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A SMALL FIRMS LEADS TO CURIOUS OUTCOMES: SOCIAL SURPLUS, CONSUMER SURPLUS, AND R&D ACTIVITIES

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A small firm leads to curious outcomes: Social surplus, consumer surplus, and R&D activities*

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Abstract

This paper investigates an asymmetric duopoly model with a Hotelling line. We find that helping a small (minor) firm can reduce both social and consumer surplus. This makes a sharp contrast to existing works showing that helping minor firms can reduce social surplus but always improves consumer surplus. We also investigate R&D competition. We find that a minor firm may engage in R&D more intensively than a major firm in spite of economies of scale in R&D activities.

JEL classification: L13, O32, R32

Key words: product selection, minor firm, R&D

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1 Introduction

Common observation suggests that firms in the same industry often differ in their market conduct and performance. Large and small firms tend to have different features in their strategies. For instance, it has been believed that small (minor) firms should differentiate their products from those of major firms because the minor firms do not have any advantage over the major firms if they do not differentiate their products. Moreover, it is also widely believed that major firms invest more in R&D. In other words, minor firms invest less in R&D.[1] After all, it is often considered that the market impact of small firms is not so large. From the view of social welfare, however, it is unclear whether or not those strategic conducts of small firms are beneficial. Therefore, investigating strategic behaviors of minor firms is an important research topic in the literature of not only industrial organization but also management strategy. Using a simple Hotelling model, we consider how a small firm affects consumer and social surplus and R&D expenditures.

In their pioneering work, Lahiri and Ono (1988) investigate an asymmetric Cournot duopoly and show that an increase in the cost of an inferior firm improves welfare when the cost difference between firms are sufficiently large, that is, helping a minor firm reduces welfare.[2] An increase in the cost of the inferior firm reduces its output, and through the strategic interaction between the firms, the output of the superior firm increases. This production substitution economizes on total production costs and thus improves welfare. This result implies that eliminating the inferior firm (minor firm) improves welfare. Wang and Zhao (2007) show that in both Cournot and Bertrand models with product differentiation, an increase in the cost of the inferior firm can improve welfare.

Although those studies of asymmetric oligopoly show that enhancing competition can

[1] As explained later, it is not always true. Cohen and Klepper (1996) mention counterexamples to this statement.

[2] Zhao (2001) provides a sufficient condition for the proposition. For the applications of this mechanism, see Lahiri and Ono (1997, 1998). Salant and Shaffer (1999) also provide important welfare implications on asymmetric Cournot models.

reduce welfare, enhancing competition improves consumer welfare in all the papers mentioned above. In contrast to the existing works, we present a situation where helping a minor firm reduces both social surplus and consumer surplus by incorporating product positions of asymmetric firms. We show this by using an asymmetric location-price model with a Hotelling line.[3] We find that a decrease in the cost of the inferior firm distorts the product selection by the superior firm and can reduce both consumer and social surplus.

We believe that investigating the relation between firm heterogeneity and social welfare in a location model is an important issue because we can investigate several important points within this framework. First, we can investigate how product varieties (the locations of firms) in a market are endogenously determined. In other words, we can investigate which firm produces a mainstream product (locates at a central point). Second, we can investigate whether these equilibrium product varieties (the locations of firms) are efficient from the viewpoint of social and consumer welfare. A typical example related to our motivation is the optimal product positioning strategies of asymmetric firms in the context of retail outlet locations in the fast food industry (see Thomadsen (2007)). In many cities, both McDonald's (the stronger competitor) and Burger King (the weaker competitor) seek optimal locations. McDonald's avoids moderate amounts of differentiation while Burger King tries to differentiate itself, and Burger King is more likely than McDonald's to choose a suboptimal location (Thomadsen (2007)). In this case, investigating the efficiency of those location strategies has the potential to provide an insight on policy for land use and regulation of zoning.

We also investigate R&D competition between asymmetric firms. The larger the firm, the greater is the output over which it can apply the fruits of its R&D and hence the greater its returns from R&D (Cohen and Klepper, 1996). Thus, it is widely believed that major firms invest more in R&D. In this paper, we show that the minor firm can have a stronger

[3] For the equilibrium location under an asymmetric cost structure in the Hotelling model, see Ziss (1993), Tyagi (2000), Liang and Mai (2006), and Matsumura and Matsushima (2009).

incentive for R&D. In our model, a decrease in the cost of the minor firm distorts the product selection of the major firm. This strategic effect can dominate the economy of scale effect mentioned above, and yields a counterintuitive result, that is, the minor firm can invest more than the major firm. Although it is widely observed that firms with larger market shares aggressively invest in R&D, it is also observed that minor firms are not always inactive and those with small market share engage in outstanding R&D (Cohen and Klepper (1996) and Rogers (2004)). Our result provides a new insight in the literature of the relation between firm size and R&D.[4]

Innovation by Harley Davidson in 1980s may be a typical example in which a minor firm can have a stronger incentive for R&D. Harley Davidson had only about a 5% share in the US motorcycle market. In this sense, it was a minor firm. Significant innovation by Harley Davidson has affected the product positioning of Japanese rivals such as Honda and Kawasaki and resulted in great success for Harley Davidson (see Reid (1991) and Kitano (2008)).[5]

Meza and Tombak (forthcoming) is closely related to our paper. They also discuss an asymmetric location-price model with a Hotelling line. They discuss three main topics: (1) endogenous timing of entries (locations), (2) mixed strategies of location, and (3) comparison between the social optimum and the equilibrium locations. They do not deal with our two main concerns: (1) the relation among cost difference, consumers and social welfare and (2)

[4] Our discussion is quite different from the well-known Arrow effect: a potential entrant obtains greater value from drastic innovation than an incumbent monopolist. In the Arrow (1962) setting, a potential entrant and an incumbent monopolist have the same opportunity to achieve a drastic innovation to reduce their production cost. Before the R&D investment, the incumbent monopolist is more efficient than the potential entrant. In other words, the entrant's additional gain from the drastic innovation is higher than that of the incumbent monopolist although the investment cost is the same for both. By assumption, the potential entrant has a better opportunity than the monopolist (see also Tirole (1988, Ch.10)). In our model, however, the firms' investment technologies are the same. That is, if the investment level of a minor firm is equal to that of a major firm, the levels of their cost reductions are the same.

[5] An older example of a small firm is Tokyo Telecommunications Engineering Corporation (forerunner of SONY) in the Japanese radio market. It competed with Matsushita (forerunner of Panasonic) and Hayakawa (forerunner of SHARP). A newer example of a small firm may be Mitsubishi Automobile in the Japanese electric automobile market. It competes with Toyota and Honda.

R&D competition among asymmetric firms. Our paper and Meza and Tombak (forthcoming) are complementary in the sense that one increases the value of the other.

The remainder of the paper is organized as follows. Section 2 presents the basic model. Section 3 provides the result for consumers and social welfare. Section 4 discusses the levels of R&D investment, and Section 5 concludes.

2 The model

Consider a linear city along the unit interval $[0, 1]$, where firm 1 is located at x_1 and firm 2 is located at $1 - x_2$. Without loss of generality, we assume that $x_1 \leq 1 - x_2$. Consumers are uniformly distributed along the interval. Each consumer buys exactly one unit of the good, which can be produced by either firm 1 or 2. Let p_i denote the price of firm i $(i = 1, 2)$. The utility of the consumer located at x is given by:

$$u_x = \begin{cases} -t(x_1 - x)^2 - p_1 & \text{if bought from firm 1,} \\ -t(1 - x_2 - x)^2 - p_2 & \text{if bought from firm 2,} \end{cases} \tag{1}$$

where t represents the exogenous parameter of the transport cost incurred by the consumer. For a consumer living at $x(p_1, p_2, x_1, x_2)$, where

$$-t(x_1 - x(p_1, p_2, x_1, x_2))^2 - p_1 = -t(1 - x_2 - x(p_1, p_2, x_1, x_2))^2 - p_2, \tag{2}$$

the utility is same whichever of the two firms is chosen. We can rewrite (2) as follows:

$$x(p_1, p_2, x_1, x_2) = \frac{1 + x_1 - x_2}{2} + \frac{p_2 - p_1}{2t(1 - x_1 - x_2)}.$$

Thus, the demand facing firm 1, D_1, and that facing firm 2, D_2, are given by:

$$\begin{aligned} D_1(p_1, p_2, x_1, x_2) &= \min\{\max(x(p_1, p_2, x_1, x_2), 0), 1\}, \\ D_2(p_1, p_2, x_1, x_2) &= 1 - D_1(p_1, p_2, x_1, x_2). \end{aligned} \tag{3}$$

We assume that one of the firms (denoted as firm 2) has a cost disadvantage. Assume the marginal costs of both firms are $c_1 = c$ and $c_2 = c + d$, where c is a positive constant and d is the cost disadvantage of firm 2.

The profit functions of the firms are:

$$\pi_1 = (p_1 - c)D_1(p_1, p_2, x_1, x_2), \quad \pi_2 = (p_2 - (c+d))D_2(p_1, p_2, x_1, x_2).$$

Consumer surplus and social welfare are written as

$$\begin{aligned} CS &= \int_0^{D_1} (v - p_1 - t(x - x_1)^2)dx + \int_{D_1}^1 (v - p_2 - t(1 - x_2 - x)^2)dx, &(4) \\ SW &= \int_0^{D_1} (v - c - t(x - x_1)^2)dx + \int_{D_1}^1 (v - (c+d) - t(1 - x_2 - x)^2)dx. &(5) \end{aligned}$$

The game runs as follows. In the first stage, firm 1 chooses its location $x_1 \in [0, 1/2]$ (by symmetry, $x_1 \geq 1/2$ is equivalent to $1 - x_1 (\leq 1/2)$). Observing the location of firm 1, firm 2 chooses its location $x_2 \in [0, 1]$. In the second stage, firm i chooses its price $p_i \in [c_i, \infty)$ simultaneously.

3 Result

In this section, we derive the equilibrium locations, profits, consumer surplus, and total social surplus.

3.1 Locations and profits

First, we investigate a price competition stage. Given the locations of firms, x_1 and x_2, the firms face Bertrand competition. Let the superscript 'B' denote the equilibrium outcome at the price-competition stage given x_1 and x_2. The equilibrium prices are:

$$\begin{aligned} p_1^B &= \begin{cases} c + d - t(1 - x_1 - x_2)(1 - x_1 + x_2) & \text{if } d \geq t(1 - x_1 - x_2)(3 - x_1 + x_2), \\ \dfrac{3c + d + t(1 - x_1 - x_2)(3 + x_1 - x_2)}{3} & \text{if } d < t(1 - x_1 - x_2)(3 - x_1 + x_2), \end{cases} \\ p_2^B &= \begin{cases} c + d & \text{if } d \geq t(1 - x_1 - x_2)(3 - x_1 + x_2), \\ \dfrac{3c + 2d + t(1 - x_1 - x_2)(3 - x_1 + x_2)}{3} & \text{if } d < t(1 - x_1 - x_2)(3 - x_1 + x_2). \end{cases} \end{aligned}$$

If $d \geq t(1 - x_1 - x_2)(3 - x_1 + x_2)$, $D_1^B = 1$ and $D_2\ B = 0$ (monopoly by firm 1 and firm 2 only serves as a potential competitor). If $d < t(1 - x_1 - x_2)(3 - x_1 + x_2)$, both firms produce in the market ($D_1^B, D_2^B > 0$).

The resulting profits of the firms are:

$$\pi_1^B = \begin{cases} d - t(1-x_1-x_2)(1-x_1+x_2) & \text{if } d \geq t(1-x_1-x_2)(3-x_1+x_2), \\ \dfrac{(d+t(1-x_1-x_2)(3+x_1-x_2))^2}{18t(1-x_1-x_2)} & \text{if } d < t(1-x_1-x_2)(3-x_1+x_2), \end{cases} \tag{6}$$

$$\pi_2^B = \begin{cases} 0 & \text{if } d \geq t(1-x_1-x_2)(3-x_1+x_2), \\ \dfrac{(t(1-x_1-x_2)(3-x_1+x_2)-d)^2}{18t(1-x_1-x_2)} & \text{if } d < t(1-x_1-x_2)(3-x_1+x_2). \end{cases} \tag{7}$$

Second, we consider the location choice of firm 2. Given the location of firm 1, x_1, firm 2 decides its location.[6] Lemma 1 states that the optimal location of firm 2 does not depend on x_1. Henceforth let the superscript'*' denote the subgame perfect Nash equilibrium outcome in the full game.

Lemma 1 *For any $x_1 (\leq 1/2)$, the optimal location of firm 2 is 1 (i.e., $x_2^* = 0$).*

As pointed out by d'Aspremont et al. (1979), to mitigate price competition, firm 2 maximizes the degree of product differentiation given the location of firm 1.

Third, we consider the location choice of firm 1. We have the following lemma:

Lemma 2 *(Meza and Tombak (forthcoming)) When the locations are sequentially determined, the optimal location of firm 1 is:*

$$x_1^* = \begin{cases} 0 & \textit{if } \ d \leq t, \\ \dfrac{t - \sqrt{(4t-3d)t}}{3t} & \textit{if } \ t < d \leq \dfrac{(29\sqrt{145}-187)t}{128} \simeq 1.267t, \\ \dfrac{1}{2} & \textit{if } \ \dfrac{(29\sqrt{145}-187)t}{128} \leq d. \end{cases} \tag{8}$$

[6] Note that, if $d > t(1-x_1)(3-x_1)$, it is indifferent what location is chosen by firm 2.

The resulting profits of the firms are:

$$\pi_1^* = \begin{cases} \dfrac{(3t+d)^2}{18t} & \textit{if} \;\; d \le t, \\ \dfrac{8(4t+3d+2\sqrt{t(4t-3d)})^2}{243(2t+\sqrt{t(4t-3d)})} & \textit{if} \;\; t < d \le \dfrac{(29\sqrt{145}-187)t}{128} \simeq 1.267t, \\ d - \dfrac{t}{4} & \textit{if} \;\; \dfrac{(29\sqrt{145}-187)t}{128} \le d. \end{cases}$$

$$\pi_2^* = \begin{cases} \dfrac{(3t-d)^2}{18t} & \textit{if} \;\; d \le t, \\ \dfrac{2(10t-6d+5\sqrt{t(4t-3d)})^2}{243(2t+\sqrt{t(4t-3d)})} & \textit{if} \;\; t < d \le \dfrac{(29\sqrt{145}-187)t}{128} \simeq 1.267t, \\ 0 & \textit{if} \;\; \dfrac{(29\sqrt{145}-187)t}{128} \le d. \end{cases}$$

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Figures 1 and 2 here

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Lemma 2 states that x_1 is increasing in d if $d > t$. A stronger advantage of firm 1 makes its location closer to the center. This result suggests that the stronger firm is more likely to choose a central position.

We now briefly mention the mechanism of the result. When the cost asymmetry is not significant, the standard intuition of d'Aspremont *et al.* (1979) works; that is, mitigating price competition between the firms is important for them and the products are maximally differentiated. When the cost asymmetry of the firms is significant, the price effect does not work. As pointed out by Ziss (1993), when the cost asymmetry is significant, the optimal location of the efficient firm is the same point chosen by the inefficient firm because monopolizing the market is the best choice for the efficient firm.[7] The mechanism described by Ziss (1993) works in our model. Because the follower escapes from the leader after the location choice by the leader, the optimal location for the leader is the center if the cost advantage is significant.

[7] If the efficient firm locates at a different point, to monopolize the market it must set its price at $c_2 - \alpha$ (α depends on the distance between the firms).

3.2 Consumer surplus and social welfare

From (6), (7), and Lemmas 1 and 2, we have the equilibrium quantity supplied by firm 1 and the equilibrium prices of the firms as follows:

$$D_1^* = \begin{cases} \dfrac{3t+d}{6t} & \text{if } d \leq t, \\ \dfrac{2(4t-\sqrt{t(4t-3d)}}{9t} & \text{if } t < d < \frac{(29\sqrt{145}-187)t}{128}, \\ 1 & \text{if } \frac{(29\sqrt{145}-187)t}{128} \leq d. \end{cases} \tag{9}$$

$$p_1^* = \begin{cases} \dfrac{3t+3c+d}{3} & \text{if } d \leq t, \\ \dfrac{27c+12d+16t+8\sqrt{t(4t-3d)}}{27} & \text{if } t < d < \frac{(29\sqrt{145}-187)t}{128}, \\ \dfrac{4c+4d-t}{4} & \text{if } \frac{(29\sqrt{145}-187)t}{128} \leq d. \end{cases} \tag{10}$$

$$p_2^* = \begin{cases} \dfrac{3t+3c+2d}{3} & \text{if } d \leq t, \\ \dfrac{27c+15d+20t+10\sqrt{t(4t-3d)}}{27} & \text{if } t < d < \frac{(29\sqrt{145}-187)t}{128}, \\ c+d & \text{if } \frac{(29\sqrt{145}-187)t}{128} \leq d. \end{cases} \tag{11}$$

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Figure 3 here

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Figure 3 indicates that the sum of the marginal costs $(2c+d)$ is not always correlated with the equilibrium prices. This property is different from the standard Cournot model (Salant and Shaffer (1999)).

Substituting (9), (10), and (11) into (4) and (5), we have the following result.

$$CS^* = \begin{cases} v - c - \dfrac{39t^2 + 18td - d^2}{36t} & \text{if } d \leq t, \\ v - c - \dfrac{t(229t + 63d) + 2(37t + 6d)\sqrt{t(4t - 3d)}}{243t} & \text{if } t < d < \frac{(29\sqrt{145} - 187)t}{128}, \\ v - c - \dfrac{6d - t}{6} & \text{if } \frac{(29\sqrt{145} - 187)t}{128} \leq d. \end{cases} \tag{12}$$

$$SW^* = \begin{cases} v - c - \dfrac{3t^2 + 18td - 5d^2}{36t} & \text{if } d \leq t, \\ v - c - \dfrac{t(65t - 9d) + 4(15d - 2t)\sqrt{t(4t - 3d)}}{243t} & \text{if } t < d < \frac{(29\sqrt{145} - 187)t}{128}, \\ v - c - \dfrac{t}{12} & \text{if } \frac{(29\sqrt{145} - 187)t}{128} \leq d. \end{cases} \tag{13}$$

Simple calculus yields the following proposition:

Proposition 1 *CS^* and SW^* are increasing in d if $t < d < (29\sqrt{145} - 187)t/128$.*

An efficiency improvement by firm 2 can harm not only social welfare but also consumer welfare if the efficiency difference between the firms is significant. We summarize the results in the following Figures:

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Figure 4 here

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When d is small, maximal differentiation holds. An increase in d increases prices of both firms and reduces consumer surplus. When $t < d < (29\sqrt{145} - 187)t/128$, an increase in d increases x_1. A smaller differentiation accelerates competition and reduces prices (Figure 3). A smaller differentiation also reduces the transportation costs of consumers. Both effects increase consumer surplus and the latter effect increases social welfare. When $d \geq (29\sqrt{145} - 187)t/128$, the market is monopolized by firm 1 and firm 2 only serves as a potential competitor. In this phase, an increase in d increases p_1 and reduces consumer surplus.

4 R&D competition

In this section, we specify how the *ex ante* cost difference affects the R&D activities of the firms. We now consider a situation in which the unit cost of the product for each firm is determined by its investment. Each firm chooses whether to invest to reduce its production costs. Through investment, firm i has to incur the fixed cost of the investment F, and it deterministically reduces its marginal production cost from c_i to $c_i - e$, where e is a positive constant and $c_1 = c$ and $c_2 = c + d$ $(d > 0)$.

The game runs as follows. In the first stage, each firm simultaneously determines whether to engage in R&D investment. In the second stage, firm 1 chooses its location $x_1 \in [0, 1/2]$ (by symmetry $x_1 \geq 1/2$ is equivalent with $1 - x_1(\leq 1/2)$). Observing the location of firm 1, firm 2 chooses its location $x_2 \in [0, 1]$. In the third stage, firm i chooses its price $p_i \in [c_i, \infty)$ simultaneously.[8]

Our main concern is whether only firm 2 (*ex ante* inefficient firm) engages in R&D investment. To investigate the matter, we examine two cases depending on the value of d: (i) $d \geq \bar{d}$ and (ii) $d < \bar{d}$, where $\bar{d} \equiv (29\sqrt{145} - 187)t/128$ is the critical value that leads to the monopoly of firm 1. As mentioned in the previous section, firm 1 monopolizes the market if and only if $d > \bar{d}$. We assume that in the first cases $d - e < \bar{d}$ is satisfied and that in the second cases $d + e \geq \bar{d}$ is satisfied. In the first case, firm 1 monopolizes the market if no firm invests, if both firms invest or only firm 1 invests (if the inequality does not hold, firm 2 does not engage in R&D investment at all). In the second case, firm 1 monopolizes the market if only firm 1 invests (if the inequality does not hold, monopolization does not occur at all).

We now consider the first case $d - e < \bar{d} \leq d$. We summarize the case with the following payoff matrix (I and N indicate 'invest' and 'not invest' respectively).

[8] The timing structure in the first stage is not essential to derive the main result in this section. Even though we assume that the firms make sequential R&D decisions, the main result does not change.

Firm 1 / Firm 2	I	N
I	$-F$ $\dfrac{4d-t}{4} - F$	0 $\dfrac{4d+4e-t}{4} - F$
N	$\pi_2^f(N,I) - F$ $\pi_1^f(N,I)$	0 $\dfrac{4d-t}{4}$

where

$$\begin{aligned}
\pi_1^f(N,I) &\equiv \frac{8(4t+3(d-e)+2\sqrt{t(4t-3(d-e))})^2}{243(2t+\sqrt{t(4t-3(d-e))})}, \\
\pi_2^f(N,I) &\equiv \frac{2(10t-6(d-e)+5\sqrt{t(4t-3(d-e))})^2}{243(2t+\sqrt{t(4t-3(d-e))})}.
\end{aligned}$$

As mentioned earlier, our main concern in this section is whether (N,I) is the unique equilibrium outcome. We now derive the condition that only (N,I) appears in equilibrium. After some calculus, we have the following proposition:

Proposition 2 *Suppose that $d-e < \bar{d} \leq d$ (the first case). Then (N,I) is the unique equilibrium outcome if and only if $e < F < \pi_2^f(N,I)$.*

If $e \geq \pi_2^f(N,I)$, (N,I) never becomes the unique equilibrium outcome. Figure 5 describes the regions where $e < \pi_2^f(N,I)$ is satisfied.

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Figure 5 here

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Proposition 2 states that the incentive of the minor firm for R&D investment can be stronger than that of the major firm. We explain the intuition in the last paragraph of this section.

We now consider the second case, $d < \bar{d} \leq d+e$. We summarize the case with the following payoff matrix (I and N indicate 'invest' and 'not invest' respectively).

Firm 1 / Firm 2	I	N
I	$\pi_2^s(N,N)-F$ $\pi_1^s(N,N)-F$	0 $\dfrac{4(d+e)-t}{4}-F$
N	$\pi_2^s(N,I)-F$ $\pi_1^s(N,I)$	$\pi_2^s(N,N)$ $\pi_1^s(N,N)$

where

$$
\begin{aligned}
\pi_1^s(N,N) &\equiv \frac{8(4t+3d+2\sqrt{t(4t-3d)})^2}{243(2t+\sqrt{t(4t-3d)})}, \\
\pi_2^s(N,N) &\equiv \frac{2(10t-6d+5\sqrt{t(4t-3d)})^2}{243(2t+\sqrt{t(4t-3d)})}, \\
\pi_1^s(N,I) &\equiv \frac{8(4t+3(d-e)+2\sqrt{t(4t-3(d-e))})^2}{243(2t+\sqrt{t(4t-3(d-e))})}, \\
\pi_2^s(N,I) &\equiv \frac{2(10t-6(d-e)+5\sqrt{t(4t-3(d-e))})^2}{243(2t+\sqrt{t(4t-3(d-e))})}.
\end{aligned}
$$

After some calculus, we have the following proposition:

Proposition 3 *Suppose that $d < \bar{d} \leq d+e$ (the second case). Then (N,I) is the unique equilibrium outcome if and only if*

$$\pi_1^s(N,N) - \pi_1^s(N,I) < F < \min\{\pi_2^s(N,N), \pi_2^s(N,I) - \pi_2^s(N,N)\}. \tag{14}$$

If $\pi_1^s(N,N) - \pi_1^s(N,I) \geq \min\{\pi_2^s(N,N), \pi_2^s(N,I) - \pi_2^s(N,N)\}$, there is no F satisfying (14). Figure 6 indicates regions where $\pi_1^s(N,N) - \pi_1^s(N,I) < \min\{\pi_2^s(N,N), \pi_2^s(N,I) - \pi_2^s(N,N)\}$ is satisfied. In regions (1), (2), and (3) in Figure 6, this inequality is satisfied.

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Figure 6 here

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As we stated in the previous section, when the cost difference d lies on $(t, (29\sqrt{145} - 187)t/128)$, a decrease in the marginal cost of firm 2 (*resp.* firm 1) is harmful (*resp.* beneficial) for consumers and social welfare. Nevertheless, it is possible that firm 2 has stronger incentives for cost-reducing R&D investment. A decrease in cost for firm 2 increases the degree of product differentiation, resulting in an increase in the market share of firm 2 and increases in the prices. Both effects enhance the profit of firm 2, thus, firm 2 has a strong incentive for R&D investment for strategic purposes. In contrast, a decrease in the marginal cost of firm 1 decreases the degree of product differentiation, resulting in an increase in the market share of firm 1 but a decrease in prices. The former effect increases the profit of firm 1 and the latter decreases the profit of firm 1. Thus, firm 1 has a smaller strategic incentive for R&D investment than firm 2.

On the other hand, there is an economy of scale in R&D. This is because the benefit of cost reduction is proportional to its market share, whereas the investment cost F does not depend on market share. For this economy of scale effect, firm 1 has a stronger incentive for R&D. The strategic effect mentioned above can dominate this economy of scale effect. This is why (N, I) can be the unique equilibrium. This result is also in sharp contrast with the existing result presented in Lahiri and Ono (1999).[9]

Remark Because of mathematical complexity, we have investigated the R&D competition with the specific form of investment technologies. To show the generality of our result, we now present the marginal gains from a marginal cost reduction. The gains are derived by the partial differentiations of π_i $(i = 1, 2)$ with respect to d. The absolute value of each firm's partial differentiation presents the significance of the marginal gain from its marginal cost reduction given the cost difference between the firms, d. To derive the marginal gain, we use π_i $(i = 1, 2)$ in Lemma 2. Simple calculus leads to the following (we ignore the discontinuities

[9] The economy of scale effect is common in other R&D literature such as Lahiri and Ono (1999). Our two strategic elements discussed in the text are quite different from those of existing works and yield our contrasting result.

of the profit functions).

$$\left|\frac{\partial \pi_1^*}{\partial d}\right| = \begin{cases} \dfrac{3t+d}{9t} & \text{if } d \leq t, \\ \dfrac{4t(4(2t+\sqrt{t(4t-3d)})^2 - 9d^2)}{27\sqrt{t(4t-3d)}(2t+\sqrt{t(4t-3d)})^2} & \text{if } t < d \leq \bar{d}, \\ 1 & \text{if } \bar{d} \leq d. \end{cases}$$

$$\left|\frac{\partial \pi_2^*}{\partial d}\right| = \begin{cases} \dfrac{3t-d}{9t} & \text{if } d \leq t, \\ \dfrac{t(6d-5(2t+\sqrt{t(4t-3d)}))(6d-7(2t+\sqrt{t(4t-3d)}))}{27\sqrt{t(4t-3d)}(2t+\sqrt{t(4t-3d)})^2} & \text{if } t < d \leq \bar{d}, \\ 0 & \text{if } \bar{d} \leq d. \end{cases}$$

This is summarized in Figure 7 (note that, t is normalized to 1 without loss of generality).

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Figure 7 here

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We easily find that the marginal gain of firm 2 from the marginal cost reduction is larger than that of firm 1 when $t < d < \bar{d}$. As mentioned earlier, this property stems from the relation between the cost difference and the degree of product differentiation.

5 Concluding remarks

In this paper, we use a location model and show that helping the minor firm reduces not only social welfare but also consumer surplus. In their pioneering work, Lahiri and Ono (1988) show that helping a minor firm can reduce welfare. However, in their models, a decrease in costs for the inferior firm increases consumer surplus. Our result makes a sharp contrast with theirs because we show that helping a minor firm can reduce consumer surplus.

We also discuss R&D incentives of firms. Lahiri and Ono (1999) and Kitahara and Matsumura (2006) have already stated that the incentive for R&D of the superior firm (inferior firm) is insufficient (excessive) from the viewpoint of social welfare. However, in their model, the superior firm has a stronger incentive for R&D than the inferior firm. We show that the incentive of the inferior firm for R&D can be larger than the superior firm when a decrease in the inferior firm's cost reduces both consumer surplus and total surplus. This also makes a sharp contrast with the previous results mentioned above.

A cost difference between these firms can appear in various contexts. One important example is international competition under import tariffs. Another example is in the context of vertical foreclosure, where a vertically integrated firm faces a smaller input price than the independent downstream firms. Applying our principle to these contexts remains an interesting research agenda for future research.

Appendix

Proof of Lemma 1: Suppose that $d \leq t(1-x_1)(3-x_1)$. Differentiating π_2^N with respect to x_2, we have:

$$\frac{\partial \pi_2^N}{\partial x_2} = -\frac{(t(1-x_1-x_2)(3-x_1+x_2)-d)(t(1-x_1-x_2)(5-3x_1-x_2)+d)}{18t(1-x_1-x_2)^2} < 0.$$

Therefore, $x_2 = 0$ is optimal, that is, the optimal location of firm 2 is 1. Suppose that $d > t(1-x_1)(3-x_1)$. Any x_2 yields zero profits for firm 2, thus any location can be optimal for firm 2. Q.E.D.

Proof of Lemma 2: From Lemma 1 we have $x_2 = 0$. Given this, the profit of firm 1 is:

$$\pi_1 = \begin{cases} d - t(1-x_1)(1-x_1) & \text{if } d \geq t(1-x_1)(3-x_1), \\ \dfrac{(d+t(1-x_1)(3+x_1))^2}{18t(1-x_1)} & \text{if } d < t(1-x_1)(3-x_1). \end{cases}$$

When $d \geq t(1-x_1)(3-x_1)$, the (local) optimal location of firm 1 is $x_1 = 1/2$. The local optimum exists if and only if $d > 5t/4$. When $d < t(1-x_1)(3-x_1)$, the (local) optimal location of firm 1 is derived by the following first-order condition:

$$\frac{\partial \pi_1}{\partial x_1} = \frac{(d+t(3+x_1)(1-x_1))(d-t(1-x_1)(1+3x_1))}{18t(1-x_1)^2}.$$

The equation is negative for any $x_1 \in [0, 1/2]$ if and only if $d < t$; it is zero when $x_1 = (t - \sqrt{t(4t-3d)})/3t$ if and only if $t \leq d \leq 4t/3$; it is positive for any $x_1 \in [0, 1/2]$ if and only if $d > 4t/3$. When $5t/4 < d < 4t/3$, there are two local optimal locations $x_1 = 1/2$ and $x_1 = (t - \sqrt{t(4t-3d)})/3t$. For the two cases, the profit of firm 1 is:

$$\pi_1 = \begin{cases} \dfrac{8(3d+4t+2\sqrt{t(4t-3d)})^2}{243(2t+\sqrt{t(4t-3d)})} & \text{when } x_1 = \dfrac{t-\sqrt{t(4t-3d)}}{3t}, \\ d - \dfrac{t}{4} & \text{when } x_1 = \dfrac{1}{2}. \end{cases}$$

The former value is larger than the latter if and only if $d < (29\sqrt{145} - 187)t/128$. This implies Lemma 2. Q.E.D.

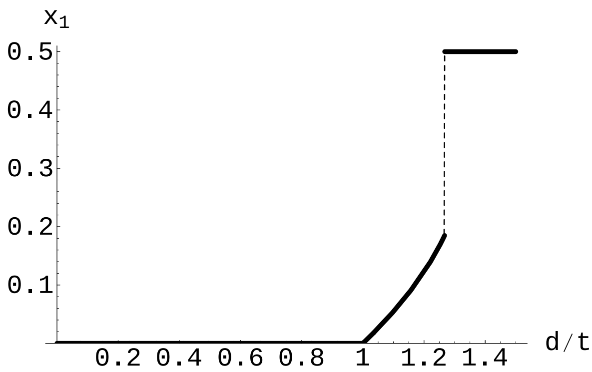


Figure 1: The optimal location of firm 1

Horizontal: d/t, Vertical: x_1

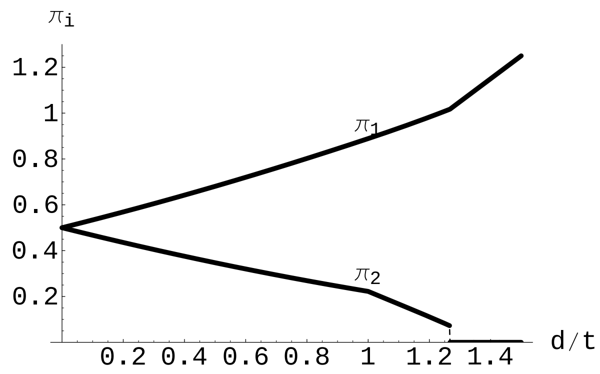
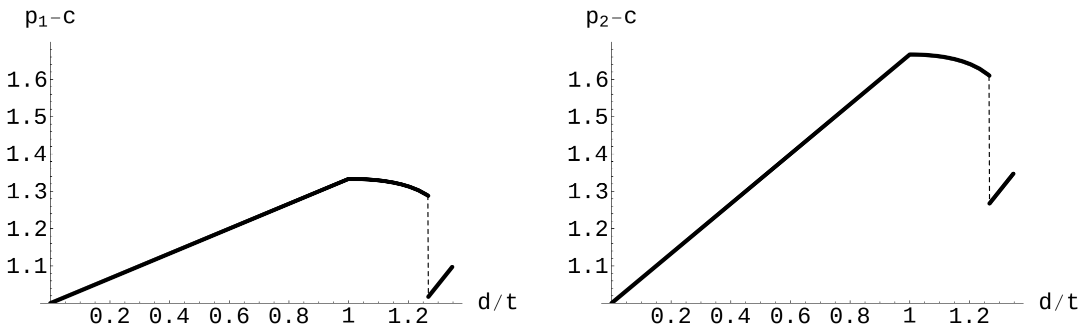


Figure 2: The profits of the firms

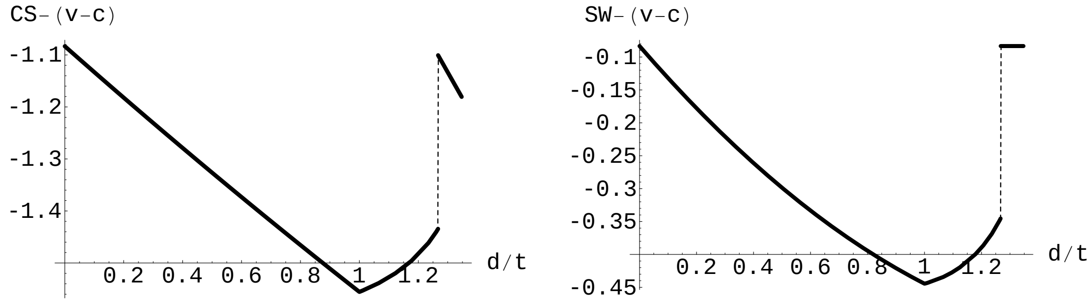
Horizontal: d/t, Vertical: π_i $(i = 1, 2)$



The price of firm 1 **The price of firm 2**

Figure 3: The equilibrium Prices

Horizontal: d/t, Vertical: $p_i^* - c$



Consumer surplus **Social welfare**

Figure 4: Consumer surplus and social welfare

Horizontal: d/t, Vertical: $CS^* - (v - c)$ and $SW^* - (v - c)$

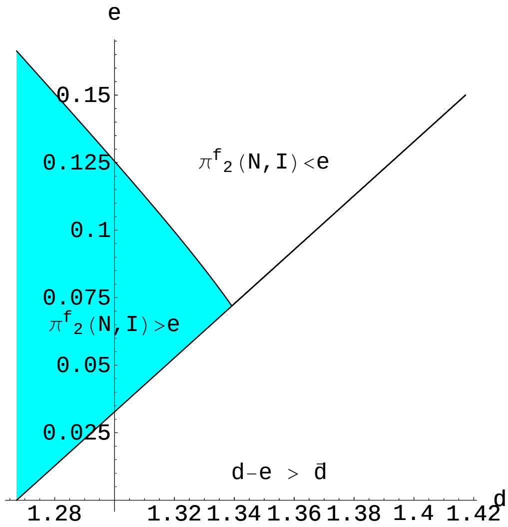


Figure 5: Parameter range within which $\pi_2^f(N, I) > e$ holds.

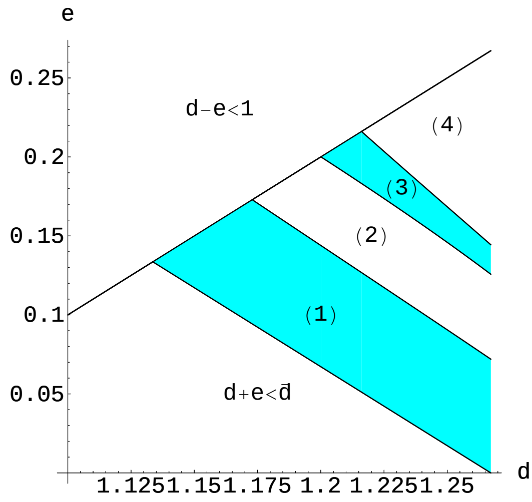


Figure 6: Parameter range within which Proposition 3 holds.

Note:

(1) and (2)	$\pi_1^s(N,N) - \pi_1^s(N,I) < \pi_2^s(N,I) - \pi_2^s(N,N) < \pi_2^s(N,N)$,
(3)	$\pi_1^s(N,N) - \pi_1^s(N,I) < \pi_2^s(N,N) < \pi_2^s(N,I) - \pi_2^s(N,N)$,
(4)	$\pi_2^s(N,N) < \pi_1^s(N,N) - \pi_1^s(N,I) < \pi_2^s(N,I) - \pi_2^s(N,N)$.

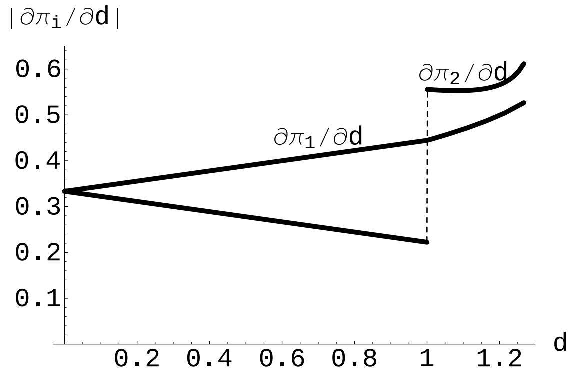


Figure 7: The marginal gain from a marginal cost reduction

Horizontal: d, Vertical: $|\partial\pi_i/\partial d|$ $(i = 1, 2)$.

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