Discussion Paper No. 946

GROWTH, SECULAR STAGNATION AND WEALTH PREFERENCE

Yoshiyasu Ono

September 2015

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

September 2015

Growth, Secular Stagnation and Wealth Preference*

by

Yoshiyasu Ono†

Abstract

In 1960s-1980s Japan enjoyed high economic growth. In the early 1990s, however, the growth rate drastically declined and thereafter Japan has been suffering secular stagnation. This paper proposes a dynamic macroeconomic model that can consistently explain such a drastic change in economic performance. Wealth preference plays an important role. In the early stage consumption grows at the same pace as productivity increases. Once consumption reaches a certain level, however, it deviates from the full-employment level and aggregate demand deficiency appears. After that the economic growth rate asymptotically approaches zero even if productivity keeps on increasing, and secular stagnation arises.

JEL: E32, E13

Keywords: Long-run stagnation, aggregate demand deficiency, wealth preference, fair wages, economic growth.

---

*This research is financially supported by the Grants-in-Aid for Scientific Research (#15H05728), the Japan Society for the Promotion of Science (JSPS). The author thanks seminar participants at Fifth Graz Schumpeter Summer School held in the University of Graz (July 2015) and Norges Bank (August 2015) for their valuable comments.

†Institute of Social and Economic Research, Osaka University, 6-1, Mihogaoka, Ibaraki, Osaka 567-0047, JAPAN. E-mail: ono@iser.osaka-u.ac.jp.

1. Introduction

Japan has been suffering secular stagnation for more than two decades since stock-price bubbles burst in the early 1990s. Before and after the bubble burst Japan's growth rate drastically decreased, the unemployment rate jumped up, and price and wage indices stopped increasing but rather declined although the Bank of Japan vastly expanded money supply, as shown in Figure 1 (Japan). Surprisingly, Japan's nominal GDP in 2014 is the same as that in 1992 (488 trillion yen). Other developed countries, such as the USA and the EU, have also faced serious business situations since the Lehman shock of 2008. They are now worrying if they would fall into such long-run stagnation as Japan's. Therefore, the central banks significantly expand money supply so that business activity should recover. However, their GDPs have not so much responded to the monetary expansions as expected. See Figure 2 for case of the USA.

In the literature such macroeconomic downturns are regarded as either declines in productivity or short-run recessions that arise during the process of price adjustment. For example, Kehoe and Prescott (2002), Hayashi and Prescott (2002) and McGratten and Prescott (2012) mention that recent stagnations are mainly due to decreases in productivity. DSGE models (e.g., Yun, 1996; Erceg et al., 2000; Smets and Wouters 2003, 2005, 2007; Christiano et al., 2005) apply the Calvo pricing (1983) to analyze macroeconomic fluctuations. Therefore, the fluctuations are due to neither aggregate demand deficiency nor involuntary unemployment but to monopolistic pricing of firms. Krugman (1989) adopts a period model and assumes price rigidity within a period.[1] Aggregate demand deficiency appears only in the period in which an unexpected shock arises; it disappears in the long run. There are also researchers who consider

[1] Eggertsson and Krugman (2012) consider a period model with sluggishness of some firms' price setting in which there are borrowers that face credit constraints and savers that do not. A sudden decline in the credit constraint makes borrowers reduce consumption and leads to aggregate demand deficiency.

a credit constraint that leads to misallocation of capital and decreases in future productivity (Kiyotaki and Moore, 1997; Hall, 2011). In this setting stagnation is interpreted to arise due to a decline in productivity.

While those approaches are useful to analyze each relevant situation, however, they may not fit the case of secular stagnation such as Japan's. In fact, it may be difficult to accept the view that the price adjustment process takes more than two decades or that in the early 1990s Japan's productivity suddenly stopped increasing and since then has never improved for more than two decades. In the IMF annual conference of 2013 Summers (2013) expressed a similar concern and criticized too much reliance on the DSGE approach in solving economic crises. He emphasized the need for researchers to work on secular stagnation rather than temporary business fluctuations. This paper shares this view and presents a model that can treat a transition from the phase of growth with full employment to that of secular stagnation with aggregate demand deficiency.

A model of stagnation that occurs in the steady state has firstly been proposed by Ono (1994, 2001) and extended by Ono and Ishida (2014). The crux of these models is the sustainment of liquidity (or wealth) preference.[2] Without it, if there is aggregate demand deficiency, wages and prices decline, real wealth increases, and hence the relative preference for consumption over wealth holding increases, stimulating consumption enough to recover

---

[2] This type of model has been extended in various directions. For example, Matsuzaki (2003) and Hashimoto (2004) examine the effect of redistribution policy on aggregate demand in the present type of stagnation. Johdo (2006) analyzes the effect of R&D subsidies on aggregate demand and employment. Rodriguez-Arana (2007) examines the dynamic path with public deficit in the present stagnation case and compares it with that in the case where full employment prevails. Ono (2006, 2014) applies the stagnation model into a two-country framework and analyzes the international spillover effects of various policies. Johdo and Hashimoto (2009) consider the role of FDI in the two-country model with the present stagnation mechanism. Using the implications of this model Ono (2010) analyzes Japan's long-run stagnation and economic policies. Murota and Ono (2011) applies the concept of status preference to explain the insatiability of wealth preference and show that aggregate demand deficiency arises in the steady state. Hashimoto and Ono (2011) examine the effects of child allowances on the population growth and aggregate demand in the stagnation steady state. Murota and Ono (2012) consider a preference for deposit holdings in this setting and find zero nominal interest rates and excess reserves held by commercial banks to appear in the persistent stagnation. Apart from this line of research, Michaillat and Saez (2014) show that aggregate demand deficiency arises in the steady state if the marginal utility of wealth holding is constant.

full employment. However, if the marginal utility of wealth holding stays positive whereas that of consumption continues to decline as it increases, consumption may not reach a level large enough to fill the deficiency of aggregate demand and then secular stagnation occurs. In those models productivity is assumed to be constant and aggregate demand deficiency always appears along the dynamic equilibrium path. In practice, however, Japan enjoyed high growth with full employment in the past and then has fallen into long-run stagnation. Moreover, this sequence is reverse to the Keynesian implication that aggregate demand deficiency occurs in the short run but eventually disappears in the long run.

This paper considers wealth preference in an economy with persistent productivity growth. Along the dynamic equilibrium path the economy initially achieves full employment and grows at the same pace as the productivity growth, but eventually falls into secular stagnation with little (or even zero) growth, steady deflation and underemployment, although the productivity keeps on increasing and the money stock continues to expand. This implication may be more in conformity with Japan's experience, shown in Figure 1, than the implications of the conventional models.

2. The model

A representative household has the following utility functional:

$$\int_0^\infty [u(c) + v(m) + \varphi(a)] \exp(-\rho t)\, dt,$$

where $u(c)$ is utility of consumption c, $v(m)$ is preference for money m due to transaction motive and $\varphi(a)$ is preference for holding wealth a. The household maximizes it subject to the flow budget equation and the stock budget constraint:

$$\dot{a} = ra + wx - c - Rm - z, \quad a = m + b, \tag{1}$$

where a is real total assets, b is interest-bearing asset whose real and nominal interest rates are

r and R respectively, w is the real wage, x is the realized labor supply and z is the lump-sum tax-cum-subsidy. The labor endowment is 1 but x may be less than 1 because there may be unemployment.

$$x \leq 1.$$

The Hamiltonian function of this problem is

$$H = u(c) + v(m) + \varphi(a) + \lambda(ra + wx - c - Rm - z),$$

and the first-order optimal conditions are

$$\lambda = u'(c),$$

$$\lambda R = v'(m),$$

$$\dot{\lambda} = (\rho - r)\lambda - \varphi'(a).$$

They are summarized as

$$\rho + \pi + \eta \frac{\dot{c}}{c} = R + \frac{\varphi'(a)}{u'(c)} = \frac{v'(m)}{u'(c)} + \frac{\varphi'(a)}{u'(c)}, \tag{2}$$

where $\pi(= R - r)$ is the inflation rate and $\eta = -u''c/u'$. The transversality condition is

$$\lim_{t\to\infty} u'(c)a \exp(-\rho t) = 0. \tag{3}$$

The monetary authority expands money supply M at an exogenous rate μ,

$$\frac{\dot{M}}{M} = \mu,$$

and hence $m(= M/P)$ satisfies

$$\frac{\dot{m}}{m} = \mu - \pi. \tag{4}$$

The government budget equation is

$$\mu m + z + \dot{b} = rb.$$

Note that the stock of government bond b equals the household's interest-bearing asset because the firm value is zero under linear technology and competitive behavior, which will be mentioned later in this section. The fiscal authority controls the bond stock to be eventually finite and thus

$$\lim_{t\to\infty} b_t = \bar{b} \ll \infty. \quad (5)$$

The representative firm is competitive and utilizes labor to produce commodities with productivity y that grows at an exogenous rate δ:

$$y = y_0 \exp(\delta t), \quad (6)$$

where y_0 is the initial level of the productivity. Because the labor endowment is normalized to 1, productivity y also represents the full-employment output while realized output yx may be less than y. Commodity price adjustment is perfect and thus always

$$c = yx. \quad (7)$$

In order to accommodate aggregate demand shortages, sluggish wage adjustment is assumed. In the recent new Keynesian literature the standard setting of price and wage adjustment is that of Calvo (1983). However, it does not suit the present analysis because it never allows aggregate demand shortages to arise. Aggregate demand shortages are explicitly analyzed by Krugman (1998) in a discrete-time setting. However, it cannot treat long-run stagnation because the length of each period is exogenously given and a demand shortage appears only in the period in which an exogenous shock occurs. Therefore, in the following analysis a dynamic version of the Akerlof-type fair wage model (Akerlof, 1982; Akerlof and Yellen, 1990), proposed by Ono and Ishida (2014), is adopted because it can deal with unemployment as well as full employment during the adjustment process and in the steady state.

There are three kinds of workers, employed, unemployed and newly hired ones. The employed randomly separate from the current job at an exogenously given Poison rate α.[3] Therefore, employment x follows

$$\dot{x} = -\alpha x + \chi, \quad (8)$$

[3] The Calvo model (1983) also assumes an exogenous Poison rate with which a firm (or a labor union) has the opportunity of revising prices (or wages) at each time point. The present α instead represents the opportunity of revising the fair wage that appears when some of the incumbent workers leave the job, as mentioned later.

where χ is the number of the newly hired per unit time. While workers are employed, they conceive fair wages in mind on the basis of their past wages, their fellow workers' fair wages and the unemployment level of the society. Formally, at the end of the previous period $(t - \Delta t)$ each employed worker conceives the rightful wage $\nu(t - \Delta t)$, which is the wage that he or she believes fair if full employment prevails. Next, he or she calculates the average of the rightful wages of the employed, which are the same to each other, and the income of the unemployed, which is assumed to be zero. The average is taken as the fair wage $W_F(t)$ at time t.

Because the number of the employed and that of the unemployed are respectively $x(t - \Delta t)$ and $1 - x(t - \Delta t)$, the employed calculate the rightful wage $\nu(t - \Delta t)$ so that it satisfies

$$\nu(t - \Delta t)x(t - \Delta t) + 0 \times [1 - x(t - \Delta t)] = W_F(t - \Delta t), \tag{9}$$

at the timing denoted by (A) in figure 3. The newly hired have no preconception about the fair wage or the rightful wage and simply follow the fair wage of the employed. When the employed calculate the fair wage $W_F(t)$ at time t, namely at the timing (B) in figure 3, the number of the employed is $x(t - \Delta t)(1 - \alpha\Delta t)$ and the total number of people that they care is $1 - \chi(t)\Delta t$ because the number of the newly hired is $\chi(t)\Delta t$. Therefore, given that the unemployed earn no income, the fair wage $W_F(t)$ is

$$W_F(t) = \frac{\nu(t-\Delta t)x(t-\Delta t)(1-\alpha\Delta t)}{1-\chi(t)\Delta t}.$$

From (9) and the above equation, one obtains

$$\frac{W_F(t)-W_F(t-\Delta t)}{\Delta t} = \chi(t)W_F(t) - \alpha W_F(t - \Delta t).$$

Reducing Δt to zero yields the dynamics of W_F:

$$\frac{\dot{W}_F}{W_F} = \chi - \alpha, \tag{10}$$

implying that the fair wage is updated at the replacement pace of employment.[4] As α is larger, the declining speed of W_F is higher. Thus, the Poison rate α is a proxy of flexibility of the fair wage.

If there is unemployment, the firm will set wage W equal to the fair wage W_F because W_F is the lowest wage under which the employees properly work. The commodity price P is always equalized to W_F/y because there is no commodity supply if $P < W_F/y$ and excess commodity supply if $P > W_F/y$. If full employment prevails, in contrast, the firm tries to pick out workers from rival firms to expand the market share by increasing W from W_F so long as the marginal profits are positive (i.e., $Py > W$). Therefore, W is set higher than the fair wage W_F that follows (10), and is instantaneously equalized to Py. Once W is thus determined, W_F is ex post updated to equal W.

In sum, P follows the movement of the fair wage W_F when there is unemployment whereas W follows the movement of P regardless of W_F when full employment maintains. Anyway one has

$$w = \frac{W}{P} = y, \tag{11}$$

under linear technology and competitive behavior. Consequently, the firm earns no profit and thus the interest-bearing assets indeed consist of only the government bond. From (6) and (11), one immediately obtains

$$\pi + \delta = \frac{\dot{W}}{W}. \tag{12}$$

Given the firm behavior mentioned above, the dynamic equations of c and m are obtained. If full employment prevails ($x = 1$), from (7),

$$y = c.$$

---

[4] This equation must be $\dot{W}_F/W_F = \chi/L_0 - \alpha$, where L_0 is the total population. However, because L_0 is normalized to unity, (10) is valid.

Then, from (2) and (6) P moves so that it satisfies

$$\pi = \frac{v'(m)+\varphi'(a)}{u'(c)} - (\rho + \eta\delta), \tag{13}$$

and W follows the movement of P so as to satisfy (11). Therefore, from (12)

$$\frac{\dot{W}}{W} = \frac{v'(m)+\varphi'(a)}{u'(c)} - \rho + (1-\eta)\delta. \tag{14}$$

When $x = 1$, from (8) $\chi = \alpha$ (i.e., the number of job separation equals that of the newly hired) and thus from (10) $\dot{W}_F = 0$. If W that follows (14) is lower than W_F, W is set equal to W_F because otherwise the employees do not properly work, and aggregate demand deficiency appears. Therefore, variables m, c and a that satisfy (14) are valid only when (14) is non-negative. Substituting π given by (13) into (4) yields $\dot{m}$. These results are summarized as

$$\frac{\dot{W}}{W} \geq 0 \leftrightarrow \frac{v'(m)+\varphi'(a)}{u'(c)} \geq \rho - (1-\eta)\delta:$$

$$\frac{\dot{c}}{c} = \frac{\dot{y}}{y} = \delta, \quad \frac{\dot{m}}{m} = \mu + \rho + \eta\delta - \frac{v'(m)+\varphi'(a)}{u'(c)}, \quad \pi = \frac{v'(m)+\varphi'(a)}{u'(c)} - (\rho + \eta\delta). \tag{15}$$

When $\dot{W}/W$ given by (14) is negative, there is aggregate demand deficiency (i.e., $x < 1$). In this case W is set equal to W_F of which the dynamics is (10) because W cannot be lower than W_F. Substituting x and $\dot{x}$ obtained from (7), which are

$$\dot{x} = \left(\frac{\dot{c}}{c} - \delta\right)\frac{c}{y}, \quad x = \frac{c}{y},$$

into (8) gives χ, the number of the newly hired. Applying this χ to (10) and utilizing (12) in which $W = W_F$ yields π:

$$\pi = \frac{\dot{W}}{W} - \delta = \dot{x} + \alpha(x-1) - \delta = \frac{\dot{c}}{y} + \alpha\left(\frac{c}{y} - 1\right) - \delta\left(\frac{c}{y} + 1\right) < 0. \tag{16}$$

π is negative because $\dot{W}/W < 0$ in the present case. Substituting this π into (2) and rearranging the result gives the dynamics of c. Because π is negative, the dynamics of m represented by (4) is always positive. The results are

$$\frac{\dot{W}}{W} < 0 \leftrightarrow \frac{v'(m)+\varphi'(a)}{u'(c)} < \rho - (1-\eta)\delta:$$

$$\frac{\dot{y}}{y} = \delta > 0, \quad \left(\eta + \frac{c}{y}\right)\frac{\dot{c}}{c} = \frac{v'(m)+\varphi'(a)}{u'(c)} - \rho - \alpha\left(\frac{c}{y} - 1\right) + \delta\left(\frac{c}{y} + 1\right),$$

$$\pi = \frac{\dot{W}}{W} - \delta < 0, \quad \frac{\dot{m}}{m} = \mu - \pi > 0. \tag{17}$$

Note that when $\dot{W}/W$ equals zero in (14), the values of the equations in (15) and (17) are

$$\frac{\dot{c}}{c} = \delta, \quad \frac{\dot{m}}{m} = \mu + \delta, \quad \pi = -\delta,$$

i.e., the dynamics in the two cases smoothly connect to each other.

3. Transition from growth to secular stagnation

In order that the dynamics is tractable the following functions are assumed:

$$u(c) = \ln c, \quad v(m) = \gamma \ln(m), \quad \varphi(a) = \beta a. \tag{18}$$

With these functions the marginal utility of m $(= \gamma/m)$ reaches zero while that of wealth $(= \beta)$ remains positive.[5] The property that the wealth preference remains positive plays the crucial role in making the economy switches from full-employment growth to secular stagnation. This section obtains the phase diagram of the dynamics of c and m when β is positive. Along the dynamic equilibrium path the economy grows at the same pace as the productivity growth before falling into secular stagnation. In the next section the case where there is no wealth preference ($\beta = 0$) is considered and persistent growth with full employment is shown to be realized.

With the functions given in (18) the $\dot{W} = 0$ curve obtained from (14) is

$$\dot{W} = 0: \quad c = \rho/\left(\frac{\gamma}{m} + \beta\right), \tag{19}$$

and the dynamic equations in (15) and (17) are

[5] Murota and Ono (2011) show that status preference with respect to money holding makes the marginal utility of mony holding constant. The present assumption applies this property to wealth holding. Michaillat and Saez (2014) consider a model in which net household wealth is always zero and hence the marginal utility of wealth holding is positive and constant. There is also an empirical research that supports the existence of such wealth preference. Using aggregate quarterly data in Japan and the Japanese survey data called NIKKEI RADAR, Ono, Ogawa and Yoshida (2004) empirically find that the preference for financial asset holding does not disappear as the money holding increases using both parametric and non-parametric methods.

$$\frac{\dot{W}}{W} \geq 0 \leftrightarrow c \geq \rho/\left(\frac{\gamma}{m} + \beta\right):$$

$$\frac{\dot{c}}{c} = \frac{\dot{y}}{y} = \delta, \quad \frac{\dot{m}}{m} = \mu + \rho + \delta - \left(\frac{\gamma}{m} + \beta\right) c, \quad \pi = \left(\frac{\gamma}{m} + \beta\right) c - (\rho + \delta); \tag{20}$$

$$\frac{\dot{W}}{W} < 0 \leftrightarrow c < \rho/\left(\frac{\gamma}{m} + \beta\right):$$

$$\frac{\dot{y}}{y} = \delta > 0, \quad \left(1 + \frac{c}{y}\right)\frac{\dot{c}}{c} = \left(\frac{\gamma}{m} + \beta + \frac{\delta - \alpha}{y}\right) c - (\rho - \alpha - \delta),$$

$$\frac{\dot{m}}{m} = \mu - \pi > 0, \quad \pi < 0. \tag{21}$$

Figure 4 illustrates the $\dot{W} = 0$ curve given by (19) and the phase diagram of m and c. In the area above the $\dot{W} = 0$ curve full employment prevails and thus (20) is valid. From (20), the boundary curve of m satisfies

$$\dot{m} = 0: \quad c = (\mu + \rho + \delta)/\left(\frac{\gamma}{m} + \beta\right) > \rho/\left(\frac{\gamma}{m} + \beta\right),$$

i.e., this curve is located above the $\dot{W} = 0$ curve given by (19). The dashed dynamic path represents the one that is tangent to the $\dot{W} = 0$ curve. Along any path located above it or above the right-hand part of the $\dot{W} = 0$ curve from point D, m will eventually reach zero within a finite time span. Therefore, any feasible path must start in the area between the dashed line and the left-hand part of the $\dot{W} = 0$ curve from D. Any path starting from that area eventually reaches the $\dot{W} = 0$ curve and enters the area with aggregate demand deficiency. In the figure point B denotes the point at which the path reaches $\dot{W} = 0$ curve. Obviously, there is a continuum of such paths and one of them must be chosen, as mentioned below.

Once the path enters the area with aggregate demand deficiency, it follows the dynamic equations in (21) and the $\dot{c} = 0$ curve is

$$\dot{c} = 0: \quad c = (\rho - \alpha - \delta)/\left(\frac{\gamma}{m} + \beta + \frac{\delta - \alpha}{y}\right). \tag{22}$$

This curve in the cases where $\delta > \alpha$ and $\delta < \alpha$ is respectively illustrated in figures 4 and 5. In order for this curve to be located within the area where c and m are both positive, it must be satisfied that[6]

$$\rho - \alpha - \delta > 0.$$

Otherwise, all paths in this area will eventually enter the area with full employment and m will reach zero within a finite time span –i.e., there is no feasible path.

In figure 4 (the case where $\delta > \alpha$) the $\dot{c} = 0$ curve shifts upward as y increases while in figure 5 (the case where $\delta < \alpha$) it shifts downward as y increases. In either case, when a path reaches the $\dot{W} = 0$ curve represented by (19), c, y and m satisfy

$$c = y = \rho / \left(\frac{\gamma}{m} + \beta\right).$$

By applying this y as a function of m to the $\dot{c} = 0$ curve given by (22), one finds that the level of c for the same m along the $\dot{c} = 0$ curve satisfies

$$c = (\frac{\rho-\alpha-\delta}{\rho-\alpha+\delta}) \left\{\rho / \left(\frac{\gamma}{m} + \beta\right)\right\} < \rho / \left(\frac{\gamma}{m} + \beta\right),$$

where the right-hand side is the $\dot{W} = 0$ curve given by (19). Therefore, the $\dot{W} = 0$ curve is located above the $\dot{c} = 0$ curve when the path reaches the area with aggregate demand deficiency. It is also located above the $\dot{c} = 0$ curve for the case where $y = \infty$ because from (22) c on this curve satisfies

$$c = (\rho - \alpha - \delta) / \left(\frac{\gamma}{m} + \beta\right) < \rho / \left(\frac{\gamma}{m} + \beta\right).$$

In the area below the $\dot{W} = 0$ curve, there is only one path along which c is finite and positive, which is BE in figures 4 and 5. The mathematical proof of the saddle-path stability is

[6] Lawrence (1991) estimates the subjective discount rate to be from 4% to 21% and Andersen et al. (2008) estimates it to be 10.1%. The average of the annual GDP growth rates of OECD member countries in 2005-2013 was 1.34% (World Bank national accounts data, and OECD National Accounts data files, 2015).

given in the appendix. From the dynamic equation of c in (21), the steady-state level of c on this path is

$$c = \frac{\rho - \alpha - \delta}{\beta}, \tag{23}$$

While productivity y keeps on expanding. Thus, from (16) and (21) $\dot{m}/m$ satisfies

$$\lim_{t \to \infty} \frac{\dot{m}}{m} = \mu + \delta + \alpha.$$

Using (5), (18), (23) and the above property one finds that the transversality condition (3) in the present case,

$$\lim_{t \to \infty} \frac{m + \bar{b}}{c} \exp(-\rho t) = \frac{\beta}{\rho - \alpha - \delta} \lim_{t \to \infty} m \exp(-\rho t) = 0,$$

is valid if

$$\rho - \alpha - \delta > \mu. \tag{24}$$

Along any other path than BE, c either approaches zero, violating the transversality condition, or continues to expand and then m reaches zero within a finite time span, as illustrated in figure 4. Therefore, BE is the unique equilibrium path and the starting point B is determined. Once B is determined, the path that reaches B in the area where $\dot{W} > 0$ is uniquely chosen and the whole shape of the dynamic equilibrium path is determined. Note that if μ is so high as to violate (24), the saddle path does not satisfy the transversality condition and then no dynamic equilibrium path exists.

As shown in the two figures, along the equilibrium path consumption c grows at the same pace as productivity y does when y is small. The economy enjoys economic growth with full employment in the developing stage with low productivity. As y reaches the level that satisfies

$$y = \rho / \left(\frac{\gamma}{m} + \beta \right),$$

which is point B in figures 4 and 5, consumption c starts to deviate from full-employment output y and the economy faces aggregate demand deficiency. Thereafter, c stays lower than the value given by (23) even though productivity y continues to increase. Thus, once the economy reaches the matured stage with high productivity, it suffers secular stagnation. Wage and price deflations continue, making m expand and $v'(m)$ equal zero eventually, and then from (2) the nominal interest rate R is

$$R = \frac{v'(m)}{u'(c)} = 0,$$

i.e., zero interest rates appear.

The steady-state level of consumption given by (23) satisfies

$$\delta \uparrow,\ \beta \uparrow,\ \alpha \uparrow\ \Rightarrow\ \ c \downarrow.$$

Higher productivity growth, greater wealth preference and more frequent job separation worsen stagnation, representing the paradoxes of toil, thrift and flexibility, respectively. In the presence of aggregate demand deficiency higher productivity enlarges the deflationary gap and makes deflation more serious, which harms the incentive to consume (toil). Greater wealth preference directly reduces the incentive to consume (thrift). More frequent job separation raises the wage adjustment speed and thus worsens deflation, which also harms the incentive to consume (flexibility). Therefore, they decrease aggregate demand and make the stagnation more serious.

4. Wealth preference and secular stagnation

Secular stagnation with aggregate demand deficiency arises due to the wealth preference βa. While the marginal utility of wealth preference is $\beta (> 0)$, the marginal utility of consumption c decreases if c increases at the same pace as productivity y does. Eventually, people want to accumulate wealth rather than to consume so much as to maintain full employment, and aggregate demand deficiency arises. Because the wealth preference remains to be β, this situation is unchanged and the economy falls into secular stagnation with aggregate demand deficiency.

Without the wealth preference, full employment prevails along the dynamic equilibrium path. To show this, let us obtain the dynamic path in the case where $\beta = 0$. From (20) in which $\beta = 0$, $s(= c/m)$ satisfies

$$\frac{\dot{s}}{s}\left(= \frac{\dot{c}}{c} - \frac{\dot{m}}{m}\right) = \gamma s - (\rho + \mu).$$

Therefore, the equilibrium path is

$$s\left(= \frac{c}{m}\right) = \frac{\rho+\mu}{\gamma}\left(> \frac{\rho}{\gamma}\right), \quad c = y = y_0 \exp(\delta t). \tag{25}$$

From (19) in the case where $\beta = 0$, one finds that

$$\dot{W} > 0 \text{ if } \frac{c}{m} > \frac{\rho}{\gamma},$$

and hence the equilibrium path represented by (25) is indeed located in the area with full employment. (25) implies that c and m expand at the same pace as y does and full employment continues to hold –i.e., aggregate demand deficiency does not arise. Applying c/m given by (25) and $\beta = 0$ to π in (20) yields

$$\pi = \mu - \delta.$$

From (6), (18) and (25), the transversality condition (3) is valid because

$$\lim_{t\to\infty} \frac{m+\bar{b}}{c} \exp(-\rho t) = \lim_{t\to\infty} \left(\frac{\gamma}{\rho+\mu} + \frac{\bar{b}}{y}\right) \exp(-\rho t) = 0.$$

When $\beta = 0$, the $\dot{m} = 0$ curve obtained from (20) is a straight line that starts from O in figures 4 and 5:

$$c = \left(\frac{\mu+\rho+\delta}{\gamma}\right) m,$$

implying that there is no upper bound of c on the curve. Therefore, there is a dynamic path that does not intersect the $\dot{m} = 0$ curve, which is given by (25). However, if $\beta > 0$, no matter how small it is, there is an upper bound of c on the $\dot{m} = 0$ curve, as shown in figure 4. A dynamic path along which full employment prevails intersects the $\dot{m} = 0$ curve, and thereafter m declines and reaches zero within a finite time span; that is, the path is infeasible. Therefore, the dynamic equilibrium path must intersect the $\dot{W} = 0$ curve and enter the area with aggregate demand deficiency, resulting in secular stagnation.

5. Conclusions

Many developed countries, such as Japan, the EU and the USA, have recently experienced drastic downturns of economic growth and lapsed into secular stagnation. Conventional macroeconomic approaches successfully deal with short-run recessions but cannot necessarily treat such a transition from economic growth to secular stagnation consistently in one unified framework. This paper considers an economy with productivity growth and wealth preference, and finds that there is a unique dynamic equilibrium path along which the economy initially enjoys steady growth but eventually falls into secular stagnation with aggregate demand deficiency.

Along the path consumption expands at the same speed as productivity does in the early stage. As productivity increases and consumption follows it, the marginal utility of consumption gradually declines and eventually the desire for consumption up to the full-employment level is dominated by the desire for wealth holding. Then, aggregated demand deficiency arises. As long as the wealth preference exists, this situation is unchanged even though productivity keeps on growing and the money stock continues to expand, and the economy suffers secular stagnation. The nominal interest rate approaches zero. This process may be in conformity with the experiences of developed countries.

Appendix: local stability

The local stability of the dynamics in the case where $\dot{W} < 0$, given by (21), around the steady state E is examined. Along the dynamics m and y continue to expand while c stays to be finite at the level given by (23). Replacing m and y by $q(= 1/m)$ and $h(= 1/y)$ respectively changes the dynamic equations in (21) to

$$\dot{c} = \big((\gamma q + \beta + (\delta - \alpha)h)c - (\rho - \alpha - \delta)\big)\frac{c}{1+ch},$$

$$\dot{q} = \left(\frac{ch\big((\gamma q+\beta)c-\rho\big)+\alpha(ch-1)}{(1+ch)} - (\mu + \delta)\right)q,$$

$$\dot{h} = -\delta h.$$

Around the steady state E:

$$(c, q, h) = \left(\frac{\rho-\alpha-\delta}{\beta}, 0, 0\right),$$

the characteristic function is

$$\begin{vmatrix} \beta c - \sigma & c^2\gamma & (\delta - \alpha)c^2 \\ 0 & -(\alpha + \delta + \mu) - \sigma & 0 \\ 0 & 0 & -\delta - \sigma \end{vmatrix} = 0.$$

This yields one positive and two negative roots for σ (i.e., eigenvalues). c is jumpable, h (or equivalently y) is historically given and q (or equivalently $m(= M/P)$) cannot jump because W is set equal to W_F and P is chosen to satisfy (11) in the area where $\dot{W} < 0$. Therefore, in this area the dynamic path is uniquely determined, which is BE in figures 4 and 5.

Among a continuum of paths in the area where $\dot{W} > 0$ the path that reaches the intersection point of the saddle path obtained above and the $\dot{W} = 0$ curve, which is B, is chosen. Because P is jumpable when $\dot{W} > 0$ while y is historically given, the initial point of the path in this area is determined, and the whole trajectory uniquely obtains.

References

Akerlof, G. A. (1982) "Labor Contracts as Partial Gift Exchange", *Quarterly Journal of*

Economics, Vol. 97, pp. 543-569.

—— and J. L. Yellen (1990) "The Fair-Wage Effort Hypothesis and Unemployment", *Quarterly Journal of Economics*, Vol. 105, pp. 255-284.

Andersen, S., G. W. Harrison, M. I. Lau and E. E. Rutström (2008) "Eliciting Risk and Time Preferences", *Econometrica*, Vol. 76, pp. 583-618.

Calvo, G. (1983) "Staggered Prices in a Utility Maximizing Framework", *Journal of Monetary Economics*, Vol. 12, pp. 383-398.

Christiano, L. J., M. Eichenbaum, and C. L. Evance (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, Vol. 113, pp. 1-45.

Eggertsson, G. B., and P. R. Krugman (2012) "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach", *Quarterly Journal of Economics*, Vol. 127, pp. 1469-1513.

Erceg, C. J., W. H. Dale and A. T. Levin (2000) "Optimal Monetary Policy with Staggered Wage and Price Contracts", *Journal of Monetary Economics*, Vol. 46, pp. 281-313.

Hall, R. (2011) "The Long Slump", *American Economic Review*, Vol. 101, pp. 431-469.

Hashimoto, K. (2004) "Intergenerational Transfer and Effective Demand", *Economics Bulletin*, Vol. 5, No. 1, pp. 1-13.

Hashimoto, K. and Y. Ono (2011) "Does Pro-population Policy Raise Per Capita Consumption?", *Japanese Economic Review*, Vol. 62, No. 2, pp. 151-169.

Hayashi, F., and E. C. Prescott (2002) "The 1990s in Japan: A Lost Decade", *Review of Economic Dynamics*, Vol. 5, pp. 206-235.

Johdo, W. (2006) "Geographical Space and Effective Demand under Stagnation", *Australian Economic Papers*, Vol. 45, No. 4, pp. 286-298.

Johdo, W., and K. Hashimoto (2009) "International Relocation, the Real Exchange Rate and

Effective Demand", *Japan and the World Economy*, Vol. 21, No. 1, pp. 39-54.

Kehoe T. J., and E. C. Prescott (2002) "Great Depressions of the 20th Century", *Review of Economic Dynamics*, Vol. 5, pp. 1-18.

Kiyotaki, N., and J. Moore (1997) "Credit Cycle," *Journal of Political Economy*, Vol. 105, pp. 211-248.

Krugman, P. R. (1998) "It's Baaack: Japan's Slump and the Return of the Liquidity Trap", *Brookings Papers on Economic Activity*, Vol. 2, pp. 137-187.

Lawrence, E. C. (1991) "Poverty and the Rate of Time Preference: Evidence from Panel Data", *Journal of Political Economy*, Vol.99, pp. 54-77.

Matsuzaki, D. (2003) "The Effects of a Consumption Tax on Effective Demand under Stagnation", *Japanese Economic Review*, Vol. 54, No. 1, pp. 101-118.

McGratten, E. R., and E. C. Prescott (2012) "The Great Recession and Delayed Economic Recovery", in *Government Policies and the DELAYED Economic Recovery*, edited by L. E. Ohanian, J. B. Taylor and I. J. Wright, Hoover Institution Press: Stanford University, 115-154.

Michaillat, P., and E. Saez (2014) "An Economic Business-Cycle Model", NBER Working Paper 19777.

Murota, R. and Y. Ono (2011) "Growth, Stagnation and Status Preference", *Metroeconomica*, Vol. 62, No. 1, pp. 112-149.

—— and —— (2012) "Zero Nominal Interest Rates, Unemployment, Excess Reserves and Deflation in a Liquidity Trap", *Metroeconomica*, Vol. 63, No. 2, pp. 335--357.

Ono, Y. (1994) *Money, Interest, and Stagnation*, Oxford University Press.

—— (2001) "A Reinterpretation of Chapter 17 of Keynes's General Theory: Effective Demand Shortage under Dynamic Optimization", *International Economic Review*, Vol. 42, pp. 207-236.

—— (2006) “International Asymmetry in Business Activity and Appreciation of a Stagnant Country's Currency”, *Japanese Economic Review*, Vol. 57, No. 1, pp. 101-120.

—— (2010) “Japan's Long-Run Stagnation and Economic Policies”, in B. Bateman, T. Hirai, M.C. Marcuzzo, eds, *The Return to Keynes*, Cambridge, Mass.: Harvard University Press, pp. 32-50.

—— (2014) “International Economic Interdependence and Exchange-rate Adjustment under Persistent Stagnation”, *Japanese Economic Review*, Vol. 65, No.1, March 2014, 70-92.

—— and J. Ishida (2014) “On Persistent Demand Shortages: A Behavioral Approach”, *Japanese Economic Review*, Vol. 65, No.1, pp. 42-69.

——, K. Ogawa and A. Yoshida (2004) “Liquidity Preference and Persistent Unemployment with Dynamic Optimizing Agents”, *Japanese Economic Review*, Vol. 55, pp. 355-371.

Rodriguez-Arana, A. (2007) “Inflation and the Public Deficit when the Utility of Money is Insatiable”, *Japanese Economic Review*, Vol. 58, No. 2, pp. 238-254.

Smets, F., and R. Wouters (2003) “An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area”, *Journal of the European Economic Association*, Vol. 1, pp. 1123-1175.

—— and —— (2005) “Bayesian New Neoclassical Synthesis (NNS) Models: Modern Tools for Central Banks”, *Journal of the European Economic Association*, Vol. 3, pp. 422-433.

—— and —— (2007) “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach”, *American Economic Review*, Vol. 97, pp. 586-606.

Summers, L. (2013) “14th Annual IMF Research Conference: Crises Yesterday and Today, Nov. 8, 2013” available at http://www.youtube.com/watch?v=KYpVzBbQIX0.

Yun, T. (1996) “Nominal Price Rigidity, Monetary Supply Endogeneity, and Business Cycles”, *Journal of Monetary Economics*, Vol. 37, pp. 345-370.

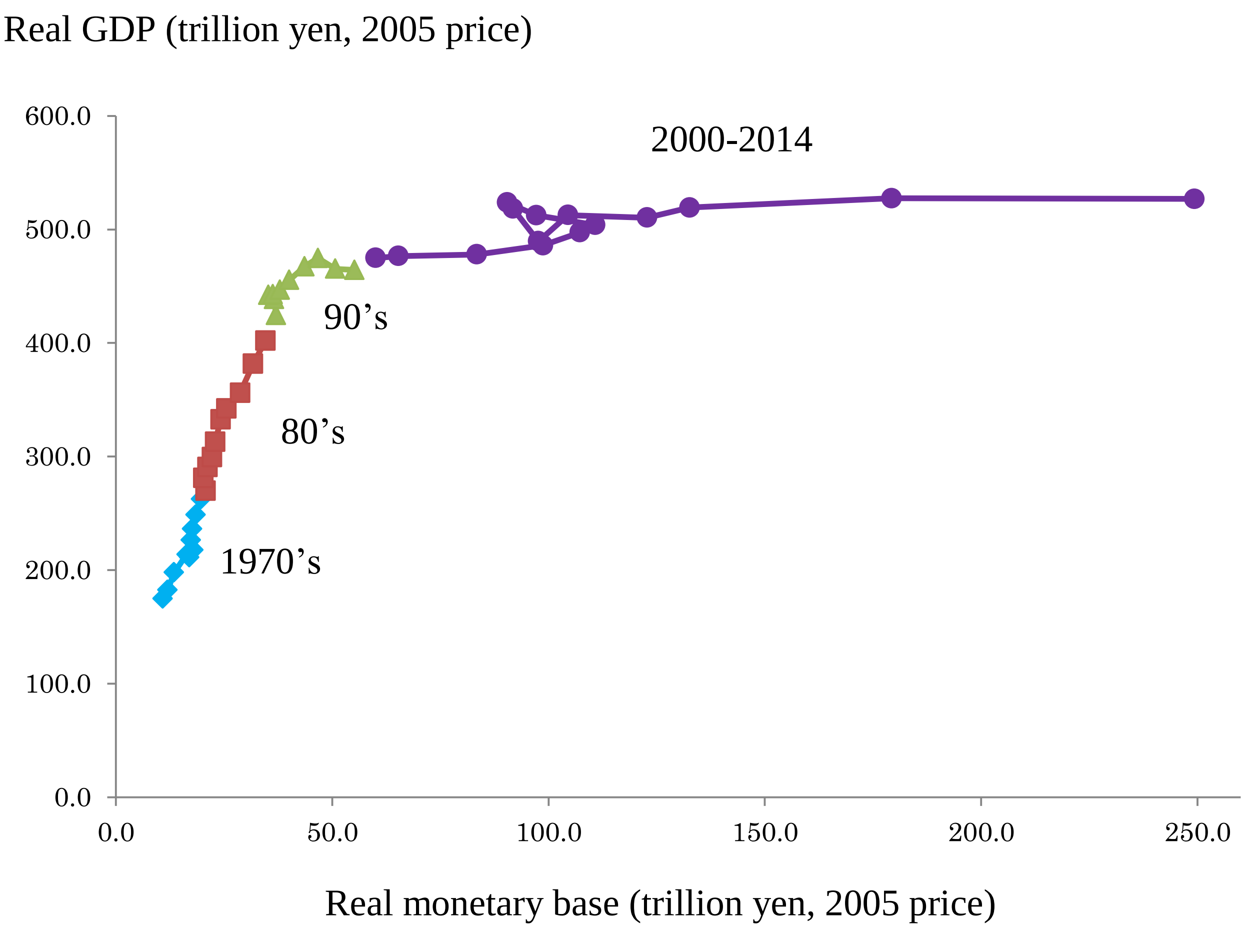

Figure 1: Money supply and GDP in Japan

(Cabinet Office, the Japanese Government; Bank of Japan)

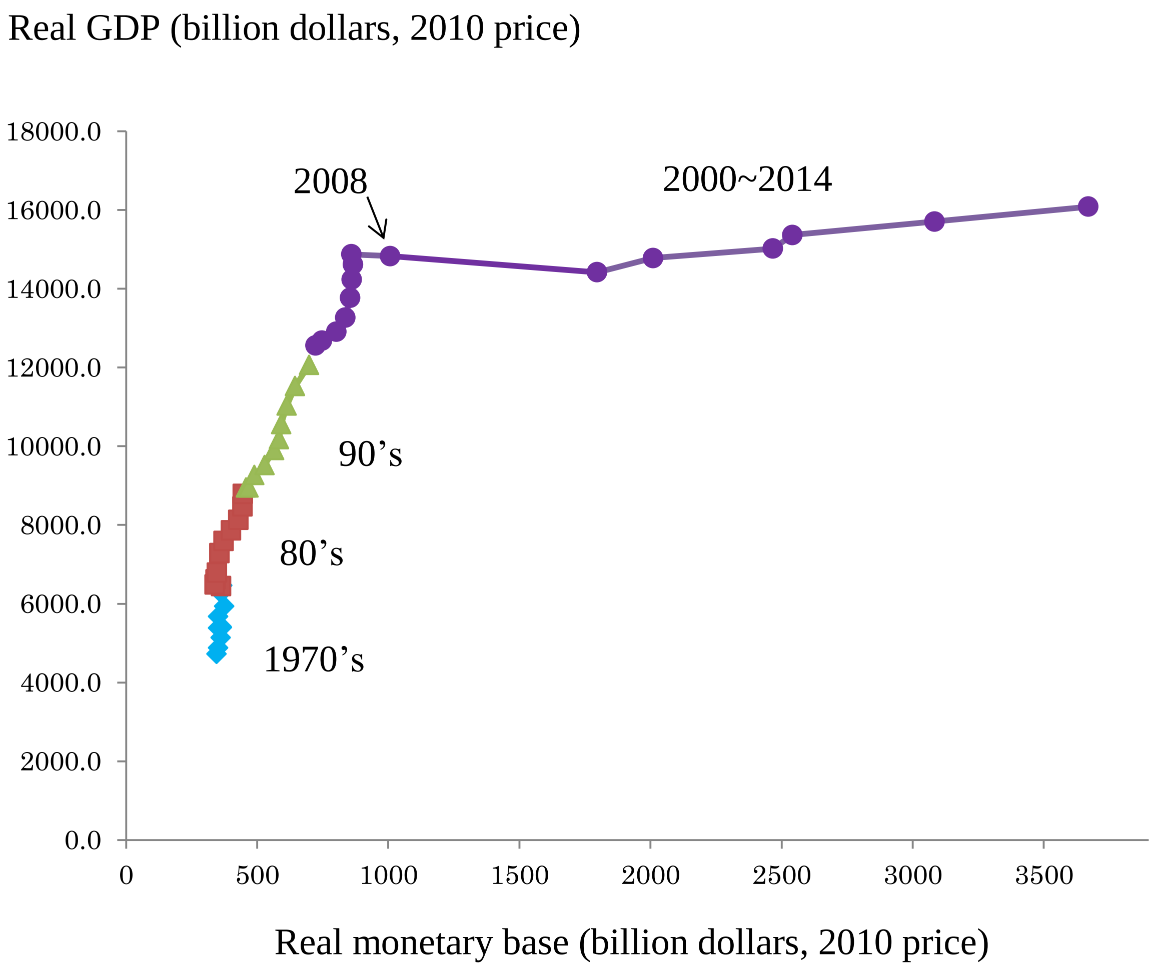

Figure 2: Money supply and GDP in the USA
(Bureau of Economic Analysis; Fed, St. Louis)

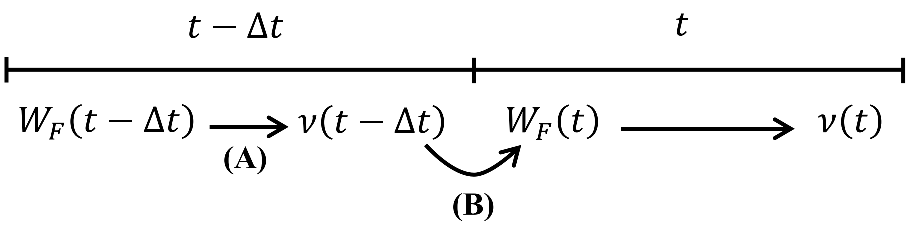

Figure 3: Determination timing of the fair and rightful wages

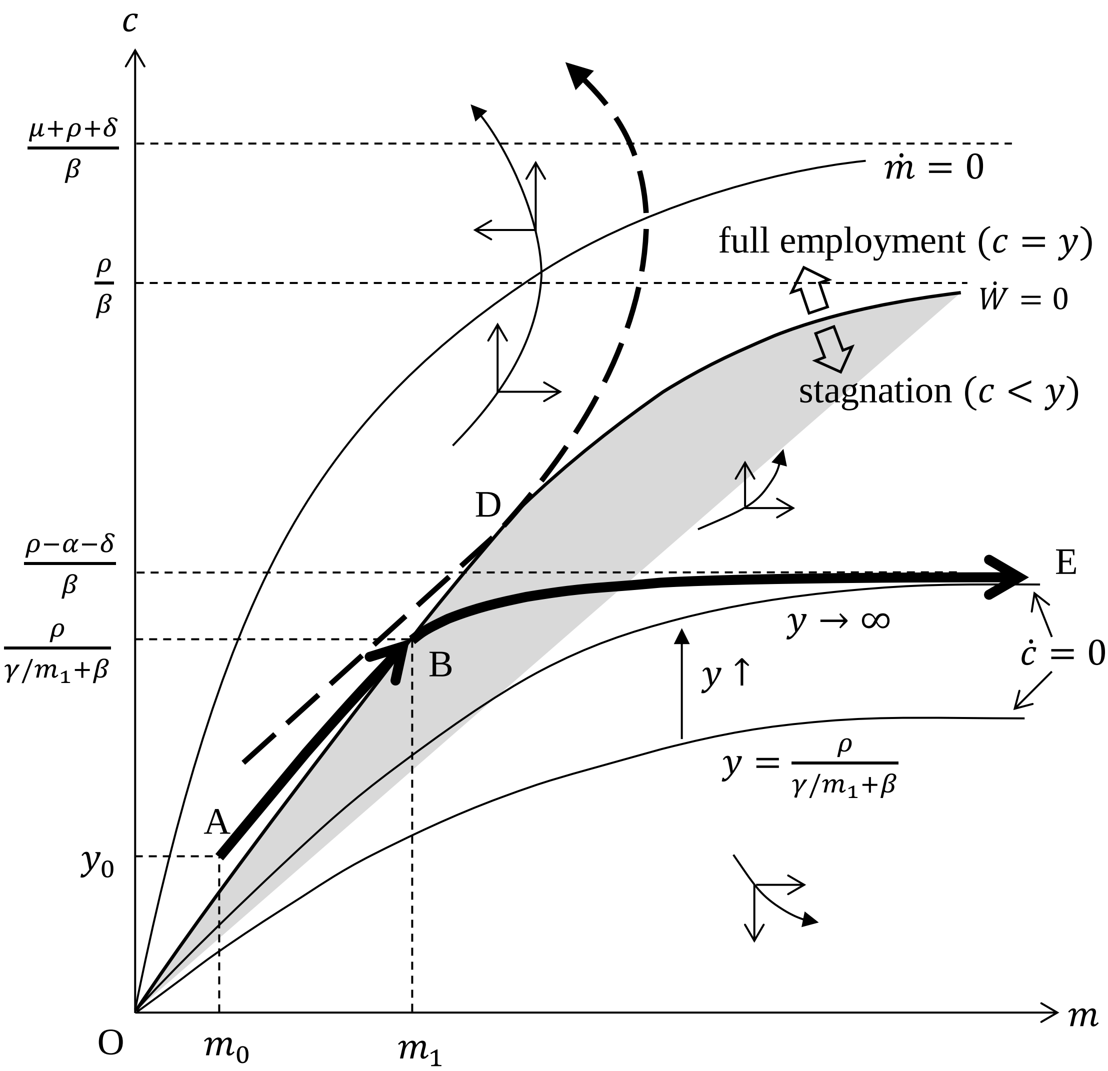

Figure 4: Dynamic equilibrium path ($\delta > \alpha$)

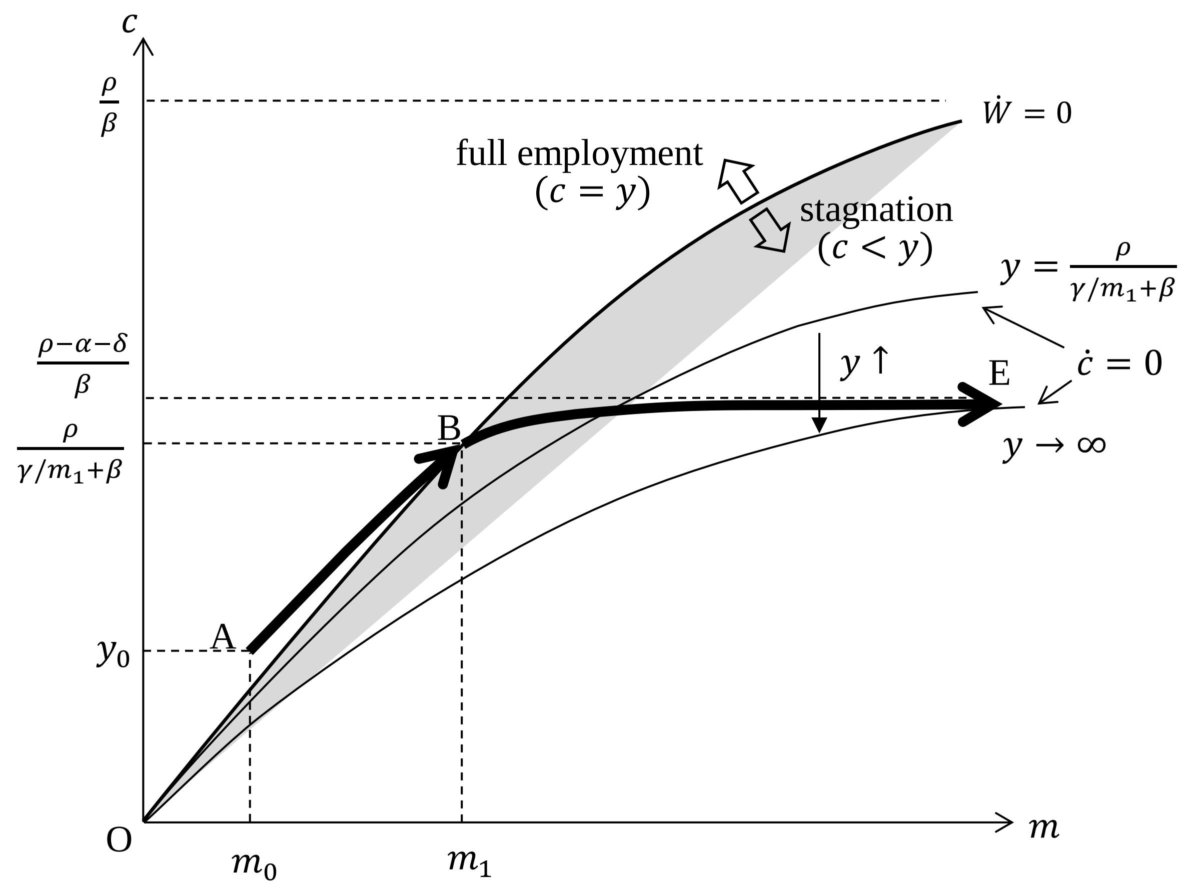

Figure 5: Dynamic equilibrium path ($\delta < \alpha$)

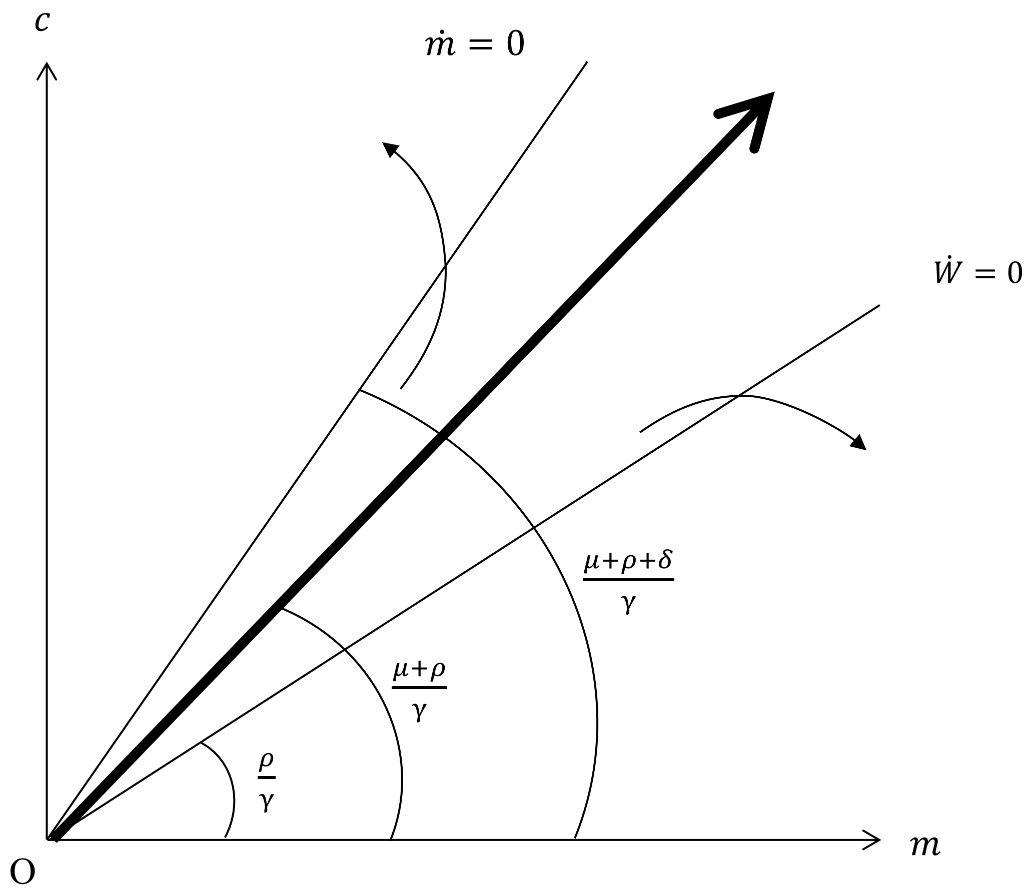

Figure 6: Persistent growth ($\beta = 0$)