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AND
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ADJUSTMENT TO INCOME SHOCKS**

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Global Habits, Habit Differentials, and International Macroeconomic Adjustment to Income Shocks¹

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Abstract

In a two-country model with habit formation, we focus on interdependent macroeconomic adjustments to global and country-specific income shocks. Global habits and habit differentials play key roles in the global equilibrium dynamics, possibly nonmonotonic, and in the determination of international asset distribution. A country's steady-state holdings of net external assets rely on (i) weighted income difference in excess of habit differentials and (ii) global income in excess of global habits. Local income shocks have greater effects on the international asset distribution than global income shocks. With habit formation, positive income shocks lower the world interest rate, thereby harming the creditor country and benefitting the debtor country due to the intertemporal terms-of-trade effect. In contrast to the case of trade theory, this intertemporal immiserizing growth effect is more likely to be caused by global income shocks than by country-specific income shocks.

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1 Introduction

The purpose of this paper is to focus on habit formation as a key factor that determines macroeconomic adjustments of an interdependent world economy. In doing so, we describe two-country equilibrium dynamics in terms of the evolution of global consumption habits and international habit differentials. When consumers in both countries are rationally habit forming, the two countries' propensities to consume depend on their individual habits. The global habits, defined as the sum of individual countries' habits, determine the aggregate preference for "present" goods, and hence decide the world equilibrium interest rate. The habit differentials between the two countries determine the difference in the consumption propensities and hence affect the dynamics of the international asset distribution. The novelty of this paper is to work out international interactions that these two forces produce.

We tackle three important issues: (i) the determinants of the equilibrium external asset holdings, (ii) the differences in the effects of global and local income shocks, and (iii) the welfare implications of habit-driven macroeconomic adjustments. We show that a country's net foreign assets depend on habit differentials and global habits. In particular, in steady state, a country's net external asset holdings rely on weighted income difference in excess of habit differentials and global income in excess of global habits. One important implication is that local income shocks have greater effects than global income shocks on the international income differential and hence on the international asset distribution.

With habit formation, positive income shocks lower the world interest rate, thereby harming the creditor country and benefitting the debtor country due to the intertemporal terms-of-trade effect. This is an intertemporal version of the immiserizing growth effect developed by, e.g., Bhagwati (1958) and Brecher and Bhagwati (1982). We show that this intertemporal immiserizing growth effect is more likely to be caused by global income shocks than by country-specific income shocks. This result contrasts to the case of trade theory: in trade theory, positive supply shocks commonly occurring in the individual countries' exporting sectors have smaller effects on the static terms of trade than positive supply shocks occurring locally in one of the two countries. In our dynamic context, when a positive income shock is global, the relative magnitudes of the harmful intertemporal terms-of-trade effect to the direct beneficial income effect is larger than they would be when the shock is country-specific.

The analytical model employs the two-country framework developed by Gombi and Ikeda (2003) and Ikeda and Gombi (2009). Those papers were concerned with the effects of fiscal policies under preference heterogeneity in habit formation, where the role of global habits in dynamic adjustment were assumed away by setting the initial holdings of net foreign assets to be zero. By assuming homogeneous preferences, the present research focuses on interactions among global habits, habit differentials, and the consumption/saving dynamics, and thereby examining their implications for international asset distribution and the adjustments to global and local income shocks.¹

The remainder of the paper is structured as follows: In Section 2, we present a two-country model and examine equilibrium dynamics of consumption, the interest rate, and net foreign assets. In Section 3, we analyze the effects of local and global income shocks on the economy and each countries' welfare. Section 4 concludes the paper.

2 The Two-Country Model

2.1 The basic framework

Consider a two-country world economy composed of home and foreign countries. Each country is populated with infinitely lived agents with homogeneous preferences. The representative agents in home and foreign countries are referred to as consumers H and F, respectively. They consume a single type of consumption goods and hold wealth in the form of bonds. Both goods and bonds are assumed to be costlessly traded in international markets. For brevity, the representative agents H and F are assumed to be endowed with constant amounts of output y and y^* , respectively. Throughout the paper, the foreign country's variables are represented with superscript asterisks.

Consumption forms habits. Letting $z_t^{(*)}$ represent the time- t *habit*, we specify $z_t^{(*)}$ as the average of the past consumption rates $c_s^{(*)}$, $s \leq t$: $z_t^{(*)} =$

¹For small country models with habit formation, see, e.g., Mansoorian (1993a, b) and Ikeda and Gombi (1999). Gruber (2002) provides empirical support to an “intertemporal current account model” with habit formation. The literature concerning two-country dynamic models includes Devereux and Shi (1991), Ikeda and Ono (1991), Bianconi and Turnovsky (1997), and Bianconi (2003).

$\alpha \int_{-\infty}^t c_s^{(*)} \exp(-\alpha(t-s)) ds$, or equivalently

$$\dot{z}_t^{(*)} = \alpha \left(c_t^{(*)} - z_t^{(*)} \right), \quad (1)$$

where \dot{x} represents the time derivative of variable x and α represents the discount rate for past consumption rates. We assume that consumers in both countries have the common discount rate α for past consumption rates. This enables us to obtain the tractable dynamics of a two-country equilibrium.

Consumers H and F are assumed to have the same lifetime utility function specified as

$$U_0^{(*)} = \int_0^{\infty} u(c_t^{(*)}, z_t^{(*)}) \exp(-\theta t) dt, \quad (2)$$

where θ represents the subjective discount rate. To ensure the steady state, the discount rate is the same in the two countries.

We focus on the typical effect of habit formation in a simple way by specifying the felicity function as

$$u \left(c_t^{(*)}, z_t^{(*)} \right) = \frac{\left(c_t^{(*)} - \gamma z_t^{(*)} \right)^{1-\varphi}}{1-\varphi}, \quad (3)$$

where the habit parameter $\gamma \in (0, 1)$ captures the strength of habit influence; and φ represents a risk aversion parameter.²

With this felicity function, consumer preferences display adjacent complementarity, wherein an increase in today's habits increases the marginal utility of today's consumption more than it increases marginal disutility of habits, thereby, *ceteris paribus*, enlarging today's optimal consumption. To be formal, Ryder and Heal (1973) define adjacent complementarity as the felicity function satisfying $u_{cz}(c, c) + \frac{\alpha}{\theta+2\alpha} u_{zz}(c, c) > 0$. To capture the intertemporal complementarity, define Ω as $-(u_{cz} + \frac{\alpha}{\theta+2\alpha} u_{zz}) / u_{cc}$. Then, with the felicity function (3), the index Ω can be computed as

$$\Omega = \gamma \left(1 - \frac{\alpha\gamma}{2\alpha + \theta} \right) > 0, \quad (4)$$

²The felicity function satisfies the regularity conditions proposed by the Ryder and Heal (1973). The same felicity function is assumed by, e.g., Constantinides (1990) and Gruber (2004).

which implies that consumer preferences in the two countries commonly display adjacent complementarity.³

Let b_t denote net foreign assets held by consumer H. The flow budget constraint for consumer H is given by

$$\dot{b}_t = r_t b_t + y - c_t. \quad (5)$$

Given the initial values (b_0, z_0) and the (perfectly predicted) time profile of the market interest rate $\{r_t\}_{t=0}^{\infty}$, consumer H chooses $C_0 = \{c_t, b_t, z_t\}_{t=0}^{\infty}$ so as to maximize (2) subject to: (i) the flow budget constraint (5); (ii) the formation of consumption habits (1); and (iii) the transversality conditions.

Letting $\lambda_t (\geq 0)$ be the shadow price of savings and $\xi_t (\leq 0)$ that of habit formation, the optimal conditions are given by

$$u_c(c_t, z_t) = \lambda_t - \alpha \xi_t, \quad (6)$$

$$\dot{\lambda}_t = (\theta - r_t) \lambda_t, \quad (7)$$

$$\dot{\xi}_t = (\theta + \alpha) \xi_t - u_z(c_t, z_t), \quad (8)$$

where $u_c(c, z) = (c_t - \gamma z_t)^{-\varphi}$ and $u_z(c_t, z_t) = -\gamma (c_t - \gamma z_t)^{-\varphi}$, together with (1), (5), and the transversality conditions for b_t and z_t . Consumer F's behavior can be specified in exactly the same way.

We assume away any role of the governments. The model is closed by introducing the market clearing conditions:

$$c_t + c_t^* = Y (\equiv y + y^*), \quad (9)$$

$$b_t + b_t^* = 0, \quad (10)$$

where Y represents the aggregate income. By Walras' law, these are not independent: (10) together with (5) and the corresponding constraint for the foreign consumer imply (9). In sum, the equilibrium time path of $(b_t, b_t^*, c_t, c_t^*, z_t, z_t^*, r_t, \lambda_t, \lambda_t^*, \xi_t, \xi_t^*)$ is determined by equations (1), (5) through (8), the corresponding equations for F, and the market equilibrium condition (9) or (10).

³Even if the risk aversion parameter φ differs between the both countries, the relation, $\Omega^H = \Omega^F = \Omega = \gamma \left(1 - \frac{\alpha\gamma}{2\alpha+\theta}\right) > 0$, is retained. The discussions in this section, therefore, can be extended without substantial changes to the heterogenous risk aversion cases.

2.2 Equilibrium dynamics

In this economy consumption dynamics in the two countries interact through international market transactions. It is useful to define *global habit* Z_t as

$$Z_t \equiv z_t + z_t^*. \quad (11)$$

Since α is assumed to be common to the two countries, the dynamics of Z_t can be expressed from (1) and the market clearing condition (9) as

$$\dot{Z}_t = \alpha (Y - Z_t). \quad (12)$$

If Z_t and z_t are given, z_t^* are determined from (11).

As in Ikeda and Gombi (2009), dynamics can be drastically simplified in the following manner. Define σ as:

$$\sigma = \frac{\lambda}{\lambda^*},$$

which is constant over time since $\dot{\lambda}_t/\lambda_t = \dot{\lambda}_t^*/\lambda_t^*$ from (7) (and the corresponding equation for F). We then construct aggregate indices for (u, u^*) and (ξ, ξ^*) as:

$$v(c, z, Z) \equiv u(c, z) + \sigma u^*(Y - c, Z - z), \quad (13)$$

$$\varsigma \equiv \xi - \sigma \xi^*, \quad (14)$$

where (9) and (11) are substituted.

As shown in Appendix A, we can reduce the equilibrium dynamics of consumption habits around a steady-state point as follows:

$$\begin{pmatrix} \dot{z} \\ \dot{\varsigma} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} -\alpha \left(1 + \frac{v_{cz}}{v_{cc}}\right) & -\frac{\alpha^2}{v_{cc}} & -\frac{\alpha v_{cz}}{v_{cc}} \\ \frac{(v_{cz})^2 - v_{cc} v_{zz}}{v_{cc}} & \theta + \frac{\alpha(v_{cc} + v_{cz})}{v_{cc}} & \frac{v_{cz} v_{cz} + v_{cc} v_{zz}}{v_{cc}} \\ 0 & 0 & -\alpha \end{pmatrix} \begin{pmatrix} z_t - \bar{z} \\ \varsigma_t - \bar{\varsigma} \\ Z_t - \bar{Z} \end{pmatrix}; \quad (15)$$

where \bar{x} denotes a steady-state value of variable x . Dynamic system (15) has two stable roots:

$$\omega \equiv \frac{\theta - \sqrt{(\theta + 2\alpha)^2 - 4\alpha(\theta + 2\alpha)\Omega}}{2} (< 0) \text{ and } -\alpha, \quad (16)$$

and one unstable root, which is conjugate with ω ; and Ω is given by (4). From the assumed property of adjacent complementarity, it can be shown that $\omega + \alpha > 0$.

As shown in Appendix B.1, the saddle plane governed by the two stable roots are expressed as

$$\dot{z} = \omega(z_t - \bar{z}) - (\omega + \alpha)(1 - \delta)(Z_t - \bar{Z}), \quad (17)$$

$$\dot{Z} = -\alpha(Z_t - \bar{Z}), \quad (18)$$

where

$$\delta \equiv \frac{u_{cc}}{u_{cc} + \sigma u_{cc}^*}.$$

By equating (17) to (1), the consumption dynamics are given by

$$c_t - \bar{c} = \left(\frac{\omega + \alpha}{\alpha} \right) [(z_t - \bar{z}) - (1 - \delta)(Z_t - \bar{Z})]. \quad (19)$$

Differentiate this by t and substitute (1), (18), and (19) successively into the result. Then, by taking (9) into account, we obtain the motion of each country's consumption as

$$\dot{c} = \omega(c_t - \bar{c}), \quad (20)$$

$$\dot{c}^* = \omega(c_t^* - \bar{c}^*). \quad (21)$$

Irrespective of the second-order habit dynamics of (17) and (18), therefore, the equilibrium consumption dynamics are of the first order. Equations (1) and (20) jointly govern the equilibrium dynamics of (c, z) ; and equations (1) and (21) do dynamics of (c^*, z^*) .

As is proven by Appendix B.2, the interest rate dynamics are given by

$$r_t - \theta = \eta \Omega (Z_t - \bar{Z}), \quad (22)$$

where η is defined as $\eta \equiv -\frac{\alpha u_{cc} u_{cc}^*}{\lambda^* u_{cc} + \lambda u_{cc}^*}$ (> 0), implying that Z_t plays a crucial role. Suppose that $Z_t > \bar{Z}$. Then, due to adjacent complementarity, i.e., $\Omega > 0$, *ceteris paribus* there prevails excess demand in the present good market. This renders r higher than its steady-state value θ . The equilibrium interest rate positively (negatively) comoves with the aggregate habit stock, which exhibits monotonic motions with stable root $-\alpha$ (see (18)). The resulting dynamics of r are given explicitly by

$$\dot{r} = -\alpha(r_t - \theta). \quad (23)$$

The transition dynamics of net foreign assets also depend on the property of the world felicity function and international heterogeneity in habit formation. As shown by Appendix B.3, by linearizing (5) and substituting (19) and (22) into the result, we can obtain

$$b_t - \bar{b} = \frac{\omega + \alpha}{\alpha(\theta - \omega)} (z_t - \bar{z}) - \left\{ \frac{(1 - \delta)(\omega + \alpha)}{\alpha(\theta - \omega)} + \frac{b_0 \eta \Omega}{\theta + \alpha} \right\} (Z_t - \bar{Z}). \quad (24)$$

Two habit stocks z_t and Z_t affect b_t by changing consumption and the interest rate. An increase in the habit stocks raises consumption from (19), which should be financed by net foreign assets greater than its steady-state level. This effect is expressed by the terms without b_0 in (24). An increase in the aggregate habit stock raises the interest rate by (22). The resulting rise in interest income alters the time profile of net foreign assets. The effect is captured by the terms associated with b_0 in (24).

In sum, the autonomous dynamics of (17) and (18) generate the evolution of two habit stocks z_t and Z_t . Given the values of the two state variables, consumption rates c_t and c_t^* , the interest rate r_t , and net foreign asset b_t are determined by (19), (22), and (24), respectively. The dynamics can be described more simply by introducing the weighted difference of the two countries' habit stocks:

$$e_t = \delta z_t - (1 - \delta) z_t^*. \quad (25)$$

We refer to e_t as weighted habit differentials between countries H and F, or simply *habit differentials*. The equilibrium net foreign assets (24) can be rewritten as

$$b_t - \bar{b} = \frac{\omega + \alpha}{\alpha(\theta - \omega)} (e_t - \bar{e}) - \frac{b_0 \eta \Omega}{\theta + \alpha} (Z_t - \bar{Z}), \quad (26)$$

which implies the following results.

Proposition 1: *Country H's net foreign assets b_t depend on habit differentials e_t and global habits Z_t in the following manners:*

- (i) b_t is larger as e_t is larger; and
- (ii) b_t is larger as Z_t is smaller (larger) if country H is the net creditor (debtor), $b_0 > (<) 0$.

With a habit differential, the corresponding optimal consumption level is high relatively to the other country's, so that net foreign assets should

be large enough to sustain it, as in (i) of Proposition 1. The effect of global habits is due to the interest rate adjustment. With adjacent complementarity, a large Z_t implies a large demand for present goods and hence a high interest rate. The resulting interest revenue induces country H to accumulate net foreign assets toward the long-run higher level. Equilibrium b thus negatively relates to global habits Z , and (ii) follows.

By differentiating (25) and (26), and substituting (11), (17), (18), (25), and (26) into the results, the equilibrium dynamic interactions of net foreign assets and habit differentials can be summarized as the autonomous system of net foreign assets and habit differentials:

$$\dot{b} = -\alpha (b_t - \bar{b}) + \frac{(\omega + \alpha)^2}{\alpha(\theta - \omega)} (e_t - \bar{e}), \quad (27)$$

$$\dot{e} = \omega (e_t - \bar{e}). \quad (28)$$

The phase diagram of (e, b) plane is depicted in Figure 1, where the $\dot{b} = 0$ schedule is positively sloping and $\dot{e} = 0$ schedule is in parallel to the vertical axis.

As seen from (19) and (25), it is also noteworthy that the associated consumption dynamics are generated by

$$c_t - \bar{c} = \left(\frac{\omega + \alpha}{\alpha} \right) (e_t - \bar{e}). \quad (29)$$

To obtain the welfare level of country H, linearize instantaneous utilities $u(c_t, z_t)$ around a steady state and substitute the result into the lifetime utility function (2) to obtain

$$U_0 = \int_0^\infty \{u(\bar{c}, \bar{z}) + u_c(c_t - \bar{c}) + u_z(z_t - \bar{z})\} \exp(-\theta t) dt.$$

After substituting (29) into the above equation, substitute solutions to (17), (18), and (28) into the result. In this way, the lifetime utilities are obtained as

$$\begin{aligned} U_0 = & \frac{u(\bar{z}, \bar{z})}{\theta} + \frac{(\omega + \alpha) u_c + \alpha u_z}{\alpha(\theta - \omega)} (z_0 - \bar{z}) \\ & - \frac{(\omega + \alpha)(1 - \delta)}{\alpha(\theta + \alpha)(\theta - \omega)} [(\theta + \alpha) u_c + \alpha u_z] (Z_0 - Y), \end{aligned} \quad (30)$$

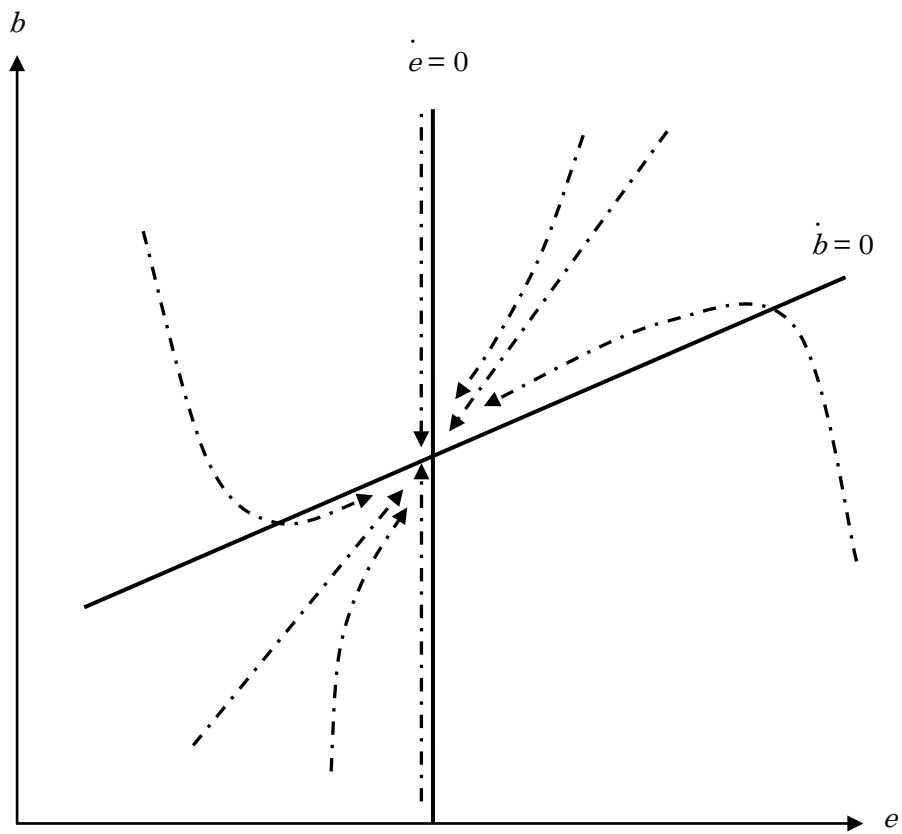


Figure 1. Equilibrium dynamics for relative habits and net foreign assets

$$\begin{aligned}
U_0^* &= \frac{u^*(\bar{z}^*, \bar{z}^*)}{\theta} + \frac{(\omega + \alpha)u_c^* + \alpha u_z^*}{\alpha(\theta - \omega)}(z_0^* - \bar{z}^*) \\
&\quad - \frac{(\omega + \alpha)\delta}{\alpha(\theta + \alpha)(\theta - \omega)} [(\theta + \alpha)u_c^* + \alpha u_z^*](Z_0 - Y),
\end{aligned} \tag{31}$$

2.3 Steady state

From (1), (5), (7), and (24), the steady-state equilibrium, $(\bar{c}, \bar{c}^*, \bar{z}, \bar{z}^*, \bar{Z}, \bar{b}, \bar{b}^*, \bar{r})$, is determined by:

$$\bar{c} = \bar{z}, \bar{c}^* = \bar{z}^*, \tag{32}$$

$$\bar{c} + \bar{c}^* = Y = \bar{Z}, \tag{33}$$

$$\bar{e} = \delta\bar{z} - (1 - \delta)\bar{z}^* \tag{34}$$

$$\bar{r} = \theta, \tag{35}$$

$$\bar{r}\bar{b} + y = \bar{c}, \tag{36}$$

$$\bar{b} - b_0 = \frac{\omega + \alpha}{\alpha(\theta - \omega)}(\bar{z} - z_0) - \left\{ \frac{(1 - \delta)(\omega + \alpha)}{\alpha(\theta - \omega)} + \frac{b_0\eta\Omega}{\theta + \alpha} \right\}(\bar{Z} - Z_0) \tag{37}$$

$$\bar{b} = -\bar{b}^*. \tag{38}$$

where (37) comes from (24) evaluated at $t = 0$.

The steady state equilibrium conditions can be reduced to:

$$CC': \bar{b} - b_0 = \frac{\omega + \alpha}{\alpha(\theta - \omega)}(\bar{e} - e_0) - \frac{b_0\eta\Omega}{\theta + \alpha}(Y - Z_0), \tag{39}$$

$$DD': \theta\bar{b} = \bar{e} + (1 - \delta)Y - y, \tag{40}$$

where the CC' schedule is obtained by evaluating (26) at $t = 0$; and DD' schedule is obtained by substituting (32) through (35) into (36). Figure 2 illustrates the two schedules, where schedule DD' is always steeper than CC' .

As shown by Figure 2, the steady-state equilibrium point (\bar{e}, \bar{b}) is given by the intersection point E of the two schedules. Given this, $(\bar{c}, \bar{c}^*, \bar{z}, \bar{z}^*, \bar{Z})$ is determined by (32), (33) and (34); and \bar{b}^* by (38).

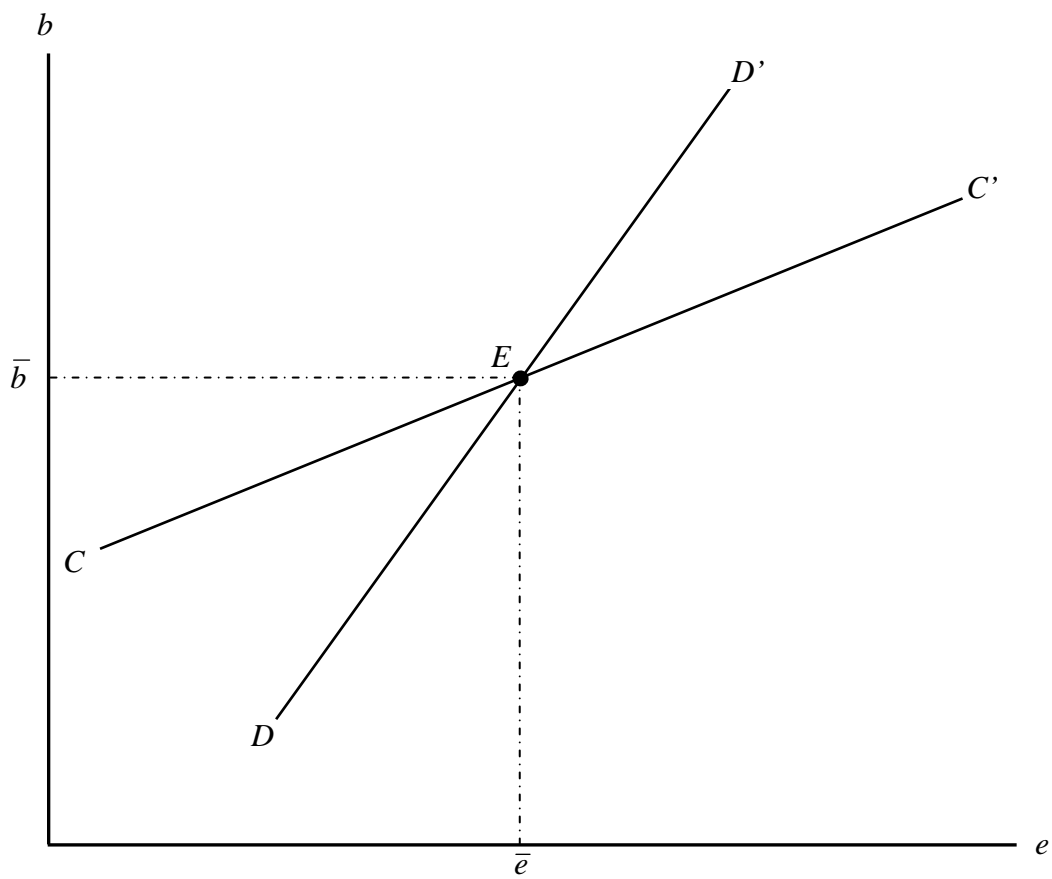


Figure 2. Steady state equilibrium

From linearity of (39) and (40), we can examine the determinants of the long-run external asset distribution by solving the two equations for \bar{b} as

$$\bar{b} = -\frac{\omega + \alpha}{\omega(\theta + \alpha)}(y_\delta - e_0) - \frac{\eta\Omega}{\theta + \alpha}b_0(Y - Z_0), \quad (41)$$

where y_δ represents the weighted difference of y and y^* :

$$y_\delta = \delta y - (1 - \delta)y^*.$$

This implies that country H's net foreign assets are determined as in Proposition 2:

Proposition 2: *Country H's steady-state holdings of external assets \bar{b} are larger as:*

1. *weighted income difference in excess of relative depth of habits, $y_\delta - e_0$, is larger and;*
2. *global income in excess of global habits, $Y - Z_0$ is smaller (resp. larger) when $b_0 > 0$ (resp. $b_0 < 0$).*

To understand the first property, suppose that income differentials y_δ exceeds habit differentials e_0 , $y_\delta - e_0$. Then, with adjacent complementarity, *ceteris paribus* country H saves the excess income and thereby holds positive external assets in the long run. We refer to this effect as the *relative surplus-income effect*. The second property represents the interest rate effect: a large global income in excess of global habits, $Y - Z_0$, implies a low interest rate, and when country H is a creditor $b_0 > 0$, H's interest rate income also decreases thereby suppressing the long-term asset holdings. This effect is called the *interest rate effect*.

2.4 Income shocks

We consider global and local income shocks by fixing constant the magnitude of the resulting increase dY in the aggregate output Y .⁴ Let us define the two kinds of income shocks as follows:

⁴This is just a simplifying assumption that eases comparison of the effects of global and local income shocks. Even when we instead assume that increases in country H's output y are the same between the two shocks, and that, for local income shocks total output increase dY equals dy , and for global income shocks dY equals dy/ε , our main results below do not change.

- global income shocks: $dy = \varepsilon dY$ and $dy^* = (1 - \varepsilon) dY$,
- local income shocks in country H: $dy = dY$ and $dy^* = 0$,

where ε denotes the proportion of the associated shock dy to country H in the total shock dY .

The effects of the global and local income shocks on country H's steady-state holdings of external assets \bar{b} can be compared easily by using Proposition 2. Note first that the interest rate effect, captured by the second term of (41), is the same for the global and local shocks. The relative surplus income effect, i.e., the first term of (41), is larger in the case of local income shocks than in the case of global shocks because the local income shock has larger effect on income difference between the two countries than the global shock. The following corollary thus obtains:

Corollary 1: *Local income shocks have greater effects than global income shocks on country H's steady-state holdings of external assets \bar{b} in the sense that:*

$$\left. \frac{d\bar{b}}{dY} \right|_{\text{local shock}} > \left. \frac{d\bar{b}}{dY} \right|_{\text{global shock}}.$$

3 Local and Global Income Shocks

Let us consider the effects of positive local and global income shocks, $dY > 0$, in order. Before discussing each shock, note that the effect on the interest rate does not depend on whether the shock is local or global. This is because, by construction, the magnitudes of the resulting increase dY in the aggregate output are the same between the two income shocks. Indeed, differentiating (22) and (33) by Y yields

$$\left. \frac{dr(0)}{dY} \right|_{\text{local shock}} = \left. \frac{dr(0)}{dY} \right|_{\text{global shock}} = -\eta\Omega < 0. \quad (42)$$

This implies that an increase in Y lowers $r(0)$. With adjacent complementarity, the positive income shock, local or global, produces excess supply in the time-zero good market, and hence lowers the equilibrium interest rate. After the initial response, the interest rate then monotonically converges toward θ as seen from (22).

Table 1. The effects of local and global income shocks

	(1) Local shocks $dy = dY$ and $dy^* = 0$	(2) Global shocks $dy = \varepsilon dY$ and $dy^* = (1 - \varepsilon) dY$	Effect differences $(2) - (1)$
$\frac{dr(0)}{dY}$	$-\eta\Omega < 0$	$-\eta\Omega < 0$	0
$\frac{d\bar{c}}{dY}$	$1 - \frac{\delta\theta(\omega + \alpha)}{\omega(\theta + \alpha)} + \frac{\alpha\theta\eta(\theta - \omega)\Omega}{\omega(\theta + \alpha)^2}b_0$	$\varepsilon + (1 - \varepsilon - \delta)\frac{\theta(\omega + \alpha)}{\omega(\theta + \alpha)} + \frac{\alpha\theta\eta(\theta - \omega)\Omega}{\omega(\theta + \alpha)^2}b_0$	$\frac{\alpha(1 - \varepsilon)(\theta - \omega)}{\omega(\theta + \alpha)} < 0$
$\frac{d\bar{c}^*}{dY}$	$\frac{\delta\theta(\omega + \alpha)}{\omega(\theta + \alpha)} - \frac{\alpha\theta\eta(\theta - \omega)\Omega}{\omega(\theta + \alpha)^2}b_0$	$(1 - \varepsilon) - (1 - \varepsilon - \delta)\frac{\theta(\omega + \alpha)}{\omega(\theta + \alpha)} - \frac{\alpha\theta\eta(\theta - \omega)\Omega}{\omega(\theta + \alpha)^2}b_0$	$-\frac{\alpha(1 - \varepsilon)(\theta - \omega)}{\omega(\theta + \alpha)} > 0$
$\frac{d\bar{e}}{dY}$	$-\frac{\alpha\delta(\theta - \omega)}{\omega(\theta + \alpha)} + \frac{\alpha\theta\eta(\theta - \omega)\Omega}{\omega(\theta + \alpha)^2}b_0$	$(1 - \varepsilon - \delta)\frac{\alpha(\theta - \omega)}{\omega(\theta + \alpha)} + \frac{\alpha\theta\eta(\theta - \omega)\Omega}{\omega(\theta + \alpha)^2}b_0$	$(1 - \varepsilon)\frac{\alpha(\theta - \omega)}{\omega(\theta + \alpha)} < 0$
$\frac{d\bar{b}}{dY}$	$-\frac{\delta(\omega + \alpha)}{\omega(\theta + \alpha)} + \frac{\alpha\eta(\theta - \omega)\Omega}{\omega(\theta + \alpha)^2}b_0$	$(1 - \varepsilon - \delta)\frac{(\omega + \alpha)}{\omega(\theta + \alpha)} + \frac{\alpha\eta(\theta - \omega)\Omega}{\omega(\theta + \alpha)^2}b_0$	$(1 - \varepsilon)\frac{(\omega + \alpha)}{\omega(\theta + \alpha)} < 0$
$\frac{d\bar{b}^*}{dY} = -\frac{d\bar{b}}{dY}$	$\frac{\lambda}{\theta}\left(1 - \frac{\theta}{\alpha + \theta}\eta\Omega b_0\right)$	$\frac{\lambda}{\theta}\left(\varepsilon - \frac{\theta}{\alpha + \theta}\eta\Omega b_0\right)$	$-\frac{\lambda}{\theta}(1 - \varepsilon) < 0$
$\frac{dU}{dY}$	$\frac{\eta\Omega\lambda^*}{\theta + \alpha}b_0 > 0$	$\frac{\lambda^*}{\theta}\left(1 - \varepsilon + \frac{\theta}{\alpha + \theta}\eta\Omega b_0\right) > 0$	$\frac{\lambda^*}{\theta}(1 - \varepsilon) > 0$

3.1 Local income shocks

We consider first the effects of a positive local income shock, $dy = dY > 0$ and $dy^* = 0$. For simplicity, let us assume that $b_0 > 0$ without further notice. From (32) through (38), we obtain the steady-state effect on each of $(\bar{c}, \bar{c}^*, \bar{e}, \bar{b})$ as in the second column of Table 1. They all are composed of the direct income effects, captured by the terms without b_0 , and the interest rate effects, represented by the terms with b_0 . Without the interest rate effect, as in the small country case (e.g., Ikeda and Gombi, 1999), the positive income shock in country H increases savings, promotes interim asset accumulation through current account surplus, and thereby increases \bar{c} , \bar{e} , and \bar{b} while decreasing \bar{b}^* and \bar{c}^* . The direct income effects represent these influences. The interest rate effects, caused by a fall in the interim market interest rate, represent the countervailing negative income effects on \bar{c} , \bar{e} , and \bar{b} under a positive b_0 , and positive effects on \bar{b}^* and \bar{c}^* . Net effects of the two have ambiguous signs.

Figure 3 illustrates the typical adjustment of the economy in the (e, b) and (e, c) plains. The local income shock shifts downward the CC' and DD' schedules defined by (39) and (40) from $C_0C'_0$ to $C_1C'_1$ and $D_0D'_0$ to $D_LD'_L$, respectively, and thereby bringing the steady-state point from E_0 to E_L , where the case in which \bar{b} and \bar{e} both increase is depicted. The interim dynamics generated by (27) and (28) are illustrated as the arrowed path from E_0 to E_L . Note that from a close look at Figure 1, the b dynamics can be nonmonotonic: Country H can initially run the current account deficit, and sooner or later it turns to surplus so as to generate a greater \bar{b} . The associated consumption dynamics are shown in the (e, c) plain, where the case in which \bar{c} increases from \bar{c}_0 to \bar{c}_L is depicted. After c jumps from F_0 up to F_{L01} instantaneously responding to the shock, it gradually rises up to F_L along the saddle trajectory (29).

The welfare effects of the local income shock, dU/dY and dU^*/dY shown in column (1) of Table 1, are obtained by differentiating (30) and (31) by Y . Two properties are noteworthy: First, the positive local income shock harms country H if and only if $1 < \theta\eta\Omega b_0/(\alpha + \theta)$. This represents the intertemporal version of immiserizing growth effect that the associated fall in the interest rate, i.e., a deterioration in the intertemporal terms-of-trade for creditor country H, harms it, irrespective of the direct beneficial effect of the income increase. Second, this interest rate fall in turn definitely benefits neighbor debtor country F.

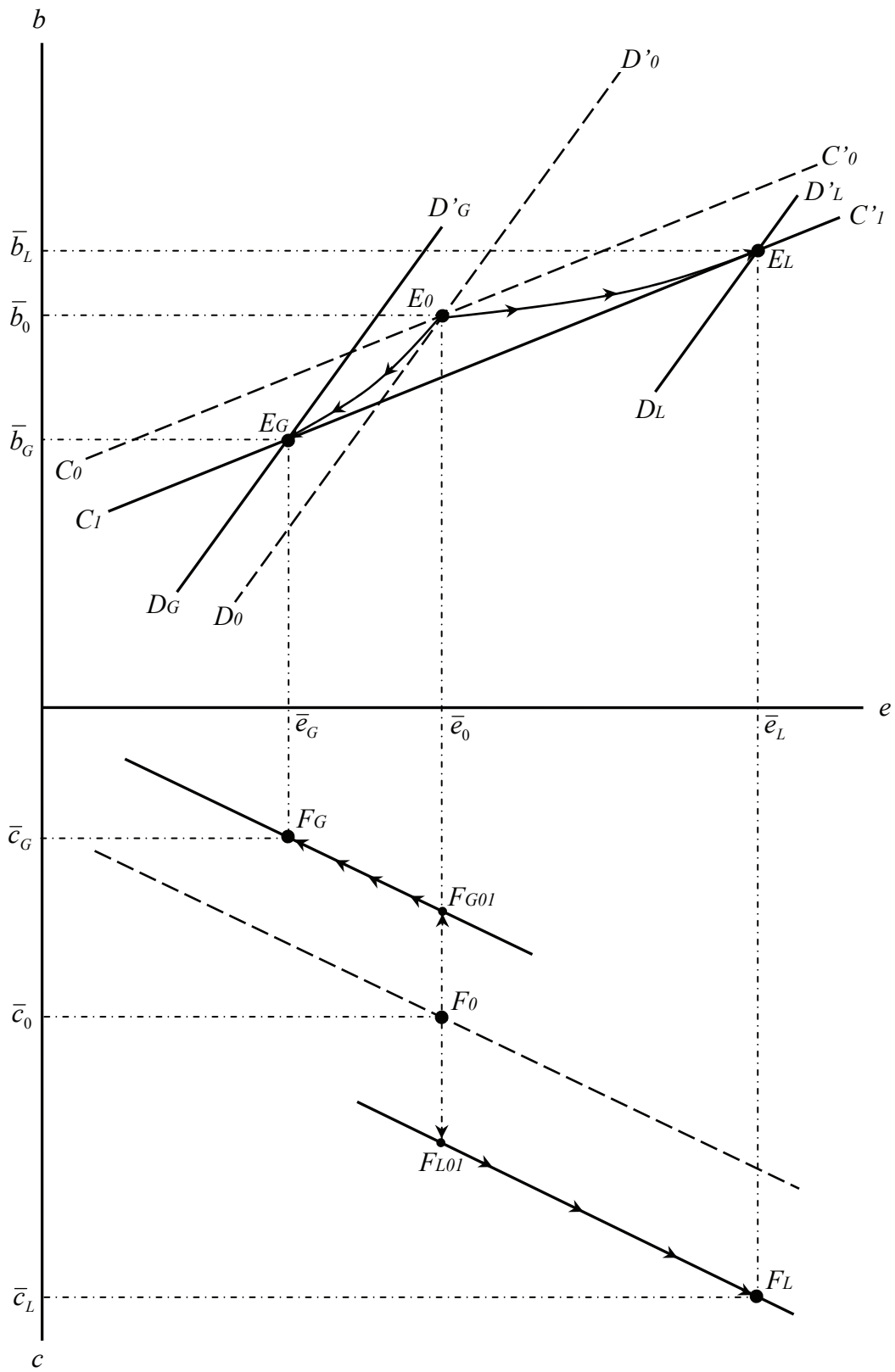


Figure 3. Adjustment Processes

Proposition 3 (the immiserizing growth effect of local income shocks):

Suppose that country H is creditor $b_0 > 0$ (resp. debtor $b_0 < 0$). Then, due to the intertemporal terms-of-trade effects, a positive local income shock in country H , $(dy, dy^*) = (dY, 0)$,

1. reduces (resp. enhances) the country's own welfare if and only if $\frac{\theta}{\alpha + \theta} \eta \Omega b_0 > 1$; and
2. definitely benefits (resp. harms) country F .

3.2 Global income shocks

Let us next consider a positive global income shock, $dy = \varepsilon dY$ and $dy^* = (1 - \varepsilon) dY$, where the magnitude dY is the same as in the previous subsection. The steady-state effect on $(\bar{c}, \bar{c}^*, \bar{e}, \bar{b})$ is shown in the third column of Table 1. As in the local shock case, the effects comprise the direct income effects of domestic and foreign income increases, represented by the terms without b_0 , and the interest rate effects, captured by the terms with b_0 . The interest rate effects are the same as in the local shock case. In contrast, the direct income effects are smaller for country H 's variables $(\bar{c}, \bar{e}, \bar{b}, U)$ and larger for country F 's (\bar{c}^*, U^*) than those in the local shock case. This is firstly because the associated increase in y is smaller under the global income shock, secondly because foreign income increase dy^* causes a negative crowding-out effect on c . As shown in the fourth column in the table, therefore the total effects are smaller for country H 's variables $(\bar{c}, \bar{e}, \bar{b}, U)$ and larger for country F 's (\bar{c}^*, U^*) .

Figure 3 depicts the typical dynamic adjustment to the global income shock. The global income shock shifts the CC' schedule downward by the same amount as in the local shock case, i.e., from $C_0C'_0$ to $C_1C'_1$, whereas the direction of the shift of the DD' schedule depends on the relative magnitudes of ε and $1 - \delta$ (see (40)). Figure 3 illustrates a typical case in which ε is smaller than $1 - \delta$, so that the DD' schedule shift upward from $D_0D'_0$ to $D_GD'_G$ with the steady-state points (\bar{e}, \bar{b}) moving downward from points E_0 to E_G . The interim time path is indicated by arrows.⁵ The corresponding consumption dynamics in the (e, c) plain are illustrated as the instantaneous downward

⁵Although Figure 3 illustrates the transition dynamics of b are monotonic, they can be nonmonotonic under certain parameter values.

jump from F_0 to F_{G01} , followed by over-time decreases along the saddle path toward F_G .

The welfare effects of the global income shock, dU/dY and dU^*/dY shown in column (2) of Table 1, can be summarized as follows:

Proposition 4 (the immiserizing growth effect of global income shocks): *Suppose that country H is creditor $b_0 > 0$ (resp. debtor $b_0 < 0$). Then, due to intertemporal terms-of-trade effects, a positive global income shock, $(dy, dy^*) = (\varepsilon dY, (1 - \varepsilon) dY)$,*

1. *reduces (resp. enhances) the country's own welfare if and only if $\frac{\theta}{\alpha + \theta} \eta \Omega b_0 > \varepsilon$; and*
2. *definitely benefits (resp. harms) country F.*

Note that the global income shock is more likely to cause the intertemporal immiserizing growth effect than the local income shock does, because compared to the local shock case, the direct income effect (positive) is small relatively to the intertemporal terms-of-trade effect (negative). Indeed, comparing Propositions 3 and 4 implies the following corollary since $\varepsilon < 1$.

Corollary 2: Intertemporal immiserizing growth effects more likely take place due to global income shocks than due to local income shocks.

Remark 1: The result in the above corollary contrasts to that in the case of static trade theory. In the typical static two-country trade model, where each country specializes production in its export goods industry, global income shocks that commonly occur in the export sectors of the two countries have smaller effects on the terms of trade, i.e., relative prices of two exporting goods, than local income shocks of the same magnitude do, and hence are less likely to cause immiserizing growth effects. In the present dynamic setting the changes in the intertemporal term of trade, i.e., the interest rate, are the same under global and local income shocks of the same magnitude, dY . Due to the direct income effects, global income shocks thus have greater effects on the two countries' welfare levels than local income shocks.⁶

⁶This reasoning is based on the simplifying assumption that increases in total output Y are the same between global and local income shocks. Corollary 2 holds valid even

4 Conclusions

In a two-country model with habit formation, we focus on interdependent macroeconomic adjustments by considering global and country-specific income shocks on each country. The global habits and habit differentials play key roles in the global economic dynamics, possibly nonmonotonic, and in the determination of international asset distribution. A country's net external asset holdings rely on weighted income difference in excess of habit differentials and global income in excess of global habits. With habit formation, positive income shocks lower the world interest rate, thereby harming the creditor country and benefitting the debtor country due to the intertemporal terms-of-trade effect. In contrast to the case of trade theory, this intertemporal immiserizing growth is more likely to be brought about by global income shocks than by country-specific income shocks.

For future research, two issues may be interesting. First, when a government can utilize the immiserizing growth effect by means of fiscal instruments, international strategic interactions would arise. It should be examined how the policy game changes our results. Secondly, extending empirical study on habit formation to the interdependent world economy model would be fruitful.

when we instead assume that increases in country H's output y are the same between the two shocks, and that, for local income shocks total output increase dY equals dy , and for global income shocks dY equals dy/ε . In that case, global income shocks have a larger intertemporal terms-of-trade effect than local income shocks, which results in Corollary 2 again. The point is that when a positive income shock is global, the relative magnitudes of the harmful intertemporal terms-of-trade effect to the beneficial direct income effect is larger than they would be when the shock is country-specific.

Appendices

A Deriving the Dynamic System (15)

Note first that λ/λ^* is constant because $\dot{\lambda}_t/\lambda_t = \dot{\lambda}_t^*/\lambda_t^*$ from (7). By eliminating c^* and z^* using (9) and (11) from the foreign counterpart of (6), combining the resulting equation and (6) yields:

$$\frac{u_c(c, z) + \alpha\xi}{u_c^*(Y - c, Z - z) + \alpha\xi^*} = \frac{\lambda}{\lambda^*} = \text{constant}.$$

By totally differentiating this equation, we obtain:

$$\hat{c} = -\frac{\lambda^*u_{cz} + \lambda u_{cz}^*}{\lambda^*u_{cc} + \lambda u_{cc}^*} \hat{z} - \frac{\alpha\lambda^*}{\lambda^*u_{cc} + \lambda u_{cc}^*} \hat{\xi} + \frac{\alpha\lambda}{\lambda^*u_{cc} + \lambda u_{cc}^*} \hat{\xi}^* + \frac{\lambda u_{cz}^*}{\lambda^*u_{cc} + \lambda u_{cc}^*} \hat{Z}.$$

We substitute this equation into (1), (8), and the foreign counterpart of (8) and eliminate c^* and z^* using (9) and (11) from the resulting equation. Then, from (12), the autonomous dynamic equation system with respect to $(\hat{z}, \hat{\xi}, \hat{\xi}^*, \hat{Z})$ is obtained as follows:

$$\begin{aligned} \dot{\hat{z}} &= -\alpha \left(\frac{\lambda^*u_{cz} + \lambda u_{cz}^*}{\lambda^*u_{cc} + \lambda u_{cc}^*} + 1 \right) \hat{z} - \frac{\alpha^2\lambda^*}{\lambda^*u_{cc} + \lambda u_{cc}^*} \hat{\xi} + \frac{\alpha^2\lambda}{\lambda^*u_{cc} + \lambda u_{cc}^*} \hat{\xi}^* \\ &\quad + \frac{\alpha\lambda u_{cz}^*}{\lambda^*u_{cc} + \lambda u_{cc}^*} \hat{Z}, \\ \dot{\hat{\xi}} &= \left\{ \frac{u_{cz}(\lambda^*u_{cz} + \lambda u_{cz}^*)}{\lambda^*u_{cc} + \lambda u_{cc}^*} - u_{zz} \right\} \hat{z} + \left(\theta + \alpha + \frac{\alpha u_{cz}\lambda^*}{\lambda^*u_{cc} + \lambda u_{cc}^*} \right) \hat{\xi} \\ &\quad - \frac{\alpha u_{cz}\lambda}{\lambda^*u_{cc} + \lambda u_{cc}^*} \hat{\xi}^* - \frac{\lambda u_{cz}u_{cz}^*}{\lambda^*u_{cc} + \lambda u_{cc}^*} \hat{Z}, \\ \dot{\hat{\xi}}^* &= -\left\{ \frac{u_{cz}^*(\lambda^*u_{cz} + \lambda u_{cz}^*)}{\lambda^*u_{cc} + \lambda u_{cc}^*} - u_{zz}^* \right\} \hat{z} - \frac{\alpha u_{cz}^*\lambda^*}{\lambda^*u_{cc} + \lambda u_{cc}^*} \hat{\xi} \\ &\quad + \left(\theta + \alpha + \frac{\alpha u_{cz}^*\lambda^*}{\lambda^*u_{cc} + \lambda u_{cc}^*} \right) \hat{\xi}^* + \left(\frac{\lambda u_{cz}^*{}^2}{\lambda^*u_{cc} + \lambda u_{cc}^*} - u_{zz}^* \right) \hat{Z}, \\ \dot{\hat{Z}} &= -\alpha \hat{Z}. \end{aligned} \tag{43}$$

From the definitions (13) and (14) of v and ς , respectively, this autonomous system reduces to (15). ■

B Equilibrium Solutions

B.1 Dynamics of habit capital z : (17)

The stable roots of dynamics (15) are given by ω and $-\alpha$ as in (16). Letting m denote $(z, \varsigma, Z)'$, the general solution to (15) can thus be expressed as

$$\hat{m}(t) = A_1 \exp(\omega t) q + A_2 \exp(-\alpha t) h,$$

where $q \equiv (q_1, q_2, q_3)'$ and $h \equiv (h_1, h_2, h_3)'$ represent the eigen vectors associated with stable roots ω and $-\alpha$, respectively. From (15), it is easy to confirm that $q_3 = 0$. By eliminating $A_1 \exp(\omega t)$ and $A_2 \exp(-\alpha t)$ from the three equations in the above vector equation, we obtain

$$\hat{\varsigma} = \frac{q_2}{q_1} \hat{z} + \frac{q_1 h_2 - q_2 h_1}{q_1 h_3} \hat{Z}, \quad (44)$$

where the coefficients of \hat{z} and \hat{Z} can be obtained by exploiting the definition of the eigenvectors q and h as

$$\begin{aligned} \frac{q_2}{q_1} &= -\frac{(\omega + \alpha) v_{cc} + \alpha v_{cz}}{\alpha^2}, \\ \frac{q_1 h_2 - q_2 h_1}{q_1 h_3} &= -\frac{v_{cc} (v_{cz} v_{ZZ} + v_{cZ} v_{zz})}{\alpha v_{cc} v_{zz} + (2\alpha + \delta) v_{cc} v_{cz}} \\ &\quad + \frac{\{(\omega + \alpha) v_{cc} + \alpha v_{cz}\} \{\alpha v_{cc} v_{ZZ} - (2\alpha + \delta) v_{cc} v_{cZ}\}}{\alpha^2 (\alpha v_{cc} v_{zz} + (2\alpha + \delta) v_{cc} v_{cz})}. \end{aligned}$$

Substituting (44) into the \dot{z} -equation in (15) yields (17).

B.2 The interest rate

From (1) and (6) through (8), the optimal consumption dynamics are given by

$$\dot{c} = -\frac{\lambda}{u_{cc}} \left(\hat{r} - \hat{\phi} \right),$$

where ϕ represents the rate of time preference,

$$\hat{\phi} = \frac{\alpha (u_{zz} + u_{zc})}{\lambda} \hat{z} - \frac{\alpha (\alpha + \theta)}{\lambda} \hat{\xi}.$$

Substitute (20) into the above Euler equation. The resulting equation can be solved for \hat{r} as

$$\hat{r} = -\frac{\omega u_{cc}}{\lambda} \hat{c} + \frac{\alpha (u_{zz} + u_{cz})}{\lambda} \hat{z} - \frac{\alpha (\alpha + \theta)}{\lambda} \hat{\xi}. \quad (45)$$

In the above, $\hat{\xi}$ can be obtained from (14),(18), (43), and (44) as

$$\hat{\xi} = -\frac{u_{cc} \Omega^H}{\theta + \alpha - \omega} \hat{c} + \frac{u_{zz}}{\theta + 2\alpha} \hat{z}. \quad (46)$$

Substituting (46) and (19) successively into (45) yields (22).

B.3 Net foreign assets

Set

$$\hat{b} = \varkappa_1 \hat{z} + \varkappa_2 \hat{Z}. \quad (47)$$

Differentiating (47) with respect to time t yields

$$\dot{b} = \varkappa_1 \dot{z} + \varkappa_2 \dot{Z}. \quad (48)$$

Since \dot{b} is given by (5), this equation implies

$$\left(\dot{b} = \right) \hat{r} b_0 + r \hat{b} - \hat{c} = \varkappa_1 \dot{z} + \varkappa_2 \dot{Z}. \quad (49)$$

Substitute (17), (18), (19), (22), and (47) into (49). By comparing the coefficients of the resulting equation, we obtain

$$\begin{aligned} (\omega - r) \varkappa_1 &= a_1 b_0 - \frac{\omega + \alpha}{\alpha}, \\ -(\omega + \alpha) (1 - \delta) \varkappa_1 - (r + \alpha) \varkappa_2 &= a_2 b_0 + \left(\frac{\omega + \alpha}{\alpha} \right) (1 - \delta). \end{aligned}$$

This simultaneous equation can be solved for \varkappa_1 and \varkappa_2 as

$$\varkappa_1 = \frac{1}{r - \omega} \left\{ \frac{\omega + \alpha}{\alpha} - b_0 a_1 \right\}, \quad (50)$$

$$\varkappa_2 = -\frac{(1 - \delta) (\omega + \alpha)}{\alpha (r - \omega)} - \frac{b_0}{r + \alpha} \left\{ \frac{(1 - \delta) (\omega + \alpha)}{(r - \omega)} a_1 + a_2 \right\}. \quad (51)$$

Substituting (50) and (51) into (47) yields (24).

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