Discussion Paper No. 893

MACROECONOMIC INTERDEPENDENCE BETWEEN A STAGNANT AND A FULLY EMPLOYED COUNTRY

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January 2014
Revised November 2014
Secondly Revised June 2017

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February 2017

Macroeconomic interdependence between a stagnant and a fully employed country*

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Abstract

This paper presents a two-country two-commodity dynamic model where one country achieves full employment and the other suffers from secular stagnation of aggregate demand. Own and spill-over effects of changes in preference, productivity and policy parameters are examined. Parameter changes that improve the stagnant country's current account, such as a reduction in government purchases, a decrease in foreign aid and an improvement in productivity, raise the relative price of the home commodity. Consequently, home employment shrinks, deflation worsens and consumption decreases. The terms of trade for the full-employment country deteriorate. Thus, income and consumption decrease in both countries.

JEL classification: F32, F41, F35

Keywords: secular stagnation, unemployment, wealth preference, current account, fiscal expansion.

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* This research was started while the author visited the Center for Economic Studies (CES) of the University of Munich (LMU). The author thanks G. Illing and the participants of the macroeconomics seminar held in LMU for their valuable comments. Helpful comments by the participants of the INFINITI Conference on International Finance and Hitotsubashi Economics Seminar are also gratefully acknowledged. This research is financially supported by the Grants-in-Aid for Scientific Research from the Japan Society for the Promotion of Science (15H05728 and 22530178) and the Joint Usage/Research Center Program by the Ministry of Education, Culture, Sports, Science and Technology, Japan.

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1. Introduction

In a closed-economy setting an increase in productivity naturally expands national income and consumption if full employment prevails. In a two-country setting it benefits not only the home country but also the foreign country because it lowers the relative price of the home commodity and improves the foreign terms of trade. The effect of an increase in home government purchases on the home commodity is the same as that of a decrease in home productivity: it harms both countries.

If the two countries suffer from aggregate demand stagnation, however, the result is quite different. Ono (2006, 2014) presents a two-country dynamic model with persistent unemployment in both countries and shows that an increase in home productivity excessively improves the current account and leads the home currency and the relative price of the home commodity to appreciate so much that home employment and consumption decrease. The rise in the relative price of the home commodity in turn causes foreign employment and consumption to increase. An increase in home government purchases works in the same manner as a decrease in home productivity. It worsens the home current account and lowers the relative price of the home commodity, which increases home employment and consumption and decreases foreign ones.

There is yet another important case: one country faces persistent deficiency of aggregate demand while the other country realizes full employment. It may be the case of Japan and Korea and that of Japan and China until recently. This paper focuses on this case and examines the own and spillover effects of changes in various parameters including productivity and government purchases. It shows that an improvement in the stagnant country's productivity reduces both countries' consumption levels and that an increase in the stagnant country's government purchases expands them.

The effect of an international transfer is also examined. It is naturally believed that foreign aid makes the donor country worse off and the recipient country better off. Due to this belief foreign-aid budgets are usually cut particularly when a country faces stagnation, as Japan did in the ‘Lost Decades’. In some important policy discussions, however, benefits of foreign aid for both recipient and donor countries in the asymmetric case have been emphasized. The Marshall Plan in 1947 was an example. George C. Marshall, the US Secretary of State at that time, proposed an aid of $20 billion to European countries that had significantly lost supply capacities in World War II and faced serious shortages of supply. He then stated that it would benefit not only European consumers by enabling them to import US commodities but also US producers and workers by expanding the market.[1] In the context of the North-South problem, the Independent Commission on International Development Issues, a panel lead by former German Chancellor Willy Brandt in the early 1980s (the Brandt commission, 1980, 1983), argued that foreign aid from the North to the South would benefit the donors as well as the recipients through not only creating better political relations but also expanding imports of the South and increasing employment in the North.[2]

This paper examines the validity of such statements in a dynamic macroeconomic framework. It is found that foreign aid expands consumption in both countries if the recipient country achieves full employment and the donor country faces persistent deficiency of aggregate demand.[3]

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[1] He stated: “The Marshall Plan, it should be noted, benefited the American economy as well. The money would be used to buy goods from the United States, and they had to be shipped across the Atlantic on American merchant vessels.” See *Congressional Record*, 30 June 1947.

[2] This issue was treated by Ono (2007) but he used CES utility functions. This paper extends that analysis to the case of general homothetic utility.

[3] In the literature such a controversial case has been mentioned as a transfer paradox. The paradox arises through not an expansion of employment but a change in the terms of trade with full employment. It arises only in the case of Walrasian instability, multiple equilibia or some distortions in a two-country case. See Bhagwati *et al* (1983, 1985) for a general analysis of the transfer paradox with distortions in static two-country and three-country frameworks. Polemarchakis (1983) extended it to an *n*-country economy. The paradoxical case in this paper arises through a change in employment.

The stagnation considered in the present analysis is not temporary but persistent and is due to aggregate demand deficiency. In the literature, however, stagnation has mostly been taken to be temporary. DSGE models, for example, regard macroeconomic fluctuations as caused by firms' and labor unions' monopolistic pricing a la Calvo (1983) and analyze short-run perturbations due to unanticipated policy and technology shocks around the steady state. They are e.g. Yun (1996), Erceg et al. (2000), Smets and Wouters (2003, 2005, 2007), Christiano et al. (2005), and Adolfson et al. (2007, 2008). In these models demand shortages appear neither during the adjustment process nor in the steady state. Stagnation is entirely generated by firms' monopolistic pricing after an exogenous economic shock. Krugman (1998) focuses on the zero lower bound of the nominal interest rate and shows that with nominal price rigidities aggregate demand deficiency appears. However, it appears only within the period in which an unanticipated exogenous shock occurs after prices are determined, and disappears in all the subsequent periods.

There are also researchers who consider a credit constraint with which capital allocation is distorted (Kiyotaki and Moore, 1997; Aghion et al., 1999; Matsuyama, 2007; Hall, 2011; Liu and Wang, 2014; Azariadis and Kaas, 2016). They take stagnation as a supply-side phenomenon: capital misallocation due to a credit constraint reduces productivity and yields productivity stagnation. By empirically analyzing the Japanese and the US economies, Kehoe and Prescott (2002), Hayashi and Prescott (2002) and McGratten and Prescott (2012) also insist that recent stagnations are due to declines in productivity. However, it may be hard to consider that in the early 1990s Japan's productivity suddenly stopped increasing and has never improved for more than two decades.

While these models may well fit short-run macroeconomic fluctuations, the present paper considers secular stagnation such as Japan's Lost Decades and possibly the EU's Great Recession triggered by the financial crisis of 2008. In the IMF conference of 2013, Summers

(2013) warned against relying too much on the DSGE approach in solving economic crises and emphasized the need to work on long-run recessions rather than short-run business fluctuations. This paper follows this line of thought and considers secular stagnation of aggregate demand.

A dynamic optimization model of secular stagnation due to aggregate demand deficiency has firstly been presented by Ono (1994, 2001). He shows that if the marginal utility of money (or wealth) holding is insatiable, aggregate demand deficiency and deflation persistently emerge. Although real money balances (or wealth) expand through the deflation, the marginal rate of substitution between money and consumption does not change because of the insatiability, implying that the wealth effect on consumption disappears. While lower prices as a result of deflation will not stimulate consumption, deflation itself makes it more advantageous for people to save more and consume less, leading to a steady state with secular deflation and stagnation.[4] This stagnation mechanism has recently been discussed also by Michaillat and Saez (2014) and Michau (2015).

There is another strand of research on secular stagnation of aggregate demand. Eggertsson and Mehrotra (2014) introduce a borrowing constraint to an OLG model and show that with the zero lower bound of the nominal interest rate the lending and borrowing market may persistently be uncleared. The intertemporal allocation of consumption is distorted by the

[4] This model has recently been developed in various ways. Matsuzaki (2003) and Hashimoto (2004) introduce heterogeneous households to the model and examine the effect of redistribution policy on aggregate demand. Johdo (2006) considers the effect of R&D subsidies on aggregate demand and employment. Ono (2010) applies the model to analyze Japan's long-run stagnation and economic policies. Murota and Ono (2011) show that if households consider money as a target of status, aggregate demand deficiency appears. Hashimoto and Ono (2011) apply this type of stagnation model to examine the effects of child allowances on the population growth and aggregate demand. Murota and Ono (2012) consider a preference for deposit holdings and show secular stagnation, zero nominal interest rates and excess reserves of commercial banks to appear. Murota and Ono (2015) reformulate the model and propose a Keynesian-multiplier-like analysis with quite different implications from the conventioanl ones. Illing et al. (2016) introduce to this model heterogenous households with different time patience and a constraint on the less patient households' borrowing and find that relaxing the borrowing constraint may temporarily expand but eventually reduces aggregate demand.

borrowing constraint, causing consumption shortages to occur.[5] Such a stagnation will disappear if the government removes the borrowing constraint and supplies enough lending to people.

In the model of secular demand stagnation due to people's insatiable wealth preference, in contrast, secular demand stagnation occurs without such market distortions as the borrowing constraint. Preference for wealth holding over consumption makes consumption too small to achieve full employment especially when the potential productivity is very large, as in developed countries. The present paper extends this model to a two-country framework and analyzes the case where the home country suffers from secular demand stagnation while the foreign country achieves full employment.

2. The model

There are two countries, home and foreign. The home production sector specializes in commodity 1 and the foreign one in commodity 2. Both of them are competitive and use only labor to produce the respective commodities with constant productivities θ_1 and θ_2^*. Variables without (or with) $*$ denote those of the home (or foreign) country below.

All households in the two countries have the same utility function of the two commodities. The function is homothetic. Therefore, for a given level of aggregate consumption c (or c^*), the function is summarized as

$$u(c^{(*)}) \equiv \hat{u}(c_1^{(*)}, c_2^{(*)}),$$

where $c_i^{(*)}$ is each country's consumption of commodity i $(i = 1, 2)$ and satisfies

$$p_1(\omega)c_1^{(*)} = \delta(\omega)c^{(*)}, \;\; p_2(\omega)c_2^{(*)} = [1 - \delta(\omega)]c^{(*)},$$

[5] Eggertsson et al. (2016) have extended this model to an open-economy setting. Eggertsson (2010) shows aggregate demand stagnation to appear in a representative agent model with the borrowing constraint. However, the stagnation is a temporary one.

$$1 > \delta(\omega) > 0, \quad \delta'(\omega) \geq 0. \tag{1}$$

$\delta(\omega)$ is the ratio of expenditure on commodity 1, ω is the relative price of commodity 2 to commodity 1, and $p_i(\omega)$ is the real price of commodity i ($i = 1, 2$). Real prices $p_1(\omega)$ and $p_2(\omega)$ satisfy

$$p_1(\omega) = \frac{P_1}{P} = \frac{P_1^*}{P^*}, \quad p_2(\omega) = \frac{P_2}{P} = \frac{P_2^*}{P^*} = \omega p_1(\omega), \quad P_1^* = \frac{P_1}{\epsilon}, \quad P_2 = \epsilon P_2^*, \tag{2}$$

where P and P^* are the home and foreign general price indices, P_i and P_i^* are the nominal prices of commodity i measured in the home currency and the foreign currency, respectively, and ϵ is the nominal exchange rate.

The first and third equations (or alternatively the second and fourth equations) in (2) yield

$$P = \epsilon P^*.$$

The time derivative of this equation gives

$$\pi = \frac{\dot{\epsilon}}{\epsilon} + \pi^*,$$

where π and π^* are respectively the two countries' inflation rates. Therefore, from the non-arbitrage condition between home and foreign assets:

$$R = \frac{\dot{\epsilon}}{\epsilon} + R^*,$$

where R and R^* respectively represent the home and foreign nominal interest rates, one obtains

$$R - \pi = r = R^* - \pi^*, \tag{3}$$

implying that the real interest rate r is internationally the same.

Because the home and foreign general price indices P and P^* satisfy

$$\frac{dP^{(*)}}{P^{(*)}} = \delta(\omega)\frac{dP_1^{(*)}}{P_1^{(*)}} + (1 - \delta(\omega))\frac{dP_2^{(*)}}{P_2^{(*)}},$$

as shown by Deaton and Muellbauer (1980, p.175), from (2) one finds

$$0 = \delta(\omega)\frac{p_1'(\omega)}{p_1(\omega)} + (1 - \delta(\omega))\frac{p_2'(\omega)}{p_2(\omega)}, \quad \frac{1}{\omega} + \frac{p_1'(\omega)}{p_1(\omega)} = \frac{p_2'(\omega)}{p_2(\omega)},$$

and hence

$$p_1'(\omega) = -\frac{(1-\delta)p_1}{\omega} < 0, \quad p_2'(\omega) = \frac{\delta p_2}{\omega} > 0. \tag{4}$$

The home and foreign representative households have the same subjective discount rate ρ and utility functional:

$$\int_0^\infty \left[u(c^{(*)}) + v(m^{(*)})\right] \exp(-\rho t)\, dt,$$

$$u' > 0, \quad u'' < 0, \quad v' > 0, \quad v'' < 0,$$

where $v(m^{(*)})$ represents utility of money. They maximize it subject to the flow budget equation and the asset constraint:

$$\dot{a}^{(*)} = ra^{(*)} + w^{(*)}x^{(*)} - c^{(*)} - R^{(*)}m^{(*)} - z^{(*)},$$

$$a^{(*)} = m^{(*)} + b^{(*)}, \tag{5}$$

where $w^{(*)}$ is the real wage, $z^{(*)}$ is the lump-sum tax, and $x^{(*)}$ is the realized employment. Real total assets $a^{(*)}$ consist of real money balances $m^{(*)}$ and real international asset $b^{(*)}$ while the firm value is zero under the linear technology and perfect competition. The real rate of interest r is internationally the same, as shown by (3). The two countries' labor endowments are normalized to 1 and may be underemployed. Therefore, realized employment $x^{(*)}$ also implies the employment rate.

The optimal household behavior is summarized as

$$\rho + \eta^{(*)}\frac{\dot{c}^{(*)}}{c^{(*)}} = \frac{v'(m^{(*)})}{u'(c^{(*)})} - \pi^{(*)}, \quad R^{(*)} = \frac{v'(m^{(*)})}{u'(c^{(*)})}, \tag{6}$$

of which the derivation is set out in Appendix I. The first property shows the Ramsey equation while the second represents the equality between the liquidity premium of money and the nominal interest rate, which gives the money demand function.

The home (or foreign) government imposes lump-sum tax $z^{(*)}$ and purchases $g_i^{(*)}$ of commodity i ($i = 1, 2$). Therefore,

$$z^{(*)} = p_1(\omega)g_1^{(*)} + p_2(\omega)g_2^{(*)}. \tag{7}$$

From the home and foreign demand functions of the two commodities in (1), world demand for commodity 1 and that for commodity 2 are respectively $\delta(\omega)(c+c^*)/p_1(\omega)$ and $[1-\delta(\omega)](c+c^*)/p_2(\omega)$. Given the outputs of the two commodities, the international competitive markets perfectly adjust so that

$$\frac{\delta(\omega)}{p_1(\omega)}(c+c^*)+g_1+g_1^*=\theta_1 x,$$

$$\frac{1-\delta(\omega)}{p_2(\omega)}(c+c^*)+g_2+g_2^*=\theta_2^* x^*, \tag{8}$$

where θ_1 is the home productivity and θ_2^* is the foreign productivity. Real balances $m^{(*)}$ satisfy

$$m^{(*)}=\frac{M^{(*)}}{P^{(*)}}, \tag{9}$$

where $M^{(*)}$ represents nominal money supply. It is assumed that $M^{(*)}$ is constant over time but may be increased in a once-and-for-all manner. Because the above equation yields

$$\dot{m}^{(*)}=-\pi^{(*)}m^{(*)},$$

and $z^{(*)}$ satisfies (7), the flow budget equation in (5) reduce to

$$\dot{b}^{(*)}=rb^{(*)}+w^{(*)}x^{(*)}-c^{(*)}-[p_1(\omega)g_1^{(*)}+p_2(\omega)g_2^{(*)}]. \tag{10}$$

International asset holdings b and b^* always satisfy

$$b+b^*=0. \tag{11}$$

3. The condition for the asymmetric steady state to arise

This section presents the condition for the asymmetric case to arise, in which the home country faces persistent unemployment while the foreign country achieves full employment. Before doing so, let us obtain the steady state in which both countries attain full employment as a benchmark.

In the steady state with full employment in both countries, where

$$x = 1, \quad x^* = 1,$$

prices, wages and consumption $c^{(*)}$ are constant, and hence from (3), (6) in which $\pi^{(*)} = 0$, and (9),

$$\rho = \frac{v'(M/P)}{u'(c)}, \quad \rho = \frac{v'(M^*/P^*)}{u'(c^*)}. \tag{12}$$

Current accounts $\dot{b}$ and $\dot{b}^*$ given in (10) are zero, $b^* = -b$ from (11), and

$$w = p_1(\omega)\theta_1, \quad w^* = p_2(\omega)\theta_2^*,$$

under perfect competition. Therefore, c and c^* are

$$c = c^f \equiv \rho b + p_1(\omega^f)\theta_1 - [p_1(\omega^f)g_1 + p_2(\omega^f)g_2],$$

$$c^* = c^{f*} \equiv -\rho b + p_2(\omega^f)\theta_2^* - [p_1(\omega^f)g_1^* + p_2(\omega^f)g_2^*], \tag{13}$$

where ω^f satisfies (8) in which $x = 1$ and $x^* = 1$ and thus

$$\frac{\omega^f \delta(\omega^f)}{1-\delta(\omega^f)} = \frac{\theta_1 - g_1 - g_1^*}{\theta_2^* - g_2 - g_2^*}. \tag{14}$$

Substituting c^f and c^{f*} given in (13) respectively to c and c^* in (12) gives the steady state levels of P and P^*.

However, in the presence of a liquidity trap the steady state presented above may not exist. In the present setting a liquidity trap emerges if the utility of money is insatiable and its marginal utility $v'(m)$ has a positive lower bound β,[6]

$$\lim_{m\to\infty} v'(m) = \beta > 0, \tag{15}$$

because under this property the home money demand function given in (6) satisfies

$$R = \frac{v'(m)}{u'(c)} \left(> \frac{\beta}{u'(c)} \text{ for any } m\right).$$

R remains to be strictly positive as m increases, implying a liquidity trap.[7] In this case, if and only if c^f, the home consumption under full employment given in (13), is so large as to satisfy

[6] Ono (1994, 2001) assumes this property in a closed-economy setting and proves that the dynamic equilibrium path uniquely exists and converges to a steady state with persistent unemployment. The validity of this property is empirically shown by Ono, Ogawa and Yoshida (2004) using parametric and non-parametric approaches.

$$\rho < \frac{\beta}{u'(c^f)} \left(< \frac{v'(M/P)}{u'(c^f)} \quad \text{for any } P \right), \tag{16}$$

there is no P that makes the first equation in (12) valid. Therefore, there is no dynamic path along which full employment is reached in the home country. Given that the left- and right-hand sides of (16) respectively represent the time preference rate and the liquidity premium, (16) shows the case where the marginal desire for holding money dominates that for consumption if the household consumes enough to realize full employment. The same argument applies to the foreign country.

If the home country cannot reach full employment while the foreign country can in the steady state, (16) is valid in the home country while foreign full-employment consumption c^{f*} given in (13) satisfies

$$\rho > \frac{\beta}{u'(c^{f*})}$$

so that there is P^* that satisfies the second equation of (12). From (13), (16) and the above inequality, the asymmetric case emerges when[8]

$$\rho < \frac{\beta}{u'(c^f)}, \quad \rho > \frac{\beta}{u'(c^{f*})},$$

where c^f and c^{f*} are given by (13). Thus, the asymmetric case in which $g_i^{(*)} = 0$ (for $i = 1, 2$) appears if

$$\rho b > \max\left(u'^{-1}\left(\frac{\beta}{\rho}\right) - p_1(\omega^f)\theta_1,\ p_2(\omega^f)\theta_2^* - u'^{-1}\left(\frac{\beta}{\rho}\right) \right). \tag{17}$$

This condition is valid when home productivity θ_1 is high while foreign productivity θ_2^* is low, and/or when the home country owns large foreign assets b. This paper focuses on this case.

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[7] If $v'(m)$ remains positive as m expands, $v(m)$ may not represent the transaction motive. It would rather imply wealth preference. In appendix II this is taken into account and the wealth preference and the narrowly-defined transaction motive are separately introduced to the present model. It is shown that the following analysis is still valid and that the home nominal interest rate is zero in the steady state.

[8] Ono (2014) deals with the case where the first inequality is valid but the second one is not; that is, the case where neither country has a full-employment steady state.

4. Wage adjustment and the steady state

In the asymmetric case the home country cannot reach full employment. Therefore, sluggish wage adjustments under labor demand shortages must be present. The dominant setting of price and wage adjustments in DSGE models is that of Calvo (1983). However, it does not suit the present analysis because it allows neither aggregate demand shortages nor involuntary unemployment any time. Aggregate demand shortages are explicitly analyzed by Krugman (1998) in a discrete-time setting. However, it cannot treat long-run shortages of aggregate demand because the length of each period is exogenously given and aggregate demand shortages appear only in the period in which an exogenous shock occurs. In contrast, Ono and Ishida (2014) propose a dynamic extension of the fair wage model by Akerlof (1982) and Akerlof and Yellen (1990), which can deal with unemployment as well as full employment during the adjustment process and in the steady state. Therefore, it is adopted.

4.1. Wage and price adjustments

There are three kinds of workers, employed, unemployed and newly hired ones. The employed randomly separate from the current job at an exogenously given Poison rate α, as in search models (e.g., Pissarides, 2000).[9] Thus, in the home country employment x follows

$$\dot{x} = -\alpha x + \chi, \tag{18}$$

where χ is the number of the newly hired. While workers are employed, they form fair wages in mind by referring to their past wages and the unemployment level of the society. More precisely, at the end of the previous period $(t - \Delta t)$ each employed worker conceives the

[9] The Calvo model also assumes an exogenous Poison rate at which a firm (or a labor union) has a chance of revising prices (or wages) at each time point. The present α instead represents a timing of revising the fair wage because some of the incumbent workers that conceive fair wages leave the job, as mentioned later.

rightful wage $\nu(t-\Delta t)$ so that the realized wage $W(t-\Delta t)$ is the average of the rightful wage of the employed, $\nu(t-\Delta t)$, and the income of the unemployed, which is zero. Because the number of the employed and that of the unemployed are respectively $x(t-\Delta t)$ and $1-x(t-\Delta t)$, the rightful wage $\nu(t-\Delta t)$ satisfies

$$\nu(t-\Delta t)x(t-\Delta t)+0\times[1-x(t-\Delta t)]=W(t-\Delta t). \tag{19}$$

Having the rightful wage calculated at the end of the previous period in mind, at the beginning of period t the employed take the average of the rightful wage $\nu(t-\Delta t)$ and the zero income of the unemployed as the fair wage $W_F(t)$. The newly hired, of which the number is $\chi(t)\Delta t$, have no preconception about the fair wage or the rightful wage and simply follow the fair wage of the employed. Therefore, when the employed calculate the fair wage $W_F(t)$ at the beginning of period t, the total number of workers that they care is $1-\chi(t)\Delta t$ while the number of the employed is $x(t-\Delta t)(1-\alpha\Delta t)$. $W_F(t)$ is then

$$W_F(t)=\frac{\nu(t-\Delta t)x(t-\Delta t)(1-\alpha\Delta t)}{1-\chi(t)\Delta t}. \tag{20}$$

If there is unemployment in the home country, the firm will set wage W equal to the fair wage W_F because W_F is the lowest wage under which the employees properly work. Therefore, from (19) and (20) W changes as follows:

$$\frac{W(t)-W(t-\Delta t)}{\Delta t}=\chi(t)W(t)-\alpha W(t-\Delta t).$$

Reducing Δt to zero yields the continuous dynamics of W:

$$\frac{\dot{W}}{W}=\chi-\alpha, \tag{21}$$

which implies that W is updated at the replacement pace of the workers. The commodity price P_1 always adjusts to equal W/θ_1 since there is no commodity supply if $P_1<W/\theta_1$ and excess commodity supply if $P_1>W/\theta_1$.

If full employment prevails ($x = 1$), in contrast, the firm tries to pick out workers from rival firms to expand the market share by raising W from W_F so long as $\theta_1 P_1 > W$, leading W to equal $\theta_1 P_1$. The same argument applies to the foreign wages.

Thus, each commodity price, P_1 in the home country and P_2^* in the foreign country, follows the movement of W (or W^*) given by (21) when there is unemployment. When full employment maintains, in contrast, W (or W^*) follows the movement of P_1 (or P_2^*). Anyway, they satisfy

$$\frac{W}{P_1}\left(=\frac{w}{p_1(\omega)}\right) = \theta_1, \quad \frac{W^*}{P_2^*}\left(=\frac{w^*}{p_2(\omega)}\right) = \theta_2^*. \tag{22}$$

4.2. The asymmetric steady state

This subsection obtains the asymmetric steady state, while the dynamic stability is analyzed in appendix III. In the asymmetric case, where

$$x < 1, \quad x^* = 1,$$

W follows (21) in the home country while W^* and P_2^* always adjust so that full employment is attained in the foreign country. Then, the market equilibrium conditions given by (8) reduce to

$$c + c^* = \frac{p_2(\omega)}{1-\delta(\omega)}(\theta_2^* - g_2^* - g_2), \quad x = \frac{1}{\theta_1}\left(\frac{\delta(\omega)}{p_1(\omega)}(c + c^*) + g_1 + g_1^*\right). \tag{23}$$

They provide x as a function of ω:

$$x = \hat{x}(\omega; \theta_1, g_1, g_2, g_1^*, \theta_2^* - g_2^*) \equiv \frac{1}{\theta_1}\left(\frac{\omega\delta(\omega)}{1-\delta(\omega)}(\theta_2^* - g_2^* - g_2) + g_1 + g_1^*\right),$$

$$\hat{x}_\omega = \left(\frac{\theta_2^* - g_2^* - g_2}{\theta_1}\right)\frac{\delta(1-\delta) + \omega\delta'}{(1-\delta)^2} = \left(\frac{c + c^*}{p_1\theta_1}\right)\frac{\delta(1-\delta) + \omega\delta'}{(1-\delta)\omega} > 0, \tag{24}$$

where $\hat{x}$ is the home employment level that clears the two commodity markets for a given ω. Note that $\hat{x}_\omega > 0$ because a rise in ω, the relative price of the foreign commodity, naturally increases world demand for the home commodity and expands home employment x.

Because the relative price ω is constant in the steady state, from the first equation of (22), the two nominal prices, general price index P and nominal wage W move in parallel. The employment rate x is also constant, and hence from (18) $\chi = \alpha x$, implying that the number of job separation equals that of new employees. Therefore, from (21) one finds

$$\pi = \frac{\dot{W}}{W} = \alpha(x-1), \tag{25}$$

i.e., the price adjustment is of the Walrasian type in the steady state. Because $x < 1$, P keeps declining, which makes $v'(m) = \beta$. Substituting this π into the Ramsey equation given in (6) and making $\dot{c} = 0$ yields

$$\frac{\beta}{u'(c)} - \rho - \alpha(x-1) = 0 \;\; \rightarrow \;\; c = \hat{c}(x;\alpha,\beta). \tag{26}$$

In the steady state the home current account $\dot{b}$ in (10) equals zero and the two equations in (6) in which $\dot{c} = 0$ yield $r(= R - \pi) = \rho$. Thus, using the first equation in (22) one obtains

$$\begin{aligned} \dot{b} &= \psi(\omega, x; b, \alpha, \beta, \theta_1, g_1, g_2) \\ &\equiv \rho b + p_1(\omega)\theta_1 x - [p_1(\omega)g_1 + p_2(\omega)g_2] - \hat{c}(x;\alpha,\beta) = 0, \\ & \qquad x = \hat{x}(\omega; \theta_1, g_1, g_2, g_1^*, \theta_2^* - g_2^*), \end{aligned} \tag{27}$$

where $\hat{x}$ and $\hat{c}$ are given by (24) and (26), respectively. Equation (27) determines ω. Consequently, from (24) x is determined. Substituting x thus obtained into (26) yields c. Foreign consumption c^* is obtained from the foreign current-account equation in (10) to which $x^* = 1$, $r = \rho$, $\dot{b}^* = 0$, (11) and the second equation in (22) are applied:

$$c^* = \hat{c}^*(\omega; b, g_1^*, \theta_2^* - g_2^*) \equiv -\rho b + p_2(\omega)(\theta_2^* - g_2^*) - p_1(\omega)g_1^*. \tag{28}$$

All the variables are thus determined.

Furthermore, from the analysis on the existence of the asymmetric steady state and the dynamics around it, presented in appendix III, one obtains the following lemma:

Lemma: The asymmetric steady state exists when (17) is valid. If changes in policy, productivity and preference parameters occur in the asymmetric steady state, a new steady state in which $\dot{c}^{(*)} = 0$, $\dot{b}^{(*)} = 0$, $\dot{m} > 0$ and $\dot{m}^* = 0$ is immediately reached without any interim process. Thus, on the new dynamic equilibrium path ω, x, c and c^* are constant while m continues to expand, keeping $v'(m) = \beta$.

There should be an adjustment process of c and b if $v'(m)$ changed as m gradually expanded with deflation. However, in the initial steady state $v'(m)$ already reached β and stays the same in the new steady state –i.e., gradual changes in m due to deflation do not affect $v'(m)$ and hence neither c nor b has any adjustment process. A new steady state is immediately reached after a parameter shift occurs.

Note that full employment cannot be reached by price and wage adjustments in the home country, as mentioned below. As shown in (22), home firms' pricing behavior leads to

$$w = p_1(\omega)\theta_1,$$

while nominal wage W and general price index P decline in parallel, yielding deflation. Because ω stays constant in the asymmetric steady state, real wage w does not change and hence full employment is not achieved by the wage adjustment. Instead, employment adjusts to aggregate demand which depends on the deflation rate.

5. Own and spillover effects of parameter changes

This section analyzes the effects of changes in preference, productivity and policy parameters on the relative price ω, home employment rate x, and the two countries' consumption c and c^* when the economy is initially in the asymmetric steady state. As declared in the lemma, a new steady state is immediately reached after a parameter shift and thus the effects on the steady-state values of the variables also represent the effects on the dynamic paths of the variables.

For simplicity, it is considered that those parameter shifts occur in the neighborhood where

$$(g_1^*, g_2^*) = (0,0), \quad (g_1, g_2) = (0,0), \quad b = 0. \tag{29}$$

Note that under (29) the condition for the asymmetric case to arise, given by (17), turns to

$$p_1(\omega^f)\theta_1 > u'^{-1}\left(\frac{\beta}{\rho}\right) > p_2(\omega^f)\theta_2^*,$$

which is the case when the value of home potential production is high enough while that of foreign potential production is low enough.

With the Marshall-Lerner condition a rise in ω improves the home current account.[10] Therefore, from (4), (24), (27) and (29), one finds

$$\frac{\partial \dot{b}}{\partial \omega} = \psi_\omega + \psi_x \hat{x}_\omega = \hat{x}_\omega\left[\left(\frac{\delta^2(1-\delta)+\omega\delta'}{\delta(1-\delta)+\omega\delta'}\right)p_1\theta_1 - \hat{c}_x\right] > 0, \tag{30}$$

$$\text{where} \quad \psi_\omega = -\theta_2^*\delta p_1 < 0, \quad \psi_x = p_1\theta_1 - \hat{c}_x,$$

which yields

$$\psi_x = p_1\theta_1 - \hat{c}_x > 0. \tag{31}$$

An increase in home employment x expands home production $p_1\theta_1 x$ by $p_1\theta_1$ and home consumption c by $\hat{c}_x$. The former effect improves, and the latter effect worsens, the home current account. The inequality in (31) implies that the current-account improvement through the production expansion dominates the current-account deterioration through the consumption expansion.

From (30), a decrease in ω worsens the home current account $\dot{b}$. From the lemma, the new steady state where $\dot{b} = 0$ is immediately reached after a parameter change. Therefore, if the home current account represented by (27) turns to be positive after a parameter change, ω must decreases so that the home current-account balance recovers. Formally, it is restated as follows:

[10] In appendix III it is proved that the Marshall-Lerner condition is valid if the asymmentric steady state exists.

Proposition 1: A parameter change that improves the home current account raises the relative price of the home commodity $1/\omega$ (or equivalently, decreases the relative price of the foreign commodity ω).

From (27), the partial derivative of the home current account $\dot{b}$ with respect to a parameter v is generally represented as

$$\frac{\partial \dot{b}}{\partial v} = \psi_v + \psi_x \hat{x}_v = \psi_v + (p_1\theta_1 - \hat{c}_x)\hat{x}_v \quad \text{for } v = b, \alpha, \beta, g_1^*, \theta_2^* - g_2^*, \theta_1, g_1, g_2, \tag{32}$$

where ψ_v and $\hat{x}_v$ are obtained from (24) and (27) as follows:

$$\psi_b = \rho,\ \ \psi_\alpha = -\hat{c}_\alpha,\ \ \psi_\beta = -\hat{c}_\beta;\ \ \hat{x}_v = 0 \ \text{ for } \ v = b, \alpha \text{ and } \beta;$$

$$\psi_v = 0 \ \text{ for } \ v = g_1^* \text{ and } \theta_2^* - g_2^*;\ \ \hat{x}_{g_1^*} = \frac{1}{\theta_1},\ \ \hat{x}_{\theta_2^* - g_2^*} = \frac{\omega\delta}{(1-\delta)\theta_1};$$

$$\psi_{\theta_1} = p_1 x,\ \psi_{g_1} = -p_1,\ \psi_{g_2} = -p_2;\ \ \hat{x}_{\theta_1} = -\frac{x}{\theta_1},\ \hat{x}_{g_1} = \frac{1}{\theta_1},\ \hat{x}_{g_2} = -\frac{\omega\delta}{(1-\delta)\theta_1}. \tag{33}$$

The first line in (33) shows that b, α and β vary the home current account $\dot{b}$ without directly changing x ($\hat{x}_v = 0$), while the second line shows that g_1^* and $\theta_2^* - g_2^*$ vary $\dot{b}$ only through changes in x ($\psi_v = 0$). The other parameters, θ_1, g_1 and g_2, vary $\dot{b}$ through both the channels, as shown on the third line.

From (27), (30) and (32), a change in parameter v affects ω as follows:

$$\frac{\partial \omega}{\partial v} = -\frac{\partial \dot{b}}{\partial v} \Big/ \frac{\partial \dot{b}}{\partial \omega} = -\frac{\psi_v + (p_1\theta_1 - \hat{c}_x)\hat{x}_v}{\hat{x}_\omega\left[\left(\frac{\delta^2(1-\delta)+\omega\delta'}{\delta(1-\delta)+\omega\delta'}\right)p_1\theta_1 - \hat{c}_x\right]}, \tag{34}$$

where the denominator is positive from (30). From (24) and (34), the effect on home employment x is

$$\frac{\partial x}{\partial v} = \hat{x}_v + \hat{x}_\omega \frac{\partial \omega}{\partial v} = -\frac{\psi_v + \left(\frac{\delta(1-\delta)^2}{\delta(1-\delta)+\omega\delta'}\right)p_1\theta_1\hat{x}_v}{\left(\frac{\delta^2(1-\delta)+\omega\delta'}{\delta(1-\delta)+\omega\delta'}\right)p_1\theta_1 - \hat{c}_x}. \tag{35}$$

From (26) the effect on c is

$$\frac{\partial c}{\partial v} = \hat{c}_v + \hat{c}_x \left(\frac{\partial x}{\partial v}\right) \quad \text{for} \;\; v = \alpha, \beta;$$

$$\frac{\partial c}{\partial v} = \hat{c}_x \left(\frac{\partial x}{\partial v}\right) \quad \text{for} \;\; v = b, g_1^*, \theta_2^* - g_2^*, \theta_1, g_1, g_2;$$

$$\hat{c}_x = \; \alpha/(\frac{\eta\beta}{u'(c)c}) > 0, \;\; \hat{c}_\alpha = \; (x-1)/(\frac{\eta\beta}{u'(c)c}) < 0, \;\; \hat{c}_\beta = -\frac{\eta\beta}{c} < 0, \qquad (36)$$

while from (4) and (28) the effect on foreign consumption c^* is

$$\frac{\partial c^*}{\partial v} = \; \hat{c}_v^* + \hat{c}_\omega^* \left(\frac{\partial \omega}{\partial v}\right) \;\; \text{for} \; v = b, \, g_1^*, \theta_2^* - g_2^*,$$

$$\frac{\partial c^*}{\partial v} = \hat{c}_\omega^* \left(\frac{\partial \omega}{\partial v}\right) < 0 \quad \text{for} \; v = \alpha, \beta, \theta_1, g_1, g_2;$$

$$\hat{c}_\omega^* = \frac{\delta p_2 \theta_2^*}{\omega} > 0, \quad \hat{c}_b^* = -\rho < 0, \quad \hat{c}_{g_1^*}^* = -p_1 < 0, \quad \hat{c}_{\theta_2^* - g_2^*}^* = p_2 > 0, \qquad (37)$$

in the neighborhood of (29). The properties in (37) are standard under full employment. An increase in ω implies an improvement in the foreign terms of trade and thus expands foreign consumption c^*. An increase in b, or equivalently a decrease in b^*, lowers foreign interest earnings and reduces c^*. An increase in g_1^* decreases foreign disposable income and consumption c^*. An increase in $\theta_2^* - g_2^*$ raises foreign disposable income and c^*.

Having these properties in mind, let us analyze the effects of various parameters on the relative price ω, home employment x and consumption c, and foreign consumption c^*.

Home foreign-asset position $\boldsymbol{b}$, wage adjustment speed $\boldsymbol{\alpha}$ and liquidity preference $\boldsymbol{\beta}$: These parameter changes directly affect the home current account without varying home employment x, as shown by (32) and (33).

A transfer from the foreign country to the home country, which is represented by an increase in b, raises home interest earnings ρb and improves the home current account. Therefore, from proposition 1, the relative price of the home commodity $1/\omega$ rises, which is explicitly obtained from (33) and (34). The decline in ω reduces world demand for the home commodity and lowers home employment x, as found from (33) and (35). It worsens deflation, making the saving desire dominate the consumption desire, and decreases home consumption c, as obtained from (36). Foreign consumption c^* also decreases because the transfer (viz., a decrease in b^*) reduces the foreign country's asset holdings and moreover the decline in ω worsens the foreign terms of trade, as is clear from (37).

An increase in the wage adjustment speed α worsens deflation and decreases home consumption c while an increase in liquidity preference β directly reduces consumption, both of which improve the home current account. Hence, from proposition 1, the relative price of the home commodity $1/\omega$ rises. This property is mathematically obtained from (33) and (34). The rise in the home relative price $1/\omega$ reduces world demand for the home commodity and home employment x, as obtained from (33) and (35), worsening deflation. Home consumption c decreases due to the worse deflation and the initial negative effects of increases in α and β, as explicitly represented by (36). Because the decline in ω worsens the foreign terms of trade, foreign consumption c^* also decreases, as seen from (37).

The above results are summarized thus:

Proposition 2: A transfer from the foreign to the home country, an improvement in the home wage adjustment speed and an increase in home liquidity preference improve the home current account and raise the relative price of the home commodity. Thus, world demand for the home commodity and home employment decrease, which in turn worsens deflation and decreases home consumption. Foreign consumption also decreases because of the decline in the relative price of the foreign commodity.

Foreign productivity θ_2^*, foreign government purchases on the home commodity g_1^* and foreign government purchases on the foreign commodity g_2^*: These parameters change home employment x and thereby vary the home current account $\dot{b}$, as shown by (32) and (33).

Because an increase in g_1^* raises demand for the home commodity, home employment x increases and hence the home current account improves. An increase in $\theta_2^* - g_2^*$ enlarges foreign disposable income and stimulates foreign demand for the home commodity, which expands home employment x and improves the home current account. Therefore, from proposition 1, the relative price of the foreign commodity ω declines, which is mathematically proved from (31), (33) and (34).

The decline in the foreign relative price ω reduces world demand for the home commodity and home employment x, while the initial increases in g_1^* and $\theta_2^* - g_2^*$ expand x. From (33) and (35), the total effects on x are positive; that is, the decline in ω is so vast that x eventually decreases, as mentioned below. Suppose that the decline in ω triggered by the policy changes is large enough to decrease x to the initial level. Then, the current account represented by (27) is still positive in the neighborhood of (29) because for the same x the decline in ω makes home total output $p_1(\omega)\theta_1 x$ higher than the initial value. Therefore, ω further declines so as to recover the current-account balance, making x and c lower than the initial levels.

Foreign consumption c^* decreases with an increase in g_1^* because an increase in g_1^* not only enlarges foreign tax burdens but also worsens the foreign terms of trade ω. An increase in $\theta_2^* - g_2^*$ also worsens the foreign terms of trade but expands foreign disposable income. Therefore, c^* may increase or decrease. The effect on foreign consumption c^* is calculated from (33), (34) and (37) as follows:

$$\frac{\partial c^*}{\partial(\theta_2^* - g_2^*)} = \hat{c}^*_{\theta_2^* - g_2^*} + \hat{c}^*_\omega \frac{\partial \omega}{\partial(\theta_2^* - g_2^*)} = \frac{p_2\left(\delta(1-\delta)^2 + \omega\delta'\right)}{(1-\delta)^2(\psi_\omega + \psi_x \hat{x}_\omega)} \left(\frac{\theta_2^*}{\theta_1}\right) \left[\left(\frac{\omega\delta'}{\delta(1-\delta)^2 + \omega\delta'}\right) p_1\theta_1 - \hat{c}_x\right].$$

Because the coefficient of $p_1\theta_1$ in the square bracket of the above equation is smaller than that in (30), as proven below,

$$\frac{\omega\delta'}{\delta(1-\delta)^2 + \omega\delta'} - \frac{\delta^2(1-\delta) + \omega\delta'}{\delta(1-\delta) + \omega\delta'} = -\frac{\delta^3(1-\delta)^3}{[\delta(1-\delta) + \omega\delta'][\delta(1-\delta)^2 + \omega\delta']} < 0,$$

$\partial c^*/\partial(\theta_2^* - g_2^*)$ can either be positive or negative under the condition of (36), depending on the magnitudes of the exogenous parameters that determine ω.

The above results are summarized as follows:

Proposition 3: An improvement in foreign productivity θ_2^* and a decrease in foreign government purchases on the foreign commodity g_2^* increase foreign disposable income and foreign import, which improves the home current account. An increase in foreign government purchases on the home commodity g_1^* straightforwardly improves the home current account. Consequently, they raise the relative price of the home commodity. The rise is so vast that home employment and consumption eventually decrease. Foreign consumption may increase or decrease with an improvement in θ_2^* and a decrease in g_2^* while it always decreases with an increase in g_1^*.

Home productivity $\boldsymbol{\theta_1}$, government purchases on the home commodity $\boldsymbol{g_1}$ and on the foreign commodity $\boldsymbol{g_2}$: These parameters vary the home current account both directly and indirectly by changing home employment x, as shown by (32) and (33).

A rise in home productivity θ_1 improves the current account, as found from (32) and (33), and hence from proposition 1 the relative price of the home commodity $1/\omega$ rises, which worsens home employment x. Furthermore, a rise in θ_1 requires less workers to produce the same amount. These two effects decrease x, which is obtained from (33) and (35), and reduce home consumption c, as is clear from (36). The deterioration in the foreign terms of trade decreases foreign consumption c^*, as shown by (37).

An increase in home government purchases on the home commodity g_1 works in the opposite manner as an increase in θ_1. It expands home employment x, mitigates deflation and stimulates home consumption c, which worsens the home current account and raises ω. The rise in ω expands home employment and consumption. It also improves the foreign terms of trade and increases foreign consumption.

If full employment prevails in the home country, an increase in home productivity θ_1 lowers, and an increase in home government purchases on the home commodity g_1 raises, the relative price of the home commodity. These properties are opposite to those in the present case. This is because in the presence of unemployment those parameter changes affect employment x without varying demand of the home commodity or production (which equals $\theta_1 x - g_1$) while under full employment they vary it (which equals $\theta_1 - g_1$) without changing employment ($x = 1$).

An increase in home government purchases on the foreign commodity g_2 directly worsens the home current account by importing more the foreign commodity. It also decreases home disposable income and hence for a given ω total demand for the home commodity declines, which reduces home employment x and worsens the home current account. These two effects worsen the home current account, as is clear from (31) and (32). Thus, from proposition 1, the relative price of the home commodity $1/\omega$ decreases. It makes home employment x increase so much as to dominate the initial decrease in x and eventually x becomes larger than the initial level, as mentioned below. Suppose that the rise in ω makes x recover the initial level. Because the rise in ω makes $p_1(\omega)$ lower than the initial level, the home current account in (27) is still negative. Therefore, ω further rises and makes c and x larger than the initial levels. These properties are mathematically obtained from (33), (35) and (36). The rise in ω improves the foreign terms of trade and increases foreign consumption c^*, as obtained from (37).

The following proposition summarizes the abovementioned results.

Proposition 4: An expansion in home productivity and decreases in home government purchases on the home and foreign commodities improve the home current account.

Consequently, they raise the relative price of the home commodity (or equivalently lower the relative price of the foreign commodity) so much that home employment and consumption eventually decrease. Foreign consumption also decreases as a result of the deterioration in the foreign terms of trade.

Homogeneous commodities: If the home and foreign commodities are perfectly substitutable, viz. $\delta'(\omega) = \infty$, or equivalently the two commodities are homogenous, then

$$p_1 = 1, \quad p_2 = 1, \quad \omega = 1, \quad g = g_1 + g_2, \quad g^* = g_1^* + g_2^*. \tag{38}$$

In this case the current account given by (27) reduces to

$$\dot{b} = \rho b + \theta_1 x - g - \hat{c}(x; \alpha, \beta) = 0,$$

which determines x and $c(= \hat{c}(x; \alpha, \beta))$. From the total differentiation of the above equation one derives

$$\frac{\partial x}{\partial b} = -\frac{\rho}{\theta_1 - \hat{c}_x} < 0, \quad \frac{\partial x}{\partial \theta_1} = -\frac{x}{\theta_1 - \hat{c}_x} < 0, \quad \frac{\partial x}{\partial g} = \frac{1}{\theta_1 - \hat{c}_x} > 0.$$

$$\frac{\partial x}{\partial \alpha} = \frac{\hat{c}_\alpha}{\theta_1 - \hat{c}_x} < 0, \quad \frac{\partial x}{\partial \beta} = \frac{\hat{c}_\beta}{\theta_1 - \hat{c}_x} < 0, \quad \frac{\partial x}{\partial v} = 0 \text{ for } v = g^*, \theta_2^*.$$

Note that these are also obtained from (33) and (35) to which $p_1 = 1$ and $\delta'(\omega) = \infty$ are applied. Because $\hat{c}(x; \alpha, \beta)$ is the same as that in the case of heterogeneous commodities, properties in (36) still hold. From (28) and (38), foreign consumption c^* reduces to

$$c^* = -\rho b + \theta_2^* - g^*,$$

which yields

$$\frac{\partial c^*}{\partial b} = -\rho < 0, \quad \frac{\partial c^*}{\partial \theta_2^*} = 1, \quad \frac{\partial c^*}{\partial g^*} = -1; \quad \frac{\partial c^*}{\partial v} = 0 \text{ for } v = \alpha, \beta, \theta_1, g_1, g_2.$$

They are the same as the effects on c^* given in (37) to which the properties in (38) are applied.

Thus, all the properties obtained above are the same as those in the case of heterogeneous commodities except that the effects through a change in ω disappear.

A numerical example: For simplicity, the utility function of consumption is assumed to be

$$u(c) = \delta \ln(c_1) + (1-\delta)\ln(c_2),$$

and hence the elasticity of consumption utility and the expenditure ratio satisfy

$$\eta = 1, \quad \delta'(\omega) = 0.$$

Applying these properties and (33) to (36) yields

$$\frac{\partial c}{\partial(\rho b)} = -\frac{1}{\delta\left(\frac{\beta p_1\theta_1}{\alpha}\right)-1}, \quad \frac{\partial c}{\partial(p_1 g_1)} = \frac{\delta}{\delta\left(\frac{\beta p_1\theta_1}{\alpha}\right)-1}, \quad \frac{\partial c}{\partial(p_2 g_2)} = \frac{1+\delta}{\delta\left(\frac{\beta p_1\theta_1}{\alpha}\right)-1},$$

$$\frac{\partial c}{\partial(p_1 g_1^*)} = -\frac{1-\delta}{\delta\left(\frac{\beta p_1\theta_1}{\alpha}\right)-1}, \quad \frac{\partial c}{\partial(p_2 g_2^*)} = \frac{\delta}{\delta\left(\frac{\beta p_1\theta_1}{\alpha}\right)-1}. \tag{39}$$

In order to calculate the multipliers in (39), α, β, $p_1\theta_1$ and δ must be numerically determined. Japanese data during the period 1994-2013 are used to do so because they are available based on 93SNA (benchmark year 2005) and the period is almost that of Japan's long-run stagnation. The average values of the relevant economic variables during the period are

$$\pi = -0.00027, \quad c = 279.7, \quad p_1\theta_1 x = 490.2,$$

$$x - 1\left(= \frac{p_1\theta_1 x}{p_1\theta_1} - 1\right) = -0.01477, \quad p_1\theta_1 = \frac{p_1\theta_1 x}{x} = 497.5,$$

where π is the average of the annual rate of change in the consumer price index for all items (source: Statistics Bureau, Ministry of Internal Affairs and Communications, Japan), c and $p_1\theta_1 x$ are the averages of real household consumption expenditure and real GDP respectively (source: National Accounts, Cabinet Office, Government of Japan), and $x-1$ is the average of the ratio of the output gap to potential output (source: IMF World Economic Outlook Database, April 2015). Following Benhabib and Farmer (1996) and Mankiw and Weinzierl (2006), ρ is assumed to be

$$\rho = 0.05.$$

From (25) and (26),

$$\pi = \alpha(x-1), \qquad \beta c = \rho + \pi.$$

Applying the above values to them yields

$$\alpha = 1.828{\times}10^{-2}, \qquad \beta = 1.778{\times}10^{-4}, \qquad \frac{\beta p_1\theta_1}{\alpha} = 4.839.$$

Table 1 shows the magnitudes of the multiplier effects given in (39) to which the above figures of α, β and $\beta p_1\theta_1/\alpha$ are applied for the cases where $\delta = 0.69$ and 0.94. From (24) and (29), δ is

$$\delta = \frac{p_1\theta_1 x}{p_1\theta_1 x + p_2\theta_2^*},$$

which is the home country's GDP share of the two countries' total GDP. Because the population sizes of the two countries are assumed to be the same, δ should be calculated from per-capita values. The two values of δ are calculated from the per-capita GDP values of Japan and Korea and from those of Japan and China, respectively. The per-capita GDP values are the averages in terms of US dollar during the period 1994-2013 (source: IMF statistics), which are \$37,322 for Japan, \$16,535 for Korea, and \$2,382 for China. Note that the Marshall-Lerner condition (30) is valid if the denominator of the multiplier is positive, which indeed holds true for the present examples.

Table 1: Multipliers on home consumption c

	$\delta = 0.69$	$\delta = 0.94$
$\rho b \uparrow$	-0.43	-0.28
$p_1 g_1 \uparrow$	0.30	0.26
$p_2 g_2 \uparrow$	0.72	0.54
$p_1 g_1^* \uparrow$	-0.13	-0.02
$p_2 g_2^* \uparrow$	0.30	0.26

The case where $\delta = 1$ represents the case of a closed economy. Then, only the second equation in (39), $\partial c/\partial(p_1 g_1)$, is relevant and equals 0.26. This value equals that obtained by Murota and Ono (2015), where they treat a closed economy and use the same data set as the present one.

6. Comparison with the symmetric cases

This section compares the results in the asymmetric case with those in the two symmetric cases: full employment in both countries and secular stagnation in both countries. In Table 2 the first column summarizes the results in the asymmetric case while columns 2 and 3 show the results in the symmetric case of secular stagnation and that of full employment, respectively. The results in the symmetric stagnation case have been presented by Ono (2014), whereas those in the symmetric full-employment case are derived from (13) and (14), as shown below. Because in the symmetric cases the effects of changes in the parameters of the foreign country, g_1^* and $\theta_2^* - g_2^*$, are symmetric to those of the home country, g_2 and $\theta_1 - g_1$, the effects of the home country's parameters α, β, b, g_1, g_2 and θ_1 are discussed below.

The effects of changes in the stagnant country's parameters α, β, b, g_1, g_2 and θ_1 on its own consumption c and the relative price ω are similar whether the other country is fully employed or underemployed, as is clear from columns 1 and 2. The parameter changes that improve the home current account $\dot{b}$, which are increases in α, β, b and θ_1 and decreases in g_1 and g_2, raise the relative price of the home commodity $1/\omega$ so that $\dot{b}$ returns to zero, and reduce home consumption c. The spillover effects on the foreign country, however, differ between the two cases. If full employment prevails in the foreign country, an increase in ω works as an improvement in the terms of trade for the foreign country and expands foreign consumption c^*. However, if the foreign country suffers from secular stagnation, an increase in ω reduces its employment and consumption c^*, as shown in column 2.

Table 2: Comparison

	ASYMMETRY	SYMMETRY (Stagnation)	SYMMETRY (Full employment)
$\alpha \uparrow$	$c\downarrow,\ c^*\downarrow,\ \omega\downarrow$	$c\downarrow,\ c^*\uparrow,\ \omega\downarrow$	No effects
$\beta \uparrow$	$c\downarrow,\ c^*\downarrow,\ \omega\downarrow$	$c\downarrow,\ c^*\uparrow,\ \omega\downarrow$	No effects
$b \uparrow$	$c\downarrow,\ c^*\downarrow,\ \omega\downarrow$	$c\downarrow,\ c^*\uparrow,\ \omega\downarrow$	$c\uparrow,\ c^*\downarrow$, no effect on ω
$g_1 \uparrow$	$c\uparrow,\ c^*\uparrow,\ \omega\uparrow$	$c\uparrow,\ c^*\downarrow,\ \omega\uparrow$	$c\downarrow,\ c^*\downarrow,\ \omega\downarrow$
$g_2 \uparrow$	$c\uparrow,\ c^*\uparrow,\ \omega\uparrow$	$c\uparrow,\ c^*\downarrow,\ \omega\uparrow$	$c\downarrow,\ c^*\uparrow,\ \omega\uparrow$
$\theta_1 \uparrow$	$c\downarrow,\ c^*\downarrow,\ \omega\downarrow$	$c\downarrow,\ c^*\uparrow,\ \omega\downarrow$	$c\uparrow,\ c^*\uparrow,\ \omega\uparrow$
$g_1^* \uparrow$	$c\downarrow,\ c^*\downarrow,\ \omega\downarrow$	—	—
$\theta_2^* - g_2^* \uparrow$	$c\downarrow,\ c^*\uparrow\downarrow,\ \omega\downarrow$	—	—

If both countries achieve full employment, from (4), (13), (14) one finds that in the neighborhood of (29)

$$dc_f = \rho db - (1-\delta)p_1\theta_1 \frac{d\omega^f}{\omega^f} - [p_1 dg_1 + p_2 dg_2],$$

$$dc_f^* = -\rho db + \delta p_2 \theta_2^* \frac{d\omega^f}{\omega^f},$$

$$\frac{\partial \omega^f(\theta_1, g_1, g_2)}{\partial \theta_1} > 0, \quad \frac{\partial \omega^f(\theta_1, g_1, g_2)}{\partial g_1} < 0, \quad \frac{\partial \omega^f(\theta_1, g_1, g_2)}{\partial g_2} > 0,$$

where the foreign parameters are fixed. These properties directly yield the results summarized in column 3 of Table 2.

First, neither α nor β affects real variables including c and c^*. A transfer from the foreign country to the home country ($b\uparrow$) naturally increases home consumption c and decreases foreign consumption c^* while ω is unaffected in the case of homothetic utility. Note that in the asymmetric case such a transfer decreases both c and c^* because it improves the home current account and lowers the foreign relative price ω, which worsens home employment and the foreign terms of trade.

Second, a decrease in home government purchases on the home commodity g_1 and an increase in home productivity θ_1 directly increase home disposable income and expand home consumption. They lower the relative price of the home commodity $1/\omega$ and hence foreign consumption c^* increases due to the improvement in the terms of trade. If the home country faces a liquidity trap, however, home consumption does not respond and unemployment deteriorates. The deterioration of unemployment worsens deflation, discourages consumption and improves the current account, which makes the relative price of the home commodity rise so that the current account returns to zero. It further decreases home employment and discourages consumption c. Moreover, it worsens the foreign terms of trade and reduces foreign consumption c^*.

Third, an increase in home government purchases on the foreign commodity g_2 reduces home consumption c by decreasing home disposable income. Foreign consumption c^* increases because it raises the relative price of the foreign commodity ω, which is an improvement in the foreign terms of trade. This result is also quite different from that in the asymmetric case. In the asymmetric case an increase in g_2 deteriorates the home current account and lowers the relative price of the home commodity $1/\omega$, which increases not only c by expanding home employment but also c^* by improving the foreign terms of trade.

Thus, the own and spillover effects in the symmetric cases are quite different from those in the asymmetric case.

7. Conclusions

A two-country economy in which the home country suffers from secular stagnation of aggregate demand while in the foreign country full employment prevails is considered.

A stagnant country often attempts to improve production efficiency, expecting that it would expand total production and home income. However, it improves the current account, which leads the home currency to appreciate, the relative price of the home commodity to rise and home employment and consumption to decrease. In order to return the current account to zero, the rise in the relative price must be so high that home employment and consumption eventually decrease. It is because even if home employment declines so much as to offset the improvement in the productivity and maintain the same production level, the current account is still positive because the reduction in home employment lowers home consumption. Therefore, the relative price of the home commodity rises and home employment further decreases. The rise in the relative price of the home commodity in turn worsens the foreign terms of trade. Therefore, if full employment prevails in the foreign country, the real value of foreign production declines and hence foreign consumption decreases.

Changes in the home country’s policy and preference parameters that worsen the home current account cause the relative price of the home commodity to decrease and thereby expand home employment and consumption. It is in turn an improvement in the terms of trade for the foreign country and thus increases foreign consumption. Typical such examples are expansions of home government purchases on home and foreign commodities. Foreign aid may be another important example. Foreign aid expenditures tend to be cut when a donor country suffers from secular stagnation. However, an expansion in foreign aid in fact benefits the donor country because it worsens the home current account, lowers the relative price of the home commodity, increases home employment, decreases deflation, and urges people to consume more.

Moreover, if the recipient country realizes full employment, it not only receives foreign aid but also benefits from the improvement in the terms of trade.

Appendix I: Optimal household behavior

From the Hamiltonian function of the optimal household behavior:

$$H^{(*)} = u\left(c^{(*)}\right) + v\left(m^{(*)}\right) + \lambda^{(*)}\left(ra^{(*)} + w^{(*)}x^{(*)} - c^{(*)} - R^{(*)}m^{(*)} - z^{(*)}\right),$$

one obtains the first-order optimal conditions:[11]

$$\lambda^{(*)} = u'\left(c^{(*)}\right), \quad \lambda^{(*)}R^{(*)} = v'\left(m^{(*)}\right), \ \frac{\dot{\lambda}^{(*)}}{\lambda^{(*)}} = \rho - r. \tag{A1}$$

From the Euler equations in (A1),

$$\frac{\lambda^*}{\lambda} = \frac{u'(c^*)}{u'(c)} = \kappa = \text{constant over time}, \quad c^* = u'^{-1}\left(\kappa u'(c)\right), \quad \dot{c}^* = \frac{\eta c^*}{\eta^* c}\dot{c}, \tag{A2}$$

$$\text{where} \quad \eta = -\frac{u''(c)c}{u'(c)}, \ \ \eta^* = -\frac{u''(c^*)c^*}{u'(c^*)}.$$

(3) and (A1) give the Ramsey equation and the money demand function in (6).

Appendix II: Wealth preference and the zero interest rate

In the text it is assumed that the marginal utility of money $v'(m)$ has a positive lower bound. This property would rather imply wealth preference than liquidity preference. Moreover, from (6) the nominal interest rate R equals $v'(m)/u'(c)$, which converges to $\beta/u'(c)$ (> 0) in the steady state. However, the rate observed in a stagnant economy is almost zero. This appendix explicitly separates liquidity preference and wealth preference and shows that the result in the text still holds while the nominal interest rate is zero.

If wealth preference $\varphi(a)$ is taken into account, the home utility functional turns to be

[11] Apparently, by replacing $u(c)$ by $\hat{u}(c_1, c_2)$ one obtains the intratemporal and intertemporal optimal conditions given by (1) and (A1) all at once.

$$\int_0^{\infty}[u(c)+v(m)+\varphi(a)]\exp(-\rho t)\,dt,$$

in which $v(m)$ now represents the transaction motive and satisfies

$$\lim_{m\to\infty} v'(m)=0. \tag{A3}$$

In this case the first-order optimal conditions of the home household in (A1) are replaced by

$$\lambda=u'(c),\quad \lambda R=v'(m),\quad \frac{\dot{\lambda}}{\lambda}=\rho-r-\frac{\varphi'(a)}{u'(c)}.$$

Therefore, the Ramsey equation in (6) turns to be

$$\eta\frac{\dot{c}}{c}=\frac{v'(m)+\varphi'(a)}{u'(c)}-(\rho+\pi),$$

while the money demand function in (6) is unchanged –i.e.,

$$R=\frac{v'(m)}{u'(c)}. \tag{A4}$$

Because all the other equations in the text are unchanged, the dynamic equations are valid by replacing $v'(m)$ by $v'(m)+\varphi'(a)$.

In the steady state deflation continues in the home country and m continues to expand. Therefore, from (A3) and (A4), one finds

$$R=\lim_{m\to\infty}\frac{v'(m)}{u'(c)}=0,$$

i.e., the nominal interest rate in the home country is zero. Furthermore, by assuming that

$$\lim_{a\to\infty}\varphi'(a)=\beta_a>0,$$

instead of (15), one obtains the same steady-state conditions except that β is replaced by β_a in (26). Therefore, all the results of the parameter changes in the text are valid, while the home nominal interest rate is zero.

Appendix III: Dynamics and the existence of the asymmetric steady state

This appendix summarizes the dynamic equations and the stability analysis, and proves the lemma in section 4.

Unemployment arises in the home country, and hence from (2), (4), (21) and the first equation of (22),

$$\chi - \alpha = \frac{\dot{W}}{W} = \frac{\dot{P}_1}{P_1} = \pi - \frac{\dot{p}_1(\omega)}{p_1(\omega)} = \pi - (1-\delta)\frac{\dot{\omega}}{\omega}. \tag{A5}$$

The time differentiation of x in (24) and that of the first equation in (23) to which c^* as a function of c given in (A2) is applied yield

$$\left(\frac{c}{\eta} + \frac{c^*}{\eta^*}\right)\eta\frac{\dot{c}}{c} = \left(\frac{\delta(1-\delta)+\delta'\omega}{1-\delta}\right)(c+c^*)\frac{\dot{\omega}}{\omega} = p_1\theta_1\dot{x}.$$

Because $\dot{x}$ and $\dot{\omega}$ are given above as functions of $\dot{c}$ and from (18) and (A5) π is

$$\pi = \dot{x} + \alpha(x-1) + (1-\delta)\frac{\dot{\omega}}{\omega},$$

π satisfies

$$\pi = \alpha(x-1) + \eta\frac{\dot{c}}{c}\left(\frac{c}{\eta} + \frac{c^*}{\eta^*}\right)\left[\frac{1}{p_1\theta_1} + \left(\frac{1}{c+c^*}\right)\frac{(1-\delta)^2}{\delta(1-\delta)+\omega\delta'}\right].$$

In the neighborhood of the steady state, deflation continues and $v'(m)$ sticks to β in the home country. Given this property, substituting the above π to the Ramsey equation in (6) yields

$$\Omega\frac{\dot{c}}{c} = \Delta(c,\kappa) \equiv \frac{\beta}{u'(c)} - \rho - \alpha(x-1), \tag{A6}$$

where Ω is

$$\Omega = \eta\left\{1 + \left(\frac{c}{\eta} + \frac{c^*}{\eta^*}\right)\left[\frac{1}{p_1\theta_1} + \left(\frac{1}{c+c^*}\right)\frac{(1-\delta)^2}{\delta(1-\delta)+\omega\delta'}\right]\right\} > 0,$$

and x is a function of c and κ from (23) to which $c^* = u'^{-1}(\kappa u'(c))$ presented in (A2) is applied.

Next, the dynamic stability of (A6) is examined in the neighborhood of (29). If c is so large that $c + c^*$ makes $x = 1$, where $c^* = u'^{-1}(\kappa u'(c))$ for a given κ, the first equation of (8) turns to be

$$x = 1, \quad \frac{\delta(\omega)}{p_1(\omega)}\left[c + u'^{-1}(\kappa u'(c))\right] = \theta_1.$$

The ω that satisfies the above equation and the second equation of (8) to which $x^* = 1$ is substituted equals ω^f given by (14) and hence c equals c^f, the full-employment consumption for the present κ. Because $x = 1$ and c^f satisfies (16), $\Delta(c, \kappa)$ defined by (A6) satisfies

$$\Delta(c^f, \kappa) = \frac{\beta}{u'(c^f)} - \rho > 0.$$

Therefore, in order for the steady-state level of c to exist in the range of $(0, c^f)$, one must have

$$\Delta(0, \kappa) < 0.$$

When $c = 0$, $c^* = u'^{-1}\big(\kappa u'(0)\big) = 0$ and thus $c + c^* = 0$, yielding $x = 0$ from the second equation of (23) in the neighborhood of (29). Therefore, from (A6) to which the property that $u'(0) = \infty$ is applied, one finds $\Delta(0, \kappa) = -(\rho - \alpha)$. In order for this value to be negative, it must be valid that

$$\rho - \alpha > 0. \tag{A7}$$

In this case $\Delta(c, \kappa)$ must be positively inclined with respect to c around the steady state, and thus

$$\Delta_c(c, \kappa) > 0,$$

i.e., the dynamics given by (A6) is unstable. Therefore, once κ is given, c jumps to the level that satisfies

$$\Delta(c, \kappa) = 0,$$

and the steady state of (A6) is immediately reached.

Let us next determine κ. Because $\kappa(= u'(c^*)/u'(c))$ is constant over time, by obtaining the steady-state levels of c and c^* one can find κ. Home consumption c in the steady state of (A6) satisfies (26). The home current account $\dot{b}$ in (27) always equals zero because otherwise b continues to either expand or decline at an accelerated pace and the non-Ponzi game condition is violated. Thus, ω that satisfies (27) is immediately reached. Consequently, from the first equation of (23), (24) and (26), D, x, c and $c^*(= D - c)$ are also determined, and hence $\kappa(=$

$u'(c^*)/u'(c)$ from (A2)) obtains. The dynamic property mentioned in the lemma of section 4 summarizes the above results.

Let us finally prove that this ω indeed exists in $(0, \omega^f)$ and then $0 < x < 1$ and $0 < c < c^f$ in the neighborhood of (29). Under (A7), when $\omega = 0$, $\hat{x}$ in (24), $\hat{c}$ in (26) and $\dot{b}$ in (27) satisfy

$$\omega = 0: \;\; \hat{x}(0; \ldots) = 0, \;\; \hat{c}(0, \ldots) = u'^{-1}\left(\frac{\beta}{\rho - \alpha}\right) > 0, \;\; \dot{b} = -\hat{c}(0, \ldots) < 0. \tag{A8}$$

When $\omega = \omega^f$ given by (14), from (24) $x = 1$ and from (26) $\hat{c}(1; \alpha, \beta) = u'^{-1}(\beta/\rho)$. Substituting these values and c^f given in (13) into $\dot{b}$ in (27) and taking into account the condition of (16) yields

$$\omega = \omega^f: \;\; \hat{x}(\omega^f; \ldots) = 1, \;\; \hat{c}(1, \ldots) = u'^{-1}\left(\frac{\beta}{\rho}\right) > 0, \;\; \dot{b} = c^f - \hat{c}(1, \ldots) > 0. \tag{A9}$$

Because $\dot{b} < 0$ when $\omega = 0$ and $\dot{b} > 0$ when $\omega = \omega^f$, as shown in (A8) and (A9), there is ω that makes $\dot{b} = 0$ within (0, ω^f):

$$0 < \omega < \omega^f, \tag{A10}$$

and around the steady state $\dot{b}$ is an increasing function of ω, implying the Marshall-Lerner condition given in (30) to hold. Because $0 < \omega < \omega^f$, $\hat{x}_\omega > 0$ from (24), and $\hat{c}_x > 0$ from (26), using the values given in (A8) and (A9) one finds

$$0 = \hat{x}(0; \ldots) < x < \hat{x}(\omega^f; \ldots) = 1, \;\; 0 < \hat{c}(0, \ldots) < c < \hat{c}(1, \ldots) < c^f.$$

Therefore, unemployment and aggregate demand deficiency appear in the home country.

In the foreign country, on the other hand, there is P^* that satisfies the second equation in (12) when (17) holds. It is because under (17)

$$\rho b > p_2(\omega^f)\theta_2^* - u'^{-1}\left(\frac{\beta}{\rho}\right) > p_2(\omega)\theta_2^* - u'^{-1}\left(\frac{\beta}{\rho}\right),$$

in which ω satisfies (A10) and hence

$$\frac{\beta}{u'(\rho b + p_2(\omega)\theta_2^*)} < \rho\left(= \frac{v'(M^*/P^*)}{u'(\rho b + p_2(\omega)\theta_2^*)}\right),$$

i.e., there is P^* with which full employment is reached in the foreign country.

Thus, the asymmetric steady state exists when (17) holds, as stated in the lemma.

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