Discussion Paper No. 735

CANDIDATE STABLE VOTING RULES FOR SEPARABLE ORDERINGS

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April 2009

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First Version: January 22, 2009
This Version: April 23, 2009

Abstract: We consider the election model in which voters choose a subset from the set of candidates. Both voters and candidates are assumed to possess preferences with separable strict orderings. We investigate a rule satisfying *candidate stability*, which is the requirement to deter any candidate from strategic withdrawal. We show that a rule satisfies *candidate stability* if and only if it satisfies *independence* of the selection for each candidate.

Keywords: multiple-winner election, strategic candidacy, separable preference, voting by committees

JEL Classification Numbers: D71, D72

*I thank Shigehiro Serizawa, William Thomson and other seminar participants at Universities of Osaka and Rochester for their helpful comments and suggestions. I gratefully acknowledge the financial support from the Japan Society for the Promotion of Science via the Research Fellowship for Young Scientist.

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1 Introduction

This paper investigates the election model in which voters can select any subset from the set of candidates. Examples are where the members of a club select new comers, or where the executive committee of a society select new members. Barberà et al. (1991) first study this model under the assumption that voters possess preferences with separable strict orderings over the power set of the sets of candidates. In this paper, we extend their model so that not only voters but candidates have preferences and the actual running candidates is variable, and we investigate a rule satisfying "*candidate stability*". *Candidate stability* requires that for any candidate, standing in the election with any set of running candidates is at least as desirable as withdrawing from it. Dutta et al. (2001) suggests the importance of this property as a basis of the study for any election model. If an election rule satisfies *candidate stability*, all candidates may stand in the election. If an election rule does not satisfy *candidate stability*, candidates' strategies whether to run the election at the pre-election stage may affect the rule and hence the rule designer has to take it into account.

Dutta et al. (2001) first study this property in the single-winner election where preferences are unrestricted, and shows that a rule satisfying *candidate stability* and *unanimity* is only *dictatorial*.[1] By comparison, our main result is a possibility result. We show that a rule satisfies *candidate stability* if and only if it satisfies the following *independence* condition. Suppose in the election with some set of running candidates and some preference profile of voters, a candidate wins. Then she also should win in the election with any set of running candidates she belongs to and any preference profile that is equivalent to the previous profile in the comparison of herself and the null outcome. This is a quite parallel work to Barberà et al. (1991). They impose *strategy-proofness*, which requires non-manipulability by voters, and derive independence of the selection for each candidate as a part of *voting by committees* that they characterize. Meanwhile, we impose non-manipulability of candidates and derive the similar independence condition.

Related literature includes Berga et al. (2004, 2006). They consider similar models to ours in which voters are existing members of a society and

[1]We omit the descriptions of standard properties in Introduction. See them in the original literature.

have exit options. They study stability of the existing members while we focus on the stability of candidates as new entrants. Literature on *candidate stability* includes Ehlers and Weymark (2003), Eraslan and McLennan (2004), Rodríguez-Álvarez (2005, 2006), and Samejima (2005, 2007).

The structure of the rest of this paper is as follows. Section 2 states the model and the main result. Section 3 gives a short discussion and conclusion. The appendix includes the proof of the main result.

2 The model and the result

Let $C \equiv \{1, 2, \cdots, c\}$ be the set of (prospective) **candidates** with $c \geq 1$. We refer to a candidate who actually stands on the election as a **running candidate**. Let $V \equiv \{c+1, c+2, \cdots, c+v\}$ be the set of **voters** with $v \geq 1$. For simplicity, we assume $C \cap V = \emptyset$. For a voter or a candidate $i \in C \cup V$, let P_i denote her **preference**, which is a complete, transitive, and asymmetric binary relation over 2^C. We assume that any preference P_i satisfies the following restrictions.

Separability: For all $i \in C \cup V$, all $x \in C$, and all $Y \subseteq C \backslash \{x\}$, we have $Y \cup \{x\}\ P_i\ Y \iff \{x\}\ P_i\ \emptyset$.

Additivity: P_i has an additive numerical representation.[2]

We refer to a preference satisfying separability and additivity as an **additively separable** preference. Let D_S denote the domain of all such preferences. All preferences of voters are assumed to be in the domain D_S.

In addition to additive separability, we impose another restriction on the preferences of the candidates: for all $x \in C$, $\{x\}\ P_x\ \emptyset$. Since without it, the concept of "candidate stability" is not meaningful, we refer to it as the **minimal** restriction for candidates' preferences. Let D_M denote the domain of all additively separable preferences satisfying the restriction. A **preference profile** is a $c+v$ tuple of preferences $P = (P_1, \cdots, P_c, P_{c+1}, \cdots, P_{c+v}) \in D_M^c \times D_S^v$. Let $i, j \in C \cup V$. Let $(P'_i, P_{-i}) \in D_M^c \times D_S^v$ denote the preference profile obtained from P by replacing P_i with P'_i, $(P''_j, P'_i, P_{\{-i,j\}}) \in D_M^c \times D_S^v$ denote the profile obtained from (P'_i, P_{-i}) by replacing P_j with P''_j, and so on. Let $P_i|_X$ denote the preference relation over $X \subseteq C$ induced by $P_i \in D_S$.

[2]Separability and additivity are independent in our definition. See Barberà et al. (1991) and Berga et al. (2004) for examples of that separability does not imply additivity.

Similarly, let $P|_X$ denote the preference profile over $X \subseteq C$ induced by $P \in D_M^c \times D_S^v$.

A **rule** is a function $\varphi : 2^C \times D_M^c \times D_S^v \to 2^C$. Following Dutta et al. (2001), a rule φ is assumed to satisfy the following three properties. First, winners should be chosen from the set of running candidates. Second, only voters' preferences matter. Third, only preferences over the running candidates matter.

Feasibility: For all $X \in 2^C$ and all $P \in D_M^c \times D_S^v$, we have $\varphi(X, P) \subseteq X$.

Independence of nonvoters' preferences: For all $X \in 2^C$ and all $P, P' \in D_M^c \times D_S^v$ such that for all $i \in V$, $P_i = P'_i$, we have $\varphi(X, P) = \varphi(X, P')$.

Independence of irrelevant alternatives: For all $X \in 2^C$ and all $P, P' \in D_M^c \times D_S^v$ such that $P|_X = P'|_X$, we have $\varphi(X, P) = \varphi(X, P')$.

The formal definition of "*candidate stability*" of a rule is given as follows.

Candidate stability: For all $X \in 2^C \backslash \{\emptyset\}$, all $x \in X$, and all $P \in D_M^c \times D_S^v$, we have $\varphi(X, P)\ P_x\ \varphi(X\backslash\{x\}, P)$ or $\varphi(X, P) = \varphi(X\backslash\{x\}, P)$.[3]

We claims that *candidate stability* is equivalent to the following independence condition. Suppose that for some set of running candidates and some preference profile, a candidate wins. Then she also should win for any set of running candidates she belongs to and any preference profile that is equivalent to the previous profile in the comparison of herself and the null outcome.

Independence: For all $X, Y \in 2^C \backslash \{\emptyset\}$, all $x \in C$ such that $x \in X$ and $x \in Y$, and all $P, P' \in D_M^c \times D_S^v$ such that $P|_{\{\{x\},\emptyset\}} = P'|_{\{\{x\},\emptyset\}}$, we have $x \in \varphi(X, P) \iff x \in \varphi(Y, P')$.

Proposition 1. *A rule satisfies candidate stability if and only if it satisfies independence.*[4]

Proof of Proposition 1. See the appendix. □

[3]The original definition of *candidate stability* in Dutta et al. (2001) is slightly weaker than ours in the sense that they consider stability only when running candidates equal candidates (*i.e.*, $X = C$). Our definition of *candidate stability* is similar to "no-exit stability" in Samejima's (2007) model with single-peaked preference domain.

[4]Ju (2003) extends Barberà et al.'s (1991) model to allow indifferences of voters' preferences. This proposition holds even in Ju's (2003) model. The proof is the same.

The intuition behind the proof is as follows. Since we impose the minimal restriction on candidate preferences, there exists a candidate who prefers another candidate to win no matter whether the other candidates, including herself, are elected. There also exists a candidate who prefers another candidate not to win no matter whether the other candidates, including herself, are elected. If *independence* is violated, a candidate with such a preference may be better off by not running. This violates *candidate stability*.

3 Discussion and concluding remarks

Before concluding this paper, we note that if we drop the minimal restriction for candidates' preferences and impose a stronger but natural restriction for them, Proposition 1 fails. Let $D_M^* \subsetneq D_M$ be the domain of all additively separable preferences satisfying the following restriction for candidates. For all $x \in C$ and all $P_x \in D_S^*$, $\{x\}\ P_x\ \emptyset$ and for all $y \in C \backslash \{x\}$, $\{x\}\ P_x\ \{y\}$. This states that any candidate prefers herself to any other candidate. A counter example to the equivalent statement of Proposition 1 in this domain is the following.

Example 1. Let $C \equiv \{1,2\}$ and $V \equiv \{3\}$. Let φ be a rule such that
(i) for any $P \in D_M^{*2} \times D_S$, $1 \in \varphi(C, P_3)$ and $1 \in \varphi(\{1\}, P)$,
(ii) if $\{1\}\ P_3\ \emptyset$, $2 \in \varphi(C,P)$ and if $\emptyset\ P_3\ \{1\}$, $2 \notin \varphi(C,P)$, and
(iii) for any $P \in D_M^{*2} \times D_S$, $2 \in \varphi(\{2\},P)$.

Then φ satisfies *candidate stability*, however, it does not satisfy *independence*.

The question how *candidate stability* works on the domain D_S^* is open. We hope that this paper will help further research on *candidate stability*.

Appendix

Before proving Proposition 1, we introduce one property and three lemmas. The next property is taken from Eraslan and McLennan (2004).

Strong candidate stability: For all $X \in 2^C \backslash \{\emptyset\}$, all $x \in X$ and all $P \in D_M^c \times D_S^v$, we have $\varphi(X,P) \backslash \{x\} = \varphi(X \backslash \{x\}, P)$.[5]

[5] The definition of *strong candidate stability* in Eraslan and McLennan (2004) is slightly different from ours since their model does not allow the outcome of empty set.

Lemma 1. A rule satisfies *candidate stability* if and only if it satisfies *strong candidate stability.*

Proof of Lemma 1. Since *the if part* of proposition is obvious, we only show *the only if part.*

Note that *strong candidate stability* is equivalent to the requirement that for all $X \in 2^C \backslash \{\emptyset\}$, all $x \in X$, and all $P \in D_M^c \times D_S^v$, (i) $\varphi(X,P)\backslash\{x\} \subseteq \varphi(X\backslash\{x\},P)$ and (ii) $\varphi(X\backslash\{x\},P) \subseteq \varphi(X,P)\backslash\{x\}$. At first, suppose, on the contrary, that φ satisfies *candidate stability* but not (i). Then there exist $X \in 2^C\backslash\{\emptyset\}$, $x \in X$, and $P \in D_M^c \times D_S^v$ such that $\varphi(X\backslash\{x\},X) \subsetneq \varphi(X,P)\backslash\{x\}$. Let $y \in \varphi(X,P)\backslash(\{x\} \cup \varphi(X\backslash\{x\},P))$. Let $P'_x \in D_M$ be such that for all $Y, Z \subseteq X$ with $y \in Y$ and $y \notin Z$, $Z \; P'_x \; Y$. Then $\varphi(X\backslash\{x\},P) \; P'_x \; \varphi(X,P)$. By *independence of nonvoters' preferences*, $\varphi(X\backslash\{x\},P'_x,P_{-x}) \; P'_x \; \varphi(X,P'_x,P_{-x})$. This contradicts *candidate stability.* Hence our supposition is incorrect.

Next, suppose, on the contrary, that φ satisfies *candidate stability* but not (ii). Then there exist $X \in 2^C\backslash\{\emptyset\}$, $x \in X$, and $P \in D_M^c \times D_S^v$ such that $\varphi(X,P)\backslash\{x\} \subsetneq \varphi(X\backslash\{x\},P)$. Let $y \in \varphi(X\backslash\{x\},P))\backslash\varphi(X,P)$. Let $P'_x \in D_M$ be such that for all $Y, Z \subseteq X$ with $y \in Y$ and $y \notin Z$, $Y \; P'_x \; Z$. Then $\varphi(X\backslash\{x\},P) \; P'_x \; \varphi(X,P)$. By *independence of nonvoters' preferences*, $\varphi(X\backslash\{x\},P'_x,P_{-x}) \; P'_x \; \varphi(X,P'_x,P_{-x})$. This contradicts *candidate stability.* Hence our supposition is incorrect. □

Lemma 2. If a rule φ satisfies *candidate stability*, then for all $X \in 2^C$ with $|X| \geq 2$, all $x, y \in X$ and all $P \in D_M^c \times D_S^v$, we have $x \in \varphi(X,P) \iff x \in \varphi(X\backslash\{y\},P)$.

Proof of Lemma 2. This is direct from Lemma 1. □

Lemma 3. If φ satisfies *candidate stability*, then for all $X \in 2^C$ with $|X| \geq 2$, all $x \in X$, all $P \in D_M^c \times D_S^v$, all $i \in V$ and all $P'_i \in D_S$ such that for a candidate $y \in X\backslash\{x\}$, $P_i|_{\{\{y\},\emptyset\}} \neq P'_i|_{\{\{y\},\emptyset\}}$ and for all other candidates $z \in X\backslash\{y\}$, $P_i|_{\{\{z\},\emptyset\}} = P'_i|_{\{\{z\},\emptyset\}}$, we have $x \in \varphi(X,P) \iff x \in \varphi(X,P'_i,P_{-i})$.

Proof of Lemma 3. Let φ be a rule satisfying *candidate stability.* Let $X \in 2^C$, $x \in X$, $P \in D_M^c \times D_S^v$, $i \in V$ and $P'_i \in D_S$ safisfy the hypothesis of Lemma 3. Suppose, on the contrary, that $x \in \varphi(X,P)$ but $x \notin \varphi(X,P'_i,P_{-i})$. By *strong candidate stability,* $\varphi(X,P)\backslash\{y\} = \varphi(X\backslash\{y\},P)$

and $\varphi(X, P'_i, P_{-i})\backslash\{y\} = \varphi(X\backslash\{y\}, P'_i, P_{-i})$. Thus, $x \in \varphi(X\backslash\{y\}, P)$ and $x \notin \varphi(X\backslash\{y\}, P'_i, P_{-i})$. However, by *independence of irrelevant alternatives*, $\varphi(X\backslash\{y\}, P) = \varphi(X\backslash\{y\}, P'_i, P_{-i})$. Thus, $x \in \varphi(X\backslash\{y\}, P'_i, P_{-i})$. It is a contradiction. Hence our supposition is incorrect. $\square$

Proof of Proposition 1. The only if part: By the iterated use of Lemma 2 on candidates and Lemma 3 on preference profiles, *independence* is obtained.
The if part: Let $X \in 2^C\backslash\{\emptyset\}$, $x \in X$ and $P \in D^c_M \times D^v_S$. For all $y \in \varphi(X, P)\backslash\{x\}$, *independence* implies $y \in \varphi(X\backslash\{x\}, P)$. Also for all $z \notin \varphi(X, P)\backslash\{x\}$, *independence* implies $z \notin \varphi(X\backslash\{x\}, P)$. Thus $\varphi(X\backslash\{x\}, P) = \varphi(X, P)\backslash\{x\}$ and φ satisfies *strong candidate stability.* Hence, it satisfies *candidate stability.* $\square$

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