Discussion Paper No. 767

ONLY THE FINAL OUTCOME MATTERS: PERSISTENT EFFECTS OF EFFORTS IN DYNAMIC MORAL HAZARD

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January 2010
Revised June 2010
Revised May 2011

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Only the Final Outcome Matters: Persistent Effects of Efforts in Dynamic Moral Hazard

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May 12, 2011

Abstract

We analyze a dynamic principal–agent problem in which the agent's effort in each period has strong persistent effects. We show that a simple contract, where the reward depends only on the final outcome, is explained as the optimal contract derived in the principal's optimization problem. The paper also discusses that the optimality of such a simple payment scheme crucially depends on the first-order stochastic dominance of the final outcome under various effort sequences.

Journal of Economic Literature Classification Numbers: C61, D82, D86.

Keywords: dynamic moral hazard; history dependence; simple contract; first-order stochastic dominance.

*This is a substantially revised paper of an earlier version (Ogawa (2003)). After the previous version was prepared, I became aware of two independent works, Mukoyama and Şahin (2005) and Kwon (2006), the results of which are incorporated in the paper. The current version is included as Chapter 2 in my Ph.D. thesis (2008), and I would like to thank my advisor, Michihiro Kandori, for his guidance and encouragement. I am grateful to the editor and anonymous referees for helpful comments. I also thank Eddie Dekel, Junichiro Ishida, Hideshi Itoh, Minoru Kitahara, Dan Sasaki, and Satoru Takahashi for comments and discussions. This research was partially supported by MEXT of the Japanese government, Grant-in-Aid for Young Scientists (B) 21730157.

1 Introduction

In long-term principal–agent relationships with complete contracts, the principal prepares payment schedules in advance and these schedules potentially depend on the agent's period-wise performance (i.e., performance related to the level of effort), in order to provide proper incentives. In the light of the celebrated *sufficient statistic theorem* (Holmström (1979)), one may expect that using a very detailed history of past performances, each of which are informative of the agent's efforts, is optimal for the principal in preparing payment schedules. In reality, however, we often observe various incentive schemes that are not necessarily dependent on the entire detailed record of performances but only on a subset of them, where such subsets are sometimes much smaller than the entire set of performances.

From the viewpoint of economic studies, such *simple* contracts are interpreted in several ways. One argument is that the principal incurs costs in preparing or enforcing complex contracts, after taking such costs into consideration, some sort of simple contract is concluded as a suboptimal solution. Another possibility, following Holmström and Milgrom (1987), is that simple payment schedules are justified by their robustness to the change of model parameters. Understanding that such interpretations provide insights that enable us to grasp important aspects of contracts in reality, the present paper aims to study a third way of explaining simple contracts.[1]

Consider an environment where the agent's current efforts have persistent effects over the future performances. In such environments, a contract that provides strong incentives in the future induces the agent to work hard in the present, and the role of such future incentives appears to be more important in such situations than in those where the agent's efforts have no persistent effects. However, this is not to say that providing incentives only in the future is sufficient: the agent's

[1] It should be noted that the current paper does not aim to provide a universal explanation for simple contracts (as in Holmström and Milgrom (1987)), but looks for an explanation that works in a particular class of environments.

current performance is *still* informative about present efforts,[2] and we may expect that every informative performance should be included in the optimal contract, no matter how low their informativeness may be. The present paper shows that this is not always the case; that is, we show that an incentive scheme that depends *only* on the final performance is optimal if the agent's effort in each period has strong persistent effects.

We provide sufficient conditions for such *simple* contracts to be optimal in dynamic moral hazard models, in which the cost of efforts is the same in all periods. The common feature of our sufficient conditions is summarized as follows: the probability distribution of the final outcome when the agent shirks only in the final period *first-order stochastically dominates* (*FOS-dominates* hereafter) the distribution when the agent shirks in any other period in such a way that *the expected number of shirking events is one*. To grasp the idea behind this condition intuitively, consider a two-period model in which the agent's first-period action also affects the probability distribution of the second-period outcome. Let (a,a') denote the action profile in which the first element (second element) indicates the agent's first-period action (second-period action, respectively), and let $\bar{a}$ ($\underline{a}$) denote a strong effort (a shirk, respectively). Then, the sufficient condition has the following two requirements (Assumption 1).

(i) The probability distribution of the second-period outcome when the agent shirks only in the second period $(\bar{a},\underline{a})$ *FOS-dominates* the distribution when the agent shirks only in the first period $(\underline{a},\bar{a})$.

(ii) The probability distribution of the second-period outcome when the agent shirks only in the second period $(\bar{a},\underline{a})$ *FOS-dominates* the half-and-half mixture of (a) the distribution when the agent shirks in *both* periods $(\underline{a},\underline{a})$ and (b) the distribution when the agent *never* shirks in any period $(\bar{a},\bar{a})$.

We can understand this sufficient condition in the following way. Suppose that

[2]In the paper, we assume that performances are statistically independent between one period and the next. See Section 2 for the formal model.

the principal can (somehow) force the agent to work hard in the first period. Then the principal's problem is to design the contract so that the agent finds it optimal to work hard in the second period (given that the agent is forced to worked hard in the first period). Such an optimal contract provides the agent with higher expected payoff (the sum of wages and effort costs) by taking $(\bar{a},\bar{a})$ than by taking $(\bar{a},\underline{a})$. Note that this optimal contract depends only on the second-period outcome so that it is a contract in which "only the final outcome matters."

Additionally suppose that this "optimal" contract is an increasing function of second-period outcomes.[3] Then requirement (i) ensures that shirking in the first period $(\underline{a},\bar{a})$ always makes the agent worse off than shirking in the second period $(\bar{a},\underline{a})$ does, since the cost of efforts is the same between the two action profiles (i.e., one effort) and FOSD ensures that the agent's expected wage by taking $(\bar{a},\underline{a})$ is larger than by taking $(\underline{a},\bar{a})$.[4] So given this contract, the agent never finds it optimal to undertake $(\underline{a},\bar{a})$, even if there is no "forced labor" in the first period. Requirement (ii), on the other hand, ensures that shirking in both periods $(\underline{a},\underline{a})$ makes the agent worse off compared with shirking in the second period $(\bar{a},\underline{a})$. The argument behind this requirement is bit more complicated since the effort costs are different between the two action profiles, $(\bar{a},\underline{a})$ and $(\underline{a},\underline{a})$. As we will see later, such a comparison is made by a FOSD between well-designed mixtures of action profiles with the same expected number of efforts. In requirement (ii), this is achieved by setting the expected number of efforts to be one on both sides $(1 = 0.5\times 2+0.5\times 0)$. Thus, under requirements (i) and (ii), the agent never finds it optimal to undertake either $(\underline{a},\bar{a})$ or $(\underline{a},\underline{a})$ even if there is no forced labor in the first period, as long as the contract induces the agent to take $(\bar{a},\bar{a})$ rather than $(\bar{a},\underline{a})$. This is how the contract in which only the final outcome matters provides a sufficient incentive to work hard in both periods.

[3] As is known in the literature, the monotone likelihood ratio property (MLRP) is required for the optimal contract to be an increasing function. Subsection 2.1 discusses the case of MLRP. In Subsection 2.2, we argue how the sufficient condition, (i) and (ii), is rewritten if MLRP is not satisfied.

[4] FOSD provides a sufficient condition for a comparison between expectations of increasing functions.

Strong persistent effects of efforts as characterized by the FOSD are the main sources of our result. Historical dependence of this sort is often seen in real economic environments. For example, if an effort has a time-lag effect into the next period, as well as the direct effect in the current period, then the probability of success in period 2 is influenced by the effort level in period 1. If the production technology involves irreversibility, then the model becomes history dependent in a similar manner. We discuss these examples briefly in Section 2.

In Section 3, we show how the result in Section 2 is extended to a general T-period setting. The extension is modestly straightforward and again the point is the FOSD between mixtures of action profiles with the same expected number of efforts. Human capital investment is discussed as an example of the sufficient condition.

1.1 Related Literature

Much simpler models than the one in the present paper have been studied in independent works by Kwon (2006), Mukoyama and Şahin (2005), and Ogawa (2003). In their studies, outcomes take only two values, "success" and "failure", and simplified versions of our requirements (i) and (ii) are presented as the sufficient condition for the simple contract.[5] The present paper studies a model with N possible outcomes and the proof is given in an organized manner using the property of FOSD, which provides clear and rigorous interpretations of the earlier works.[6]

Studies on the dynamic models of the moral hazard problem date back to Lam-

[5]Although the distinction between MLRP and FOSD is important in the incentive theories with hidden actions (Mas-Colell, Whinston, and Green (1995, Section 14.B)), the two conditions are given by the same inequality

$$\Pr[\text{“success”} \mid \bar{a}] > \Pr[\text{“success”} \mid \bar{a}]$$

in the 2-outcome models.

[6]The earlier works have another contributions. Mukoyama and Şahin (2005) provided some numerical analyses when the theoretical approach is difficult. The highlight of Kwon (2006) is the empirical analysis using health insurance data. The "nonincreasing marginal returns" condition by Kwon (2006) is a special case of our sufficient condition (see Section 3).

bert (1983) and Rogerson (1985). Their studies are on the *repeated* model in the sense that there is no persistent effects as studied in the present paper. In repeated moral hazard, it is shown that the optimal contract depends on the entire history of outcomes (*memory* effect in Rogerson (1985)). Our paper, in contrast, shows that the optimal contract depends only on the final outcome under certain sufficient conditions.

Dynamic moral hazard problems then have been studied in various extensions. Problems in which the agent can access a bank are analyzed by Fudenberg, Holmstrom, and Milgrom (1990) and Chiappori, Macho, Rey, and Salanié (1994) among others. Renegotiation problems in dynamic moral hazard are studied by Fudenberg and Tirole (1990), Ma (1991), Ma (1994), and Matthews (1995) among others. In the present paper, we assume that there is no access to banks and no renegotiation.

The relationship between *sufficient statistic theorem* (Holmström (1979)) and our result casts an interesting light on the interpretation of "informativeness" in economic studies. In the view of the sufficient statistic theorem, every statistically informative signal is useful (and should be used) in the optimal contract, whereas in our model, the principal sometimes finds it optimal to "ignore" some statistically informative signals.

Recent theoretical studies on repeated moral hazard problems include Jarque (2010), who considers a similar problem with infinite horizon, continuum effort. The current paper studies the model with two effort levels and we do not need to replace incentive constraints with first-order conditions.

2 The Basic Model

We study a simple dynamic moral hazard model with "history dependence." The relationship between a principal (she) and an agent (he) lasts for two periods ($t = 1,2$).

In each period, the agent chooses his action a^t from the action space $A =$

$\{\underline{a}, \bar{a}\}$. These actions are unobservable to the principal. We may find it convenient to interpret these actions as effort levels and say that the agent works hard (shirks) when he chooses $\bar{a}$ ($\underline{a}$).

In period t, after the agent has chosen his action a^t, the outcome $x^t \in \{x_1, \cdots, x_N\} \equiv X$ is realized according to probabilities that depend on the history of the agent's actions; that is, the distribution of x^1 depends on a^1, whereas that of x^2 depends on the pair (a^1, a^2). These outcomes are immediately observed by both parties (and assumed to be verifiable to third parties, such as a court). We may regard these outcomes as performances and identify each of them with a corresponding revenue received by the principal.

We assume that x^1 and x^2 are independently distributed;[7] hereafter, we write the distributions as given below:

$$\begin{aligned} p_i^1(a^1) &= \Pr\left[x^1 = x_i \mid a^1\right] && (i = 1, \cdots, N), \\ p_i^2(a^1, a^2) &= \Pr\left[x^2 = x_i \mid (a^1, a^2)\right] && (i = 1, \cdots, N). \end{aligned}$$

Throughout the paper, we assume that the distributions have full support:

$$\begin{aligned} p_i^1(a^1) &> 0 && \text{for all } (i, a^1) \in \{1, \cdots, N\} \times A, \\ p_i^2(a^1, a^2) &> 0 && \text{for all } (i, a^1, a^2) \in \{1, \cdots, N\} \times A^2. \end{aligned}$$

At the beginning of the game (i.e., before $t = 1$), the principal and the agent sign a contract in the manner detailed below.

First, the principal offers a long-term contract $\mathbf{w} = (\mathbf{w}^1, \mathbf{w}^2)$, where $\mathbf{w}^1 = (w^1(x^1))_{x^1 \in X}$ and $\mathbf{w}^2 = (w^2(x^1, x^2))_{(x^1, x^2) \in X^2}$ are payment schedules for periods 1 and 2, respectively, under outcome realizations (x^1, x^2). Such a contract stipulates $N + N^2$ possible payments, depending on the realizations of outcomes. Next, the agent decides whether to accept or reject the contract offered by the principal.

[7] Under this assumption, the realized value of x^1 does not influence the distribution of x^2, so the former yields no information on the current likelihood of any particular production levels in period 2. The "history dependence" discussed in this paper deals with the case where x^2 is affected by a^1, but not by the realization of x^1.

If the agent refuses the offered contract, both parties receive their reservation utilities and the game comes to an end. If the agent accepts the contract, the game enters into the two-time moral hazard repetition discussed above.

We assume that the principal can commit to the long-term contract that she has offered before $t = 1$, therefore, once the contract is accepted by the agent, the principal cannot change the payment schedule $\mathbf{w}$ and must make the payment in each period according to the history of outcome realizations up to the date of payment. In addition, we assume that the agent must commit to his participation in the game, therefore, once he accepts the contract, he cannot exit in the midst of the game and must participate in it until the end of period 2.

In each period, the agent attains a payoff of $u(w) - c(a)$, where u is strictly increasing and strictly concave (the agent is risk averse) and $c(\underline{a}) < c(\bar{a})$ (harder work involves a greater cost). We normalize this as $c(\underline{a}) = 0$ and $c(\bar{a}) = C$.

Given a long-term contract $\mathbf{w}$, the agent's strategy consists of two parts: one is the action he takes in the first period, a^1, and the other is the action schedule for the second period $a^2 = (a_i^2)_{i=1}^N$, each of which specifies the action he takes in period 2 under the outcome realization of x^1 in period 1.[8] Let $U_i(a^1, a_i^2, \mathbf{w}^2)$ denote the expected utility in period 2 for the agent when he chose a^1 and the outcome was x_i in the first period:

$$U_i(a^1, a_i^2, \mathbf{w}^2) = \sum_{j=1}^{N} p_i^2(a^1, a_i^2) u(w^2(x_i, x_j)) - c(a_i^2).$$

Using this notation, the intertemporal expected utility for the agent $U(a^1; a^2; \mathbf{w})$ under the agent's strategy $(a^1; a^2)$ is written as

$$U(a^1; a_1^2, \cdots, a_N^2; \mathbf{w}) = \sum_{i=1}^{N} p_i^1(a^1) \left[u(w^1(x_i)) + U_i(a^1, a_i^2, \mathbf{w}^2) \right] - c(a^1).^9$$

[8] Accordingly, we allow the agent to change his action in period 2 after he observes the outcome realization in period 1, which is one of the standard assumptions in this literature. Once we stop making this assumption and assume that the agent has to commit to a pair of actions (a^1, a^2) *ex ante*, then the model reduces to a one-shot multitask incentive problem. We will make the sequentiality assumption to focus on the dynamics of the model, but note that the main result of the paper also applies to the one-shot multitask model.

[9] We assume that both the principal and the agent have a common discount factor of one. If

The optimization problem for the principal when she wishes to implement an action profile (a^1, a^2) is written as

$$\min_{\mathbf{w}} \sum_{i=1}^{N} p_i^1(a^1) \left[w^1(x_i) + \sum_{j=1}^{N} p_j^2(a^1, a_i^2) w^2(x_i, x_j) \right], \tag{P}$$

subject to

$$U(a^1, a^2, \mathbf{w}) \geq U(a', a'', \mathbf{w}), \quad a' \neq a^1, \quad \forall a'' \in A^N, \tag{IC1}$$

$$U_i(a^1, a_i^2, \mathbf{w}^2) \geq U_i(a^1, a', \mathbf{w}^2), \quad a' \neq a_i^2, \quad i = 1, \cdots, N, \tag{IC2}$$

$$U(a^1, a^2, \mathbf{w}) \geq 2\bar{u}, \tag{PC}$$

where $\bar{u}$ denotes the reservation utility for the agent.[10]

2.1 Simple Contract

In this subsection, we show that the optimal long-term contract is dependent only on the second-period outcome if the probability distribution of the second-period outcome satisfies certain conditions, as briefly discussed in the Introduction. The result (Theorem 1) contrasts with that of the *repeated* moral hazard literature, where the optimal long-term contract is always dependent on the complete history of past outcomes.

Throughout this subsection, we assume that

$$\frac{p_1^2(\bar{a}, \bar{a})}{p_1^2(\bar{a}, \underline{a})} < \cdots < \frac{p_N^2(\bar{a}, \bar{a})}{p_N^2(\bar{a}, \underline{a})} \tag{2}$$

the common discount factor was less than one (but positive) and the outcome space consisted of three elements or more, we could not attain the plausible sufficient conditions as in Assumption 1, which relies on the nature of $(p_i^1(\cdot))$ and $(p_i^2(\cdot,\cdot))$. Mukoyama and Şahin (2005) showed that when $N = 2$, an extension of Assumption 1 is a sufficient condition for $w^1(x^1)$ to be constant, in a similar model in which both players have a common discount factor of less than one.

[10] When the model is just a repetition of two moral hazard stages (as in Lambert (1983) and Rogerson (1985)), the action taken in period 1, a^1, does not affect the probability distribution of outcomes in period 2, such that $U_i(a', a_i^2, \mathbf{w}^2) = U_i(a'', a_i^2, \mathbf{w}^2)$ for any $a' \neq a''$. This reduces the incentive constraints for the first period (IC1) to

$$U(a^1, a^2, \mathbf{w}) \geq U(a', a^2, \mathbf{w}), \quad (a' \neq a^1), \tag{1}$$

under which we must only take into account the deviation strategies from a^1 to the other a', with a^2 being fixed.

for the sake of a simple exposition. (2) is referred to as the *monotone likelihood ratio property* (MLRP), and plays an important role in the monotonicity of optimal contracts in the basic moral hazard model.[11] In Subsection 2.2, we discuss how the sufficient condition (Assumption 1 below) is described if (2) is not satisfied. At this point we should note that the discussion in Subsection 2.2 is not restrictive, and there we can describe an analogue of Assumption 1 for *any* given pairs of $p_j^2(\bar{a},\bar{a})$ and $p_j^2(\bar{a},\underline{a})$ which do not satisfy MLRP.

The following assumption provides a sufficient condition for such simple contracts. We may regard this assumption as relating to "strong persistent effects" in the sense that the action chosen in period 1 has a stronger influence on the outcome in period 2 than does the action chosen in period 2.

Assumption 1. $p_j^2(a^1,a^2)$ satisfies the following conditions:

(i) $\sum_{k=1}^{j} p_k^2(\underline{a},\bar{a}) \geq \sum_{k=1}^{j} p_k^2(\bar{a},\underline{a})$ for all $j=1,\cdots,N$.

(ii) $\frac{1}{2}\sum_{k=1}^{j}\left(p_k^2(\underline{a},\underline{a})+p_k^2(\bar{a},\bar{a})\right) \geq \sum_{k=1}^{j} p_k^2(\bar{a},\underline{a})$ for all $j=1,\ldots,N$.

Condition (i) states that $(\bar{a},\underline{a})$ (first-order) *stochastically dominates* $(\underline{a},\bar{a})$, so it asserts that, for any increasing function $\gamma\colon\{1,\ldots,N\}\to\mathbb{R}$,

$$\sum_{j=1}^{N} p_j^2(\bar{a},\underline{a})\gamma(j) \geq \sum_{j=1}^{N} p_j^2(\underline{a},\bar{a})\gamma(j).$$

Condition (ii) is the stochastic dominance between $(\bar{a},\underline{a})$ and $\frac{1}{2}((\bar{a},\bar{a})+(\underline{a},\underline{a}))$, and hence, again,

$$\sum_{j=1}^{N} p_j^2(\bar{a},\underline{a})\gamma(j) \geq \frac{1}{2}\sum_{j=1}^{N}\left(p_j^2(\bar{a},\bar{a})+p_j^2(\underline{a},\underline{a})\right)\gamma(j),$$

for any increasing function γ.[12] Given a pair of $p_j^2(\bar{a},\bar{a})$ and $p_j^2(\bar{a},\underline{a})$, conditions (i) and (ii) are mutually independent.

[11] MLRP's implications in the principal-agent problem are discussed in Milgrom (1981).

[12] For comprehensive discussions of stochastic orders, see Mas-Colell et al. (1995, Section 6.D), Shaked and Shanthikumar (2007) for instance.

A different interpretation of condition (ii) can be done in the following manner. Since $p_j(\bar{a},\bar{a})/p_j(\bar{a},\underline{a})$ is increasing in j and therefore $\sum_{k=1}^{j} p_k^2(\bar{a},\underline{a}) \geq \sum_{k=1}^{j} p_k^2(\bar{a},\bar{a})$ for all j, condition (ii) implies that

$$\sum_{k=1}^{j} \left(p_k^2(\underline{a},\underline{a}) - p_k^2(\bar{a},\underline{a})\right) \geq \sum_{k=1}^{j} \left(p_k^2(\bar{a},\underline{a}) - p_k^2(\bar{a},\bar{a})\right) \geq 0.$$

Therefore, condition (ii) implies that $(\bar{a},\underline{a})$ has to be "*closer*" to $(\bar{a},\bar{a})$, compared with $(\underline{a},\underline{a})$.

Here, we provide two examples of probability distributions that satisfy Assumption 1. In both examples, it is assumed that the outcome is either "success" or "failure" ($N = 2$).

Example 1 (Time lag)**.** There is a time lag between the effort and its effect.

If the agent works hard in period t, it increases the probability of success not only in the same period by α but also in the following period by β. We assume that $0 < \alpha < \beta$, and regard β as a "full effect" of the effort and α as a "partial effect" of the effort. Let $\underline{\pi}$ denote the probability of success when the agent has never taken any positive efforts. Then, we have

$$\begin{aligned}
p^1_{\text{success}}(\underline{a}) &= \underline{\pi}, & p^1_{\text{success}}(\bar{a}) &= \underline{\pi} + \alpha, \\
p^2_{\text{success}}(\underline{a},\underline{a}) &= \underline{\pi}, & p^2_{\text{success}}(\underline{a},\bar{a}) &= \underline{\pi} + \alpha, \\
p^2_{\text{success}}(\bar{a},\underline{a}) &= \underline{\pi} + \beta, & p^2_{\text{success}}(\bar{a},\bar{a}) &= \underline{\pi} + \alpha + \beta.
\end{aligned}$$

Example 2 (Irreversibility)**.** The agent has to make a positive effort in every period to maintain the highest probability of success $\bar{\pi}$. If he shirks, the probability of success declines by γ and this degree of success is not recovered even if the agent makes a positive effort in the following period:

$$\begin{aligned}
p^1_{\text{success}}(\underline{a}) &= \bar{\pi} - \gamma, & p^1_{\text{success}}(\bar{a}) &= \bar{\pi}, \\
p^2_{\text{success}}(\underline{a},\underline{a}) &= \bar{\pi} - 2\gamma, & p^2_{\text{success}}(\underline{a},\bar{a}) &= \bar{\pi} - \gamma \\
p^2_{\text{success}}(\bar{a},\underline{a}) &= \bar{\pi} - \gamma, & p^2_{\text{success}}(\bar{a},\bar{a}) &= \bar{\pi}.
\end{aligned}$$

The main result is as follows.

Theorem 1. *Suppose that the probability distribution of the second-period outcome satisfies Assumption 1. Then, the optimal long-term contract* $\mathbf{w}$ *that implements* $a^1 = \bar{a}$ *and* $a^2 = (\bar{a}, \ldots, \bar{a})$ *is such that*

(a) $w^1(x^1)$ *is a constant for all* x^1 *and*

(b) $w^2(x^1, x^2)$ *is independent of* x^1 *and depends only on* x^2.

Proof. The proof proceeds in two steps. In the first step, we solve a "relaxed" optimization problem as follows:

$$\min_{\mathbf{w}} \sum_{i=1}^{N} p_i^1(a^1) \left[w^1(x_i) + \sum_{j=1}^{N} p_j^2(a^1, a_i^2) w^2(x_i, x_j) \right], \tag{P'}$$

subject to

$$U_i(a^1, a_i^2, \mathbf{w}^2) \geq U_i(a^1, a', \mathbf{w}^2), \quad a' \neq a_i^2, \quad i = 1, \cdots, N, \tag{IC2}$$

$$U(a^1, a^2, \mathbf{w}) \geq 2\bar{u}, \tag{PC}$$

and show that the solution satisfies the properties (a) and (b). In the second step, we verify that (any of the) contract satisfying properties (a) and (b) are always compatible with the constraint (IC1). By these two steps, we conclude that the solution to the "original" optimization problem (P) satisfies properties (a) and (b).

1. The first-order condition for $w^1(x_i)$ in the "relaxed" problem (P′) is

$$\frac{1}{u'(w^1(x_i))} = \nu \quad \text{for all } x_i,$$

where ν is the Lagrange multiplier with respect to (PC). Thus, $w^1(x_i)$ is a constant for all x_i.

The first-order condition for $w^2(x_i, x_j)$ is

$$\frac{1}{u'(w^2(x_i, x_j))} = \frac{\mu_i}{p_i(\bar{a})} \left[1 - \frac{p_j^2(\bar{a}, \underline{a})}{p_j^2(\bar{a}, \bar{a})} \right] + \nu, \tag{3}$$

where μ_i is the Lagrange multiplier with respect to (IC2) for the corresponding i. Here, $w^2(x_i, x_j)$ is independent of i (otherwise, the principal could be strictly better off by offering the certainty equivalence $\tilde{w}'_j$, such that $u(\tilde{w}'_j) = \sum_i p^1_i(\bar{a})u(w^2(x_i,x_j))$, without affecting the remaining constraints (IC2) and (PC)). Hence, the ratio $\mu_i/p_i(\bar{a})$ is a constant for all i.

If $\mu_i = 0$, then $w^2(x_i,x_j)$ is a constant for all j, which violates (IC2) for i. Hence, $\mu_i > 0$ should be satisfied for all i, which means that (IC2) is binding for all i in the optimum. Therefore, $w^2(x_i,x_j)$ depends only on j. In particular, from the concavity of the utility function and the definition of j, $w^2(x_i,x_j)$ is increasing in j .

2. First, we check that (IC1) is satisfied for two deviation strategies, $(a^1;a^2) = (\underline{a};\bar{a},\cdots,\bar{a})$ and $(a^1;a^2) = (\underline{a};\underline{a},\cdots,\underline{a})$, under the optimal contract derived in step 1. Here, we write $w^1(x_i) = w^1$ and $w^2(x_i,x_j) = w^2_j$, as the contract is not dependent on x_i.

As shown in step 1, (IC2) is binding at the optimum. Therefore,

$$C = \sum_{j=1}^{N} p^2_j(\bar{a},\bar{a})u(w^2_j) - \sum_{j=1}^{N} p^2_j(\bar{a},\underline{a})u(w^2_j). \tag{4}$$

From Assumption 1 (i), we have

$$\sum_{j=1}^{N} p^2_j(\bar{a},\underline{a})u(w^2_j) \geq \sum_{j=1}^{N} p^2_j(\underline{a},\bar{a})u(w^2_j), \tag{5}$$

since $u(w_j)$ is an increasing function of j. (5) together with (4) implies (IC1) with $a' = \underline{a}$ and $a'' = \bar{a}$. Similarly, from Assumption 1 (ii), we have

$$2\sum_{j=1}^{N} p^2_j(\bar{a},\underline{a})u(w^2_j) \geq \sum_{j=1}^{N} p^2_j(\bar{a},\bar{a})u(w^2_j) + \sum_{j=1}^{N} p^2_j(\underline{a},\underline{a})u(w^2_j),$$

which implies (IC1) with $a' = a'' = \underline{a}$.

Finally, we check that (IC1) is satisfied for any deviation strategies, $(a^1;a^2) = (\underline{a};a^2_1,\cdots,a^2_N)$. Suppose the agent undertakes $a^2_i = \bar{a}$ if $i \in \bar{I} \subset \{1,\cdots,N\}$ and $a^2_i = \underline{a}$ if $i \in \underline{I} = \{1,\cdots,N\} \setminus \bar{I}$. The intertemporal payoff to the agent following this deviation strategy satisfies

$$u(w^1) + \sum_{i\in\bar{I}} p^1_i(\underline{a}) \left[\sum_{j=1}^{N} p^2_j(\underline{a},\bar{a})u(w^2_j) - C\right] + \sum_{i\in\underline{I}} p^1_i(\underline{a}) \left[\sum_{j=1}^{N} p^2_j(\underline{a},\underline{a})u(w^2_j)\right]$$

$$
\begin{aligned}
&\leq u(w^1) + \max\left\{ \sum_{j=1}^{N} p_j^2(\underline{a},\bar{a})u(w_j^2) - C,\ \sum_{j=1}^{N} p_j^2(\underline{a},\underline{a})u(w_j^2) \right\} \\
&= \max\{U(\underline{a};\bar{a},\cdots,\bar{a};\mathbf{w}),\ U(\underline{a};\underline{a},\cdots,\underline{a};\mathbf{w})\} \\
&\leq U(\bar{a};\bar{a},\cdots,\bar{a};\mathbf{w}),
\end{aligned}
$$

where the last inequality is derived from the previous result that (IC1) is satisfied both for $(a^1;a^2) = (\underline{a};\bar{a},\cdots,\bar{a})$ and for $(a^1;a^2) = (\underline{a};\underline{a},\cdots,\underline{a})$. Hence, (IC1) is satisfied for any deviation strategy $(a^1;a^2) = (\underline{a};a_1^2,\cdots,a_N^2)$. □

The intuition behind the proof is as follows. For the principal who intends to induce the agent to exert the positive effort $\bar{a}$ in period 2, it is necessary to make the second-period payment $w^2(x_i,x_j)$ dependent on the second-period outcome x_j, as this is the only source of incentive power available. However, such a payment schedule induces the agent to work hard in period 1 because the distribution of second-period outcomes is affected not only by a^2 but also by a^1. Assumption 1 (i) ensures that the agent always obtains a larger *gross* expected payoff in terms of wages by undertaking action profile $(\bar{a},\underline{a})$ than by undertaking $(\underline{a},\bar{a})$ as a result of the FOSD. In addition, as the cost of effort, C, is the same in both periods, the agent obtains a larger *net* expected payoff as well. Thus, if the contract is to induce hard work by the agent in the second period, it automatically provides the agent with the incentive to work hard in the first period. Assumption 1 (ii), on the other hand, ensures that the agent does not deviate to a strategy of shirking in both periods (i.e., to $(\underline{a},\underline{a})$). Half-and-half mixture of the two probability distributions, $(\bar{a},\bar{a})$ and $(\underline{a},\underline{a})$, gives the agent's *gross* expected payoff from taking $(\underline{a},\underline{a})$ in accordance with the benefit of effort cost reduction normalized to C (a one-time shirk). Thus, if the contract is to induce hard work in the second period, it automatically makes the agent worse off if he shirks in both periods, $(\underline{a},\underline{a})$.

To summarize, if the probability distribution of the second-period outcome when the agent shirks only in the second period $(\bar{a},\underline{a})$ *FOS-dominates* the distribution when the agent shirks in any other periods in such a way that *the expected number of shirkings is one*, providing incentives to work hard in the second period becomes sufficient to induce the agent to make strong efforts in both periods. As we will see in Section 3, such arguments regarding *FOSD* and *one-time shirking* play central roles in T-period models as well. We also discuss the sufficient conditions for simple contracts for T-period models in a similar manner.

2.2 Non-MLRP Case

In this subsection, we discuss how an analogue of Assumption 1 is described if $p_j^2(\bar{a},a)$ and $p_j^2(\bar{a},\underline{a})$ do not satisfy MLRP.

For making the problem non-trivial, we suppose that

$$\frac{p_j^2(\bar{a},\bar{a})}{p_j^2(\bar{a},\underline{a})} \neq \frac{p_k^2(\bar{a},\bar{a})}{p_k^2(\bar{a},\underline{a})} \quad \text{for all } j \neq k$$

is satisfied.[13] Then we have a unique rearrangement (permutation) $\kappa(1),\ldots,\kappa(N)$ of $1,\ldots,N$ so that

$$\frac{p_{\kappa(1)}^2(\bar{a},\bar{a})}{p_{\kappa(1)}^2(\bar{a},\underline{a})} < \cdots < \frac{p_{\kappa(N)}^2(\bar{a},\bar{a})}{p_{\kappa(N)}^2(\bar{a},\underline{a})}.$$

MLRP in the previous subsection is equivalent to $(\kappa(1),\ldots,\kappa(N)) = (1,\ldots,N)$.

Then an analogue of Assumption 1 (i) is written as

$$\sum_{k=1}^{j} p_{\kappa(k)}^2(\underline{a},\bar{a}) \geq \sum_{k=1}^{j} p_{\kappa(k)}^2(\bar{a},\underline{a}) \quad \text{for all } j = 1,\ldots,N,$$

which implies, for any increasing function $\gamma\colon \{1,\ldots,N\} \to \mathbb{R}$,

$$\sum_{j=1}^{N} p_{\kappa(j)}^2(\bar{a},\underline{a})\gamma(j) \geq \sum_{j=1}^{N} p_{\kappa(j)}^2(\underline{a},\bar{a})\gamma(j).$$

The difference from the normal stochastic dominance cases is that the function γ should be increasing in terms of the permutated numbers $\kappa(1),\ldots,\kappa(N)$.

Then (3) in the proof of Theorem 1 is rewritten as

$$\frac{1}{u'(w^2(x_i,x_{\kappa(j)}))} = \frac{\mu_i}{p_i(\bar{a})}\left[1 - \frac{p_{\kappa(j)}^2(\bar{a},\underline{a})}{p_{\kappa(j)}^2(\bar{a},\bar{a})}\right] + \nu,$$

[13] Equalities can be allowed for a subset of $\{1,\ldots,N\}$, but we avoid such cases for the sake of a brief exposition.

and, following the similar argument as in the proof, $w^2(x_i, x_{\kappa(j)})$ is increasing in j (and independent of i). Then (5) is rewritten as

$$\sum_{j=1}^{N} p^2_{\kappa(j)}(\bar{a},\underline{a})u(w^2_{\kappa(j)}) \geq \sum_{j=1}^{N} p^2_{\kappa(j)}(\underline{a},\bar{a})u(w^2_{\kappa(j)}),$$

which is satisfied since $u(w^2_{\kappa(j)})$ is increasing in j and $(\bar{a},\underline{a})$ FOS-dominates $(\underline{a},\bar{a})$ in terms of $\kappa(j)$. Similar argument holds for Assumption 1 (ii) and its implications in the proof.

The following example depicts the role of $\kappa(j)$ in the case where MLRP is not satisfied. Suppose $N = 3$ (e.g., a performance is either low, middle, or high), and the distributions are given as in the table.

j	1	2	3
$(\bar{a},\bar{a})$	0.3	0.2	0.5
$(\bar{a},\underline{a})$	0.3	0.4	0.3
$(\underline{a},\bar{a})$	0.4	0.4	0.2
$(\underline{a},\underline{a})$	0.3	0.6	0.1
LR	1.0	0.5	1.6

$\Rightarrow$

$\kappa(1) = 2$	$\kappa(2) = 1$	$\kappa(3) = 3$
0.2	0.3	0.5
0.4	0.3	0.3
0.4	0.4	0.2
0.6	0.3	0.1
0.5	1.0	1.6

The likelihood ratio (LR in the table) is not monotone in the original numbers $1,2,3$, but we have a permutation $\kappa(1), \kappa(2), \kappa(3)$ so that LR is monotone. It is easy for the reader to check that $(\bar{a},\underline{a})$ FOS-dominates both $(\underline{a},\bar{a})$ and $\frac{1}{2}((\bar{a},\bar{a}) + (\underline{a},\underline{a}))$ (in terms of $\kappa(1),\ldots,\kappa(N)$), and hence, only the final outcome matters in the optimal contract.

3 Extension

In this section, we extend the basic model to a T-period setup and show that a similar result as in Theorem 1 are obtained. As in Section 2, we let $a^t \in \{\bar{a},\underline{a}\} = A$ and $x^t \in \{x_1,\ldots,x_N\} = X$ denote the agent's actions and outcomes in each period $t = 1,\cdots,T$, respectively. We also let $\mathbf{a}^t = (a^1,\ldots,a^t)$ and $\mathbf{x}^t = (x^1,\ldots,x^t)$. The distribution of each outcome x^t is dependent on the whole past history of actions $\mathbf{a}^t$. We write the distributions as follows:

$$p_i^t(\mathbf{a}^t) = \Pr\left[x^t = x_i \mid \mathbf{a}^t\right], \quad i = 1, \cdots, N.$$

Throughout this section, we assume the *monotone likelihood ratio property* (MLRP) for the final period outcome:

$$\frac{p_1^T(\bar{a}, \ldots, \bar{a}, \bar{a})}{p_1^T(\bar{a}, \ldots, \bar{a}, \underline{a})} < \cdots < \frac{p_N^T(\bar{a}, \ldots, \bar{a}, \bar{a})}{p_N^T(\bar{a}, \ldots, \bar{a}, \underline{a})}.$$

This is just for a simplification as in Subsection 2.2. If the model does not satisfy MLRP, we can rearrange the order of outcomes so that the sufficient condition for the simple contracts as given below is written in a consistent way (see Subsection 2.3). It should be noted that MLRP is imposed only on the distributions by $(\bar{a}, \ldots, \bar{a}, \bar{a})$ and $(\bar{a}, \ldots, \bar{a}, \underline{a})$.

We also assume that the agent decides his entire action profile $\mathbf{a}^T = (a^1, \cdots, a^T)$ at the beginning of period 1 and that he never changes this profile after observing the outcomes in each period.[14] We split the agent's action space $A^T = A \times \cdots \times A$ into partitions $(\mathscr{A}_0, \ldots, \mathscr{A}_T)$, where

$$\mathscr{A}_k = \left\{(a^1, \ldots, a^T) \mid \#\left\{t \mid a^t = \underline{a}\right\} = k\right\}, \quad k = 0, \ldots, T.$$

That is, $\mathscr{A}_k$ is the set of the action profile $\mathbf{a}^T$ in which there are k weak efforts $\underline{a}$ (and hence, $T-k$ strong efforts $\bar{a}$).[15] For instance, if $T = 3$, then we have $\mathscr{A}_0 = \{(\bar{a}, \bar{a}, \bar{a})\}$, $\mathscr{A}_1 = \{(\bar{a}, \bar{a}, \underline{a}), (\bar{a}, \underline{a}, \bar{a}), (\underline{a}, \bar{a}, \bar{a})\}$, etc.

Let $V(\mathbf{a}, \mathbf{w})$ denote the agent's *gross* expected payoff from payment schedule $\mathbf{w}$ when he takes action profile $\mathbf{a}$. Then, the agent's *net* expected payoff is written

[14] In the basic model presented in Section 2, it is assumed that the agent makes his second-period action a^2 after observing the first-period outcome x^1; therefore, the action profile consists of $N+1$ components $(a^1; a_1^2, \ldots, a_N^2)$, where a_i^2 denotes the second-period action when the first-period outcome is x_i. For the T-period model in this section, we may also consider the possibility that the agent's actions depend on past outcomes (the action profile in such a model consists of $(N^T-1)/(N-1)$ components), but such a consideration does not change the result in Theorem 2. See the Appendix for more on this point.

[15] It is obvious that $(\mathscr{A}_0, \ldots, \mathscr{A}_T)$ satisfies $\mathscr{A}_k \cap \mathscr{A}_l = \emptyset$ for any k, l $(k \neq l)$, and $\bigcup_k \mathscr{A}_k = A$. Hence, $(\mathscr{A}_0, \cdots, \mathscr{A}_T)$ is a partition of A.

as

$$V(\mathbf{a},\mathbf{w}) - C \cdot m(\mathbf{a}),$$

where C is the cost of a strong effort $\bar{a}$ (as in Section 2) and $m(\mathbf{a})$ is the number of strong efforts in action profile $\mathbf{a}$. Then, the incentive compatibility constraint in the T-period model is simplified as follows: for $k = 1, \ldots, T$,

$$V(\bar{a}, \ldots, \bar{a}, \mathbf{w}) - C \cdot k \geq V(\mathbf{a}', \mathbf{w}), \quad \text{for all } \mathbf{a}' \in \mathscr{A}_k. \tag{IC$_k$}$$

Now, we have an extension of Assumption 1.

Assumption 2. For all $k = 1, \ldots, T$ and all $\mathbf{a}' \in \mathscr{A}_k$,

$$\frac{\sum_{i=1}^{j} \left[(k-1)p_i^T(\bar{a}, \ldots, \bar{a}, \bar{a}) + p_i^T(\mathbf{a}')\right]}{k} \geq \sum_{i=1}^{j} p_i^T(\bar{a}, \ldots, \bar{a}, \underline{a}) \tag{6}$$

holds for all $j = 1, \ldots, N$.

The following is an important example of Assumption 2.

Example 3 (Human capital investment)**.** Suppose that the distributions of outcomes in each period are dependent not on the detail of past actions, but on the number of strong efforts that the agent has taken to date. To be specific, we let $q_i(k)$ denote the probability distribution when the agent has undertaken k strong efforts, and provide $p_i^t(\cdot)$ as

$$p_i^t(a^1, \ldots, a^t) = q_i(\#\{\tau \leq t \mid a^\tau = \bar{a}\}).$$

Such distributions depict the agent's human capital investment or the learning-by-doing effect of the agent's effort.

In this example, the requirement of Assumption 2 is the following: For all $k = 2, \cdots, T$, the inequality

$$\frac{\sum_{i=1}^{j} [(k-1)q_i(T) + q_i(T-k)]}{k} \geq \sum_{i=1}^{j} q_i(T-1)$$

holds for all $j = 1, \dots, N$. This condition seems rather artificial, but it includes an important economic environment in which the investment has *nonincreasing marginal returns*:

$$\sum_{i=1}^{j} [q_i(k+1) - q_i(k)] \geq \sum_{i=1}^{j} [q_i(k) - q_i(k-1)]. \tag{7}$$

This inequality states that the marginal "benefit" in the probability distribution of one additional effort is decreasing in k. It is easy to show that (7) is a special case of condition (ii).[16]

Theorem 2. *Suppose that $p_i^T(\cdot)$ satisfies Assumption 2. Then, payments in the optimal long-term contract are dependent only on the final outcome x^T.*

Remark. Assumption 2 is imposed only on the distribution of outcomes in the final period ($t = T$), and how the efforts of the agent affect the outcomes in other periods ($t = 1, \dots, T-1$) is irrelevant to the result of Theorem 2. We should discuss the reason briefly.

As long as the principal wants to make the agent work hard in the final period ($t = T$), the optimal wage should depend (at least) on the final outcome, x^T. If the economic environment satisfies Assumption 2, then the agent works hard in other periods ($t = 1, \dots, T-1$) under the wage scheme that gives enough incentives to work hard in the final period. As long as the principal finds it optimal to implement high efforts (as usually assumed in studies on moral hazard, including the present

[16]Kwon (2006) investigated a similar model with binary outcomes ($N = 2$) and showed that the optimal long-term contract is dependent only on the final outcome under the assumption of "nonincreasing marginal returns." Although nonincreasing marginal returns is an easy assumption to interpret economically, it is a rather strong assumption if T is a substantially large number, as it requires that the inequality (7) be satisfied for *all* possible numbers of high efforts, $k = 1, \dots, T-1$. On the other hand, consider the following distributions when $T = 3$ and $N = 2$:

$$q_1(0) = 0.9, \quad q_1(1) = 0.8, \quad q_1(2) = 0.2, \quad q_1(3) = 0.1,$$

($q_1(\cdot)$ represents the probability of "failure" when $N = 2$). This is not "nonincreasing returns," but it satisfies our FOSD condition. Our result suggests that what is central to the incentives in simple contracts is the FOSD relationship, and nonincreasing marginal returns is just one example of the condition. In addition, note that the two conditions coincide if (and only if) $T = 2$.

one), other distributions are irrelevant in the principal's cost minimization problem and only the final distribution is central to the contract in which only the final outcome matters.

We should also discuss that Assumption 2 is close to a "necessary" condition in the following informal manner. Suppose that $k = 1$ and only $\mathbf{a}' = (\underline{a}, \bar{a}, \ldots, \bar{a}) \in \mathscr{A}_1$ violates (6). If the contract gives enough incentive to work hard in the final period (and the IC in the final period is binding), the agent finds it optimal to work hard in periods $t = 2, \ldots, T$, but *may* not in the first period.[17] If the utility function of the agent is such that the incentive to work hard in the first period is not provided by the simple contract, the principal needs to rewrite the contract. However, the precise form of the optimal contract is complicated in dynamic models, so we cannot argue whether or not only the final outcome matters in the optimal contract in the way using the FOSD condition as shown in the present paper.

Sketch of the proof. As in the proof of Theorem 1, we can show that the optimal long-term contract is independent of outcome history $\mathbf{x}^{T-1}$ up to the period $T-1$, if all incentive constraints are not binding with the exception of

$$V(\bar{a}, \ldots, \bar{a}, \bar{a}, \mathbf{w}) - C \geq V(\bar{a}, \ldots, \bar{a}, \underline{a}, \mathbf{w}) \tag{8}$$

In the following, we show that the derived contract, which is dependent only on the final outcome x^T, satisfies all incentive compatibility constraints.

As the derived contract $\mathbf{w}^*$ satisfies (8) with equality, we have

$$C = V(\bar{a}, \ldots, \bar{a}, \bar{a}, \mathbf{w}^*) - V(\bar{a}, \ldots, \bar{a}, \underline{a}, \mathbf{w}^*). \tag{9}$$

For each $k = 1, \ldots, T$, substituting (9) into (IC_k) yields:

$$k \cdot V(\bar{a}, \ldots, \bar{a}, \underline{a}, \mathbf{w}^*) \geq (k-1) \cdot V(\bar{a}, \ldots, \bar{a}, \bar{a}, \mathbf{w}^*) + V(\mathbf{a}', \mathbf{w}^*).$$

Condition (6) ensures that this inequality holds. □

[17] FOSD between random variables X and Y is a sufficient condition for that $E[u(X)] \geq E[u(Y)]$ holds for an exogenously given increasing function $u(\cdot)$, but not a necessary one (Shaked and Shanthikumar (2007)).

4 Concluding Remarks

This paper explores the role of history dependence in a dynamic moral hazard model. It is shown that, under certain conditions on the probability distributions of outcomes, the optimal long-term contract is such that the payment schedules are not contingent upon the realization of past outcomes. This finding contrasts strikingly with the results in *repeated* moral hazard models, where the optimal long-term contracts are generally dependent on the complete history of past outcomes.

An important point of the results is the relationship between statistical informativeness of the signals (outcomes) and its effects on incentive problems in the optimal contracts. In the light of statistical inference of agent's past efforts, the history of all outcomes must be valuable to the principal (i.e., informative), because the distributions of outcomes are assumed to be independent over time in the present analysis (Apart from the information provided by x^2 about a^1, x^1 provides a statistically independent information about a^1). The principal, however, sometimes finds it optimal to ignore such x^1 in the present paper. We believe the result that the principal finds it optimal to "ignore" informative signals is found in other (non-trivial) contract theory models, but we leave this for future research.

This paper demonstrated a particular class of dynamic moral hazard models in which the optimal contract is written in a simple manner. This extra conclusion brings some new insights to the incentive provision in moral hazard problems, but does not extend to general models on which we pose no conditions on the information structure. Perhaps more important question is to explore the general structure of dynamic incentives in which efforts have persistent effects over time. The general theoretical examination of these problems remains to be done.

Appendix

In this Appendix, we provide some mathematical arguments for footnote 14 in Section 3.

Suppose that the agent is to undertake actions dependent on past outcomes. We think of such an agent's strategy as a sequence of "behavior strategy"; for example,

$$\alpha = (\alpha^1, \alpha^2(x^1), \alpha^3(x^1, x^2), \ldots, \alpha^T(x^1, \ldots, x^{T-1})),$$

where each $\alpha^t : \{1, \ldots, N\}^{t-1} \to A$ is a mapping from the history of past outcomes (up to period $t-1$) to the action in period t.

The problem in footnote 14 is whether the agent improves his payoff by selecting such a history-dependent strategy α (rather than a history-independent strategy $\mathbf{a} = (a^1, a^2, \ldots, a^T)$).

Theorem 3. *If the contract is* simple*, then the agent cannot improve his payoff by selecting a history-dependent strategy* α*.*

Proof. The expected payoff to the agent undertaking such a strategy α is written as

$$\sum_{\mathbf{a} \in A^T} \left\{ \left(\sum_{(x^1, \ldots, x^{T-1}) \in I(\mathbf{a}, \alpha)} \prod_{t=1}^{T-1} \Pr[x^t \mid \alpha^t(x^1, \ldots, x^{t-1})] \right) \times \left(\sum_{t=1}^{T-1} u(w^t) + \sum_{j=1}^{N} p_j^T(\mathbf{a}) u(w_j^T) - C \cdot m(\mathbf{a}) \right) \right\} \tag{10}$$

where

$$I(\mathbf{a}, \alpha) = \left\{ (x^1, \ldots, x^{T-1}) \mid \alpha^t(x^1, \ldots, x^{t-1}) = a^t \text{ for } t = 1, \ldots, T \right\};$$

that is, $I(\mathbf{a}, \alpha)$ is the set of historical outcomes with the positive probability that action profile $\mathbf{a}$ is played under strategy α.

Because every history $(x^1, \ldots, x^{T-1})$ generates exactly one action profile, $\mathbf{a}$ given α, the first parentheses in (10) is seen as a probability distribution of $\mathbf{a}$ over

A^T; that is,

$$\sum_{\mathbf{a}\in A^T}\left(\sum_{(x^1,\ldots,x^{T-1})\in I(\mathbf{a},\alpha)}\prod_{t=1}^{T-1}\Pr[x^t \mid \alpha^t(x^1,\ldots,x^{t-1})]\right)=1 \text{ for any } \alpha.$$

As the expected value of random variables does not exceed the maximum of the variables, we establish that

$$\begin{aligned}(10) &\leq \max_{\mathbf{a}\in A^T}\left(\sum_{t=1}^{T-1}u(w^t)+\sum_{j=1}^{N}p_j^T(\mathbf{a})u(w_j^T)-C\cdot m(\mathbf{a})\right)\\ &= \max_{\mathbf{a}\in A^T}\left(V(\mathbf{a},\mathbf{w})-C\cdot m(\mathbf{a})\right).\end{aligned}$$

□

References

CHIAPPORI, P.-A., I. MACHO, P. REY, AND B. SALANIÉ (1994): "Repeated Moral Hazard: The Role of Memory, Commitment, and the Access to Credit Markets," *European Economic Review*, 38, 1527–1553.

FUDENBERG, D., B. HOLMSTROM, AND P. MILGROM (1990): "Short-Term Contracts and Long-Term Agency Relationships," *Journal of Economic Theory*, 51, 1–31.

FUDENBERG, D., AND J. TIROLE (1990): "Moral Hazard and Renegotiation in Agency Contracts," *Econometrica*, 58, 1279–1319.

HOLMSTRÖM, B. (1979): "Moral Hazard and Observability," *Bell Journal of Economics*, 10, 74–91.

HOLMSTRÖM, B., AND P. MILGROM (1987): "Aggregation and Linearity in the Provision of Intertemporal Incentives," *Econometrica*, 55, 303–328.

JARQUE, A. (2010): "Repeated Moral Hazard with Effort Persistence," *Journal of Economic Theory*, 145, 2412–2423.

KWON, I. (2006): "Incentives, Wages, and Promotions: Theory and Evidence," *RAND Journal of Economics*, 37, 100–120.

LAMBERT, R. A. (1983): "Long-Term Contracts and Moral Hazard," *Bell Journal of Economics*, 14, 441–452.

MA, C.-T. A. (1991): "Adverse Selection in Dynamic Moral Hazard," *Quarterly Journal of Economics*, 106, 255–275.

——— (1994): "Renegotiation and Optimality in Agency Contracts," *Review of Economic Studies*, 61, 109–129.

MAS-COLELL, A., M. WHINSTON, AND J. GREEN (1995): *Microeconomic Theory*. Oxford: Oxford University Press.

MATTHEWS, S. A. (1995): "Renegotiation of Sales Contracts," *Econometrica*, 63, 567–589.

MILGROM, P. R. (1981): "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, 12, 380–391.

MUKOYAMA, T., AND A. ŞAHIN (2005): "Repeated Moral Hazard with Persistence," *Economic Theory*, 25, 831–854.

OGAWA, R. (2003): "Memoryless Contract in Repeated Moral Hazard with History Dependence," Master's thesis, The University of Tokyo.

ROGERSON, W. P. (1985): "Repeated Moral Hazard," *Econometrica*, 53, 69–76.

SHAKED, M., AND J. SHANTHIKUMAR (2007): *Stochastic Orders*. New York: Springer Science+Business Media.