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LOCAL HOME ENVIRONMENT EXTERNALITY IS A SOURCE OF INDETERMINACY

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Abstract

We address *the local home environment externality* conceptualized by Galor and Tsiddon (1997a; 1997b) in the two sector growth model of Lucas (1988). We show that this version of externality related to human capital accumulation process can be a source of indeterminacy.

Keywords: Local home environment externality; human capital; indeterminacy

JEL classifications: O41; E32

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1 Introduction

As clearly documented in Benhabib and Perli (1994), dynamic equilibrium can be indeterminate in models with externalities. Recently, Chen and Hsu (2007) showed that a version of consumption externalities, admiration, is a source of indeterminacy. In this paper, following the spirit of Chen and Hsu (2007), we present another clear-cut and plausible source of indeterminacy.

The externality we address here is related to human capital; the local home environment externality (LHEE) conceptualized by Galor and Tsiddon (1997a; 1997b). With this externality, newly born agents are affected by parents with home education and they are endowed with positive amount of human capital before they enter production process.[1] Hence, in our model there is inter-generational spillover of human capital while parents cannot recognize the influence of themselves on their children. A priori, it is plausible that this version of externality can be another source of indeterminacy. We show that it is the case. While Benhabib and Perli (1994) show that increasing returns scale technology in human capital production process is a source of indeterminacy, the externality considered here provides theoretically simpler explanation for business fluctuations through distortions in human capital production sector.

In section 2 we provide our model. Section 3 investigates the stability of the equilibrium. Section 4 concludes the paper.

2 The Model

We study the two sector growth model of Lucas (1988) with LHEE. Specifically, addressing a Cobb-Douglas production function on final output as

$$Y(t) = Ak(t)^{\beta}(u(t)h(t))^{1-\beta}, \quad A > 0, \quad \beta \in (0,1)$$

the model is given as

$$\max_{c(t),u(t)} \int_0^{\infty} \frac{c(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \tag{P}$$

subject to

$$\dot{k}(t) = Ak(t)^{\beta}(u(t)h(t))^{1-\beta} - c(t) - nk(t)$$
$$\dot{h}(t) = \delta(1-u(t))h(t) - nh(t) + \alpha h(t),$$

[1]Behrman et al. (1999) argue that the educational achievements of mothers are positively correlated with the intensity of home schooling towards their children.

where c is consumption, k is physical capital, h is human capital, and u is the fraction of human capital devoted to production of final output. Note here that human capital per capita is diluted because of the population growth, $nh(t)$, but augmented with LHEE, $\alpha h(t)$. We assume that agents cannot internalize the effect of LHEE on human capital augmentation. $\sigma(>0)$, $\delta(>0)$, $n(>0)$, $b(>0)$, and $\alpha(>0)$ are parameters.

The problem (P) is solved by defining the current value Hamiltonian[2]

$$H \equiv \frac{c(t)^{1-\sigma}-1}{1-\sigma}+\lambda_1(t)[Ak(t)^{\beta}(u(t)h(t))^{1-\beta}-c(t)-nk(t)]+\lambda_2(t)[\delta(1-u(t))h(t)-nh(t)]$$

and deriving the optimal conditions;

$$c(t)^{-\sigma} = \lambda_1(t)$$

$$\lambda_1(t)(1-\beta)Ak(t)^{\beta}u(t)^{-\beta}h(t)^{1-\beta} = \lambda_2(t)\delta h(t)$$

$$\dot{\lambda_1}(t) = [\rho+n-\beta Ak(t)^{\beta-1}u(t)^{1-\beta}h(t)^{1-\beta}]\lambda_1(t)$$

$$\dot{\lambda_2}(t) = -\lambda_1(t)(1-\beta)Ak(t)^{\beta}u(t)^{1-\beta}h(t)^{-\beta}+[\rho+n-\delta(1-u(t))]\lambda_2(t)$$

and the usual two transversality conditions,

$$\lim_{t\to\infty}\lambda_1(t)k(t)e^{-\rho t} = 0$$

$$\lim_{t\to\infty}\lambda_2(t)h(t)e^{-\rho t} = 0,$$

where λ_1 and λ_2 represent shadow prices of physical and human capital, respectively.[3]

We define new two variables which are stationary on the balanced growth path in order to investigate the stability of the equilibrium path; $q \equiv c/k$ and $x \equiv k/h$. Then, with a little algebra, the intensive form dynamical system consisting of $(u(t), q(t), x(t))$ can be obtained as

$$\frac{\dot{u}(t)}{u(t)} = \delta u(t) - q(t) + \frac{\delta}{\beta} - \delta - \alpha,$$

$$\frac{\dot{q}(t)}{q(t)} = \left(\frac{\beta}{\sigma}-1\right)Ax(t)^{\beta-1}u(t)^{1-\beta}+q(t)+n-\frac{n}{\sigma}-\frac{\rho}{\sigma}$$

[2] Although $\alpha h(t)$ appears in the low of motion for human capital, it does not taken into account by agents so that in the Hamiltonian $\alpha h(t)$ is not included. Alternatively, one can think that $\alpha h(t)$ is an uncontrollable constant term for agents.

[3] With a little algebra, we can see that the TVCs can be written as $(1-\sigma)(\Lambda-n)-\rho<0$. This condition does not depend on α so that we innocuously assume it holds throughout our analyses below.

$$\frac{\dot{x}(t)}{x(t)} = Ax(t)^{\beta-1}u(t)^{1-\beta} + \delta u(t) - q(t) - \delta - \alpha,$$

together with stationary variables of

$$u^* = 1 - \frac{(\delta - \rho - n)/\sigma + n - \alpha}{\delta}$$

$$q^* = \delta u^* + \frac{\delta}{\beta} - \delta - \alpha$$

$$x^* = \left(\frac{q^* + \delta(1-u^*) + \alpha}{A(u^*)^{1-\beta}}\right)^{\frac{1}{\beta-1}}.$$

Inner solution conditions of $u^* \in (0,1)$ and $q^* > 0$ require that parameters satisfy the following conditions

$$\Lambda - \delta < \alpha < \Lambda, \tag{C1}$$

and

$$\Lambda < \frac{\delta}{\beta}, \tag{1}$$

where $\Lambda \equiv (\delta - \rho - n)/\sigma + n$ must be positive to ensure a positive growth rate of the economy.[4]

For later use, here we put another condition on Λ as

$$\Lambda - \delta > 0. \tag{2}$$

(1) and (2) provide a parameter restriction on Λ as

$$\delta < \Lambda < \frac{\delta}{\beta}. \tag{C2}$$

3 Stability Analysis

The linearized dynamical system around the steady state, (u^*, q^*, x^*), can be derived as

$$\begin{pmatrix} \dot{u} \\ \dot{q} \\ \dot{x} \end{pmatrix} = \begin{pmatrix} \delta u^* & -u^* & 0 \\ \left(\frac{\beta}{\sigma} - 1\right)(1-\beta)\frac{\Upsilon^*}{u^*}q^* & q^* & \left(\frac{\beta}{\sigma} - 1\right)(\beta - 1)\frac{\Upsilon^*}{x^*}q^* \\ \left\{(1-\beta)\frac{\Upsilon^*}{u^*} + \delta\right\}x^* & -x^* & (\beta-1)\Upsilon^* \end{pmatrix} \begin{pmatrix} u - u^* \\ q - q^* \\ x - x^* \end{pmatrix},$$

[4]We find that the dynamical system for the corner solution case ($u^* = 1$) is always saddle-stable. Proof is available upon requests from the authors.

where $\Upsilon^* \equiv A(u^*/x^*)^{1-\beta}$. Then, we define the characteristic equation of the dynamical system as

$$\Omega(\omega) = -\omega^3 + Tr\omega^2 + B\omega + Det,$$

where ω denotes eigenvalues, Tr is the trace and Det is the determinant of the Jacobean matrix of the dynamical system. We can easily see that Det is always negative as

$$Det = -(1-\beta)\delta\Upsilon^* q^* u^* < 0.$$

With $Det < 0$, we have the following lemma.

Lemma 1:

Indeterminacy of the dynamical system requires $Tr < 0$ and $B < 0$.

Proof:

See the proof in Chen and Hsu (2007). Q.E.D.

We then investigate if there are parameter sets to satisfy these requirements. Now we can obtain B as

$$B = (1-\beta)\frac{\delta}{\beta}(\delta u^* + q*) - \delta u^* q^*$$

The following lemma shows that there exists a parameter set where $B < 0$.

Lemma 2:

There exists a threshold value of β, $\beta^* \in (0,1)$, such that $B < 0$ with $\beta \in (\beta^*, 1)$.

Proof:

First, $sign(\lim_{\beta\to 1} B) = sign(-\delta u^* q^*)$, which is negative since u^* and q^* are positive and bounded. It is easy to see that $lim_{\beta\to 0} B = +\infty$. Also, with the expressions of u^* and q^*, B can be expressed as

$$\begin{aligned} B &= (1-\beta)\frac{\delta}{\beta}(\delta u^* + q*) - \delta u^* q^* \\ &= \frac{\delta^2}{\beta^2} - \frac{(\delta^2 + \delta\Lambda)}{\beta} - \Lambda^2 + (\alpha - \delta)\Lambda - \delta^2 - \delta\alpha \equiv B(\beta). \end{aligned}$$

Obviously, the function $B(\beta)$ is continuous and monotone with $\beta \in (0,1)$. Also, with (1) we can see that

$$\begin{aligned}\frac{\partial B(\beta)}{\partial \beta} &= -\frac{2\delta^2}{\beta^3} + \frac{\delta^2 + \delta\Lambda}{\beta^2} \\ &= \frac{\delta^2}{\beta^2}(1 - \frac{2}{\beta} + \frac{\Lambda}{\delta}) \\ &< \frac{\delta^2}{\beta^2}(1 - \frac{2}{\beta} + \frac{1}{\beta}) \\ &= \frac{\delta^2}{\beta^2}(1 - \frac{1}{\beta}) < 0.\end{aligned}$$

Hence, from the Intermediate value theorem, we can see that there exists a threshold value of β, $\beta^* \in (0,1)$, such that $B < 0$ with $\beta \in (\beta^*, 1)$. B then will be negative when β is close to one.

With respect to Tr, it is derived as

$$Tr = 2\delta u^* - \alpha,$$

which is always positive when we do not consider LHEE ($\alpha = 0$). With u^*, we can see that $Tr < 0$ if and only if

$$\alpha < 2(\Lambda - \delta). \tag{C3}$$

Now we have the proposition as

Proposition:

Local home environment externality is a source of Indeterminacy.

Proof:

Together with Lemma 2, a simple algebra shows that parameter region satisfying (C1) – (C3) is not empty as shown in Figure 1. Hence, the dynamical system can be indeterminate around the steady state. Q.E.D.

To finish this section, we recap that there is no possibility of indeterminacy if we do not consider any externalities in this type of growth models (Mulligan and Sala-i-Martin; 1993). The intuition of the local indeterminacy from LHEE is as follows. Now suppose that the representative household increases the amount of investment on human capital creation from an optimal level. This increase reduces current consumption and utility, which without LHEE cannot be compensated from the increased stream of future consumption. With LHEE, however, the reduction of current consumption (and the increase in human capital investment) brings unexpected increse in human capital due to LHEE. This unexpected future reward may compensate the current reduction of utility level, and hence, local indeterminacy will be possible.

4 Conclusion

Following the spirit of Chen and Hsu (2007), we present a plausible source of indeterminacy related to human capital accumulation. The merit in considering LHEE is that it has empirical supports such as Behrman et al. (1999). We argue that while the existence of LHEE enhances the per capita GDP growth rate, it can be a source of business fluctuations, which is a dilemma.

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Figure 1: Parameter restrictions for indeterminacy