (\omega\alpha, (n-1)\omega(1-\alpha))$ for $i \in N$. Therefore, the difference between the utility level when all agents participate in the mechanism and that when all agents except *i* participate in it is given by

(4) $$u_i^{\alpha}(x_i^N, y^N) - u_i^{\alpha}(\omega_i, y^{N-\{i\}}) = \alpha \ln \alpha + (1-\alpha)[\ln n - \ln(n-1)] \equiv f(\alpha, n)$$

for $i \in N$. We prove that the sign of $f(\alpha, n)$ is negative when $\alpha = 0.6$ and $n \geq 3$.[12] Note that the function $\ln n - \ln(n-1)$ is decreasing in *n*. Therefore, for $n \geq 3$, $f(0.6, n) \leq f(0.6, 3) = 0.6 \ln 0.6 + 0.4[\ln 3 - \ln 2] < -0.3 + 0.2 < 0$. This implies that the voluntary participation condition is violated. Q.E.D.

*Remark:* By using an argument similar to the proofs of Theorems 1 and 2, we can show that the condition of Pareto efficiency only for participants can be replaced by a weaker condition in Theorem 2: if a mechanism satisfies non-emptiness, feasibility, symmetry, and Pareto efficiency *only with respect to n-1 participants* (i.e., for all $T \in P(N)$ with $\#T = n-1$ and all $u_T \in U_T$, $g^T \circ \mu_\Gamma(u_T) \subseteq \mathcal{P}(u_T)$), then it fails to satisfy the voluntary participation condition. This result holds when there are at least two agents and hence Theorem 1 on the two-agent case is a corollary of it. Although the result is logically better than Theorem 2, the condition of Pareto efficiency only with respect to *n*-1 participants would not have a meaningful economic interpretation, except the case of two agents in which the condition is equivalent to the Robinson Crusoe condition.

## 6. The Voluntary Contribution Mechanism

In the previous section, we found negative results on voluntary participation for any mechanism satisfying non-emptiness, feasibility, symmetry, and Pareto efficiency only for participants. However, Pareto efficiency only for participants is not necessary to obtain such results. In this section, we study the *voluntary contribution mechanism* that does not satisfy Pareto efficiency only for participants when the equilibrium concept is

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[12] By using an argument similar to the below, it is not difficult to check that the sign of $f(\alpha, n)$ is negative

Nash equilibrium. To our surprise, this mechanism does not satisfy the voluntary participation condition, even though the name of the mechanism contains the term "voluntary".

*Definition 7.* The *voluntary contribution mechanism* is a mechanism such that for all $T \in P(N)$ and $i \in T$, $S_i^T = [0, \omega_i]$ and $g_i^T(s) = (\omega_i - s_i, \sum_{i \in T} s_i)$ for $s \in S^T$.

The above definition of the voluntary contribution mechanism is a generalization of the usual one, in which all agents are supposed to participate, to the case in which voluntary participation is allowed.

When the equilibrium concept is Nash equilibrium, each agent *i* selects her contribution out of her endowment to the provision of the public good, $s_i$, to maximize her utility $u_i(\omega_i - s_i, \sum_{j \in T} s_j)$, given contributions of the other agents in *T*, $(s_j)_{j \in T - \{i\}}$ in the voluntary contribution mechanism.

*Theorem 3. Let $n \geq 3$. Suppose that (i) $U \supseteq U^{SCD}$; (ii) for all $i \in N$, $\omega_i = \omega > 0$; and (iii) $\mu$ is a Nash equilibrium correspondence. Then the voluntary contribution mechanism fails to satisfy the voluntary participation condition.*

*Proof.* Take $(u_i^{\alpha})_{i \in N} \in U^{SCD}$. Let $(x_i^N, y^N)$ be the consumption bundle that each agent *i* receives at the unique symmetric Nash equilibrium if all agents in *N* decide to participate

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for any $\alpha \in (0.203, 1)$ and any $n \geq 3$.

in the mechanism. It is easy to see that ( $x_i^N$ , $y^N$ ) = ( $\omega\alpha n / (1+\alpha(n-1))$ ,

$\omega(1-\alpha)n / (1+\alpha(n-1))$ ). Also, let $y^{N-\{i\}}$ be the public good level at the unique Nash equilibrium allocation of the mechanism played among $n$-1 participants in $N-\{i\}$ . It is straightforward to check that $y^{N-\{i\}}$ = $\omega$(1-α) $(n-1)$ / (1+α($n$-2)). Thus, for $i \in N$ ,

(5) $$u_i^{\alpha}(x_i^N, y^N) - u_i^{\alpha}(\omega_i, y^{N-\{i\}})$$

$$= \alpha\ln\alpha + (1-\alpha)[\ln n - \ln(n-1)] + \alpha\ln n + (1-\alpha)\ln[1+\alpha(n-2)] - \ln[1+\alpha(n-1)]$$

$$\equiv h(\alpha,n).$$

We show that the sign of $h(\alpha,n)$ is negative when $\alpha = 0.6$ and $n \geq 3$ .[13] By partially differentiating $h(\alpha,n)$ with respect to $n$, we have

$$\frac{\partial h(\alpha,n)}{\partial n} = \frac{-(1-\alpha)[1-2\alpha+\alpha n(1+\alpha-\alpha n)]}{n(n-1)[1+\alpha(n-2)][1+\alpha(n-1)]}.$$

If $\alpha = 0.6$ and $n \geq 3$ , then

$$\frac{\partial h(\alpha,n)}{\partial n} = \frac{0.4[9n(n-3)+3n+5]}{n(n-1)(3n-1)(3n+2)} > 0 .$$

Moreover, $\lim_{n\to\infty} h(\alpha,n) = 0$ . Hence, for any finite number $n \geq 3$ , $h(0.6,n)$ < 0. This implies that the voluntary participation condition is violated. Q.E.D.

## 7. Concluding Remarks

We see that the solutions to the free-rider problem, which have been proposed in mechanism design theory, are not necessary solutions to the free-rider problem when

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[13] By using an argument similar to the below, it is not difficult to check that the sign of $h(\alpha,n)$ is negative for any $\alpha \in (0.25,1)$ and any $n \geq 3$ .

participation in mechanisms is voluntary. Furthermore, we show that it is quite difficult or impossible to design a mechanism with voluntary participation.

As Olson (1965) argued, a public good would be less likely provided as the number of agents becomes large. Saijo and Yamato (1999) confirmed this conjecture by proving that in a two-stage game with voluntary participation, the measure of the set of symmetric Cobb-Douglas economies for which every agent chooses participation at equilibrium becomes smaller as the number of agents grows large. In a similar way, we can show that the measure of the set of economies for which the voluntary participation condition is satisfied is strictly decreasing as the number of agents increases. This would be another result supporting Olson's conjecture.

In the voluntary participation condition defined above, it is implicitly assumed that each agent has the most optimistic conjecture on the number of other agents who will not participate in the mechanism if she does not, that is, she expects no agent other than her to choose non-participation in the mechanism. On the other hand, in the individually rational condition usually discussed in the literature on mechanism design, it is assumed that each agent has the most pessimistic conjecture on that number, that is, she expects all other $n-1$ agents to select non-participation, too. However, an agent might have an intermediate conjecture: her conjecture on the number of other non- participants can take on a whole range of values from 0 to $n$-1. An open question is to examine other conditions on voluntary participation taking account of these possible conjectures.

Saijo, Yamato, Yokotani, and Cason (1998), and Cason, Saijo, and Yamato (2001) observed that cooperation has emerged though spiteful behavior in their experiments on the voluntary contribution mechanism with voluntary participation. Our theory in this paper suggests that no cooperation will emerge. Reconciling theoretical results to experimental results is an open area of our future research.

## References


BRAMS, S. J., AND P. C. FISHBURN (1983): "Paradoxes of Preferential Voting," *Mathematics Magazine*, 56, 207-214.

CASON, T., T. SAIJO, AND T. YAMATO (2001): “Voluntary Participation and Spite in Public Good Provision Experiments: An International Comparison,” Purdue University Center for International Business Education and Research (CIBER) working paper.

DIXIT, A. AND M. OLSON (2000): "Does Voluntary Participation Undermine the Coase Theorem?” *Journal of Public Economics,* 76, 307-335.

GROVES, T., AND J. LEDYARD (1977): "Optimal Allocation of Public Goods: A Solution to the 'Free Rider' Problem," *Econometrica*, 45, 783-811.

GROVES, T., AND J. LEDYARD, "Incentive Compatibility Since 1972," in *Information, Incentives, and Economic Mechanisms: Essays in Honor of Leonid Hurwicz*, eds., T. Groves, R. Radner, and S. Reiter, (Minneapolis, University of Minnesota Press, 1987), 48-111.

HURWICZ, L. (1972): "On Informationally Decentralized Systems," in *Decision and Organization: A Volume in Honor of Jacob Marschak*, eds., R. Radner and C. B. McGuire, (Amsterdam, North-Holland), 297-336.

HURWICZ, L. (1979): "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points," *Review of Economic Studies*, 46, 217-225.

HURWICZ, L. (1994): "Economic Design, Adjustment Process, Mechanisms, and Institutions," *Economic Design*, 1, 1-14.

HURWICZ, L., E. MASKIN, AND A. POSTLEWAITE (1984): "Feasible Implementation of Social Choice Correspondences by Nash Equilibria," mimeo.

LEDYARD, J. (1984): "The Pure Theory of Large Two-Candidate Elections," *Public Choice*, 44, 7-41.

LEDYARD, J. AND J. ROBERTS (1974): "On the Incentive Problem with Public Goods," mimeo, Northwestern University.

MOULIN, H. (1986): "Characterizations of the Pivotal Mechanism," *Journal of Public Economics*, 31, 53-78.

OKADA, A. (1996): "The Organization of Social Cooperation: A Noncooperative Approach," KEIR Discussion Paper, Kyoto University.

OLSON, M. (1965): "The Logic of Collective Action: Public Goods and the Theory of Groups," Cambridge: Harvard University Press.

PALFREY, T., AND H. ROSENTHAL (1983): "A Strategic Calculus of Voting," *Public Choice,* 41, 7-53.

PALFREY, T., AND H. ROSENTHAL (1984): "Participation and the Provision of Discrete Public Goods: A Strategic Analysis," *Journal of Public Economics*, 24, 171-193.

PALFREY, T., AND H. ROSENTHAL (1985): "Voter Participation and Strategic Uncertainty," *American Political Science Review*, 79, 62-78.

SAIJO, T. (1991): "Incentive Compatibility and Individual Rationality in Public Good Economies," *Journal of Economic Theory*, 55, 203-212.

SAIJO, T., AND T. YAMATO (1999): "A Voluntary Participation Game with a Non-excludable Public Good," *Journal of Economic Theory*, 84, 227-242.

SAIJO, T, T. YAMATO, K. YOKOTANI, AND T. N. CASON (1998): "Voluntary Participation in Public Good Provision Experiments: Is Spitefulness a Source of Cooperation?" mimeo, Osaka University.

SAMUELSON, P. A. (1964): "The Pure Theory of Public Expenditure," *The Review of Economics and Statistics*, 36, 387-389.

SELTEN, R. (1973): "A Simple Model of Imperfect Competition, where 4 Are Few and 6 Are Many," *International Journal of Game Theory*, 2, 141-201.

TIAN, G. (1990): "Completely Feasible Continuous Implementation of the Lindahl Correspondence with a Message Space of Minimal Dimensions," *Journal of Economic Theory,* 51, 443-452.

WALKER, M. (1981): "A Simple Incentive Compatible Scheme for Attaining Lindahl Allocations," *Econometrica*, 49, 65-71.

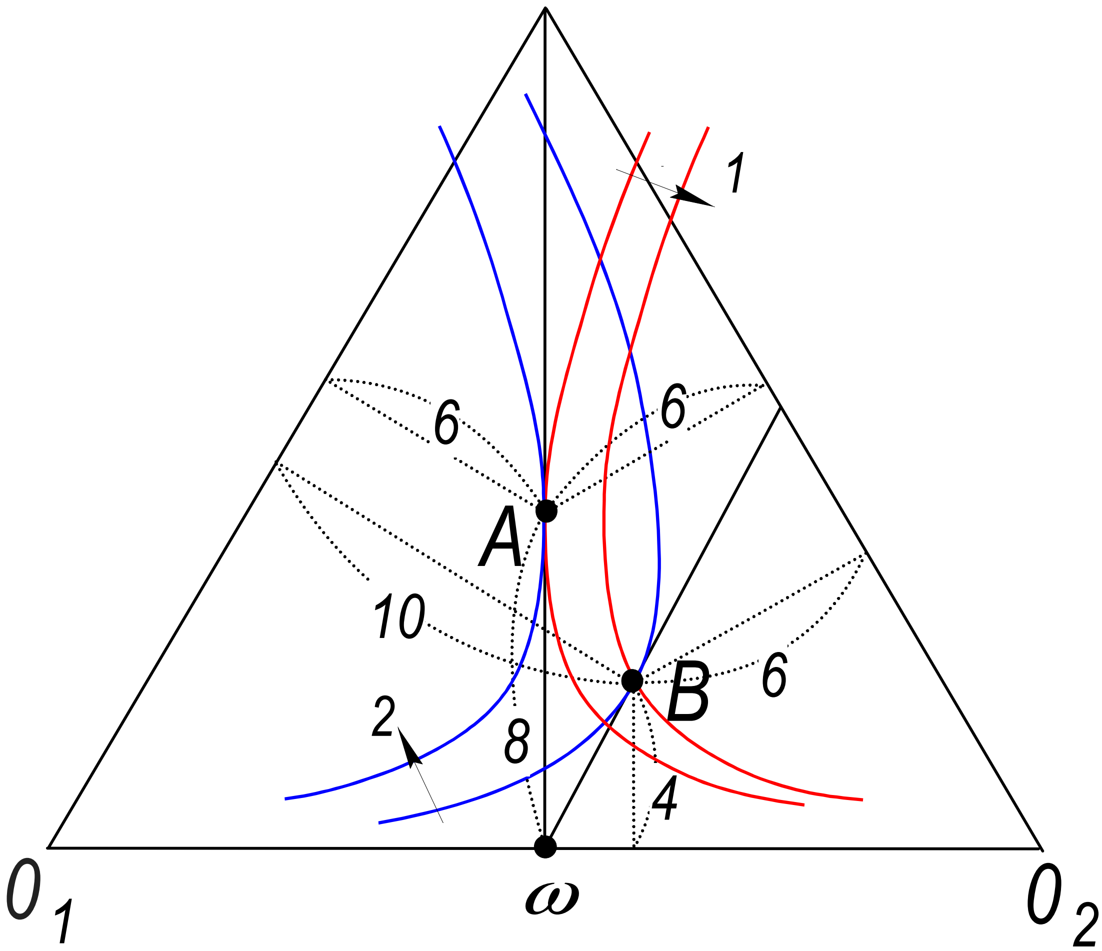


Figure 1. A Lindahl mechanism does not satisfy the voluntary participation condition.

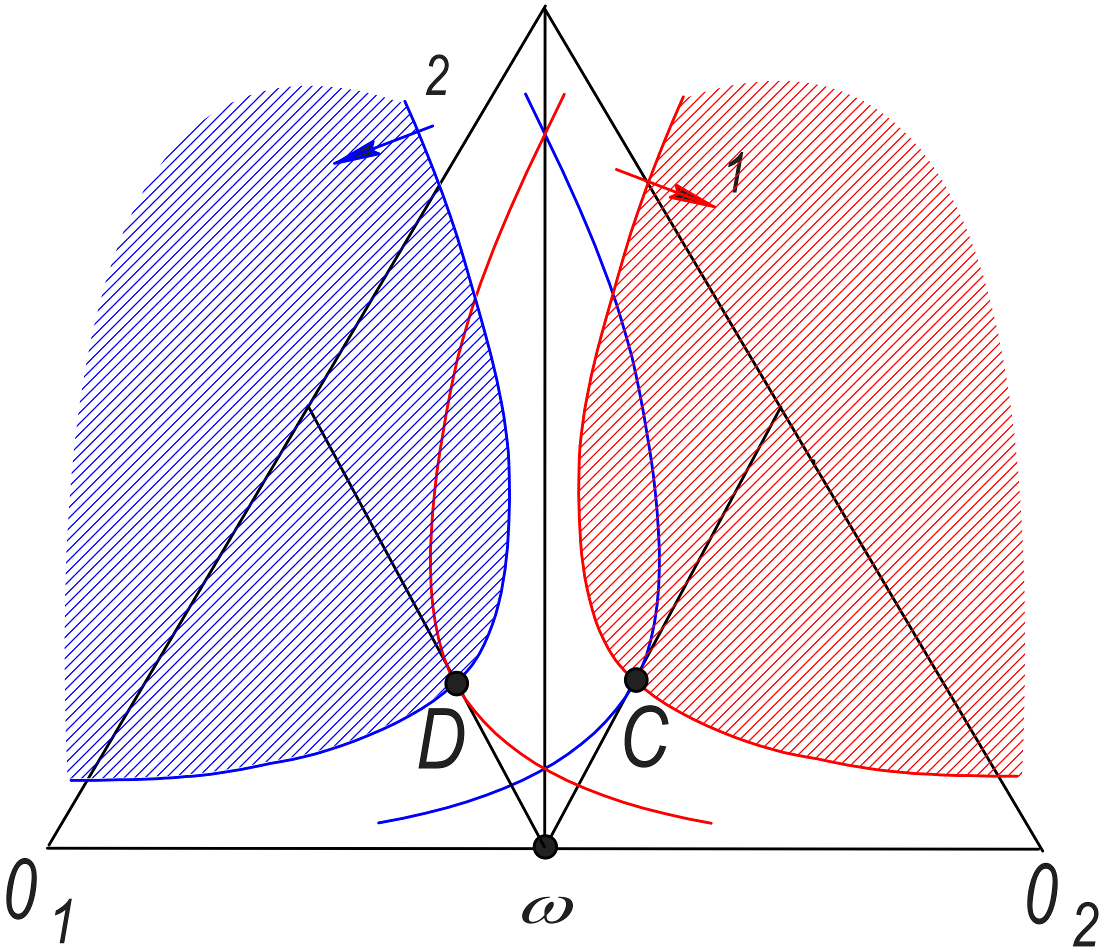


Figure 2. No mechanism satisfies the the voluntary participation condition for the two-agent case.