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DETERMINACY OF EQUILIBRIUM UNDER VARIOUS PHILLIPS CURVES

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Determinacy of Equilibrium under Various Phillips Curves

by

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Abstract

Determinacy of equilibrium under the original, the backward-looking, the forward-looking and the hybrid Phillips curves is examined. If the monetary authority keeps the nominal money stock to be constant, the equilibrium path is always determinate under the original Phillips curve and the forward-looking one. Under the backward-looking one and the hybrid one, however, the path can be non-existent. The case of a Taylor rule is also examined. Under any of the four curves the path is always determinate if the monetary policy is active but is never determinate if it is passive.

Keywords: Phillips curve, indeterminacy, non-existence, Taylor rule.

JEL Classification Numbers: E52, E31

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1. Introduction

This paper examines the existence of a unique equilibrium path under various Phillips curves, viz. the original Phillips curve, the backward-looking one, the forward-looking one and the hybrid of the backward- and forward-looking ones. It does so under a money stock control rule and interest-rate feedback rules (Taylor rules) and finds that the path is non-existent or indeterminate under various situations.

Since Phillips (1958) found a negative relationship between the inflation rate and the unemployment rate, various attempts to elaborate the Phillips curve have been made. Friedman (1968) and Phelps (1967) introduce the expected inflation rate into the original Phillips curve and present a backward-looking Phillips curve, called the New Classical Phillips Curve.[1] More recently microeconomic foundations of nominal price adjustment have been analyzed and a forward-looking Phillips curve, called the New Keynesian Phillips Curve, is proposed instead of the backward-looking one. Roberts (1995) derives it from the models of staggered contracts developed by Taylor (1979, 1980), Rotemberg (1982) and Calvo (1983).[2]

However, empirical fitting of the New Keynesian Phillips Curve is found to be unsuccessful. Thus, a hybrid type of the backward- and forward-looking Phillips curves is proposed and shown to fit data better than the others. Fuhrer and Moore (1995), Gali and Gartler (1999), Christiano et al. (2005), Rudd and Whelan (2005a, 2005b), Sawyer (2007), Smith and Wickens (2007) are such examples.

Apart from those empirical analyses Benhabib et al. (2001a) theoretically examine the stability of macroeconomic dynamics under the forward-looking Phillips curve when the monetary authority follows a Taylor rule. They find that the equilibrium path is determinate if the authority moves the nominal interest rate so much that the real interest rate also moves in the same direction as the inflation rate does (viz. active feedback) but indeterminate if the feedback is not so much (viz. passive feedback). Benhabib et al. (2001b) analyze the stability in the case of multiple steady states and find that the equilibrium path is indeterminate even under the active feedback. Woodford (2003) extensively studies this issue under the

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[1] This type of Phillips curve is adopted by Kydland and Prescott (1977) and Barro and Gordon (1983) when analyzing the effects of monetary policies.

[2] Kimball (1995) and Yun (1996) incorporate Calvo's price-setting model into a dynamic general equilibrium framework.

forward-looking Phillips curve and various Taylor rules and shows that the path can be determinate or indeterminate, depending on the parameter values of the Taylor rules.[3]

Since they focus on the dynamic performance of Taylor rules rather than that of the Phillips curve, they treat only the cases of flexible prices and the forward-looking Phillips curve:[4] the cases of the other Phillips curves are not examined. In contrast, this paper focuses on the dynamic stability under various Phillips curves, viz. the original, the backward-looking, the forward-looking and the hybrid ones, and explores under which type of Phillips curve the equilibrium path is determinate. Furthermore, since a money stock control rule instead of a Taylor rule is mostly assumed in the monetary-growth literature, it begins with the case of a fixed amount of nominal money stock. It is found that there is a case where the path is non-existent under the backward-looking Phillips curve and the hybrid one whereas it is always determinate under the original Phillips curve and the forward-looking one.

Determinacy of equilibrium under a Taylor rule is also examined in each case. It is shown that, as long as the Taylor rule is passive, the equilibrium path is never determinate under any of the four Phillips curves. It is either non-existent or indeterminate. If it is active, the equilibrium path is always determinate under any of them. Thus, if a Taylor rule is adopted, the monetary policy must be active in order for the equilibrium path to be determinate.

2. The Model

The household sector receives income y, which equals actual total production, and holds real balances m. For simplicity, it is assumed that m is the only storable asset. The household sector determines c and m so as to maximize lifetime utility U:

$$U = \int_0^\infty u(c,m)e^{-\rho t}dt,$$

subject to the flow budget equation:

$$\dot{m} = y - c - \pi m + \tau, \tag{1}$$

where ρ is the subjective discount rate, π is the inflation rate of commodity price P, y is

[3] The possibility of equilibrium indeterminacy under a Taylor rule is also explored in the presence of capital accumulation. See Itaya and Mino (2004) and Meng and Yip (2004) for the case of flexible prices and Dupor (2001) and Carlstrom and Fuerst (2005) for the case of the forward-looking Phillips curve in a continuous-time and discrete-time setting respectively.

[4] Schmitt-Grohé and Uribe (2000) analyze the dynamic stability under a Taylor rule and flexible prices and find that there are the case with a continuum of equilibrium paths and that with a unique equilibrium path.

aggregate demand, and τ is the government's lump-sum transfer.

Naturally, $u(c, m)$ is assumed to satisfy the following properties:

Assumption 1. $u_c > 0,\ u_m > 0,\ u_{cc} < 0,\ u_{mm} < 0,\ \Phi = u_{mm}u_{cc} - u_{cm}^2 > 0.$

Assumption 2.

$$\partial(u_m/u_c)/\partial c = (u_{mc}u_c - u_{cc}u_m)/u_c^2 > 0,$$

$$\partial(u_m/u_c)/\partial m = (u_{mm}u_c - u_{cm}u_m)/u_c^2 < 0,$$

$$1 + u_{cm}m/u_c > 0.$$

Assumption 1 presents the standard conditions, including concavity, that a utility function must satisfy. The first two properties of assumption 2 imply normality of consumption c and of real balances m, respectively. The third property means that u_{cm} is not significantly negative. It is definitely valid either if c and m are Edgeworth substitutes (viz. $u_{cm} > 0$) or if $u(c, m)$ is additive separable (viz. $u_{cm} = 0$).

From the Hamiltonian function of this problem:

$$H = u(c, m) + \lambda(y - c - \pi m + \tau),$$

one derives the first-order optimal conditions that are summarized as follows:

$$R = \frac{u_m(c,m)}{u_c(c,m)} = \rho + \pi - \frac{u_{cc}(c,m)\dot{c} + u_{cm}(c,m)\dot{m}}{u_c(c,m)}, \tag{2}$$

where a dot represents a time derivative, each subscript implies the partial derivative with respect to it, and R is the nominal interest rate. The transversality condition is

$$\lim_{t\to\infty} \lambda_t m_t \exp(-\rho t) = 0 \quad \text{where} \quad \lambda_t = u_c(c_t, m_t). \tag{3}$$

The equilibrium of the money market is given by

$$M/P = m, \tag{4}$$

where M is the nominal money stock. If commodity price P is flexible, the present model is the same as that of Obstfeld and Rogoff (1983) and thus the unique equilibrium path obtains.[5] Alternatively, this paper assumes P to be sluggish and considers the following four types of commodity price adjustment:

$$\text{Original:} \quad \pi = f_1(y/y^n), \tag{5a}$$

$$\text{Backward-looking:} \quad \dot{\pi} = f_2(y/y^n), \tag{5b}$$

[5] The path is such that P initially jumps to the steady-state level and stays there.

$$\text{Forward-looking:} \quad \dot{\pi} = \rho\pi - f_3(y/y^n), \tag{5c}$$

$$\text{Hybrid:} \quad \ddot{\pi} = \rho\dot{\pi} - f_4(y/y^n), \tag{5d}$$

$$\text{where } f_i(1) = 0 \text{ and } f_i' > 0 \text{ for } i = 1, 2, 3, 4.$$

In the above equations y^n is the natural rate of output and ρ is the subjective discount rate.

(5a) is clearly the original Phillips curve. Note that y and π have a one-to-one correspondence and hence the degree of freedom is 1 when choosing the initial levels of the two variables. (5b) is the continuous version of the backward-looking adjustment (viz. the Neoclassical Phillips Curve) proposed by Phelps (1967) and Friedman (1968):

$$\pi_t = \pi_{t-1} + \gamma\ln(y_t/y^n).$$

It reduces to

$$\pi_t - \pi_{t-1} = \gamma\ln(y_t/y^n),$$

which is essentially the same as (5b). Since π_{t-1} is predetermined, π_t is determined once y_t is determined –i.e., the degree of freedom is 1 when choosing the initial levels of y and π.

(5c) is the continuous version of the forward-looking adjustment (viz. the New Keynesian Phillips Curve) presented by Roberts (1995):

$$\pi_t = \beta\mathrm{E}_t[\pi_{t+1}] + \kappa\ln(y_t/y^n),$$

where β is the subjective discount factor. This equation reduces to

$$\pi_{t+1} - \pi_t = [(1-\beta)/\beta]\pi_t - (\kappa/\beta)\ln(y_t/y^n),$$

which is essentially the same as (5c). Note that discount factor β equals 1/(1 + ρ) and hence the coefficient of π_t in the right-hand side equals ρ, as presented in (5c). Since there is π_{t+1} in this equation, y_t and π_t are both jump variables.

The hybrid model by Christiano et al. (2005) (see also Woodford, 2003, p.215) is

$$\pi_t - \pi_{t-1} = \beta\mathrm{E}_t[\pi_{t+1} - \pi_t] + \kappa\ln(y_t/y^n),$$

which reduces to

$$(\pi_{t+1} - \pi_t) - (\pi_t - \pi_{t-1}) = [(1-\beta)/\beta](\pi_t - \pi_{t-1}) - (\kappa/\beta)\ln(y_t/y^n).$$

(5d) is its continuous version. Since π_{t-1} is predetermined, π_t and y_t are jump variables. However, once they are chosen, $\pi_t - \pi_{t-1}$ and $\pi_{t+1} - \pi_t$ are determined. Thus, the degree of freedom is 2 when choosing the initial levels of y, π and $\dot{\pi}$ in this case.

Needless to say, each $f_i(y/y^n)$ (for i = 1, 2, 3, 4) is a generalized form of $\gamma\ln(y/y^n)$ or $(\kappa/\beta)\ln(y/y^n)$ discussed above.

3. Money Stock Control Rule

This section assumes a simple money stock control rule, viz. keeping money stock M to be constant. Then, from (4),

$$\dot{m} = -\pi m. \tag{6}$$

Fiscal spending is assumed to be zero and thus transfer τ is zero. In this case (1) and (6) imply

$$y = c. \tag{7}$$

Applying (6) and (7) to (2) gives the dynamic equation of y:

$$\dot{y} = \left(-\frac{u_c(y,m)}{u_{cc}(y,m)}\right)\left[\frac{u_m(y,m)}{u_c(y,m)} - \rho - \left(1 + \frac{u_{cm}(y,m)m}{u_c(y,m)}\right)\pi\right]. \tag{8}$$

Equations (6), (8) and one of the four equations in (5) formulate an autonomous dynamic system. Note that the four scenarios have the same steady state represented by

$$(\pi, y, m) = (0, y^n, m^*), \quad \text{where } \frac{u_m(y^n, m^*)}{u_c(y^n, m^*)} = \rho. \tag{9}$$

In this state transversality condition (3) is obviously valid.[6]

3.1. Original Phillips curve

Substituting π given by (5a) into (6) and (8) yields

$$\dot{y} = \left(-\frac{u_c(y,m)}{u_{cc}(y,m)}\right)\left[\frac{u_m(y,m)}{u_c(y,m)} - \rho - \left(1 + \frac{u_{cm}(y,m)m}{u_c(y,m)}\right)f_1(y/y^n)\right],$$

$$\dot{m} = -f_1(y/y^n)m.$$

Therefore, around the steady state given by (9) the characteristic equation of the (y, m) dynamics is

$$\begin{vmatrix} \dot{y}_y - \lambda & \dot{y}_m \\ -\dfrac{f_1' m^*}{y^n} & -\lambda \end{vmatrix} = 0,$$

where under assumption 2 $\dot{y}_m$ satisfies

[6] There may be another steady state in which m expands to infinity, $u_m = 0$, and y takes the value that makes $\dot{y}$ given by (8) equal zero. In the case where $u_{cm} = 0$, for example, such a state indeed obtains when $\rho + \pi = 0$ and y takes the value that makes π equal $-\rho$ under each Phillips curve (5a) – (5d). However, from (2), λ in (3) and (6), $\dot{\lambda}/\lambda + \dot{m}/m - \rho = u_m/u_c = 0$ in this state, implying that transverslity condition (3) is invalid. Therefore, the path that approaches this state cannot be an equilibrium path.

$$\dot{y}_m = \left(-\frac{u_c}{u_{cc}}\right)\frac{\partial(u_m / u_c)}{\partial m} < 0. \tag{10}$$

The characteristic function reduces to

$$\lambda^2 + A\lambda + B = 0,$$

where using (10) one finds

$$B = \lambda_1\lambda_2 = f_1' \, \dot{y}_m \, m^*/y^n < 0,$$

and λ_1 and λ_2 are the two characteristic roots. Therefore, the two roots are both real numbers, among which one is positive and the other is negative. Since y is jumpable while m is not, the equilibrium path is uniquely determined.

3.2. Backward-looking adjustment

Around the steady state given by (9) the characteristic function of the (π, y, m) dynamics formulated by (5b), (6) and (8) is

$$\begin{vmatrix} -\lambda & f_2'/y^n & 0 \\ \dot{y}_\pi & \dot{y}_y - \lambda & \dot{y}_m \\ -m^* & 0 & -\lambda \end{vmatrix} = 0, \tag{11}$$

where $\dot{y}_m$ is the same as given by (10). Under assumptions 1 and 2 $\dot{y}_\pi$ and $\dot{y}_y$ satisfy

$$\dot{y}_\pi = \left(\frac{u_c}{u_{cc}}\right)\left(1 + \frac{u_{cm} m}{u_c}\right) < 0,$$

$$\dot{y}_y = \left(-\frac{u_c}{u_{cc}}\right)\frac{\partial(u_m / u_c)}{\partial c} > 0. \tag{12}$$

The characteristic function (11) reduces to

$$\lambda^3 + A\lambda^2 + B\lambda + D = 0,$$

where using (10) and (12) one finds

$$A = -(\lambda_1 + \lambda_2 + \lambda_3) = -\dot{y}_y < 0,$$

$$B = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = -f_2' \, \dot{y}_\pi /y^n > 0,$$

$$D = -\lambda_1\lambda_2\lambda_3 = f_2' \, \dot{y}_m \, m^*/y^n < 0. \tag{13}$$

First, suppose that all the roots are real numbers. Since neither π nor m is jumpable whereas y is jumpable, in order that there is a unique equilibrium path one of the roots is positive and the other two are negative. Calling the positive root λ_1 and the two negative roots λ_2 and λ_3, the first property of (13) gives

$$0 < -(\lambda_2 + \lambda_3) < \lambda_1,$$

which implies

$$B = \lambda_1(\lambda_2 + \lambda_3) + \lambda_2\lambda_3 < \lambda_2\lambda_3 - (\lambda_2 + \lambda_3)^2 < 0.$$

This property contradicts the second property of (13), and hence the case of one positive and two negative roots never occurs. From this result and the third property of (13), all roots are positive, implying that there is no equilibrium path.

If two of the three roots are complex numbers, which must be conjugate, the third property of (13) implies the real root to be positive. From the first and second properties the real part of the complex roots can be either positive or negative. If it is positive, all paths are unstable and hence there is no equilibrium path. If it is negative, the equilibrium path is determinate. At the end of this section it is shown that the two cases in fact arise.

3.3. Forward-looking adjustment

Around the steady state given by (9) the characteristic function of the (π, y, m) dynamics formulated by (5c), (6) and (8) is

$$\begin{vmatrix} \rho - \lambda & -f_3'/y^n & 0 \\ \dot{y}_\pi & \dot{y}_y - \lambda & \dot{y}_m \\ -m^* & 0 & -\lambda \end{vmatrix} = 0.$$

This equation reduces to

$$\lambda^3 + A\lambda^2 + B\lambda + D = 0,$$

where from (10) and (12)

$$A = -(\lambda_1 + \lambda_2 + \lambda_3) = -(\rho + \dot{y}_y) < 0,$$

$$D = -\lambda_1\lambda_2\lambda_3 = -f_3'\, \dot{y}_m\, m^*/y^n > 0. \tag{14}$$

If all the roots are real numbers, from the second property of (14) λ_i's (i = 1, 2, 3) satisfy

$$\text{either} \quad \lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 < 0;$$

$$\text{or} \quad \lambda_1 < 0, \quad \lambda_2 < 0, \quad \lambda_3 < 0.$$

Since the second case contradicts the first property of (14), only the first case is valid, implying that there are two positive roots and a negative one. Since π and y are jump variables whereas m is not, there is a unique equilibrium path in this case.

If two of the three roots are complex numbers, which must be conjugate, from the second property of (14) the real root is negative and from the first property of (14) the common real part of the complex roots is positive. Therefore, the equilibrium path is determinate whether or not all roots are real numbers.

3.4. Hybrid adjustment

Since (5d) is a second-order differential equation, it is decomposed to the following two equations:

$$\dot{\pi} = z,$$

$$\dot{z} = \rho z - f_4(y/y^n). \tag{15}$$

The (π, z, y, m) dynamics is given by (6), (8) and the two equations in (15). Around the steady state given by (9) the characteristic function is

$$\begin{vmatrix} -\lambda & 1 & 0 & 0 \\ 0 & \rho-\lambda & -\dfrac{f_4'}{y^n} & 0 \\ \dot{y}_\pi & 0 & \dot{y}_y - \lambda & \dot{y}_m \\ -m^* & 0 & 0 & -\lambda \end{vmatrix} = 0,$$

where $\dot{y}_\pi$, $\dot{y}_y$ and $\dot{y}_m$ are given by (10) and (12). It reduces to

$$\lambda^4 + A\lambda^3 + B\lambda^2 + D\lambda + E = 0,$$

where using (10) and (12) one finds

$$A = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) = -(\rho + \dot{y}_y) < 0,$$

$$B = (\lambda_1 + \lambda_3)(\lambda_2 + \lambda_4) + \lambda_1\lambda_3 + \lambda_2\lambda_4 = \rho\,\dot{y}_y > 0,$$

$$D = -(\lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\lambda_4 + \lambda_3\lambda_4\lambda_1 + \lambda_4\lambda_1\lambda_2) = f_4'\,\dot{y}_\pi/y^n < 0,$$

$$E = \lambda_1\lambda_2\lambda_3\lambda_4 = -f_4'\,\dot{y}_m m^*/y^n > 0. \tag{16}$$

First, suppose that all roots are real numbers. Since m is not jumpable and only two of y, π and z are jumpable, as discussed at the end of section 2, two of the four roots (called λ_1 and λ_2) are positive and the other two are negative (called λ_3 and λ_4) in order that the equilibrium path is determinate. In this case $\lambda_1\lambda_3 + \lambda_2\lambda_4 < 0$ and hence from the first and the second properties of (16) one finds

$$\lambda_1 + \lambda_3 > 0,\ \ \lambda_2 + \lambda_4 > 0.$$

Since both $\lambda_1\lambda_3$ and $\lambda_2\lambda_4$ are negative, the above property yields

$$\lambda_1\lambda_3(\lambda_2 + \lambda_4) + \lambda_2\lambda_4(\lambda_1 + \lambda_3) < 0,$$

which contradicts the third property of (16). Therefore, the case of two positive and two negative roots never occurs. Moreover, from the first and last properties of (16) it follows that all roots are positive, implying that there is no equilibrium path.

Next, when two roots (λ_1 and λ_2 without loss of generality) are real numbers and the other two are mutually conjugate complex numbers ($\delta \pm \varepsilon i$), (16) reduces to

$$- A = \lambda_1 + \lambda_2 + 2\delta > 0,$$

$$B = \lambda_1\lambda_2 + 2\delta(\lambda_1 + \lambda_2) + \delta^2 + \varepsilon^2 > 0,$$

$$- D = 2\delta\lambda_1\lambda_2 + (\lambda_1 + \lambda_2)(\delta^2 + \varepsilon^2) > 0,$$

$$E = \lambda_1\lambda_2(\delta^2 + \varepsilon^2) > 0. \tag{17}$$

From the fourth property of (17), λ_1 and λ_2 are either both negative or both positive. If the former is right, the first property implies

$$\delta > - (\lambda_1 + \lambda_2)/2 > 0.$$

From this property one obtains

$$2\delta\lambda_1\lambda_2 + (\lambda_1 + \lambda_2)(\delta^2 + \varepsilon^2) < \delta[2\lambda_1\lambda_2 + (\lambda_1 + \lambda_2)\delta]$$

$$< \delta[2\lambda_1\lambda_2 - (\lambda_1 + \lambda_2)^2/2] = - \delta(\lambda_1 - \lambda_2)^2/2 < 0,$$

which contradicts the third property of (17) and hence both λ_1 and λ_2 are positive. The value of δ that is consistent with all the four properties can be either positive or negative. The equilibrium path is non-existent if δ is positive, and determinate if it is negative. It is later shown that the two cases indeed occur.

Finally, suppose that all roots are conjugate complex numbers:

$$\delta \pm \varepsilon i \quad \text{and} \quad \theta \pm \nu i.$$

Then, the first and second properties of (16) reduce to

$$(\rho + \dot{y}_y)/2 = \delta + \theta,$$

$$\rho\, \dot{y}_y = (\theta + \delta)^2 + \nu^2 + 2\delta\theta + \varepsilon^2.$$

Since they yield

$$(\rho - \dot{y}_y)^2/4 + \nu^2 + 2\delta\theta + \varepsilon^2 = 0,$$

δ and θ must have mutually different signs –i.e., the equilibrium path is determinate.

The results of this section are summarized as follows:

Proposition 1. Suppose that the monetary authority keeps the nominal money stock to be constant. Under the backward-looking Phillips curve and the hybrid one the equilibrium path is either non-existent or determinate. Under the forward-looking Phillips curve and the original Phillips curve the equilibrium path is always determinate.

As mentioned in Proposition 1, the equilibrium path may be non-existent under the backward-looking Phillips curve and the hybrid one. Using an additive separable utility function in c and m (viz. $u_{cm} = 0$) one can explicitly obtain the condition with respect to η_m, η_c and f_i' (i = 2 or 4) for the path to be non-existent (or determinate) under the backward-looking Phillips curve and the hybrid one, where $\eta_m = -\, u_{mm}m/u_m$, the elasticity of the marginal utility of money, and $\eta_c = -\, u_{cc}c/u_c$, the elasticity of the marginal utility of consumption. It is formally stated in proposition 2, of which the proof is set out in the appendix.

Proposition 2. Suppose that the monetary authority keeps the nominal money stock to be constant and that the utility function is additive separable. Under the backward-looking Phillips curve the equilibrium path is non-existent if and only if $\eta_m < 1$. Under the hybrid Phillips curve the path is non-existent if and only if $\eta_m < 1/2 - (f_4'/\eta_c)/(4\rho^3)$ (see figure 1).[7]

Since nominal interest rate R satisfies (2), if the utility function is additive separable, one finds

$$(m/R)\partial R/\partial m = \eta_m.$$

If R has a positive lower bound R_0, as is the case under a liquidity trap, η_m approaches zero as m increases.[8] Thus, η_m is very small if m is large enough in the steady state. Proposition 2 implies that if the equilibrium path is determinate in this case, the Phillips curve is neither the backward-looking type nor the hybrid type.

4. Taylor Rule

This section examines the dynamic stability under a Taylor rule. Nominal money stock M is controlled so that nominal interest rate R given by (2) should be a function of π:

[7] Note that f_4' has the same order as ρ^3 and $\ddot{\pi}$, as is clear from (5d).

[8] By applying both parametric and nonparametric methods to Japanese data in the 1990s, Ono, Ogawa and Yoshida (2004) find that u_m/u_c (= R) has indeed a positive lower bound.

$$\frac{u_m(y,m)}{u_c(y,m)} = R(\pi), \quad R'(\pi) > 0. \tag{18}$$

It is also assumed that the monetary authority sets R equal to the subjective discount rate when $\pi = 0$:

$$R(0) = r(0) = \rho,$$

so that nominal price P will be stabilized when the natural output is realized and the steady state is reached. In this case the steady state is the same as that under the money stock control rule, which is given by (9). Obviously, transversality condition (3) is valid there.

The increment of M is transferred to the household sector in a lump-sum manner and thus

$$\mu m = \tau \quad \text{where} \quad \mu = \frac{\dot{M}}{M}.$$

Note that μ is continuously controlled so that (18) is valid. Given that under (18) real interest rate r is a function of π:

$$r(\pi) = R(\pi) - \pi, \tag{19}$$

the monetary policy is called

$$\text{active when } r'(\pi) > 0,$$

$$\text{passive when } r'(\pi) < 0.$$

From (2), the time derivative of (18) and (19), one derives

$$\Phi\,\dot{y} = -\,u_{cm}u_cR'\,\dot{\pi} - (u_{mm}u_c - u_{cm}u_m)[r(\pi) - \rho], \tag{20}$$

where Φ (> 0) is given in assumption 1.

4.1. Original Phillips curve

Substituting into (20) the time derivative of (5a):

$$\dot{\pi} = \dot{y}\; f_1'(y/y^n)/y^n,$$

and rearranging the result yields

$$-\left(\frac{\Phi + u_{cm}u_cR'f_1'/y^n}{[\partial(u_m/u_c)/\partial m]u_c^2}\right)\dot{y} = r(f_1(y/y^n)) - \rho, \tag{21}$$

where the coefficient of $\dot{y}$ is positive under assumptions 1 and 2 unless u_{cm} is significantly negative. Thus,

$$\dot{y}_y \;\begin{matrix} > \\ = \\ < \end{matrix}\; 0 \;\Leftrightarrow\; r' \;\begin{matrix} > \\ = \\ < \end{matrix}\; 0.$$

Since y is a jump variable, this property shows that under an active monetary policy (viz. $r' > 0$) the dynamics of (21) is unstable and that the equilibrium path is uniquely determined: the steady state is chosen from the beginning. Under a passive policy ($r' < 0$), however, the dynamics is stable and hence there is a continuum of equilibrium paths –i.e., the path is indeterminate.

4.2. Backward-looking adjustment

By substituting (5b) into (20) one obtains

$$\Phi \dot{y} = -u_{cm}u_cR'f_2(y/y^n) - (u_{mm}u_c - u_{cm}u_m)[r(\pi) - \rho]. \tag{22}$$

Around the steady state given by (9) the characteristic function of the (π, y) dynamics formulated by (5b) and (22) is

$$\begin{vmatrix} -\lambda & \dfrac{f_2'}{y^n} \\ -\dfrac{(u_{mm}u_c - u_{cm}u_m)r'}{\Phi} & \dfrac{-u_{cm}u_cR'f_2'}{\Phi y^n} - \lambda \end{vmatrix} = 0.$$

It is rewritten as

$$\lambda^2 + A\lambda + B = 0,$$

where under assumptions 1 and 2

$$A = -(\lambda_1 + \lambda_2) = -u_{cm}u_cR'f_2'/(\Phi y^n) \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0 \quad \Leftrightarrow \quad u_{cm} \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0,$$

$$B = \lambda_1\lambda_2 = r'f_2'(u_{mm}u_c - u_{cm}u_m)/(\Phi y^n) \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0 \quad \Leftrightarrow \quad r' \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0.$$

From these properties, the two roots satisfy

$$r' > 0: \ \lambda_1 > 0 \ \text{ and } \ \lambda_2 < 0;$$

$$r' < 0: \ \lambda_1 \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0 \text{ and } \lambda_2 \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0 \quad \Leftrightarrow \quad u_{cm} \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0 \qquad \text{if } A^2 - 4B > 0,$$

$$\text{the real part of } \lambda_1 \text{ and } \lambda_2 \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0 \quad \Leftrightarrow \quad u_{cm} \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0 \quad \text{if } A^2 - 4B < 0.$$

Since y is jumpable while π is not, under an active monetary policy ($r' > 0$) there is a unique equilibrium path. Under a passive policy ($r' < 0$) the equilibrium path is non-existent if m and c are Edgeworth substitutes ($u_{cm} > 0$) and indeterminate if $u_{cm} \leq 0$.

4.3. Forward-looking adjustment

Substituting (5c) into (20) yields

$$\Phi \dot{y} = -u_{cm}u_c R'[\rho\pi - f_3(y/y^n)] - (u_{mm}u_c - u_{cm}u_m)[r(\pi) - \rho]. \tag{23}$$

Since $r'(\pi) = R'(\pi) - 1$ from (19), around the steady state given by (9) the characteristic function of the (π, y) dynamics formulated by (5c) and (23) is

$$\begin{vmatrix} \rho - \lambda & -\dfrac{f_3'}{y^n} \\ -\dfrac{u_{cm}u_m + u_{mm}u_c r'}{\Phi} & \dfrac{u_{cm}u_c R' f_3'}{\Phi y^n} - \lambda \end{vmatrix} = 0.$$

It reduces to

$$\lambda^2 + A\lambda + B = 0,$$

where under assumptions 1 and 2

$$A = -(\lambda_1 + \lambda_2) = -[\rho + u_{cm}u_c R' f_3'/(\Phi y^n)],$$

$$B = \lambda_1\lambda_2 = (u_{cm}u_m - u_{mm}u_c)r' f_3'/(\Phi y^n) \gtreqless 0 \quad \Leftrightarrow \quad r' \gtreqless 0.$$

Thus, under an active monetary policy ($r' > 0$) the characteristic equation has either two positive roots or two conjugate complex numbers with a positive real part as long as u_{cm} is not so significantly negative as to make A positive. Since both y and π are jumpable and the dynamics is unstable, it has a unique equilibrium path on which the steady state is reached from the beginning. Note that this is valid whether the two roots are real numbers or complex numbers.

Under a passive monetary policy ($r' < 0$) it has two real roots among which one is positive and the other is negative. The equilibrium path is indeterminate since both y and π are jump variables.[9]

4.4. Hybrid adjustment

Substituting (5d) into (20) yields

$$\Phi \dot{y} = -u_{cm}u_c R' z - (u_{mm}u_c - u_{cm}u_m)[r(\pi) - \rho], \tag{24}$$

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[9] The result of this subsection is consistent with that of Benhabib et al. (2001a) in the case where real balances are held for nonproductive purposes.

where z represents $\dot{\pi}$, as given by (15). The two equations in (15) and (24) formulate an autonomous dynamic system with respect to π, z and y. Around the steady state given by (9) the characteristic function is

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & \rho-\lambda & -f_4'/y^n \\ -(u_{mm}u_c-u_{cm}u_m)r'/\Phi & -u_{cm}u_cR'/\Phi & -\lambda \end{vmatrix} = 0.$$

It is rewritten as

$$\lambda^3 + A\lambda^2 + B\lambda + D = 0,$$

where under assumptions 1 and 2

$$A = -(\lambda_1 + \lambda_2 + \lambda_3) = -\rho < 0,$$

$$B = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = -\frac{u_{cm}u_cR'f_4'}{\Phi y^n} \overset{<}{\underset{>}{=}} 0 \iff u_{cm} \overset{>}{\underset{<}{=}} 0,$$

$$D = -\lambda_1\lambda_2\lambda_3 = -\frac{(u_{mm}u_c-u_{cm}u_m)r'f_4'}{\Phi y^n} \overset{>}{\underset{<}{=}} 0 \iff r' \overset{>}{\underset{<}{=}} 0. \tag{25}$$

In this dynamics m is not jumpable and only two of y, π and z are jumpable under (5d), as mentioned at the end of section 2.

Under an active policy ($r' > 0$) D is positive. Therefore, from the first and third properties of (25), one of the three roots is negative and the other two are positive if all the three roots are real numbers. If one of the three roots is a real number and the other two are complex numbers, from the third property of (25) the real root is negative and hence from the first property of (25) the real part of the complex roots must be positive. Therefore, the equilibrium path is anyway determinate.

Under a passive monetary policy ($r' < 0$) D in (25) is negative. Therefore, when all roots are real numbers, the case where two of them are positive and the rest is negative is invalid. The case where all roots are positive and the case where one is positive and the other two are negative are both possible if $u_{cm} < 0$, and hence the equilibrium path is either non-existent or indeterminate. If $u_{cm} \geq 0$, however, B in (25) is non-positive –i.e., only the latter case is valid and then the path is indeterminate.

If two of the three roots are mutually conjugate complex numbers, $\delta \pm \varepsilon i$, the third property of (25) implies the real root λ to be positive. Since the first and second properties reduce to

$$\lambda + 2\delta > 0,$$

$$2\lambda\delta + \delta^2 + \varepsilon^2 \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0 \quad \Leftrightarrow \quad u_{cm} \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0,$$

δ is either positive or negative if $u_{cm} < 0$ and hence the equilibrium path is either non-existent or indeterminate. If $u_{cm} \geq 0$, however, δ is negative and hence the path is indeterminate.

Thus, under a passive monetary policy the equilibrium path is either non-existent or indeterminate if $u_{cm} < 0$, whether the characteristic roots are all real numbers or not. However, if m and c are Edgeworth substitutes ($u_{cm} > 0$) or if the utility function is additive separable in c and m ($u_{cm} = 0$), the path exists but is indeterminate.

The results of this section are summarized as follows:

Proposition 3. If the monetary authority follows an active interest-rate feedback rule, the equilibrium path is always determinate under any of the four Phillips curves. If a passive interest-rate feedback rule is adopted, under any of the four Phillips curves the equilibrium path is never determinate. It is always indeterminate under the original Phillips curve and the forward-looking one whereas under the backward-looking one and the hybrid one it is either non-existent or indeterminate.

5. Conclusion

This paper examines equilibrium determinacy under four types of the Phillips curve, viz. the original, the backward-looking, the forward-looking and the hybrid ones, when the monetary authority adopts a money stock control rule or an interest-rate feedback rule (a Taylor rule). The result is summarized in table 1.

If the monetary authority keeps the nominal money stock to be constant, the equilibrium path is always determinate under the original Phillips curve and the forward-looking one. Under the backward-looking Phillips curve and the hybrid one, however, the path can be non-existent. It is indeed the case if the utility function is additive separable and the elasticity of the marginal utility of money (η_m) is small, as is under a liquidity trap.

The forward-looking Phillips curve is often criticized because of poor empirical fitting. The original Phillips curve is also criticized because of the lack of a microeconomic foundation and empirical fitting. However, as is seen from figure 2, the original one seems to fit the

Japanese data quite well.[10] Such an empirical analysis is beyond the scope of the present paper but the present theoretical result and figure 2 may suggest that we should reexamine the empirical validity of the original Phillips curve and explore a microeconomic foundation for it.[11]

Turning to an interest-rate feedback rule, if the monetary policy is passive, the equilibrium path is never determinate under any of the four Phillips curves: it is either non-existent or indeterminate. If it is active, the equilibrium path is always determinate under any one of them. Thus, the monetary authority must actively control the interest rate in order for the equilibrium path to be determinate, as long as it follows a Taylor rule.

Appendix: Proof of Proposition 2

The case of the backward-looking Phillips curve:

From (9), (10), (12) and (13) in which $u_{cm} = 0$, the characteristic function is then

$$F(\lambda) = \lambda^3 - \rho\lambda^2 + (f_2'/\eta_c)\lambda - \rho\eta_m f_2'/\eta_c = 0, \tag{A1}$$

where $\eta_c = -u_{cc}c/u_c$ and $\eta_m = -u_{mm}m/u_m$. As proven in subsection 3.2, if it has two complex roots ($\delta \pm \varepsilon i$), the real root λ_1 is positive. Since the first equation of (13) and (A1) yield

$$\rho = \lambda_1 + 2\delta,$$

the common real part of the complex roots δ satisfies

$$\delta \begin{array}{c} > \\ = \\ < \end{array} 0 \quad \Leftrightarrow \quad \rho \begin{array}{c} > \\ = \\ < \end{array} \lambda_1 \ (> 0). \tag{A2}$$

Since λ_1 is the only real root in the present case, function $F(\cdot)$ given by (A1) intersects the horizontal axis only once and from below. Moreover, since $F(\rho)$ is

$$F(\rho) = (\rho f_2'/\eta_c)(1 - \eta_m) \begin{array}{c} > \\ = \\ < \end{array} 0 \ \Leftrightarrow \ \eta_m \begin{array}{c} < \\ = \\ > \end{array} 1,$$

and $F(\lambda_1) = 0$ by the definition of λ_1, using (A2) one finds that δ satisfies

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[10] Using Japanese quarterly data (1981-2000) Kitaura et al. (2002) find that the backward-looking Phillips curve is empirically rejected and that the original Phillips curve fits the data well. The yearly data used in figure 2 are consistent with it, as shown by figure 3 –i.e., there is almost no correlation between a yearly change in the inflation rate and the unemployment rate.

[11] Akerlof et al. (1996) is an important attempt to present such a foundation.

$$\delta \gtreqless 0 \quad \Leftrightarrow \quad \eta_m \lesseqgtr 1. \tag{A3}$$

Next, it is shown that (A1) indeed has only one real root if $\eta_m > 1$. From (A1),

$$F'(\lambda) = 3\lambda^2 - 2\rho\lambda + (f_2'/\eta_c).$$

Therefore, if $\rho^2 - 3f_2'/\eta_c > 0$, the two extrema of $F(\lambda)$ exist and the local maximum is

$$F((\rho - \sqrt{\rho^2 - 3f_2'/\eta_c}\,)/3)$$

$$= -[2(\rho^2 - 3f_2'/\eta_c)(\rho - \sqrt{\rho^2 - 3f_2'/\eta_c}\,) + 24\rho f_2'/\eta_c]/3^3 - (\eta_m - 1)\rho f_2'/\eta_c,$$

which yields

$$F((\rho - \sqrt{\rho^2 - 3f_2'/\eta_c}\,)/3) < 0 \quad \text{if} \quad \eta_m > 1.$$

This property shows that there is indeed only one real root if $\eta_m > 1$ and $\rho^2 - 3f_2'/\eta_c > 0$. If $\rho^2 - 3f_2'/\eta_c < 0$, $F(\lambda)$ has no extremum and hence there is only one real root, whether $\eta_m > 1$ or not. Therefore, if $\eta_m > 1$, (A1) has only one real root, which is λ_1 (> 0). From (A3) δ is negative in this case and hence the path is determinate.

The above result also implies that η_m must be less than 1 in order for the three roots of (A1) to be real numbers and then the path is non-existent, as shown in subsection 3.2. Furthermore, if two of the three roots are complex numbers and λ_1 (> 0) is the only real root in the case where $\eta_m < 1$, from (A3) δ is positive –i.e., the path is non-existent. Thus, if $\eta_m < 1$, the path is non-existent, whether (A1) has two complex roots or not.

If $\eta_m = 1$, δ is zero as shown by (A3) and hence the characteristic roots are a positive real number (viz. λ_1) and two pure imaginary ones. Since y is the only jump variable, it jumps to one of the cyclical paths. Although the steady state given by (9) is not reached along any cyclical path, all variables cyclically move within some finite ranges and transversality condition (3) is valid. Therefore, the path is determinate.

The case of the hybrid Phillips curve:

Since it has already been found in subsection 3.4 that the path is non-existent when all roots are real numbers and determinate when all roots are complex numbers, the case where two roots are real numbers, λ_1 and λ_2, and the other two are complex numbers, $\delta \pm \varepsilon i$, is examined. Note that the path is non-existent if and only if $\delta > 0$ since λ_1 and λ_2 satisfy

$$\lambda_1 > 0, \quad \lambda_2 > 0,$$

as shown in subsection 3.4.

From (9), (10), (12) and (16) in which $u_{cm} = 0$, the characteristic function is

$$G(\lambda) = \lambda^4 - 2\rho\lambda^3 + \rho^2\lambda^2 - (f_4'/\eta_c)\lambda + \rho\eta_m f_4'/\eta_c = 0. \tag{A4}$$

Since λ_1 and λ_2 are real roots of (A4), from the first property of (17) and (A4) they satisfy

$$\lambda_1 + \lambda_2 = 2(\rho - \delta) > 0,$$
$$(\lambda_1 - \rho)[\lambda_1^2(\lambda_1 - \rho) - f_4'/\eta_c] = (f_4'/\eta_c)(1 - \eta_m)\rho,$$
$$(\lambda_2 - \rho)[\lambda_2^2(\lambda_2 - \rho) - f_4'/\eta_c] = (f_4'/\eta_c)(1 - \eta_m)\rho. \tag{A5}$$

By taking the difference between the second and third equations above and substituting the first equation into the result one obtains

$$f_4'/\eta_c + 2\delta\,(\rho - 2\delta)^2 = 2(\rho - 2\delta)[(\rho - 2\delta)\rho - \lambda_1\lambda_2]. \tag{A6}$$

Since from the first equation of (A5) $\lambda_1\lambda_2$ satisfies

$$\lambda_1\lambda_2 = (\lambda_1 + \lambda_2 - \rho)\rho - (\lambda_1 - \rho)(\rho - \lambda_2) = (\rho - 2\delta)\rho - (\lambda_1 - \rho)(\rho - \lambda_2),$$

substituting this value into $\lambda_1\lambda_2$ in (A6) and rearranging the result gives

$$(\lambda_1 - \rho)(\rho - \lambda_2) = [f_4'/\eta_c + 2\delta\,(\rho - 2\delta)^2]/[2(\rho - 2\delta)]. \tag{A7}$$

Multiplying the second equation of (A5) by $(\lambda_2 - \rho)$ and the third one by $(\lambda_1 - \rho)$, summing them up and substituting the first equation into the result yields

$$(\lambda_1 - \rho)(\rho - \lambda_2)[(\rho - 2\delta)^2 + (\lambda_1 - \rho)(\rho - \lambda_2)] = (1 - \eta_m)\rho f_4'/\eta_c.$$

Substituting $(\lambda_1 - \rho)(\rho - \lambda_2)$ given by (A7) into the above equation and rearranging the result leads to

$$2\rho^3(1 - 2\eta_m) - f_4'/\eta_c = 4\delta(\rho - \delta)[2\rho(1 - 2\eta_m) + (\rho - 2\delta)^4/(f_4'/\eta_c)]. \tag{A8}$$

If $2\rho^3(1 - 2\eta_m) - f_4'/\eta_c > 0$, then $1 - 2\eta_m > 0$. From the first property of (A5), $\rho - \delta > 0$. Using these properties and (A8) one finds $\delta > 0$ if $2\rho^3(1 - 2\eta_m) - f_4'/\eta_c > 0$.

As the converse, if $\delta > 0$, then $1 - 2\eta_m > 0$ since (A8) is rewritten as

$$2\rho(\rho^2 - 2\delta)^2(1 - 2\eta_m) = 4\delta(\rho - \delta)(\rho - 2\delta)^4/(f_4'/\eta_c) + f_4'/\eta_c.$$

Therefore, the right-hand side of (A8) is positive so that the left-hand side is also positive, i.e., $2\rho^3(1 - 2\eta_m) - f_4'/\eta_c > 0$.

In sum,

$$\delta > 0 \quad \Leftrightarrow \quad 2\rho^3(1 - 2\eta_m) - f_4'/\eta_c > 0, \tag{A9}$$

and then the path is non-existent, as long as (A4) has two real and two complex roots.

Next, it is shown that the case of four complex roots does not arise in the area given by (A9). $G(\lambda)$ given by (A4) satisfies

$$G(\rho\eta_m) = [\eta_m(\eta_m - 1)]^2\rho^4 > 0,$$

$$G(\rho) = (f_4'/\eta_c)(\eta_m - 1)\rho < 0 \quad \text{if} \quad \eta_m < 1. \tag{A10}$$

When the condition given by (A9) is valid, $\eta_m < 1/2$ and hence the two properties of (A10) show that $G(\lambda)$ has at least one real root. Thus, the case of four complex roots (in which the path is determinate) does not arise in the area given by (A9). Either the case with two real and two complex roots or the case of four real roots occurs.

The area in which the four roots are real numbers is included by the area given by (A9), as shown below. From (A4),

$$G'(\lambda) = 2\lambda(2\lambda - \rho)(\lambda - \rho) - f_4'/\eta_c,$$

$$G''(\lambda) = 2[\lambda - (1/2 - \sqrt{3}/6)\rho][\lambda - (1/2 + \sqrt{3}/6)\rho]. \tag{A11}$$

Since $\eta_m < 1/2$ in the area given by (A9), from (A10) and (A11) $G(\rho)$ satisfies

$$G(\rho) < 0, \quad G''(\lambda) > 0 \ \text{ for } {}^{\forall}\lambda > \rho,$$

implying that there is a positive real root that is larger than ρ and that there is no other real root larger than it. Therefore, from (A11), if the four roots are real numbers,

$$G(\theta_1) < 0, \quad G(\theta_2) > 0,$$

$$\text{where } G'(\theta_1) = G'(\theta_2) = 0, \quad 0 < \theta_1 < (1/2 - \sqrt{3}/6)\rho < \theta_2 < \rho. \tag{A12}$$

Note that $0 < \theta_1$ since all the four roots are positive, as shown in subsection 3.4. Since the second property of (A11) implies

$$G''(\lambda) > 0 \ \text{ for } {}^{\forall}\lambda < (1/2 - \sqrt{3}/6)\rho,$$

using the first equation in (A11) and (A12) one finds

$$0 = G'(\theta_1) < G'((1/2 - \sqrt{3}/6)\rho) = (\sqrt{3}/9)\rho^3 - f_4'/\eta_c,$$

i.e., f_4'/η_c must satisfy

$$f_4'/\eta_c < (\sqrt{3}/9)\rho^3. \tag{A13}$$

Furthermore, from the first equation in (A11) and (A12),

$$G(\theta_1) < 0, \quad G'(\theta_1) = 2\theta_1(2\theta_1 - \rho)(\theta_1 - \rho) - f_4'/\eta_c = 0.$$

Therefore, replacing f_4'/η_c by $2\theta_1(2\theta_1 - \rho)(\theta_1 - \rho)$ in (A4) and rearranging the result yields

$$G(\theta_1) = \theta_1(\rho - \theta_1)H(\theta_1) < 0,$$

$$\text{where } \ H(\theta_1) = 3\theta_1^2 - (1 + 4\eta_m)\rho\theta_1 + 2\eta_m\theta_1^2.$$

Since $\theta_1 < \rho$ from (A12), the above property shows $H(\theta_1)$ to be negative and hence θ_1 is located between the two solutions that makes $H(\theta_1) = 0$:

$$[1 + 4\eta_m - (16\eta_m^2 - 16\eta_m + 1)^{1/2}]\rho/6 < \theta_1 < [1 + 4\eta_m + (16\eta_m^2 - 16\eta_m + 1)^{1/2}]\rho/6, \tag{A14}$$

where η_m satisfy

$$16\eta_m^2 - 16\eta_m + 1 \equiv 16[\eta_m - (2 - \sqrt{3})/4][\eta_m - (2 + \sqrt{3})/4] > 0.$$

Since $\theta_1 < (1/2 - \sqrt{3}/6)\rho$, as shown in (A12), in order for θ_1 that satisfies (A14) to exist η_m must satisfy

$$[1 + 4\eta_m - (16\eta_m^2 - 16\eta_m + 1)^{1/2}]\rho/6 < (1/2 - \sqrt{3}/6)\rho.$$

Since this property reduces to

$$\{[\eta_m - (2 - \sqrt{3})/4][\eta_m - (2 + \sqrt{3})/4]\}^{1/2} > \eta_m - (2 - \sqrt{3})/4,$$

η_m must satisfy

$$\eta_m < (2 - \sqrt{3})/4. \tag{A15}$$

Therefore, if the four roots are real numbers, both (A13) and (A15) are valid.

When f_4'/η_c and η_m satisfy (A13) and (A15) respectively, it is easily found that the condition given by the right-hand side of (A9) is valid. Thus, the area of four real roots is included by the area given by (A9). Since the path is non-existent when all roots are real numbers, as shown in subsection 3.4, and (A9) gives the condition for the path to be non-existent in the case of two complex roots, (A9) in fact shows the condition under which the path is non-existent, whether all roots are real numbers or not.

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	Original	Backward-looking	Forward-looking	Hybrid
Fixed money stock	Determinate	Non-existent or determinate	Determinate	Non-existent or determinate
Active Taylor rule	Determinate	Determinate	Determinate	Determinate
Passive Taylor rule	Indeterminate	Non-existent or indeterminate	Indeterminate	Non-existent or indeterminate

Table 1: Equilibrium Determinacy under Various Phillips Curves

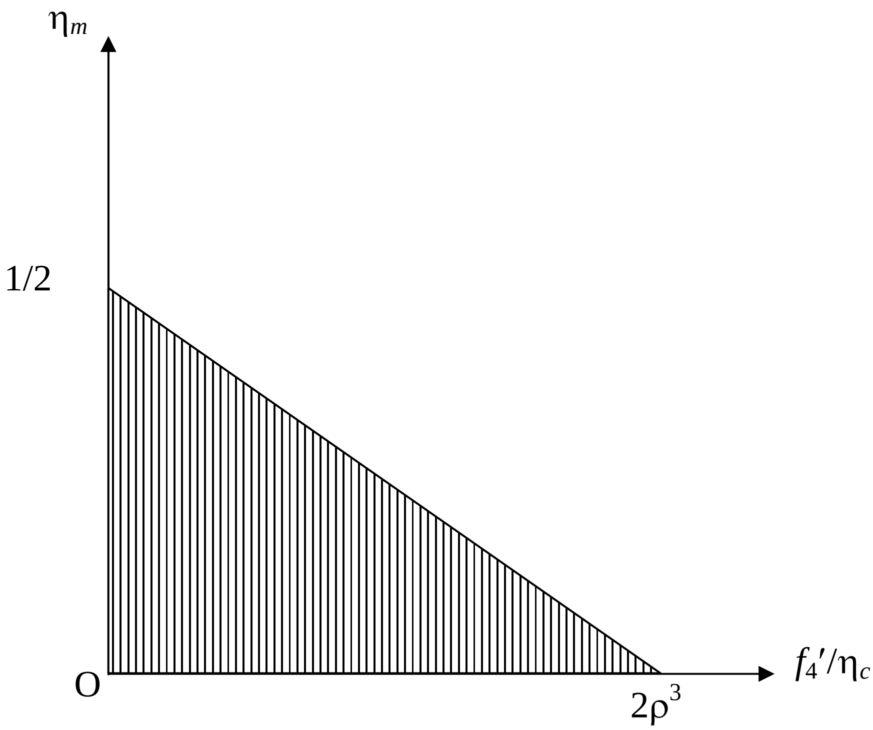
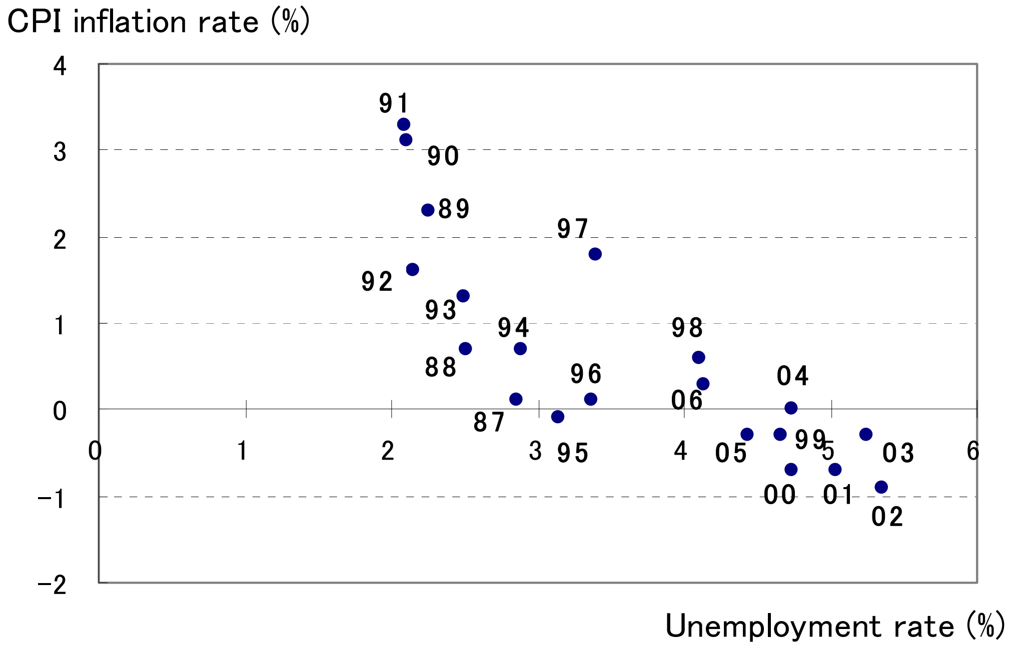


Figure 1: η_m and f_4'/η_c That Make the Path Non-existent



Statistics Bureau, Ministry of Internal Affairs and Communications, Japan.

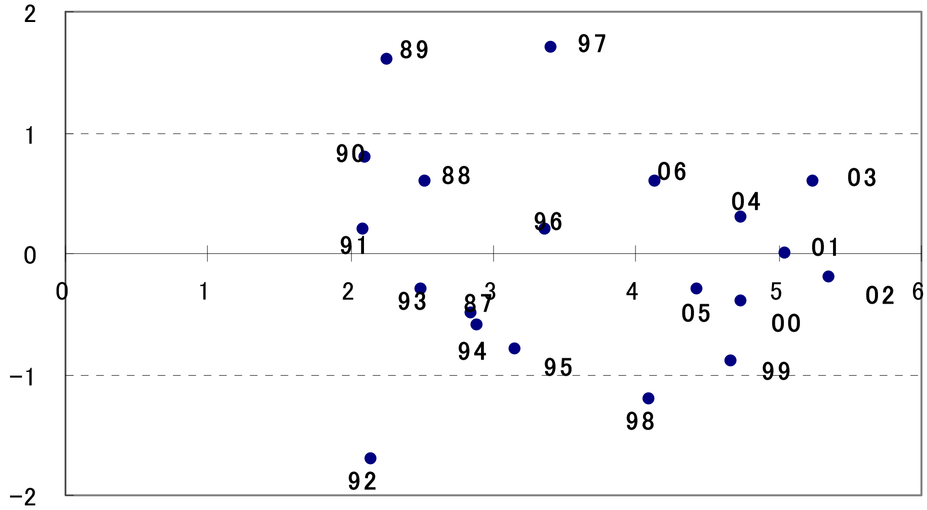
Unemployment rate: http://www.stat.go.jp/data/roudou/longtime/03roudou.htm.

CPI: http://www.stat.go.jp/data/cpi/200707/index.htm

and http://www.stat.go.jp/data/cpi/longtime/index.htm.

Figure 2: Phillips Curve in Japan (1987-2006)

A yearly change in CPI inflation rate



Unemployment rate (%)

Statistics Bureau, Ministry of Internal Affairs and Communications, Japan.

Unemployment rate: http://www.stat.go.jp/data/roudou/longtime/03roudou.htm.

CPI: http://www.stat.go.jp/data/cpi/200707/index.htm

and http://www.stat.go.jp/data/cpi/longtime/index.htm.

Figure 3: Change in Inflation and Unemployment Rate