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HABIT FORMATION IN AN INTERDEPENDENT WORLD ECONOMY

Shinsuke Ikeda
and
Ichiro Gombi

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The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

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Shinsuke Ikeda[2] and Ichiro Gombi[3]
Osaka University and Ritsumeikan University

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[2]Corresponding author: S. Ikeda, The Institute of Social and Economic Research, Osaka University, Mihogaoka, Ibaraki, Osaka 567-0047, Japan. Telephone: 81-6-6879-8568, Facsimile: 81-6-6878-2766, Email: <ikeda@iser.osaka-u.ac.jp>.
[3]I. Gombi, The Faculty of Economics, Ritsumeikan University, Kusatsu, Shiga 525-8577 Japan. Telephone: 81-77-561-4840, Email: <gombi@ec.ritsumei.ac.jp>.

Abstract

In a two-country world economy, consumption-habit dynamics in one country are affected, due to endogenous interest rate adjustments, by the other country's habits and preferences. External indebtedness depends crucially on international differences in habit-adjusted net output less habitual living standard. Interest rate adjustments enlarge the consumption impact of an income shock. Consistently with the empirical facts, the habit parameter of a large country, therefore, would be underestimated, and the current account volatility overestimated, if estimated using a small-country model. An increase in fiscal spending in one country can benefit the country and harm the neighbor due to intertemporal terms-of-trade effects.
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1 Introduction

An important stylized fact is that consumers' habit forming behavior has a significant effect on intertemporal choices, and hence macroeconomic phenomena (e.g., Constantinides, 1990, Campbell and Cochrane, 1999, Carroll et.al., 2000, Díaz et.al., 2003). Accordingly, there have been numerous attempts to better understand open-macroeconomic phenomena by incorporating habit formation. For example, by introducing consumers' habit-persistent behavior to small-country models, Mansoorian (1993a, b) and Ikeda and Gombi (1999) derive its implications for current account dynamics and macroeconomic policies. Gruber (2002) provides empirical support to an "intertemporal current account model" with habit formation as explaining the actual current account behavior of the G-7 countries.

However, in the existing studies, including those small-country analyses, habit formation is examined within the representative-agent framework, in which consumers' interactions due to pecuniary externalities are completely neglected. When the interest rate is endogenously determined in a world economy with two heterogeneous agents (countries), one consumer's (or country's) habit forming behavior is affected by other consumers' (or countries') habits and preferences. It has a particular importance to examine how this produces different (or same) results from what the literature has, theoretically or empirically, predicted using small-country and/or representative-agent models; and to figure out what kind of bias would arise when a large country's consumption behavior is estimated by using a small-country model. The two-country analysis would also enable us to investigate how habit formation in consumption affects each consumer's (country's) long-run indebtedness and the transmission mechanism of various policies.

This paper presents a tractable two-country world economy model with heterogeneous habit forming consumers. The purposes of the paper are: (i) to describe interactive habit-forming consumer behavior in a world economy; (ii) to examine long-run external indebtedness; (iii) to address how fiscal policies in one country affect the two countries' consumptions, wealth holdings, and welfare levels; and (iv) to figure out empirical implications. Issues (ii) and (iii) are also examined by Devereux and Shi (1991) by using a variable discount rate model with capital accumulation.[1] At the cost of assuming away capital

[1] The other literature concerning two-country dynamic models include Turnovsky and Bianconi (1992), Kalayalcin (1996), and Bianconi and Turnovsky (1997), which focus on supply side adjustments such as capital accumulation and/or leisure/labor choice. See

accumulation, our main interest is to focus on habit formation as a more empirically relevant component of the macroeconomic adjustment process.

Under habit formation, consumption dynamics depend crucially on intertemporal complementarities. For example, when the marginal utility of consumption is strongly and positively related to habits, which Ryder and Heal (1973) designate adjacent complementarity, optimum consumption is tied tightly to and comoves positively with current habits. When there are heterogeneous consumers or countries, the behavior of individual consumers is affected by other consumers' habits and preferences, as well as their own, due to interest rate adjustments. Equilibrium intertemporal complementarity in consumption can thus differ from what is defined by individual consumers' subjective preferences.

To examine habit forming consumer behavior in an interdependent world economy, we propose to simplify the general equilibrium system by using two aggregations. First, we construct a *world felicity function* from both countries' felicity functions. One appealing aspect of this approach is the ability to thereby characterize intertemporal complementarity in each country's *equilibrium* consumption in a similar way as in the small-country case. Even though habit preferences in one country display distant (resp. adjacent) complementarity, its equilibrium consumption behavior may exhibit adjacent (resp. distant) complementarity. A corollary is that even if one country has no preference for habits, the time series of consumption rates will display habit-persistent patterns.

Second, we define an aggregate habit capital as the sum of individual countries' habit capitals. When aggregate output is constant, the aggregate habit capital is also constant since, from the market clearing condition, it equals aggregate output. When aggregate output changes at a point in time due to exogenous shocks, the aggregate habit formation affects consumption dynamics as an additional dynamic source. The resultant consumption behavior in each country is then affected by the aggregate habit capital, as well as its own habit capital.

We characterize long-run external indebtedness by defining the effective net output that can be allocated for saving under habit formation as *surplus income*. It is shown that a country is more likely to be a long-run creditor the larger surplus income it has. Surplus income depends on net output (i.e.,

also Ikeda and Ono (1991) for an analysis of non-monotonic multi-country dynamics under heterogeneous consumer preferences.

output income less government spending), the initial stock of habit capital, and the strength of adjacent complementarity. When a country's net output is larger (resp. smaller) than its habitual consumption level, we show that the country has larger (resp. smaller) surplus income and hence is more likely to be a creditor (debtor), the stronger adjacent complementarity it exhibits. This result is shown to be consistent with the usual proposition that a country tends to be a long-run creditor the more patience it has. The novelty of this paper is that which country is more patient depends on the relative strengths of adjacent complementarity and on how much greater net output than the habitual consumption standard they have.[2]

We address macroeconomic adjustments in response to an increase in country H's fiscal spending financed with lump-sum taxes. As shown in the literature (see, e.g., Mansoorian 1993a, b), in the small-country case, negative income shocks, such as an increase in fiscal spending, necessarily lowers both initial and steady-state consumption levels; and decreases (resp. increases) net foreign assets under adjacent (resp. distant) complementarity. By assuming that the initial holding of net foreign assets, and hence the income re-transfer effect due to interest rate changes are negligible, we show that an increase in a country's fiscal spending (i) can increase the initial or steady-state consumption level of that country, with the other country's consumption being crowded out; (ii), as in the small country case, necessarily worsens (resp. improves) the steady-state external asset position if the country's preference displays adjacent (resp. distant) complementarity, irrespective of the other country's preferences; and (iii) has long-run and short-run spill-over effects on the other country's consumption, depending crucially on whether the policy implementing country displays adjacent or distant complementarity.

Our two-country model helps to explain two puzzling facts that Gruber (2004) reports by using a small-country model. First, his GMM estimates of the habit preference parameter for two large countries, United States and Japan, are smaller than those for smaller countries, e.g. Italy, Spain, and Netherlands. Second, in the case of the two large countries, his habit model overpredicts the actual volatility of the current account, unlike for in the smaller country case. To explain these, say, "large-country puzzles", we

[2] For the effect of habit formation on wealth distribution, see Díaz et al. (2003). They show that habit formation decreases wealth inequality by raising wealth-poor consumers's precautionary savings more than wealth-rich ones'.

show that, under adjacent complementarity, interest rate adjustments enlarge the consumption effect of an income shock and thereby mitigate its current account effect. As an important empirical implication, the habit parameter of a large country would be underestimated, and the current account volatility overestimated, if they were estimated, as in Gruber (2004), using a small-country model.

With non-negligible initial net foreign assets, an increase in fiscal spending induces additional income effects by changing the world interest rate. Even though fiscal spending is assumed to generate no direct utility, it is shown, as an important welfare implication, that due to the intertemporal terms-of-trade effect, an increase in fiscal spending financed by lump-sum taxation can benefit the policy implementing country and harm the neighboring country. This is an intertemporal version of (reversed) "immiserizing growth" discussed by, e.g., Bhagwati (1958) and Brecher and Bhagwati (1982).

The remainder of the paper is structured as follows: In Section 2, we present a two-country model. In Section 3, we examine equilibrium dynamics of consumption, the interest rate, and net foreign assets. Section 4 analyzes the effects of an increase in one country's fiscal spending. In Section 5, welfare enhancing government spending is discussed. Section 6 concludes the paper.

2 The Model

Consider a two-country world economy composed of home and foreign countries. Each country is populated with infinitely lived identical agents. The representative agents in home and foreign countries are referred to as consumers H and F, respectively. They consume a single consumption good and hold wealth in the form of bonds. Both goods and bonds are assumed to be costlessly traded in international markets. For brevity, the representative agents H and F are assumed to be endowed with constant amounts of output y and y^*, respectively. Throughout the paper, the foreign country's variables are represented with superscript asterisks.

Consumption forms habits. Letting $z_t^{(*)}$ represent the time-t *habit,* we specify $z_t^{(*)}$ as the average of the past consumption rates $c_s^{(*)}, s \leqq t$: $z_t^{(*)} = \alpha \int_{-\infty}^{t} c_s^{(*)} \exp\left(-\alpha\left(t-s\right)\right) ds$, or equivalently

$$\dot{z}_t^{(*)} = \alpha\left(c_t^{(*)} - z_t^{(*)}\right), \qquad (1)$$

where $\dot{x}$ represents the time derivative of variable x and α represents the discount rate for past consumption rates. We assume that consumers in both countries have the common discount rate α for past consumption rates. This enables us to obtain the tractable dynamics of a two-country equilibrium.

Consumers H and F have different preferences over consumption and habits. Consumer H's preferences are specified as

$$U_0 = \int_0^\infty u(c_t, z_t) \exp(-\theta t) \, \mathrm{d}t, \tag{2}$$

where θ represents the subjective discount rate, which is assumed constant. Following Ryder and Heal (1973), function u is assumed to satisfy the following regularity conditions: (C1) $u_c > 0$; (C2) $u_z \leqq 0$; (C3) $u_c(c,c) + u_z(c,c) > 0$; (C4) u is concave in (c, z); (C5) $\lim_{c \to 0} u_c(c, z) = \infty$ uniformly in z; and (C6) $\lim_{c \to 0} [u_c(c,c) + u_z(c,c)] = \infty$. Consumer F's utility U_0^* is specified in the same way.

Due to habit formation, consumer preferences are intertemporally dependent. Within the representative agent framework, Ryder and Heal (1973) characterize the resulting intertemporal complementarity in consumption by adjacent and distant complementarities: With adjacent complementarity, an increase in today's habits increases the marginal utility of today's consumption more than it increases marginal *disutility* of habits, thereby enlarging today's optimal consumption. Under distant complementarity, an increase in today's habits increases marginal *disutility* of habits so much that it reduces today's consumption rate. We apply this idea to characterize each country's preferences: Consumer H's preference is said to display *adjacent (distant) complementarity* when $u_{cz}(c,c) + \frac{\alpha}{\theta + 2\alpha} u_{zz}(c,c) > (<) 0$. Similarly, adjacent and distant complementarities in country F are defined in terms of u^*. In the case of the representative agent economy, intertemporal complementarities displayed by consumers' preferences definitely determine the equilibrium dynamics (see, e.g., Ryder and Heal 1973 and Ikeda and Gombi 1999). In contrast, when the economy consists of heterogeneous agents, as in the present case, pecuniary externalities through markets divert each agent's consumption saving behavior from what is predicted from each agent's own preference, as shown in the following section.

To ensure the steady state, let us assume that

$$\theta = \theta^*.$$

Unlike in the single agent model, the interest rate r can deviate from this subjective discount rate.

Let b_t denote net foreign assets held by consumer H. The flow budget constraint for consumer H is given by

$$\dot{b}_t = r_t b_t + y - c_t - \tau, \tag{3}$$

where τ represents a lump-sum tax levied by the government in country H. Given the initial values (b_0, z_0), consumer H chooses $C_0 = \{c_t, b_t, z_t\}_{t=0}^{\infty}$ so as to maximize (2) subject to: (i) the flow budget constraint (3); (ii) the formation of consumption habits (1); and (iii) the transversality conditions.

Letting $\lambda_t\,(\geqq 0)$ be the shadow price of savings and $\xi_t (\leqq 0)$ that of habit formation, the optimal conditions are given by

$$u_c\,(c, z) = \lambda_t - \alpha \xi_t, \tag{4}$$

$$\dot{\lambda}_t = (\theta - r_t)\,\lambda_t, \tag{5}$$

$$\dot{\xi}_t = (\theta + \alpha)\,\xi_t - u_z\,(c_t, z_t)\,, \tag{6}$$

together with (1), (3), and the transversality conditions for b_t and z_t. Consumer F's behavior can be specified in exactly the same way.

The governments in the two countries follow the balanced budget principle. Their fiscal spendings g and g^* equal tax revenues from lump-sum taxes τ and τ^*, respectively. By substituting this into (3) we obtain the balance of payment equation,

$$\dot{b}_t = r_t b_t + y - c_t - g. \tag{7}$$

The model is closed by introducing the market clearing conditions:

$$c_t + c_t^* = Y\,(\equiv y + y^* - g - g^*)\,, \tag{8}$$

$$b_t + b_t^* = 0, \tag{9}$$

where Y represents the disposable aggregate income. By Walras' law, these are not independent: (9) together with (7) and the corresponding constraint for the foreign consumer imply (8). In sum, the equilibrium time path of $(b_t, b_t^*, c_t, c_t^*, z_t, z_t^*, r_t, \lambda_t, \lambda_t^*, \xi_t, \xi_t^*)$ is determined by equations (1), (4) through (7), the corresponding equations for F, and the market equilibrium condition (8) or (9).

3 Two-Country Equilibrium

3.1 The world felicity function

In this economy consumption dynamics in the two countries interact through international market transactions. To ease the analysis, it is useful to define *aggregate habit capital* Z_t as

$$Z_t \equiv z_t + z_t^*. \tag{10}$$

Since α is assumed to be internationally identical, the dynamics of Z_t can be expressed from (1) and the market clearing condition (8) as

$$\dot{Z}_t = \alpha \left(Y - Z_t \right). \tag{11}$$

Note that this is an autonomous dynamic equation. If Z_t and z_t are given, z_t^* are determined from (10) as

$$z_t^* = Z_t - z_t. \tag{12}$$

By using (12), dynamics can be drastically simplified in the following manner. Define σ as:

$$\sigma = \frac{\lambda}{\lambda^*},$$

which is constant over time since $\dot{\lambda}_t/\lambda_t = \dot{\lambda}_t^*/\lambda_t^*$ from (5) (and the corresponding equation for F). We then construct aggregate indices for (u, u^*) and (ξ, ξ^*) as:

$$v\left(c, z, Z\right) \equiv u\left(c, z\right) + \sigma u^*\left(Y - c, Z - z\right), \tag{13}$$

$$\varsigma \equiv \xi - \sigma \xi^*, \tag{14}$$

where (8) and (12) are substituted.

The new felicity function v represents an aggregate utility function with weights being given by relative individual marginal utilities. We could call it the *world felicity function*. The new shadow price ς is the weighted difference in the shadow prices of habits. From the definition of v, ς could be regarded as net marginal gains of transferring consumption capital from consumer F to H. With these definitions, we can reduce the equilibrium dynamics of consumption habits around a steady-state point as follows:

Proposition 1: *In equilibrium, consumer H's habits around a steady-state point are governed by*

$$\begin{pmatrix} \dot{z} \\ \dot{\varsigma} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} M & \begin{matrix} -\frac{\alpha v_{cz}}{v_{cc}} \\ \frac{v_{cz}v_{cz}+v_{cc}v_{zz}}{v_{cc}} \end{matrix} \\ 0 \quad 0 & -\alpha \end{pmatrix} \begin{pmatrix} \hat{z} \\ \hat{\varsigma} \\ \hat{Z} \end{pmatrix}; \tag{15}$$

$$M = \begin{pmatrix} -\alpha\left(1+\frac{v_{cz}}{v_{cc}}\right) & -\frac{\alpha^2}{v_{cc}} \\ \frac{(v_{cz})^2 - v_{cc}v_{zz}}{v_{cc}} & \theta + \frac{\alpha(v_{cc}+v_{cz})}{v_{cc}} \end{pmatrix},$$

where $\hat{x}$ denotes a deviation of variable x from its steady-state value $\bar{x}$: $\hat{x} \equiv x_t - \bar{x}$.

Proof. See Appendix A. ■

Remark 1: The dynamic equation (15) is block recursive, where coefficient matrix M for habit and its shadow price $\left(\hat{z}, \hat{\zeta}\right)$ is exactly the same as in the case of the small-country model (e.g., Ikeda and Gombi 1999 and Mansoorian 1993a, b), except that v and ς represent the world average preferences, instead of individual countries' preferences.

Remark 2: As easily seen, Proposition 1 is an application of the second theorem of welfare economics. Without any distortion, the Pareto optimal resource allocation attained as a complete market equilibrium can be duplicated as a solution to a social welfare maximization problem:

$$\max\ U_0 + \beta U_0^*,$$

$$\text{subject to } Y = c_t + c_t^*,\ Z_t = z_t + z_t^*,\ \dot{z}_t = \alpha\left(c_t - z_t\right),$$
$$\text{and } \dot{Z}_t = \alpha\left(Y - Z_t\right).$$

Indeed, the corresponding Hamiltonian function,

$$H_t = u(c_t, z_t) + \beta u^*(Y - c_t, Z_t - z_t) + \varsigma_t \alpha\left(c_t - z_t\right) + \iota_t \alpha\left(Y - Z_t\right),$$

produces the same optimal condition as in Proposition 1 when weight β is set equal to σ.

Since the dynamic equation (11) of aggregate habit Z is autonomous, it is noteworthy that, if the economy is initially in steady-state equilibrium, and if the steady-state value of Z, which equals Y from (11), does not change, the aggregate habit stays at value Y. In other words, in the absence of aggregate income shocks, the aggregate habit dynamics degenerate at $\bar{Z}(=Y)$, so that the equilibrium dynamics in (15) are completely described by the $\left(\hat{z},\hat{\zeta}\right)$ dynamics, as in the small-country case. Note, however, that the transition matrix here is defined in terms of the world felicity function.

3.2 Equilibrium dynamics

3.2.1 Habit and consumption

Dynamic system (15) has two stable roots:

$$\omega \equiv \frac{\theta - \sqrt{\left(\theta+2\alpha\right)^2 - 4\alpha\left(\theta+2\alpha\right)\Omega}}{2} \; (<0) \text{ and } -\alpha, \tag{16}$$

and one unstable root, which is conjugate with ω, where ω is a characteristic root of M; and $\Omega \equiv -\left(v_{cz} + \frac{\alpha}{\theta+2\alpha}v_{zz}\right)/v_{cc}$. In parallel with intertemporal complementarities with respect to the individual consumers' preferences, Ω captures *equilibrium* intertemporal complementarities: we refer to the world felicity function as displaying adjacent (distant) complementarity when $v_{cz}\left(c,c\right) + \frac{\alpha}{\theta+2\alpha}v_{zz}\left(c,c\right) > (<)0$, i.e., $\Omega > $ (resp. $<$)0.

Let us introduce complementarity indices Ω^H and Ω^F for the individual countries' preferences, in parallel with Ω, as

$$\Omega^H \equiv -\left(u_{cz} + \frac{\alpha}{\theta+2\alpha}u_{zz}\right)/u_{cc} \text{ and } \Omega^F \equiv -\left(u^*_{cz} + \frac{\alpha}{\theta+2\alpha}u^*_{zz}\right)/u^*_{cc},$$

where, e.g., a positive Ω^H represents adjacent (distant) complementarity for H. The indices, Ω^H and Ω^F, capture the strength of habit formation of the individual consumers H and F. When Ω^H is larger than Ω^F, for example, complementarity between consumption and habit is stronger for consumer H than for consumer F.

Definition: Consumer H is *more (less) in the habit of consuming* than consumer F when $\Omega^H > (<)\Omega^F$.

From the definition (13) of v, intertemporal complementarities with respect to the individual countries' preferences can be related to those prevailing in equilibrium by:

$$\Omega = \varepsilon\Omega^H + \left(1-\varepsilon\right)\Omega^F; \tag{17}$$

$$\varepsilon \equiv \frac{u_{cc}}{u_{cc} + \sigma u^*_{cc}}.$$

Therefore, adjacent (distant) complementarity in both countries implies adjacent (distant) complementarity in the two-country equilibrium. When the individual countries' preferences display different intertemporal complementarities, equilibrium intertemporal complementarity is determined by the relative strength of intertemporal complementarities regarding the individual preferences.

As shown in Appendix B.1, the saddle plane governed by the two stable roots are expressed as

$$\dot{z} = \omega \hat{z} - (\omega + \alpha)(1 - \delta)\hat{Z}, \tag{18}$$

$$\dot{Z} = -\alpha \hat{Z}, \tag{19}$$

where

$$\delta \equiv \varepsilon \Omega^H / \Omega.$$

By equating (18) to (1), the consumption dynamics are given by

$$\hat{c} = \left(\frac{\omega + \alpha}{\alpha}\right) \left(\hat{z} - (1 - \delta)\hat{Z}\right). \tag{20}$$

Differentiate this by t and substitute (1), (19), and (20) successively into the result. Then, by taking (8) into account, we obtain the motion of each country's consumption as

$$\dot{c} = \omega \hat{c}, \tag{21}$$

$$\dot{c}^* = \omega \hat{c}^*. \tag{22}$$

Irrespective of the second-order habit dynamics of (18) and (19), therefore, the equilibrium consumption dynamics are of the first order. Equations (1) and (21) jointly govern the equilibrium dynamics of (c, z); and equations (1) and (22) do dynamics of (c^*, z^*). The resulting phase diagrams are illustrated in the (c, z) and (c^*, z^*) planes of Figure 1.

To understand the consumption-habit dynamics given by (20), consider first the case without any aggregate income shocks, $\hat{Z} = 0$, in which case the equilibrium $\hat{c}$-$\hat{z}$ relationship reduces to:

$$\hat{c}_t = \left(\frac{\omega + \alpha}{\alpha}\right) \hat{z}_t \text{ and } \hat{c}^*_t = \left(\frac{\omega + \alpha}{\alpha}\right) \hat{z}^*_t. \tag{23}$$

From equations (16) through (23), when the world felicity function displays adjacent complementarity ($\Omega > 0$), $\omega + \alpha$ is positive and thus the stable arms (23) are positively sloping, whereas under distant complementarity with respect to v ($\Omega < 0$) the trajectories have a negative slope with negative $\omega + \alpha$. That is, in equilibrium, both countries experience positive or negative comovements between consumption and habits, i.e., $\mathrm{d}c_t^{(*)}/\mathrm{d}z_t^{(*)} > (<)\, 0$, as the world felicity function displays adjacent or distant complementarity. This property is the same as in the small-country case except that intertemporal complementarities here are defined in terms of the world felicity function, instead of each country's felicity function.

When aggregate habit capital $\hat{Z}$ varies, the equilibrium consumption dynamics depend on $\hat{Z}$. The second term of (20) captures the effect of an increase in $\hat{Z}$ for given $\hat{z}_t$. Alternatively, we substitute the definition of δ, (17), and (10) successively into (20) to obtain

$$\hat{c} = \left(\frac{\omega + \alpha}{\alpha\Omega}\right)\left(\varepsilon\Omega^H \hat{z} - (1-\varepsilon)\,\Omega^F \hat{z}^*\right). \tag{24}$$

Since $(\omega + \alpha)/\Omega$ is positive irrespective of whether the world felicity function displays adjacent or distant complementarity,[3] this reveals that the sign of $\hat{c}$ is determined by the relative magnitudes of $\varepsilon\Omega^H\hat{z}$ and $(1-\varepsilon)\,\Omega^F\hat{z}^*$. $\hat{c}$ can thus be negative even when $\varepsilon\Omega^H\hat{z}$ is positive if $(1-\varepsilon)\,\Omega^F\hat{z}^*$ is large enough. Suppose that both the countries' preferences display adjacent complementarity, $\Omega^H, \Omega^F > 0$. If $\hat{z}$ and $\hat{z}^*$ are positive, each country's consumer plans to consume a larger quantity than the steady-state level. Since a positive $\hat{c}^*$ implies a negative $\hat{c}$ in equilibrium, the sign of equilibrium $\hat{c}$ should be negative when $(1-\varepsilon)\,\Omega^F\hat{z}^*$ is larger than $\varepsilon\Omega^H\hat{z}$. As will be shown later, the market will be cleared by a rise in the interest rate.

It is important to note that even if one country has no preference for habits, the time series of consumption rates will display seemingly habit-persistent patterns. For example, suppose that country H has no preference for habits: $u_z = 0, u_{zz} = 0$, $u_{cz} = 0$, and hence $\Omega^H = 0$. From (20), the *equilibrium* consumption of country H still depends on its own habit capital, with root ω being determined solely by country F's preferences for habits (see (16) and (17)). Given that c is related to c^* by market equilibrium condition (8), and that z is related to z^* and Z by (10), the dependence of c^* on z^*

[3] When $\Omega = 0$, we have $\omega + \alpha = 0$. It can be shown from (16) that $\lim_{\Omega\to 0}(\omega+\alpha)/\,\Omega = \alpha$.

due to country F's habit preferences induces the dependence of c on (z, Z) even though c does not depend directly on z.

3.2.2 The interest rate

As is proven by Appendix B.2, the interest rate dynamics are given by

$$\hat{r} = \kappa \left(\Omega^H - \Omega^F\right) \hat{z} + \left[\frac{(1-\varepsilon)\,\Omega^F \left(\Omega^F - \Omega^H\right) \kappa}{\Omega} + \frac{\Omega^H \Omega^F}{\Omega} \eta\right] \hat{Z}, \tag{25}$$

where κ and η are defined as

$$\kappa \equiv \frac{\omega u_{cc} u_{cc}^* (\theta + 2\alpha)}{\lambda^* (\alpha + \theta - \omega)(u_{cc} + \sigma u_{cc}^*)} (> 0)\,; \eta \equiv -\frac{\alpha u_{cc} u_{cc}^*}{\lambda^* u_{cc} + \lambda u_{cc}^*} (> 0)\,.$$

Equation (25) reveals that the equilibrium interest rate is driven by country H's habit capital $\hat{z}$ and the aggregate habit capital $\hat{Z}$. Since $\hat{z} - \hat{z}^* = 2\hat{z} - \hat{Z}$, a change in $\hat{z}$ with a given $\hat{Z}$ can be taken as a change in difference between $\hat{z}$ and $\hat{z}^*$. Equation (25) thus implies that the dynamics of $\hat{r}$ rely on the difference between, and the sum of, $\hat{z}$ and $\hat{z}^*$. Given that the dynamics of z and Z are jointly generated by (18) and (19), it is easy to show that the resulting dynamics of the interest rate can be non-monotonic. As seen from (25), $\hat{r}$ also depends on the difference in Ω^H and Ω^F as well as on the average Ω of the two.

It is too complicated to analyze $\hat{r}$ by using (25) directly. Instead, we focus on two special cases: that (i) the aggregate habit stock stays constant at the initial steady-state value, i.e., $\hat{Z}_t = 0$ for all t; and that (ii) the two countries' preferences exhibit identical degrees of adjacent complementarity, i.e., $\Omega^H = \Omega^F = \Omega$.[4]

Case (i): $\hat{Z}_t = 0$ for all t. In this case, equation (25) reduces to:

$$\hat{r} = \kappa \left(\Omega^H - \Omega^F\right) \hat{z}, \tag{26}$$

[4] This case takes place (i) when the values of Ω^i $(i = H, F)$ happen to be the same in steady state; (ii) when the felicity functions take quadratic forms with constant and internationally identical second-order derivatives; or (iii) when the two countries are perfectly identical with respect to preferences, endowments, and the initial stocks of habit capitals and net foreign assets. In case (iii), although the Ω^i values are globally identical, we cannot examine the case of non-zero b_0.

implying that $\hat{r}$ positively (negatively) comoves with $\hat{z}$ as consumer H is more or less in the habit of consuming than consumer F, i.e., $\Omega^H - \Omega^F$ is positive or negative.[5] To understand this, suppose that both consumers H and F display adjacent complementarity ($\Omega^H > 0$, $\Omega^F > 0$); that $\Omega^H > \Omega^F$; and that $z(t) > \bar{z}$, and hence, from (12), $z^*(t) < \bar{z}^*$. Then, if r equaled θ, consumer H would consume more now than in the future steady state, thereby producing excess demand in the "present good" market, irrespective of that consumer F would consume less now than in the future (note: $\Omega^H > \Omega^F$). The market clearing r should thus be higher than its steady-state level θ. As shown later, the dynamics of $\hat{z}$ are monotonically driven by stable root ω when $\hat{Z}_t = 0$. From (26), therefore, the interest rate in this case displays monotonic dynamics,

$$\dot{r} = \omega \hat{r}.$$

Case (ii): $\Omega^H = \Omega^F = \Omega$. When the two countries' preferences exhibit the identical degrees of adjacent complementarity, the equilibrium interest rate is determined from (25) as

$$\hat{r} = \eta \Omega \hat{Z}, \tag{27}$$

implying that $\hat{Z}$ and Ω play crucial roles in this case. Suppose that $Z(t) > \bar{Z}$. Then, if consumers exhibit adjacent (distant) complementarity, i.e., $\Omega > 0$ ($\Omega < 0$), *ceteris paribus* there prevails excess demand (supply) in the present good market. This renders r higher (lower) than its steady-state value θ. The equilibrium interest rate positively (negatively) comoves with the aggregate habit stock, which exhibits monotonic motions with stable root $-\alpha$ (see (19)). The resulting dynamics of r are given explicitly by

$$\dot{r} = -\alpha \hat{r}. \tag{28}$$

3.2.3 Net foreign assets

The transition dynamics of net foreign assets also depend on the property of the world felicity function and international heterogeneity in habit formation.

[5] If country H is identical to country F ($\Omega^H = \Omega^F$), or if one of the two countries is small in the sense that λ or λ^* is infinitely large (λ or $\lambda^* \to \infty$), for which case κ is zero, the interest rate dynamics degenerate at θ as in the small-country case (e.g., Ikeda and Gombi 1999).

As shown by Appendix B.3, by linearizing (7) and substituting (24) and (25) into the result, we can obtain

$$\begin{aligned}\hat{b}_t &= \frac{1}{\theta-\omega}\left(\frac{\omega+\alpha}{\alpha}-a_1 b_0\right)\hat{z}_t+\left[-\frac{(\omega+\alpha)(1-\varepsilon)\Omega^F}{\alpha(\theta-\omega)\Omega}\right.\\ &\quad\left.+\frac{b_0}{\theta+\alpha}\left\{\frac{a_1(1-\varepsilon)\Omega^F(\omega+\alpha)}{(\theta-\omega)\Omega}-a_2\right\}\right]\hat{Z}_t, \qquad (29)\end{aligned}$$

where a_1 and a_2 represent the coefficients of $\hat{z}$ and $\hat{Z}$ in (25):

$$\begin{aligned}a_1 &= \kappa\left(\Omega^H-\Omega^F\right),\\ a_2 &= \frac{(1-\varepsilon)\Omega^F\left(\Omega^F-\Omega^H\right)\kappa}{\Omega}+\frac{\Omega^H\Omega^F}{\Omega}\eta,\end{aligned}$$

respectively.

Two habit stocks $\hat{z}_t$ and $\hat{Z}_t$ affect $\hat{b}_t$ by changing consumption and the interest rate. Changes in the habit stocks affect consumption through (20) and thereby influence the accumulation of net foreign assets. This effect is expressed by the terms without b_0 in (29), which depend on $\omega+\alpha$, i.e., whether the world felicity function displays distant or adjacent complementarity. The habit stocks also affect the interest rate by (25). The resulting change in interest income alters the time profile of net foreign assets. The effect is captured by the terms associated with b_0 in (29). These income retransfer effects through interest rate changes may well have various adverse effects. To avoid analytical complexity, we get rid of the effects by assuming that the initial amounts b_0 of net foreign assets for both countries are negligible, as in Devereux and Shi (1991). The implications of a nonzero b_0 will be discussed briefly in Section 5 and in Appendix C.

Assumption 1: $b_0^* \approx 0$.

With Assumption 1, equation (29) reduces to

$$\hat{b}_t = \frac{\omega+\alpha}{\alpha(\theta-\omega)}\left(\hat{z}_t-(1-\delta)\hat{Z}_t\right).$$

Combining (20) with the above equation yields the saddle arm in the (b,c) plane:

$$\hat{b}(t) = \frac{1}{\theta-\omega}\hat{c}(t). \qquad (30)$$

Since from (21) the dynamics of $\hat{c}(t)$ are monotonic, those of $\hat{b}(t)$ are also monotonic. Figure 1 depicts the saddle arm in the (b, c) plane as upward sloping schedule DD'.

3.3 Steady state

From (1), (5), (7), and (29), the steady-state equilibrium, $(\bar{c}, \bar{c}^*, \bar{z}, \bar{z}^*, \bar{Z}, \bar{b}, \bar{b}^*, \bar{r})$, is determined by:

$$\begin{aligned}
\bar{c} &= \bar{z}, \bar{c}^* = \bar{z}^*, && (31)\\
\bar{c} + \bar{c}^* &= Y = \bar{Z}, && (32)\\
\bar{r} &= \theta, && (33)\\
\bar{r}\bar{b} + y &= \bar{c} + g, && (34)\\
\bar{b} &= \frac{\omega + \alpha}{\alpha(\theta - \omega)}(\bar{z} - z_0) + \frac{(\omega + \alpha)(1 - \varepsilon)\Omega^F}{\alpha(\theta - \omega)\Omega}\left(\bar{Z} - Z_0\right), && (35)\\
\bar{b} &= -\bar{b}^*. && (36)
\end{aligned}$$

together with (8), (9) and (12), where (35) comes from (29) evaluated at $t = 0$.

By substituting (31) and (33) into (34), we obtain

$$\theta\bar{b} + y = \bar{z} + g. \quad (37)$$

Combining (32) and (35) yields

$$\bar{b} = \frac{\omega + \alpha}{\alpha(\theta - \omega)}(\bar{z} - z_0) + \frac{(\omega + \alpha)(1 - \varepsilon)\Omega^F}{\alpha(\theta - \omega)\Omega}(Y - Z_0). \quad (38)$$

Equations (37) and (38) jointly determine $\left(\bar{b}, \bar{z}\right)$. Figure 2 depicts the determination of $\left(\bar{b}, \bar{z}\right)$: schedule BB' depicted with a positive slope represents (37), whereas schedule SS', which represents (38), can be either positively or negatively sloping as the world felicity function displays either adjacent or distant complementarity. Even when schedule SS' is positively sloping, schedule BB' is always steeper than SS'. The steady-state equilibrium point $\left(\bar{b}, \bar{z}\right)$ is given by the intersection point E of the two schedules. Given this, $\left(\bar{c}, \bar{c}^*, \bar{z}^*, \bar{Z}\right)$ is determined by (31) and (32); and $\bar{b}^*$ by (36).

From linearity of (37) and (38), we can examine the determinants of the long-run external asset distribution by solving the two equations for $\bar{b}$ as

$$\bar{b} = -\frac{\omega + \alpha}{\omega(\theta + \alpha)\Omega}(I - I^*), \quad (39)$$

where

$$
\begin{aligned}
I &= \varepsilon \Omega^H \left(y - g + \frac{\omega(\theta+\alpha)}{\alpha(\theta-\omega)} z_0 \right), \\
I^* &= (1-\varepsilon)\, \Omega^F \left(y^* - g^* + \frac{\omega(\theta+\alpha)}{\alpha(\theta-\omega)} z_0^* \right).
\end{aligned}
$$

Roughly, terms I and I^* represent net outputs $y^{(*)} - g^{\ (*)}$in excess of habitual consumption levels $\frac{\omega(\theta+\alpha)}{\alpha(\theta-\omega)} z_0^{(*)}$, adjusted by the relative strengths of adjacent complementarities $\varepsilon\Omega^H$ and $(1-\varepsilon)\,\Omega^F$. For short, we call I and I^* surplus incomes.

Definition: I and $I^*$are referred to as *surplus incomes.*

Since $(\omega + \alpha)/\Omega$ is positive, (39) implies that $\bar{b}$ is positively proportionate to the international difference $I - I^*$ in surplus income. This implies the following proposition:

Proposition 2: *With Assumption 1, a country is a creditor (resp. debtor) in steady state when its surplus income is larger (resp. smaller) than the other country's:* $\bar{b} \gtreqless 0 \Leftrightarrow I \gtreqless I^*$.

To understand this result, suppose that a country's net output exceeds the habitual consumption level. Then, if country H's preferences are of adjacent (resp. distant) complementarity, *ceteris paribus* it saves (resp. consumes) the excessive income and thereby holds positive (resp. negative) external assets in the long run. The same is true for the other country. In equilibrium, the country with a larger (resp. smaller) surplus income accumulates (resp. decumulates) external assets.

From Proposition 2, long-run indebtedness depends on three determinants: (i) net output; (ii) the strength of habit formation; (iii) the initial habit stock. Net output is also emphasized as a key determinant by Frenkel and Razin (1996) and Devereux and Shi (1991). Unlike in our model, in the Frenkel and Razin model with intertemporally separable preferences, net output affects net foreign assets only when time preference differs internationally; and in the Devereux and Shi model of endogenous discounting, larger net output implies smaller net foreign assets.

Proposition 2 implies the three parts of Corollary 1 below. First, from (39), the effects of increases in fiscal spending are obtained as

$$\frac{\mathrm{d}\bar{b}}{\mathrm{d}g} = -\frac{\mathrm{d}\bar{b}^*}{\mathrm{d}g} = \frac{\varepsilon\Omega^H}{(\theta+\alpha)\,\omega}\left(\frac{\omega+\alpha}{\Omega}\right), \tag{40}$$

$$\frac{\mathrm{d}\bar{b}^*}{\mathrm{d}g^*} = -\frac{\mathrm{d}\bar{b}}{\mathrm{d}g^*} = \frac{(1-\varepsilon)\,\Omega^F}{(\theta+\alpha)\,\omega}\left(\frac{\omega+\alpha}{\Omega}\right), \tag{41}$$

implying that an increase in a country's fiscal spending reduces (resp. enlarges) its long-run external assets and hence enlarges (resp. reduces) the other country's when consumer preferences in the policy implementing country display adjacent (resp. distant) complementarity. For example, an increase in g reduces or enlarges $\bar{b}$ as Ω^H is positive or negative. Irrespective of the two-country setting, note that this property is the same as in the small-country case.

Second, a permanent income transfer from country H to country F, $-dy = dy^* > 0$, affects each country's external asset position as

$$\left.\frac{\mathrm{d}\bar{b}}{\mathrm{d}y}\right|_{dy=-dy^*} = -\left.\frac{\mathrm{d}\bar{b}^*}{\mathrm{d}y}\right|_{dy=-dy^*} = -\frac{\omega+\alpha}{\omega(\alpha+\theta)}. \tag{42}$$

It follows that the income transfer reduces or enlarges country H's net foreign assets as the world felicity function exhibits adjacent or distant complementarity.

Third, the initial stock z_0 of habit capital has qualitatively the same implication for the external asset position $\bar{b}$ as fiscal spending g. These discussions can be summarized in the following corollary.

Corollary 1: *With Assumption 1,*
(i) an increase in a country's fiscal spending reduces (resp. enlarges) its long-run external assets and hence enlarges (resp. reduces) the other country's when consumer preferences in the policy implementing country display adjacent (resp. distant) complementarity: $\mathrm{d}\bar{b}/\mathrm{d}g = -\mathrm{d}\bar{b}^*/\mathrm{d}g \lesseqgtr 0 \Leftrightarrow \Omega^H \gtreqless 0$;
(ii) a permanent income transfer from country H to country F reduces (resp. enlarges) H's net foreign assets when the world felicity function exhibits adjacent (resp. distant) complementarity: $\left.\mathrm{d}\bar{b}/\mathrm{d}y\right|_{dy=-dy^*} = -\left.\mathrm{d}\bar{b}^*/\mathrm{d}y\right|_{dy=-dy^*} \gtreqless 0 \Leftrightarrow \Omega \gtreqless 0$*; and*

(iii) the initial stocks of habit capitals have qualitatively the same implication for external asset positions as fiscal spending.

Remark 3: The existing literature (e.g. Devereux and Shi (1991), Frenkel and Razin (1996), and Ikeda and Ono (1992)) shows that a country will tend to be a long-run creditor the more patient it is and/or the greater is its net output. Our results can be related to their discussion as follows. Starting from the initial situation that net output equals the habitual consumption standard in each country, consider a marginal increase in country H's net output, making its net output greater than the habitual consumption standard. As conjectured from the small-country analysis (and as will be indeed shown later), when country H's preferences are more of adjacent complementarity than country F's, then, in response to the positive income shock, country H is more reluctant to increase consumption in the short run and hence more likely to be a creditor in the long run. In this case, country H can be regarded as more patient than F in the short run and, as in the literature, the more patient country becomes a long-run creditor. The point is that which country is more patient depends on the relative strengths of adjacent complementarity and on how much greater net output than the habitual consumption standard they have. To see this, consider, instead, a marginal *decrease* in country H's net output, in which case the new level of net output is lower than the habitual consumption standard. When country H's preferences are more of adjacent complementarity than country F's, country H is more reluctant to decrease consumption in response to the negative income shock and becomes a debtor in the long run. It is true that the less patient country is a long-run debtor. But it is country H, in this case, that is the less patient.

3.4 Welfare

From the linearized equilibrium dynamics obtained above, Appendix B.4 shows that the welfare levels of countries H and F are given by

$$\begin{aligned} U_0 &= \frac{u(\bar{z},\bar{z})}{\theta} + \frac{(\omega+\alpha)\,u_c + \alpha u_z}{\alpha\,(\theta-\omega)}\,(z_0-\bar{z}) \\ &\quad - \frac{(\omega+\alpha)\,(1-\varepsilon)\,\Omega^F}{\alpha(\theta+\alpha)\,(\theta-\omega)\,\Omega}\,[(\theta+\alpha)\,u_c+\alpha u_z]\,(Z_0-Y)\,, \end{aligned} \tag{43}$$

$$U_0^* = \frac{u^*(\bar{z}^*, \bar{z}^*)}{\theta} + \frac{(\omega+\alpha)\,u_c^* + \alpha u_z^*}{\alpha(\theta-\omega)}(z_0^* - \bar{z}^*)$$
$$-\frac{(\omega+\alpha)\,\varepsilon\Omega^H}{\alpha(\theta+\alpha)(\theta-\omega)\,\Omega}\left[(\theta+\alpha)\,u_c^* + \alpha u_z^*\right](Z_0 - Y)\,, \qquad (44)$$

respectively.

4 Effects of government spending

Let us next consider an increase in fiscal spending g in country H, financed by lump-sum taxes.[6]

4.1 Consumption and net foreign assets

As shown by Corollary 1(i), an increase in g reduces or increases $\bar{b}$ as the policy-implementing country H exhibits adjacent or distant complementarity, as in the small-country case. From (8), (37) and (38), the effects on steady-state consumption are obtained as follows:

$$\frac{\mathrm{d}\bar{c}}{\mathrm{d}g} = \frac{\mathrm{d}\bar{z}}{\mathrm{d}g} = \theta\frac{\mathrm{d}\bar{b}}{\mathrm{d}g} - 1 = \frac{\varepsilon\theta\Omega^H}{(\theta+\alpha)\,\omega}\left(\frac{\omega+\alpha}{\Omega}\right) - 1, \qquad (45)$$

$$\frac{\mathrm{d}\bar{c}^*}{\mathrm{d}g} = -\frac{\varepsilon\theta\Omega^H}{(\theta+\alpha)\,\omega}\left(\frac{\omega+\alpha}{\Omega}\right) \gtreqless 0 \Leftrightarrow \Omega^H \gtreqless 0. \qquad (46)$$

From (45), an increase in g decreases $\bar{c}$ whenever Ω^H is positive. Figure 2 illustrates the typical case that $\Omega^H > 0; \Omega^F > 0$; and $\Omega > 0$, for which case the slope of the SS' schedule is positive but gentler than that of BB'. An

[6] We could consider three types of shocks: a shock that affects both of the BB' and SS' schedules in Figure 2 (e.g., an increase in fiscal spending in country H); one that shifts the BB' schedule, leaving the SS' schedule unchanged (e.g., an income transfer between the two counties, $-dy = dy^*$); and one that shifts the SS' schedule, leaving BB' unchanged (e.g., productivity shocks dy^* in country F). For the effects of income transfers, see Gombi and Ikeda (2003). The effects of productivity shocks dy^* in country F can be obtained from the effects of fiscal spending in country H by replacing country H's variables with country F's.

increase in g shifts schedule BB' to the left whereas SS' to the right, thereby reducing $\bar{b}$ and $\bar{z}$ (and hence $\bar{c}$).[7]

From (46), the spill-over effect on country F's consumption $\bar{c}^*$ depends crucially on whether country H's preferences display adjacent or distant complementarity, $\Omega^H \gtreqless 0$. This is because the effects on $\bar{b}$ and hence on $\bar{b}^*$ rely crucially on the sign of Ω^H (see (40)). When country H's preferences exhibit adjacent complementarity ($\Omega^H > 0$), an increase in g decreases $\bar{b}$, increases $\bar{b}^*$ and hence increases $\bar{c}^*$, and vice versa.

From (20) and (45) the impact effect on country H's consumption is derived as

$$\frac{\mathrm{d}c\,(0)}{\mathrm{d}g} = \frac{\varepsilon\Omega^H}{\theta+\alpha}\left(\frac{\omega+\alpha}{\Omega}\right) - 1. \tag{47}$$

This implies that, as in the small-country case, $c\,(0)$ plumps less (resp. more) than its net output as the preferences exhibit adjacent (resp. distant) complementarity: $\mathrm{d}c\,(0)\,/\mathrm{d}g \gtrless -1 \Leftrightarrow \Omega^H \gtrless 0$.[8]

Given this, in turn, market-clearing condition (8) implies that $c^*\,(0)$ decreases or increases as Ω^H is positive or negative. Formally, from (47) and (8), we obtain that

$$\frac{\mathrm{d}c^*\,(0)}{\mathrm{d}g} = -\frac{\varepsilon\Omega^H}{\theta+\alpha}\left(\frac{\omega+\alpha}{\Omega}\right) \lesseqgtr 0 \Leftrightarrow \Omega^H \gtreqless 0. \tag{48}$$

Therefore, when H's preferences display adjacent (resp. distant) complementarity, country F's consumption jumps down (or up) on impact. Comparing (46) with (48) yields

$$\frac{\mathrm{d}c^*\,(0)}{\mathrm{d}g} \lesseqgtr 0 \Leftrightarrow \frac{\mathrm{d}\bar{c}^*}{\mathrm{d}g} \gtreqless 0 \left(\Leftrightarrow \Omega^H \gtreqless 0\right),$$

[7]When Ω^H is negative, however, $\bar{c}$ can *increase* due to an increase in interest income $\theta\bar{b}$ in excess of the lump-sum tax increase if $\Omega \geqq 0$ and hence $\Omega^F > 0$. Formally, from (17) and (45), we can obtain

$$\frac{\mathrm{d}\bar{c}}{\mathrm{d}g} > 0 \Leftrightarrow -\frac{1-\varepsilon}{\varepsilon}\Omega^F \leqq \Omega^H < \frac{\omega\,(1-\varepsilon)\,(\theta+\alpha)}{\alpha\varepsilon\,(\theta-\omega)}\Omega^F.$$

From this relation, it can be seen that necessary conditions for $\mathrm{d}\bar{c}/\mathrm{d}g$ to be positive are: $\Omega^H < 0$, $\Omega^F > 0$, and $\Omega \geqq 0$ by noting that $\frac{\omega(1-\varepsilon)(\theta+\alpha)}{\alpha\varepsilon(\theta-\omega)} \gtreqless -\frac{1-\varepsilon}{\varepsilon} \Leftrightarrow \Omega \gtreqless 0$.

[8]We cannot exclude the pathological case that equilibrium consumption $c\,(0)$ *increases* on impact when consumer H's preferences display strong adjacent complementarity.

implying that the impact effect of an increase in g on c^* is necessarily opposite in signs to its steady-state effect. This is because, without direct income shocks in country F, any increases or decreases in $\bar{c}^*$ should be financed by increases or decreases in $\bar{b}^*$, which are only brought about by decreases or increases in $c^*(0)$ to cause current account surplus or deficits in transition.

When consumer H's preferences are of adjacent complementarity, $\Omega^H > 0$, the transition dynamics are illustrated in Figure 3 by using the phase diagrams introduced in Figure 1. From (40), an increase in g reduces $\bar{b}$ and hence $\bar{c}$ (see (45)). Although $c(0)$ may decrease or increase (see (47)), savings necessarily reduce to generate the current account deficits, as in the small-country case. The figure depicts the normal case that $c(0)$ decreases, so that saddle arm DD' in the (b, c) plane shifts downward. Net foreign assets b gradually decumulate from b_0 to $\bar{b}$ along the new saddle arm, D_1D_1'. Since $\bar{b}^*(= -\bar{b})$ increases, $\bar{c}^*$ increases (see (46)). To realize this, $c^*(0)$ necessarily drops to induce the current account surplus (see (48)). As c^* monotonically increases toward $\bar{c}^*$, z^* sooner or later stops decreasing and starts rising toward $\bar{z}^*(= \bar{c}^*)$.

4.2 The interest rate

The effect on the interest rate can be computed by differentiating (25) with respect to g and by substituting (45) into the result as

$$\begin{aligned}\frac{dr(0)}{dg} &= \frac{\alpha\varepsilon u_{cc}^*\Omega^H}{\lambda^*(\theta+\alpha)(\alpha+\theta-\omega)(u_{cc}+\sigma u_{cc}^*)\Omega}\left\{u_{cc}(\omega-\theta)(\theta+2\alpha)\Omega^H\right.\\ &\quad \left.-\left\{\alpha(\omega+\alpha)u_{cc}+(\theta+\alpha)(\alpha+\theta-\omega)\sigma u_{cc}^*\right\}\Omega^F\right\}. \qquad (49)\end{aligned}$$

Instead of discussing the general properties of this complex result, we restrict our attention to the special case (ii) discussed in Section 3.2.2.

Consider case (ii), in which consumer preferences exhibit identical degrees of adjacent complementarity: $\Omega^H = \Omega^F = \Omega$. Equation (49) then reduces to

$$\left.\frac{dr(0)}{dg}\right|_{\Omega^H=\Omega^F=\Omega} = \eta\Omega \gtreqless 0 \Leftrightarrow \Omega \gtreqless 0. \qquad (50)$$

This implies that an increase in g raises (or lowers) $r(0)$ as the world felicity function displays adjacent (or distant) complementarity. Such a policy affects the commodity market on both the supply and demand sides: it reduces the supply of the good that should be consumed; and it decreases disposable income in country H. In the case of adjacent complementarity, the decrease in disposable income would lower $c(0)$, but not so much as it would lower $\bar{c}$ if $r(0)$ is constant at θ. This means that there would be excess demand in the time-zero good market. To clear the market, $r(0)$ rises above θ, which, in turn, decreases aggregate consumption to the lowered aggregate supply level. Alternatively, when the world felicity function exhibits distant complementarity, an increase in g produces excess supply in the present good market, thereby lowering the interest rate. After the initial response, the interest rate then monotonically converges toward θ as seen from (28).

4.3 Welfare

Irrespective of how each country's consumption responds over time, an increase in government spending g necessarily reduces country H's welfare. To see this, we substitute (8) into (43) and (44), differentiate the results by g, and incorporate (45) and (46). The results are simply given by

$$\frac{dU_0}{dg} = -\frac{\lambda}{\theta}\,(<0) \text{ and } \frac{dU_0^*}{dg} = 0.$$

The increase in g harms country H's welfare by the amount equal to the present value of the marginal utility. The marginal change in g, however, does not affect country F's welfare in this non-distortionary competitive equilibrium.[9] However, these properties are limited to the case of $b_0 = 0$, as shown later in Section 5.

4.4 Empirical implications

By using a small-country model, Gruber (2004) shows that incorporating habit formation improves the explanatory power of the intertemporal current account model in advanced countries. However, we can find two puzzling differences between his result for two large countries, the United States and Japan, and that for smaller countries, such as Italy, Spain, and Netherlands. First, the GMM estimates of the habit preference parameter for the two countries are smaller than those for the smaller countries. Second, in the case of the two large countries, the habit model overpredicts the actual volatility

[9] The same result can be obtained from Bianconi and Turnovsky (1997, p.80) by assuming that g does not generate direct utility and that there is no initial external indebtedness.

of the current account, unlike in the smaller country case. We shall show that these "large-country puzzles" could be attributed to biases that will arise when, with the small country assumption, the interest-rate adjustment is ignored.

To consider the large-country puzzles in this deterministic framework, we examine impacts of an income shock on consumption and on the current account, instead of directly treating their volatilities. Consider an increase in g, a negative income shock, in two different economies: (i) the two-country world economy with the internationally same strength of habit formation, $\Omega^H = \Omega^F = \Omega > 0$ and (ii) the small open economy (SOE) with the same Ω.

In the SOE, the consumption impact of an increase in g can be obtained as[10]

$$\left.\frac{dc(0)}{dg}\right|_{\text{SOE}} = \frac{\alpha+\omega}{\alpha+\theta} - 1, \tag{51}$$

whereas in the two-country economy, from (47), we obtain

$$\left.\frac{dc(0)}{dg}\right|_{\Omega^H=\Omega^F=\Omega} = \frac{\epsilon(\alpha+\omega)}{\alpha+\theta} - 1, \tag{52}$$

implying that the consumption impact is larger in the two-country world economy than it would be in the SOE:

$$\left|\left.\frac{dc(0)}{dg}\right|_{\text{SOE}}\right| < \left|\left.\frac{dc(0)}{dg}\right|_{\Omega^H=\Omega^F=\Omega}\right|. \tag{53}$$

This comes from the fact that in the two-country case an interest-rate hike induced by the increase in g (see (50)) enlarges the decrease of $c(0)$. Empirically, inequality (53) implies that consumption is more volatile in large countries than it would be in small countries if the habit parameter value is the same.

From the viewpoint of estimating the habit parameter value, captured by ω, from a given observed volatility of consumption, in turn, inequality (53) implies that the ω estimate is larger when it is estimated by using the large country model than by using the SOE model: Explicitly, from the comparison of (51) and (52), the ω value required to explain a given value of the consumption change is larger in (52) than in (51) since $\epsilon \in (0,1)$. The

[10] See Ikeda and Gombi (1999).

habit parameter value of a large country would thus be underestimated if it is estimated using a small country model. This consistently explains the puzzle that the estimates of the habit parameters for the United States and Japan are smaller than the smaller countries.[11]

As for the current account effect of an increase in g, inequality (53) implies that the direct negative effect of the g-increase on the current account is offset by a larger decrease in consumption in the two-country setting than in the SOE setting. Formally, we can derive

$$\left.\frac{d\dot{b}(0)}{dg}\right|_{\text{SOE}} = -\frac{\alpha+\omega}{\alpha+\theta},$$

whereas in the two-country case, from (7) and (47), we have

$$\left.\frac{d\dot{b}(0)}{dg}\right|_{\Omega^H=\Omega^F=\Omega} = -\frac{\epsilon(\alpha+\omega)}{\alpha+\theta},$$

implying

$$\left|\left.\frac{d\dot{b}(0)}{dg}\right|_{\text{SOE}}\right| > \left|\left.\frac{d\dot{b}(0)}{dg}\right|_{\Omega^H=\Omega^F=\Omega}\right|.$$

As a result, the current account volatility of a large country would be overestimated under the small country assumption since the interest-rate effect is ignored in the setting. This consistently explains Gruber's overprediction of the current account volatility for Japan and the United States.[12]

These empirical implications are summarized as follows:

Proposition 3: *The habit parameter of a large country would be underestimated, and the current account volatility overestimated, if they were estimated using a small-country model.*

The point is that, in a large country, the interest rate adjustment exaggerates consumption impact of an income shock, so that neglecting this effect

[11]Gruber (2004) assumes linear habit formation in specifying felicity function, as we shall define as $u(c,z) = \frac{(c-\gamma z)^{1-\varphi}}{1-\varphi}$ below in subsection 5.2, and thereby estimates γ as the habit parameter. It is easy to confirm that ω is positively related with γ.

[12]Gruber (2004) gives the conjecture that the overpredictions of the current account volatility for United States and Japan could be attributed to the small-country assumption. Our discussion provides a formal proof for it.

under the small country assumption brings upward bias to the impacts on saving and on the current account for the large country and brings downward bias to the estimated habit parameter of the large country.

5 Welfare-enhancing fiscal spending: An implication of $b_0 \neq 0$

5.1 Reversed immiserizing growth effects

We have so far assumed that b_0 equals zero by Assumption 1. When $b_0 \neq 0$, however, an increase in government spending g has additional welfare effects by causing income re-transfers through interest rate changes, i.e., changes in the intertemporal terms of trade. Even though government spending is assumed to provide no direct utility, we shall show that an increase in g can enhance country H's welfare level due to favorable changes in the intertemporal terms of trade, i.e., rises (resp. falls) in the world interest rate if the country is a creditor (resp. debtor).

Let us now abandon Assumption 1 to allow b_0 to take a non-zero value. For expository purposes, we focus on case (ii), $\Omega^H = \Omega^F = \Omega$. As shown in Appendix C, an increase in g affects country H's welfare by

$$\left.\frac{\mathrm{d}U}{\mathrm{d}g}\right|_{\Omega^H=\Omega^F=\Omega} = -\frac{\lambda}{\theta}\left(1 - \frac{\theta}{\alpha+\theta}\eta\Omega b_0\right), \tag{54}$$

implying that the increase in g improves country H's welfare if $\frac{\theta}{\alpha+\theta}\eta\Omega b_0 > 1$. With a positive b_0, the policy is welfare enhancing if Ω is a sufficiently large positive number, i.e., if consumers' preferences are of sufficiently strong adjacent complementarity. Note that the term $\frac{\theta}{\alpha+\theta}\eta\Omega b_0$ represents an income re-distribution effect due to interest rate changes since, as derived in Appendix C, the effect on the interest rate is given by

$$\left.\frac{\mathrm{d}r(0)}{\mathrm{d}g}\right|_{\Omega^H=\Omega^F=\Omega} \gtreqless 0 \Leftrightarrow \Omega \gtreqless 0.$$

With adjacent complementarity ($\Omega > 0$), *ceteris paribus* an increase in g causes excess demand for the present good, raises the interest rate, and enlarges the net output and the welfare level if b_0 is positive and large. This is an intertemporal version of (reversed) immiserizing growth discussed by the trade theory: a creditor country, i.e., an exporter of the present good, can be better off due to intertemporal terms-of-trade improvements by decreasing the supply of the present good.

These changes in the intertemporal terms of trade have spill-over effects on country F. Appendix C derives

$$\left.\frac{\mathrm{d}U^*}{\mathrm{d}g}\right|_{\Omega^H=\Omega^F=\Omega} = -\frac{\eta\Omega\lambda^*}{\theta+\alpha}b_0,$$

which reveals that the increase in g makes country F worse (resp. better) off when $\Omega b_0 > 0$ (resp. $\Omega b_0 < 0$). With a positive $b_0 (= -b_0^*)$, for example, an increase in g induces a deterioration (resp. an improvement) in country F's intertemporal terms of trade, thereby harming (resp. benefiting) country F.

These results can be summarized as follows:

Proposition 4: *Suppose that country H is as much in the habit of consuming as country F: $\Omega^H = \Omega^F = \Omega$. Then, due to intertemporal terms-of-trade effects, an increase in fiscal spending g in country H*

(i) *enhances the country's own welfare if and only if* $\frac{\theta}{\alpha+\theta}\eta\Omega b_0 > 1$*; and*

(ii) *harms country F if and only if* $\Omega b_0 > 0$.

Remark 4: By using a two-country model, Bianconi (2003) shows the possibility that an increase in fiscal spending has a reversed immiserizing growth effect due to changes in the *intratemporal* terms of trade. He shows that an increase in a country's fiscal spending financed by capital taxation contracts the production frontier in the country, and thereby appreciates the real exchange rate, which can improve the asset position and the welfare level of the country. However, as pointed out in Bianconi (2003, p.26) this reversed immiserizing growth effect cannot take place when government spending is financed by a lump-sum tax, and/or when there is only one consumption good. Proposition 4 shows that with habit formation, fiscal spending can make the policy implementing country better off by improving the *intertemporal* terms of trade even when the policy is financed by lump-sum taxation in a single good economy.

5.2 Example

To clarify the possibilities of the reversed immiserizing growth effect caused by fiscal spending, assume that country H is the creditor: $b_0 > 0$, and that the two countries have the same felicity functions specified, as in the literature (e.g., Constantinides (1990), Gruber (2004)), as:

$$u(c, z) = \frac{(c - \gamma z)^{1-\varphi}}{1 - \varphi}, \tag{55}$$

where the habit parameter $\gamma \in (0, 1)$ captures the strength of habit influence; and φ represents a risk aversion parameter. It is then valid that $\Omega^H = \Omega^F = \Omega = \gamma \left(1 - \frac{\alpha\gamma}{2\alpha+\theta}\right) > 0$.[13] Therefore, from Proposition 4 (ii), an increase in g necessarily harms the debtor country F.

As for Proposition 4 (i), the condition for the reversed immiserizing growth effect, i.e., $\frac{\theta}{\alpha+\theta}\eta\Omega b_0 > 1$, reduces to

$$\frac{\theta}{\alpha + \theta} \frac{\alpha\varphi\gamma}{(1 - \gamma)\left(1 - \frac{\alpha\gamma}{\alpha+\theta}\right)} \left(1 - \frac{\alpha\gamma}{2\alpha + \theta}\right) \left(\frac{b_0}{y + \theta b_0 + z_0^*}\right) > 1.$$

Note that, as $\gamma \to 1$, the left hand side of this inequality diverges to infinity for any parameter values. From Proposition 4 (i), this implies that an increase in g definitely causes the reversed immiserizing growth effect under a sufficiently large habit parameter γ.[14]

		γ											
		0.70		0.75		0.80		0.85		0.90		0.95	
		z_0^*		z_0^*		z_0^*		z_0^*		z_0^*		z_0^*	
		0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5
φ	2.0	2.88	3.92	1.92	2.62	1.21	1.66	0.70	0.98	0.34	0.47	0.11	0.16
	3.0	0.11	2.51	1.24	1.70	0.79	1.09	0.46	0.64	0.23	0.32	0.08	0.11
	4.0	1.35	1.84	0.92	1.26	0.59	0.81	0.35	0.48	0.17	0.24	0.06	0.08
	5.0	1.07	1.46	0.73	1.00	0.47	0.65	0.28	0.38	0.14	0.19	0.05	0.07

Table 1: The net foreign assets-GDP ratio b_0 required for the reversed immiserizing growth effect

[13] Even if the risk aversion parameter φ differs between the both countries, the relation, $\Omega^H = \Omega^F = \Omega = \gamma \left(1 - \frac{\alpha\gamma}{2\alpha+\theta}\right) > 0$, is retained. The discussions in this section, therefore, can be extended without substantial changes to the heterogenous risk aversion cases.

[14] Gruber (2004) obtains the significant estimates of γ as 0.789 to 0.920 from the current account data of five advanced countries.

Corollary 2: *Suppose that two countries have the same felicity function (55); and that country H is the creditor. Then, an increase in country H's fiscal spending*

(i) *benefits country H under sufficiently strong habit formation; and*

(ii) *necessarily harms country F.*

To evaluate tentatively how likely the reversed immiserizing growth effect occurs, we figure out the critical values of the net foreign assets-GDP ratio b_0 under alternative values of the habit parameter γ, the risk aversion parameter φ, and the initial habit stock z_0^* of country F, with fixing the parameters y, α, and θ to $y = 1$, $\alpha = 0.9$, and $\theta = 0.05$, respectively. Table 1 summarizes the results, where each value represents the minimal b_0 value required for the effect under different (γ, φ, z_0^*) values. As seen from the table, the reversed immiserizing growth effect is not so implausible. For example, when $(\gamma, \varphi) = (0.90, 3.0)$, the reversed immiserizing growth effect takes place if the net foreign assets-GDP ratio b_0 is higher than 0.23 when $z_0^* = 0.1$ and 0.32 when $z_0^* = 0.5$, both of which are smaller than the Japanese actual data (0.35).[15] The critical condition can be even weaker if the risk aversion parameter φ is larger; and/or if the speed of habit formation α is larger.[16]

6 Conclusions

We have examined macroeconomic adjustments in a tractable two-country model of heterogeneous habit forming consumers, thereby figuring out implications for welfare and for the understanding of some empirical facts. Our conclusions can be summarized as follows:

1. To focus on interactive consumption-habit dynamics, we have proposed to construct the world felicity function and the aggregate habit capital, which successfully help to characterize intertemporal complementarities in the two-country equilibrium.

[15] Although Gruber (2004) estimates Japan's γ as 0.789, it is based on the small country assumption. Compared with Gruber's other estimates, the value 0.90 is not unrealistically high.

[16] When the felicity function is specified in the alternative form as $u(c, z) = (c/z^{\gamma})^{1-\varphi} / (1-\varphi); \gamma \in (0, 1)$, however, the conditions for the reversed immiserizing effect are much more stict than in the case of (55).

2. External indebtedness depends crucially on international difference in surplus income, i.e., the effective net output that can be allocated for saving under habit formation. This is shown to be consistent with the existing proposition (e.g., Devereux and Shi 1991) that a country tends to be a long-run creditor the more patient it is. Our contribution is that which country is more patient is determined endogenously from the relative strengths of adjacent complementarity and how much greater net output than the habitual consumption standard they have.

3. When the initial holding of net foreign assets, and hence the income re-transfer effect due to interest rate changes are negligible, an increase in a country's fiscal spending (i) can increase the initial or steady-state consumption level of that country, with the other country's consumption being crowded out; (ii), as in the small country case, necessarily worsens (resp. improves) the steady-state external asset position if the country's preference displays adjacent (resp. distant) complementarity, irrespective of the other country's preferences; and (iii) has long-run and short-run spill-over effects on the other country's consumption, depending crucially on whether the policy implementing country displays adjacent or distant complementarity.

4. With non-negligible initial net foreign assets, an increase in fiscal spending in the creditor country can make the country better off and the neighbor debtor worse off due to the intertemporal terms-of-trade effect. With a specific felicity function, this necessarily occurs under sufficiently strong habit formation.

5. Interest rate adjustments enlarge the consumption effect of an income shock and thereby mitigate its current account effect. The habit parameter of a large country, therefore, would be underestimated, and the current account volatility overestimated, if they were measured using a small-country model as in the literature.

For future research, two issues may be interesting. First, to evaluate explicitly the effect of habit formation on the current account volatility of a large country, the analysis should be extended to a stochastic model. Second, when a government knows that fiscal spending causes the reversed immiserizing growth effect at the loss of the neighbor country's welfare, some strategic policy game naturally arises. It should be examined how the game changes our basic results.

Appendices

A Proof of Proposition 1

Note first that λ/λ^* is constant because $\dot{\lambda}_t/\lambda_t = \dot{\lambda}^*_t/\lambda^*_t$ from (5). By eliminating c^* and z^* using (8) and (12) from the foreign counterpart of (4), combining the resulting equation and (4) yields:

$$\frac{u_c(c,z)+\alpha\xi}{u_c^*(Y-c,Z-z)+\alpha\xi^*}=\frac{\lambda}{\lambda^*}=\text{constant.}$$

By totally differentiating this equation, we obtain:

$$\hat{c}=-\frac{\lambda^* u_{cz}+\lambda u_{cz}^*}{\lambda^* u_{cc}+\lambda u_{cc}^*}\hat{z}-\frac{\alpha\lambda^*}{\lambda^* u_{cc}+\lambda u_{cc}^*}\hat{\xi}+\frac{\alpha\lambda}{\lambda^* u_{cc}+\lambda u_{cc}^*}\hat{\xi}^*+\frac{\lambda u_{cz}^*}{\lambda^* u_{cc}+\lambda u_{cc}^*}\hat{Z}.$$

We substitute this equation into (1), (6), and the foreign counterpart of (6) and eliminate c^* and z^* using (8) and (12) from the resulting equation. Then, from (11), the autonomous dynamic equation system with respect to $\left(\hat{z},\hat{\xi},\hat{\xi}^*,\hat{Z}\right)$ is obtained as follows:

$$\begin{aligned}
\dot{z} &= -\alpha\left(\frac{\lambda^* u_{cz}+\lambda u_{cz}^*}{\lambda^* u_{cc}+\lambda u_{cc}^*}+1\right)\hat{z}-\frac{\alpha^2\lambda^*}{\lambda^* u_{cc}+\lambda u_{cc}^*}\hat{\xi}+\frac{\alpha^2\lambda}{\lambda^* u_{cc}+\lambda u_{cc}^*}\hat{\xi}^* \\
&\quad +\frac{\alpha\lambda u_{cz}^*}{\lambda^* u_{cc}+\lambda u_{cc}^*}\hat{Z}, \\
\dot{\xi} &= \left\{\frac{u_{cz}\left(\lambda^* u_{cz}+\lambda u_{cz}^*\right)}{\lambda^* u_{cc}+\lambda u_{cc}^*}-u_{zz}\right\}\hat{z}+\left(\theta+\alpha+\frac{\alpha u_{cz}\lambda^*}{\lambda^* u_{cc}+\lambda u_{cc}^*}\right)\hat{\xi} \\
&\quad -\frac{\alpha u_{cz}\lambda}{\lambda^* u_{cc}+\lambda u_{cc}^*}\hat{\xi}^*-\frac{\lambda u_{cz}u_{cz}^*}{\lambda^* u_{cc}+\lambda u_{cc}^*}\hat{Z}, \qquad (56)\\
\dot{\xi}^* &= -\left\{\frac{u_{cz}^*\left(\lambda^* u_{cz}+\lambda u_{cz}^*\right)}{\lambda^* u_{cc}+\lambda u_{cc}^*}-u_{zz}^*\right\}\hat{z}-\frac{\alpha u_{cz}^*\lambda^*}{\lambda^* u_{cc}+\lambda u_{cc}^*}\hat{\xi} \\
&\quad +\left(\theta+\alpha+\frac{\alpha u_{cz}^*\lambda^*}{\lambda^* u_{cc}+\lambda u_{cc}^*}\right)\hat{\xi}^*+\left(\frac{\lambda u_{cz}^{*2}}{\lambda^* u_{cc}+\lambda u_{cc}^*}-u_{zz}^*\right)\hat{Z}, \\
\dot{Z} &= -\alpha\hat{Z}.
\end{aligned}$$

From the definitions (13) and (14) of υ and ς, respectively, this autonomous system reduces to (15). ■

B Equilibrium solutions

B.1 Dynamics of habit capital z: (18)

The stable roots of dynamics (15) are given by ω and $-\alpha$ as in (16). Letting m denote $(z, \varsigma, Z)'$, the general solution to (15) can thus be expressed as

$$\hat{m}(t) = A_1 \exp(\omega t) q + A_2 \exp(-\alpha t) h,$$

where $q \equiv (q_1, q_2, q_3)'$ and $h \equiv (h_1, h_2, h_3)'$ represent the eigen vectors associated with stable roots ω and $-\alpha$, respectively. From (15), it is easy to confirm that $q_3 = 0$. By eliminating $A_1 \exp(\omega t)$ and $A_2 \exp(-\alpha t)$ from the three equations in the above vector equation, we obtain

$$\hat{\varsigma} = \frac{q_2}{q_1}\hat{z} + \frac{q_1 h_2 - q_2 h_1}{q_1 h_3}\hat{Z}, \tag{57}$$

where the coefficients of $\hat{z}$ and $\hat{Z}$ can be obtained by exploiting the definition of the eigenvectors q and h as

$$\begin{aligned} \frac{q_2}{q_1} &= -\frac{(\omega+\alpha) v_{cc} + \alpha v_{cz}}{\alpha^2}, \\ \frac{q_1 h_2 - q_2 h_1}{q_1 h_3} &= -\frac{v_{cc}(v_{cz} v_{ZZ} + v_{cZ} v_{zz})}{\alpha v_{cc} v_{zz} + (2\alpha+\delta) v_{cc} v_{cz}} \\ &\quad + \frac{\{(\omega+\alpha) v_{cc} + \alpha v_{cz}\}\{\alpha v_{cc} v_{ZZ} - (2\alpha+\delta) v_{cc} v_{cZ}\}}{\alpha^2 (\alpha v_{cc} v_{zz} + (2\alpha+\delta) v_{cc} v_{cz})}. \end{aligned}$$

Substituting (57) into the $\dot{z}$-equation in (15) yields (18).

B.2 The interest rate

From (1) and (4) through (6), the optimal consumption dynamics are given by

$$\dot{c} = -\frac{\lambda}{u_{cc}}\left(\hat{r} - \hat{\phi}\right),$$

where ϕ represents the rate of time preference,

$$\hat{\phi} = \frac{\alpha(u_{zz} + u_{zc})}{\lambda}\hat{z} - \frac{\alpha(\alpha+\theta)}{\lambda}\hat{\xi}.$$

Substitute (21) into the above Euler equation. The resulting equation can be solved for $\hat{r}$ as

$$\hat{r} = -\frac{\omega u_{cc}}{\lambda}\hat{c} + \frac{\alpha\left(u_{zz} + u_{cz}\right)}{\lambda}\hat{z} - \frac{\alpha\left(\alpha + \theta\right)}{\lambda}\hat{\xi}. \tag{58}$$

In the above, $\hat{\xi}$ can be obtained from (14),(19), (56), and (57) as

$$\hat{\xi} = -\frac{u_{cc}\Omega^{H}}{\theta + \alpha - \omega}\hat{c} + \frac{u_{zz}}{\theta + 2\alpha}\hat{z}. \tag{59}$$

Substituting (59) and (20) successively into (58) yields (25).

B.3 Net foreign assets

Set

$$\hat{b} = \varkappa_1\hat{z} + \varkappa_2\hat{Z}. \tag{60}$$

Differentiating (60) with respect to time t yields

$$\dot{b} = \varkappa_1\dot{z} + \varkappa_2\dot{Z}. \tag{61}$$

Since $\dot{b}$ is given by (7), this equation implies

$$\left(\dot{b} =\right) \hat{r}b_0 + r\hat{b} - \hat{c} = \varkappa_1\dot{z} + \varkappa_2\dot{Z}. \tag{62}$$

Substitute (18), (19), (20), (25), and (60) into (62). By comparing the coefficients of the resulting equation, we obtain

$$\begin{aligned} \left(\omega - r\right)\varkappa_1 &= a_1 b_0 - \frac{\omega + \alpha}{\alpha}, \\ -\left(\omega + \alpha\right)\left(1 - \delta\right)\varkappa_1 - \left(r + \alpha\right)\varkappa_2 &= a_2 b_0 + \left(\frac{\omega + \alpha}{\alpha}\right)\left(1 - \delta\right). \end{aligned}$$

This simultaneous equation can be solved for $\varkappa_1$ and $\varkappa_2$ as

$$\varkappa_1 = \frac{1}{r - \omega}\left\{\frac{\omega + \alpha}{\alpha} - b_0 a_1\right\}, \tag{63}$$

$$\varkappa_2 = \frac{\left(1 - \delta\right)\left(\omega + \alpha\right)}{\alpha\left(r - \omega\right)} + \frac{b_0}{r + \alpha}\left\{\frac{\left(1 - \delta\right)\left(\omega + \alpha\right)}{\left(r - \omega\right)}a_1 + a_2\right\}. \tag{64}$$

Substituting (63) and (64) into (60) yields (29).

B.4 Welfare

To obtain the welfare level of country H, linearize instantaneous utilities $u(c_t, z_t)$ around a steady state and substitute the result into the lifetime utility function (2) to obtain

$$U_0 = \int_0^\infty \{u(\bar{c}, \bar{z}) + u_c \hat{c}_t + u_z \hat{z}_t\} \exp(-\theta t)\, dt.$$

Since, from (21) and (1), $\hat{c}_t$ and $\hat{z}_t$ are solved as

$$\hat{c}_t = \hat{c}_0 \exp(\omega t) \text{ and } \hat{z}_t = \left(\hat{z}_0 - \frac{\alpha \hat{c}_0}{\omega + \alpha}\right) \exp(-\alpha t) + \frac{\alpha \hat{c}_0}{\omega + \alpha} \exp(\omega t),$$

the lifetime utility is expressed as

$$\begin{aligned} U_0 &= \int_0^\infty \{u(\bar{c}, \bar{z}) + u_c \hat{c}_0 \exp((\omega - \theta)t) + \\ &\quad + u_z \left(\hat{z}_0 - \frac{\alpha \hat{c}_0}{\omega + \alpha}\right) \exp(-(\alpha + \theta)t) + \frac{\alpha u_z \hat{c}_0}{\omega + \alpha} \exp((\omega - \theta)t)\right\} dt \\ &= \frac{u(\bar{c}, \bar{z})}{\theta} + \frac{\hat{c}_0}{\omega - \theta}\left(u_c + \frac{\alpha u_z}{\omega + \alpha}\right) + \frac{u_z}{\alpha + \theta}\left(\hat{z}_0 - \frac{\alpha \hat{c}_0}{\omega + \alpha}\right). \end{aligned}$$

The initial optimal consumption $\hat{c}_0$ is obtained by setting $t = 0$ in (20). Substituting it into the above equation yields (43). Equation (44) can be obtained in the same way.

C Dynamics with non-zero b_0: The identical degrees of adjacent complementarity

C.1 Solutions

By assuming that the two countries' preferences exhibit the identical degrees of adjacent complementarity, $\Omega^H = \Omega^F = \Omega$, we shall derive equilibrium solutions in the case of a nonzero b_0. In this case, from (18), (19), (20), (25), and (29), the equilibrium dynamics are generated by the following equations

$$\dot{z} = \omega \hat{z} - (\omega + \alpha)(1 - \delta)\hat{Z}, \tag{65}$$

$$\dot{Z} = -\alpha\hat{Z}, \tag{66}$$

$$\hat{c} = \left(\frac{\omega+\alpha}{\alpha}\right)\left(\hat{z} - (1-\delta)\hat{Z}\right),$$

$$\dot{c} = \omega\hat{c},$$

$$\hat{r} = \eta\Omega\hat{Z},$$

$$\hat{b} = \frac{\omega+\alpha}{\alpha\left(\theta-\omega\right)}\hat{z} - \left\{\frac{\left(1-\delta\right)\left(\omega+\alpha\right)}{\alpha\left(\theta-\omega\right)} + \frac{b_0\eta\Omega}{\theta+\alpha}\right\}\hat{Z}. \tag{67}$$

$$\begin{aligned} U_0 &= \frac{u\left(\bar{z},\bar{z}\right)}{\theta} + \frac{\left(\omega+\alpha\right)u_c + \alpha u_z}{\alpha\left(\theta-\omega\right)}\left(z_0-\bar{z}\right) \\ &\quad - \frac{\left(\omega+\alpha\right)\left(1-\delta\right)}{\alpha(\theta+\alpha)\left(\theta-\omega\right)}\left[\left(\theta+\alpha\right)u_c + \alpha u_z\right]\left(Z_0-Y\right), \end{aligned} \tag{68}$$

$$\begin{aligned} U_0^* &= \frac{u^*\left(\bar{z}^*,\bar{z}^*\right)}{\theta} + \frac{\left(\omega+\alpha\right)u_c^* + \alpha u_z^*}{\alpha\left(\theta-\omega\right)}\left(z_0^*-\bar{z}^*\right) \\ &\quad - \frac{\left(\omega+\alpha\right)\delta}{\alpha(\theta+\alpha)\left(\theta-\omega\right)}\left[\left(\theta+\alpha\right)u_c^* + \alpha u_z^*\right]\left(Z_0-Y\right), \end{aligned} \tag{69}$$

respectively.

Define the *effective habit* stock e as

$$e \equiv z - (1-\delta)Z. \tag{70}$$

By using the effective habit stock, the equilibrium dynamics can be summarized as

$$\dot{b} = -\alpha\hat{b} + \frac{\left(\omega+\alpha\right)^2}{\alpha\left(\theta-\omega\right)}\hat{e}, \tag{71}$$

$$\dot{e} = \omega\hat{e}. \tag{72}$$

Figure 4 illustrates the phase diagram in the (e,b) plane.

Note that (67) can be rewritten in terms of $\hat{e}$ as

$$\hat{b} = \frac{\omega+\alpha}{\alpha\left(\theta-\omega\right)}\hat{e} - \frac{b_0\eta\Omega}{\theta+\alpha}\hat{Z}. \tag{73}$$

The steady state equilibrium is determined by the following two schedules, CC' and DD':

$$CC'\colon\ \bar{b}-b_0=\frac{\omega+\alpha}{\alpha\left(\theta-\omega\right)}\left(\bar{e}-e_0\right)-\frac{b_0\eta\Omega}{\theta+\alpha}\left(Y-Z_0\right), \tag{74}$$

$$DD'\colon\ \theta\bar{b}=\bar{e}+(1-\varepsilon)Y+g-y, \tag{75}$$

where schedule CC' is obtained by evaluating (73) at $t=0$; and DD' is obtained by substituting (70) for $\bar{c}\,(=\bar{z})$ in (34). The two schedules are illustrated in Figure 4, where schedule CC' is positively- (resp. negatively-) sloping when the world felicity function displays adjacent (resp. distant) complementarity, $\Omega>0$ (resp. $\Omega<0$). It can be easily shown that the slope of schedule DD' is larger than that of CC';[17] and that, when $\Omega>0$, schedule CC' is steeper than the $\dot{b}=0$ schedule which is obtained by setting $\dot{b}=0$ in (71).[18] The steady-state equilibrium is determined at the intersection of the two schedules.

C.2 The effect of fiscal spending

The effects of an increase in fiscal spending g on $\bar{b}$ and $\bar{e}$ are obtained from (74) and (75) as

$$\frac{\mathrm{d}\bar{b}}{\mathrm{d}g}=-\frac{\mathrm{d}\bar{b}^{*}}{\mathrm{d}g}=\frac{\delta\left(\omega+\alpha\right)}{\omega\left(\theta+\alpha\right)}-\frac{\alpha\eta\left(\theta-\omega\right)\Omega}{\omega\left(\theta+\alpha\right)^{2}}b_0, \tag{76}$$

$$\frac{\mathrm{d}\bar{e}}{\mathrm{d}g}=\frac{\alpha\delta\left(\theta-\omega\right)}{\omega\left(\theta+\alpha\right)}-\frac{\alpha\theta\eta\left(\theta-\omega\right)\Omega}{\omega\left(\theta+\alpha\right)^{2}}b_0. \tag{77}$$

As seen by comparing (40) and (76), the first terms on the right hand sides in these equations represent the same effects as are discussed in Section 4.2 whereas the second terms capture income effects that arise under non-zero b_0 due to interest rate changes. In fact, differentiating (50) by g yields

$$\left.\frac{\mathrm{d}r\left(0\right)}{\mathrm{d}g}\right|_{\Omega^{H}=\Omega^{F}=\Omega}=\eta\Omega, \tag{78}$$

[17]The slope of schedule DD', $1/\theta$, minus that of CC', $\left(\omega+\alpha\right)/\alpha\left(\theta-\omega\right)$, equals $-\omega\left(\theta+\alpha\right)/\alpha\theta\left(\theta-\omega\right)$, which is positive.

[18]The slope of schedule CC', $\left(\omega+\alpha\right)/\alpha\left(\theta-\omega\right)$, minus that of the $\dot{b}=0$ schedule, $\left(\omega+\alpha\right)^{2}/\alpha^{2}\left(\theta-\omega\right)$, equals $-\omega\left(\omega+\alpha\right)/\alpha^{2}\left(\theta-\omega\right)$, which is positive (resp. negative) when $\omega+\alpha>0$ (resp. <0).

implying that, with adjacent (resp. distant) complementarity, $\Omega >$ (resp. $<$) 0, *ceteris paribus* an increase in g causes excess demand (resp. supply) for the present good, and raises (resp. lowers) the interest rate. From (76), therefore, when country H is a creditor, $b_0 > 0$, the increase (resp. decrease) in the interest rate has an additional positive (resp. negative) effect on steady-state external asset holding of country H. Consequently, in contrast to the case of a zero b_0, even under adjacent complementarity, an increase in g can enlarge steady-state external asset holding of country H by increasing interest income if b_0 is positive and sufficiently large.

Figure 5 illustrates such a case by assuming that $\Omega > 0$ and $b_0 > 0$. An increase in fiscal spending shifts both the CC' and DD' schedules upward. Due to a dominant interest-income effect, these shifts bring the steady-state point from E_0 to E_1 with a higher $\bar{b}$ and a lower $\bar{e}$. In transition the effective habit stock e decreases monotonically whereas net foreign assets increase non-monotonically.

From (70) and (77), the effects on consumptions and habit capitals are given by

$$\begin{aligned}\frac{\mathrm{d}\bar{c}}{\mathrm{d}g} &= \frac{\mathrm{d}\bar{z}}{\mathrm{d}g} = \frac{\mathrm{d}\bar{e}}{\mathrm{d}g} + (1-\delta)\frac{\mathrm{d}Y}{\mathrm{d}g} \\ &= \frac{\delta\theta(\omega+\alpha)}{\omega(\theta+\alpha)} - 1 - \frac{\alpha\theta\eta(\theta-\omega)\Omega}{\omega(\theta+\alpha)^2}b_0, \end{aligned} \tag{79}$$

$$\begin{aligned}\frac{\mathrm{d}\bar{c}^*}{\mathrm{d}g} &= \frac{\mathrm{d}\bar{z}^*}{\mathrm{d}g} = 1 - \frac{\mathrm{d}\bar{c}}{\mathrm{d}g} \\ &= -\frac{\delta\theta(\omega+\alpha)}{\omega(\theta+\alpha)} + \frac{\alpha\theta\eta(\theta-\omega)\Omega}{\omega(\theta+\alpha)^2}b_0, \end{aligned} \tag{80}$$

in which the last terms on the right hand sides represent the income effect of interest rate changes. When $\Omega b_0 > 0$, an increase in g increases country H's interest income, thereby having a positive effect on $\bar{c}$ and a negative one on $\bar{c}^*$.

The welfare effects are obtained from (68), (69), (79) and (80) as

$$\begin{aligned}\left.\frac{\mathrm{d}U}{\mathrm{d}g}\right|_{\Omega^H=\Omega^F=\Omega} &= -\frac{\lambda}{\theta}\left(1 - \frac{\theta}{\alpha+\theta}\eta\Omega b_0\right), \\ \left.\frac{\mathrm{d}U^*}{\mathrm{d}g}\right|_{\Omega^H=\Omega^F=\Omega} &= -\frac{\eta\Omega\lambda^*}{\theta+\alpha}b_0,\end{aligned}$$

implying that

$$\left.\frac{\mathrm{d}U}{\mathrm{d}g}\right|_{\Omega^H=\Omega^F=\Omega} \gtreqless 0 \Leftrightarrow \frac{\theta}{\alpha+\theta}\eta\Omega b_0 \gtreqless 1. \tag{81}$$

$$\left.\frac{\mathrm{d}U^*}{\mathrm{d}g}\right|_{\Omega^H=\Omega^F=\Omega} \gtreqless 0 \Leftrightarrow \Omega b_0 \lesseqgtr 0. \tag{82}$$

Equations (78) and (81) reveal that an increase in g benefits country H when the income effect of interest rate changes is sufficiently large. From (78) and (82), the spill-over effect on country F's welfare is positive or negative as the interest rate revenue of country F is enlarged or reduced by the policy. For example, when country H is a creditor ($b_0 > 0$) and when preferences display adjacent complementarity ($\Omega > 0$), an increase in g raises the interest rate and causes income transfers from country F to H. This makes country F worse off. If b_0 and/or Ω are/is large enough, the increase in fiscal spending makes country H better off by raising her interest revenues.

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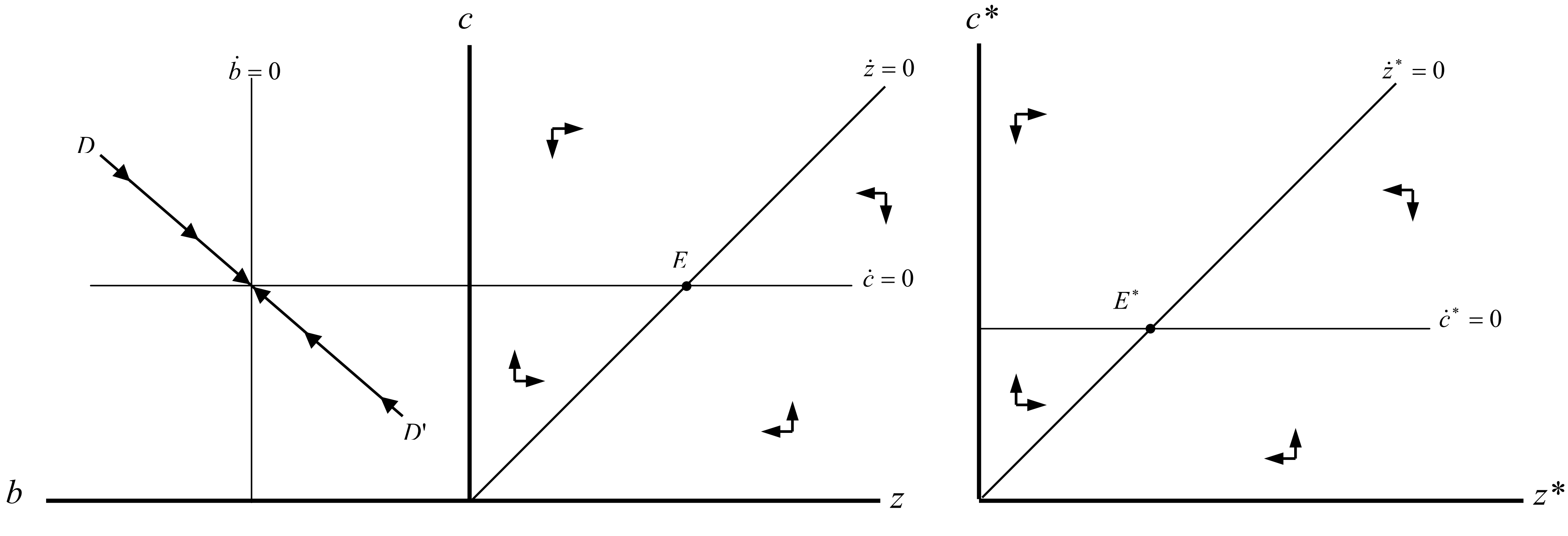

Figure 1. Equilibrium dynamics

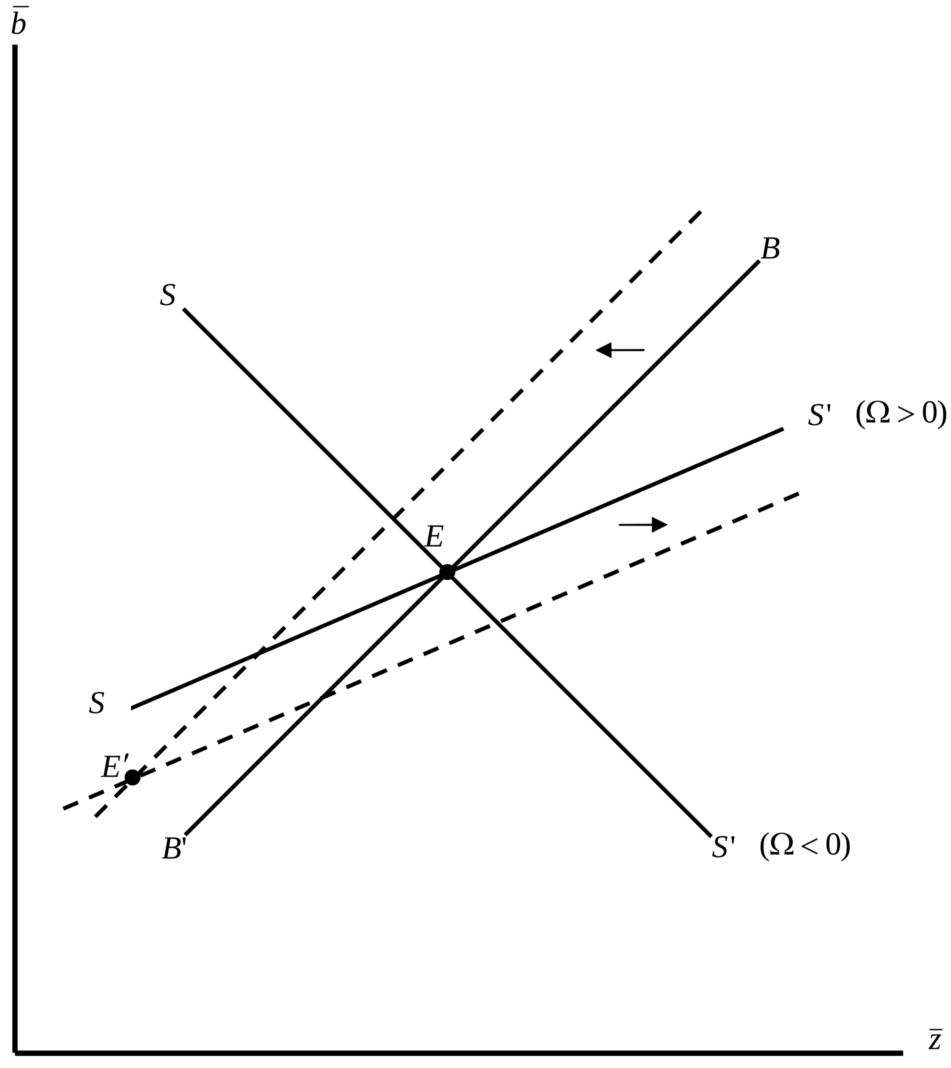

Figure 2. Steady-state equilibrium and the effects of an increase in fiscal spending

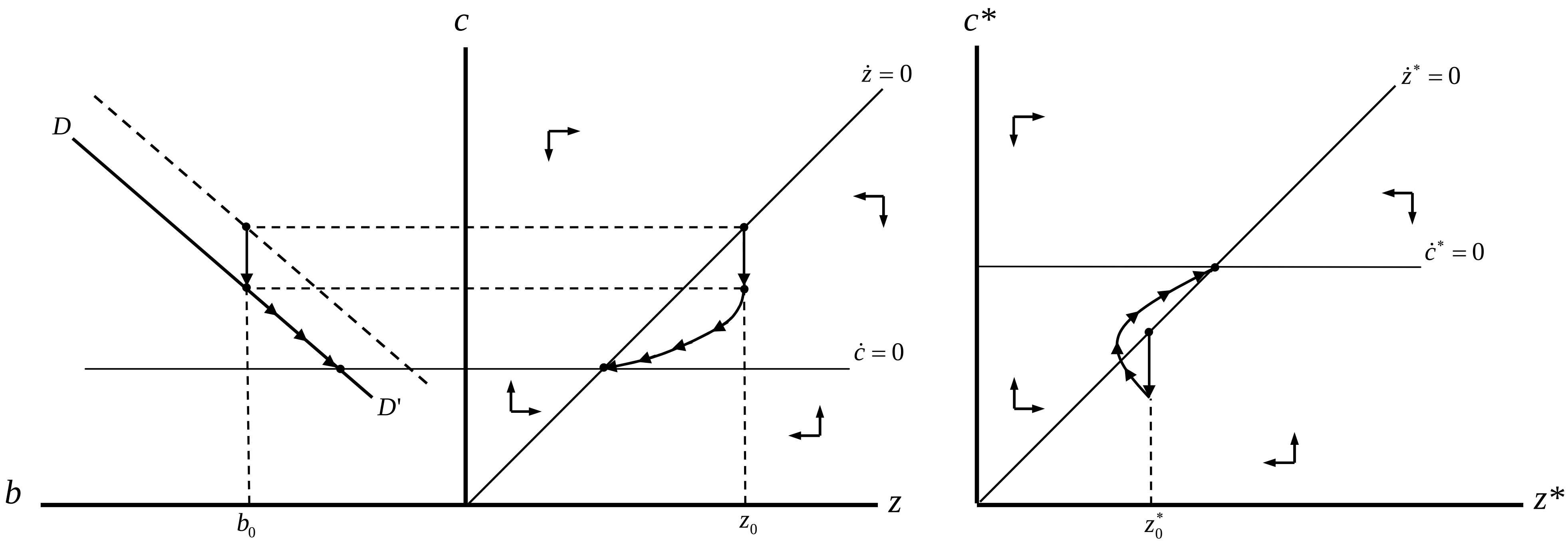

Figure 3. The effects of an increase in fiscal spending when $\Omega_H > 0$.

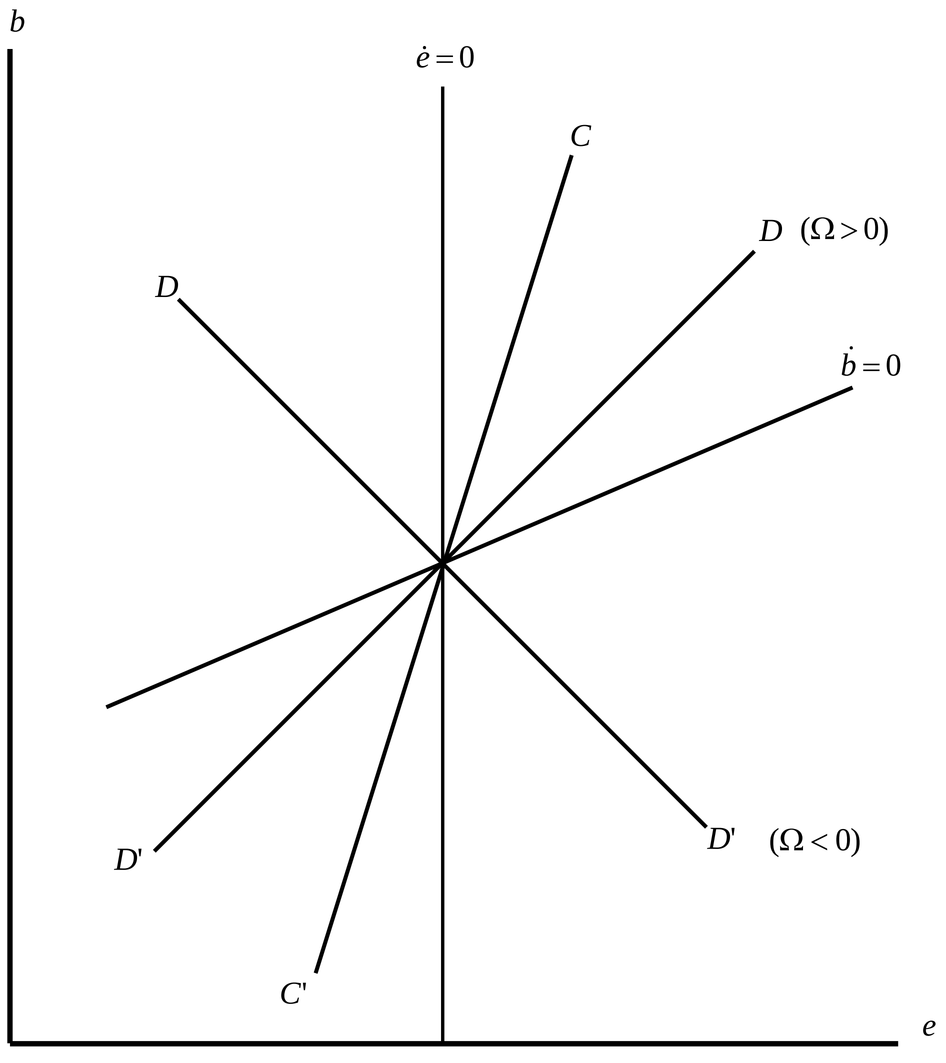

Figure 4. Equilibrium dynamics

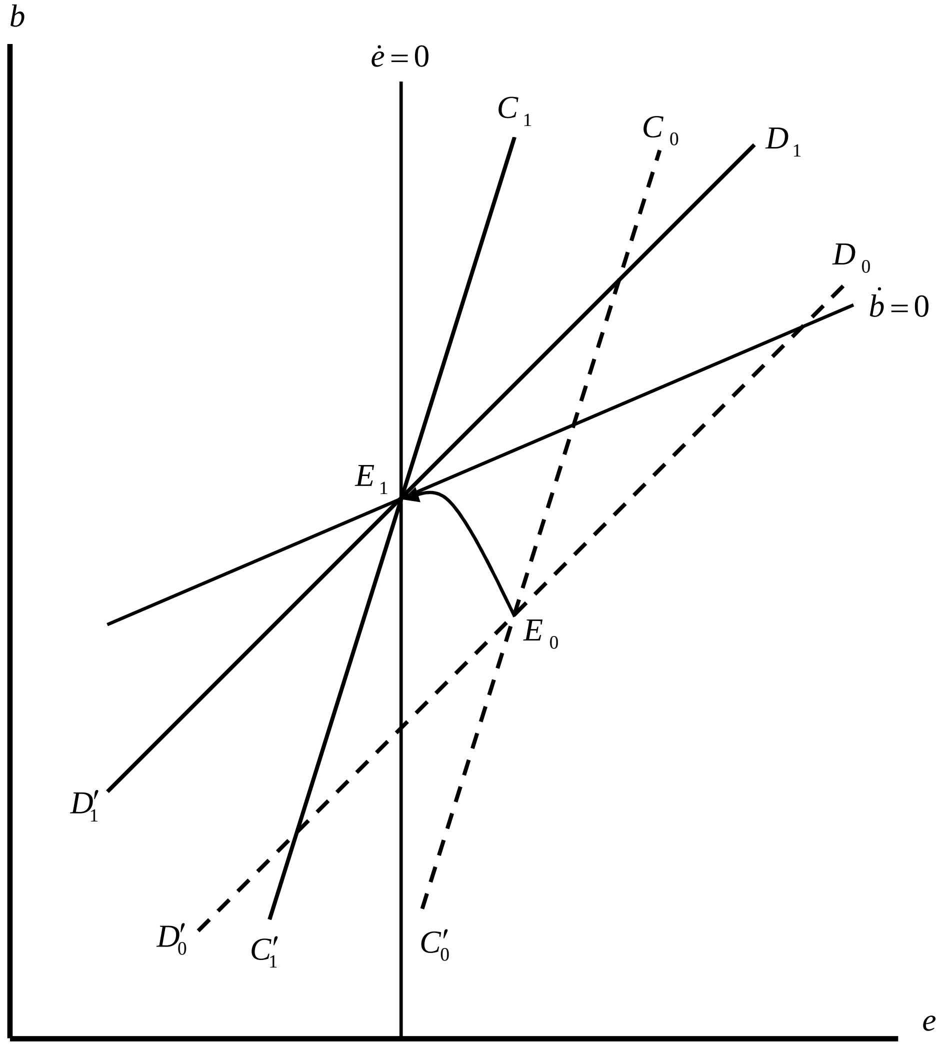

Figure 5. Effects of an increase in fiscal spending when $\Omega > 0$ and $b_0 > 0$.