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OPTIMAL EMISSION TAX WITH ENDOGENOUS LOCATION CHOICE OF DUOPOLISTIC FIRMS

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Optimal emission tax with endogenous location choice of duopolistic firms*

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Abstract

This paper explores optimal environmental tax policy under which duopoly firms strategically choose the location of their plants in a simple three-stage game. We examine how the relationship between the optimal emission tax and the choice of location of duopoly firms affects the welfare of the home country. We characterize the relationship between the optimal emission tax and the fixed cost, depending on the degree of environmental damage from production. Finally, we show the existence of asymmetric equilibrium in which either firm chooses relocation of its plant even if the duopoly firms are identical ex ante.

keywords: Environmental policy, Relocation, Welfare

JEL classification: H 23; L 13;

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1 Introduction

The environmental policy in a game theoretic model has been receiving much attention in environmental economics. In order to reduce local pollution, the government imposes a tax on emissions, which may influence the location strategies of polluting firms. This paper describes the interaction of the government and duopoly firms and examines the optimal environmental policy when the location choice of the firms is endogenous in a three-stage game.

When the environmental policy is stringent, the domestic firms have an incentive to relocate their plants abroad to reduce the marginal production cost. The firms face a trade-off between relocation costs and tax payments. The improved environment increases welfare of the country, while at the same time the reduction of tax revenue and profits of domestic firms decreases welfare. Therefore, the government sets the environmental policy considering the location decisions of the domestic firms.

The optimal tax of policy makers who face mobile polluting firms has been extensively studied in the literature on international trade, tax competition among governments, and relocation decisions of firms. Among others, Markusen *et al.* (1993) analyse decisions of firms on the number and location of their plants in response to environmental policies which is given exogenously. Markusen *et al.* (1995) examine tax competition in a two-region model. To determine region's optimal tax rate, they assume that plant locations are exogenous. Petrakis and Xepapadeas (2003) analyze relocation decisions of a monopolist under an emission tax and compare welfare under two policy regimes in which the government can commit to its policy and it cannot in a three-stage model. They show that welfare under government commitment to an-ex ante emission tax is higher because the government can affect the monopolist's location decision.

Looking at the real world, many firms face the same environmental policy in one country, but nevertheless some firms shift their plants abroad and the others remain in home country. A model of monopolist cannot explain a phenomenon in which ex-ante homogeneous firms choose different decision *ex post*.[1] Our objective is to explain this asymmetric outcome found in the real world.

The study of an asymmetric outcome, in which firms have different choices under the same environmental policy even though they have identical, is sparse. Some papers have studied the hybrid outcomes in a framework of industrial economics and organization and international trade theory. Mills

[1]Katsoulacos and Xepapadeas (1995) analyze emission taxes under both fixed-number oligopoly and endogenous market structure, but not location decisions of firms.

and Smith (1996) establish that asymmetric equilibria can exist in a duopoly with Cournot-Nash game in which firms choose technologies. In the context of international trade, Yomogida (2007) models a duopoly setting in which firms strategically choose whether to export or to undertake foreign direct investments and shows the emergence of hybrid equilibrium even if they have identical cost structure *ex ante*. These studies, however, assume that the parameter determining technological choices and tariff rate are given exogenously.

By introducing endogenous determination of tariff rate, Ohkawa *et al.* (2009) examine the relationship between the optimal tariff policy of the host country and the strategic location choice of foreign duopoly firms. Based on their model, we assume that the government set its emission tax level taking into account the strategical behaviour of duopoly firms and commits to the policy, which gives firms endogenous choice of whether to remain in domestic country and pay for emission tax or to relocate the plant to foreign country and pay for fixed relocation cost.

In this paper we examine how the domestic government determines the optimal environmental tax when the government can strategically set its emission tax rate considering location decisions of duopoly firms in a simple three-stage game. Another question we address is whether asymmetric equilibrium emerges. Also we examine how the relationship between the optimal emission tax and location choice of duopoly firms affects welfare of home country. Moreover, we describe the relationship between the optimal emission tax and the fixed cost that is one of determinants of relocation of production.

We derive that the optimal emission tax and welfare resulted from location choices of firms depend on the degree of damage from pollution so that there are various pattern of equilibrium set. When the degree of damage is sufficiently small, the optimal tax is zero irrespective of the relocation cost. Subsidies can be optimal without environmental damage from production. As the degree of damage from pollution increases, asymmetric equilibrium emerges. That is, the location pattern appears, in which one firm relocates its plant to foreign country while another remains in home country. When the environmental damage is large enough, the government attempts to drive both firms out by setting relative higher emission tax.

Since welfare of domestic country depends on consumer surplus, profits of domestic firms, tax revenues and environmental damage, the level of welfare varies corresponding to the relocation patterns. When both firms remain in domestic country, the government enjoys profits and tax revenue from two firms as well as consumer surplus, while it suffers from environmental damage. When either firm relocates its production abroad, benefits from tax revenue and profits decrease while benefits from reduction in damage and increases in consumer surplus because of price reduction. Therefore, the size of these opposite force on benefits affects the optimal emission tax, which results in various relocation patterns including asymmetric equilibrium.

The remainder of the paper is structured as follows. Section 2 presents the basic model and characterizes each stage to derive equilibrium. Section 3 derives the optimal environmental tax and the relationship between resulted welfare and the fixed cost in various degrees of environmental damage. Section 4 concludes.

2 The Model

Consider two countries, country H and country F. We assume two identical firms produce and sell a homogeneous good in the market of country H. The production cost function is assumed to be the same in both countries and given by $C(q_i) = cq_i$, where c is parameter and q_i is output of firm $i = 1, 2$. The firms face a demand, $P(q) = a - q$, where a denotes the market size and q is the demand for the product. Pollution is produced per unit of output in its production process. The damage function is given by $D(x) = dx^2/2$, where d is a damage coefficient and x is the total emission in which firms locate.

Both firms are subject to the environmental policy in country H. Since we focus on the optimal environmental tax policy under which duopoly firms choose the location of their plants, without loss of generality, we assume that there is no active environmental policy in country F. Following Petrakis and Xepapadeas (2003), we assume that both firms can relocate their production plants from country H to country F with the cost of relocation, f, and that the firm exports its product without transportation cost and sells in country H after the relocation. Its profits are assumed to remain in country F.

We consider a three-stage game by firms 1 and 2, and the government of country H. In the first stage, the government sets an emission tax rate so as to maximize the welfare of country H. We assume that the government can commit to an emission tax *ex ante*. Given the tax rate, in the second stage, duopoly firms choose the location of their plants. In the third stage, firms compete *á la* Cournot in the market, given an emission tax rate and their location choice. We derive a Nash equilibrium of this three-stage game by using backward induction.

2.1 Output decision stage

In the last stage, given an emission tax set by the government and the location choices of each firm in the previous stages, each firm chooses its output to maximize profits. First, assume that both firms remain in country H. Firm i's profit maximization problem is:

$$\max_{q_i} \quad (a-q)q_i - cq_i - tq_i, \quad i = 1, 2,$$

where t denotes the tax per unit of emissions. From the first-order condition and the symmetry assumption, the optimal output of the duopoly firms are given by:

$$q_1 = q_2 = \frac{1}{3}(A-t), \tag{1}$$

where $A \equiv a - c > 0$ represents the scale of market. The aggregate output is $q = 2(A-t)/3$, so that the equilibrium profit of firm $i = 1, 2$ is:

$$\pi^i_{HH} = \frac{1}{9}(A-t)^2 \equiv \pi_{HH}. \tag{2}$$

If both firms relocate their production plants to country F and supply their products in the market of country H, firm i's profit maximization problem is given by:

$$\max_{q_i} \quad (a-q)q_i - cq_i - f, \quad i = 1, 2.$$

Similarly, the optimal outputs of the duopoly firms are given by:

$$q_1 = q_2 = \frac{1}{3}A. \tag{3}$$

The aggregate output is $q = 2A/3$ and the equilibrium profit of firm $i = 1, 2$ is:

$$\pi^i_{FF} = \frac{1}{9}A^2 - f \equiv \pi_{FF}. \tag{4}$$

Finally, if firm 1 remains in country H while firm 2 has decided to relocate its plant in country F, the first-order conditions of each firm are:

$$A - t - (q_1 + q_2) - q_1 = 0,$$
$$A - (q_1 + q_2) - q_2 = 0.$$

Solving the above two equations for q_1 and q_2 yields the optimal outputs of the respective firm:

$$q_1 = \frac{1}{3}(A - 2t) \quad \text{and} \quad q_2 = \frac{1}{3}(A+t). \tag{5}$$

In this case, the aggregate output is $q = (2A - t)/3$ and the equilibrium profits of the respective firms are given by:

$$\pi^1_{HF} = \frac{1}{9}(A - 2t)^2 \equiv \pi_{HF} \quad \text{and} \quad \pi^2_{HF} = \frac{1}{9}(A + t)^2 - f \equiv \pi_{FH}. \qquad (6)$$

We assume that $0 \leq t < A/2$ and $f < A^2/9$ in order to ensure that both firms can coexist in the market.

2.2 Location choice stage

In this stage, the duopoly firms make the relocation decisions given an emission tax rate in country H and its rival's strategies about the location of production: strategy H (remain in country H) and strategy F (relocate in country F). From (2), (4), and (6), we obtain the payoff matrix depicted in Table 1.

firm 1 \ firm 2	H	F
H	$\pi_{HH}, \quad \pi_{HH}$	$\pi_{FH}, \quad \pi_{HF}$
F	$\pi_{HF}, \quad \pi_{FH}$	$\pi_{FF}, \quad \pi_{FF}$

Table 1: The payoff matrix

There are three Nash equilibria: HH, FF, HF (or FH). Let us compare payoffs to derive the conditions for Nash Equilibrium of each case. Given firm 2's strategy H, the condition under which firm 1 has no incentive to relocate its plant to country F is:

$$\pi_{HH} \geq \pi_{FH} \quad \Leftrightarrow \quad f \geq \frac{4}{9}At.$$

Given firm 2's strategy F, the condition under which firm 1 has no incentive to remain in country H is:

$$\pi_{FF} \geq \pi_{HF} \quad \Leftrightarrow \quad f \leq \frac{4}{9}(A - t)t.$$

Given firm 2's strategy F, the condition under which firm 1 decides to remain in country H is:

$$\pi_{HF} \geq \pi_{FF} \quad \Leftrightarrow \quad f \geq \frac{4}{9}(A - t)t.$$

Finally, when firm 2 chooses strategy H, the condition under which firm 1 decides to relocate its plant to country F is:

$$\pi_{FH} \geq \pi_{HH} \quad \Leftrightarrow \quad f \leq \frac{4}{9}At.$$

Taken together, we can identify the following three Nash equilibria depending on the size of f:

Lemma 1 *(i) If $f < (4/9)(A-t)t$, then both firms relocate their plants to country F. (ii) If $(4/9)(A-t)t \leq f < (4/9)At$, then one firm remains in country H, while another relocates to country F. (iii) If $(4/9)At \leq f$, then both firms remain in country H.*

Figure 1 shows the possibilities of three Nash equilibria. Define $F_{HH}(t) \equiv 4At/9$ and $F_{HF}(t) \equiv 4(A-t)t/9$. Hence, the curve $t_{HH} \equiv F_{HH}(t)^{-1}$ denotes the maximum tax rate at which both firms remain in country H and the curve $t_{HF} \equiv F_{HF}(t)^{-1}$ denotes the maximum tax rate at which either firm relocates to country F. When an environmental tax is low and the fixed cost is high, both firm remain in country H, while when an environmental tax is high and the fixed cost is low, both firm relocate their plants to country F. Since an emission tax increases the marginal cost of production, it is profitable for a firm with low fixed cost to relocate in country F. On the other hand, since the cost of relocation becomes sunk, it is profitable for a firm with high relocation cost to remain in country H if an emission tax is low. When both an emission tax and the fixed cost are given at the medium levels, either of the duopoly firms relocates in country F. In this case, if one firm relocates abroad, relocation of the rival firm reduces the market price, which in turn reduces the revenue. Hence, the rival firm decides to remain in country H. In contrast, if one firm chooses production in country H and decreases its output due to the tax burden, the rival firm can increase its output, and thus, the revenue resulting from relocation of its production in country F. In what follows, when a firm is indifferent between two locations (i.e. the home country and the foreign country), we assume that this firm chooses a location that gives a higher welfare for the home country.

2.3 Environmental policy decision stage

The government of country H chooses an emission tax rate t so as to maximize its economic welfare. However, the welfare is determined by the location choices of the duopoly firms, which depends on the environmental policy set by the government. To derive the optimal emission tax rate, firstly, let us characterize the welfare of home country associated with Nash equilibrium in location choices. The welfare of home country when both firms remain in country H is defined as:

$$W_{HH} = CS_{HH} + \pi^1_{HH} + \pi^2_{HH} - D(q) + tq,$$

where $CS_{HH} \equiv \int_0^q p(s)ds - p(q)q$ is consumer surplus in Nash equilibrium HH. The welfare when one of firms remains in country H and another relocates in country F is given by:

$$W_{HF} = CS_{HF} + \pi^i_{HF} - D(q_i) + tq_i, \quad i = 1, 2.$$

Note that profit and emission tax of only one firm that remains in country H and damage cost from emissions of its production in country H are included. The welfare in case that both firms relocate to country F is:

$$W_{FF} = CS_{FF}.$$

Since the profits of relocated firms remain abroad, the welfare function does not include the fixed cost f.

Substituting (1)–(6) into the definitions of welfare above, we obtain three reduced forms for the welfare in each equilibrium:

$$W_{HH}(t;d) = \frac{2}{9}\Big(-(1+d)t^2 - (1-2d)At + (2-d)A^2\Big), \qquad (7)$$

$$W_{HF}(t;d) = \frac{1}{18}\Big((6-d)A^2 - (6-4d)At - (3+4d)t^2\Big), \qquad (8)$$

$$W_{FF}(t;d) = \frac{2}{9}A^2. \qquad (9)$$

The welfare functions (7) and (8) are a concave function of t given d, whereas (9) is constant. The intuition of the figures of welfare functions (7) and (8) is as follows. Both welfare functions consist of four components: consumer surplus, profit(s) of domestic firm(s), tax revenue, and damage cost, three of which are a function of t and damage cost is indirectly affected by the emission tax through a change in domestic production. On one hand, given d, an increase in t reduces consumer surplus and domestic production, thus, damage cost and the gross profits monotonically and its reduction rate is accelerated as t becomes higher. On the other hand, an increase in t raises tax revenue when the tax rate is low, and reduces it after the tax rate exceeds $A/4$ in the case of HF. When the tax rate is low, the positive effect of an increase in t on tax revenue dominates the marginal reduction of other components. As the tax rate becomes high, other components decrease at an accelerated rate and tax revenue changes to decrease after the tax rate reaches its threshold, which leads to the concave welfare functions with respect to the tax rate for any d.

As we will see later, the welfare-maximizing rate in the case HH is higher than one in the case HF given d. That is, the peak of concave function W_{HH} is the right of the peak of concave function W_{HF}. Because the government in the case HH attempts to obtain higher tax revenue and lower damage cost, the government sets a higher tax rate until the negative effects of reduction of domestic production and consumer surplus dominate the positive effect of a higher tax rate. In the case HF, the government bears damage cost and gains tax revenue and profit from only one firm in addition to consumer surplus affected by the aggregate output. In this case, the negative effect of higher tax on consumer surplus dominates the positive effect in the smaller level of tax rate than in the case HH. Thus, given d, the optimal tax rate in the case HF is smaller than in the case HH.

Consider the effect of a change in damage coefficient on W_{HH} and W_{HF} given t. Since damage cost is directly affected by d, an increase in d must shift the curves of W_{HH} and W_{HF} downwards. The government with a higher d wants to reduce pollution more so as to require lower level of domestic production than the government with small d . Therefore, the government attempts to set the higher optimal emission tax, which implies that the welfare functions shift rightward. Combining these effect of a change in d on the welfare, the welfare functions shift downwards to the right as d increases. The positional relationships among three welfare functions depending on d are given by Lemma 6-9 (See in Appendices) and illustrated in Figures 2-5.

3 Optimal emission tax and welfare

We now want to figure out the relationship of three welfare functions (7)–(9) with respect to an emission tax rate, given values of damage coefficient d. The welfare W_{HH} and W_{HF} maximizing-taxes are respectively given by

$$\widetilde{t}_{HH}(d) = \frac{(2d-1)A}{2(1+d)} \quad \text{and} \quad \widetilde{t}_{HF}(d) = \frac{(2d-3)A}{3+4d}. \tag{10}$$

Let us compare the welfare. Subtracting (7) from (9) yields $W_{FF} - W_{HH} = \frac{2}{9}g_1(t)$, where

$$g_1(t) \equiv (1+d)t^2 - (2d-1)At + (d-1)A^2. \tag{11}$$

When $d < 5/4$, $g_1(t) = 0$ has two different real roots, t_2 and $t_5(> t_2)$: $A\left(2d - 1 \pm \sqrt{5-4d}\right)/2(1+d)$. Subtracting (8) from (9), we obtain $W_{FF} -$

$W_{HF} = \frac{1}{18} g_2(t)$, where

$$g_2(t) \equiv (4d+3)t^2 - (4d-6)At + (d-2)A^2. \tag{12}$$

When $d < 15/7$, $g_2(t) = 0$ has two different real roots, one of which is negative, t_1 and another is positive, t_4: $A\left(2d - 3 \pm \sqrt{15 - 7d}\right)/2(1+d)$. Comparing (8) with (7) yields $W_{HH} - W_{HF} = \frac{1}{18} g_3(t)$, where

$$g_3(t) \equiv -t^2 + (4d+2)At - (3d-2)A^2. \tag{13}$$

In this case, $g_3(t) = 0$ has always two different real roots, t_3 and $t_6(> t_3)$: $A\left(2d + 1 \pm \sqrt{4d^2 + d + 3}\right)$.

3.1 The case of small d

Let us first examine the case where the damage coefficient is small or equivalently, the marginal damage from pollution is relatively low. Figure 2 illustrates the optimal emission tax and the resulting welfare that country H obtains for $0 \le d \le 2/3$. Since W_{HH} is always the largest for $0 \le d \le 2/3$, the government of country H chooses an emission tax rate so as to maximize W_{HH}. Thus we have found:

Proposition 2 *When $0 \le d < 1/2$, the negative tax rate (subsidy) on emission is optimal: $t = -A/2$ for $d = 0$ and $t = t_{HH}(f)$ for $0 < d < 1/2$. When $d = 1/2$, the optimal tax rate is zero. When $1/2 < d \le 2/3$, the government sets the optimal tax rate according to $t_{HH}(f)$ in the range $f \in [0, \widetilde{f}]$, where $\widetilde{f}$ satisfies $\widetilde{t}_{HH} = t_{HH}(\widetilde{f})$, and it sets a constant tax rate, $\widetilde{t}_{HH}$, for the range $f \in [\widetilde{f}, 2A^2/9)$.*

If emission has no damage on the environment, the government attempts to attract both firms by subsidizing emission at the welfare-maximizing rate so as to obtain the gain from profits of two firms and a price reduction effect. If environmental damage is not zero, it is not optimal for the government to subsidize pollution any longer. When the environmental damage from emission is small enough, W_{HH} is the largest. Therefore, the government imposes an emission tax according to $t_{HH}(f)$ that is the highest rate for firms to remain in country H, from which tax revenue contributes to the welfare of country H. When the fixed cost reaches $\widetilde{f}$, the governments sets the welfare-maximizing rate, $\widetilde{t}_{HH}$ and maintains the level for $f > \widetilde{f}$.

3.2 The case of medium d

Next, let us look at the case when a damage coefficient is not sufficiently large. Define the critical value d^* such that $W_{HH}(t; d^*) = W_{HF}(t; d^*) = W_{FF}$. That is, when the damage coefficient is d^*, the welfare takes the same value for each location pattern. Figure 3 illustrates how the optimal emission tax is determined in the left panel and the relationship between the optimal emission tax and the fixed cost and the relationship between the maximized welfare and the fixed cost in the right panel when $2/3 < d \leq d^*$. Note that the pattern of the maximized welfare depending on the fixed cost in this case is different from one in the case of $0 \leq d \leq 2/3$.

Suppose that the fixed cost is f_0. If the government sets t_0, both firms remain in country H, which gives country H the welfare $W_{HH}(t_0)$. If the government increases the tax and sets any rate $t \in [t_{HH}(f_0), t_{HF}(f_0)]$, either firm chooses relocation so that the government gains W_{HF} that is larger than $W_{HH}(t_0)$. However, if it sets $t > t_{HF}(f_0)$, the welfare reduces to W_{FF}. Hence, the best strategy is to obtain W_{HF} for country H when f_0. Since W_{HF} is decreasing with t, the government desires to leave a tax rate as lower as possible so as to attain larger W_{HF}. Thus, the optimal tax is $t_{HH}(f_0)$, under the assumption that the firm chooses the location that gives the government the largest welfare when a firm is indifferent between two locations.

Suppose that the fixed cost is f_3. If the government sets $t > t_{HH}(f_3)$, the welfare becomes $W_{HF}(t)$ or W_{FF}, which is strictly smaller than $W_{HH}(t_3)$. Hence, the optimal tax rate is $t_3 = t_{HH}(f_3)$. Until the fixed cost approaches $\widetilde{f}$ that satisfies $\widetilde{t}_{HH} = t_{HH}(\widetilde{f})$, it is optimal to set a tax according the curve $t_{HH}(f)$. Since the welfare W_{HH} decreases with $t > \widetilde{t}_{HH}$, it is optimal to maintain an emission tax at $\widetilde{t}_{HH}$ for $f \geq \widetilde{f}$. Thus we have found:

Proposition 3 *For $2/3 < d \leq d^*$, the optimal tax rate is (i) $t_{HH}(f)$ if $f \in [0, \widetilde{f}]$; (ii) $\widetilde{t}_{HH}$ if $f \in [\widetilde{f}, 2A^2/9)$.*

Until the degree of damage reaches the critical value at which the welfare of all patterns of location are the same, $W_{HF}(t)$ is the largest for $f \in [0, f_3]$. The government sets an emission tax to induces either firm to relocate its plant to country F. When the tax rate is in the range, $[0, t_{HH}(f_3)]$, W_{HF} decreases with t for any f. Hence, the government sets $t_{HH}(f)$, at which two locations are indifferent for each firm. In the location pattern of HF with small degree of environmental damage, an increase in consumer surplus of price reduction due to low tax rate dominates a reduction in profits and tax revenue from one firm. For $f \geq f_3$, the government sets tax rate according to the curve $t_{HH}(f)$ so as to attract both firms to country H. In this range, the government enjoys tax revenue and a reduction in damage cost due to an increased emission tax corresponding to an increased fixed cost unless either firm decides to relocate its production. When the fixed cost reaches $\widetilde{f}$, the government fixes emission tax at $\widetilde{t}_{HH}$ that maximizes welfare in the case of HH. If the fixed cost is higher than $\widetilde{f}$, both firms are reluctant to relocate their production even if the tax rate increases. However, setting tax rate too high reduces the profit of firms. This reduction in the profit dominates the benefit of the increased tax revenue. Thus, it is optimal for the government to fixed emission tax at $\widetilde{t}_{HH}$ for $f \geq \widetilde{f}$.

Consider next the case of $d^* < d < 5/4$. Note that when $d < 5/4$, there exists tax rates such that $W_{FF} < W_{HH}$ and $W_{FF} < W_{HF}$ from (11) and (12). As Figure 4 shows, for $f \in [0, f_4]$, where f_4 satisfies $t_4 = t_{HH}(f_4)$, the government obtains the largest welfare W_{HF} by setting tax rate $t_{HH}(f)$, under which only one firm relocate to suit the government's purpose. Once the fixed cost exceeds f_4, W_{HF} is smaller than W_{FF} for $f \in (f_4, f_2]$, where f_2 satisfies $t_2 = t_{HH}(f_2)$. Hence, the government attempts to keep both firms away from home country. The government suddenly increases the tax rate up to $t_{HF}(f)$ (at the point a in Figure 4). When the fixed cost reaches f_2, the government desires both firms to stay in home country. The government suddenly reduces the tax to $t_{HH}(f)$ (at the point b). For $f \in [f_2, \widetilde{f}]$, the government sets a tax rate along the curve $t_{HH}(f)$ and fixes the welfare-maximizing rate $\widetilde{t}_{HH}$ for $f > \widetilde{f}$ due to decreasing W_{HH} with respect to t. We may summarize the above:

Proposition 4 *For $d^* < d < 5/4$, the optimal tax rate is (i) $t_{HH}(f)$ if $f \in [0, f_4]$; (ii) $t_{HF}(f)$ if $f \in [f_4, f_2]$; (iii) $t_{HH}(f)$ if $f \in [f_2, \widetilde{f}]$; (iv) $\widetilde{t}_{HH}$ if $f \in [\widetilde{f}, 2A^2/9)$.*

When the degree of damage is medium, the relationship between the optimal tax rate and the fixed cost is complicated. For relatively small fixed costs, $f \in [0, f_4]$, the government attracts only one firm to bear damage from its production and to obtain benefits from tax revenue and profits of one firm. Since a higher emission tax rate reduces consumer surplus and domestic production and this cost dominates the positive effect of an increased tax on a reduction in damage cost, the government sets the lowest tax rate such that only one firm relocates, which is $t_{HH}(f)$ from the assumption. For the middle level of the fixed cost, $f \in [f_4, f_2]$, the government sets $t_{HF}(f)$ to keep both firms away from the home country. The government enjoys benefits from the improved environment and a price reduction although it sacrifices tax revenues and profits. Once the fixed cost exceed f_2, the government can set a relatively high emission tax $t_{HH}(f)$ on both firms in country H to obtain increased tax revenue because the large cost of relocation becomes heavy burden for the firms so that both firms are reluctant to relocate their production under $t \leq t_{HH}(f)$. Since $t > t_{HH}(\widetilde{f})$ reduces W_{HH} through reduction in aggregate production, the government maintains $\widetilde{t}_{HH}$ for $f > \widetilde{f}$.

Let us consider the case of $5/4 < d \leq 15/7$. Figure 5 illustrates the relationships between the optimal tax, the maximized welfare and the fixed cost of two cases in which the degrees of damage is relatively large. The upper panel shows the case of $5/4 \leq d \leq 2$. In this case, for $f \in [0, f_4]$, the government attempts to induce only one firm to relocate because W_{HF} is the largest. While W_{HF} increases with t for $t \in [0, \widetilde{t}_{HF}]$, it decreases with t for $t \in [\widetilde{t}_{HF}, t_4]$, where $t_4 = t_{HH}(f_4)$. Hence, until the fixed cost reaches $\widetilde{f}_{HF}$, which satisfies $\widetilde{t}_{HF} = t_{HF}(\widetilde{f}_{HF})$, it is desirable for the government to set an emission tax according to the curve $t_{HF}(f)$ and maintain the welfare-maximizing rate $\widetilde{t}_{HF}$ for $f \in [\widetilde{f}_{HF}, \widetilde{f}_{HH}]$, where $\widetilde{f}_{HH}$ satisfies $\widetilde{t}_{HF} = t_{HH}(\widetilde{f}_{HH})$. However, when the fixed cost exceeds $\widetilde{f}_{HH}$, the welfare W_{HF} decreases with t due to the negative effect on production, thus on profit of domestic firm, which induces the government to lower the tax to $t_{HH}(f)$. Note that if both firms remain in country H, the welfare drastically decreases to W_{HH} so that the government avoids both firms remain in the home country. Once the fixed cost exceeds f_4, the welfare W_{HF} is smaller than W_{FF}. The government attempts to drive both firms out so as to raise the tax up to $t_{HF}(f)$.

The lower panel shows the case of $2 < d \leq 15/7$. For $f \in [0, f_1]$, W_{FF} is the largest. Hence, the government sets $t_{HF}(f)$ to drive both firms out. For $f \in [f_1, \widetilde{f}_{HF}]$, the government sets tax rate according to the curve $t_{HF}(f)$, which yields W_{HF}. For $f \in [\widetilde{f}_{HF}, \widetilde{f}_{HF}]$, the government sets $\widetilde{t}_{HF}$ to obtain the maximized W_{HF}. For the cases of $f \in [\widetilde{f}_{HF}, f_4]$ and $f \in [f_4, 2A^2/9]$, the government takes the same strategies to those in the case of $5/4 \leq d \leq 2$ as described above. We may summarize the above argument as the following proposition:

Proposition 5 *For $5/4 \leq d \leq 15/7$, the optimal tax rate is (i) $t_{HF}(f)$ if $f \in [0, \widetilde{f}_{HF}]$, where $\widetilde{f}_{HF}$ satisfies $\widetilde{t}_{HF} = t_{HF}(\widetilde{f}_{HF})$; (ii) $\widetilde{t}_{HF}$ if $f \in [\widetilde{f}_{HF}, \widetilde{f}_{HH}]$, where $\widetilde{f}_{HH}$ satisfies $\widetilde{t}_{HF} = t_{HH}(\widetilde{f}_{HH})$; (iii) $t_{HH}(f)$ if $f \in [\widetilde{f}_{HH}, f_4]$; (iv) $t_{HF}(f)$ if $f \in [f_4, 2A^2/9)$.*

In the case of large degree of damage, W_{HH} is the lowest. The environmental damage from the aggregate production of two firms is so large that the government avoids to attract both firms. When damage is enough small for country H to allow one firm to remain and the fixed cost is relative low, the government sets $t_{HF}(f)$ to attract only one firm and enjoys the benefit of tax revenue and a profit from one domestic firm and reduction in price. Until the fixed cost reaches $\widetilde{f}_{HF}$, an increase in tax has positive effect on tax revenue and a reduction in damage cost, the government raises emission tax corresponding to increasing fixed cost. As the burden of the fixed cost becomes heavier, a domestic firm does not choose relocation when an emission tax is relatively low, which gives the government an incentive to raise the tax in order to obtain W_{HF}. Once the fixed cost exceeds $\widetilde{f}_{HF}$, the government enjoys the maximized welfare W_{HF}. For the larger fixed costs than $\widetilde{f}_{HH}$, the welfare-maximizing tax $\widetilde{t}_{HF}$ induces both firms to remain in country H. Since damage cost from the production of two firms is so large, the government raises the emission tax in order to drive only one firm out. The welfare W_{HF} decreases as tax rate increases, the government sets the lowest tax that induces one firm to relocate its production, according to the curve $t^{HH}(f)$. When the fixed cost reaches f_4, the government sets $t_{HF}(f)$ to drive two firms out. In the lower case of Figure 5, for the lower fixed cost, the government attempts to drive both firms out because tax revenue is too small to compensate large damage from pollution.[2]

4 Globalization and economic welfare

We may consider the location cost, f, as a degree of globalization of world economy. That is, the more globalization grows, the lower f is. When the damage coefficient, d, is small, the location pattern of two firms, (H, H), remains unchanged even if f decreases. Given $d \in [0, 1/2]$, the welfare that the country H gains is maximized for a nonpositive tax rate. When d is in the range of $(1/2, 2/3]$, the welfare, W_{HH}, decreases as f becomes smaller. In this case, if the government sets a welfare-maximizing tax, a firm with small f relocates its production to the foreign country, which reduces the welfare. To avoid the reduction in the welfare due to relocation of firms, the government sets a lower tax according to the curve, $t^{HH}(f)$.

When d is large, the welfare, W_{FF} is the largest for various f. Hence, the governments raises the tax to gain W_{FF}, which is independent of f. For the governments with high sensibility to the environmental quality, the production of polluting firms outside gives the largest welfare regardless of

[2]In the case of large damage, $d > 15/7$, the government sets $t_{HF}(f)$ for any f to lead both firms to relocate their production abroad, because benefits from tax revenue and profits no longer offset huge environmental damage.

the degree of globalization.

When the damage coefficient is medium, the relationship between globalization and economic welfare shows various patterns with respect to d. Firstly, when d is in the range of $(2/3, d^*]$, the equilibrium location pattern changes from (H, H) to (H, F) as the globalization proceeds. If t is small, the effect of tax revenue is small (tax revenue effect is zero when $t = 0$) so that the effect of a reduction in environmental damage due to relocation of one firm dominates reductions in tax revenue and profits. As a result, $W_{HF} > W_{HH}$. When the relocation cost decrease cross the $\tilde{f}$, the welfare-maximizing tax, $\tilde{t}_{HH}$ induces firms to relocate abroad. Hence, the government lowers the tax rate corresponding to the globalization grows so as to attract both firms in country H. When the location cost decreases cross f_3, the location pattern that maximizes the welfare is (H, F). As f decreases, the tax rate that induces only one firm to relocate shifts lower. Since W_{HF} is a decreasing function of $t > 0$, a reduction in the tax rate increases the welfare. When d is in the range of $(d^*, 5/4)$, the equilibrium location pattern (F, F) appears during it changes from (H, H) to (H, F) as f_2 decreases to f_4. In the case of $2/3 < d < 5/4$, the welfare decreases first, then increases as the globalization proceeds.

When the damage coefficient is relatively large and in the range of $[5/4, 2]$, the equilibrium location pattern changes from (F, F) to (H, F) as the globalization grows. The environmental damage is too large to maximize W_{HH}. When the tax rate is small, the marginal cost of a domestic firm is not so high, which contribute to consumer surplus. In addition, under the location pattern (H, F), tax revenue contributes to the welfare. The benefit of a small tax on consumer surplus and a profit besides tax revenue dominates the environmental damage, which leads to a larger W_{HF} than W_{FF}. According to the growing globalization, the government lowers the tax so as to obtain W_{HF}. Since the welfare-maximizing tax rate for the location pattern (H, F) is in the range of $[0, t_4]$, the equilibrium welfare increases and then decreases as the globalization proceeds. It is interesting to note that the degree of globalization reverses the change of welfare.

5 Conclusions

This paper explores the optimal environmental tax policy under which duopoly firms strategically choose the location of their plants in a simple three-stage game. In the first stage, the government sets an emission tax rate to maximize the country's welfare. Given the tax rate, in the second stage, duopoly firms choose the location of their plants. In the third stage, firms compete *á la* Cournot in the market, given an emission tax rate and their location choice. In this setting, we show how the optimal environmental tax is determined when the government can affect the strategies of duopoly firms about relocation which, in turn, depends on endogenous emission tax rate.

We derive that the optimal emission tax and welfare resulted from location choices of firms depend on the degree of damage from pollution, which affects benefits from profits of firms, tax revenue, and price reduction and cost from environmental damage. Under some range of damage coefficient, hybrid equilibrium emerges. That is, either firm chooses relocation of its plant even if duopoly firms are identical ex ante.

Appendices

Lemma 6 *For $0 \leq d \leq 2/3$, W_{HH} is the largest if $0 \leq t \leq A/2$. For $0 \leq d \leq 1/2$, W_{HH} is maximized at $t = 0$. For $1/2 < d \leq 2/3$, W_{HH} is maximized at $t = \widetilde{t}_{HH}$.*

Proof. When $d = 0$, we have $W_{FF} < W_{HF}(0) < W_{HH}(0)$ and $W_{HF}(A/2) < W_{FF} < W_{HH}(A/2)$. Since welfare-maximizing taxes, $\widetilde{t}_{HH}$ and $\widetilde{t}_{HF}$ are negative, W_{HH} is largest in $t \in [0, A/2)$ and is maximized at $t = 0$ for this range. In the case that $d = 1/2$, W_{HH} is maximized at $\widetilde{t}_{HH} = 0$. Since $\widetilde{t}_{HF} < 0$ and $W_{HF}(A/2) < W_{FF} < W_{HH}(A/2)$, W_{HH} is largest in $0 < t < A/2$. In the case that $d = 2/3$, W_{HH} and W_{HF} intersect at $t = 0$, hence, $t_3 = 0$ with $\widetilde{t}_{HH} > 0$ and $\widetilde{t}_{HF} < 0$. Since $W_{HF}(A/2) < W_{FF} < W_{HH}(A/2)$, W_{HH} is largest in $0 < t < 2/3$. ■

Lemma 7 *For $2/3 < d \leq d^*$, W_{HF} is largest if $0 \leq t \leq t_3$, while W_{HH} is the largest if $t_3 < t < t_5$ and is maximized at $t = \widetilde{t}_{HH}$.*

Proof. This follows directly from Lemma 6. ■

Lemma 8 *For $d^* \leq d < 5/4$, W_{HF} is largest if $0 \leq t \leq t_4$, W_{FF} is largest in $t_4 < t < t_2$, W_{HH} is largest if $t_2 < t < t_6$ and is maximized at $t = \widetilde{t}_{HH}$.*

Proof. Since $g_3(t) = 0$ if only if $W_{HH} = W_{HF}$, we can rewrite as:

$$G\left(t(d), d\right) = -t^2 + (4d + 2)At - (3d - 2)A^2 \equiv 0.$$

By the implicit function theorem, we have:

$$t^{'}(d) = -\frac{G_d}{G_t}, \tag{14}$$

where the subscripts denote partial derivatives. Because $g_3(t) = 0$ is quadratic function of t and t_3 is its smaller roots, the sign of $G_t(\cdot)$ at $t = t_3$ must be positive. By assumption of $t < A/2$, we have $G_d = 4A\left(t - (3/4)A\right) < 0$. Hence, from (14), $t^{'}(d) > 0$.

Next, looking at the slope of W_{HF} at $t = t_3$, again by assumption of $t < A/2$, $\partial WHF/\partial d = -(2t - A)^2 < 0$. Combining this with $t^{'}(d) > 0$, for $d > d^*$, we have:

$$W_{FF} = W_{HF}\left(t(d^*), d^*\right) > W_{HF}\left(t(d), d^*\right) > W_{HF}\left(t(d), d\right).$$

This shows that the intersection of W_{HH} and W_{HF} is below the function W_{FF}. ■

Lemma 9 *For* $5/4 \leq d \leq 2$, W_{HF} *is the largest if* $0 \leq t \leq t_4$, *while* W_{FF} *is the largest if* $t_4 < t < A^2/2$. *For* $2 < d \leq 15/7$, W_{FF} *is the largest if* $0 < t \leq t_1$ *or* $t_4 < t < A^2/2$, *while* W_{HF} *is the largest if* $t_1 < t \leq t_4$.

Proof. In this case, there is no possibility that W_{HH} is maximum since W_{HH} does not intersect W_{FF} from (11). When $5/4 \leq d \leq 2$, W_{HF} is largest because $t_1 \leq 0$ and $0 < t_4 < A/2$. When $2 < d \leq 15/7$, W_{FF} is largest for the ranges of $t \in [0, t_1]$ or $t_4 < t < A/2$, and W_{HF} is largest for $t_1 < t < t_4$. ■

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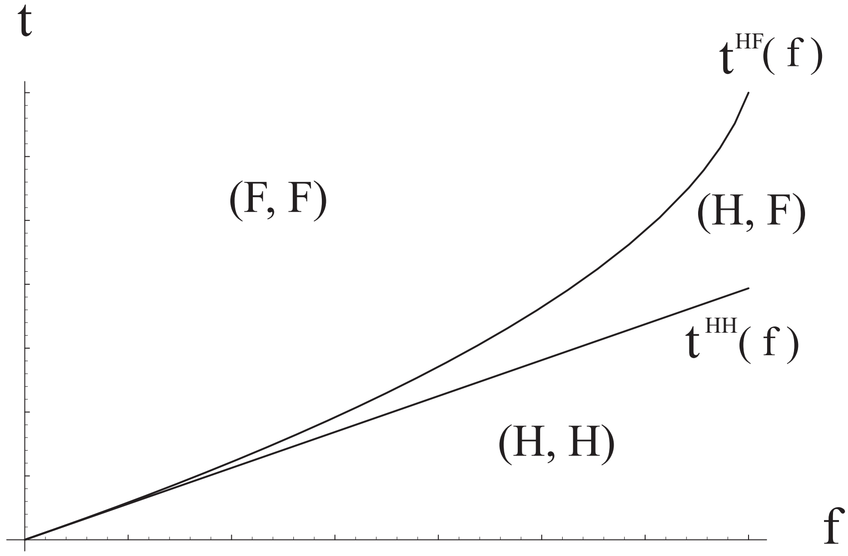


Figure 1: Possibilities of three Nash equilibria

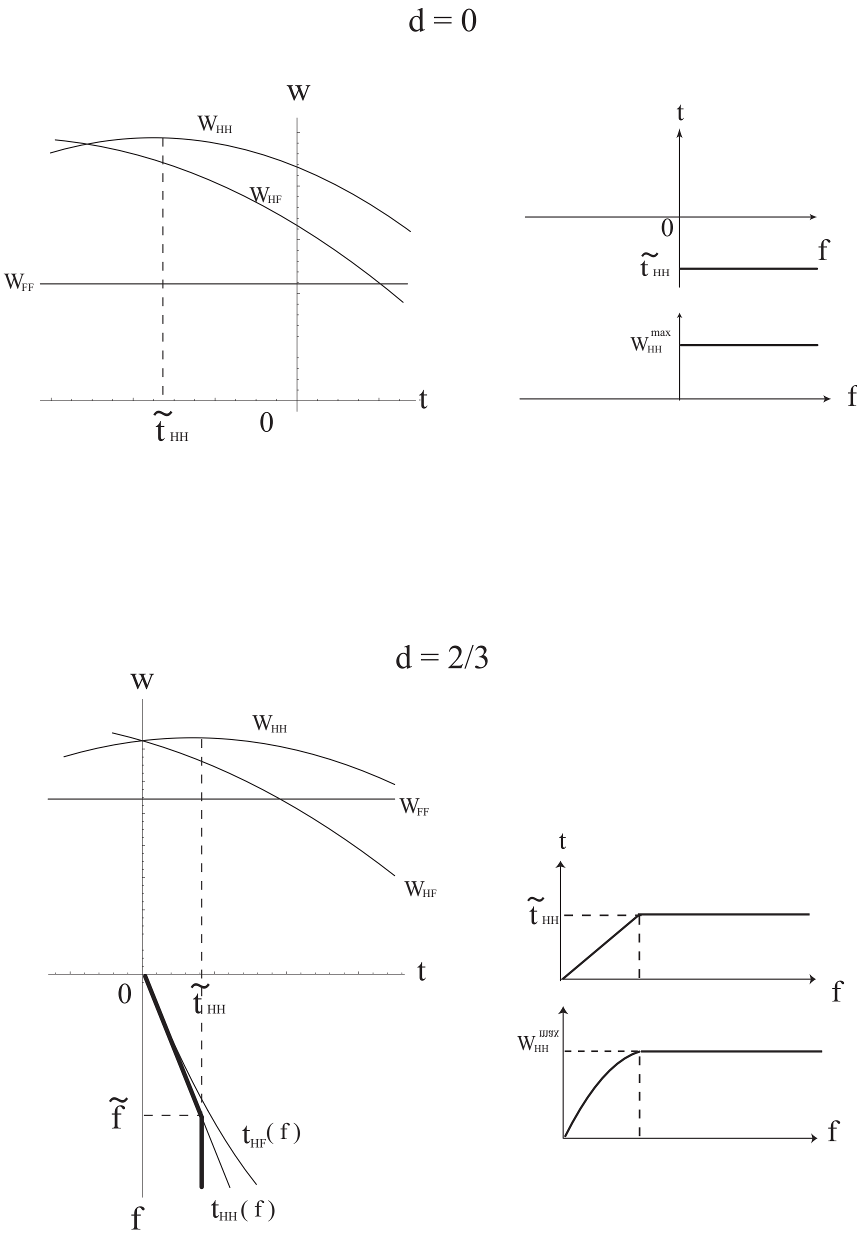


Figure 2: The case of $0 \leq d \leq 2/3$.

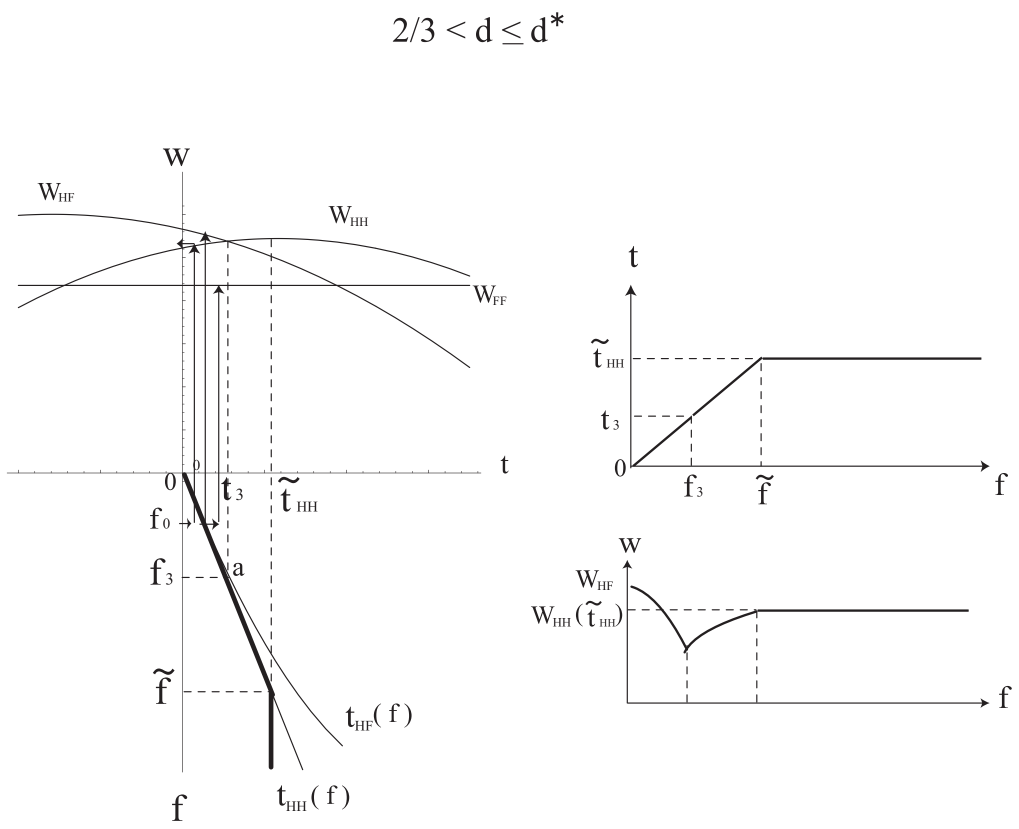


Figure 3: The case of $2/3 < d \leq d^*$.

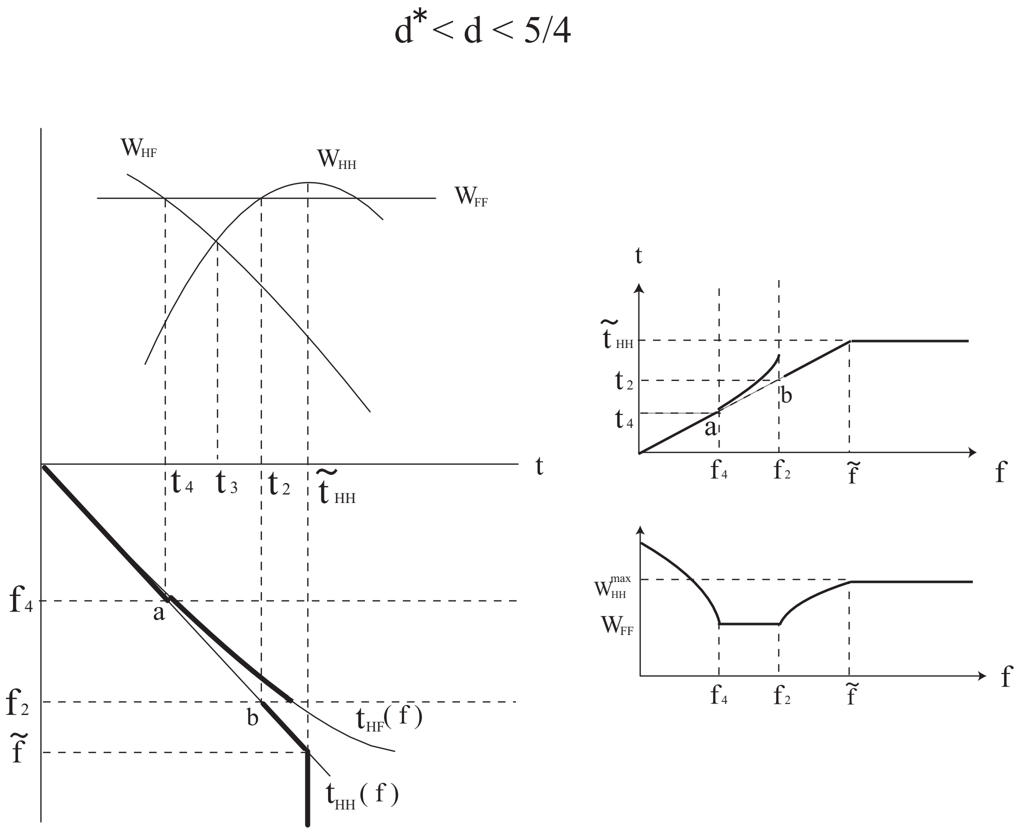
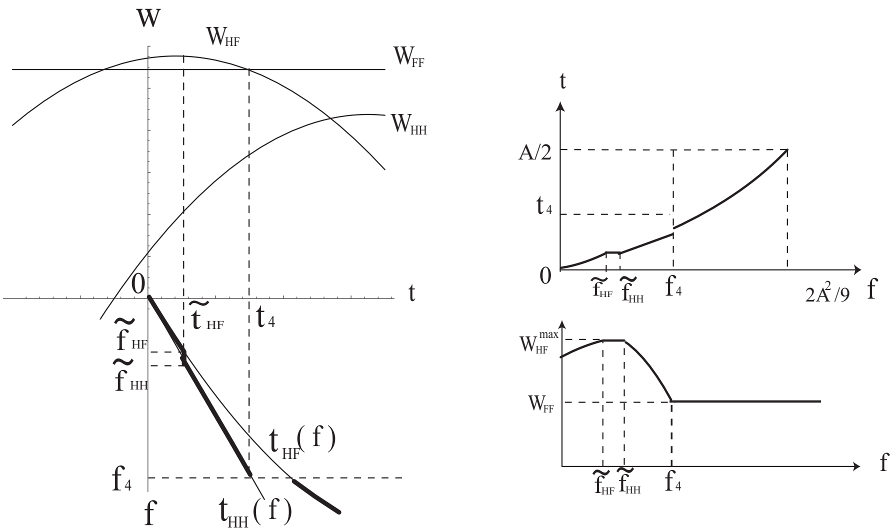


Figure 4: The case of $d^* < d < 5/4$.

$5/4 \leq d \leq 2$



$2 < d \leq 15/7$

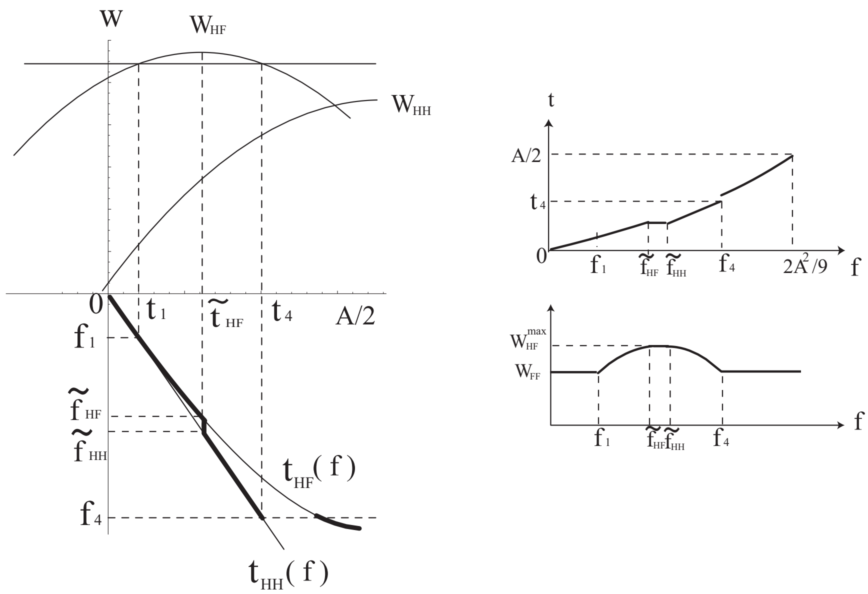


Figure 5: The cases of $5/4 \leq d \leq 2$ and $2 < d \leq 15/7$