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SHOULD BRAND FIRMS ALWAYS TAKE PIONEERING POSITION?

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Abstract

This paper discusses brand firms' endogenous timing problem when facing nonbrand firms under quantity competition. We study a market comprising brand and nonbrand products. There exist heterogeneous consumer groups-one group buys only brand products while the other one cares little about the brand. These two consumer groups constitute the high- and low-end markets respectively. The brand firms' moving order is endogenized, whereas the nonbrand firms are restricted to move in a later period. We show that if the low-end market is of an intermediate size, the leader-follower equilibrium outcome occurs, and the follower obtains second mover advantage which diminishes when the number of nonbrand firms increases. These results follow from the fact that each brand firm's best response function has an upward jump if the rival's output exceeds a particular level. Thus, the leader's profit function has a downward jump at some particular point while the follower's profit does not.

1 Introduction

When an established firm creates a new product, it gets a chance to choose a pioneering position over its competitor if it presents this new product for sale as early as possible. However, there are some real-world cases in which firms with branded products gave up their pioneering position and intentionally released their new product after their competitors. For example, Adidas created Adidas 1, its high-end sports equipment, in 2005. However, Adidas postponed its release and kept silent until it created its next generation product in 2006, right after Nike released a similar high-end product, Nike+.[1] In the SmartWatch market, rumors about Apple's iWatch had been spread on the Internet for years. Even the testing machine of iWatch had been spotted along with its detailed description long before Samsung established its similar product-Galaxy Gear-in Berlin (September, 2013). Unexpectedly, Apple then released their iWatch one year later.[2] In the above two cases, we see that the brand firms delayed the release of new, market-ready products. These cases are abnormal because people conventionally think that a firm should take action as early as possible to gain advantage.[3] One specific commonality in the above two cases is that while big firms with brand products compete for the high-end market, there also exist relatively small firms supplying nonbrand products to the low-end market.[4] Therefore, whether conventional thinking ("seeking pioneering position is necessary") still makes sense under the above specificity remains unknown.

This paper discusses brand firms endogenous timing problem when they face nonbrand firms. We consider a three-period game with two brand firms and n nonbrand firms. In the pre-determinate period (period 0), each brand is authorized to choose between moving in period 1 or 2. Each brand firm then decides its production level according to the timing choices made in period 0. The nonbrand firms are restricted to take action only in a later stage (period 3).[5]

We became inspired by a distinctive market structure introduced by Ishibashi and Matsushima (2009). Here, two heterogeneous consumer groups exist: high-end consumers, who buy only brand products, and low-end consumers, who care little about the brands. These two consumer groups respectively constitute the high- and low-end markets. There are three types of exogenous parameters: market size, consumer heterogeneity in valuing different products, and the number of nonbrand firms. There are two potential market statuses: brand firms supply the high-end market at a higher price and nonbrand firms supply the low-end market at a lower price separately; otherwise, brand and nonbrand products are sold at the same price and the high- and low-end markets integrate. The market structure is depicted in Figure 1. In this setting, two types of symmetric equilibria outcomes can exist under different market statuses: when market separation takes place, both firms choose small outputs; when the two markets integrate, both firms choose large outputs.[6] Endogenizing brand firms' moving orders enables the timing equilibrium outcomes to interact with different market statuses correspondingly, giving rise to significantly different results compared with the standard endogenous timing game under quantity-setting. I also focus on the strategic relation between brand and nonbrand firms. The number of nonbrand firms greatly affects the price in the low-end market and is the basis on which brand firms decide their outputs. Thus, the activities of nonbrand firms indirectly affect brand firms' profits as well. There are two main results of this paper. First, if the low-end market is of an intermediate size, the asymmetric timing outcome (i.e., the leader-follower equilibrium outcome) occurs, and the leader has a lower profit than the follower. Second, the leader's profit increases but the follower's decreases with the increasing number of nonbrand firms.

[Figure 1 about here]

The logic behind the first result is as follows. Due to the existence of the low-end market, brand firms may exercise their option to overproduce and drive the market price low enough to enter the low-end market. Therefore, each brand firms best response function has an upward jump if its rival's output exceeds a certain level q_J. From this property, when brand firms move sequentially, the leader has to restrict its output to q_J in order to maintain the high-end market price at an adequate level. A tiny excess would make the follower viciously raise its output so that the market price collapses rapidly, thus decreasing the leader's profit. However, when brand firms move simultaneously, because their respective outputs are not restricted by each other, overproducing and low-end market entrance due to price collapse are more likely to occur (if one brand firm chooses a large output, its rival will also choose a large output as a response). When the profitability of the low-end market is low, sequential moving enables brand firms to keep the high-end market price high enough to avoid entering the low-end market, whereas simultaneous moving leads to a less profitable market entrance. If this is the case, the leader-follower equilibrium outcome occurs. At equilibrium, although the leader restricts output to q_J, due to the upward shifting best response function, the follower need not worry about the price collapsing because it chooses a high output immediately to compensate for the losses from the falling price. Therefore, it chooses an output that is larger than the leader's and has a profit higher than the leader.

The second result shows that the competition in the low-end market affects brand firms' profits even when brand firms supply only the high-end market. The intuition is as follows. As the number of nonbrand firms increases, competition in the low-end market becomes more intensive, depressing the low-end market price. Therefore, a price collapse in the high-end market, which leads to market integration, is less likely to occur and the leader's output constraint imposed by the follower is alleviated. As a consequence, the leader chooses a higher threshold output q_J, and obtains a higher profit. Due to the strategic substitutability, the follower chooses less output and obtains a lower profit.

These results have two implications for timing strategy. First, new product release timing is closely related to the market's structural elements such as market size. Second, for greater concern, a brand firm may abandon the pioneering position to mitigate head-to-head competition and achieve a higher profit.

Some empirical works seem to be consistent with the above arguments. First, Mahajan and Muller (1996) study the IBM case and find that decisions to introduce a new generation product as soon as possible or to delay it until the maturity stage comes are closely affected by the relative size of the potential market.[7] Second, Krider and Weinberg (1995) show that in the film industry, some movie companies would rather delay debut and let their rivals take action first to avoid head-to-head competition.[8] Although the main structures of the PC and film industries are not exactly the same as those discussed in the present paper, there is some common logic to gain with modification.[9]

In reviewing the relevant literature, I first discuss the leader-follower equilibrium outcome in the present paper. Hamilton and Slutsky (1990) show that in the strand of observable delay, the asymmetric timing outcome occurs only with strategic complementarities. However, this paper provides evidence that the leader-follower outcome can also exist with strategic substitutability. Another related work is by Normann (2002), who discusses the extended game of observable delay with strategic substitutability under asymmetric information. Although leader-follower equilibrium outcomes also exist in that work, the result is largely based on the assumption of asymmetric information. Therefore, the incentives behind this result are different from that of the present paper.[10]

Our first result also displays the evidence of the second mover advantage. Two related papers typically discuss this point. Amir and Stepanova (2006) discusses the endogenous timing problem in a Bertrand-Nash equilibrium case with two firms: one efficient and one inefficient. They find that the inefficient firm always has second mover advantage, whereas the efficient one can also have second mover advantage only when the efficiency difference is not that large. Julien (2011) studies a Cournot sequential moving case with many leaders and followers and without considering the endogenous timing problem. When the players have equal marginal costs, each follower has a higher profit than each leader only if the follower's reaction function increases in the leader's quantity. In both these works, the increasing reaction function of the follower is a necessary condition for second mover advantage. However, in this paper, the follower has a reaction function that is decreasing almost everywhere, except for the upward jumping point at equilibrium, although it obtains a higher profit than the leader.

As this paper refers often to Ishibashi and Matsushima (2009), we share the common conclusion that "coordination failure" occurs when both brand firms choose large outputs in order to enter the low-end market. However, the focus of Ishibashi and Matsushima is the strategic interaction between brand firms, which is different from the focus of the present paper.[11] The main focus here is on brand firms' endogenous timing problem and the strategic interaction between brand and nonbrand firms. We extend Ishibashi and Matsushima's (2009) basic model by adding several new insights.

The remaining paper is organized as follows. Section 2 introduces the kinked inverse demand functions. Section 3 derives nonbrand firms' equilibrium outcomes in period 3 and two cases of subgames in which brand firms obtain different equilibrium outcomes according to their endogenous timing choices in period 0. In each subgame, parameter ranges within which two types of equilibrium outcomes exist are derived. Then a subgame perfect Nash equilibrium in which the leader-follower timing outcome occurs is derived and the relating propositions are elaborated on. Section 4 concludes the paper and the Appendix is presented in Section 5.

2 The Model

The basic model takes the spirit of Ishibashi and Matsushima (2009). We consider a demand function with strategic substitutable products. There are two brand firms and n nonbrand firms producing brand products and nonbrand products respectively. All of these products are homogenous for use. For simplicity, we assume the marginal cost to be zero, and there is no fixed cost. Here, only the quantity competition is considered. Let q_i^B $(i = 1, 2)$ and q_j^N $(j = 1, ..., n)$ be the output level of brand firm and nonbrand firm, respectively. Denote $Q^B \equiv \sum_i q_i^B, Q^N \equiv \sum_j q_j^N$.

We assume that there exists one group of consumers B, who only buy brand products, and another group of consumers N, who buy whatever product is cheaper. We also define the market facing consumer B as the high-end market and that facing consumer N the low-end market. We assume consumer B's valuation toward brand products to be uniformly distributed on $[0, 1]$ and the size of the high-end market to be 1; consumer N's valuation is uniformly distributed on $[0, a]$, where $0 < a < 1$. The low-end market size is assumed to be b.

Let P^B be the price of brand products, which is decided by the demand from the high-end market. Let P^N be the price of brand products, which is decided by the demand from low-end market. The demand functions of the high-end and low-end markets are as follows:

$$D^B(P^B) = \begin{cases} 0 & \text{if } P^B \in (1, \infty) \\ 1 - P^B & \text{if } P^B \in [0, 1]. \end{cases}$$

$$D^N(P^N) = \begin{cases} 0 & \text{if } P^N \in (a, \infty) \\ b(1 - \dfrac{P^N}{a}) & \text{if } P^N \in [0, a]. \end{cases}$$

If $P^B \geq P^N$, then brand firms supply the high-end market and nonbrand firms supply the low-end market separately. If $P^B < P^N$, then brand and nonbrand products are sold at the same price, which is decided by the aggregate demand from the high- and low-end markets.

We define $q = (q_1^B, q_2^B; q_1^N, ..., q_n^N)$. The inverse demand functions are as follows:

$$P^B(q) = \begin{cases} 1 - Q^B & \text{if } 1 - Q^B \geq a(1 - \frac{Q^N}{b}) \\ \frac{a[1 + b - (Q^B + Q^N)]}{a + b} & \text{otherwise.} \end{cases}$$

$$P^N(q) = \begin{cases} a(1 - \frac{Q^N}{b}) & \text{if } 1 - Q^B \geq a(1 - \frac{Q^N}{b}) \\ \frac{a[1 + b - (Q^B + Q^N)]}{a + b} & \text{otherwise.} \end{cases}$$

Let $\pi_i^B(q)$ and $\pi_j^N(q)$ denote the profit function of brand firm i, and nonbrand firm j, respectively. Each firm's profit function can be expressed as follows:

$$\pi_i^B(q) = \begin{cases} (1 - Q^B)q_i^B & \text{if } 1 - Q^B \geq a(1 - \frac{Q^N}{b}) \\ \frac{a[1 + b - (Q^B + Q^N)]}{a + b} q_i^B & \text{otherwise,} \end{cases} \tag{1}$$

$$\pi_j^N(q) = \begin{cases} a(1 - \frac{Q^N}{b})q_j^N & \text{if } 1 - Q^B \geq a(1 - \frac{Q^N}{b}) \\ \frac{a[1 + b - (Q^B + Q^N)]}{a + b} q_j^N & \text{otherwise.} \end{cases} \tag{2}$$

Let's consider a game with 4 periods. In period 0, each brand firm is authorized strategic options between moving in period 1 or 2. Each firm then decides its production level according to the timing choices made in period 0. It is noteworthy that when sequential moving is decided, the brand firm moving in period 2 observes the outcome in period 1. Nevertheless, the remaining nonbrand firms can only move in period 3.

3 Result

The different timing choices give rise to three different cases as follows: I, one brand firm moves in period 1 and the other moves in period 2; II, both brand firms move in period 1; III, both brand firms move in period 2. For each case, in equilibrium, both firms can choose either small outputs to maintain market separation, or large outputs which cause the high- and low-end markets to integrate. Case II and III are the same situations, as brand firms engage in the simultaneous moving game and nonbrand firms move afterwards. In the next subsection, we derive the equilibrium outcomes of nonbrand firms.

We first introduce the following assumption:

Assumption 1 *In a subgame there could exist multiple equilibria within the same parameter range. We apply payoff dominance to select the unique equilibrium outcome. That means that we only select the equilibrium outputs that bring brand firms higher profits.*

3.1 Nonbrand Firms' Equilibrium Outcomes

From Eq. (2), we solve each nonbrand firm's maximization problem in the simultaneous moving case. Here, due to kinked inverse demand, we solve the problem by two market statuses: separate or integrated. It is noteworthy that, it is also possible that the interior solution we obtain in each market status can not perfectly make the corresponding conditional inequality hold in the profit function (2), which means that we do not have either type of interior solution (we have a corner solution). Therefore, we need to consider the threshold value when P^B is equal to P^N, or, the case of a corner solution. We denote the three cases, "separate," "integrate," or "corner solution," by superscripts "S," "I," and "C," respectively. There are three types of symmetric equilibrium outcomes according to the above three cases respectively:

$$q_j^{NS}(Q^B) = \frac{b}{n+1};\ q_j^{NI}(Q^B) = \frac{1+b-Q^B}{n+1};\ q_j^{NC}(Q^B) = \frac{b}{n}(1 - \frac{1-Q^B}{a}).$$

$q_j^{NS}(Q^B)$ (respectively $q_j^{NI}(Q^B)$) is the interior solution for the maximization problem of $\pi_j^{NS}(q) = a(1 - Q^N/b)q_j^N$ (respectively $\pi_j^{NI}(q) = a[1 + b - (Q^B + Q^N)]q_j^N/(a + b)$). $q_j^{NC}(Q^B)$ is the corner solution that makes $P^B = P^N$. We need to check the condition under which each type of the above equilibrium outcomes is globally optimal. After several calculations, we obtain each nonbrand firms' equilibrium outcome in period 3 (see Appendix 5.1 for detailed calculations).

$$q_j^N(Q^B) = \begin{cases} q_j^{NS}(Q^B) & if\ Q^B \leq 1 - \dfrac{a}{n+1}, \\ q_j^{NI}(Q^B) & if\ Q^B > 1 - \dfrac{ab}{an + bn + b}, \\ q_j^{NC}(Q^B) & otherwise. \end{cases} \tag{3}$$

3.2 Brand Firms' Equilibrium Outcomes

In period 3, each nonbrand firm's best response is as derived by Eq. (3). The corresponding price of brand products is as follows:

$$P^B(Q^B) = \begin{cases} 1 - Q^B & if\ Q^B \leq q_K \equiv 1 - \dfrac{ab}{an + bn + b}, \\ \dfrac{a(1 + b - Q^B)}{(n + 1)(a + b)} & otherwise. \end{cases} \tag{4}$$

Case I: Either Brand Firm Moves in Period 1

We call the brand firm moving in period 1 the leader and the one moving in period 2 the follower. The leader chooses its output level q_l^B and the follower chooses its output level q_f^B.

The subscript l (f) denotes the leader (follower).

In period 2, we define $q^B \equiv (q_l, q_f)$. By substituting the nonbrand firms' best response function, the follower solves its maximization problem by

$$\max_{q_f^B} \pi_f^B(q^B) = \begin{cases} (1 - Q^B)q_f^B & if\ Q^B \leq q_K, \\ \dfrac{a(1 + b - Q^B)}{(n + 1)(a + b)} q_f^B & Otherwise, \end{cases} \tag{5}$$

and obtains two types of the best response functions. x (y) denotes the type of solution derived from the first (second) maximization problem of Eq. (5). Please see Appendix 5.2 for how to derive the follower's best response function.

$$q_f^B(q_l^B) = \begin{cases} q_f^{Bx}(q_l^B) \equiv \dfrac{1}{2}(1 - q_l^B) & if\ q_l^B \leq q_J \equiv 1 - b[a + \sqrt{a(a + b)(n + 1)}]/(an + bn + b), \\ q_f^{By}(q_l^B) \equiv \dfrac{1}{2}(1 + b - q_l^B) & Otherwise. \end{cases} \tag{6}$$

The corresponding profit is as follows:

$$\pi_f^B(q_l^B, q_f^B(q_l^B)) = \begin{cases} \pi_f^{Bx}(q_l^B, q_f^{Bx}(q_l^B)) \equiv \dfrac{1}{4}(1 - q_l^B)^2 & if\ q_l^B \leq q_J, \\ \pi_f^{By}(q_l^B, q_f^{By}(q_l^B)) \equiv \dfrac{a(1 + b - q_l^B)^2}{4(n + 1)(a + b)} & Otherwise, \end{cases} \tag{7}$$

where q_J is a threshold value of q_l^B that satisfies $\pi_f^{Bx}(q_l^B, q_f^{Bx}(q_l^B)) = \pi_f^{By}(q_l^B, q_f^{By}(q_l^B))$. When the leader's output q_l^B reaches q_J, the follower raises its own output from $(1 - q_J)/2$ to $(1 + b - q_J)/2$, which leads to a sudden decrease in the high-end market price. By doing so, the follower enables low-end consumers to afford brand products and thereby enters the low-end market. Although losing from the price collapse, the follower maintains the same profit level by the increase in sales when the leader chooses q_J.

In period 1, based on the follower's best response function, the leader's inverse demand function is as follows:

$$P^B(q_l^B, q_f^B(q_l^B)) = \begin{cases} P^x(q_l^B) \equiv \dfrac{1}{2}(1 - q_l^B) & if\ q_l^B \leq q_J, \\ P^y(q_l^B) \equiv \dfrac{a(1 + b - q_l^B)}{2(a+b)(n+1)} & otherwise. \end{cases} \tag{8}$$

The resulting profit function is as follows:

$$\pi_l^B(q_l^B) = \begin{cases} \pi_l^x(q_l^B) \equiv \dfrac{1}{2}(1 - q_l^B)q_l^B & if\ q_l^B \leq q_J, \\ \pi_l^y(q_l^B) \equiv \dfrac{a(1 + b - q_l^B)}{2(a+b)(n+1)}q_l^B & otherwise. \end{cases} \tag{9}$$

We can confirm that $P^x(q_J) > P^y(q_J)$ for any $0 < a < 1$, $b > 0$, $n > 1$. The inverse demand curve has a downward jump at q_J, which is caused by the follower who viciously raises its output, thus causing the leader to suffer a sudden drop in profit. Since the leader loses in profit while the follower does not, whether to be a leader or a follower matters a lot to the brand firm's profit when the leader reaches the output level $q_l^B = q_J$.

It is straightforward to find potentially three types of leader's equilibrium candidates: two types of locally optimal solutions ($q_l^{x*} \equiv 1/2$ and $q_l^{y*} \equiv (1 + b)/2$), which are derived from $\max_{q_l^B} \pi_l^{Bx}(q_l^B)$ and $\max_{q_l^B} \pi_l^{By}(q_l^B)$, respectively, and the corner solution at the jump-point q_J. The resulting profits are $\pi_l^{Bx}(q_l^{x*}) \equiv 1/8$, $\pi_l^{By}(q_l^{y*}) \equiv a(1 + b)^2/[8(a + b)(n + 1)]$ and $\pi_l^{Bx}(q_J) \equiv (1 - q_J)q_J/2$, respectively. Therefore, we need to check not only the global optimality at the locally optimal point, but also compare the leader's locally optimal profits, with the profit gained at q_J to obtain the global optimality. After several calculations, we obtain three conditions (A, B, and C), under which each type of equilibrium outcome exists. Please see Appendix 5.3 for how to derive the equilibrium conditions.

Definition 1 $A \equiv \{(a, b, n) \mid q_J > q_l^{x*}\}$; $B \equiv \{(a, b, n) \mid q_J \leq q_l^{x*}\ and\ \pi_l^{Bx}(q_J) \geq \pi_l^{By}(q_l^{y*})\}$; $C \equiv \{(a, b, n) \mid q_J \leq q_l^{x*}\ and\ \pi_l^{Bx}(q_J) < \pi_l^{By}(q_l^{y*})\}$.

Condition A can be rearranged to $b < \underline{b}_1$; condition B can be rearranged to $\underline{b}_1 \leq b < \overline{b}_1$; and condition C can be rearranged to $b \geq \overline{b}_1$, where $\underline{b}_1$ and $\overline{b}_1$ are decided given a and n.

Lemma 1 *When either brand firm moves in period 1, brand firms choose* $(q_l^{x*}, q_f^{x*}) \equiv (1/2, 1/4)$ *if* $b < \underline{b}_1$; $(q_J, (1-q_J)/2)$ *if* $\underline{b}_1 \leq b < \overline{b}_1$; *or* $(q_l^{y*}, q_f^{y*}) \equiv ((1+b)/2, (1+b)/4)$ *if* $b \geq \overline{b}_1$.

The numericized $\underline{b}_1$ and $\overline{b}_1$ are depicted in Table 1.

[Table 1 about here]

When either firm moves in period 1, (q_l^{x*}, q_f^{x*}) is picked up as an equilibrium outcome if the low-end market size is small enough (i.e. $b < \underline{b}_1$). Under this condition, we always have $\pi_l^{Bx}(q_l^{x*}) \geq \max\{\pi_l^{Bx}(q_J), \pi_l^{By}(q_l^{y*})\}$. $\pi_l^{Bx}(q_l^{x*})$ is larger than $\pi_l^{Bx}(q_J)$ because q_l^{x*} is the locally optimal value. The leader will not expand its output from q_l^{x*} to q_l^{y*}, or $\pi_l^{Bx}(q_l^{x*}) \geq \pi_l^{By}(q_l^{y*})$, because the decrease in price outweighs the increase in quantity. When the low-end market size is relatively large (i.e., $b \geq \underline{b}_1$), if the leader chooses q_l^{x*}, then the high- and low-end markets integrate and the price is given by $P^y(q_l^{x*})$. q_l^{x*} will not be selected by the leader because q_l^{y*} is the locally optimal value for $\pi_l^y(q_l^B)$. Whether the leader chooses q_J or q_l^{y*} is decided by comparing the values of $\pi_l^{Bx}(q_J)$ and $\pi_l^{By}(q_l^{y*})$. Due to the downward jump point in the inverse demand curve, the leader has a sudden drop in profit when q_l^B just exceeds q_J, although its profit rises gradually as it further increases output until q_l^{y*}. If b is smaller than $\overline{b}_1$, the sudden decrease in profit at q_J cannot be compensated by the gradual rise as q_l^B further increases until q_l^{y*} (i.e., $\pi_l^{Bx}(q_J) \geq \pi_l^{By}(q_l^{y*})$). Then, q_J is picked up by the leader. If the low-end market size is large (i.e., $b \geq \overline{b}_1$), then brand firms always choose to enter the low-end market. Thus, q_l^{y*} is picked up by the leader.

Case II or Case III: Both Brand Firms Move in Period 1 or Period 2

For simplicity, we only consider the case when both brand firms move in period 1. In period 1, based on the inverse demand in Eq. (4), each brand firm's best response function is the same as given in Eq. (6) except that we substitute the subscript l and f with i and $-i$, respectively. Then, we get

$$q_i^B(q_{-i}^B) = \begin{cases} q_i^{Bx}(q_{-i}^B) \equiv \dfrac{1}{2}(1 - q_{-i}^B) & if\ q_{-i}^B \leq q_J, \\ q_i^{By}(q_{-i}^B) \equiv \dfrac{1}{2}(1 + b - q_{-i}^B) & Otherwise. \end{cases} \tag{10}$$

Based on the inverse demand function in Eq. (4), brand firm i's profit function is as follows:

$$\pi_i^B(q^B) = \begin{cases} \pi_c^{Bx}(q^B) \equiv (1 - Q^B)q_i^B & if\ Q^B \leq q_K, \\ \pi_c^{By}(q^B) \equiv \dfrac{a(1 + b - Q^B)}{(a + b)(n + 1)}q_i^B & Otherwise. \end{cases} \tag{11}$$

We use subscript "c" to denote the Cournot case. Because of the kinked inverse demand function, brand firms' profit functions are decided by their aggregate output. We have the following two types of equilibrium outcome candidates: $(q_c^{x*}, q_c^{x*}) \equiv (1/3, 1/3)$ with the equilibrium profit $\pi_c^{Bx}(q_c^{x*}) \equiv 1/9$; $(q_c^{y*}, q_c^{y*}) \equiv ((1 + b)/3, (1 + b)/3)$ with the equilibrium profit, $\pi_c^{By}(q_c^{y*}) \equiv a(1 + b)^2/[9(a + b)(n + 1)]$. We need to ensure that each type of equilibrium outcome satisfies the corresponding conditional inequality of best response function (10). It is noteworthy that multiple equilibria could exist within the same parameter range. Then, we use Assumption 1 to select the unique equilibrium outcome. Please see Appendix 5.4 for how to derive the equilibrium conditions. After several calculations, we obtain the following condition D in which the equilibrium outcome (q_c^{Bx*}, q_c^{Bx*}) exists.

Definition 2 $D \equiv \{(a, b, n) \mid q_J \geq q_c^{x*}\}$; $E \equiv \{(a, b, n) \mid q_J < q_c^{x*}\}$.

Condition D can be rearranged to $b \leq b_2$; condition E can be rearranged to $b > b_2$, where b_2 is decided given a and n.

Lemma 2 *When both brand firms move in period 1 or period 2, they choose* (q_c^{x*}, q_c^{x*}) *if* $b \leq b_2$ *or* (q_c^{y*}, q_c^{y*}) *if* $b > b_2$.

The numericized b_2 is depicted in Table 2.

[Table 2 about here]

From Lemma 2, we see that when both brand firms move in period 1 or period 2, if the low-end market size is small enough, brand firms choose small equilibrium outputs so as to supply only the high-end market. This is the first part of Lemma 2. Conversely, if the low-end market size is large enough, brand firms choose large equilibrium outputs to enter the low-end market.

3.3 Endogenous Timing

We focus on the market condition when both conditions B and E hold. To explicitly clarify the condition, we rearrange the inequalities in these two conditions and obtain the following simplified condition:

$$b_2 < b \leq \overline{b}_1.$$

Under the above condition, when both brand firms move in period 1 or period 2, they choose (q_c^{y*}, q_c^{y*}) and when either of them moves in period 1, they choose $(q_J, (1-q_J)/2)$. The game in period 0 is depicted in Table 3.

[Table 3 about here]

We discuss brand firm 2's best timing response given brand firm 1's timing choice. Given brand firm 1 moving in period 2, we consider the leader's profit $\pi_l^{Bx}(q_J)$. From condition B, $\pi_l^{Bx}(q_J) \geq \pi_l^{By}(q_l^{y*})$. As $\pi_l^{By}(q_l^{y*})$ is larger than $\pi_c^{By}(q_c^{y*})$ for any $0 < a < 1$, $b > 0$ and $n > 1$, we obtain

$$\pi_l^{Bx}(q_J) > \pi_c^{By}(q_c^{y*}). \tag{12}$$

Thus, firm 2 reacts by moving in period 1. Given firm 1 moving in period 1, we consider the follower's profit $\pi_f^{Bx}(q_J, q_f^{Bx}(q_J))$. From condition E, q_J is less than $1/3$. In this inequality, we see that the leader's output q_J is smaller than that of the follower's $(1 - q_J)/2$, which implies that the follower obtains a higher profit than the leader, or $\pi_f^{Bx}(q_J, q_f^{Bx}(q_J)) \geq \pi_l^{Bx}(q_J)$. Together with inequality (12), we obtain

$$\pi_f^{Bx}(q_J, q_f^{Bx}(q_J)) > \pi_c^{By}(q_c^{y*}). \tag{13}$$

Therefore, brand firm 2 reacts by moving in period 2. By symmetry, brand firm 1 will respond the same way given firm 2's timing choice. Sequential timing equilibria thus exist. Table 4 depicts the corresponding values of $\pi_l^{Bx}(q_J)$, $\pi_f^{Bx}(q_J, q_f^{Bx}(q_J)$, and $\pi_c^{By}(q_c^{y*})$ given $n = 20$, $a = 0.6$ and $b_2 < b \leq \overline{b}_1$.

[Table 4 about here]

Next, we show how the number of nonbrand firms affects the profits of brand firms if $b_2 < b \leq \overline{b}_1$. As the number of nonbrand firms n increases, q_J becomes larger. Since $q_J \leq q_l^{x*}$, the leader's profit, $\pi_l^{Bx}(q_J) = (1 - q_J)q_J/2$ increases in n as well. Conversely, since $\pi_f^{Bx}(q_J, q_f^{Bx}(q_J)) = (1 - q_J)^2/4$ deceases in q_J, the increasing number of nonbrand firms has a counter effect on the follower's profit.

Proposition 1 *If $b_2 < b \leq \overline{b}_1$, then (i) the leader-follower equilibrium outcome exists, (ii) the leader obtains less profit than the follower, and (iii) the leader's profit increases but the follower's profit decreases as the number of nonbrand firms increases.*

Figure 2 depicts brand firms' best response functions and isoprofit curves. Due to strategic substitutability, each brand firm has a higher profit level toward the coordinate axis. If $b_2 < b \leq \overline{b}_1$, then the brand firms' reaction curves only intersect at (q_c^{y*}, q_c^{y*}). The Pareto superior set relative to the Cournot equilibrium output level is denoted by the shaded area. Since each brand firm's reaction curve enters the Pareto superior set at $q_i^B \leq q_J$, either brand firm would be happy to give the pioneering position to its rival. When brand firm 1 achieves the pioneering advantage over brand firm 2, it chooses to produce at the output level q_J, which brings it the highest profit along firm 2's reaction curve inside the Pareto superior set. It is noteworthy that Assumption 1 diminishes the possibility on which asymmetric timing appears. This is because Assumption 1 selects the type of equilibrium outcome that brings brand firms higher profits. This increases the brand firms' profits when they move simultaneously, which enhances the incentives for firms to deviate from the asymmetric timing situation. However, Proposition 1 shows that asymmetric timing appears even though the brand firms earn higher profits picked up by Assumption 1 under the simultaneous moving situation.

[Figure 2 about here]

The intuition of (*ii*) is as follows. When brand firms move sequentially, because of the existence of the low-end market, the follower has the option of overproducing to make the market price low enough so that it can enter the low-end market. Therefore, the follower's best response function has an upward jump when the leader's output surpasses a certain level q_J. A tiny output excess over q_J by the leader would cause the follower to viciously raise its output and cause the market price to collapse rapidly. Although this would bring the leader demand from the low-end market, because the low-end market is small (i.e., $b \leq \overline{b}_1$), the increase in sales would be outweighed by the decrease in price, which does harm the leader. The leader thus restricts its output to q_J to maintain the high-end market price. Conversely, the follower does not need to worry about the price collapsing because it chooses a high output at one stroke to compensate for loses from the dropping prices. Therefore, it chooses an output that is larger than the leader and has a profit higher than the leader.

In (*iii*), the more intensive competition in the low-end market increases the leader's profit while decreasing the follower's, although brand firms do not supply the low-end market directly. The logic behind this result is as follows. The increasing number of nonbrand firms makes the low-end market more competitive, which depresses the price P^N. Therefore, the low-end market becomes less profitable and the market integration is less likely to occur. Thus, the leader's output constraint imposed by the follower is alleviated so that the leader can choose a higher threshold value q_J and obtain a higher profit. Due to the strategic substitutability, the follower obtains a lower profit.

4 Conclusion

This paper shows how brand firms' endogenous timing decisions are affected by nonbrand firms. If the low-end market is of an intermediate size, then brand firms may choose to move sequentially with an outcome that the follower obtains a higher profit than the leader. Furthermore, the nonbrand firms positively affect the leader's profit but negatively affect the follower's. Due to the existence of the low-end market, each brand firm is given an option to choose a large output and depress the price of brand products such that it can enter the low-end market. Thus, each brand firm's best response function has an upward jump if its rival oversupplies beyond a certain level. This property significantly affects the equilibrium outcome when brand firms move sequentially. The leader has to restrict its output lower than that of the follower, or the follower retaliates by raising its output viciously to cause a price collapse. Thus, the brand firm who acts as a follower obtains a higher profit than its rival. As the competition in the low-end market intensifies, it becomes harder for the brand firms to enter the low-end market, which alleviates the leader's output restriction imposed by the follower, thus, the leader has a higher profit. Due to strategic substitutability, the follower's profit decreases.

In this paper, the brand firm's discontinuous best response function plays an important role in the main results. The upward shift of the best response function gives rise to the sudden drop of price along the inverse demand function of brand products, which gives rise to the consequence of second mover advantage. This property is derived from a particular kinked linear demand function, whose price elasticity is larger on the right hand side of the kink point. One necessary condition of this property is the existence of a low-end market. We believe that other demand function shapes, from which the resulting profit function is globally nonconcave and has multiple locally optimal output levels, can also give rise to this kind of best response function. The cubic type equation $P(Q) = 0.5 + (1.6 - Q)^3$ is a good example. Therefore, this paper may be a good complement to the study of second mover advantage.

The present paper can be a complementary explanation as to why some brand firms postpone the release of new products. From our main results, a brand firm does this to prevent intensive competition from ensuing counterfeit producers (i.e., nonbrand firms) in the low-end market, because the resulting low total outputs give rise to high pricing of brand products and remove low-end consumers' incentives for purchasing brand products. Therefore, when deciding anti-counterfeiting policies, a brand firm may consider an indirect method such as postponing release rather than face head-to-head competition. From this aspect, this research can also be extended to counterfeit deterrence.

Notes

[1]Businessweek (May, 2006) mentioned that Nike and Apple started to supply their first jointly produced Nike+ Ipod kit from that year.

[2]Yahoo News (September, 2013); CBS News (September, 2014).

[3]Stackelberg (1934) gives one type of explanation to this argument: under duopoly and quantity settings, a firm that moves earlier than its competitor can get a larger market share than when it moves simultaneously with its competitor. It is straightforward that this conclusion holds not only under duopoly but also oligopoly. Therefore, the pioneering position is advantageous to some extent. Urban et al. (1986) present an empirical analysis about how pioneering position matters for brand firms. They find that the earlier the brand firm's order of entry, the greater the brand firms long-term market share.

[4]For the first case, Smith (2010) analyzes how the cluster of local small firms in China challenged the

might of Nike and Adidas. He describes the sportswear market in China as "toe-to-toe," with Adidas and Nike competing with numerous small domestic firms. For the second case, Forbes Business News (November, 2013) reported that some mobile phone companies in China are heading for the SmartWatch field.

[5]Hamilton and Slutsky (1990) insightfully discuss this problem from two viewpoints: the extended game with observable delay and that with action commitment. The present paper follows the first viewpoint. In the extended game with action commitment, instead of simply announcing the period that each firm would like to choose, it chooses an action to which it is then committed. This strand has been widely discussed for quite some time (e.g., Mailath, 1993; Sadanand and Sadanand, 1996; van Damme and Hurkens, 1999). Since it is not the main content of the present paper, I refer to this point just in sentences.

[6]Ishibashi and Matsushima (2009) consider a similar market structure with the present paper. The brand and nonbrand firms are named as high-end and low-end firms, respectively. However, they do not explicitly consider the strategic interaction between high-end and low-end firms and simply discuss the entry of low-end firms by assuming that the low-end market is fully supplied by them and the resulting price in that market is assumed to be zero.

[7]As summarized in that study, empirical analysis finds that IBM postponed the release of its two mainframe computer generations-360 family (integrated circuits) and 370 family (silicon chips)-to a very late stage in the mid-20th century. Although their main results are based on the monopolistic assumption, the authors mention that similar results can also be deemed under oligopoly settings.

[8]Statistical analysis on the box office data in the 1990s finds that for avoiding head-to-head competition in the summer and Christmas holidays, some movie companies let their competitors go first.

[9]Unfortunately, there are limited empirical works investigating the direct relation between firm's pioneering position in releasing new products and market profitability in the market size and consumer heterogeneity in valuating different products. For one difficulty, as stated in Ishibashi and Matsushima (2009), it is hard for researchers to access firms' profit data; thus, it is hard to measure the magnitude of the profitability of a market. For another difficulty, it is ambiguous that to what extent a brand firm postpones its action can be defined as giving up pioneering position. In the present paper, it is the brand firm that chooses to follow its competitor. However, Urban et al. (1986) find that in some industries, brand firms can delay their actions to different extents, i.e., being the first follower or the second follower means a different market share. Despite these difficulties, some indirect empirical works are consistent with the present paper in some related results. Axarloglou (2003) analyzes the cyclical nature of the timing of new product introductions in U.S. manufacturing. Mukherjee and Kadiyali (2008) discusses the release timing in the DVD market and Engelstatter and Ward (2013) study the

entry timing in the video games market.

[10]There are other works discussing endogenous timing under incomplete information. Unfortunately, I find Normann (2002) to be the only work that belongs to the extended game with observable delay. Mailath (1993) is the early work of Normann (2002) that studies almost the same market model but applies the extended game with action commitment. Several others consider the situation where market uncertainty vanishes if firms choose to delay actions (e.g., Spencer and Brander, 1992; Sadanand and Sadanand, 1996). For other parallel works, Amir and Grilo (1999), Yang et al. (2009), and Julien (2011) discuss the strategic rivalry between firms; Matsumura and Ogawa (2009) study the relationship between payoff and risk dominance.

[11]The "high-end firms" and "low-end" firms in Ishibashi and Matsushima (2009) are referred to as "brand firms" and "nonbrand firms", respectively, in this paper.

5 Appendix

5.1 Equilibrium Outcomes of Nonband Firms:

The nonbrand firms' inverse demand function is denoted as follows:

$$P^N(Q^B, Q^N) = \begin{cases} a(1 - \dfrac{Q^N}{b}) & \text{if } 1 - Q^B \geq a(1 - \dfrac{Q^N}{b}), \\ \dfrac{a[1 + b - (Q^B + Q^N)]}{a + b} & \text{otherwise.} \end{cases} \tag{14}$$

We denote $Q^N_{-j} = \sum_{k \neq j} q^N_k$. In period 3, we derive nonbrand firm j's best response function taking Q^B and Q^N_{-j} as given.

We first consider type S case, $P^B \geq P^N$. j's best response function is given as follows:

$$q^{NS}_j(Q^B, Q^N_{-j}) = \arg\max_{q^N_j} \pi^N_j(q) = a[1 - \frac{1}{b}(Q^N_{-j} + q^N_j)]q^N_j = \frac{1}{2}(b - Q^N_{-j}).$$

Since $q^{NS}_j(Q^B, Q^N_{-j})$ does not always satisfy $P^B \geq P^N$, the existence of an interior solution needs to satisfy

$$1 - Q^B \geq a[1 - \frac{1}{b}(q^{NS}_j(Q^B, Q^N_{-j}) + Q^N_{-j})].$$

Next we consider the type I case, $P^B < P^N$. j's best response function is given as follows:

$$q_j^{NI}(Q^B, Q_{-j}^N) = \arg\max_{q_j^N} \pi_j^N(q) = \frac{a(1 + b - Q^B - Q_{-j}^N - q_j^N)}{a + b} q_j^N = \frac{1}{2}(1 + b - Q^B - Q_{-j}^N).$$

We also need to make the result satisfy $P^B < P^N$. We obtain

$$1 - Q^B < a[1 - \frac{1}{b}(q_j^{NI}(Q^B, Q_{-j}^N) + Q_{-j}^N)].$$

For $a[1 - (q_j^{NI}(Q^B, Q_{-j}^N) + Q_{-j}^N)/b] < 1 - Q^B \leq a[1 - (q_j^{NS}(Q^B, Q_{-j}^N) + Q_{-j}^N)/b]$, neither $q_j^{NS}(Q^B, Q_{-j}^N)$ nor $q_j^{NI}(Q^B, Q_{-j}^N)$ exists. Instead we have corner solutions within the range here. From $1 - Q^B = a[1 - (q_j^{NC} + Q_{-j}^N)/b]$, we obtain

$$q_j^{NC}(Q^B, Q_{-j}^N) = b[1 - \frac{1}{a}(1 - Q^B)] - Q_{-j}^N.$$

Thus, we derive nonbrand firm j' best response function as follows:

$$q_j^N(Q_B^N, Q_{-j}^N) = \begin{cases} q_j^{NS}(Q_B^N, Q_{-j}^N) \equiv \frac{1}{2}(b - Q_{-j}^N) & if\ 1 - Q^B \geq a[1 - \frac{1}{b}(q_j^{NS}(Q^B, Q_{-j}^N) + Q_{-j}^N)], \\ q_j^{NI}(Q_B^N, Q_{-j}^N) \equiv \frac{1}{2}(1 + b - Q^B - Q_{-j}^N) & if\ 1 - Q^B < a[1 - \frac{1}{b}(q_j^{NI}(Q^B, Q_{-j}^N) + Q_{-j}^N)], \\ q_j^{NC}(Q_B^N, Q_{-j}^N) \equiv b[1 - \frac{1}{a}(1 - Q^B)] - Q_{-j}^N & otherwise. \end{cases} \tag{15}$$

We assume that each nonbrand firm has the same equilibrium outcome and will prove each firm sticks to this outcome later. Under this assumption, given brand firms' aggregate output, we obtain nonbrand firm j's equilibrium outcome in period 3 as follows:

$$q_j^N(Q^B) = \begin{cases} q_j^{NS}(Q^B) \equiv \dfrac{b}{n+1} & if\ Q^B \leq 1 - \dfrac{a}{n+1}, \\ q_j^{NI}(Q^B) \equiv \dfrac{1+b-Q^B}{n+1} & if\ Q^B > 1 - \dfrac{ab}{an+bn+b}, \\ q_j^{NC}(Q^B) \equiv \dfrac{b}{n}(1 - \dfrac{1-Q^B}{a}) & otherwise. \end{cases} \tag{16}$$

Nonbrand firm j's corresponding profit is as follows:

$$\pi_j^N(Q^B) = \begin{cases} \pi_j^{NS}(Q^B) \equiv \dfrac{ab}{(n+1)^2} & if\ Q^B \leq 1 - \dfrac{a}{n+1}, \\ \pi_j^{NI}(Q^B) \equiv \dfrac{a(1+b-Q^B)^2}{(a+b)(n+1)^2} & if\ Q^B > 1 - \dfrac{ab}{an+bn+b}, \\ \pi_j^{NC}(Q^B) \equiv \dfrac{b}{n}(1-Q^B)(1 - \dfrac{1-Q^B}{a}) & otherwise. \end{cases}$$

We prove the equilibrium outcome denoted by Eq. (16) is globally optimal. We first derive the condition under which $q_j^{NS}(Q^B)$ is an equilibrium outcome. Given Q^B and $Q_{-j}^{NS}(Q^B) \equiv \sum_{k \neq j} q_k^{NS}(Q^B) = (n-1)q_j^{NS}(Q^B)$, and based on nonbrand firm j's best response function in (15), if j deviates from $q_j^{NS}(Q^B, Q_{-j}^{NS}(Q^B))$ to $q_j^{NC}(Q^B, Q_{-j}^{NS}(Q^B))$, then it will choose a deviating outcome $q_j^{NC\prime}(Q^B) \equiv b[1-(1-Q^B)/a] - Q_{-j}^{NS}(Q^B)$ with a profit $\pi_j^{NC\prime}(Q^B) \equiv (1-Q^B)q_j^{NC\prime}(Q^B)$. j would not deviate in this manner if it can only choose $q_j^{NS}(Q^B)$ or choosing $q_j^{NS}(Q^B)$ is more profitable than choosing $q_j^{NC\prime}(Q^B)$. Then we need either one of the following two inequalities to hold:

$$1 - Q^B \geq a[1 - \frac{1}{b}(q_j^{NS}(Q^B) + Q_{-j}^{NS}(Q^B))], \tag{17}$$

$$a[1 - \frac{1}{b}(q_j^{NI\prime}(Q^B) + +Q_{-j}^{NS}(Q^B))] \leq 1 - Q^B < a[1 - \frac{1}{b}(q_j^{NS}(Q^B) + Q_{-j}^{NS}(Q^B))]$$
$$and\ \pi_j^{NC\prime}(Q^B) \leq \pi_j^{NS}(Q^B). \tag{18}$$

Eq. (17) is the conditional formula under which $q_j^{NS}(Q^B)$ exists. The first inequality of (18) is the conditional formula under which $q_j^{NC\prime}(Q^B)$ exists and the second inequality ensures that j gets less profit when it chooses $q_j^{NC\prime}(Q^B)$. By arranging Eqs. (17) and (18), we obtain $Q^B \leq 1 - a/(n+1)$ and $1 - a/(n+1) < Q^B \leq 1 - 2ab/[(n+1)(a+2b)]$. Having either of these two inequalities hold, we obtain

$$Q^B \leq 1 - \frac{2ab}{(n+1)(a+2b)}. \tag{19}$$

If j deviates from $q_j^{NS}(Q^B, Q_{-j}^{NS}(Q^B))$ to $q_j^{NI}(Q^B, Q_{-j}^{NS}(Q^B))$, then it will choose a deviating outcome $q_j^{NI\prime}$
$(Q^B) \equiv [1 + 2q_j^{NS}(Q^B) - Q^B]/2$ with a profit $\pi_j^{NI\prime}(Q^B) \equiv a(q_j^{NI\prime}(Q^B))^2/(a+b)$. j would not deviate in this manner if it can only choose $q_j^{NS}(Q^B)$ or choosing $q_j^{NS}(Q^B)$ is more profitable than choosing $q_j^{NI\prime}(Q^B)$. Then we need either one of the following two inequalities to hold:

$$1 - Q^B \geq a[1 - \frac{1}{b}(q_j^{NS}(Q^B) + Q_{-j}^{NS}(Q^B))], \tag{20}$$

$$1 - Q^B < a[1 - \frac{1}{b}(q_j^{NI\prime}(Q^B) + Q_{-j}^{NS}(Q^B))] \; and \; \pi_j^{NI\prime}(Q^B) \leq \pi_j^{NS}(Q^B). \tag{21}$$

Arranging Eq. (20), we obtain $Q^B \leq 1 - a/(n+1)$. By arranging the first inequality of Eq. (21), we obtain $Q^B > 1 - ab/[(n+1)(a+2b)]$. This contradicts Eq. (19), which ensures that j does not deviate from $q_j^{NS}(Q^B)$ to $q_j^{NC\prime}(Q^B)$. Therefore, Eq. (21) is ruled out and $Q^B \leq 1 - a/(n+1)$ is a necessary condition to ensure that j does not deviate from $q_j^{NS}(Q^B)$ to $q_j^{NI\prime}(Q^B)$. Since $1 - a/(n+1) \leq 1 - 2ab/[(n+1)(a+2b)]$, the condition under which $q_j^{NS}(Q^B)$ is globally optimal given Q^B and $Q_{-j}^{NS}(Q^B)$ is

$$Q^B \leq 1 - \frac{a}{n+1}. \tag{22}$$

Then, we derive the condition under which $q_j^{NI}(Q^B)$ is an equilibrium outcome. Given Q^B and $Q_{-j}^{NI}(Q^B) \equiv \sum_{k \neq j} q_k^{NI}(Q^B) = (n-1)q_j^{NI}(Q^B)$ and based on nonbrand firm j's best response function in Eq. (15), if j deviates from $q_j^{NI}(Q^B, Q_{-j}^{NI}(Q^B))$ to $q_j^{NC}(Q^B, Q_{-j}^{NI}(Q^B))$, then it will choose a deviating outcome $q_j^{NC\prime}(Q^B) \equiv b[1-(1-Q^B)/a] - Q_{-j}^{NI}(Q^B)$ with a profit $\pi_j^{NC\prime}(Q^B) \equiv (1-Q^B)q_j^{NC\prime}(Q^B)$. j would not deviate in this manner if it can only choose $q_j^{NI}(Q^B)$ or choosing $q_j^{NI}(Q^B)$ is more profitable than choosing $q_j^{NC\prime}(Q^B)$. Then we need either one of the following two inequalities to hold:

$$1 - Q^B < a[1 - \frac{1}{b}(q_j^{NI}(Q^B) + Q_{-j}^{NI}(Q^B))], \tag{23}$$

$$a[1 - \frac{1}{b}(q_j^{NI}(Q^B) + Q_{-j}^{NI}(Q^B))] \leq 1 - Q^B < a[1 - \frac{1}{b}(q_j^{NS\prime}(Q^B) + Q_{-j}^{NI}(Q^B))]$$
$$\mathit{and}\ \pi_j^{NC\prime}(Q^B) \leq \pi_j^{NI}(Q^B). \tag{24}$$

By arranging Eq. (23), we obtain $Q^B > 1 - ab/(an + bn + b)$. By arranging Eq. (24), we obtain $1 - 2ab/(-a + 2b + an + 2bn) < Q^B \leq 1 - ab/(an + bn + b)$. Having either of theses two inequalities hold, we obtain

$$Q^B > 1 - \frac{2ab}{-a + 2b + an + 2bn}. \tag{25}$$

If j deviates from $q_j^{NI}(Q^B, Q_{-j}^{NI}(Q^B))$ to $q_j^{NS}(Q^B, Q_{-j}^{NI}(Q^B))$, then it will choose a deviating outcome $q_j^{NS\prime}$
$(Q^B) \equiv [b - Q_{-j}^{NI}(Q^B)]/2$ with a profit $\pi_j^{NS\prime}(Q^B) \equiv a(q_j^{NS\prime}(Q^B))^2/b$. j would not deviate in this manner if it can only choose $q_j^{NI}(Q^B)$ or choosing $q_j^{NI}(Q^B)$ is more profitable than choosing $q_j^{NS\prime}(Q^B)$. Then we need either one of the following two inequalities to hold:

$$1 - Q^B < a[1 - \frac{1}{b}(q_j^{NI}(Q^B) + Q_{-j}^{NI}(Q^B))], \tag{26}$$

$$1 - Q^B \geq a[1 - \frac{1}{b}(q_j^{NS\prime}(Q^B) + Q_{-j}^{NI}(Q^B))]\ \mathit{and}\ \pi_j^{NS\prime}(Q^B) \leq \pi_j^{NI}(Q^B). \tag{27}$$

By arranging Eq. (26), we obtain $Q^B > 1 - ab/(an + bn + b)$. By arranging the first inequality of Eq. (27), we obtain $Q^B \leq 1 - 2ab/(-a + 2b + an + 2bn)$. This contradicts Eq. (25), which ensures that j does not deviate from $q_j^{NI}(Q^B)$ to $q_j^{NC\prime}(Q^B)$. Therefore, Eq. (27) is ruled out and $Q^B > 1 - ab/(an + bn + b)$ is a necessary condition to ensure that j does not deviate from $q_j^{NI}(Q^B)$ to $q_j^{NS\prime}(Q^B)$. Since $1 - ab/(an + bn + b) > 1 - 2ab/(-a + 2b + an + 2bn)$, the condition under which $q_j^{NI}(Q^B)$ is globally optimal given Q^B and $Q_{-j}^{NI}(Q^B)$ is

$$Q^B > 1 - \frac{ab}{an + bn + b}. \tag{28}$$

Finally, we derive the condition under which $q_j^{NC}(Q^B)$ is an equilibrium outcome. Given Q^B and $Q_{-j}^{NC}(Q^B) \equiv \sum_{k \neq j} q_k^{NC}(Q^B) = (n-1)q_j^{NC}(Q^B)$ and based on nonbrand firm j's best response function in Eq. (15), if j deviates from $q_j^{NC}(Q^B, Q_{-j}^{NC}(Q^B))$ to $q_j^{NS}(Q^B, Q_{-j}^{NC}(Q^B))$, then it will choose a deviating outcome $q_j^{NS\prime}(Q^B) \equiv [b - Q_{-j}^{NC}(Q^B)]/2$ with a profit $\pi_j^{NS\prime}(Q^B) \equiv a(q_j^{NS\prime}(Q^B))^2/b$. j would not deviate in this manner if it can only choose $q_j^{NC}(Q^B)$ or choosing $q_j^{NC}(Q^B)$ is more profitable than choosing $q_j^{NS\prime}(Q^B)$. Then we need either one of the following two inequalities to hold:

$$a[1 - \frac{1}{b}(q_j^{NI\prime}(Q^B) + Q_{-j}^{NC}(Q^B))] \leq 1 - Q^B < a[1 - \frac{1}{b}(q_j^{NS\prime}(Q^B) + Q_{-j}^{NC}(Q^B))], \tag{29}$$

$$1 - Q^B \geq a[1 - \frac{1}{b}(q_j^{NS\prime}(Q^B) + Q_{-j}^{NC}(Q^B))] \ and\ \pi_j^{NS\prime}(Q^B) \leq \pi_j^{NC}(Q^B). \tag{30}$$

By arranging Eq. (29), we obtain $1 - a/(n+1) < Q^B \leq 1 - ab/(an + bn + b)$. By arranging Eq. (30), we obtain $Q^B = 1 - a/(n+1)$. Having either of these two conditions hold, we obtain

$$1 - \frac{a}{n+1} \leq Q^B \leq 1 - \frac{ab}{an + bn + b}. \tag{31}$$

If j deviates from $q_j^{NC}(Q^B, Q_{-j}^{NC}(Q^B))$ to $q_j^{NI}(Q^B, Q_{-j}^{NC}(Q^B))$, then it will choose a deviating outcome $q_j^{NI\prime}(Q^B) \equiv [1 + b - Q^B - Q_{-j}^{NC}(Q^B)]/(n+1)$ with a profit $\pi_j^{NI\prime}(Q^B) \equiv a(q_j^{NI\prime}(Q^B))^2/(a + b)$. j would not deviate from $q_j^{NC}(Q^B)$ to $q_j^{NI\prime}(Q^B)$ if it can only choose $q_j^{NC}(Q^B)$ or choosing $q_j^{NC}(Q^B)$ is more profitable than choosing $q_j^{NI\prime}(Q^B)$. Then, we need either one of the following two inequalities to hold:

$$a[1-\frac{1}{b}(q_j^{NI\prime}(Q^B)+Q_{-j}^{NC}(Q^B))] \le 1-Q^B < a[1-\frac{1}{b}(q_j^{NS\prime}(Q^B)+Q_{-j}^{NC}(Q^B))], \tag{32}$$

$$1-Q^B < a[1-\frac{1}{b}(q_j^{NI\prime}(Q^B)+Q_{-j}^{NC}(Q^B))] \; and \; \pi_j^{NI\prime}(Q^B) \le \pi_j^{NC}(Q^B). \tag{33}$$

By arranging Eq. (32), we obtain $1-a/(n+1) < Q^B \le 1-ab/(an+bn+b)$. By arranging the first inequality of Eq. (33), we obtain $Q^B > 1-ab/(an+bn+b)$. This contradicts Eq. (31), which ensures that j does not deviate from $q_j^{NC}(Q^B)$ to $q_j^{NS\prime}(Q^B)$. Therefore, Eq. (33) is ruled out and $1-a/(n+1) < Q^B \le 1-ab/(an+bn+b)$ is a necessary condition to ensure that j does not deviate from $q_j^{NC}(Q^B)$ to $q_j^{NI\prime}(Q^B)$. Together with Eq. (31), the condition under which $q_j^{NC}(Q^B)$ is globally optimal given Q^B and $Q_{-j}^{NC}(Q^B)$ is

$$1-\frac{a}{n+1} < Q^B \le 1-\frac{ab}{an+bn+b}. \tag{34}$$

5.2 Follower's Best Response Functions:

By solving the profit maximization problem in Eq. (5), we obtain the follower's best response function as follows:

$$q_f^B(q_l^B) = \begin{cases} q_f^{Bx}(q_l^B) & if \; Q^B \le q_K, \\ q_f^{By}(q_l^B) & Otherwise. \end{cases} \tag{35}$$

Since $q_f^{Bx}(q_l^B)$ does not always satisfy the first conditional formula in Eq. (35), the existence type x outcome needs to satisfy $q_f^{Bx}(q_l^B)+q_l^B \le q_K$. By arranging the inequality, we obtain

$$q_l^B \le 1-\frac{2ab}{an+bn+b}.$$

For the existence of type y outcome, we need $q_f^{By}(q_l^B)$ to satisfy the second conditional formula in Eq. (35): $q_f^{By}(q_l^B) + q_l^B > q_K$.By arranging the inequality, we obtain

$$q_l^B > 1 - b - \frac{2ab}{an + bn + b}.$$

For $1-b-2ab/(an+bn+b) < q_l^B \leq 1-2ab/(an+bn+b)$, the follower chooses $q_f^{Bx}(q_l^B)$ if this brings it a higher profit, or $\pi_f^{Bx}(q_l^B, q_f^{Bx}(q_l^B)) \geq \pi_f^{By}(q_l^B, q_f^{By}(q_l^B))$. By arranging the inequality, we obtain[12]

$$q_l^B \leq q_J.$$

Since $1 - b - 2ab/(an + bn + b) < q_J \leq 1 - 2ab(an + bn + b)$ for any $0 < a < 1$, $b > 0$ and $n > 1$, the follower chooses $q_f^{Bx}(q_l^B)$ for q_l^B smaller than q_J. Thus, we obtain the follower's best response function as in Eq. (6).

5.3 Equilibrium Conditions When Either Brand Firm Moves in Period 1:

We consider three cases: (i)$q_l^{x*} < q_l^{y*} < q_J$, (ii)$q_J \leq q_l^{x*} < q_l^{y*}$, and (iii)$q_l^{x*} < q_J \leq q_l^{y*}$.

First, we consider case (i). For $q_i^B \leq q_J$, the optimization problem of $\pi_l^{Bx}(q_l^B)$ has the locally optimal solution q_l^{x*}. The equilibrium profit is $\pi_l^{Bx}(q_l^{x*})$. For $q_l^B > q_J$, leader's profit decreases in q_l^B. The optimization problem has the corner solution at q_l^B, which is close enough to q_J. Within this range, brand firm 1 obtains a profit that is strictly less than $\pi_l^{By}(q_J)$. Due to the jump at q_J, $\pi_l^{Bx}(q_J) > \pi_l^{By}(q_J)$. Therefore, $\pi_l^{Bx}(q_l^{x*}) > \pi_l^{By}(q_J)$, from which we confirm that q_l^{x*} is the globally optimal solution when the following condition holds:

$$q_l^{By*} < q_J. \tag{36}$$

Next we consider case (ii). For $q_l^B > q_J$, the optimization problem of $\pi_l^{By}(q_l^B)$ has the locally optimal solution q_l^{y*}. The equilibrium profit is $\pi_l^{By}(q_l^{y*})$. For $q_l^B \leq q_J$, brand firm 1's profit increases in q_l^B. The optimization problem has the corner solution at point q_J and the maximized profit is $\pi_l^{Bx}(q_J)$. We compare $\pi_l^{Bx}(q_J)$ with $\pi_l^{By}(q_l^{y*})$. q_J is the globally optimal solution when the following condition holds:

$$q_l^{x*} \geq q_J \text{ and } \pi_l^{Bx}(q_J) \geq \pi_l^{By}(q_l^{y*}). \tag{37}$$

q_l^{y*} is the globally optimal solution with a profit when the following condition holds:

$$q_l^{x*} \geq q_J \text{ and } \pi_l^{Bx}(q_J) < \pi_l^{By}(q_l^{y*}). \tag{38}$$

Finally, we consider case (iii). For $q_l^B \leq q_J$, the optimization problem has the locally optimal solution q_l^{x*}. For $q_l^B > q_J$, the optimization problem has the locally optimal solution q_l^{y*}. Brand firm 1 chooses the solution which brings it a higher profit. Therefore, q_l^{x*} is the globally optimal solution with a profit $\pi_l^{Bx}(q_l^{x*})$ when the following condition holds:

$$q_l^{x*} < q_J \leq q_l^{y*} \text{ and } \pi_l^{Bx}(q_l^{x*}) \geq \pi_l^{By}(q_l^{y*}). \tag{39}$$

q_l^{y*} is globally optimal with a profit $\pi_l^{By}(q_l^{y*})$ when the following condition holds:

$$q_l^{x*} < q_J \leq q_l^{y*} \text{ and } \pi_l^{Bx}(q_l^{x*}) < \pi_l^{By}(q_l^{y*}). \tag{40}$$

We arrange the above three situations of q_J's value and derive the conditions under which each type of equilibrium exists.

First, we consider the situation when Eq. (36) or Eq. (39) holds. From the first inequality of (39), we arrange the necessary condition of $q_l^{x*} < q_J \leq q_l^{y*}$, $q_J > q_l^{x*}$ and obtain $b < b_3 \equiv$

$[n+1-4a+\sqrt{(16a^2-8a+n+1)(n+1)}]/(8a)$. By arranging the second inequality of Eq. (39), we obtain $b \leq b_4 \equiv [n+1-2a+\sqrt{(4a^2-4a+n+1)(n+1)}]/(2a)$. Since $b_3 \leq b_4$, Eq. (39) can be simplified as $q_l^{x*} < q_J \leq q_l^{y*}$. Let either Eq. (36) or Eq. (39) hold, we obtain condition A, under which equilibrium outcome (q_l^{x*}, q_f^{x*}) exists.

Next, we consider the situation when Eq. (37) holds: we obtain condition B under which equilibrium outcome $(q_J, (1-q_J)/2)$ exists.

Finally, we consider the situation when Eq. (38) or Eq. (40) holds. It is straight forward to see that the solution of Eq. (40) is an empty set.[13] Therefore, we obtain condition C under which equilibrium outcome $(q_l^{y*}, q_f^{y*}) \equiv ((1+b)/2, (1+b)/4)$ exists.

5.4 Equilibrium Conditions When Both Brand Firms Move in Period 1 or Period 2:

We derive the condition under which the x and y type equilibrium outcomes exist. For type x equilibrium, we first consider $1/3 \leq q_J$. Given $q_1^B = 1/3$, brand firm 2's best response function is $q_2^{Bx}(q_1^B) = 1/3$. Then we consider $1/3 > q_J$. Given $q_1^B = 1/3$, brand firm 2's best response function is $q_2^{By}(q_1^B) = 1/3 + b/2 > 1/3$. Thus, the condition for the existence of type x equilibrium is $1/3 \leq q_J$.

For type y equilibrium, we first consider $(1+b)/3 > q_J$. Given $q_1^B = (1+b)/3$, brand firm 2's best response function is $q_2^{By}(q_1^B) = (1+b)/3$. Then, we consider $(1+b)/3 \leq q_J$. Given $q_1^B = (1+b)/3$, brand firm 2's best response function is $q_2^{Bx}(q_1^B) = 1/3 - b/6 < (1+b)/3$. Thus, the condition for the existence of type y equilibrium is $(1+b)/3 > q_J$.

For $1/3 \leq q_J < (1+b)/3$, type x and type y equilibria exist. From Assumption 1, brand firms choose a type x equilibrium outcome if $\pi_c^{Bx}(q_c^{x*}) \geq \pi_c^{By}(q_i^{y*})$. By arranging the necessary condition of $1/3 \leq q_J < (1+b)/3$, $1/3 \leq q_J$, we obtain $b \leq 2[(n+1-3a)+\sqrt{(n+1)(9a^2-6a+n+1)}]/(9a) \equiv b_1$. Arranging $\pi_c^{Bx}(q_c^{x*}) \geq \pi_c^{By}(q_c^{y*})$, we obtain $b \leq [(n+$

$1 - 2a) + \sqrt{(n+1)(4a^2 - 4a + n + 1)}]/(2a) \equiv b_2$. Since $b_2 \geq b_1$, type x is always picked up for $1/3 \leq q_J < (1+b)/3$. In other words, as long as (q_c^{x*}, q_c^{x*}) exists, it is picked up as an equilibrium outcome. Thus, we obtain Lemma 2.

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n	a	$\underline{b}_1$	$\overline{b}_1$	n	a	$\underline{b}_1$	$\overline{b}_1$	n	a	$\underline{b}_1$	$\overline{b}_1$
3	0.2	4.1795	11.2049	10	0.2	12.9432	33.6281	20	0.2	25.4465	65.6339
	0.35	2.1983	5.8694		0.35	7.2042	18.6928		0.35	14.3485	36.9841
	0.4	1.8956	5.0093		0.4	6.2735	16.2299		0.4	12.5242	32.2348
	0.6	1.2613	3.1035		0.6	4.1812	10.5719		0.6	8.3489	21.2392

Table 1: Numericizing Conditions *A*, *B* and *C*

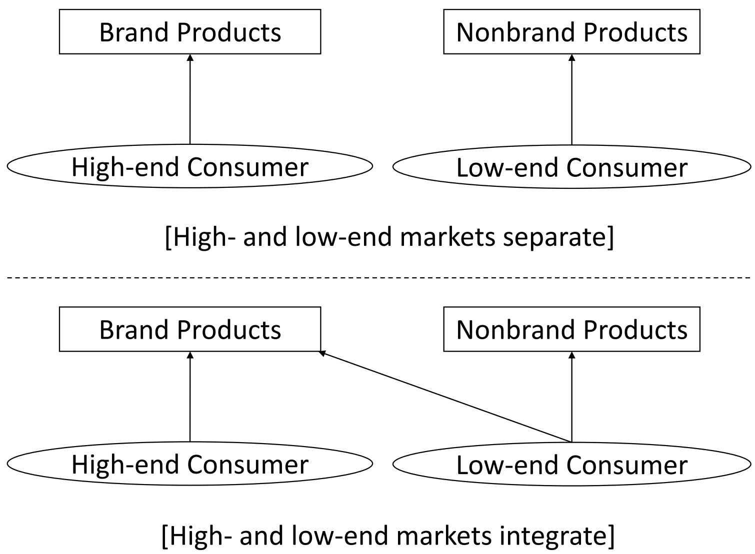

Figure 1: The Market Structure

n	a	b_2	n	a	b_2	n	a	b_2
3	0.2	7.7281	10	0.2	23.3018	20	0.2	45.5286
	0.35	4.0734		0.35	12.9774		0.35	25.6795
	0.4	3.4928		0.4	11.2828		0.4	22.3969
	0.6	2.2281		0.6	7.4143		0.6	14.8219

Table 2: Numericizing Conditions D and E

brand firm 1 \ brand firm 2	leader	follower
leader	$(\pi_c^{By}(q_c^{y*}), \pi_c^{By}(q_c^{y*}))$	$(\pi_l^{Bx}(q_J), \pi_f^{Bx}(q_J, q_f^{Bx}(q_J))$
follower	$(\pi_f^{Bx}(q_f^{Bx}(q_J), q_J), \pi_l^{Bx}(q_J))$	$(\pi_c^{By}(q_c^{y*}), \pi_c^{By}(q_c^{y*}))$

Table 3: When B and E hold

b	$\pi_l^{Bx}(q_J)$	$\pi_f^{Bx}(q_J, q_f^{Bx}(q_J))$	$\pi_c^{By}(q_c^{y*})$
15.0	0.1104	0.1124	0.0521
16.0	0.1065	0.1199	0.0553
18.0	0.0976	0.1348	0.0616
20.0	0.0876	0.1496	0.0680

Table 4: $n = 20$, $a = 0.6$, $14.8219 \le b < 21.2392$

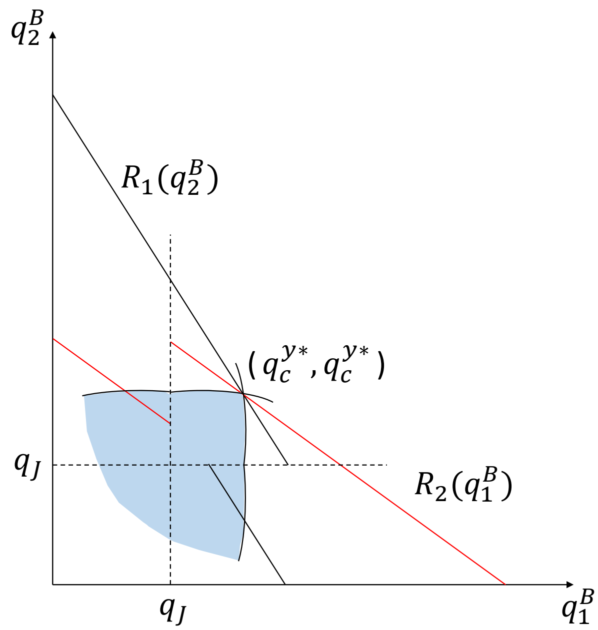

Figure 2: Brand Firms' Reaction Functions and Pareto Superior Set