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Ken-Ichi Hirose
and
Shinsuke Ikeda

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The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

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Ken-Ichi Hirose[1] and Shinsuke Ikeda[2]
Otaru University of Commerce and Osaka University

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[1]The Faculty of Commerce, Otaru University of Commerce, 3-5-21 Midori, Otaru, Hokkaido 047-8501, Japan. Telephone: +81(134)27-5365. E-mail: hirose@res.otaru-uc.ac.jp

[2]Corresponding author. The Institute of Social and Economic Research, Osaka University, 6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan. Telephone: +81(6)6879-8568. Fax: +81(6)6878-2766. E-mail: ikeda@iser.osaka-u.ac.jp

Abstract

Unlike the standard assumption that the degree of impatience, measured by the rate of time preference, is increasing in wealth, empirical studies support that impatience is marginally decreasing. By introducing decreasing marginal impatience into the neoclassical monetary growth model á la Sidrauski, we show that (i) consistently with empirical results, an increase in the core rate of inflation reduces capital stocks in a steady state; and that (ii) its long-run welfare cost is larger than predicted with increasing or constant marginal impatience, implying that estimates of the inflation cost which have so far been obtained by assuming constant time preference may be underestimates.
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JEL classification: D90, E00.

1 Introduction

In the theory of endogenous time preference, one of the most controversial assumptions is that the degree of impatience, measured by the rate of time preference, is marginally increasing in wealth. Although existing empirical studies, e.g., Lawrance (1991), Becker and Mulligan (1997), Samwick (1997), and Barsky *et. al.* (1997), commonly report that the degree of impatience is marginally decreasing in wealth or income, few research has so far been conducted on implications of decreasing marginal impatience. This contrasts to the huge accumulation of research on increasing marginal impatience (see, e.g., Epstein (1987)). The main reason for this is that marginal decreasing impatience *ceteris paribus* destabilizes optimal consumption dynamics. However, Das (2003) and Hirose and Ikeda (2004) recently show that, even with decreasing marginal impatience, equilibrium economic dynamics can be stable in neoclassical growth models where capital input displays decreasing return. By using the resulting, dynamically well-behaved model, it is of particular importance to work out policy implications of decreasing marginal impatience.

By introducing decreasing marginal impatience into the neoclassical monetary growth model á la Sidrauski (1967), the purpose of this exercise is to re-examine the real effects of an increase in the core rate of inflation. Many empirical studies have reported that inflation harms capital accumulation (e.g., Fischer (1993), De Gregorio (1992), Barro (1995), and Jones and Manuelli (1995)). Theoretical studies, however, have shown different results since Tobin (1965) claimed that inflation enlarges capital and production in the long run by causing portfolio shifting away from real money balances to real capital. Especially Epstein and Hynes (1983) reproduced the Tobin effect by using a general equilibrium model with endogenous time preference.[1] However, the result might depend crucially on their assumption that impatience is marginally increasing.

With decreasing marginal impatience, we indeed show that, consistently with the empirical stylized fact, a higher inflation rate results in smaller long-run capital stocks. Intuitively, higher inflation reduces real money balances,

[1] The Tobin effect has been also obtained by using overlapping generations models (e.g., Weil (1991)). The Tobin effect in these models is associated with the nonneutrality of debts, which differs from Tobin's argument. Introducing explicitly a transaction-facilitating function of money (e.g., Stockman (1981)) weakens the validity of the Tobin effect. For a survey, see Orphanides and Solow (1990).

which raises the steady-state time preference and hence the real interest rate under decreasing marginal impatience. The long-run capital stock should thus be reduced.

As an important welfare implication, it is also shown that the resulting long-run welfare cost of inflation is larger than predicted with increasing or constant marginal impatience. If actual time preference displays decreasing marginal impatience as empirically reported, the result implies that misspecifying impatience as marginally increasing or constant leads to underestimating the long-run welfare costs of inflation. For example, by using constant time preference models, Jones and Manuelli (1995) and Lucas (2000) report that the welfare cost of inflation is quantitatively very modest. They may, however, underestimate the actual values.

The rest of the paper is organized as follows. Section 2 presents a monetary growth model with decreasing marginal impatience. Section 3 examines the effects of inflation on capital accumulation. Section 4 concludes the paper.

2 The model with decreasing marginal impatience

We consider an infinitely-lived representative agent in the "money-in-the-utility-function" framework. He or she holds wealth a in the form of real capital k and real money balances m; supplies one unit of labor inelastically; and maximizes lifetime utility by choosing the time profiles of consumption c, real money balances, and the real capital stock. His or her problem is specified as follows:

$$\max \int_0^\infty u\left(c_t, m_t\right) \exp(-\Delta_t) \mathrm{d}t, \tag{1}$$

subject to:

$$\dot{\Delta}_t = \delta\left(u\left(c_t, m_t\right)\right), \Delta_0 = 0, \tag{2}$$

$$\dot{a}_t = r_t a_t + w_t - c_t - \left(r_t + \pi_t\right) m_t + x_t, a_t \geq 0, \tag{3}$$

$$a_t = k_t + m_t, k_t > 0, k_0 = \text{given},$$

where a dot represents the time derivative; $u(\bullet,\bullet)$ represents the felicity function; Δ denotes a cumulative discount rate with the instantaneous discount rate $\delta(u)$ á la Uzawa (1968): $\Delta_t = \int_0^t \delta(u_\tau)\mathrm{d}\tau$; r the real rate of interest; w the wage rate; π the rate of inflation; and x lump-sum transfer payments from the government.

For intertemporal preferences to be well-defined, we follow the literature (e.g., Epstein (1987, p.75)) in assuming that the following standard regularity conditions are valid: (C1) $u < 0$; (C2) u is increasing and concave; (C3) u is log-convex; and (C4) $\delta(u(c,m))$ is concave in (c,m).

In contrast to the existing literature (e.g., Uzawa (1968) and Epstein (1987)), we assume that $\delta'(u) < 0$, for which case, as shown later, the degree of impatience, measured by the pure rate of time preference is marginally decreasing in wealth.[2]

Let λ and ϕ represent the current-value shadow prices for savings and the discount factor $\Omega = \exp(-\Delta)$, respectively. Let $g(c,m,\phi)$ represent generating function $u(c,m) - \phi\delta(u(c,m))$. Then, the marginal utilities are given by first derivatives g_c and g_m, where $g_c \equiv \partial g/\partial c$, etc. It is assumed that g_c and g_m are all positive.[3] [4]For analytical simplicity, we also assume that c and m are separable: $g_{cm} = 0$.[5]

The optimal consumption should satisfy:

$$g_c(c_t, \phi_t) = \lambda_t, \tag{4}$$

$$\chi(c_t, m_t) \equiv \frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = r_t + \pi_t, \tag{5}$$

[2]Das (2003) proposes an alternative set of regularity conditions in which $\partial\delta/\partial c < 0$ itself is imposed as a part of the reguralITY condition. Our main result below does not change at all even when we adopt his regularity conditions, instead of conditions (C1) through (C4).

[3]This assumption is necessary because, from regularity condition (C1), ϕ, which equals the utility index as shown later, is negative. Since ϕ equals u/δ in steady state, the positivity of g_c and g_m is satisfied near the steady-state point only when the elasticity of the discount rate δ with respect to felicity u is smaller than unity: $u\delta'/\delta < 1$. Similar assumptions are needed in the case of increasing marginal impatience when u is assumed to be positive (see, e.g., Uzawa (1968) and Gootzeit et al (2002)).

[4]If we adopted Das's reguralITY conditions, instead of our conditions (C1) through (C4), the positivity of g_c and g_m would be ensured. But, unlike what we do later, we could not consider the case of increasing marginal impatience under the Das reguralITY conditions in which $\delta' < 0$ is imposed as a reguralITY condition.

[5]The separability $g_{cm} = 0$ is satisfied when u and δ are both separable, $u_{cm} = 0$ and $\delta_{cm} = 0$.

(6) $$\dot{\lambda}_t = (\delta(u(c_t, m_t)) - r_t)\lambda_t,$$

(7) $$\dot{\phi}_t = -g(c_t, m_t, \phi_t),$$

and the transversality conditions. Note that the optimal ϕ_t equals the lifetime utility obtained from the optimal consumption stream after time t, as can be seen by solving differential equation (7) under transversality condition $\lim_{t\to\infty} \exp(-\Delta_t)\phi_t\Delta_t = 0$.

Define the rate of time preference ρ as $\rho(c, m, \phi) \equiv -\mathrm{d}\ln\Gamma_t/\mathrm{d}t|_{\dot{c}=0}$, where $\Gamma \equiv g_c(c, \phi)\exp(-\Delta)$ represents the present-value marginal utility of c.[6] Differentiate (4) and (5) by t and substitute the results into (6) to obtain:

(8) $$\frac{\dot{c}_t}{c_t} = -\frac{g_c(c_t, \phi_t)}{cg_{cc}(c_t, \phi_t)}(r_t - \rho(c_t, m_t, \phi_t)),$$

(9) $$\frac{\dot{m}_t}{m_t} = -\frac{g_m(m_t, \phi_t)}{mg_{mm}(m_t, \phi_t)}(r - \rho(c_t, m_t, \phi_t)).$$

where

$$\rho(c_t, m_t, \phi_t) = \delta(u(c_t, m_t)) - \frac{\delta'(u(c_t, m_t))}{1 - \phi\delta'(u(c_t, m_t))}g(c_t, m_t, \phi_t).$$

Money is supplied by the government in the form of "helicopter money:"

(10) $$\mu m_t = x_t,$$

where μ denotes the growth rate of nominal money supply.

Goods are produced by competitive firms with a constant-to-scale technology. Let f denote a per capita production function satisfying $f_k > 0$ and $f_{kk} < 0$. From the profit-maximizing behavior of firms, r and w are given as

(11) $$w_t = f(k_t) - k_t f_k(k_t); r_t = f_k(k_t).$$

By substituting equations (10) and (11) into (3), we obtain

(12) $$\dot{k}_t = f(k_t) - c_t.$$

[6] The rate of time preference can also be defined by using the marginal utility of real money balances. Due to weak separability of the Uzawa-type intertemporal preferences, however, the time preferece rate with respect to m equals that with respect to c. For the detailed discussions, see Shi (1994).

Equations (3) through (12) jointly determine the equilibrium dynamics for $(a_t, c_t, k_t, m_t, r_t, w_t, , x_{t,} \lambda_t, \phi_t)$. The steady-state solution (c^*, m^*, k^*, ϕ^*) is determined by

$$\delta\left(u\left(c^*, m^*\right)\right) = f_k(k^*),$$

$$\chi(c^*, m^*) = f_k(k^*) + \mu, \tag{13}$$

$$f\left(k^*\right) = c^*,$$

$$\phi^* = \frac{u\left(c^*, m^*\right)}{f_k(k^*)}.$$

To obtain a linearized autonomous dynamic system, combine $\dot{m}/m = \mu - \pi$ with (5) and (9) to get

$$\frac{g_m}{m_t g_{mm}}(\rho(c_t, m_t, \phi_t) - f_k(k_t)) = \mu + f_k(k_t) - \chi(c_t, m_t),$$

which can be solved for m_t as

$$m_t = \upsilon(c_t, \phi_t, k_t; \mu). \tag{14}$$

From (11) and (14), the dynamic equations (7), (8), and (12) then reduce to:

$$\dot{c}_t = -\frac{g_c\left(c_t, \phi_t\right)}{g_{cc}\left(c_t, \phi_t\right)}\left(f_k(k_t) - \rho\left(c_t, \upsilon(c_t, \phi_t, k_t; \mu), \phi_t\right)\right),$$

$$\dot{\phi}_t = -g\left(c_t, \upsilon(c_t, \phi_t, k_t; \mu), \phi_t\right), \tag{15}$$

$$\dot{k}_t = f\left(k_t\right) - c_t.$$

Letting z represent $(c, \phi, k)'$, this dynamic system can be linearized around the steady state as $\dot{z}_t = A\left(z_t - z^*\right)$;

$$A = \begin{pmatrix} 0 & \frac{r\delta' u_c}{g_{cc}} & -\frac{g_c}{g_{cc}} f_{kk} \\ -g_c - g_m(-\frac{\chi_m}{\chi_c}) & r - g_m(-\frac{r\delta' u_m}{m g_{mm}\chi_m}) & -g_m(\frac{f_{kk}}{\chi_m} + \frac{g_m}{m g_{mm}\chi_m} f_{kk}) \\ -1 & 0 & r \end{pmatrix},$$

where the coefficient matrix A is evaluated at the steady-state point.

The steady-state equilibrium point is locally saddle-point stable if and only if $\det(A) < 0$ and trace $(A) > 0$, for which case matrix A has two positive and one negative characteristic roots. It can be easily shown that the saddle-point stability condition is satisfied if and only if

$$\Lambda \equiv (1 - \chi \frac{\chi_c}{\chi_m}) r\delta' u_c - f_{kk} + \frac{\chi}{\chi_m}\delta' u_c f_{kk} > 0,$$

$$2r + \frac{g_m r \delta' u_m}{m g_{mm} \chi_m} > 0.$$

In what follows, these conditions are assumed to be met.[7] [8]

3 The effect of inflation

Let us now examine the effect of a permanent increase in the core rate of inflation μ. From (13), the steady-state equilibrium is determined by

$$\delta\left(u\left(f\left(k^*\right), m^*\right)\right) = f_k(k^*), \tag{16}$$

$$\chi(f\left(k^*\right), m^*) = f_k(k^*) + \mu. \tag{17}$$

Figure 1 depicts the determination, where schedule DD' represents (16) and MM' (17). The MM' schedule is positively sloping and, under the assumption that Λ is positive, the slope of the DD' schedule is also positive but gentler than that of MM'. The steady-state equilibrium is determined at the intersection of the two schedules. An increase in μ shifts the MM' schedule upward, bringing the steady state point from E_0 to E_1. As illustrated, the steady state capital stock decreases in response to the rise in the inflation rate. In fact, from (16) and (17), we can derive

$$\frac{\mathrm{d}k^*}{\mathrm{d}\mu} = -\frac{\delta' u_m}{\chi_m \Lambda} < 0. \tag{18}$$

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[7] We have

$$\det(A) = \frac{g_m}{g_{cc}\chi_m} r\delta' u_c f_{kk} + \frac{g_c r}{g_{cc}}\left[(1 - \frac{g_m}{g_c}\frac{\chi_c}{\chi_m}) r\delta' u_c - f_{kk}\right],$$

$$\operatorname{trace}(A) = 2r + \frac{g_m r \delta' u_m}{m g_{mm} \chi_m}.$$

It can easily be shown that $\det(A) < 0 \Leftrightarrow \Lambda > 0$.

[8] It is a standard procedure to assume saddle-point stability for comparative dynamics. See, e.g., Heijdra (1998).

Intuitively, higher inflation reduces real money holdings, which, under decreasing marginal impatience, *ceteris paribus* raises the discount rate and hence the interest rate in the steady state. This reduces the capital stock in the long run. This result contrasts to the case of increasing marginal impatience, in which case (16) is depicted as negatively sloped as illustrated as schedule II' in Figure 1, so that the increase in μ enlarges k^* as illustrated by the movement from E_0 to E_2.

Proposition 1: *When impatience is marginally decreasing, an increase in the core rate of inflation reduces the capital stock.*

Remark: As seen from (18), whether an increase in μ increases or decreases k^* depends crucially on the sign of $\partial\delta/\partial m$ ($= \delta' u_m$ in present case), but not on $\partial\delta/\partial c$, that is on whether impatience with respect to m is marginally increasing or decreasing, but not on whether impatience with respect to c is marginally increasing or decreasing. When δ depends directly on (c, m) independently of u, and hence when preferences are weakly non-separable in the sense of Shi (1994), the same result as in Proposition 1 can be valid even if impatience is marginally increasing in c, insofar as it is marginally decreasing in m.

From (13), jointly with the decrease in k^*, steady-state consumption is also reduced. Since consumption and real money balances decrease and hence the discount rate rises in the long run, the steady-state welfare level ϕ^* is definitely lowered. Note that the new steady-state point E_1, obtained under decreasing marginal impatience, is located in the south-west to, and hence dominated by, point E_2, attained under increasing marginal impatience. This implies that the long-run welfare deterioration caused by an increase in μ is larger than it would when impatience is marginally increasing. More precisely, we have $u\left(c^*, m^*\right)|_{\text{point } E_1} < u\left(c^*, m^*\right)|_{\text{point } E_2}$ and $f_k\left(k^*\right)|_{\text{point } E_1} > f_k\left(k^*\right)|_{\text{point } E_2}$, so that, from (13), $\phi^*|_{\text{point } E_1}$ $(= u\left(c^*, m^*\right)/f_k\left(k^*\right)|_{\text{point } E_1})$ is smaller than $\phi^*|_{\text{point } E_2}$ $(= u\left(c^*, m^*\right)/f_k\left(k^*\right)|_{\text{point } E_2})$. Similarly it is easy to see that the long-run welfare deterioration is also larger than it would under constant marginal impatience.

Proposition 2: *The long-run welfare cost of inflation under decreasing marginal impatience is larger than predicted with increasing or constant marginal impatience.*

If actual time preference displays decreasing marginal impatience as often empirically reported, Proposition 2 implies that estimates of the welfare cost of inflation which are obtained by using models of increasing or constant marginal impatience are underestimates of the actual cost. For example, by using constant time preference models, Jones and Manuelli (1995) and Lucas (2000) obtain quantitatively very modest estimates for the welfare cost of inflation. They may, however, underestimate the actual costs.

4 Conclusions

By introducing decreasing marginal impatience into a neoclassical monetary growth model, we have shown that, consistently with many empirical studies, a rise in the core rate of inflation reduces the steady-state capital stock. The resulting long-run welfare cost of inflation is larger than it would if impatience is marginally increasing or constant. This implies that if actual impatience is marginally decreasing as empirically reported, misspecifying it as marginally increasing or constant as in the literature results in underestimating the long-run welfare costs of inflation.

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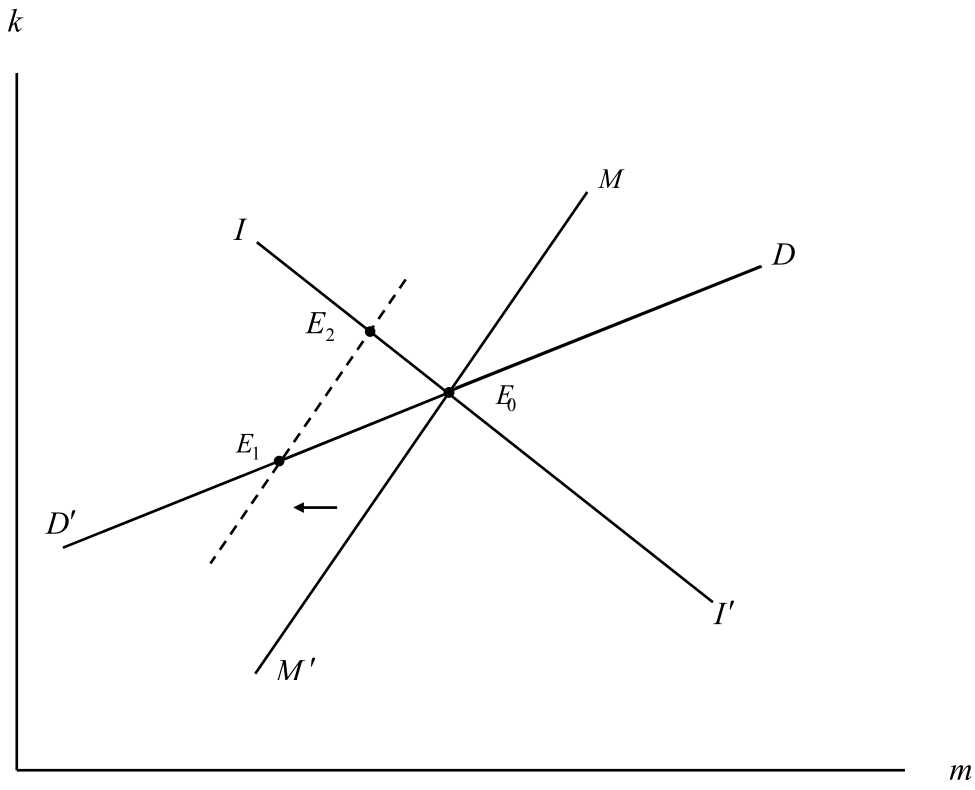

Figure 1. Inflation and capital stock under decreasing marginal impatience