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# RELATIVE PERFORMANCE AND R&D COMPETITION

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**Abstract**

This paper formulates a duopoly model in which firms care about relative profits as well as their own profits. Our purpose is to investigate the relationship between the weight of relative performance and R&D expenditure. We find a non-monotone relationship between the weight of relative performance in their objectives and their R&D levels. Both highly reciprocal (altruism) and negative reciprocal attitudes yield high levels of R&D, while the intermediate situations yield low levels of R&D.

**JEL classification numbers:** L21, O31, L13

**Key words:** relative profit, innovation, spite, altruism

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# 1 Introduction

One tends to care about the performance of other people as well as one's own performance. This concern may either stem from the available incentive schemes or from just one's intrinsic interest. For instance, evaluations of managers' performances are often based on their relative performance as well as their absolute performance (Murphy (1998)). Outperforming managers often obtain good positions in management job markets. In this case, these managers act in a way that allows them to meet the relevant incentive schemes. Moreover, a considerable amount of laboratory (experimental) research has pointed out spiteful behavior as well as reciprocal or altruistic behavior, which is closely related to the objective functions of agents based on relative performance (Brandts *et al.* (2004), Cason *et al.* (2002), and Coats and Neilson (2005)). These reciprocal and spiteful preferences often stem from genuine emotions or incentive schemes. Therefore, incorporating these preferences into a model is an important research topic.

The purpose of this paper is to investigate the relationship between the payoff functions of firms incorporating both positive and negative reciprocal preferences and the R&D expenditures of the firms. The outline of the model treated here is as follows. Firm $i$'s payoff is its relative profit $\pi_i - \alpha\pi_j$, where $\pi_i$ is its own profit, $\pi_j$ is the rival's profit, and $\alpha \in (-1, 1)$. The parameter $\alpha$ represents the degree of reciprocal preference. If $\alpha$ is positive, the firms envy the rivals' success. If $\alpha$ is negative, the firms have reciprocal (altruism) payoff functions.[1]

We find a non-monotone (U-shaped) relationship between the degree of reciprocal preference $\alpha$ and the innovation activities. Given a nonpositive $\alpha$, an increase in $\alpha$ reduces R&D. When $\alpha$ reaches a critical value (which is strictly larger than 0), the relationship is inverted. From the critical value of $\alpha$, an increase in $\alpha$ increases the levels of R&D. This result indicates that R&D activities are quite active in both highly cooperative ($\alpha$ is close to $-1$) and highly noncooperative

[1] The parameter $\alpha$ is closely related to the "coefficient of effective sympathy" used by Edgeworth (1881) and "coefficient of cooperation" used by Cyert and DeGroot (1973). For discussions on non-profit maximizing preferences in this context, see Bolton and Ockenfels (2000) and Fehr and Schmidt (1999).

($\alpha$ is close to 1) societies, while they are less active in the intermediate cases.[2]

Through the direct interpretation of $\alpha$, our result sheds some light on how reciprocal and negative reciprocal attitudes affect R&D. Given the pure selfish preferences of firms, an introduction of minor spiteful preferences reduces their innovative activities, but an introduction of significant spiteful preferences stimulates their innovation.[3]

We extend our basic analysis in two directions. First, we consider a joint R&D implementation where firms cooperatively choose their R&D levels and then compete in the product markets. In general, collusion in the product market is *per se* illegal, while it is possible for R&D cooperation to be allowed. Thus, this situation is worth discussing. It is shown that in this situation, an increase in $\alpha$ reduces R&D. Second, an oligopoly model is considered. It is shown that an increase in $\alpha$ is less likely to stimulate R&D when the number of firms is larger.

We add some comments on rationales for discussing objective functions on the basis of relative performance. First, relative performance, especially in a positive $\alpha$ case, is quite important from the viewpoint of evolutionary stability.[4] Second, owners of firms often have incentives for adopting a positive $\alpha$ in the context of strategic commitment games (Kockesen et al., 2000).

[2] Although the literature on strategic R&D competition is fairly abundant, most papers assume Cournot competition, where firms maximize their own profits. See, among others, Brander and Spencer (1983), Spence (1984), d'Aspremont and Jacquemin (1988), Suzumura (1992), Kamien et al. (1992), Matsumura (1995), and Lahiri and Ono (1999).

[3] The payoff functions that are based on relative wage or relative wealth status have also been intensively discussed in the macroeconomics context. Keynes (1936) discussed the rigidity of nominal wage based on relative wage. See also Akerlof and Yellen (1988), Corneo and Jeanne (1997, 1999), and Futagami and Shibata (1998). The relative performance approach is important in political science. Obviously, a party cares about the number of votes obtained not in absolute terms but in relative terms. In addition, in the context of international policies, the possibility that governments care about their relative performance as well as absolute performance is pointed out. See, among others, Grieco et al. (1993) and Mastanduno (1991).

[4] See Alchian (1950) and Vega-Redondo (1997). Vega-Redondo (1997) also shows that Cournot competition with relative performance objectives yields the Bertrand outcome even in duopoly, and this outcome is evolutionary stable.

Third, as mentioned in Symeonidis (2008), we can interpret $\alpha$ as a parameter indicating severity of competition.[5] $\alpha = 0$ indicates the standard Cournot case; $\alpha = 1$, the perfectly competitive case (related to the Bertrand case); and $\alpha = -1$, the monopoly case. Thus, a larger $\alpha$ indicates a more competitive market.[6] The relative performance approach enables us to treat competitiveness as a continuous variable, and this model contains three standard models—Cournot, Bertrand, and monopoly—as special cases. We believe that our formulation has sufficient importance for an investigation into innovations and that our relative profit approach is applicable to many other problems.

The rest of this paper is organized as follows. Section 2 formulates our basic model. In Section 3, we demonstrate the U-shaped relationship between $\alpha$ and R&D expenditures. Section 4 provides the analysis of joint R&D implementation. In Section 5, the basic model is extended to the case of oligopoly. We discuss welfare implications in Section 6. Section 7 concludes this paper. All proofs are relegated to the Appendix.

## 2 The Basic Model

We formulate a two-stage symmetric duopoly model. In the first stage, firm $i$ ($i = 1, 2$) chooses its R&D level $I_i$. At the beginning of the second stage, each firm observes the rival's R&D. In the second stage, firms produce perfectly substitutable commodities for which the market demand function is given by $p = a - Y$ (price as a function of quantity), where $Y$ is the total output of the firms. Let $y_i$ denote the output of firm $i$. Firm $i$'s marginal production cost $c_i$ depends on $I_i$. Each firm $i$ chooses $y_i$ independently.

The payoff of firm $i$ ($i = 1, 2$) is given by $U_i = \pi_i - \alpha\pi_j$ ($i \neq j$), where $\pi_i$ is the profit of firm

[5] See also Shubik (1980), Brod and Shivakumar (1999), and Symeonidis (2000).

[6] Under the standard conditions in Cournot, the ratio of the profit margin (the price minus the marginal cost) and the price, called the Lerner index, is decreasing in $\alpha$. This index is intensively used in the empirical literature as a measure of competitiveness in product markets.

$i$ and $\alpha \in (-1, 1)$. The parameter $\alpha$ indicates the importance of relative performance for firm $i$'s management. The firm $i$'s profit $\pi_i$ is given by $\pi_i = (a - Y)y_i - c_i(I_i)y_i - I_i$. It is assumed that $c_i' \leq 0$ and $c_i''$ is positive and sufficiently large so as to satisfy the second-order condition at the first stage. We also assume that $\lim_{I \to 0} c'(I) = -\infty$ and $\lim_{I \to \infty} c'(I) = 0$ so as to ensure the interior solution at the first stage.

## 3 Equilibrium Analysis and the U-shaped Relationship

In this section, we study a situation where two firms maximizing relative profits compete in the market. The game is solved by backward induction. In the second stage competition, given the investments $I_i$ of two firms, each firm independently chooses its output to maximize the relative profit $U_i$. The first-order condition is as follows:

$$a - 2y_i - (1 - \alpha)y_j = c_i \quad (i = 1, 2, i \neq j). \tag{1}$$

Obviously, the second-order condition is satisfied. Arranging this equation, we obtain the following reaction function:

$$y_i = R_i(y_j : \alpha) = \frac{a - c_i - (1 - \alpha)y_j}{2}. \tag{2}$$

By solving the first-order condition, the second stage equilibrium outputs are obtained:

$$y_1^E = \frac{(1 + \alpha)a - 2c_1 + (1 - \alpha)c_2}{(1 + \alpha)(3 - \alpha)}, \quad y_2^E = \frac{(1 + \alpha)a - 2c_2 + (1 - \alpha)c_1}{(1 + \alpha)(3 - \alpha)}. \tag{3}$$

The resulting profit of firm $i$ is given by

$$\pi_i^E(c_i, c_j) = \frac{[(1 - \alpha)a - (2 - \alpha)c_i + c_j][(1 + \alpha)a - 2c_i + (1 - \alpha)c_j]}{(3 - \alpha)^2(1 + \alpha)} - I_i. \tag{4}$$

Next, we consider the first stage R&D competition. In this stage, each firm $i$ independently chooses $I_i$ so as to maximize $U_i = \pi_i^E - \alpha\pi_j^E$. We restrict our attention to the symmetric equilibrium because the sufficiently large $c_i''$ guarantees that the unique equilibrium is symmetric. The first-order condition is

$$-\frac{[(1+\alpha)a - 2c_i + (1-\alpha)c_j][4 - 3\alpha + \alpha^2]}{(3-\alpha)^2(1+\alpha)} c_i' = 1. \tag{5}$$

The second-order condition is satisfied. On substituting $c_1 = c_2 = c$ into (5), we have that $-G(\alpha)c' = 1$ must be satisfied, where

$$G(\alpha) := \frac{(a-c)(4 - 3\alpha + \alpha^2)}{(3-\alpha)^2}.$$

Let $I^E$ denote the equilibrium R&D investment level. A larger $G$ and a larger $|c'|$ imply a higher marginal benefit of R&D. Since the payoff function is assumed to be concave with respect to $I_i$ ($c''$ is large enough), a larger $G$ (as well as larger $|c'|$) yields a higher level of the equilibrium R&D investment.

We discuss how $\alpha$ affects the equilibrium R&D level. We find a non-monotone relationship between the equilibrium level of R&D ($I^E$) and the weight of relative performance ($\alpha$).

**Proposition 1** *Suppose that two firms compete in a product market, and they make their R&D investments independently. Then, the equilibrium R&D investment level $I^E$ is decreasing in $\alpha$ for $\alpha < 1/3$ and is increasing in $\alpha$ for $\alpha > 1/3$.*

We explain the intuition behind Proposition 1. Since $y_i^E$ is increasing in $\alpha$, the cost-minimizing level of R&D, which is derived by minimizing $c_i y_i^E + I_i$, is increasing in $\alpha$. Thus, for the purpose of cost minimization (minimization of production cost plus R&D cost), firm $i$ has a stronger incentive for R&D when $\alpha$ is larger. On the other hand, firm $i$ has a weaker incentive for innovation for strategic purposes.[7] From (2), we have that $|R_i'|$ is decreasing in $\alpha$. This implies that the strategic value of R&D is decreasing in $\alpha$. The former dominates when

[7] From (3) we see that an increase in $I_i$ (a decrease in $c_i$) decreases $y_j^E$, and it results in an increase in $\pi_i$ ($i, j \in \{1, 2\},\ i \neq j$). This is the strategic value of R&D. For this strategic effect of R&D, see Brander and Spencer (1983).

$\alpha$ is large, while the latter dominates when $\alpha$ is small. This yields the U-shaped relationship between $\alpha$ and the equilibrium R&D level.

The implications of Proposition 1 are as follows. Both aggressive competition (the case where $\alpha$ is close to 1) and a collusive situation (the case where $\alpha$ is close to $-1$) yield high levels of R&D.[8] Our result is related to two influential views on the relationship between competitiveness and R&D.[9] One view is the monopoly view. The monopoly yields intensive R&D investments. The other is the competition view. Severe competition accelerates innovation. Researchers have presented many theoretical foundations and empirical (or anecdotal) evidences supporting both views. Using a single model, we explain that both views can be accurate.

In our setting, R&D is minimized when $\alpha = 1/3$. Starting Cournot competition ($\alpha = 0$), a slight increase in $\alpha$ decreases R&D investments, while a large increase in $\alpha$ increases them. Thus, an envy society (positive $\alpha$) in which people care about their relative performances as well as their absolute performances can yield either more or less aggressive R&D activities.

# 4 Joint R&D Implementation

The question is whether or not our result depends on the assumption about the formation of R&D activities. Then, we consider the case in which two firms[10] cooperatively determine their investment level $I$ to maximize their joint profits, while they noncooperatively compete in the

[8] For the relationship between $\alpha$ and market competition, see the rationale in the second last paragraph of the introduction.

[9] Economists have long been interested in the relationship between product market competition and innovation. See Aghion et al. (2005). They show an inverse U-shaped relationship between toughness of competition and the equilibrium level of R&D. Traditionally, many researchers believe that monopoly yields intensive R&D (monopoly view), while many others believe that competition yields intensive R&D (competition view). Both have presented many theoretical foundations and empirical (or anecdotal) evidences supporting their views. See, for example, Schumpeter (1950) and Arrow (1962). See also Mateus and Moreira (2007), Cabral (2000), and the works cited in these books.

[10] We can show that Proposition 2 holds true in $n$-firm oligopoly too.

product market.[11] Some points of the basic model are modified. Each firm has a common marginal cost $d(I)$ that depends on the joint R&D investment $I$. We assume that each firm pays half of the joint R&D cost $I$. Thus, firm $i$'s profit is given by

$$\Pi_i = (a - Y)y_i - d(I)y_i - \frac{I}{2}.$$

It is assumed that $d'(I) < 0$ and $d''(I)$ is positive and sufficiently large. We also assume that $\lim_{I \to 0} d'(I) = -\infty$ and $\lim_{I \to \infty} d'(I) = 0$ so as to ensure the interior solution at the first stage.

We consider a two-stage game. In the first stage, the firms choose their R&D level cooperatively. In the second stage, they noncooperatively produce perfectly substitutable commodities.

We now solve the game. The second stage competition has already been discussed in the previous section: all that needs to be done now is replace $c(I)$ with $d(I)$. In the first stage, the firms choose the investment level $I$. The first-order condition is as follows:

$$-\frac{2(a-d)(1-\alpha)}{(3-\alpha)^2} d'(I) = 1. \tag{6}$$

The left-hand side in (6) is the marginal benefit of their joint R&D investment and the right-hand side in (6) is the marginal cost of the investment. Let $I^C$ denote the equilibrium R&D investment level. Define $H(\alpha) := 2(a-d)(1-\alpha)/(3-\alpha)^2$. Because of the concavity of the payoff function, which is guaranteed by the assumption of sufficiently large $d''$, a larger $H$ implies a higher level of investment. We investigate how $\alpha$ affects the equilibrium R&D level. In contrast to the case of noncooperative investment, we find a monotone relationship between the R&D level and the weight of relative performance $\alpha$.

**Proposition 2** *Suppose that two firms compete in a product market, and they make their R&D investments cooperatively. Then, the equilibrium R&D investment level $I^C$ is decreasing in $\alpha$ for $-1 < \alpha < 1$.*

[11] Explicit collusion in product markets is usually illegal, while cooperation at the R&D stage is often permitted by anti-monopoly legislations.

We explain the intuition behind Proposition 2. A decrease in $d$ (the common cost of two firms) lowers the price. Since $dp/dd = 2/(3-\alpha)$, $dp/dd$ is increasing in $\alpha$. In other words, when $\alpha$ is large, a cost reduction by R&D reduces the equilibrium price substantially; thus, firms lose incentives for R&D. This yields a smaller R&D level as $\alpha$ is larger.

Proposition 2 suggests that under joint implementation of R&D, spiteful preference can be obstacles to innovations.

# 5 Oligopoly

In this section, we move back to the model with noncooperative investment and discuss an oligopoly version. Regarding the payoff function based on relative performance, there are several different formulations: (a) each firm cares about the average profits of the rivals (the other firms); (b) each firm cares about the highest profit firms among the rivals; and (c) each firm has one specific rival as a benchmark firm and cares about its profit only. Since all formulations yield exactly the same results in our model, we adopt the first formulation.

There are $n(\geq 3)$ symmetric firms and they engage in the two-stage game formulated in Section 2. According to formulation (a), the payoff of firm $i$ $(i = 1, 2, ...n)$ is given by:

$$U_i = \pi_1 - \frac{\alpha}{n-1} \sum_{i \neq i} \pi_j.$$

Let $\beta := \alpha/(n-1)$. If we interpret $\beta$ as an indicator of the degree of envy or altruism, it is natural to assume that $\beta \in (-1, 1)$. On the other hand, if we interpret $\beta$ as an indicator of the degree of competition, it is natural to assume that $\beta \in (-1, 1/(n-1))$, because the case where $\beta = 1/(n-1)$ corresponds to the perfect competition.

Consider the second stage competition (quantity competition). The first-order condition of each firm is as follows:

$$a - 2y_i - (1-\beta) \sum_{j \neq i} y_j = c_i. \tag{7}$$

The reaction function of firm $i$ is as follows:

$$y_i = R_i(Y_{-i} : \beta) = \frac{a - c_i - (1-\beta)Y_{-i}}{2}, \tag{8}$$

where $Y_{-i} = \sum_{j \neq i} y_j$. Summing each side in (7) for all firms, we have

$$na - 2Y - (1-\beta)(n-1)Y = \sum_i c_i. \tag{9}$$

Rearranging (9), the second stage total output is obtained:

$$Y = \frac{na - \sum_i c_i}{2 + B}, \tag{10}$$

where

$$B := (1-\beta)(n-1).$$

Note that $Y$ is increasing in $\beta$. From (7) and (10), we have the second stage output of each firm:

$$y_i^E = \frac{(1+\beta)a - (1+\beta+B)c_i + (1-\beta)\sum_{j \neq i} c_j}{(1+\beta)(2+B)}.$$

The equilibrium profit of firm $i$ given the first stage actions of firms is denoted by

$$\pi_i^E = \frac{\left\{a[1-(n-1)\beta] - (1+B)c_i + \sum_{j \neq i} c_j\right\}\left[(1+\beta)a - (1+\beta+B)c_i + (1-\beta)\sum_{j \neq i} c_j\right]}{(1+\beta)(2+B)^2} - I_i.$$

Consider the first stage competition. Each firm $i$ maximizes $U_i = \pi_i^E - \beta \sum_{j \neq i} \pi_j^E$ with respect to $I_i$. We again restrict our attention to the symmetric equilibrium. Differentiating $U_i$ with $I_i$, and then substituting $c_j = c$ for all firms we obtain

$$-J(\beta, n)c' = 1,$$

where

$$J(\beta, n) := \frac{(a-c)(\beta + \beta^2 + 2n - 2\beta^2 n - \beta n^2 + \beta^2 n^2)}{(2+B)^2}.$$

Then, we have the following result.

**Proposition 3** *Suppose that $n(\geq 3)$ firms compete in a market, and they make their R&D investments independently. Then, the equilibrium R&D investment level $I^E$ is decreasing in $\beta$ for $\beta < \bar{\beta}$ and is increasing in $\beta$ for $\beta > \bar{\beta}$, where $\bar{\beta} := (n-1)/(n+1)$.*

The threshold value of $\beta$ in Proposition 3 is increasing in $n$. This indicates that an increase in $\beta$ (and so $\alpha$) is more likely to decrease R&D investments when $n$ is large. As we discussed in Section 3, an increase in $\alpha$ has two countervailing effects. On the one hand, an increase in $\alpha$ increases the equilibrium output of each firm and stimulates R&D. On the other hand, an increase in $\alpha$ reduces $|R_i'|$ and so reduces the strategic value of R&D. The former effect becomes weaker when $n$ is large, and thus, the latter effect more likely dominates the former when $n$ is large.

## 6 Welfare Analysis

In this section, we examine welfare implications in duopoly with non-cooperative investment. The equilibrium investment level is compared to the second-best investment level. Social welfare $W$ is the consumer surplus plus profit of firms:

$$W := \int_0^Y p(y)dy - p(Y)Y + \pi_1 + \pi_2.$$

Suppose that the social planner cannot control the competition in the second stage and can control only $I$, the R&D investment level of each firm (the second-best problem). This problem is intensively discussed in the literature of R&D competition. Let $I^*$ denote the second-best R&D level.[12] We compare $I^*$ with $I^E$, the equilibrium R&D investment level.

The first-order condition for the social planner is given by

$$\frac{dW^E}{dI_1} = -\Big[\frac{4a + 2a\alpha - 2a\alpha^2 - 11c_1 + 5\alpha c_1 + 7c_2 - 7\alpha c_2 + 2\alpha^2 c_2}{(3-\alpha)^2(1+\alpha)}\Big]c_1' - 1 = 0. \qquad (11)$$

[12] We implicitly assume that the second-best outcome is symmetric. This holds true for sufficiently large $c_i''$.

Since $c_i''$ is positive and sufficiently large, the second-order condition is satisfied. Substituting $c_1 = c_2 = c$ into (11), we have that $-K(\alpha)c' = 1$ must be satisfied, where

$$K(\alpha) := \frac{2(a-c)(2-\alpha)}{(3-\alpha)^2}.$$

A larger $K$ implies a higher level of $I^*$, and $I^* > (<) I^E$ if and only if $K(\alpha) > (<) G(\alpha)$ (the definition of $G(\alpha)$ is given in Section 3).

We now state welfare implications.

**Proposition 4** *Suppose that two firms compete in a market, and they make their R&D investments independently. Then,*
*(i) $I^*$ is increasing in $\alpha$ for $-1 < \alpha < 1$.*
*(ii) $I^* < I^E$ for $-1 < \alpha < 0$, and $I^* > I^E$ for $0 < \alpha < 1$.*

Proposition 4(i) states that the second-best R&D level is increasing in the reciprocal parameter $\alpha$. This result is very intuitive. The larger the output each firm yields, the greater is the social benefit of R&D; further a larger $\alpha$ yields the larger output.

Proposition 4(ii) states that when $\alpha$ is negative (the market is collusive), the equilibrium R&D is excessive, while a positive $\alpha$ yields insufficient R&D. We explain the intuition. When $\alpha$ is negative, the output level is low. Thus, the social gain of R&D is small, while firms make relatively large investments in R&D (see Proposition 1). This yields excessive investment. The social optimal level of R&D is increasing in $\alpha$ and its equilibrium level is decreasing in $\alpha$ for $\alpha \leq 1/3$, $I^E - I^*$ is decreasing in $\alpha$; it happens to be zero when $\alpha = 0$. An increase in $\alpha$ from zero yields the insufficient investment ($I^E < I^*$). See Figure 1.

[Figure 1 AROUND HERE]

# 7 Concluding Remarks

This paper develops a relative performance approach in the context of R&D competition; we investigate the two-stage R&D game in oligopoly markets where firms' payoffs depend on both absolute and relative profits. Firm $i$'s payoff is $\pi_i - \alpha\pi_j$, where $\pi_i$ is its own profit, $\pi_j$ is the rival's profit, and $\alpha \in (-1, 1)$. We find the U-shaped relationship between $\alpha$ and the levels of R&D investments when the firms choose the levels of R&D investments independently. This result indicates that innovation is quite active in both altruistic and spiteful preferences, while it is less active in the intermediate situations. We also show that the second-best R&D level is increasing in $\alpha$, and it is more likely to exceed the equilibrium R&D level when $\alpha$ is large.

The relative performance approach smoothly connects industrial organizations and preferences of firms. Our approach contains Bertrand and Cournot competition as special cases, and a larger $\alpha$ indicates tougher competition in product markets. Therefore, this approach connects the standard Cournot situation with the standard Bertrand situation in a peculiar way. Moreover, it has the potential to reveal important properties between toughness of competition and market performance that have thus far been overlooked. To investigate a general property of the relationship between market competition and performance remains an issue for future research.

## Appendix: Proofs

**Proof of Proposition 1:** Differentiating $G(\alpha)$ with respect to $\alpha$, we have

$$\frac{dG(\alpha)}{d\alpha} = \frac{(a-c)(3\alpha-1)}{(3-\alpha)^3}. \tag{12}$$

(12) is negative for $\alpha < 1/3$ and positive for $\alpha > 1/3$. This implies that given $c_1 = c_2 = c$, the marginal benefit of R&D is decreasing (increasing) in $\alpha$ for $\alpha <\ (>)\ 1/3$. This yields Proposition 1. **Q.E.D**.

**Proof of Proposition 2:** Differentiating $H(\alpha)$ with respect to $\alpha$, we have

$$\frac{dH(\alpha)}{d\alpha} = -\frac{2(a-d)(1-\alpha)}{(3-\alpha)^3}. \tag{13}$$

(13) is negative for $-1 < \alpha < 1$. **Q.E.D.**

**Proof of Proposition 3:** We prove this proposition along the same lines as Proposition 1. Differentiating $J(\beta, n)$ with respect to $\beta$, we have

$$\frac{\partial J(\beta,n)}{\partial \beta} = \frac{(a-c)(n-1)^2(1+\beta-n+\beta n)}{[2+(1-\beta)(n-1)]^3}. \tag{14}$$

Note that $(a-c)$ and $2+(1-\beta)(n-1)$ is always positive. Hence, (14) is negative for $1+\beta-n+\beta n < 0$ and positive for $1+\beta-n+\beta n > 0$. A simple calculation leads us to

$$\frac{\partial J(\beta,n)}{\partial \beta} < 0 \text{ for } \beta < \frac{n-1}{n+1} \text{ and } \frac{\partial J(\beta,n)}{\partial \beta} > 0 \text{ for } \beta > \frac{n-1}{n+1}.$$

This implies that given $c_j = c$ for all $j$, the marginal benefit of R&D is decreasing (increasing) in $\beta$ for $\beta <\ (>)\ (n-1)/(n+1)$. This yields Proposition 3. **Q.E.D**.

**Proof of Proposition 4:** (i) Differentiating $K(\alpha)$ with respect to $\beta$, we have

$$\frac{dK(\alpha)}{d\alpha} = \frac{2(1-\alpha)(a-c)}{(3-\alpha)^3}.$$

This implies that $dK/d\alpha > 0$ for all $\alpha \in (-1, 1)$. Thus, we have Proposition 4(i).

(ii) Note that $I^* > (<) I^E$ if and only if $K(\alpha) > (<) G(\alpha)$. The simple calculation leads us to

$$K(\alpha) - G(\alpha) = \frac{(a-c)(1-\alpha)\alpha}{(3-\alpha)^2}.$$

This implies Proposition 4(ii). **Q.E.D**.

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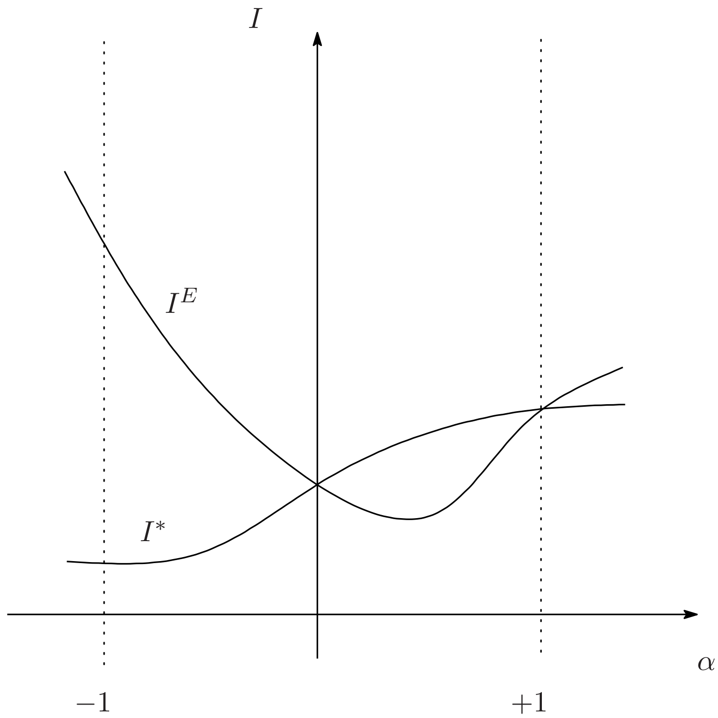

Figure 1: Welfare Implication ($n = 2$)